Multi-Sector Sustainability in Agroecosystem Environments: Using Value Function Iteration for Numerical Solutions

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Selected Paper prepared for presentation at the American Agricultural Economics Association Annual Meeting, Long Beach, California, July 23-26, 2006

The study of sustainability in the economics literature has concentrated almost solely on macroeconomic considerations, with dynamic, economy-wide growth models demonstrating optimal paths of resource extraction and investment in order to generate final goods and productivity-augmenting capital stocks (Woodward and Bishop 2003; Solow 1974a,b; Hartwick 1977; Chichilnisky 1997; Asheim, Buchholz, and Tungodden 2001). Under many sets of assumptions, it has been shown that these present-value optimizing paths result in declining instantaneous utility and/or consumption paths at some point during the planning horizon, suggesting that future generations may be worse off than those preceding them (see, for e.g., Dasgupta and Heal 1974, 1979; Solow 1974b; Stiglitz 1974; Knapp 2004). Irreversibilities in the time dimension thus imply that actions taken by current generations will affect the opportunities of future generations through the endowments of resources left to the latter by the former.

Precisely what is meant by "opportunities" is the subject of considerable debate, as assumptions about technology and the substitutability of resources can drastically affect the characterization of an economy as sustainable or not. Strict complementarities in production, for example, might imply preservation of a physical resource is required to sustain welfare opportunities, while increased substitution possibilities might imply that complete extraction of any given resource is not only feasible and optimal from a present value perspective, but also consistent with sustainable welfare paths. Furthermore, the very definition of which measure of welfare is to be sustained over what time period is contested, with suggestions ranging from the maintenance of the maximum long-run utility level (Chichilnisky's Green Golden Rule, or GGR), to very strict notions of intergenerational equity (Rawlsian maximin paths), to the maintenance of aggregate national wealth (accounting for externalities and resource depletion), to less strict definitions of non-declining utility (either instantaneous or summed over time) (Chichilnisky, Heal, and Beltratti 1995; Chichilnisky 1997; Solow 1974a,b, 1986; Hartwick 1990; Asheim 1994, 1997; Farzin 2002, 2004; Woodward 2000; Knapp 2004).

While sustainable development at the global level is certainly of interest, many of these same concerns naturally extend to the sector level where most decisions about resource use are actually made (Woodward and Bishop 2003). At this level, however, there are often competing uses for a particular resource or set of resources, or incomplete property rights or markets result in external effects across sectors, resources, and stakeholders. Indeed, much of the non-economic literature on sustainable development seems particularly focused on broad issues of environmental degradation caused by otherwise productive uses of natural resources (see, for e.g., Tillman, et. al. 2002). This paper addresses these sector-level concerns by investigating the ramifications of alternative interpretations of agricultural sustainability on a simple dynamic model of an agroecosystem with externalities. Using numerical dynamic programming techniques, the present value problem of maximizing the discounted sum of instantaneous welfare over time is amended by imposing alternative sustainability constraints, each implying a particular definition of intergenerational equity for a subset of agents. Retention of the assumption of maximization behavior allows for considerations of efficiency as well.

A General Sector Model with Externalities

Consider a model in which there are two vectors of resources, $\mathbf{x}_t \in \Re_+^{N_1}$ and $\mathbf{z}_t \in \Re_+^{N_2}$, which are sources of (dis)utility, either directly (through immediate consumption) or indirectly (using the resource as an input into a production process). There are two sectors for each generation t: the first sector chooses controls $\mathbf{c}_t \in C(\mathbf{x}_t)$ [1] to achieve instantaneous utility $u(\mathbf{c}_t, \mathbf{x}_t)$, while the second sector is passive (in the sense that they make no controlling choices in the model) and derives utility $v(\mathbf{x}_t, \mathbf{z}_t)$ from each of the stocks. For example, the first sector could represent a firm or collection of firms involved in transforming the resource vector $\mathbf{x}_t$ into a final good (e.g., mining or agriculture), while the second sector derives utility from this final good or the resource stock itself, but is negatively affected by a (related) stock of pollution $\mathbf{z}_t$; for example, a subset of individuals engaged in non-consumptive water recreation in a small lake. Each stock evolves according to the equations of motion

$$\begin{aligned} \mathbf{x}_{t+1} &= \mathbf{x}_t + g\left(\mathbf{c}_t, \mathbf{x}_t, \varepsilon_t\right) \\ \mathbf{z}_{t+1} &= \mathbf{z}_t + h\left(\mathbf{c}_t, \mathbf{z}_t\right), \end{aligned} \tag{1}$$

with $\varepsilon \sim iid(\mu, \sigma^2)$ an instantaneous shock to $\mathbf{x}_t$ that is realized after the control decisions are made.[2] Note that the choices of the active sector affect future representative agents' utility, but the effects of the choices on $\mathbf{z}_t$ do not directly enter his utility function. In addition, we assume the evolution of $\mathbf{z}_t$ is only indirectly influenced by the stochastic shock through choice of controls. In other words, $\mathbf{x}$ and $\mathbf{z}$ are disjoint in the sense that the evolution of each of the stocks does not depend on the stock of the other. The linkage is instead through the control variable $\mathbf{c}_t$, which affects the time path of both stocks, albeit in possibly different ways. This specification is appropriate when analyzing the effects of the actions of “upstream” agents on “downstream” agents, such as a farm that uses a soil resource for production purposes, but in so doing pollutes a waterway downstream that is used for recreation.

Assuming that each generation of the active agent has property rights over $\mathbf{x}_t$ and maximizes the expected discounted sum of instantaneous utility over an infinite time horizon, $E_t \sum_{s=t}^{\infty} \rho^{s-t} u(\mathbf{c}_s, \mathbf{x}_s, \varepsilon_s)$, $0 < \rho < 1$, private welfare can be recursively defined as

$$V^p(\mathbf{x}_t) = \max_{\mathbf{c}_t \in C(\mathbf{x}_t)} u(\mathbf{c}_t, \mathbf{x}_t) + \rho E_t \left[V^p(\mathbf{x}_{t+1}) \right], \tag{2}$$

subject to (1), while if the externality were internalized, the social planner’s solution could be defined as

$$V^s(\mathbf{x}_t, \mathbf{z}_t) = \max_{\mathbf{c}_t \in C(\mathbf{x}_t)} u(\mathbf{c}_t, \mathbf{x}_t) + v(\mathbf{x}_t, \mathbf{z}_t) + \rho E_t \left[V^s(\mathbf{x}_{t+1}, \mathbf{z}_{t+1}) \right], \tag{3}$$

subject to (1). Note that the decision rule for the private solution and social solution will differ due to the dependence of $\mathbf{z}_{t+1}$ on $\mathbf{c}_t$. This model is similar to the one described in Woodward (2000), except that a stock externality is explicitly incorporated into the specification. We next turn to amending these utilitarian problems with sustainability constraints.

Sustainability Constrained Models

One of the key contributions of this paper is to characterize paths of development under alternative interpretations of the concept of sustainability. We begin by assuming that sustainability implies some degree of intergenerational equity, or “fairness”, in resource allocations across time, but vary the assumptions about what exactly is to be sustained (physical stocks, alternative interpretations of welfare) to explore the consequences of these alternative interpretations. As such, sustainability in whatever incarnation is represented by a constraint on the present-value optimizing problem discussed in section 2, rather than redefinition of the objective of the maximizing agent. More specifically, the sustainability constraints used here will take the form

$$E_t\left[f(\mathbf{c}_t,\mathbf{x}_t,\mathbf{z}_t)\right] \leq E_t\left[f(\mathbf{c}_{t+1},\mathbf{x}_{t+1},\mathbf{z}_{t+1})\right], \tag{4}$$

where the expectation is taken with respect to the information available in time t.[3] In this sense, we assume that a benevolent social planner has restricted the choice set via a self-imposed constraint, as discussed in Sen (1997). A discussion of this assumption can be found in Bond (2006), which summarizes the "pro" arguments of Woodward (2000) and Sen (1997), and the "con" arguments of Dasgupta and Mäler (1994) and Knapp (2004). We recognize that constraints of the form (4) are not unique in representing a sustainability concept, and that alternative specifications could be justified on many different grounds. Nevertheless, as the purpose of the analysis is to explore the implications of alternative interpretations of intergenerational equity according to what exactly is to be sustained over generations, we maintain the expected value formulation as a reasonable representation of a self-imposed obligation to succeeding generations.

An additional consideration is the extent to which allocations over time can be considered "fair". For example, a minimum standard type of criterion would allow time paths of some measure of interest to decline over some sub-interval of the planning horizon, so long as it remained above a predefined threshold, and still be consistent with sustainability. Similarly, a Green Golden Rule type of criterion would not necessarily preclude any path that converged to the GGR solution in the long run, regardless of intergenerational distribution. Here, however, we follow both Woodward (2000) and Knapp (2004) and define criteria which assume non-declining paths between consecutive generations, disallowing a declining path at any point over the planning horizon.[4] Woodward (2000) shows that such a constraint, when combined with the assumption of each generation behaving in an analogous way with respect to welfare, results in "fair" distributions in which future generations do not envy the present.

We focus on two specific forms of (4), each of which implicitly embodies assumptions about the opportunities available to future generations. A strong sustainability constraint assumes that future opportunity is directly linked to resource stocks, so that "fairness" implies

$$\mathbf{q}_t \leq E_t\left[\mathbf{q}_{t+1}\right] \tag{5}$$

where $\mathbf{q}_t \in \left[\mathbf{x}_t, \mathbf{z}_t\right]$ is a (sub)vector of the resource stocks in the model. Depending on the sector(s) of interest, constraint (5) might include any number of stocks in either resource vector, and there may be no admissible solutions that satisfy the constraint. Furthermore, even if a solution exists, the expected value framework does not guarantee that each succeeding generation will actually be at least as endowed as the preceding one, as stochastic realizations of ε_t may result in $\mathbf{q}_{t+1} < \mathbf{q}_t$. Nevertheless, the expectations framework presents a means of mathematically representing a concern for future generations (at least in terms of the first moment of a stochastic distribution) based on the best information available at the time of the control choice (Woodward 2000).[5]

Rather than sustaining expected physical stocks (akin to the "strong" sustainability concept), economics has tended to focus on intergenerational equity in consumption or utility across time as measures of economic opportunity (Solow 1974a,b Hartwick 1977). Following this lead, we consider a representation of sustainability based on Woodward's (2000, 2003) interpretation of intergenerational welfare, also advocated by Arrow, Dasgupta, and Mäler (2003). We assume instantaneous welfare for the private problem, in which externalities are not taken into account when choosing the controls, is defined by $U^P(\mathbf{c}_t,\mathbf{x}_t,\mathbf{z}_t)=u(\mathbf{c}_t,\mathbf{x}_t)$, while the social problem considers $U^S(\mathbf{c}_t,\mathbf{x}_t,\mathbf{z}_t)=u(\mathbf{c}_t,\mathbf{x}_t)+v(\mathbf{x}_t,\mathbf{z}_t)$.[6] Depending on the problem under consideration, we define each generations' welfare as the discounted sum of the infinite stream of utility from all succeeding generations, or $\sum_{s=t}^{\infty}\rho^{s-t}U^j(\mathbf{c}_s,\mathbf{x}_s,\mathbf{z}_s)$. Realizing that we have written this measure recursively for each problem in (2) and (3), the sustainability constraint in terms of non-decreasing welfare over time can thus be written

$$V^j(\mathbf{x}_t,\mathbf{z}_t)\leq E_t\left[V^j(\mathbf{x}_{t+1},\mathbf{z}_{t+1})\right] \tag{6}$$

where V^j is the value function for each constrained optimization problem. Note that (6) does generally allow for declining instantaneous utility over some portion of the planning horizon, but explicitly incorporates the utility of future generations into the welfare specification (Knapp 2004). Clearly, the set of admissible paths that satisfy each constraint will likely differ, highlighting the importance of a multi-sector specification in sustainability analysis.

Each sustainability constrained problem is thus defined as a present value optimization problem with an additional sustainability constraint, or

$$\begin{aligned}
&V^j(\mathbf{x}_t,\mathbf{z}_t)=\max_{\mathbf{c}_t}\ U^j(\mathbf{c}_t,\mathbf{x}_t,\mathbf{z}_t)+\rho E_t\left[V^j(\mathbf{x}_{t+1},\mathbf{z}_{t+1})\right]\\
&\textit{subject to}\\
&\mathbf{c}_t\in C(\mathbf{x}_t)\\
&\mathbf{x}_{t+1}=\mathbf{x}_t+g\left(\mathbf{c}_t,\mathbf{x}_t,\varepsilon_t\right)\\
&\mathbf{z}_{t+1}=\mathbf{z}_t+h\left(\mathbf{c}_t,\mathbf{z}_t\right)\\
&E_t\left[f(\mathbf{c}_t,\mathbf{x}_t,\mathbf{z}_t)\right]\le E_t\left[f(\mathbf{c}_{t+1},\mathbf{x}_{t+1},\mathbf{z}_{t+1})\right].
\end{aligned}\tag{7}$$

where V^j is the value function for each constrained problem. Depending on the specific form of the sustainability constraint, there may not be a solution to (7); if there is, however, the problem can be solved using numerical techniques (Judd 1998). The specification in (7) provides the framework for analyzing sustainable outcomes in a two sector framework in which one sector imposes a stock externality on the other.[7] While many specific problems fit into this general framework, the application presented here will concentrate on the conflict between the agricultural sector (as the active sector with soil stocks $\mathbf{x}_t$ and fertilizer input use $\mathbf{c}_t$) and a passive, non-consumptive downstream sector (say, water-based recreation) that is damaged by this sectors' input use. The results highlight the importance of including sectoral linkages in analyses of problems where intergenerational equity is of interest, and the need for policy makers to explicitly characterize the goals of "sustainable" systems.

Agroecosystem Model Specification

The basic present-value agroecosystem model used for this analysis is the discrete time analog of the model presented in Bond (2006). Omitting many of the details here, the model includes a mass-balance representation of productive land and the cycle of a crop-limiting nutrient, generalized from Baisden and Amundson (2003), with an additional state variable representing environmental damage from the use of inorganic inputs. These resources evolve according to the following equations:

$$N_{1t+1} = (1-\alpha_{11})N_{1\mathrm{t}} + \alpha_{12}N_{2t} + \alpha_{13}F_t + \alpha_\gamma \gamma_3 + \gamma_{atm} \tag{8}$$

$$N_{2t+1} = \alpha_{21}N_{1t} + (1-\alpha_{22})N_{2t} \tag{9}$$

$$X_{t+1} = (1-\delta)X_t + \eta F_t, \tag{10}$$

where N_1 and N_2 are annual and decadal nutrient pools, respectively, that represent nutrient cycling behavior, X is the stock of pollution, F is an index of inorganic fertilizer used as an input into the production system, γ_{atm} and γ_3 are exogenous nutrient deposits from the atmosphere and the millennial nutrient pool (assumed constant here), and the remaining parameters (α_{ij}, α_γ, δ, and η) govern the behavior of the dynamic system. Technology is assumed constant, and yields are a function of the perfectly substitutable indigenous nutrients and inorganic fertilizer inputs, so that $Y_t = f\left(\beta_1 N_{1t} + \beta_2 N_{2t} + \beta_F F_t + \beta_0\right), f' > 0, f'' < 0.$ We assume two representative agents in the model: a producer whose utility depends on profit derived from yields and fertilizer use, such that $U_t^P = \pi_t = pY_t - cF_t,$ and a representative consumer of non-productive environmental services damaged by fertilizer use such that $v_t = g\left(X\right),\ g' \le 0,\ g'' \le 0.$ For simplicity, we assume a risk-neutral producer who is not directly affected by environmental damage. Instantaneous social welfare is assumed separable and defined as the sum of the welfare of the representative agents such that $U_t^S = \pi_t + v_t$. Note that this specification implicitly assumes that the change in consumer surplus as a result of changes in agricultural output equals zero. A sufficient condition for this interpretation is that the productive agricultural agent is small relative to the overall market, and changes in production do not affect market prices for the agricultural good (i.e., perfect competition in the agricultural market). As such, the model is most appropriate when viewed in the context of a small region (for example, either one or a small group of homogenous producers whose production negatively affects a neighborhood or community recreation resource). In certain cases, the geographical interpretation may be broadened, but with special attention to the homogeneity of the land resource. Furthermore, we assume for simplicity that initial environmental damage is scaled to be zero at time *t*=0, and we focus on the intensive, rather than extensive, margin.

The use of numerical analysis requires specific functional forms for the yield and utility functions, as well as parameter values for the biogeochemical model and these functions. A Spillman-Mitscherlich yield function of the form $Y = A\left(1 - e^{-b(N_s)}\right)$, widely used in agronomic research, is assumed for the yield function, with N^s representing the linear combination of state and control variables that comprise nutrient availability (Lanzer, Paris, and Williams 1987; Frank, Beattie, and Ernbleton 1990; Llewelyn and Featherstone 1997; Kastens, Schmitdt, and Dhuyvetter 2003).[8] The function exhibits diminishing returns in the nutrient over the domain of N_s, and asymptotes at parameter A with curvature determined by parameter *b.* The utility function for the representative consumer affected by the externality is assumed to be a simple quadratic of the form $v = -dX^2$. All parameters take on the values in Table 1 unless otherwise indicated.

One key advantage of the numerical specification is the ease to which one can introduce uncertainty into the analysis. In the case of the agroecosystem model, a primary source of uncertainty is the atmospheric deposition of the limiting nutrient from year to year, γ_{atm}, due to variations in weather. Randomness in this parameter directly affects the growth of the active pool, and indirectly affects the other stocks, yields, and profits through the cycling behavior and input choice. For the stochastic simulations below, it is assumed that $\gamma_{atm} \sim N\left(20, 5^2\right)$, making the active pool a controlled Markov process.

In addition to estimating the present value solution, we consider the sustainability-constrained problems for both the private and social models. Strong sustainability for the private model could be represented in a number of ways depending on one's assumption about the intergenerational obligation with regard to each nutrient stock; however, we take as an example the constraint

$$N_{1t} \le E_t\left[N_{1t+1}\right], \tag{11}$$

as the active pool is the primary source of soil nutrients and is likely highly correlated with available soil test measures. In the social model, we would ideally like to preserve both the productive capacity of the soil and the initial level of damage to the environmental resource.[9] However, augmentation of the nutrient stocks through fertilizer use will increase environmental damage, and thus there are likely no admissible paths for which $X_t \geq E(X_{t+1})$ and (11) are both satisfied.[10] Thus, the sustainability problem becomes one of trading off endowments not only between *successive* generations, but *current* generations as well.

Feasible solutions do exist, however, when welfare, rather than physical stocks, is the object of the sustainability criterion, but tradeoffs in welfare between current generations are still necessary.[11] Sustaining welfare (in the sense of (2) or (3), as opposed to instantaneous utility) for the agricultural sector requires

$$\pi_t(F, N_t, N_{2t}) + \rho V^P\left(N_{1t+1}, N_{2t+1}\right) \leq E_t\left[V^P\left(N_{1t+1}, N_{2t+1}\right)\right], \tag{12}$$

while the analogous constraint for social welfare is

$$\begin{aligned} \pi_t(F, N_{1t}, N_{2t}) + v(X_t) + \rho V^S\left(N_{1t+1}, N_{2t+1}, X_t\right) \\ \leq E_t\left[V^S\left(N_{1t+1}, N_{2t+1}, X_{t+1}\right)\right], \end{aligned} \tag{13}$$

in accordance with (6). Again, while (12) is essentially a problem of intergenerational allocation within one sector, taking the externality into account involves substitutions between sectors as well. We wish to characterize the optimal paths of the solution to each constrained problem relative to the PV-optimal solution, and examine the welfare tradeoffs involved with each.

Numerical Dynamic Programming Approach

Numerical solutions to the dynamic programming specification require estimation of the continuous value function $V^j\left(\cdot\right)$ in (7) for each problem under consideration. We follow Judd (1998) and Howitt, et al. (2003) and use orthogonal polynomials to approximate this unknown function.[12] In short, this method utilizes the Contraction Mapping property of the value function, which assures convergence of each successive approximation of the value function $V^{S+1} = TV^{S}$ where T is a mapping function (Howitt, et al. 2003). The problem under consideration here is either two dimensional if only private welfare is considered, but three-dimensional in the case of the multiple sector specification. Illustrating the three dimensional problem, a Chebyshev polynomial functional form of each of the approximated functions $\hat{G}$ is used and is defined by

$$\hat{G}\left(N^1, N^2, X\right) = \sum_i \sum_j \sum_k a_{ijk} \phi_i\left(M(N_1)\right) \phi_j\left(M(N_2)\right) \phi_k\left(M(X)\right), \tag{14}$$

where $M(\cdot)$ transforms each state variable to the $[-1,1]$ interval, ϕ_m is the m^{th} term ($m = 1,...,M$)of the polynomial defined by $\phi_m(z) = \cos\left(m \cdot \cos^{-1}(z)\right)$, and the a_{ijk} are the coefficients to be estimated (Judd 1998; Howitt, et al. 2003).[13] In practice, the coefficients for the deterministic problem are obtained by evaluating (14) at a user-specified number of n nodes given by $z_n = \cos\left(\pi \frac{2n-1}{2N}\right)$ for $n = 1,...,N$ and $N \geq M + 1$, then regressing these values on the ϕ_m (Judd, 1998; Howitt, et al. 2003). For the stochastic representation, the expected function is found by taking a discrete approximation of the underlying continuous distribution of the random variable, then calculating the probability-weighted sum of the functions evaluated at each of these discrete points.

Optimal Present Value Solutions

Deterministic and stochastic estimations of the unconstrained (PV-optimal) solution with only producer welfare taken into account during the optimization are shown in Figure 1 for various nutrient stock initial conditions, and assuming zero initial damage.[14] The certainty equivalent solutions are shown as solid lines, while a particular stochastic realization for each initial value is represented by the dashed line. Introducing risk into the analysis does not substantially change the qualitative conclusions about the PV-optimal paths, but does introduce noise into the system, and results in a steady state distribution for each variable, rather than a particular steady state level.

Consistent with the analysis in Bond (2006), low initial nutrient stocks imply high fertilizer use in the short run, while high initial values result in an optimal fertilization path that is gradually increasing over time. Note that the presence of the decadal pool N_2 and the assumed unidirectional flow of the nutrients through the model, as well as the irreversibilities in investment, result in an asymmetry in response between these cases, with fertilization rapidly approaching the steady state level in the former as stocks are built up quickly, while the relatively gradual depletion of the decadal stock with high initial nutrient levels results in a more gradually increasing fertilization path. The higher is the initial value of N_1, the lower is short run fertilization, with little effect on rate of convergence to the steady state. A higher initial N_2, however, results in both lower short-run fertilization as well as a longer rate of convergence to the steady state. This translates into profit paths that rapidly increase to the steady state in the case of severely degraded initial soil, but gradually decreasing profit paths in the case of relatively undisturbed land.

As seen in Figure 2, optimizing over social welfare through internalization of the externality has the expected effects of lowering input use at all points over the planning horizon, including the steady state. As a result, steady-state levels of both the active and decadal pools are lower, as is steady-state environmental damage. In essence, private profits are thus traded off with increased social welfare. It is important to note, however, that internalization of the externality in a Pareto-efficient manner does *not* automatically result in intergenerationally equitable welfare paths for either of the two sectors, nor for society as a whole. Profits are slowly declining over the planning horizon before reaching the steady state, damage is monotonically increasing to the steady state, and social welfare thus declines as well. Thus, internalization of an externality *does not* result in sustainable welfare paths under any of the criteria examined in this paper, but does, of course, improve resource allocation efficiency in the presence of the externality. We postpone analysis of the intergenerational equity effects of externality internalization until section 9, after presenting the results of the sustainability-constrained solutions.

Strong Sustainability Solutions

We next turn to the sustainability constrained solutions to explore the tradeoffs of following a sustainability optimal path, beginning with a representation of a "strong sustainability" solution in which physical stocks, rather than welfare flows, are sustained. For the private problem, this implies maintaining active pool nutrient stock levels between subsequent time periods, although the distribution of the total stock through the two pools can differ. Alternatively, one could sustain a non-increasing path of damage for the sector affected by the externality, at the expense of agricultural production. As noted previously, the parameterization of the problem ensures that there are no admissible paths for which both of these conditions can be satisfied; as such, the paths described below can be interpreted as the bounds on the continuum of strongly sustainable criteria.

Strong sustainability paths that maintain active nutrient stocks, and thus satisfy constraint (11), for the private model are shown in Figure 3, and are termed N_1-Sustainable (N_1-sus).[15][16] As the PV-optimal solution exhibits an "overshooting" characteristic due to the presence of the decadal pool when initial stocks are low (i.e., the ending stock in the first period is greater than the steady state), the constraint binds at most stock levels. As such, the optimal N_1-Sus control strategy is to maintain active pool stock levels at approximately the PV steady state level for the entire planning horizon, while the decadal pool stocks gradually adjust to some new, non-unique steady-state level which is increasing in the initial stocks. If initial stocks in each pool are greater than the PV-optimal steady state, then these levels are maintained over the entire planning horizon through fertilization levels greater than the PV-optimal solution.

While the solutions to this problem ensure that subsequent generations of producers in the active sector are endowed with at least the same physical resource stocks as the preceding one, this does not guarantee either non-declining welfare paths over time nor that the welfare of later generations is enhanced relative to the PV-optimal solution. In fact, because maintenance of the resource is costly and the marginal benefits are small due to concavity of the yield function, the value function $V^p_{strong}(N_1, N_2)$ is negatively sloped with respect to N_1 for active pool stock levels greater than the PV-optimal level, as seen in Figure 4. As $\frac{\partial V^j}{\partial N_k}$ is the current value shadow price of N_k, the negative slope implies that additional soil nutrients are negatively valued, and welfare could be increased by "mining" the soil through reduced fertilizer applications. Because this is inconsistent with the sustainability constraint, however, the optimal strongly sustainable solution results in declining and lower steady-state profit levels than the PV-optimal solution, and increased fertilization rates results in greater damage to the resource used by the passive sector, resulting in severely downward sloping paths of social welfare and low levels of steady-state social utility. Physically sustaining the productive capacity of the soil at these high levels, then, is directly at odds with the economic goals of the producer in both the short and long runs, as would be expected *a priori* with a positive discount rate.

If, on the other hand, the resource used by the passive sector is to be preserved, the strong sustainability constraint can be written as

$$X_t \geq X_{t+1}, \tag{15}$$

and assuming $X_0 = 0,$ the only feasible solution to either the private or social problem is to set $F_t = 0 \ \forall t.$ We term these paths X-sustainable (X-sus). As such, private and social welfare are equal with paths determined entirely by the initial stocks of the nutrients, as shown in Figure 5. While this sustains both the resource, and therefore utility, of the passive sector, any initial stock values above the zero fertilizer steady-state result in declining nutrient stocks and thus declining profits and social welfare over the planning horizon. Furthermore, steady state profits and social welfare in the long run are lower than the social PV-optimal solution, and this inefficiency does not result in more equitable welfare paths over time.

Value Sustainability Solutions

Solutions to the private and social problems that interpret the obligation between generations as one of non-declining welfare streams consistent with (12) are presented in Figures 6 and 7, respectively, and termed value sustainable (V-sus). According to Assumption B in Woodward (2000), a sufficient condition to ensure a solution to (7) with the value sustainability constraint is the existence of some control such that a lower bound on instantaneous welfare can be reached for any values of the states; in other words, a free disposability assumption. In the case of the agroecosystem model, this assumption is satisfied if one assumes that revenue in the active sector can be "wasted", particularly when initial nutrient stocks are high.[17] In the numerical simulations, this is accomplished by adding a non-negative multiplicative control variable to active sector revenues that takes on a value less than one if revenues are wasted, but equal to one otherwise. Woodward (2000) terms the former "stepwise inefficiency", and it is characterized by the ability to increase the welfare of the generation in time *t* without adversely affecting the subsequent generation in *t*+1, but only by violating the sustainability constraint. In the analysis that follows, we assume that the Pareto-superior choice is chosen by the actor, and the welfare streams that are reported assume that the generation in *t* keeps, rather than disposes of, any revenue that must be discarded in order to satisfy the sustainability constraint.

Qualitatively, the paths of the state and control variables for the private V-sus solution are very similar to the PV-optimal solution, despite the fact that the sustainability constraint is binding at a minimum of one point for each path shown.[18] As seen in Figure 8, the value function for the PV-optimal solution is monotonically increasing in each stock, with the marginal value of each additional unit of stock independent of the stock level. In the value constrained problem, however, this is only true up to a point, as the value function is restricted from above at $S_{value}^{P} = \bar{\pi}/(1-\rho)$, where $\bar{\pi}$ is the maximum profit level that can be maintained for an infinite time period. [19] Thus, the shadow value of additional nutrients is state-dependent: constant and positive for relatively low stock levels, but zero for higher stock levels. Note, however, that these values are defined in terms of the future value of the optimal path, and do not include the additional value for the current generation that can be appropriated (through not disposing of profits) from choosing the Pareto-optimal solution as a result of the stepwise inefficiency.

In accordance with Figure 8, the major difference in the optimal paths between the PV-optimal and V-sus solutions occur when stocks are high relative to the PV-optimal steady state level. For example, Figure 9 shows the optimal fertilization schedule and total N stocks for selected years beginning with initial stocks of (140,140). Overall, both paths are described by increasing fertilizer use compensating for the removal of indigenous nutrients that occurs from harvesting and leaching as a result of cultivation. As suggested in the previous section, it is neither necessary nor optimal to sustain the physical nutrient stocks to maintain welfare between generations, and due to the stepwise inefficiencies, it is actually optimal for stock levels in early time periods for the constrained problem to be *less* than the PV-opt levels, accompanied by lower fertilization rates. In the long run, however, this pattern is reversed, and the sustainable steady state level is slightly higher than the PV solution, as defined by V_{value}^{P}. Of course, in order to maintain these stocks, a higher fertilization level is required, which in turn results in greater steady-state damage in a social sense. This runs counter to many claims that reduced fertilizer use is consistent with sustainable outcomes (e.g., Rigby, et al. 2001).

In many cases, however, these claims are likely due to the external damage caused by what may be perceived as excessive fertilizer use that affects primarily downstream assets. The social value sustainability problem admits paths that preserve opportunity in the sense of (13) with $U^s(\mathbf{c}_t, \mathbf{x}_t, \mathbf{z}_t, \varepsilon_t) = pY_t - cF_t - dX^2$, i.e., the utility of both the active and passive sector is recognized and the welfare of all successive generations is recognized in the sustainability constraint. Recall that internalization of the externality is not sufficient to sustain social welfare over time, so that additional tradeoffs between both sectors and generations are necessary.

In contrast to the PV-optimal social solution, which prescribes a fertilization strategy that provides for gradually declining nutrient availability over time as damage grows (as indicated through declining profit levels with relatively stable fertilization rates), the social V-sus solution presented in Figure 7 is to gradually *increase* nutrient availability from relatively low levels over the intermediate time periods, providing the opportunity for subsequent generations of producers to obtain higher yields (and profits). This is similar to the finding in Woodward (2000) that value sustainable extraction in a renewable resource economy is associated with growth in the resource stock.[20] It should be noted, however, that this growth follows a small number of adjustment periods during which fertilization levels that are greater than the new steady state are optimal. This odd result occurs as a result of the low level of initial damage assumed, and the fact that utility for the passive sector is decreasing and concave in the stock of damage. As such, fertilization can be used at low damage levels without significantly decreasing the welfare of future generations, allowing the productive sector to reap the benefits. At sufficiently high stocks of nutrients, stepwise inefficiencies might occur as well.[21] As the externality accumulates and the nutrient balances in the soil adjust according to the dynamics of the natural system, higher levels of fertilization more severely degrade the welfare of future generations, and the optimal solution is to gradually increase fertilization rates. Thus, for intermediate levels of damage below the steady state, a social V-sus solution is characterized by lower fertilizer use than the PV-optimal solution, but this level actually *increases* over time.

Another key result of interest is the tradeoff between sectors as a result of the imposition of this particular form of intergenerational equity. In the absence of stepwise inefficiencies, the social utility path has a slope of essentially zero, much like the profitability path in the private problem, and is very close in value to the corresponding steady state level in the present value problem. In essence, then, social utility is maintained by exactly balancing the growth in the disutility of the passive sector with the growth in the profitability of the productive sector. Thus, even in the presence of an externality, an intergenerationally equitable path from the standpoint of society as a whole can be characterized by *increasing* producer profits *and* damage of the sector affected by the externality, albeit the latter occurs at a lower growth rate than in the PV-optimal solution.

Welfare Implications of S-Optimal Solutions

In this section, we further examine the welfare implications of following the sustainable paths as compared to the standard PV-optimal solutions for each problem under consideration. As the sum of discounted welfare is bounded from above by the present value solution, following any of the sustainable paths will necessitate a loss of total welfare (defined as the discounted sum of instantaneous utility streams) in any case in which a sustainability constraint binds. However, the distribution of this loss across generations and/or sectors is generally not known *a priori*, though either physical endowments or overall instantaneous utility are guaranteed to be non-decreasing over time.

Figure 10 shows the total value of the optimal solutions over the infinite planning horizon, with the values for the private problem representing total discounted profits, while the social solution values show the discounted sum of overall social utility.[22] Perhaps the most striking feature of the private problem graph is that with the exception of the X-sus solution (which severely restricts the ability of the productive agricultural sector to generate profits), overall value is very similar across solutions. Differences are most severe in the case of high initial stocks, suggesting that policy intervention on behalf of satisfying a given sustainability constraint would be most justified in cases such as a previously unexploited (or underexploited) resource. When an externality is taken into account, however, the differences between the PV-optimal and value sustainable solutions are magnified.

Figure 11 partially decomposes these welfare measures by examining the intergenerational and intersector variation of these utility differences for the first, fifth, tenth, twentieth, and fortieth generations, with relatively high initial stocks such that any sustainability constraint will be binding at some point during the planning horizon. In contrast to Figure 10, however, the gains from stepwise inefficiencies are taken into account. From the figure, it is clear that sustaining physical stocks is not only inefficient, as there is at least one option for each sector which would be preferred by every generation, but it can also be more inequitable than the preferred choice. In the private sector, profits for the private V-sus solution are higher than those for N_1-sus, with the inequality between the first and subsequent periods a result of stepwise inefficiencies (recall that the steady-state profits for this problem are $\overline{\pi}$). In terms of social welfare, sustaining the high initial soil nutrient stocks results in far greater disutility for the passive sector over the long run than the other potential paths, while limiting damage to zero results in high passive sector utility only at great expense to the productive sector. Neither solution is as equitable across both sectors as even the PV-soc result.

Several other interesting points can be made from the figure. First, if an unregulated productive sector (such that the externality was not taken into account) made the decision to follow a sector-specific value sustainable policy (V-sus private) to enhance intergenerational equity, private profits will be slightly greater than the PV-optimal solution in the long run, but the resultant social welfare path favors earlier generations over later ones. Second, as already noted, internalization of the externality does not result in intergenerationally equitable welfare paths for either sector, nor in total. In fact, it actually *increases* the disparity in profit levels across generations of producers. However, equitability increases in the passive sector, and the resultant social welfare paths decline by only 21% from the first to fortieth generation, as opposed to the 92% under the private PV-optimal solution.

Finally, it is interesting to note that the value sustainability solution for the social problem (V-Sus Soc) results in only slightly greater social utility in the fortieth generation than the corresponding PV-optimal solution, but does so though a reallocation of overall social utility from intermediate to later generations. In this sense, the solution to the sustainability-constrained problem bears similarities to that of a PV-optimal solution with a relatively lower discount rate.[23] The analogy is not perfect, however, as the V-sus solution results in gradually *increasing*, rather than decreasing, profit paths over time, unlike the PV-opt (or any other) solution.

Conclusions

Sustainability is an oft-cited resource and environmental management goal in policy circles around the globe, despite a multiplicity of meanings across individuals, disciplines, or problems. While it may be desirable to maintain or increase economic vitality, environmental quality, and intergenerational equity across all future generations, doing so may impose significant costs on the present or even future generations, or may even be impossible. On the other hand, in some cases a sustainable solution may be identical (or, at the least, very close) to the standard present-value welfare maximizing paradigm commonly assumed in economics.

This paper illustrates some of these issues in the context of a simple dynamic model of agriculture with an externality. We assume that the agricultural sector chooses inputs (fertilizer) that directly contribute to crop growth and indirectly contribute to the quality of the soil (nutrient levels), but that runoff and leaching pollutes a downstream sector in the form of a stock externality. Without taking the pollution into account, the present value solution results in increasing profitability and nutrient stocks over time for low initial stocks, but the converse for a relatively high initial endowment. Social welfare, defined as the sum of utility from both sectors, declines in both cases. Internalization of the externality results in a more equitable social welfare distribution over time, but at the expense of the profitability and equitability of the agricultural sector.

Two sustainability criteria were analyzed: a "strong" constraint that maintains physical stocks over time, and a "value" constraint that ensures that future generations' welfare is non-decreasing over time. A key result is that physically sustaining a particular resource is not a necessary condition for sustaining welfare, nor does sustaining welfare require maintenance of any particular stock. In fact, in many cases, physically sustaining a stock is not only Pareto-dominated by other feasible paths, but also results in a *less* equal distribution of welfare across generations.

In the specific case examined here, the PV-optimal solution to the private problem results in a fairly equitable distribution of profits over time, although the existence of stepwise inefficiencies due to the fact that the value function is bounded from above allows for earlier generations to enjoy higher profit levels than those that follow. Although this violates the sustainability constraint, it is Pareto improving, and thus future generations would have no reason to object were they able to do so. From a social perspective, the value sustainable solution results in an equitable distribution of total welfare over time (again, except for stepwise inefficiencies), but implies a relatively slow rate of growth (rather than decline) in the agricultural sectors' profits coupled with a slow decrease in utility for the passive sector. The trade-off, however, is that profits for each generation under the value sustainable solution are less than the social planners' PV-optimal solution (as well as the private PV-optimal solution), but the disutility generated by the externality is also reduced. Hence, the value sustainability constraint in the presence of an externality essentially acts as a welfare transfer mechanism from the productive to the passive sector, especially in the short to medium run, but also induces a type of deadweight loss in that the discounted sum of social welfare is lower than that under a present value criterion.

The results of this paper highlight the fact that many dynamic resource allocation problems may involve multi-dimensional tradeoffs between contemporaneous resource-consuming sectors, tradeoffs between subsequent generations in the same sector, and tradeoffs between actors in different sectors in different time periods. As scientists, policy makers, and the larger society as a whole debate the extent of the obligation of the present generation towards those in the future, the economics discipline can contribute by using models and techniques such as these to inform others about the relative costs and benefits of each potential choice.

Footnotes

[1] It is assumed that $\mathbf{c}_t$ is piecewise continuous and that the control set $C(\mathbf{x}_t)$ is defined as $C(\mathbf{x}_t) = \left\{\mathbf{c}_t : h^k(\mathbf{x}_t, \mathbf{c}_t) \geq 0, k = 1 \ldots K\right\}$ for any K time-invariant inequality constraints on the control (Caputo 2005).

[2] A deterministic specification is a special case of the problem considered here, and would result from assuming $\mu = \sigma = 0$. We note here that the incorporation of stochastic shocks is a) relatively simple in a dynamic programming, as opposed to an analytical optimal control, framework in most cases; b) more realistic from the standpoint of a decision maker, in that future values of the state variables will not likely be known with certainty; and c) is of general interest in the sustainability literature (Woodward, 2000).

[3] Note that each time period *t* is implicitly associated with a generation and a particular value of the control variable, and that the stochastic shock is realized after the choice decision $\mathbf{c}_t$ is made.

[4] The authors differ in their definitions of what is to be sustained, however. In addition, as discussed below, the presence of a particular type of inefficiency may result in declining welfare paths if present generations are assumed to retain utility that would otherwise have to be disposed.

[5] If resiliency issues were particularly relevant for a given problem, constraints involving higher order moments might be in order. Note that there is nothing inherent in the framework thus far that precludes a utilitarian problem from being "sustainable" in the sense that the constraint is not binding.

[6] The additive framework is justified by the fact that the first-order conditions for a Pareto efficient allocation of resources between representative agents can be represented as a weighted sum of utilities (Varian, 1992). Implicit welfare weights can be changed through alternative parameterizations of the instantaneous utility functions.

[7] In this context, a stock externality is particularly interesting in that the damage persists between generations, rather than simply affecting the present. A flow externality could easily be incorporated by adding a term reflecting the increased contemporaneous damage (i.e., decrease in instantaneous utility) as a result of the external effects.

[8] This functional form can easily incorporate additional factors of production as well.

[9] Recall that we have normalized initial damage to zero for the purposes of analysis.

[10] The parameters of the model are chosen such that a positive level of fertilizer is optimal in the long run for the unconstrained problem. Note that since *X* is a source of disutility, the inequality is reversed.

[11] As discussed later in the paper, an assumption of free disposability might be necessary to ensure a solution to the value sustainable problems.

[12] Estimation of the value function alone has been termed "value iteration", while estimation algorithms that include an approximation of the policy function has been termed "policy iteration" (Judd, 1998; Howitt, et al., 2003).

[13] As the state space expands, the dimensionality of the polynomials quickly becomes unruly. As such, we use an M=15 degree polynomial in each dimension for the private, 2-state problem, and M=9 for the social, 3-state problem. Programming was done in using the GAMS modeling system, with varying run times ranging from just over one hour to several days for the 9 coefficient, 3 state function.

[14] For illustrative purposes, results are presented for initial (N_1, N_2, X) stock levels of (20, 20, 0), (60, 60, 0), (100,100, 0), and (140, 140, 0).

[15] The introduction of risk in the strong sustainability case presented numerical difficulties as estimation of the value function at high stock levels with high realizations of the stochastic inflow requires evaluation at points outside of the state-space bounds of the Chebychev polynomial. As such, only the deterministic case is considered in this section. Note that this is a numerical, rather than conceptual, difficulty, and that certainty equivalent solutions are presented for all of the scenarios analyzed.

[16] Strong sustainability solutions for the social model are qualitatively similar, in that initial N_1stocks below the social PV-optimal solution are built up to slightly less than the steady state level, while stocks greater than this level are maintained indefinitely.

[17] Profits are monotonically increasing in nutrient stocks, and the non-negativity constraint on fertilizer application precludes controls which "waste" the physical nutrient stocks.

[18] This is again due to the "overshooting" of the PV steady state level of the active pool in the unconstrained problem.

[19] This corresponds to stock and control values of $\bar{N}^1 = 90.9$, $\bar{N}^2 = 31.2$, and $\bar{F} = 3.74$, while the PV optimal values are 90.1, 30.9, and 3.69, respectively. PV optimal stock levels and fertilization at the steady state are declining in the discount rate.

[20] In that model, the single renewable resource was a direct source of utility.

[21] At relatively high levels of nutrient stocks with low levels of damage, future generations' welfare, as represented by the value function at the end of period stocks, is relatively flat with respect to low levels of fertilization, though the numerical approximation is slightly downward sloping. It is unclear if this negative slope is due solely to the numerical algorithm, or if the slope of the true curve is zero. In any case, stepwise inefficiencies are frequent at stock levels less than the PV-optimal steady state. Interestingly, multiple Pareto-optima (and thus multiple optimal fertilization levels) can exist in cases when the current generations' value function is positively sloped with respect to the control, but future generations' function is negatively sloped. We follow the convention here that in such cases, the highest obtainable level of welfare for future generations is chosen (in other words, zero fertilization), as strict adherence to the sustainability criterion requires disposal of "excess" utility and equivalence of the welfare of subsequent generations.

[22] These figures assume disposal of any profits and/or utility arising from stepwise inefficiencies for any generation.

[23] Although not as extreme, this interpretation is similar to the Green Golden Rule solution to a dynamic resource problem in which one takes the limit as the discount rate tends to zero.

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Table 1. Model Parameters for Simulation

Biogeochemical Parameters		*Yield Parameters*	
α_{11}	1.5	A	400
α_{12}	0.122	b	0.030
α_{13}	0.409	β_0	0.166
α_γ	0.500	β_1	0.901
γ_3	0.177	β_2	0.244
$E[\gamma_{atm}]$	20	β_F	0.817
$Var[\gamma_{atm}]$	25	*Economic Parameters*	
α_{21}	0.089	p	1
α_{22}	0.260	c	0.2
δ	0.100	d	0.02
η	0.183		

Note: Based on Baisden and Amundson (2003) for their $600x10^3$ year old soil

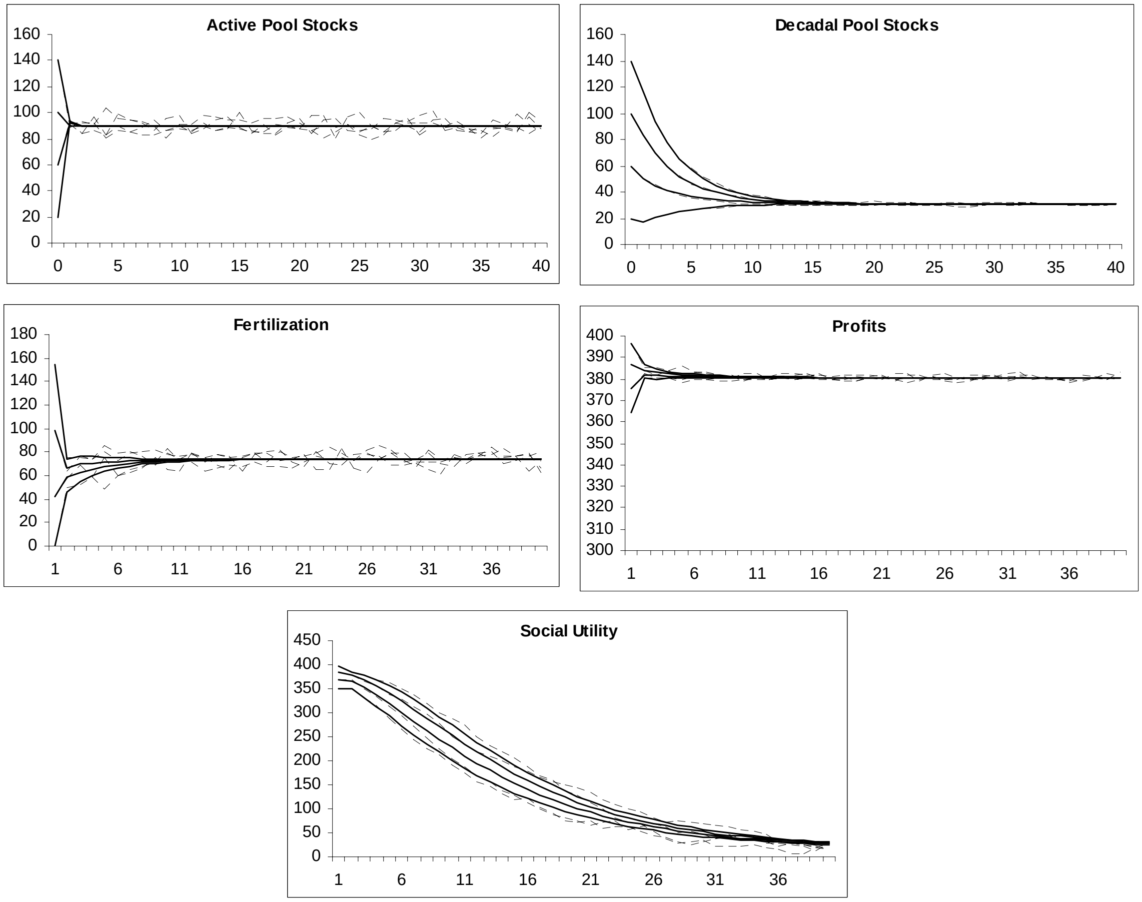

Figure 1. Selected simulated paths, private PV-optimal model as a function of time

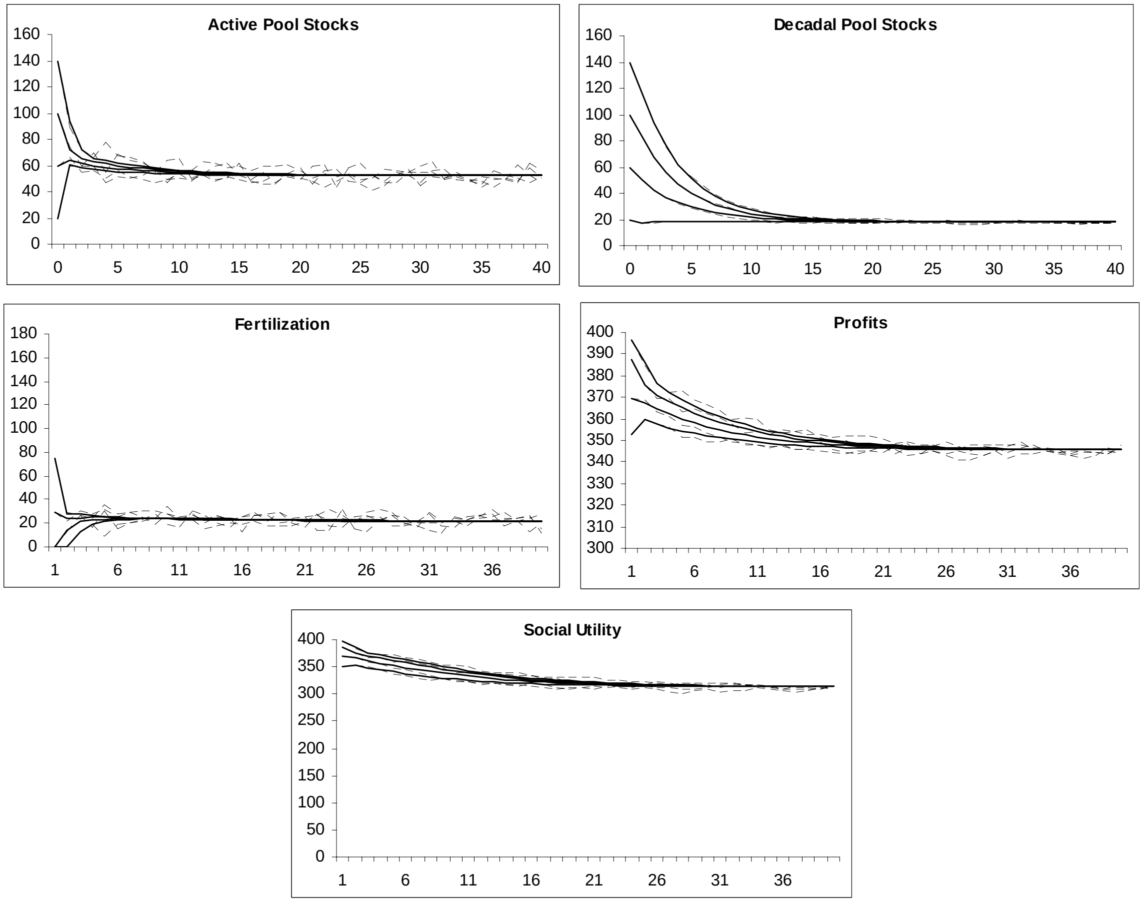

Figure 2. Selected simulated paths, social PV-optimal model as a function of time with internalized externality

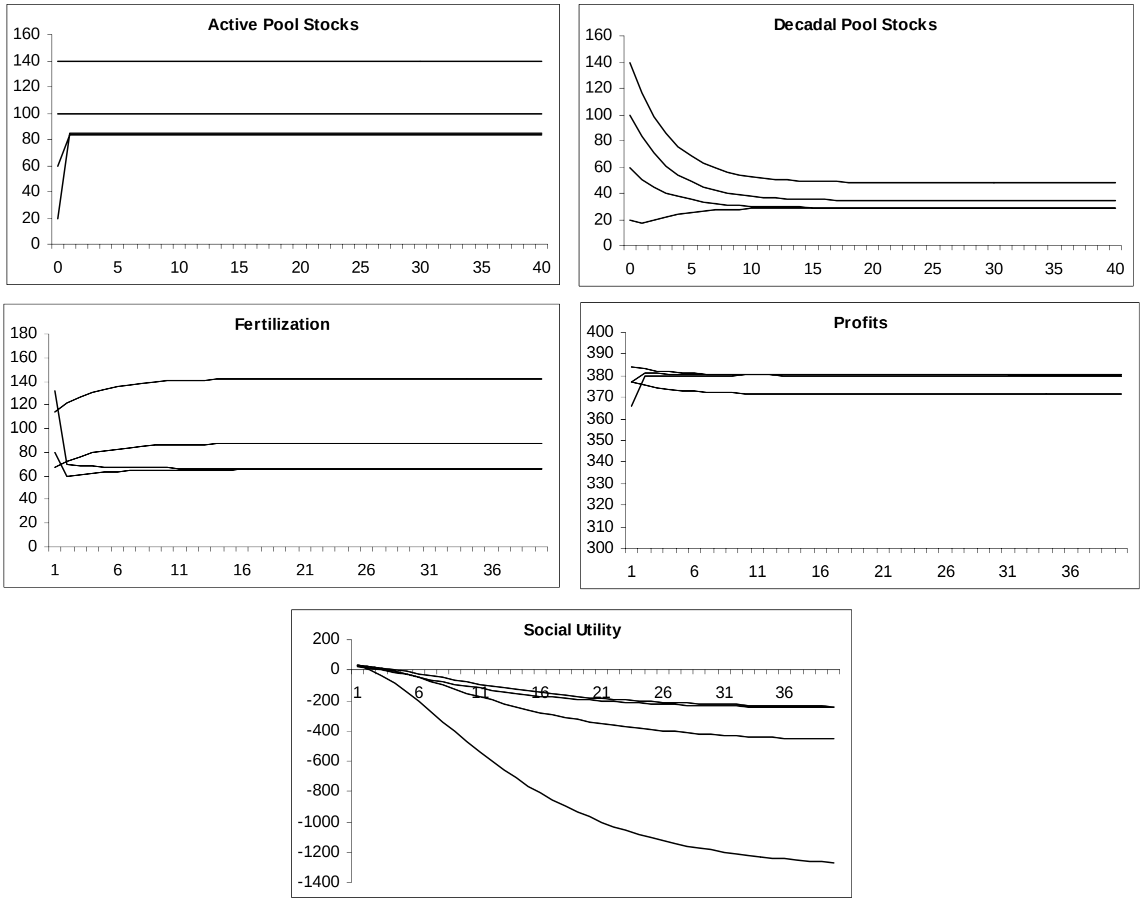

Figure 3. Selected simulated paths, strong N_1-sus model as a function of time

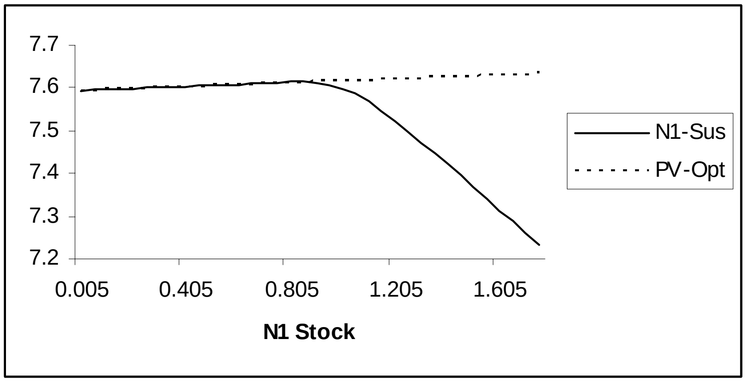

Figure 4. Private PV-optimal and N_1-sus value functions at steady state N_2

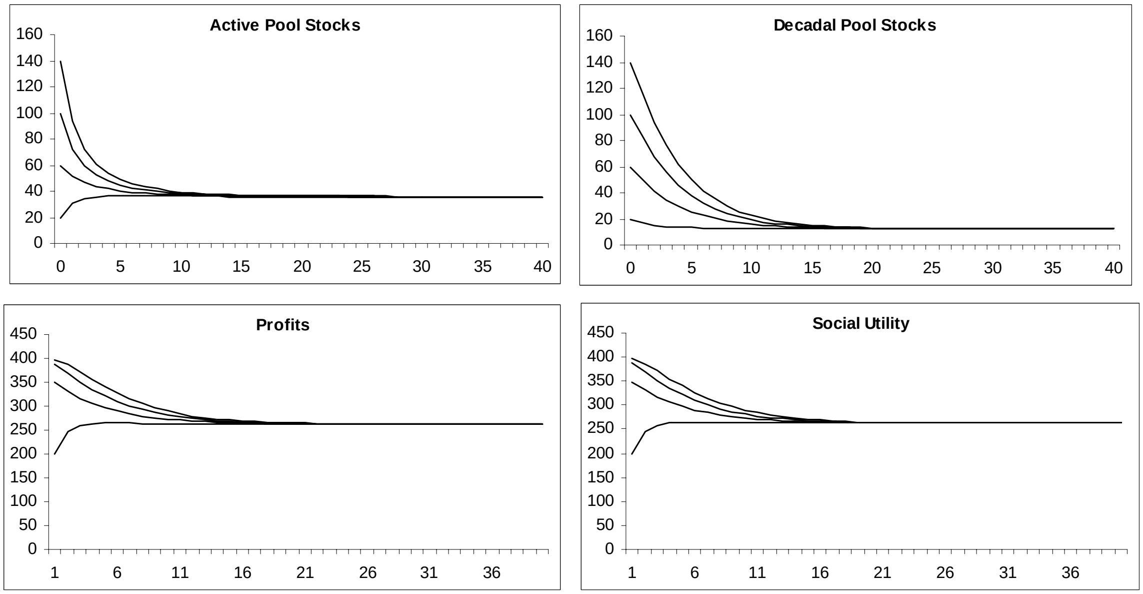

Figure 5. Selected simulated paths, strong X-sus model as a function of time

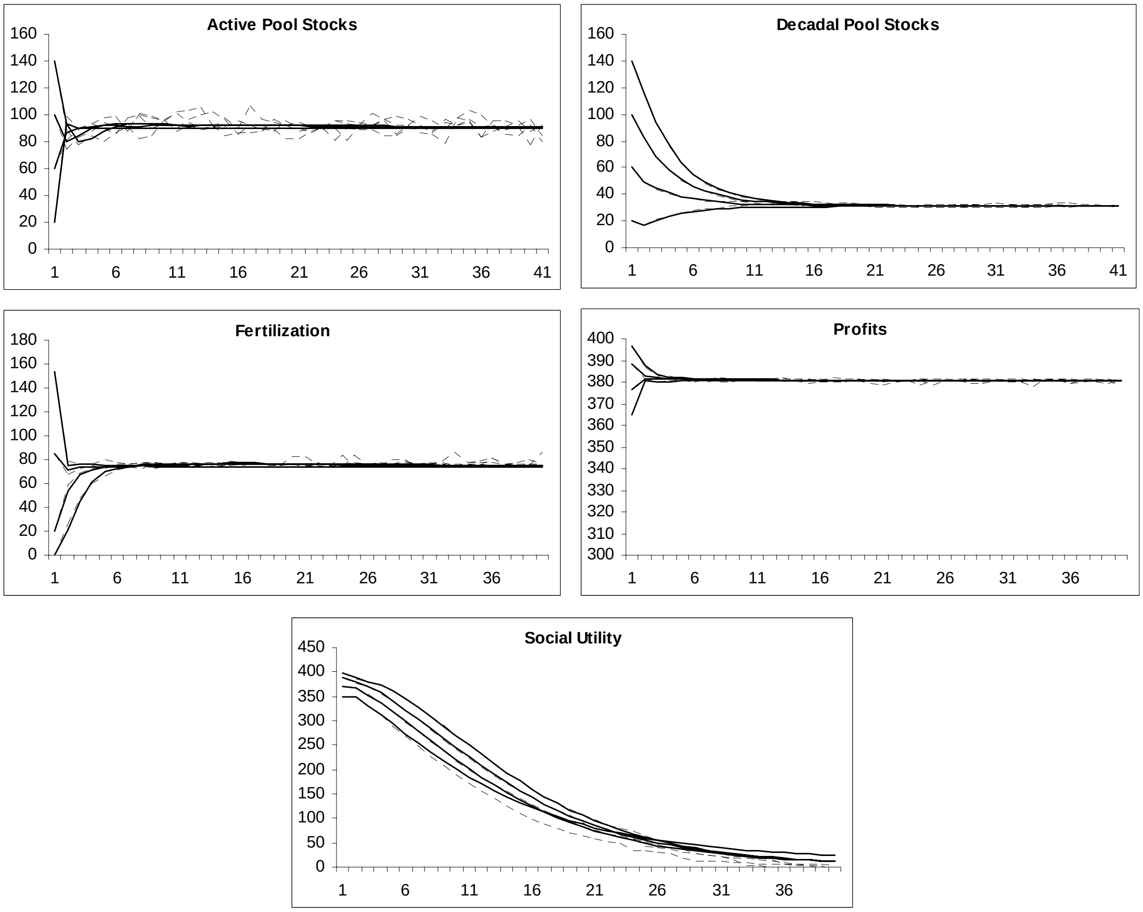

Figure 6. Selected simulated paths, private V-sus model as a function of time

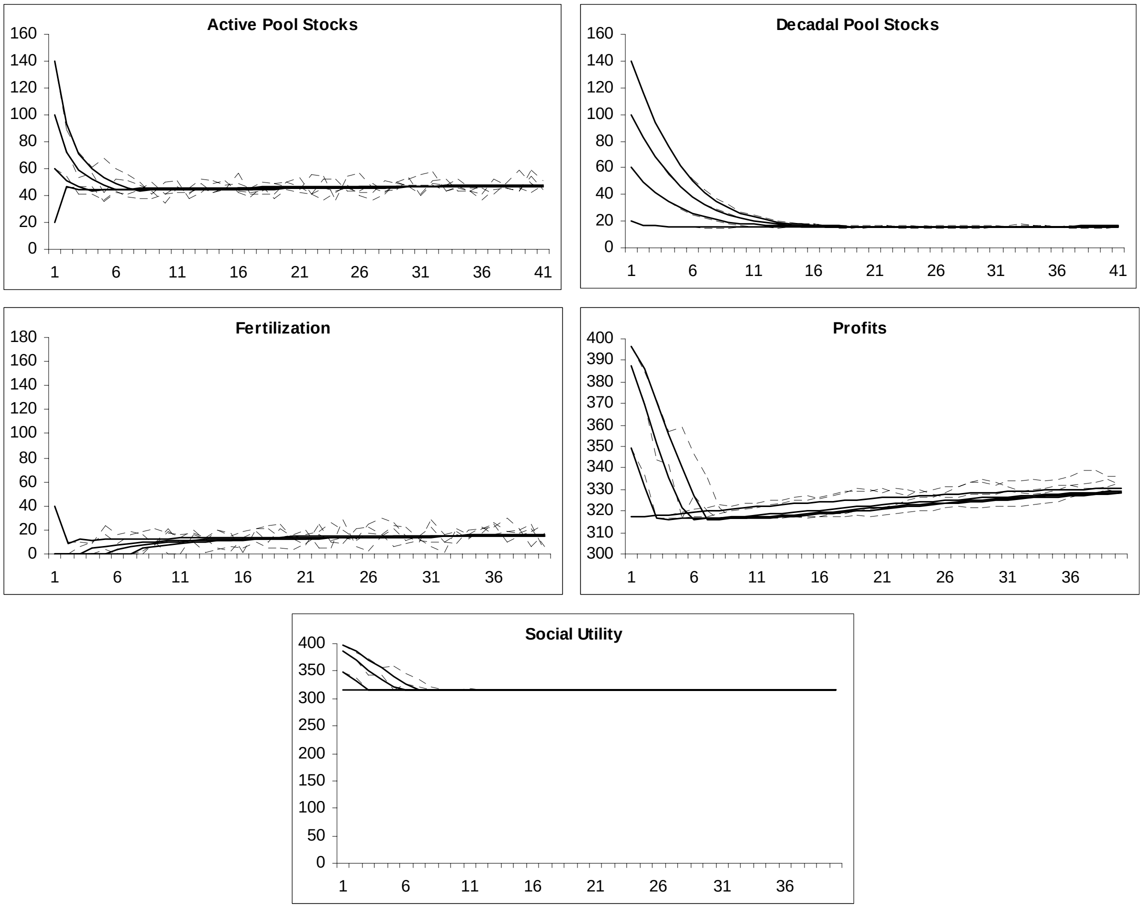

Figure 7. Selected simulated paths, social V-sus model as a function of time

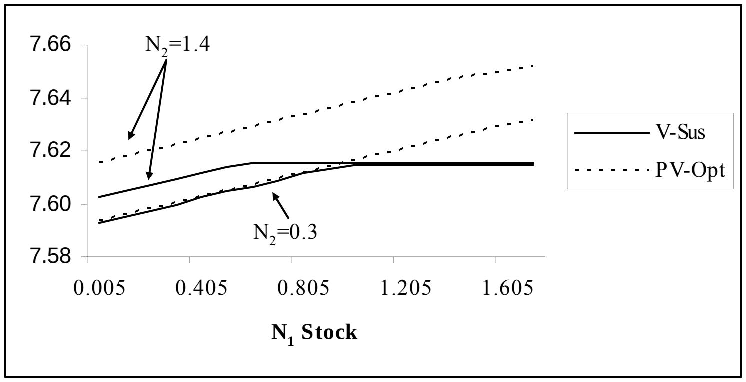

Figure 8. Private PV-optimal and V-sus value functions at various N_2 levels

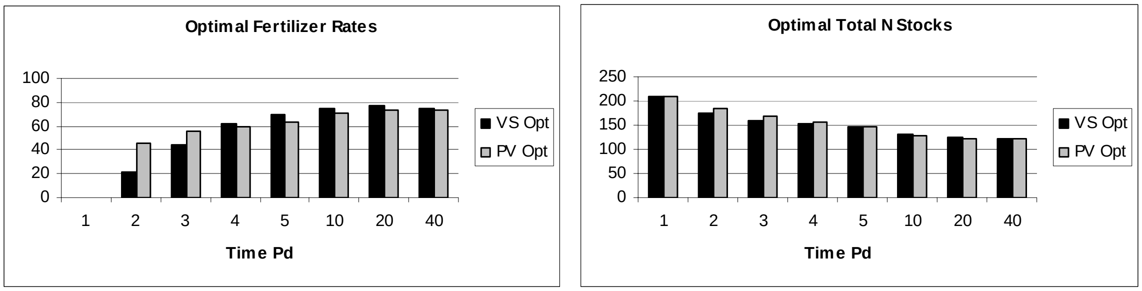

Figure 9. Optimal fertilization and total N stocks for selected time periods with initial stock levels (140, 140)

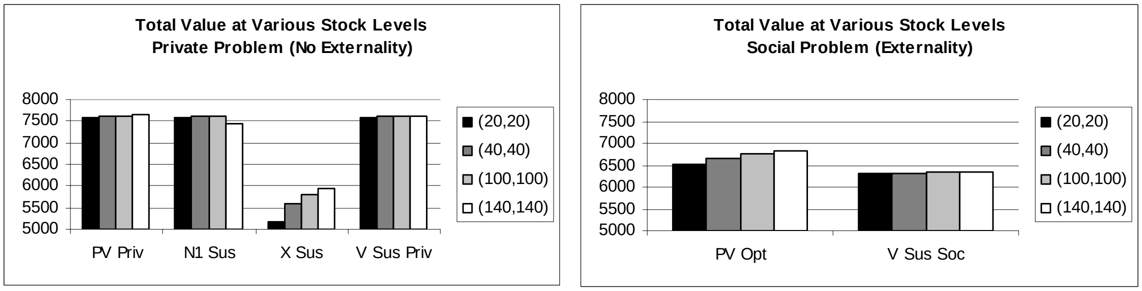

Figure 10. Total value at various initial stock levels (not including stepwise inefficiencies)

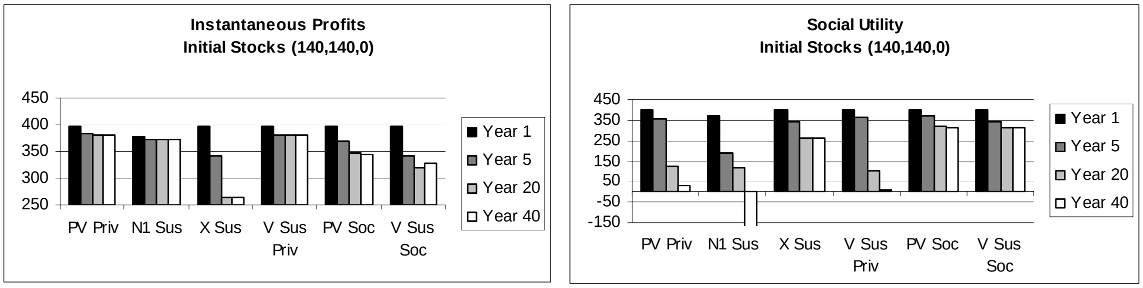

Figure 11. Welfare decomposition across time and sector (including stepwise inefficiencies)