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Monte Carlo Estimation of Project
Volatility for Real Options Analysis

Monte Carlo Estimation of Project Volatility for Real Options Analysis[*]

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ABSTRACT

Volatility is a fundamental parameter for option valuation. In particular, real options models require project volatility, which is very hard to estimate accurately because there is usually no historical data for the underlying asset. Several authors have used a method based on Monte Carlo simulation for estimating project volatility. In this paper we analyse the existing procedures for applying the method, concluding that they will lead to an upward bias in the volatility estimate. We propose different procedures that will provide better results, and we also discuss the business consequences of using upwardly biased volatility estimates in real options analysis.

1. INTRODUCTION

Real options analysis has led to very important theoretical advances in the field of project valuation. However, its practical application to real life projects presents some serious difficulties that have hindered its success. Estimating underlying asset volatility is one of the most important problems faced by practitioners wanting to use real options models. Sometimes, the only significant source of uncertainty for the project is the price of a commodity and, in such cases, market data can be used to estimate volatility (for example, Kelly, 1998, and Smit, 1997). However, most projects contain multiple sources of uncertainty, and historical data do not exist for some significant sources of volatility. For such projects, it may be useful to estimate the volatility for the project without options, and use the project without options as the underlying asset for the analysis (see Copeland and Antikarov, 2001, for example).

Some authors have tackled the problem of estimating the volatility of the project without options. Davis (1998) provides a closed-form expression for the volatility of projects that can be fitted to a particular production model. Copeland and Antikarov (2001) propose a general method that uses Monte Carlo simulation for estimating project volatility. This method has been used or recommended by some other authors – Herath and Park (2002), Munn (2002), Cobb and Charnes (2004), for example.

In this paper we will analyse the method proposed by Copeland and Antikarov, and the two procedures for its application that can be found in the literature. We argue that both procedures will overestimate project volatility, show examples of this bias and discuss its implications. We also propose alternative procedures that will provide better results.

The paper is structured as follows. After this introduction, Section 2 outlines the Copeland and Antikarov method for estimating project volatility with Monte Carlo simulation, and the different procedures for its application. In Section 3 it is argued that these procedures overestimate the true volatility, and an example of such bias is presented. Section 4 proposes different procedures for an unbiased estimation of volatility, and Section 5 gives an example of applying the proposed procedures. Section 6 analyses the business consequences of using upwardly biased volatility estimates. Finally, some concluding remarks are presented in Section 7.

2. THE USE OF MONTE CARLO SIMULATION TO ESTIMATE PROJECT VOLATILITY

In most projects there are multiple sources of uncertainty, and it becomes impossible to find market-traded securities that can be used as underlying assets for all of them. In such cases, it is useful to estimate the volatility for the project without options, and use the project without options as the underlying asset for the analysis. Copeland and Antikarov (2001) propose a Monte Carlo method for estimating project volatility. This method is outlined in this section, along with the procedures for its implementation that can be found in the literature.

We start by defining some important concepts. Let us consider an investment project with a known initial investment cash flow F_0 and a series of future uncertain cash flows F_t, $t = 1,\ldots,T$, and a continuously compounded discount rate r. Define the market value of the project at time n (MV_n) as the value of the cash flows that will occur after time n, discounted to time n:

$$MV_n = \sum_{t=n+1}^{T} F_t \cdot e^{-r(t-n)} \tag{1}$$

Define the present worth of the project at time n (PW_n) as market value at time n plus the current cash flow:

$$PW_n = MV_n + F_n = \sum_{t=n}^{T} F_t \cdot e^{-r(t-n)} \tag{2}$$

The present worth at time 0, PW_0, is the net present value (NPV) of the project. Although it is not apparent from equations (1) and (2), both MV_n and PW_n are expectations over the future cash flows, calculated at time n (this point will be made clearer in section 3). Let k_n be a random variable that represents the continuously compounded rate of return on the project between time n-1 and time n. Then:

$$PW_n = MV_{n-1} \cdot e^{k_n} \tag{3}$$

3

From expression (3) it follows that k_n can be written as:

$$k_n = \ln\left(\frac{PW_n}{MV_{n-1}}\right) \tag{4}$$

The project volatility is the uncertainty over expected project returns from period to period. Since k_n is the rate of return on the project between time n-1 and time n, the volatility of project value will thus be the volatility of k_n. Copeland and Antikarov propose using Monte Carlo simulation to build a probability distribution for k_1. The standard deviation of this simulated distribution may then be used as the volatility of the project.

It is easily seen that it does not make sense to calculate a single value for the project volatility if this volatility changes with time or with project value. So an implicit assumption of this method is that project volatility is constant. If the volatility changes with time, the method can be easily adapted by estimating the standard deviation of k_n for different values of n, instead of only using k_1 – this way, a term structure of volatility would be estimated, instead of only one project volatility. However, this method will be much more difficult (or even impossible) to apply if the volatility also changes with project value. Therefore, we will only consider the application of this method to projects whose volatility does not change with project value.

It is also clear that k_n can only be defined for projects whose present worth and market value do not change signs (typically, for projects whose present worth is always positive). In fact, this definition of the rate of return is particularly suited to market-traded assets, and may be unsuitable for many investment projects. A deeper analysis of this problem is beyond the scope of this paper.

Two different procedures for developing the probability distribution of k_1 can be found in the literature, both of them based on Monte Carlo simulation. Copeland and Antikarov (2001) (CA) suggest that MV_0 should be estimated with the expected cash flows and subsequently held constant, and so only PW_1 would be iterated in the simulation. Herath and Park (2002) (HP) use a different procedure, also followed by Cobb and Charnes (2004). These authors handle MV_0 and PW_1 as random variables, so both their values are simulated. They also consider that the values of these variables

should be generated independently, using different sets of random variates. In the next Section, we will argue that both these procedures overestimate the true volatility.

3. THE UPWARD BIAS IN VOLATILITY ESTIMATION

We will now show that both the CA and the HP procedures overestimate the project volatility. We will illustrate our argument with the case of a very simple project. The project consists of producing 100 units of a market-traded commodity that has a current price of 1. The continuously compounded rate of return on the commodity is normally distributed with a mean $\mu = 10\%$/year and a standard deviation $\sigma = 15\%$/year[1]. The only cost is the initial investment, whose value is irrelevant (since we are only interested in estimating volatility). We also consider that the 100 units of the commodity will only be available 2 years after starting the project, and that the rate of return shortfall is null for the commodity – so the correct risk-adjusted discount rate is the average annual commodity price increase, $r = \mu + 0.5 \cdot \sigma^2 = 11.125\%$ (see Dixit and Pindyck, 1994, Chapter 3, for more details on the expression used for this calculation).

Notice that this is a well-behaved project with constant volatility, particularly suited to the Copeland and Antikarov volatility estimation method. The present value of this project is perfectly correlated with the price of the commodity, and follows a geometric Brownian motion. Therefore, k_1 will have a normal distribution (as assumed in Herath and Park, 2002). The estimation method should thus provide an unbiased estimate for the project volatility, which is, by construction, 15% (since the project value is perfectly correlated with the price of the commodity).

It is very easy to build a simulation model for this project. Define x_1 and x_2 as two independent samples from a normal distribution with an average of 10% and a standard deviation of 15%. The initial price of the commodity is $P_0 = 1$ and the year-1 price can be simulated as:

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[1] This continuously compounded rate of return on the commodity is the annualized change in the price logarithm. If the price process is seen as a geometric Brownian motion $dP/P = \alpha \cdot dt + \sigma \cdot dz$, where dz is the increment of a standard Wiener process, then the mean of this continuously compounded rate of return is $\mu = \alpha - 0.5 \cdot \sigma^2$ (see Dixit and Pindyck, 1994, Chapter 3, for more details). Should we want to define $\alpha = 10\%$ instead of $\mu = 10\%$, then the expressions used in the simulation would have to be adapted, but the conclusions would not change.

$$P_1 = P_0 \cdot e^{x_1} \tag{5}$$

From P_1, the year-2 price of the commodity can be simulated as:

$$P_2 = P_1 \cdot e^{x_2} \tag{6}$$

Since the year-1 cash flow is $F_1 = 0$ and the year-2 cash flow is $F_2 = 100 \cdot P_2$, the year-1 present worth and the year-0 market value of the project are:

$$PW_1 = F_1 + F_2 \cdot e^{-r} = \left(100 \cdot P_2\right) \cdot e^{-0.11125} \tag{7}$$

$$MV_0 = F_1 \cdot e^{-r} + F_2 \cdot e^{-2 \cdot r} = \left(100 \cdot P_2\right) \cdot e^{-2 \cdot 0.11125} \tag{8}$$

The simulation will provide an expected year-2 commodity price of approximately 1.249 that, when discounted at the 11.125% discount rate, will lead to a present value of 1 – in fact, since the rate of return shortfall on the commodity was null, the present value of the expected future price should equal the current price. Using the expected year-2 price, we get a current project market value of $MV_0 = 100$.

The CA procedure uses this market value $MV_0 = 100$, and iterates only the value of PW_1. A simulation based on this procedure should lead to an estimated volatility of $\sigma \approx 21.21\%$. The HP procedure uses independent sets of random variates to generate MV_0 and PW_1. A simulation based on this procedure should lead to an estimated volatility of $\sigma \approx 30\%$ (the interested reader can easily confirm these values using standard simulation software like @Risk™ or Crystal Ball™, or even using only the random number generator of Microsoft Excel™).

The estimated values for the volatility seem particularly odd since it was assumed that $\sigma = 15\%$. However, it was not necessary to perform any simulation to calculate the volatility estimated by these procedures, and to conclude that they are estimating a wrong volatility. In order to perform the analysis, let us write k_1 as:

$$k_1 = \ln\left(\frac{PW_1}{MV_0}\right) = \ln(PW_1) - \ln(MV_0) \tag{9}$$

Let us analyse the CA procedure. Since MV_0 is a constant, k_1 is the sum of a random variable with a constant. Adding a constant to a random variable does not change the standard deviation, so:

$$\sigma_{k_1} = \sigma_{\ln(PW_1)} \tag{10}$$

From (7), $\ln(PW_1) = \ln(100) + \ln(P_2) - 0.11125$. Therefore the standard deviation of k_1 is equal to the standard deviation of $\ln(P_2)$. From (5) and (6), $\ln(P_2) = \ln(P_0) + x_1 + x_2$. Since x_1 and x_2 are samples from independent random variables with a standard deviation of 15%, it follows that:

$$\sigma_{k_1} = \sigma_{\ln(P_2)} = \sqrt{0.15^2 + 0.15^2} = \sqrt{0.045} \approx 21.21\% \tag{11}$$

This proves that, for this project, the CA procedure leads to an estimated volatility of approximately 21.21%, while the true volatility is 15%.

The HP procedure generates independent sets of random variates to calculate MV_0 and PW_1, so the volatility that is being estimated by this procedure is:

$$\sigma_{k_1} = \sqrt{\left(\sigma_{\ln(PW_1)}\right)^2 + \left(\sigma_{\ln(MV_0)}\right)^2} \tag{12}$$

It has been seen that the standard deviation of $\ln(PW_1)$ is $0.045^{\frac{1}{2}}$ and, using a similar argument, it can also be seen that the standard deviation of $\ln(MV_0)$ is identical. It follows that:

$$\sigma_{k_1} = \sqrt{0.045 + 0.045} = 30\% \tag{13}$$

So, the values that the procedures try to estimate are not the correct volatility of the project and are, in fact, considerably larger than the true volatility. The problem with these procedures concerns the calculation of cash flows. By using the ex post cash flows to calculate PW1, instead of their expected values, these procedures consider more sources of possible changes in PW_1 than those that exist in the first year of the project,

therefore leading to an increased volatility. The HP procedure increases the bias by using simulated cash flows, instead of their expected values, in the calculation of MV_0.

In the previous Section, the time at which the cash flows are estimated was deliberately left ambiguous when the concepts of market value and present worth were defined; now, in order to make the problem clear, we will redefine these concepts, explicitly incorporating the moment when the cash flows are estimated, and the information that is available at that moment.

The market value MV_n is the value at which a market-traded investment with similar cash flows would trade at time n. This market value is calculated at time n, using the information available at that moment. Let $E_n(F_t)$ be the expected value of the time t cash flow, calculated at time n with all the information available at that time. Then the market value can be defined as:

$$MV_n = \sum_{t=n+1}^{T} E_n(F_t) \cdot e^{-r(t-n)} \tag{14}$$

Similarly, the present worth at time n can be defined as:

$$PW_n = \sum_{t=n}^{T} E_n(F_t) \cdot e^{-r(t-n)} \tag{15}$$

By using simulated samples of all cash flows, instead of making the simulation just until year 1 and then calculating the expected value of future cash flows according to the information available at year 1, both the CA and the HP procedures are really defining the present worth as:

$$PW_1 = \sum_{t=1}^{T} F_t \cdot e^{-r(t-1)} = \sum_{t=1}^{T} E_t(F_t) \cdot e^{-r(t-1)} \tag{16}$$

This definition is not correct, since any measure of the project value in a given moment should only use information that is available at that moment. Expression (16) uses the values of future cash flows (unknown at year 1), instead of using only information available at year 1 (that is, the expected values of the cash flows), so it does not measure the year-1 project value, but it calculates an ex post value for a given

scenario. Moreover, the HP procedure also considers an incorrect definition of PW_0, which includes the simulated cash flows instead of the expected value of the cash flows calculated at time 0 (the CA procedure correctly considers PW_0 to be the expected project value at the beginning of the project).

Now, let us examine why these incorrect definitions of PW_1 and MV_0 lead to a significant upward bias in the estimation of volatility. Starting with PW_1, it is possible to see that, by using the ex post cash flows instead of their expected values, we are including more sources of possible changes in PW_1 than those that exist in the first year of the project – we are including sources of variation in project value that are subsequent to the first year and, therefore, we are artificially increasing the annualized project volatility. It is as if, to estimate the volatility of weekly returns on a stock, we would calculate the distribution of the stock prices one year from now, discount them to the end of the next week and then calculate the standard deviation of the resulting distribution of returns – this way we would surely obtain an upwardly biased estimate of the volatility.

Figure 1 illustrates this point. In this figure, we can see the difference between the distribution of the year-2 cash flow (F_2) and its expected value at the end of year 1 ($E_1(F_2)$), for the example considered above. The distribution of F_2 clearly shows a larger dispersion. Therefore, if F_2 is used instead of $E_1(F_2)$ to calculate PW_1, we get a more disperse distribution of project value, and a larger volatility.

The definition of MV_0 in the HP procedure creates another artificial source of variability in k_1, since the time 0 present worth of the project must be calculated with the time 0 expected cash flows, and is therefore constant. By simulating the future cash flows and using them in the definition of k_1, the HP procedure increases the upward bias in the volatility estimate[2].

Therefore, it is possible to conclude that both the HP and the CA procedures systematically overestimate the project volatility (with the exception of very simple projects whose only cash flows are the initial investment and a year-1 cash flow). In the next Section, some solutions to avoid the overestimation of volatility are discussed.

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[2] Brealey and Myers (2000), page 275, point out a similar problem in the use of simulation in project analysis. The authors state that it is wrong to calculate a distribution of values of PW_0, since there is only one PW_0, representing the price at which the project would be traded in a competitive capital market. Such a distribution would thus be meaningless.
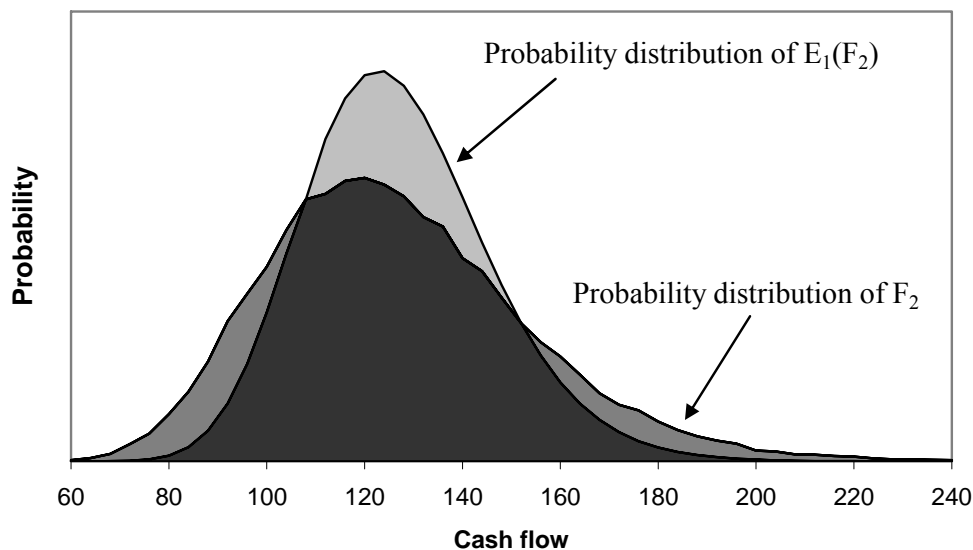
Figure 1 – Simulated probability distributions of $E_1(F_2)$ (light grey area) and F_2 (grey area; the dark grey area is common to both distributions).

4. ALTERNATIVE PROCEDURES FOR VOLATILITY ESTIMATION

In the previous Section it was shown that the existing procedures for applying the Copeland and Antikarov method will usually produce upward biases in estimating volatility. It was argued that the problem stems from the cash flows that are being used to calculate PW_1 and MV_0 – samples of the cash flows are used by the procedures for calculating each PW_1, while their expected values at year 1 should be used instead; furthermore, the HP procedure also uses samples of the cash flows to calculate MV_0, instead of their expected value at the beginning of the project.

$E_0(F_t)$, the expected value of the future cash flows needed to calculate MV_0, is easy to estimate. In fact, even if an analytical expression cannot be determined, it is very easy to calculate these values by simulation. Simulating the cash flows until the end of the project, and calculating the average of the values obtained will lead to an unbiased estimation of the expected cash flows.

To calculate PW_1, we want to simulate the project's behaviour in the first year and, for each iteration, estimate the expected future cash flows ($E_1(F_t)$) according to the information available at the end of this first year (that is, according to the values of the state variables in the end of the first year). For some projects, it may be possible to

determine analytically how these expected values can be calculated. When practicable, this is the fastest and most accurate procedure to calculate the expected future cash flows according to the year-1 information.

In the example presented in the previous Section, it would be possible to perform such an analysis. Since the rate of return shortfall on the commodity was null, the current price will be the present value of the expected future price. So, given a simulated year-1 price P_1, the expected year-2 cash flow is:

$$E_1(F_2) = 100 \cdot P_1 \cdot e^{0.11125} \tag{17}$$

If we use this expected value in the simulation, we will be estimating the true project volatility. In fact, a simulation performed with @Risk™ software, using 50000 iterations[3] and Latin Hypercube sampling, led to an estimated volatility $\sigma = 15.000\%$. The problem with this procedure is that it is often very hard, or even impossible, to find analytical expressions for the expected value of future cash flows, given the information available at year 1. So, we now turn to other alternative procedures.

As explained before, when an analytical expression for $E_0(F_t)$ cannot be found, simulation may be used to estimate that expected value. So, the use of simulation may also seem a good alternative to estimate $E_1(F_t)$. However, there is one problem: the estimation of $E_0(F_t)$ is based on the information available at the beginning of the project, and that information is known and does not change; on the other hand, the estimation of $E_1(F_t)$ must be based on the year-1 information, which changes in each iteration of the simulation.

In order to handle this problem, the estimation of project volatility may be based on a two-level simulation procedure. Each iteration of the first level simulates the project behaviour during the first year. When the end of the first year is reached by the iteration, it is then necessary to estimate the expected future cash flows, given the first year events. In order to obtain this estimation, a new simulation (the second level simulation) is performed. The starting point of this second level simulation is the information generated by the first level iteration, and each iteration will calculate a

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[3] In any simulation-based estimation there will be estimation error even if there is no systematic bias. This estimation error will decline with the number of iterations. Given the closeness between the theoretically expected values and the estimated values, it was considered that this number of iterations was appropriate to provide sufficiently accurate values in the cases that were considered.

sample of each cash flow until the end of the project. When the second level simulation is completed, the samples of the cash flows are averaged for each year, and this average will be the estimate of the expected value of the cash flows, given the year-1 information.

Algorithm for the two-level simulation procedure

numiter1 ← number of iterations of the first level simulation

numiter2 ← number of iterations of the second level simulation

For i = 1 to numiter1

    Simulate the project behaviour in the first year

    For j = 1 to numiter2

        Simulate the project behaviour after the first year, until the end

    Next j

    Calculate the average cash flows after the first year

    Use the average cash flows after the first year to calculate PW_1

    Use PW_1 to calculate a sample of k_1

Next i

Calculate the volatility as the standard deviation of k_1

This means that each iteration of the first level must be followed by a complete simulation of the second level, leading us to the most important shortcoming of this procedure – the length of computer time that is required. Assume that we intend to use 50000 iterations for the first level simulation and the same number of iterations for each second level simulation. This will require a total of $50000^2 = 25 \times 10^8$ iterations for the second level, which may be impracticable. This exponential growth of the total number of second level iterations with the number of iterations used in each level may quickly render the use of large numbers of iterations impossible. If the user is forced to reduce the number of iterations, then the accuracy of the results may also be reduced.

We used this procedure in the example given in the previous Section, using 50000 iterations in the first level and 5000 in the second. The computations required significant computational time (about forty minutes, while the other procedures only required a few seconds), and led to an estimated volatility $\sigma = 14.999\%$. Since it was possible to calculate the value of PW_1 analytically, we also analysed the error introduced into each iteration by the use of the second level simulation to estimate that

value. We found that the error had a mean $\mu_\varepsilon = 0.0004$ and a standard deviation $\sigma_\varepsilon = 0.269$, so this error is not very significant (notice that the average value of PW_1 is approximately 112).

In order to avoid the shortcomings of the previous procedure, we developed another estimation procedure, inspired in part by the Least Squares Monte Carlo approach for American option valuation (Longstaff and Schwartz, 2001). Instead of using one two-level simulation, whose complexity grows exponentially with the number of iterations, this approach just considers two single level simulations. The first one simulates the behaviour of the project during its whole life, and it is used to estimate a model that allows the calculation of the conditional expectation of PW_1, given the information available at year 1. The second only simulates the first year of the project, and uses the estimated model to calculate a value for PW_1 in each iteration.

Algorithm for the regression procedure

numiter1 ← number of iterations of the first simulation

numiter2 ← number of iterations of the second simulation

For i = 1 to numiter1

      Simulate the project behaviour for all the years

Next i

From the results of the first simulation, estimate a model (for example, using

      linear regression) that calculates the conditional expectation of PW_1 given

      year-1 information

For i = 1 to numiter2

      Simulate the project behaviour in the first year

      Use the estimated model to calculate the expected value of PW_1

      Use the expected value of PW_1 to calculate a sample of k_1

Next i

Calculate the volatility as the standard deviation of k_1

Linear regression may be a convenient way to estimate the model in the first simulation, although more sophisticated methods may sometimes provide better results. The dependent variable will be the year-1 discounted sum of the cash flows, which will be regressed on several functions of the project state variables – for example, those state

variables and some of their powers and cross-products. This procedure, termed the regression procedure, is less computationally demanding than the two-level simulation. However, the accuracy of results will usually be very dependent on the choice of the functions of the state variables that are to be used, and it may often be difficult to determine which ones produce the best model.

The regression procedure was also used in the example of the previous section. The first simulation had 50000 iterations, and was used to build a regression model of PW_1 that included the year-1 commodity price and its squared value[4], as well as a constant. We arrived at the following model:

$$PW_1 = -0.151 + 100.245 \cdot P_1 - 0.095 \cdot P_1^{2}$$ (18)

Notice that the exact model would be:

$$PW_1 = 100 \cdot P_1$$ (19)

The second simulation used 50000 iterations, and the model (18) was used to estimate PW_1 from the year-1 data. This simulation led to an estimated volatility of 15.005%. Since it was possible to calculate the value of PW_1 in each iteration, we also estimated the error introduced by expression (18). The mean error was $\mu_\varepsilon = 0.0013$ with a standard deviation $\sigma_\varepsilon = 0.006$, so it is not very significant. The volatility estimates provided by the different procedures are depicted in the graph of figure 2, along with the correct volatility, in order to make their comparison easier.

5. AN APPLICATION EXAMPLE

This section presents an application example of the volatility estimation procedures. An example originally analysed by Cobb and Charnes (2004) will be used, in order to compare the results provided by the new procedures with the results obtained by those authors.

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[4] It becomes clear from an analysis of the project that neither the squared commodity price nor the constant were required – the value of PW_1 can be computed solely from the year-1 price. However, we wanted to tackle this problem just as we would if it was impossible to reach that conclusion.
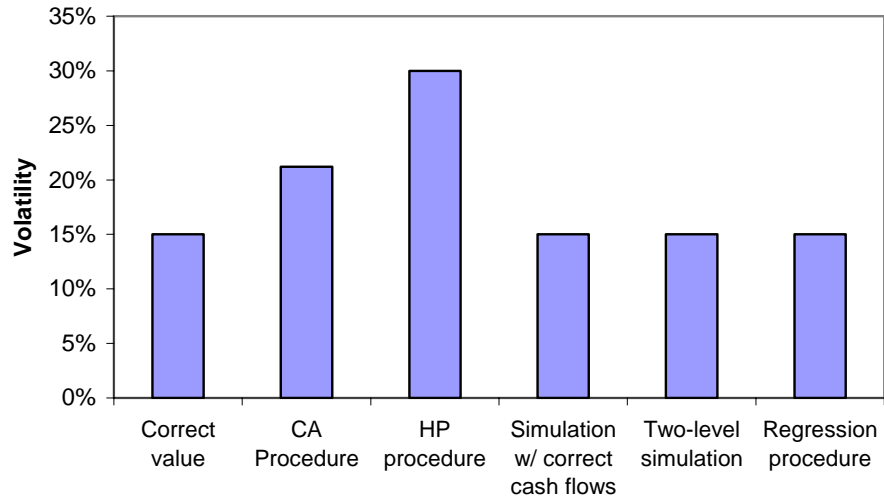
Figure 2 – Correct volatility and estimates provided by the different procedures for the example presented in Section 3.

Following Cobb and Charnes, it will be assumed that an investment project produces cash flows for five years. Each year t (t = 1,..,5) the relevant sources of uncertainty are the unit contribution margin X_t and the annual demand D_t. Let $N(\mu,\sigma^2)$ represent the normal distribution with mean μ and variance σ^2, let $T(a,b,c)$ represent the triangular distribution with minimum a, mode b and maximum c, and define the following distributions for X_t and D_t: $X_1 \sim N(50,10)$; $X_2 \sim N(60,15)$; $X_3 \sim N(70,21)$; $X_4 \sim N(80,28)$; $X_5 \sim N(90,36)$; $D_1 \sim T(95,100,105)$; $D_2 \sim T(82.5,100,117.5)$; $D_3 \sim T(70,100,130)$; $D_4 \sim T(57.5,100,142.5)$; $D_5 \sim T(45,100,155)$. The discount rate is 12%, the tax rate is 40% and the fixed expenses are 4250 in the first year and rise by 250 each year (all these values are non-stochastic). The required initial investment is irrelevant for the estimation of project volatility. The volatility of the project is not constant over time, so we will only address the first year volatility (as, in fact, Cobb and Charnes do).

Cobb and Charnes consider several different scenarios for the correlations among the random variables X_t and D_t. Since the focus of the present paper is quite different, only two scenarios will be considered here to exemplify the use of the volatility estimation procedures: (i) all random variables are uncorrelated and (ii) there is serial correlation in the unit price (and consequently in the unit contribution margin) with a correlation factor of 0.6 between X_t and X_{t+1} (t = 1,...,4).

The simulations were performed with @Risk™ software, using Latin Hypercube sampling. 50000 iterations were used both in single-level simulations and in the first level of two-level simulations; the second level of two-level simulations was based on 5000 iterations. The regression model (used in the regression procedure) included a constant, X_1, D_1, their product and their squared values.

Cobb and Charnes estimate a volatility $\sigma = 35.44\%$ for scenario (i), using the HP procedure. The same procedure led us to a volatility estimate $\sigma = 35.36\%$, and the CA procedure estimated a volatility $\sigma = 25.05\%$.

Since all random variables are independent in this scenario, the distributions of X_t and D_t for $t>1$ are not influenced by the events of the first year. So, in order to estimate the volatility in the first year of the project, it is only necessary to simulate the values of X_1 and D_1, and calculate PW_1 with the unconditional mean of the distribution of the remaining variables. By doing so, we arrived at a volatility estimate of $\sigma = 4.03\%$. This value may be considered surprisingly low but, in fact, it is easy to see that the considered price process may be seen as a mean reverting process with an infinite rate of mean reversion, and this mean reversion leads to a very small price risk. Figure 3 depicts the simulated probability distributions of project return considered by the HP and the CA procedures, and the distribution generated by the correct model. From this figure it becomes clear that the return distributions considered by the CA and the HP procedures show excessive dispersion, leading to the overestimation of volatility.

As explained in the previous section, the analytical calculation of PW_1 using the information available at the end of the first year leads to the most accurate results. However, to assess whether the other procedures suggested in the previous Section would lead to significantly different volatility estimates, they were also used. The two-level simulation procedure produced a volatility $\sigma = 4.04\%$ and the regression procedure led to $\sigma = 3.94\%$. Figure 4 compares the volatility estimates obtained with the different procedures.

Figure 3 – Simulated probability distributions of the continuously compounded rate of return on the project, for the models considered by the CA and HP procedures and for the correct model.

Figure 4 – Volatility estimates provided by the different procedures for scenario (i).

Since it was possible to analytically calculate the expected values of the cash flows subsequent to year 1, we were able to examine the errors that the procedures

introduced into PW_1. We concluded that the error introduced by the second level simulation had an average $\mu_\varepsilon = 0.005$ and a standard deviation $\sigma_\varepsilon = 16.12$, while the error introduced by the regression procedure had a mean $\mu_\varepsilon = -0.283$ and a standard deviation $\sigma_\varepsilon = 6.96$. Since the average value of PW_1 is nearly 5000, these errors can be considered quite small.

For scenario (ii), Cobb and Charnes report an estimated volatility of $\sigma = 40.69\%$, obtained with the HP procedure. The same procedure led us to an estimate of $\sigma = 40.85\%$, while the CA procedure led to $\sigma = 28.89\%$.

In this scenario, it is also possible to analytically calculate the expected cash flows given year-1 information. The demand is not serially correlated and is independent of the unit contribution margin. So, using $E_t(Y)$ to denote the expected value of the random variable Y given all the information available at year t:

$$E_1(D_t) = E_0(D_t) = 100, t = 2,..,5 \tag{20}$$

As for the unit contribution margin X_t, using some elementary statistics it is easy to conclude that it is related to the simulated year-1 contribution margin X_1 by the following expression:

$$E_1(X_t) = E_0(X_t) + 0.6^{t-1} \frac{\sigma_{X_t}}{\sqrt{10}}(X_1 - 50), t = 2,..,5 \tag{21}$$

Expressions (20) and (21) allow the calculation of PW_1 given the values of X_1 and D_1 simulated for the first year. Using these analytical results to calculate PW_1, the simulation produced an estimated volatility $\sigma = 9.63\%$. Once again, we chose to use the other procedures to assess whether or not the volatility estimates were significantly different. The two-level simulation procedure led to a volatility $\sigma = 9.64\%$ and the regression procedure led to $\sigma = 9.49\%$. Figure 5 compares the volatility estimates obtained with the different procedures.
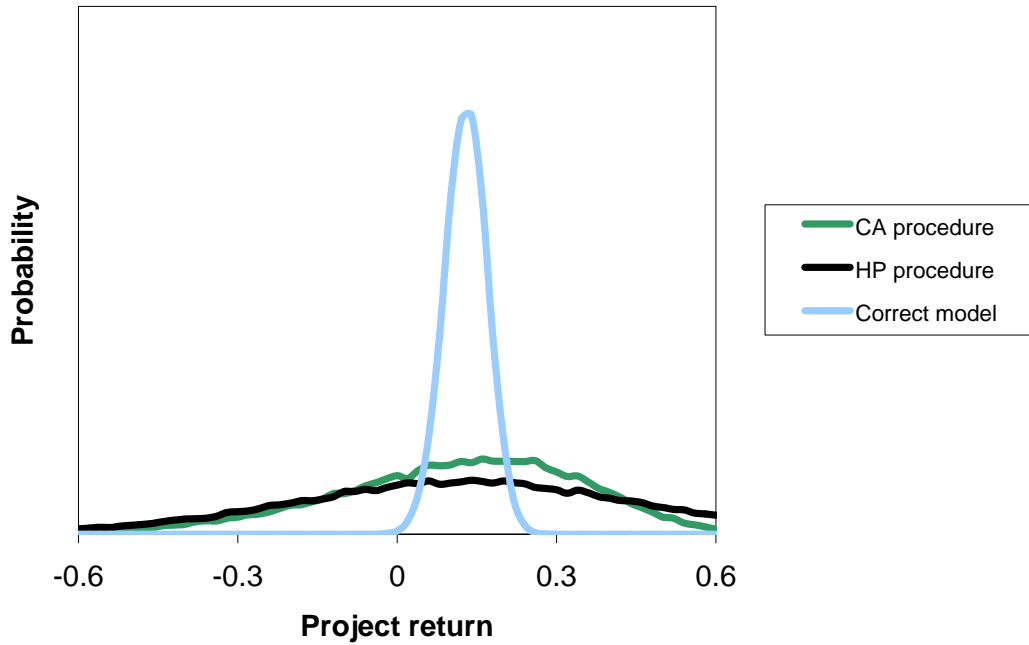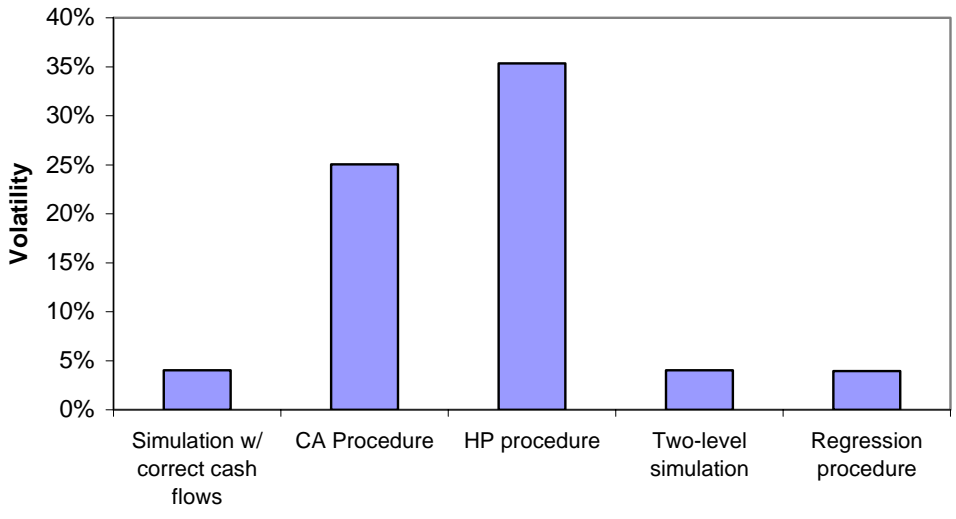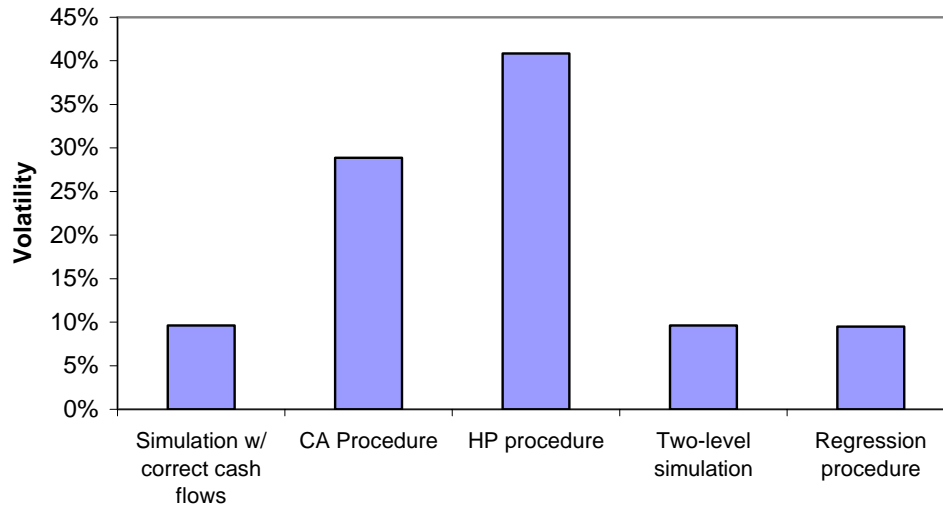
Figure 5 – Volatility estimates provided by the different procedures for scenario (ii).

The error introduced by the second level simulation had an average $\mu = -0.005$ and a standard deviation $\sigma = 17.25$, while the error introduced by the regression procedure had a mean $\mu = -0.615$ and a standard deviation $\sigma = 8.85$. The average value of PW_1 is nearly 5000, therefore these errors can be considered quite small.

So, in both scenarios the three procedures proposed in this paper were applicable, and led to similar estimates. These estimates were significantly lower than the ones obtained through the CA and HP procedures, confirming that the latter procedures introduce significant upward biases into the volatility.

6. IMPLICATIONS OF USING UPWARDLY BIASED VOLATILITY ESTIMATES

In this section, we will discuss the implications of using volatility estimation procedures that lead to upwardly biased estimates. We will argue that such estimates will lead to over-investment – that is, they will lead firms to invest in unprofitable projects – and we will show an example in which real options analysis, performed with upwardly biased volatility estimates, incorrectly recommends that a project should be undertaken.

In the approach proposed by Copeland and Antikarov (2001), the estimated project volatility is used as an input for the construction of lattices. These lattices intend to approximate the dynamics of project value, and are used to calculate the value of the real options embedded in the project. If the estimated volatility is incorrect, the lattices will approximate the wrong dynamics of project value, and lead to the calculation of incorrect real option values. In particular, the use of a larger volatility will lead to a distribution of project value with heavier tails. The value of real options is usually driven by the tails of the distribution: either the upper tail (in the case of expansion and growth options, for instance) or the lower tail (in the case of contraction and abandonment options, for instance). Heavier tails lead to larger real option values, and therefore they make the project look better than it really is.

Let us consider an example from Copeland and Antikarov (2001), Chapter 9. Copeland and Antikarov consider the 7-year project whose data is presented in table 1, and they assume that the initial investment is $1600, and that the discount rate is 12%. These values lead to an initial market value $MV_0 = \$1506.8$, and a net present value $NPV = -1600+1506.8 = -93.2$. So, without operational flexibility the project is unprofitable. Later on we will add operational flexibility to the project, to explain the influence of project volatility in the value of real options. For now, we turn to the estimation of the volatility of this project.

Copeland and Antikarov assume that there is uncertainty in the price estimates, that prices are estimated with a 10% standard deviation and that estimation errors have a 90% autocorrelation coefficient. Using the CA procedure in Crystal Ball™, the authors reach a volatility $\sigma = 21\%$ for this project.

Since this project was originally analysed with Crystal Ball™, we used the same software to estimate the project volatility. The CA procedure produced an estimated volatility $\sigma = 20.76\%$ and the HP procedure led to $\sigma = 29.28\%$. In order to get a better volatility estimate, we used the regression procedure. The regression model included a constant, the year-1 price and its squared value. This procedure estimated a volatility $\sigma = 17.60\%$[5]. So, once more, the CA and the HP procedures lead to higher volatility estimates.

---

[5] We were unable to use the two-level simulation procedure, with 5000 second-level iterations, in the Crystal Ball™ software, due to the software limit on the number of admissible correlations. However, we performed two-level simulation in the @Risk™ software, and it produced $\sigma = 17.67\%$ (very close to the volatility obtained with the regression procedure, as expected).

Table 1 – Calculation of the cash flows for the example (data from Copeland and Antikarov, 2001, pag. 247).

Year	1	2	3	4	5	6	7
Price/unit	10	10	9.5	9	8	7	6
Quantity	100	120	139	154	173	189	200
Variable cost /unit	6.0	6.0	5.7	5.4	4.8	4.2	3.6
Revenue	1000	1200	1321	1386	1384	1323	1200
- Variable cash costs	-600	-720	-792	-832	-832	-790	-711
- Fixed cash costs	-20	-20	-20	-20	-20	-20	-20
- Depreciation	-229	-229	-229	-229	-229	-229	-229
EBIT	151	231	280	305	303	284	240
- Cash taxes	-61	-93	-112	-122	-121	-114	-96
+ Depreciation	229	229	229	229	229	229	229
- Increase in working capital	-200	-40	-24	-13	0	13	24
Cash flow	119	327	373	399	411	412	397

The different volatility estimates lead to different probability distributions of project value. The lattices used by the Copeland and Antikarov approach assume a lognormal risk-neutral distribution of the project value. At the end of the first year, we reach the risk-neutral distributions depicted in figure 6, using the volatilities estimated by the previous procedures (in order to reach these distributions, we must additionally assume that the annually compounded risk-free rate of return is 5%.). Analysing this figure, it becomes obvious that the tails of the distributions are heavier for the larger volatilities (that is, for the ones estimated by the HP and CA procedures), meaning that project values far from the mean are more likely to occur if the volatility is higher, and less likely to occur if it is lower.

We will now analyse the impact of different volatilities in the value of real options. In order to keep the analysis simple, we will restrict the analysis to real options that expire at the end of the first year, immediately before the first cash flow is received (similar results would be reached if we considered real options with longer lives).

Assume that, in the first year of the project, there are the following options:

- the option to expand the project, making an additional investment of \$1820 in order to double the project value (that is, its present worth);
- the option to abandon the project, selling the assets for \$1410.
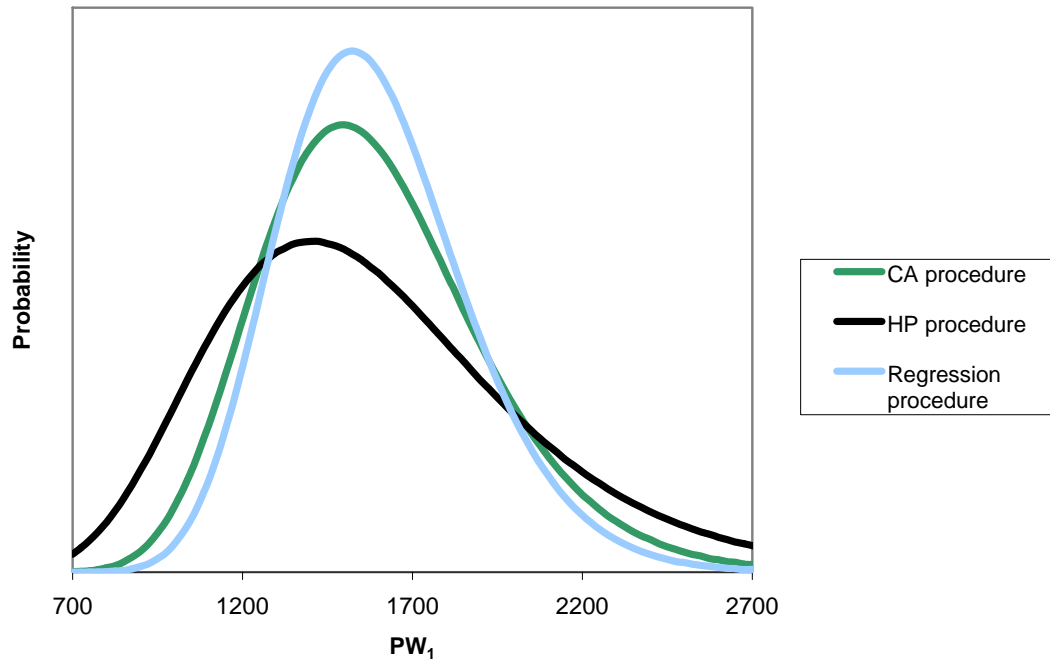
Figure 6 – Simulated risk-neutral distributions of the year-1 present worth of the project (PW_1), for the volatilities estimated by the CA, HP and regression procedures. It is assumed that the risk-free rate of return is 5%.

The first option is only valuable in the upper tail of the project value distribution (when present worth is above \$1820), and the second is only valuable in the lower tail of the distribution (when present worth is below \$1410). As it was seen, in figure 6, that the tails of the distribution are heavier when the volatility is larger, we may conclude that the HP volatility estimates will lead to larger option values, followed by the CA volatility estimates. Since both these procedures introduce upward biases in the volatility estimates, the option values that are reached when these estimates are used will also be upwardly biased. This means that the options will seem more attractive than they really are, and sometimes projects may be undertaken on account of those overvalued options.

In order to analyse the effect of overvaluing real options, we used a lattice with four steps per year to represent the dynamics of project value. The lattices obtained with

the previous volatility estimates are depicted in figure 7. In this example the options do not interact, so we can analyse them separately, and then add their values.

Dynamics of project value (present worth)

σ = 20.76% (CA procedure):

σ = 29.28% (HP procedure):

σ = 17.60% (regression procedure):
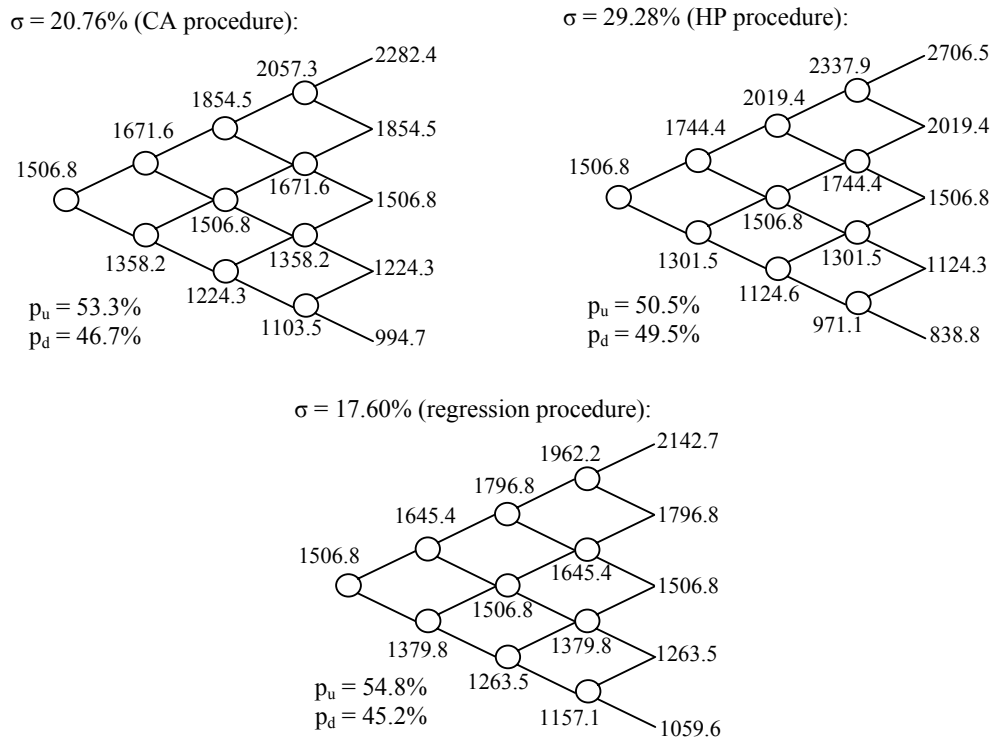
Figure 7 – Lattices depicting the first year dynamics of project value, under the volatilities estimated by the CA, HP and regression procedures. The risk-neutral probabilities of an up movement (p_u) and of a down movement (p_d) are presented below the lattices. It is assumed that the risk-free rate of return is 5%.

The lattices in figure 8 approximate the dynamics of the value of the expansion option under the different estimated volatilities. When we use either the CA or the HP estimates, it is profitable to exercise the option in one more node of the lattice than when we use the regression procedure estimate (which is the most correct estimate). It can also be seen that, when the volatility is larger, the value of the option, when it is

exercised, is also larger. So, the expansion option is overvalued by both the HP and the CA procedures, making it seem more attractive than it really is[6].

Value of the expansion option

σ = 20.76% (CA procedure):

```
                              259.4 (NE)    462.4 (E)
                     145.0 (NE)    ◯
             80.8 (NE)    ◯             34.5 (E)
     44.8        ◯              18.2 (NE)    0 (NE)
     (NE) ◯          ◯    9.6 (NE)    ◯
         5.0 (NE)    ◯         0 (NE)
              ◯         0 (NE)         0 (NE)
                  0 (NE)    ◯
                       0 (NE)    0 (NE)
```

σ = 29.28% (HP procedure):

```
                              539.9 (NE)    886.5 (E)
                     318.1 (NE)    ◯
             183.1 (NE)    ◯             199.4 (E)
     103.5       ◯              99.5 (NE)    0 (NE)
     (NE) ◯          ◯    49.7 (NE)    ◯
         24.8 (NE)   ◯         0 (NE)
              ◯         0 (NE)         0 (NE)
                  0 (NE)    ◯
                       0 (NE)    0 (NE)
```

σ = 17.60% (regression procedure):

```
                          174.6 (NE)    322.7 (E)
                 94.4 (NE)    ◯
         51.1 (NE)    ◯             0 (NE)
     27.6     ◯              0 (NE)    0 (NE)
     (NE) ◯        ◯    0 (NE)    ◯
         0 (NE)   ◯         0 (NE)
              ◯        0 (NE)         0 (NE)
                 0 (NE)    ◯
                      0 (NE)    0 (NE)
```

E : exercise the option
NE : do not exercise the option

Figure 8 – Lattices depicting the dynamics of the value of the expansion option and the optimal exercise decision, under the volatilities estimated by the CA, HP and regression procedures.
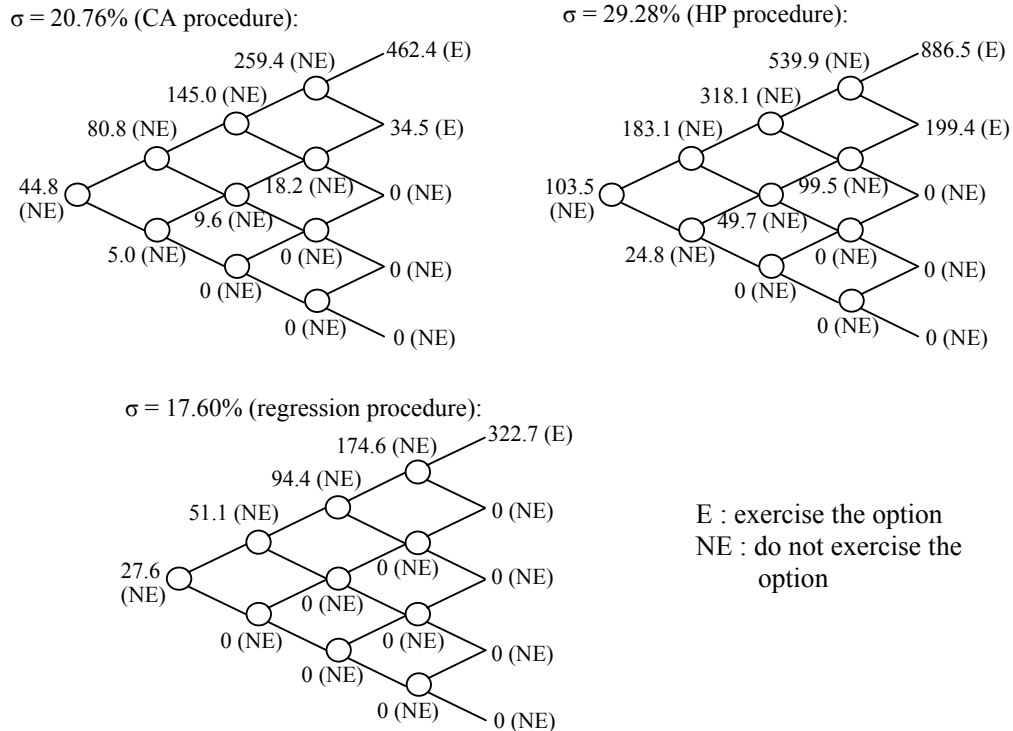
Figure 9 represents the value of the abandonment option. Now, the option is exercised in the same nodes of the lattice when we use the CA and the regression procedure estimates of the volatility. However, the CA estimate always leads to a larger value when the option is exercised. The HP estimate makes it profitable to exercise the option in one additional node of the lattice, and always leads to larger option values than the other estimates.

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[6] In order to make a complete analysis, we should also take the risk-neutral probabilities into account, since these probabilities are changed by the different volatilities. We kept the risk-neutral probabilities out of the analysis because, in this situation, they do not change the results, and in this way we were able to keep the analysis much simpler.

Value of the abandonment option

σ = 20.76% (CA procedure):

0 (NE)   0 (NE)
0 (NE)
18.2 (NE)   0 (NE)
58.9 (NE)   0 (NE)   0 (NE)
39.5 (NE)
106.8 (NE)   85.7 (NE)   185.7 (E)
186.5 (NE)
306.5 (E)   415.3 (E)

σ = 29.28% (HP procedure):

0 (NE)   0 (NE)
0 (NE)
33.4 (NE)   0 (NE)
101.6 (NE)   0 (NE)   0 (NE)
68.3 (NE)
173.7 (NE)   139.7 (NE)   285.7 (E)
285.7 (E)
438.9 (E)   571.2 (E)

σ = 17.60% (regression procedure):

0 (NE)   0 (NE)
0 (NE)
13.1 (NE)   0 (NE)
43.8 (NE)   0 (NE)   0 (NE)
29.2 (NE)
82.2 (NE)   65.4 (NE)   146.5 (E)
148.4 (NE)
252.9 (E)   350.4 (E)

E : exercise the option
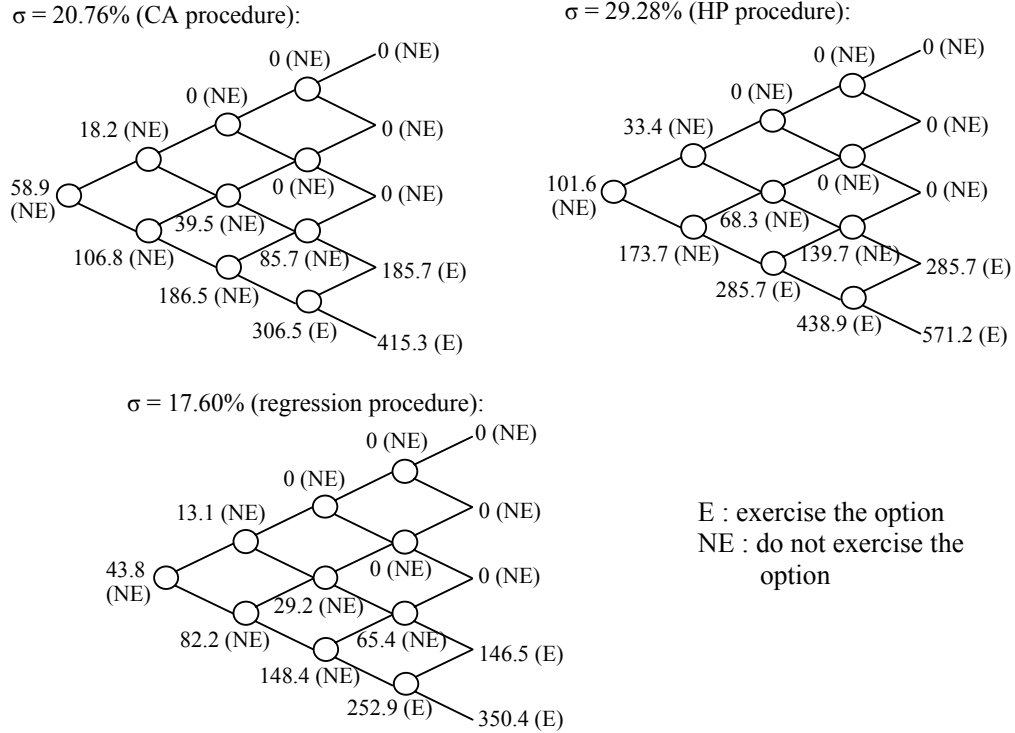NE : do not exercise the option

Figure 9 – Lattices depicting the dynamics of the value of the abandonment option and the optimal exercise decision, under the volatilities estimated by the CA, HP and regression procedures.

In order to calculate the project NPV, incorporating the value of the flexibility, we may simply add the value of both options to the initial NPV (calculated without incorporating flexibility). Notice that we may only proceed in this way because the options do not interact (see Trigeorgis, 1993, for more details).

The HP procedure leads to NPV = -93.2+103.5+101.6 = $111.9. With this NPV, the project would probably be undertaken without hesitation. If the CA procedure is used, we have NPV = -93.2+44.8+58.9 = $10.5. So, the NPV is much smaller than with the HP procedure. Since the NPV is positive, the project is still undertaken. If we use the regression procedure, we reach NPV = -93.2+27.6+43.8 = -$21.8. Since the latter procedure produces the best volatility estimate, the latter NPV is also the best estimate of the project NPV. This means that both the HP and the CA procedures lead to a wrong

investment decision – both procedures produce a positive NPV for an unprofitable project (a project that really has a negative NPV)[7].

In the previous example, the use of the HP and the CA procedures produced upward biases in the project NPV. This situation will be repeated in most projects. Since these procedures lead to overestimation of project volatility, and since option values are driven by the tails of the risk-neutral distribution of project value, the use of these procedures will overvalue the real options embedded in the project and the NPV of the project with flexibility. So, if these procedures are used, some unprofitable projects may be undertaken.

Another consequence of using biased volatility estimates is the deferment of projects beyond the optimal initiation time. If deferment options are present they will also be over-valued, and so it may happen that the value of keeping the option alive is incorrectly considered larger than the value of immediately starting the project. This means that upwardly biased estimates of volatility may also result in excessively delayed projects. The new procedures proposed in this paper will lead to better estimates of project volatility, and consequently to more accurate calculations of the value of real options.

7. CONCLUDING REMARKS

The application of real options analysis to the valuation of real-life projects presents some serious difficulties. A general approach proposed by Copeland and Antikarov (2001) seems to make the practical application of real options analysis much more accessible. However, this general approach has some important weaknesses. One of these weaknesses lies in the procedure for volatility estimation. This paper shows that both the procedure originally proposed by Copeland and Antikarov and another procedure used by Herath and Park (2002) and Cobb and Charnes (2004) introduce significant upward biases in the estimates of volatility. Given the importance of this parameter in option analysis, it can be expected that such biases will sometimes lead to the overvaluation of investment projects, and to over-investment. It may also happen

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[7] Notice that these conclusions do not change when we use larger lattices. Lattices with 100 levels produced NPVs of $104.2, $14.3 and -$16.9 for the volatility estimates produced by the HP, CA and regression procedures, respectively.

that projects are excessively delayed due to over-valuation of deferment options. This paper proposes alternative procedures that will lead to better estimates of project volatility, thus allowing more accurate valuations.

Some other weaknesses of the approach have not been addressed by this paper. For example, expression (4) will be undefined when the present worth of the project becomes negative, and it is even doubtful that the approach can be applied when the project value does not follow a geometric Brownian motion. Given the importance of this approach for the practical application of real options analysis, it is important that other future works address these weaknesses in order to make the approach more widely applicable.

REFERENCES

- Brealey, R. and Myers, S., 2000, *Principles of Corporate Finance*, 6th edition, Boston: Irwin/McGraw-Hill.
- Cobb, B.R. and Charnes, J.M., 2004, "Real options volatility estimation with correlated inputs", *The Engineering Economist* (49), 119-137.
- Copeland, T. and Antikarov, V., 2001, *Real options: a practitioners guide*, New York: Texere.
- Davis, G.A., 1998, "Estimating volatility and dividend yield when valuing real options to invest or abandon", *The Quarterly Review of Economics and Finance* (38) 725-754.
- Dixit, A.K. and Pindyck, R.S., 1994, *Investment under uncertainty*, Princeton University Press.
- Herath, H.S.B. and Park, C.S., 2002, "Multi-stage capital investment opportunities as compound real options", *The Engineering Economist* (47), 1-27.
- Kelly, S., 1998, "A binomial lattice approach for valuing a mining property IPO", *The Quarterly Review of Economics and Finance* (38), 693-709.
- Longstaff, F.A. and Schwartz, E.S., 2001, "Valuing American options by simulation: A simple least-squares approach", *The Review of Financial Studies* (14), 113-147.

- Munn, J., 2002, *Real options analysis: Tools and techniques for valuing strategic investments and decisions*, John Wiley and Sons.

- Smit, H.T.J., 1997, "Investment analysis of offshore concessions in the Netherlands", *Financial Management* (26), 5-17.

- Trigeorgis, L., 1993, "The nature of option interactions and the valuation of investments with multiple real options", *Journal of Financial and Quantitative Analysis* (28), 1-20.

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