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GAMMA, a Simulation Model for Ageing, Pensions and Public Finances

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Abstract in English

To answer policy questions that have intergenerational implications, a computable simulation model should obey four conditions: it should incorporate long-term demographic developments, it should include a detailed modelling of the public sector, it should decompose the population into several generations and it should account for the behaviour of the various economic agents. This document describes and illustrates a model that meets all these conditions. It is an applied general equilibrium model that is based on generational accounting principles named GAMMA (Generational Accounting Model with Maximizing Agents).

Abstract in Dutch

De intergenerationele implicaties van beleid kunnen worden ingeschat met een model dat de lange-termijnbevolkingsontwikkeling, de bevolkingssamenstelling, de publieke sector en het gedrag van de economische agenten beschrijft. Dit document bespreekt en illustreert een model dat deze karakteristieken heeft. Het is een toegepast, algemeen-evenwichtsmodel met overlappende generaties, waarbij alle tranasacties met de publieke sector en de pensioensector per generatie worden bepaald (generationele rekeningen). We noemen het GAMMA (Generational Accounting Model with Maximizing Agents).

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Preface

The need to make accurate long-run economic and fiscal projections has become increasingly important in recent years in light of the demographic changes occuring in The Netherlands. The changing population structure will have profound impacts on the Dutch economy, public finances and pension system in the years ahead. To simulate long-run developments and assess the intergenerational implications of reforms that address the effects of demographic changes, the CPB uses the computable general equilibrium model GAMMA (Generational Accounting Model with Maximizing Agents) as an instrument. This document gives an accessible description of GAMMA. In addtition to a system of generational accounts for the government sector, the model includes a detailed elaboration of the private pension sector, overlapping generations of households and sub-models for household and firm behaviour. Integration of these sub-models guarantees consistency, accounts for endogenous behavioural reactions after policy changes and makes an evaluation of the welfare effects of policy measures possible.

Over the past five years, several people have contributed to the development and application of GAMMA. Nick Draper was primarily responsible for the model building project. The other contributors were Alex Armstrong, Jan Bonenkamp, Andre Nibbelink, Harry ter Rele, Richard Rosenbrand and Ed Westerhout. Gilbert van Hagen and Thijs Knaap were involved in the project at an earlier stage. This document was edited by Alex Armstrong and Nick Draper; Jan Bonenkamp, Harry ter Rele and Ed Westerhout also contributed to the text. Both Andre Nibbelink and Richard Rosenbrand gave technical support.

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Coen Teulings

Director

1 Introduction

Changes in the demographic structure of The Netherlands will profoundly impact the economy in the coming decades. Low fertility and increasing life expectancy will combine to produce a dramatic increase in the elderly dependency ratio. A higher proportion of retirees to workers will result in increased demands on the public pension and health care systems as well as in a relative shrinkage in the tax base. The Dutch case is not typical. Many industrialized countries face the same issues, although they may differ in the timing of the developments and their severity. The reason is that the demographic developments that underlie population ageing as well as the financing method of social security deviate from each other. A long list of studies now exists that quantify the long-run development of public finances in a typical country (Van Ewijk et al. (2006) for the Netherlands, Guest (2006) for Australia) or several countries (Raffelhueschen (1999a), European Commission (2006)).

The problems caused by demographic changes can be addressed in a number of ways. Policies of increased taxation or reduced government expenditure can help to alleviate the pressure on public finances. The same holds true for institutional reforms of the pension sector, the health care sector or the sector that provides long-term care services. Despite the fact that all these policies contribute to an improved fiscal outlook, they differ in a number of aspects. First, policies that affect the behaviour of households differently may have different macroeconomic effects. An obvious example is a reform that raises taxes versus a reform that reduces public expenditure. Second, various policies may have differing effects on different generations of households, such as a reform that is implemented immediately versus a reform that delays the adjustment until some later time. Third, policy reforms may be different in their direct fiscal impact. Policies that reduce expenditure on national defense, for example, may have an effect on the primary surplus that grows in line with GDP. Whereas the effect of policies that reduce health care consumption may grow in line with the much higher increase of health care consumption.

In order to make long-run fiscal projections and answer policy questions that have intergenerational implications, a computable simulation model should obey four conditions: it should incorporate long-term demographic developments, it should include a detailed modelling of the public sector, it should decompose the population into several generations and it should account for the behaviour of the various economic agents. This document describes and illustrates a model that meets these conditions. It is an applied general equilibrium model that is based on generational accounting principles named GAMMA (Generational Accounting Model with Maximizing Agents). Hence it is a new member of the family of overlapping generation (OLG) models, which we take to consist of generational accounting (GA) models and applied general equilibrium (AGE) models that distinguish different generations of households.

Obviously, the GAMMA model is not the first in the field of models that feature overlapping generations (OLG) of households. Following Auerbach and Kotlikoff (1987) for the U.S., models have been constructed for individual European countries (Lassila and Valkonen (2001), Jensen et al. (2002), Lassila (1999)) and for multiple countries (Borsch-Supan et al. (2006) and INGENUE (2005)). Neither is GAMMA the first OLG model for the Netherlands (Broer (2001)). However, it differs from most other OLG models mainly on three accounts. The first is the inclusion of behaviour of economic agents like households, firms and pension funds, which distinguishes GAMMA from the class of generational accounting (GA) models. The second is the detailed modelling of institutions in the public sector, which distinguishes GAMMA from many applied general equilibrium (AGE) models. Third, a characteristic of GAMMA is the specific version of the life-cycle approach that the model adopts to describe the saving and labour supply behaviour of households which is, as far as we know, unique in models with overlapping generations of households.

GA models have been quite successful since their introduction in the early nineties (Auerbach and Kotlikoff (1991)). As they make so few specific assumptions, they can be applied in a variety of circumstances. Indeed, GA models are nowadays applied worldwide by researchers to assess the sustainability and intergenerational implications of fiscal policies in various countries, two issues that are highly relevant for policy making (Auerbach and Leibfritz (1999), Raffelhueschen (1999a), Raffelhueschen (1999b)). By their nature, GA models are well suited for analyzing the implications of long-term trends, such as demographic changes. However, when it comes to the analysis of policy reforms, GA models are less suitable due to their neglect of economic behaviour. For instance, they may give false answers to questions about the incidence of taxation (Haveman (1994), Buiter (1995)).

Due to its inclusion of economic behaviour, GAMMA improves GA models on both accounts. In particular, accounting for economic behaviour implies that age profiles of various tax items are endogenous. Policy reforms that affect tax bases may change these age profiles as well. Hence, accounting for economic behaviour may also affect the assessment of the fiscal stance. Furthermore, GAMMA improves upon the first generation of GA models by accounting for changes in age profiles that result from future trends. As Bovenberg and ter Rele (2000) have shown, it is particularly relevant to account for likely changes of labour market participation rates and for the increasing scope (maturing) of pension schemes when making fiscal projections.

A final point on which the GAMMA model improves on GA models is the scope of generational accounting. Where GA calculations tend to focus on the transfers to and from the public sector, our model also accounts for transfers to and from pension funds that provide supplementary pensions. This is important in cases where policy reforms affect not only net benefits from the public sector, but also those from the pension sector.

AGE models are constructed precisely for simulating the effects of policy reforms (Auerbach and Kotlikoff (1987), Altig et al. (2001), Bovenberg and Knaap (2005)). Models of this type that focus on ageing populations distinguish between different age cohorts, allowing them to assess the intergenerational implications of government policies. When it comes to making future projections and assessing fiscal sustainability, these models are less suitable, however. The reason is not so much fundamental as practical. AGE models often lack the kind of institutional details that may be particularly relevant for predicting the future development of the economy. GAMMA improves on these models due to its detailed description of institutions in the public and private pension sectors.

This document gives a full description of GAMMA. It describes its component modules (households, firms, pension funds, the government), and it explains the calibration procedure and the solution method. In addition it illustrates the working of the model with four sets of simulations:

- A projection of the likely development of the Dutch economy under the hypothetical assumption of unchanged government policies;
- A set of scenarios that describe the effects of alternative demographic developments;
- A set of scenarios that illustrate the effects of tax policy reforms;
- A set of scenarios that illustrate the effects of pension reforms.

That GAMMA can be used both for projections and simulations was illustrated before in two other publications (Westerhout et al. (2004) and Van Ewijk et al. (2006)). The simulations presented here show the effects of policy reforms upon the Dutch economy, upon the welfare of different generations and upon the time path of the government deficit and the government debt.

The structure of the document is as follows. Chapter 2 describes the building blocks of GAMMA. Chapter 3 discusses the calibration of the model. Chapter 4 presents the baseline projection. Chapter 5 presents several simulations that illustrate the working of the model: including both sensitivity scenarios and policy reform scenarios. Five appendices give information about the bookkeeping system, the derivations of the houshold model, the derivations of the firm model, the growth characteristics of the model and the definitions of the symbols used in this document.

2 A dynamic open-economy overlapping generation model

This chapter presents a technical description of the GAMMA model. The model distinguishes markets for goods, capital, labour and a ‘market’ for income transfers from the government to households. GAMMA identifies the following agents: households, pension funds, the government, firms and the foreign sector. Households are divided up into one hundred age cohorts. The government and private pension sectors are elaborated in detail, and a comprehensive set of generational accounts for all current and future generations is given.

GAMMA goes beyond the traditional generational accounting framework by incorporating the economic behaviour of households, firms and pension funds. Households decide on labour supply and private saving, firms decide on demand for labour and capital, and pension funds decide on pension contributions and benefit levels. Agents are rational and forward looking, and optimise in a consistent microeconomic framework. GAMMA thus allows for welfare analysis of policy reforms. However, since perfect labour and capital markets are assumed, the model is not equipped to describe short- and medium-term dynamics.

The following features are attached to the Dutch economy. First, the economy is small relative to the outside world. That is, domestic policies do not affect the interest rate, which is determined on world capital markets. Second, goods produced at home are perfect substitutes for those produced abroad: prices are given and terms-of-trade effects are absent. This fits in with the long-term horizon of the model. Third, the model is deterministic. Lifetime uncertainty is recognised, but perfect capital markets enable households to insure against longevity risk.

The population model in section 2.1 outlines the development of the demographic distribution over one hundred age cohorts. Aggregation rules are discussed in section 2.2. The household, government and pension models make use of the generated population distributions. The household model (section 2.3) is based on the life-cycle theory for individual behaviour. Section 2.4 discusses the firm model and factor demand relations. The government model (section 2.5) is a generational accounting framework in which expenditures and revenues are linked to the age of individuals. No economic behaviour is assumed on the part of the government.

The pension model (section 2.6) does include economic behaviour. Indeed contribution rates and indexation factors are functions of the coverage ratio of pension funds, i.e. the amount of financial wealth in terms of pension rights. Finally, 2.7 explains the equilibrium conditions and solution method for GAMMA. Appendix A presents a bird’s-eye view of the interactions between the various components by summarizing the bookkeeping system as a circular flow of the macroeconomy.

2.1 Demography

GAMMA does not directly plug in projections of the age structure of the population but derives them from a demographic model that links the age structure to the development of fertility rates, mortality rates and rates of immigration and emigration. This is done for two reasons. The first is that the Statistics Netherlands demographic forecast on which the calculations with the GAMMA model are based run only to 2100, while our calculations need projections over a longer time horizon.[1] The second is that we need a population model in order to be able to simulate alternative demographic developments. [2]

The demographic model of GAMMA splits the population up into one hundred age cohorts $j \in (0,...,99)$, each of which is subdivided into men and women. The gender distinction is appropriate because of the differing mortality probabilities by sex: women live longer than men on average. The population vector $\mathbf{n}_g^s(t)$[3] at the end of year t, which gives the distribution over the age cohorts and over gender groups, develops according to

$$\mathbf{n}_g^s(t) = \mathbf{T}(t)\mathbf{n}_g^s(t-1) + \mathbf{n}_i(t) \tag{2.1}$$

In this equation $\mathbf{T}$ is the time dependent transformation matrix which describes the influences of emigration, death and birth and $\mathbf{n}_i$ is the time dependent immigration vector. The population vector is 200×1: the first hundred elements consist of women aged 1-100 and the last hundred of men of the same age. The transformation matrix $\mathbf{T}$ is large (200×200), but sparse. That is, most elements are zero.

The relationships for men and women are similar, so here we restrict ourselves to the description of the female population. Aside from the first row (and row hundred and one) all elements of matrix $\mathbf{T}$, except the elements $(j, j-1)$ are zero. Element $(j, j-1)$ describes the probability that a woman of cohort $j-1$ will remain in the country and survive into the next period. That is, element $(j, j-1)$ is the complement of emigration probability q_e and the probability of death q_d (*i.e.* $T(j, j-1) = 1 - q_e(j) - q_d(j)$). The first row describes the number of births in a period. The number of births depends on the size of the fertile age cohorts: ages 16 up to 50. So in the first row, elements 17 up to 51 consist of the fertility rates multiplied by the probability that the baby is a girl.

Unlike the demographic model of GAMMA, the economic part of the model does not distinguish between men and women. Only the age distribution of individuals (men and women

[1] The reason for a longer forecast period is a technical one. We need a forecast end year with a steady (growth) demographic and economic development. Only in a steady-state year we can break off the calculations due to a perfect foresight assumption in the economic model.

[2] In the long run, demographic developments are subject to a high level of uncertainty. The GAMMA model is used to assess this uncertainty, for instance, in the DEMWEL project (See: Armstrong et al. (2007))

[3] A superscript s denotes a stock variable.

together) plays a role, which is obtained as $\mathbf{n}_h^s(t) = \mathbf{E}\mathbf{n}_g^s(t)$ with $\mathbf{E}$ being the aggregation matrix (two side-by-side unit matrices). Element j of vector $\mathbf{n}_h^s$ is the number of individuals of age j.

2.2 Aggregation

In this document we use suffixes as indicators for variables that refer to specific time periods or ages. For individual variables we use only the age suffix j, for intergenerational variables we use both the age suffix j and the time suffix t, for aggregated (macro) variables we use only time suffix t. At the individual level time and age are related on a one-to-one basis, so using the age indicator j is sufficient.

GAMMA assumes that all individuals of the same age are equal. Stocks are measured at the end of the period and are indicated with a superscript s. A stock variable, $x^s(j,t)$, related to an individual of age j multiplied by the population size of the same age $n_h^s(j,t)$ results in the cohort variable $n_h^s(j,t)x^s(j,t)$. The population size of age j at the end of period t is relevant for the flows in period $t+1$. A flow variable, $x(j,t)$, related to an individual of age j multiplied by the population size of the previous year $n_h^s(j-1,t-1)$ results in the cohort variable $n_h^s(j-1,t-1)x(j,t)$. One population unit represents one thousand people. Stocks and flows per population unit are also measured in thousands. This results in total values per cohort in millions. Macro variables are obtained by aggregating over the cohorts $x^s(t) = 10^{-3}\sum_j n_h^s(j,t)x^s(j,t)$ and $x(t) = 10^{-3}\sum_j n_h^s(j-1,t-1)x(j,t)$, so macro values are in billions. In the following sections we make use of these aggregation rules without presenting them explicitly.

2.3 Households

This section discusses one of the model's main building blocks, the module concerning the consumption-saving and labour supply behaviour of households. Like most AGE models, GAMMA bases consumption-saving and labour supply behaviour on life-cycle theory (*e.g.* Auerbach and Kotlikoff (1987), Altig et al. (2001), Broer (2001), Lassila and Valkonen (2001) and Bovenberg and Knaap (2005)). Unlike most of these models, our instantaneous utility specification is such that leisure is independent of household wealth. This seems to be in line with econometric evidence, which suggests small wealth effects on the demand for leisure.[4] Moreover, this utility specification implies a positive correlation between commodity consumption and labour supply, which is consistent with excess sensitivity (a positive correlation between commodity consumption and expected income changes aside from the influence of the interest rate) found in the econometric literature (see Flavin (1981)).

[4] Lumsdaine and Mitchell (1999) conclude in their survey article on early retirement that the wealth effect on labour supply is small relative to the price effect. The estimates in Blundell and Macurdy (1999) also point to small wealth effects on labour supply.

The life-cycle model is made data consistent using 'taste shifters': age-specific and time-specific parameters in the utility function which account for the effects of age and household situation on commodity consumption and labour supply. This section derives the indirect utility specification which leads in a natural way to the equivalent variation definition that we use for welfare analysis. Lastly, we elaborate on the relationship between different pension systems and household behaviour in appendix B.4.

Sub-section 2.3.1 presents the assumptions. The implied consumption-saving and labour supply behaviour is discussed in sub-section 2.3.2. Sub-section 2.3.3 gives an graphical exposition of the workings of the model. A money measure of utility, which can be used for welfare analysis, is derived in sub-section 2.3.4. We end with some concluding remarks. The appendices present the main technicalities.

2.3.1 Assumptions

An individual of age j maximizes his expected remaining lifetime utility U, which depends on per-period utility u and on the subjective discount factor d_s (as opposed to the objective discount factor which is introduced below). Expectations have to be formed because life expectancy is uncertain, so it is assumed that individuals weigh their future per-period utility with survival probabilities. The lifetime utility function reads as

$$U(j) = \sum_{i=j}^{j_e} u(i) d_s(i) \text{ with,} \tag{2.2}$$
$$d_s(i) = [1+\delta]^{-(i-j)} \prod_{l=j}^{i-1} \zeta(l).$$

In this equation j_e (=99) is the maximum attainable age[5], δ the time preference parameter, which measures the impatience to consume, and $\zeta(j)$ the conditional (upon being alive at $j-1$) probability of living through the next year. Households derive no utility from leaving bequests. The subjective discount factor consists of two elements. The first element is the already mentioned survival probability which gives a lower weight to per-period utility in more distant years. This survival probability equals the accumulated conditional survival rates ζ.[6] The second element of the subjective discount factor gives a lower weight to per-period utility further in the future due to the impatience of individuals. This impatience element depends on the time preference parameter δ. Per period utility, u, is a function of the consumption of commodities,

[5] The minimum age of independent decision making is 20. Children do not supply labour and their material consumption is attributed to their parents. This, in part, accounts for the hump-shaped life-cycle consumption profile used for calibration. See Figure 3.2 in section 3.3.

[6] Note, we use as convention $\prod_{l=j}^{j-1} \zeta(l) = 1$

c_h, and leisure, v:

$$u(i) = \frac{1}{1-\gamma}\left(\alpha_c(i)c_h(i) + \alpha_v(i)\frac{v(i)^{1-\beta}}{1-\beta}\right)^{1-\gamma} \text{ with} \tag{2.3}$$
$$\alpha_c,\ \alpha_v > 0\ ,\ \beta > 1,\ \gamma > 0 \text{ and } 0 \leq v \leq 1.$$

In this equation $1/\gamma$ is the intertemporal substitution elasticity, $1/\beta$ is the price elasticity of leisure demand and α_c and α_v are positive age and time[7] dependent taste shifters which are used for calibration purposes (see section 3.3). The marginal per-period utility of leisure (u_v: the derivative of u with respect to v) becomes infinite as leisure approaches zero. This guarantees positive leisure. The restriction that leisure must be equal to or smaller than the maximum available time (normalized to unity) has to be explicitly enforced. This utility specification implies a restriction for the commodity consumption

$$c_h(i) > -\frac{\alpha_v(i)}{\alpha_c(i)}\frac{v(i)^{1-\beta}}{1-\beta} \equiv c_l(i). \tag{2.4}$$

Indeed, the marginal utility of the per-period commodity consumption (u_c) becomes infinite as the commodity consumption approaches this minimum level.[8] The positive leisure consumption guarantees positive commodity consumption as long as the price elasticity of leisure demand is smaller than one ($\beta > 1$).

What is the interpretation of this restriction? Define the right hand side of 2.4 as *labour induced commodity consumption*, c_l. Labour induced commodity consumption is decreasing in leisure. Thus it attains its lowest value in case all available time is spent on leisure, *i.e.* at $v = 1$.[9] So the restriction motivates the positive correlation between commodity consumption and labour time. In other words, the commodity consumption is large in periods that households supply more labour. A possible interpretation for the positive correlation between commodity consumption and labour time may be professional costs. Another interpretation may be shifts from market goods to home produced goods over the life cycle due to changes in leisure.[10]

This specification has as another characteristic that the per-period marginal rate of substitution between the commodity consumption and leisure (u_c/u_v) depends on the latter only: that is, leisure is determined independently of the intertemporal consumption savings choice. As a consequence, there is no intertemporal substitution effect of leisure. This specification (the per-period composite good linear in commodity consumption and concave in leisure) was proposed by Greenwood et al. (1988) and was also used by Heijdra (1998).

[7] For convenience we suppress the time indicator.

[8] Since households are assumed to be optimizing, it will never be the case that $c_h = c_l$. As c_h approaches c_l, the household can increase its utility by an infinite magnitude by consuming a marginally higher level of commodities.

[9] Alternatively using a Box-Cox transformation in the instantaneous utility function ($\frac{v(i)^{1-\beta}}{1-\beta} \Longrightarrow \frac{v(i)^{1-\beta}-1}{1-\beta}$) the restriction becomes $c_h(i) > 0$ for $v = 1$.

[10] Note, however, that neither professional costs nor home production are modelled in GAMMA.

Individuals maximize lifetime utility given their budget and leisure constraints using leisure and commodity consumption as instruments. The budget constraint equates total wealth W^s to the discounted value of total consumption X_h

$$W^s(j-1) = \sum_{i=j}^{j_e} d_o(i) X_h(i) \text{ and} \tag{2.5}$$
$$X_h(i) = c_h(i) p_c(i) + v(i) p_v(i)$$
$$d_o(i) = [1 + r_h]^{-(i-j)} \prod_{l=j}^{i-1} \zeta(l)\ .$$

In this equation d_o is the objective discount factor, p_c, p_v are the prices for commodity and leisure consumption[11] respectively and r_h is the net rate of return on wealth. The objective discount factor[12] consists of two elements. The first element makes all future income and expenditure streams comparable with current financial wealth, by discounting it by the accumulated net rate of return. The second element is the survival probability that also gives a lower weight to more distant years. This implies that expected income and expenditure streams are relevant in the budget equation.[13]

Total wealth is the sum of financial wealth W_f^s and human wealth (the discounted value of future full, after-tax non-capital income) Y_f. Total wealth is defined as

$$W^s(j-1) = [1 + r_h(j)]\, W_f^s(j-1) + \sum_{i=j}^{j_e} d_o(i) Y_f(i). \tag{2.6}$$

Full after-tax non-capital income is defined as the aggregate of net transfer income and labour income in case all available time is spent working [14]. The price of leisure is the marginal reward of supplying labour, taking into account not just the net wage but also future pension income to the extent that it can be imputed to current labour (see appendix B.3 for the imputation method and the influence of a pension system on the leisure price). This implies that financial wealth W_f^s consists of financial wealth actually in the hands of households as well as households' pension rights.

The problem of the individual of age j is now to maximize lifetime utility (equation (2.2)) subject to the intertemporal budget constraint (2.5) and the restriction $v \leq 1$ on leisure. The next

[11] Note: the value of consumption in consumer prices is defined as $C_h(i) = c_h(i) p_c(i)$. The consumption of households according to the circular flow (see Table A.1) is obtained after aggregation over the age cohorts.

[12] Note the difference with the subjective discount factor in utility function (2.2).

[13] Expected future income and expenditures are relevant in the budget equation due to a perfect market assumption. Individuals can diversify their mortality risk on the capital market. More precisely there is a market that transfers the wealth of the fraction of each cohort that dies in each period to the remaining fraction of that cohort. (This assumption was introduced by Yaari (1965). Appendix B.1 elaborates).

[14] See appendix B.1 for details about the definition of full income, and appendix B.2 for the different income sources GAMMA distinguishes. Both appendices also give information on how the individual income sources are related to the circular flow variables of Table A.1.

section presents the leisure and commodity consumption relations for his remaining lifetime that are consistent with utility maximization. Appendix B.4 gives the derivations.

2.3.2 Consumption of commodities and leisure

The assumptions of previous section determine the allocation of total wealth to commodity and leisure consumption for all periods ($s \in \{j, \ldots, j_e\}$) over the remaining life cycle. Optimal behaviour implies that the per-period marginal rate of substitution between commodity and leisure consumption (i.e. the per-period utility ratio u_c/u_v) equals the price ratio (p_c/p_v) in every period of the life cycle. The marginal rate of subsitution does not depend on the commodity consumption by assumption. So we get the leisure demand relation out of this optimality condition:

$$\begin{aligned} & v(s) = \widetilde{v}(s) \ \text{ if } \ \widetilde{v}(s) \leq 1 \\ & v(s) = 1 \ \text{ if } \ \widetilde{v}(s) > 1 \ \text{ and} \\ & \widetilde{v}(s) = \left(\frac{\alpha_v(s)}{\alpha_c(s)} \frac{p_c(s)}{p_v(s)} \right)^{\frac{1}{\beta}}. \end{aligned} \tag{2.7}$$

This relation implies that a commodity price rise makes working less attractive relative to leisure, while the reverse holds when wages increase. Retirees are always at the corner solution because for them $p_v = 0$ which implies that $v = 1$. Note that total expenditure does not appear in the leisure equation (2.7) and wealth changes do not have effects on leisure and labour supply.[15] Another aspect of our version of the life-cycle model is that the wage elasticity of labour supply is age-dependent. In particular the labour supply elasticity in time s equals $v(s)/(1-v(s))/\beta$. This specification implies that the labour supply elasticity of older people (who supply relatively little labour) is large. The reason is that we take the price elasticity of leisure constant: that is, independent of age.

Given the optimal levels of leisure $v(s)$ commodity consumption demand is given by

$$\begin{aligned} c_h(s)p_c(s) = c_l(s)p_c(s) + & \left[W^s(j-1) - \beta \sum_{i=j}^{j_e} d_o(i)c_l(i)p_c(i) \right] \times \\ & \left(\frac{d_o(s)}{d_s(s)} \right)^{-\frac{1}{\gamma}} \left(\frac{p_c(s)}{\alpha_c(s)p_W(j-1)} \right)^{\frac{\gamma-1}{\gamma}}, \qquad s \in \{j, \ldots, j_e\} \end{aligned} \tag{2.8}$$

[15] Note: the price of leisure increases in the long-run in proportion with the general price level and labour productivity. This implies that the price of leisure relative to the consumption price increases with productivity leading to a continuous decline in leisure, which is not consistent with empirical evidence. A CES instanteneous utility function should more than counteract this effect, because leisure becomes linear homogeneous in total wealth, unless the intratemporal substitution elasticity equals one (Cobb-Douglas utility). This would imply a continuous increase of leisure which seems more consistent with data, but excludes long-run equilibrium growth. Introduction of a "keeping up with the Jones" effect (see for instance Gelauff and Graafland (1994)) meets this problem but has as a drawback that welfare analyses become path dependent. The utility function used by King et al. (1988a) (see also: King et al. (1988b)) makes leisure dependent on the price of leisure relative to the value of consumption but makes the consumption equation cumbersome. Taking this into consideration we prefered the specification presented here provided that we also let the parameter α_v grow with productivity.

with

$$p_W(j-1) = \left[\sum_{i=j}^{j_e} \left(\frac{p_c(i)}{\alpha_c(i)} \right)^{\frac{\gamma-1}{\gamma}} d_o(i)^{\frac{\gamma-1}{\gamma}} d_s(i)^{\frac{1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}} . \tag{2.9}$$

In equation (2.9) p_W is the price index of total wealth, *i.e.* the composite price of future consumption, W^s total wealth and c_l the labour induced commodity consumption.[16] After subtraction of the discounted value of the labour induced commodity consumption and the leisure consumption[17] a free disposable level of total wealth results (the first term between brackets in equation (2.8)), which is allocated between the different periods dependent on relative prices (the second and third term between brackets in equation (2.8)).

The interpretation of the commodity consumption equation (2.8) is easiest in the simplified case that the taste shifters α_c, the intertemporal substitution elasticity $(1/\gamma)$ and the subjective and objective discount factors are all equal to one (*i.e.* a zero time preference parameter, rate of return and death rates). In this case the relative price term (second and third term between brackets on the right hand side of equation 2.8) becomes equal to the inverse of the potential number of remaining periods in the household's lifetime $(j_e - s)$.[18] The free disposable level of total wealth is then divided equally over the rest of the life cycle, *i.e.* commodity consumption minus the labour induced commodity consumption is completely smoothed over the life cycle.

How does the allocation of total consumption over the life-cycle respond to the values in the parameters of equation (2.8)?

- All things equal except for a positive value of the time preference parameter δ means that the subjective discount factor d_s becomes smaller for more distant years. In particular a positive time preference parameter leads to front loading of commodity consumption, *i.e.* individuals become more impatient (the second term between brackets decreases over time).
- All things equal except for a higher net rate of return r_h leads to a smaller objective discount factor d_o for more distant years. This change has two different effects. First, the free disposable level of total wealth increases because future labour induced commodity consumption is discounted by a larger discount rate. Second, saving becomes more attractive due to the larger return on financial wealth (the second term between brackets increases over time).
- All things equal except for a price change in one period changes the value of the labour induced commodity consumption in that period but not the division over time of the free disposable level of the total wealth. However this result does not hold if the intertemporal substitution elasticity deviates from one. The last term between brackets in equation (2.8) then induces price effects

[16] Note, the expression for the labour induced commodity consumption includes an additional term if the leisure restriction holds, *i.e.* if $v = 1$. Appendix B.4 elaborates.

[17] The leisure consumption explains the coefficient β in the first term between brackets.

[18] Subtitute the expression (2.9) for p_W into $p_W(s)^{\frac{1}{\gamma}-1}$ and set the intertemporal substitution elasticity at one afterwards.

over time.[19]

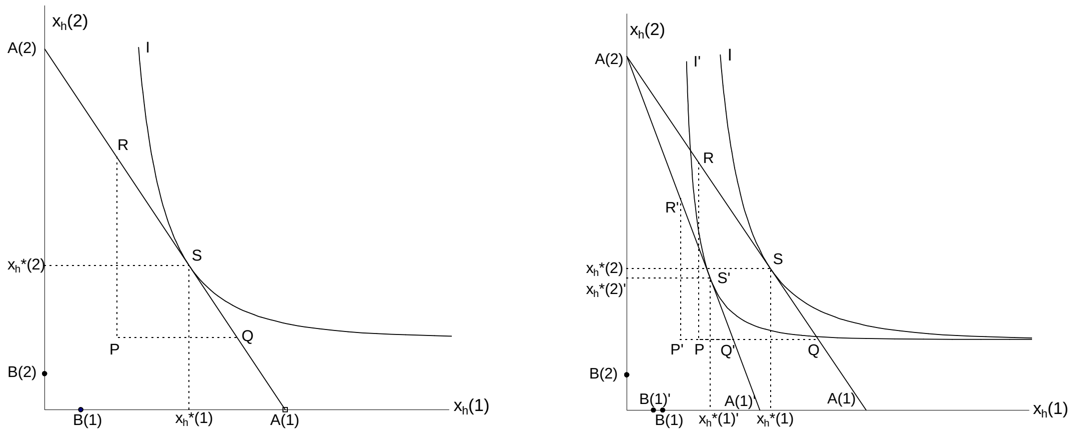
The following section gives a further interpretation of these results and some graphical illustrations. These graphs help to give a better insight into the income and substitution effects of various parametric changes.

2.3.3 Interpretation

To demonstrate some of the intuition behind the model, here we present graphs in which the life-cycle of individuals is split up into two periods (*i.e.* $j_e = 2$ in the set of equations presented in last section). For convenience we abstract from the probability of dying after the first period ($\zeta(1) = 1$). We start with a graphical description of the basic decision problem. Then we graphically analyse behavioural reactions due to changes in the consumption price, the leisure price and the interest rate respectively.

The left side of Figure 2.1 presents the basic decision problem. Real total consumption ($x_h = X_h/p_c$, *i.e.* total consumption divided by the consumption price) in periods 1 and 2 is depicted on the horizontal and vertical axes respectively. Individuals face a budget constraint, which is determined by their total wealth W^s. The budget constraint crosses the horizontal axis at point $A(1)$. At that point the individual consumes his total wealth completely in the first period ($x_h(2) = 0$ and $x_h(1) = W^s/p_c(1)$). The budget constraint crosses the vertical axis at point $A(2)$. At that point the individual consumes his total wealth (which grows at the rate of return, r_h) completely in the second period ($x_h(1) = 0$ and $x_h(2) = (1+r(2))W^s/p_c(2)$). The

Figure 2.1 Basic decision problem (left); a consumption price rise (right)



left side of Figure 2.1 presents one indifference curve I of the utility function U, connecting points with the same utility level. The indifference curves do not have the horizontal and vertical

[19] This utility specification can be considered to be an extended Stone-Geary utility function (See: Stone (1954) and Geary (1950)). The extensions involve a CES instead of a Cobb-Douglas form and the endogeneity of the minimum value of total consumption.

axes as lower bounds but parallel lines that go through point P on each axis. Indeed, only commodity consumption above labour induced consumption (defined in 2.4) provides utility. The same holds for total consumption: only total consumption above labour induced total consumption gives utility, *i.e.* $x_h - x_l = c_h - c_l$ gives positive utility, with x_l the labour induced total consumption. The maximum is obviously reached at point S because points to the left of the indifference curve I have lower utility. The optimum is projected onto the axis as x_h^*. Households allocate their total consumption to leisure and commodity consumption. For each period the real value of leisure ($v p_v / p_c$) is indicated as point B, while the real value of commodity consumption (c_h) is measured by the difference between the point x_h^* and the point B. Labour induced commodity consumption (c_l) is the difference between the projections of point P and point B on the axes. Leisure v and the labour induced commodity consumption c_l determine the labour induced total consumption level x_l above which total consumption x_h generates utility, *i.e.* the labour induced total consumption level x_l determines point P, the intersection of the asymptotes of the indifference curve.

The left hand side of Figure 2.1 is constructed for the case that the rate of return equals the subjective discount factor. In that case total consumption will be the same in both periods in spite of the larger consumption possibilities in period 2.

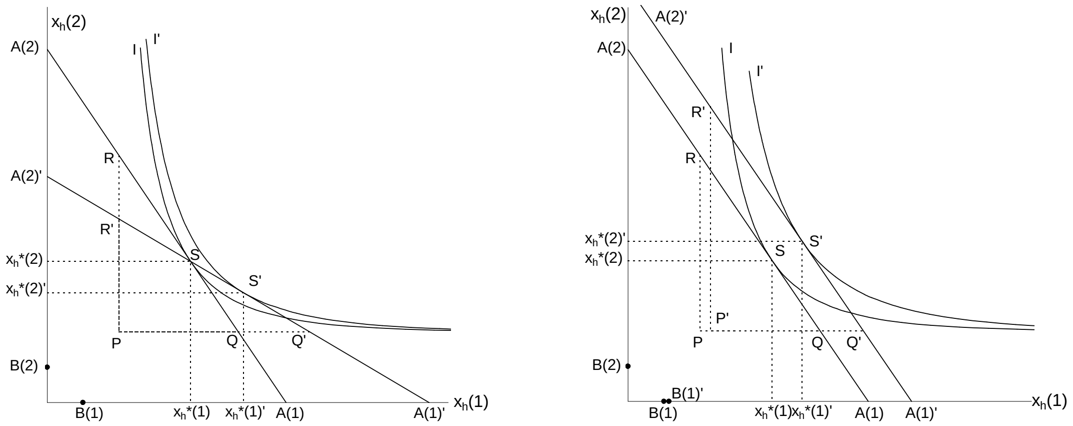
The right hand side of Figure 2.1 presents the effects of a consumption price rise in the first period only. The equilibrium changes from S to S'. The initial and new situation are indicated without and with a prime respectively. The budget constraint turns around point $A(2)$, because the total consumption possibilities in the second period remain the same while the possibilities in the first period decline. It becomes attractive to increase leisure in the first period[20]. As a result, the labour induced total consumption level declines, leading to a shift to the left of point P. The consumption of commodities will decline, mainly in the first period. Note that leisure does not change in the second period (P and P' have the same value on the y axes) and also that there is a positive correlation between commodity consumption and labour supply.

The left hand side of Figure 2.2 gives the effects of a decrease in the net rate of return. Saving becomes less attractive so total consumption increases in the first period, while it declines in the second period (the optimum shifts from S to S'). Labour supply does not change because the price of leisure and commodities remain the same (Point P and B are fixed).

The right hand side of Figure 2.2 gives the effects of a leisure price increase in the first period. Labour supply will increase in the first period. Leisure decreases in the first period. However, due to a subsitution effect smaller than one $B(1)$ shifts to the right. Labour induced commodity consumption increases, too, leading to a shift to the right of point P. The budget line shifts parallel to the right because it is determined by lifetime total income, so total consumption

[20] Note, leisure v increases due to the consumption price rise, but point $B(1) = v p_v / p_c$ shifts to the right because of a substitution effect smaller than one

Figure 2.2 An interest rate decline (left); an increase of the price of leisure (right)



increases in both periods. However, the increase in the first period is larger than in the second period due to the shift of point P, the labour induced total consumption. This leads to a larger consumption of commodities in the first period than in the second period. This illustrates the positive correlation between commodity consumption and labour supply.

2.3.4 A money measure of utility

Equivalent variations are used to evaluate government policies with respect to their effects on efficiency and equity. An equivalent variation for a household is defined as the money amount that gives that household the same utility change as some policy measure. The advantage of using this measure rather than simply calculating the utility changes lies in the obvious way that equivalent variations can be aggregated across generations that live in different periods of time. At any point in time the equivalent variations of different individuals can be aggregated, while equivalent variations of individuals (or, if aggregated, cohorts) that live in different periods are comparable after discounting with the market rate of return. The total sum of the discounted aggregated equivalent variations over individuals and cohorts can be used as an efficiency measure, while a comparison between the individual or cohort aggregated equivalent variations can be used as an equity measure. On the other hand, aggregation of individual utility changes is not possible, because utility is not comparable between individuals due to its ordinal character.

The indirect utility function can be used to implement equivalent variations. The indirect utility function is obtained after substitution of the solution for leisure and commodity consumption into the utility function (2.2), which leads to

$$U(j) = \frac{1}{1-\gamma}\left(\frac{W^s(j-1) - \beta \sum_{i=j}^{j_e} d_o(i)c_l(i)p_c(i)}{p_W(j-1)}\right)^{1-\gamma} \tag{2.10}$$

Indirect utility is a function of real total wealth diminished by the discounted value of the labour induced commodity consumption. The equivalent variation based on this utility function reads as

$$EV(j) = \left(\frac{U_1'(j)}{U_0'(j)} - 1\right)\left[W^s(j-1) - \beta \sum_{i=j}^{j_e} d_o(i) c_l(i) p_c(i)\right]_0 \tag{2.11}$$

$$U' \equiv [(1-\gamma)U]^{\frac{1}{1-\gamma}}$$

In this function the subscript 1 denotes the situation after a policy change and subscript 0 the situation before the policy change, U' is a monotone transformation of U and represents thus the same utility ordering. Utility function U' has as an advantage its linearity in wealth. The term within brackets is total wealth, net of labour induced commodity consumption. This wealth measure multiplied by the percentage change of utility, presents us with the equivalent variation. This formula is intuitively clear because it corresponds to the equivalent variation definition: the money amount that gives the same utility change as the considered policy measure.

This theoretically derived welfare measure is not used in Van Ewijk et al. (2006). In that study, welfare was measured as the sum of lifetime income, the net benefits from the government and the net benefits from pension funds. The indicator discussed here is roughly the same as that used in Fehr and Kotlikoff (1997).

The equivalent variations of future cohorts can be made comparable with current cohorts by discounting with the gross rate of return. After which aggregation over all current and future cohorts can occur. A positive aggregated equivalent variation indicates that a Pareto improvement is possible.

2.4 Firms

Private sector production in GAMMA is characterized by a simple neo-classical model for a representative firm.[21] The model's perspective is long-run and so it is reasonable to assume perfect competition in the goods, labour and capital markets. Hence there is one price p for goods, which is established on the world market. Perfect competition on the labour market gives the wage rate of an efficiency unit of labour[22] p_{le} which is determined by the ability of the firm to pay, which is determined in turn by the production price, the user cost of capital p_k and the technology of the firm. Perfect competition on the capital market suggests that the rate of return is also determined on the world market: that is, the bond rate r_b and the rate of return on equity r_s are given. The model abstracts from adjustment costs, an assumption that also seems reasonable in a long-run context.[23]

[21] The model follows in broad lines Draper and Huizinga (2001).

[22] Individual productivity (labour efficiency) changes over the life cycle and the productivity at the aggregated level changes over time. For these reasons we measure labour in standard efficiency units.

[23] The model does not take into account the disutility of risk. So the model can not explain the risk premium which is a compensation for the disutility of risk. To prevent erroneous interpretations, the model uses one uniform market rate of return. This market rate of return is also used as discount rate.

GAMMA assumes that the firm maximizes its value given a budget restriction, technology constraints and the capital accumulation equation. We will now present the details of this budget restriction, the determinants of firm's value and its production technology. Subsequently the implied capital demand and wage equations will be presented. The derivations are relegated to appendix C.

2.4.1 The budget, value and technology of the firm

The value of the representative firm is determined by a budget restriction indicating how much it can pay out in dividends each year and an arbitrage equation which values this stream of dividends on the capital market. The budget restriction of the firm can be written as:

$$D_{iv}(t) + p(t)i_e(t) = [p(t)y_{ge}(t) - p_{le}(t)l_e(t) - r_b W^s_{be}(t-1)] - T_p(t) - G_{cb}(t) \\ - G_{pg}(t) + \Delta W^s_{be}(t) \tag{2.12}$$

Dividend payments D_{iv} and investment pi_e can be financed out of profits (the term between brackets) net of taxes T_p, net of central bank profits payed to the government G_{cb} and net of government income through leasing of land G_{pg}, or financed out of an increase of debt ΔW^s_{be}. Profits equal revenue py_{ge} minus the wage bill $p_{le}l_e$ and interest payments on debt $r_b W^s_{be}(t-1)$. Employment l_e is measured in efficiency units as is the wage rate p_{le}. Taxes consist of the corporate tax rate τ_p times taxable revenue:

$$T_p(t) = \tau_p \left[p(t)y_{ge}(t) - p_{le}(t)l_e(t) - r_b W^s_{be}(t-1) - G_{pg}(t) - A_f(t)\right] \tag{2.13}$$

with A_f the fiscal depreciation allowance. Fiscal depreciation is based on the historical cost price of investment and is geometric with a fiscal depreciation rate v.[24] In period t the firm is allowed to deduct $v(1-v)^{i-1}p(t-i)i_e(t-i)$ for the investment purchased in period $t-i$, for all $\tau i \geq 1$ according to this depreciation rule. Assume a fraction ρ_0 of the principal of the debt is repaid in each period and a fraction ρ_1 of new investment is financed with new debt. The firm's financing decision is not modelled, that is, the debt to equity ratio is exogenous. Debt payment, therefore, equals $(r_b + \rho_0) W^s_{be}(t-1)$, and total debt evolves as

$$W^s_{be}(t) = (1-\rho_0)W^s_{be}(t-1) + \rho_1 p i_e(t) \tag{2.14}$$

Equations (2.12), (2.13) and (2.14) determine the firm's budget.

The value of the firm is determined by this budget equation and an arbitrage equation that indicates how this dividend stream is valued on the capital market. The firm is valued such that the return $r_s W^s_{se}(t-1)$ from having invested $W^s_{se}(t-1)$ in alternative assets equals the return on

[24] Fiscal depreciation may be linear or degressive. Fiscal depreciation equal to a fixed percentage of the book value is allowed if the original investment becomes less productive with age. Since we assume that physical depreciation is exponential, a degressive fiscal depreciation scheme indeed seems most appropriate.

owning the firm which consists of a capital gain of ΔW_{se}^{s} and a dividend D_{iv}:

$$r_s W_{se}^{s}(t-1) = \Delta W_{se}^{s}(t) + D_{iv}(t). \tag{2.15}$$

In this equation r_s is the nominal return on shares and $W_{se}^{s}(t)$is the value of the firm at the end of period t. Forward solution of this equation, assuming that the transversality condition ($\lim_{i\to\infty}(1+r_s)^{-i}W_{se}^{s}(t+i)=0$) holds, results in an explicit expression for the value of the firm.

The firm produces with capital and labour. Output is produced according to a CES production function with labour and capital as production factors

$$y_{ge}(t) = \left(\kappa k_e^{s}(t-1)^{\frac{\sigma-1}{\sigma}} + \theta l_e(t)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{2.16}$$

with $k_e^{s}(t-1)$ the relevant capital stock and σ the absolute value of the substitution elasticity between capital and labour. Investments are necessary for capital growth and for replacement of scrapped capital

$$i_e(t) = \Delta k_e^{s}(t) + \phi\, k_e^{s}(t-1) \tag{2.17}$$

with ϕ the technical rate of deterioration. Note that technical depreciation and fiscal depreciation do not necessarily coincide. This completes the description of the assumptions.

2.4.2 Factor demand and factor prices

The firm maximizes its value W_{se}^{s}, subject to the budget constraints and its available technology. Its instruments are investment and employment.

The value of the firm reaches its maximum in case all available labour supply is employed. Given this labour supply, the necessary capital stock to produce optimally:

$$k_e^{s}(t) = l_e(t+1)\left(\frac{\theta}{\kappa}\frac{p_k(t+1)}{p_{le}(t+1)}\right)^{-\sigma}, \tag{2.18}$$

depends on the user cost of capital relative to the wage rate.

The user cost of capital (see for a derivation in appendix C equation (C.16)) reads as

$$\begin{aligned} p_k(t) = \frac{1}{1-\tau_p}&\left[1+r_s-(1-\phi)\frac{p(t)}{p(t-1)}\right] \\ &\times\left[1-\tau_p\frac{v}{v+r_s}-\rho_1\frac{r_s-(1-\tau_p)r_b}{r_s+\rho_0}\right]p(t-1). \end{aligned} \tag{2.19}$$

The first term before the brackets, the inverse of the complement of the profit tax, makes capital costs comparable to wage costs. It defines the user cost of capital as a before-tax variable. The first term between brackets is the user cost of capital as a percentage of the effective investment price in case the firm is financed by shares only. Let us illustrate this argument. Assume a firm starts its business for one year at time $t-1$ with one unit of capital. This capital unit was purchased at the price $p(t-1)$. After one year the business can sell the remaining capital, i.e. $1-\phi$ at the larger price $p(t)$. The shareholders will finance this whole business in case they obtain a rate of return r_s. On balance the total capital costs equal (after linearizing) the rate of return on shares plus the rate of deterioration minus the inflation rate if all investments are financed with share issues. The second term between brackets consists of two correction factors which lead to a deviation of the effective investment price from the general price of goods. The first factor is a correction for the fiscal depreciation allowance. Indeed, the total discounted value of the geometric depreciation allowance is $v(v+r_s)^{-1}$. The effective price of the investment goods diminishes through this fiscal facility. The second factor of the second term between brackets is also a fiscal facility which concerns the deductibility of the debt services of the firm. A fraction ρ_1 of the new investments can be invested with debt. The discounted value of the net interest costs are then $\rho_1(r_s+\rho_0)^{-1}(1-\tau_p)r_b$. Without debt financing the discounted value of the financing costs (with share issues) becomes $\rho_1(r_s+\rho_0)^{-1}r_s$. The difference between those two finance forms determine the second correction factor.[25]

The profit tax rate enters the user cost of capital in three different ways. The net effect of these three different effects is in general ambiguous. The profit tax is neutral if[26]

$$1=\frac{v}{v+r_s}+\rho_1\frac{r_s}{r_s+\rho_0}. \tag{2.20}$$

The tax system encourages investments if the right hand side is larger than the left hand side.

The wage rate is determined by the production price, the user costs of capital and the available technology

$$p_{le}(t)=\left[\theta^{-\sigma}p(t)^{1-\sigma}-\left(\frac{\kappa}{\theta}\right)^{\sigma}p_k(t)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \tag{2.21}$$

This relation gives the maximum wage the firm can pay, given the world market price of goods, the user costs of capital and the available technology.

Different age cohorts j have different productivity levels, which can be represented by their productivity profile $e_f(j,t)$. More precisely, we assume that productivity increases over the life cycle. This assumption links age j's wages $p_l(t)$ to the macro wage in efficiency units p_{le}

$$p_l(j,t)=p_{le}(t)e_f(t)e_f(j,t) \tag{2.22}$$

with e_f the general productivity index. Employment in efficiency units, l_e, is the aggregate over the different cohorts

$$l_e(t)=\sum_j l_e(j,t)=\sum_j l_d(j,t)e_f(j,t)e_f(t) \tag{2.23}$$

with $l_d(j,t)$ employment of age j in period t.

[25] Note, the user costs of capital are not inflation neutral. An inflation increase given constant real rates of return influences the user costs of capital due to historical cost price depreciation allowances and due to the specific finance structure.

[26] This restriction follows by writing the second term between brackets of the user costs of capital as $a-b\tau_p$, and setting $a=b$.

2.5 The government

This section, which draws heavily upon Ter Rele (1998), discusses how the government sector is modelled in GAMMA. Using generational accounts both the future budget of the government as well as the influence of the government on individual welfare can be determined. This section discusses first how government expenditures and revenues are projected into the future in order to construct future government budgets. Next the net benefits calculation for separate cohorts is discussed.

2.5.1 Projecting public finances into the future

Primary expenditure

We distinguish between two types of primary government expenditures. The first consists of the expenditures of which the benefits can be attributed to beneficiaries, *i.e.* age-related expenditures. This category consists of expenditures on social security (public pensions B_{gp}, disability benefits B_{da}, unemployment benefits B_{un}, social assistance B_{sa} and other expenditure on social security B_{ot}), health care B_h and education B_e, and totals about 26% of GDP. The second type of primary expenditure consists of the expenditures which can not be attributed to beneficiaries; they are non-age-related. This category consists of expenditure on defense X_d, general government X_a, transfers abroad X_{fo} and subsidies X_{su} and amounts to around 19% of GDP.

Age-related expenditure

For most expenditure items in this category, future expenditure levels are constructed by using the assumption common in GA analysis that, apart from indexation to the wage rate (wage in efficiency units p_{le} multiplied by the productivity level e_f) in the private sector, age-specific benefits per person from these expenditures remain unchanged. Average public expenditures related to a person of a certain age, e.g. a 30- or 70-year old, will thus increase each year at a rate that corresponds to the increase of wages in the private sector. This form of extrapolation, which is considered to be a reasonable approximation of present public arrangements, leads to the following projection of the average expenditure on (age related) budget item i for a person of age j in period t:

$$B_i(j,t) = B_i(j,t-1)\frac{e_f(t)}{e_f(t-1)}\frac{p_{le}(t)}{p_{le}(t-1)} \tag{2.24}$$

The variable $B_i(j,t_0)$ represents the average expenditure on item i per person of age j, or the age profile, in the base year t_0. This is generally the last year for which a short term forecast is available. The aggregate expenditure in period t on age related expenditure item i can be calculated using the aggregation rules as explained in section 2.2.

There are three exceptions to this method of extrapolation. The first concerns disability benefits. To derive the future numbers of beneficiaries, we include the effects of a number of recent reforms that aim to curb the inflow into these schemes. These reforms are discussed in Van Ewijk et al. (2006). The second exception relates to unemployment benefits. In this case we take into account the effect of the business cycle (see below). In these two cases we add an additional factor to equation (2.24) that accounts for the impact of these factors and affects all age groups equally in relative terms.

The third exception concerns health care expenditure. In this case we also follow an extended procedure. In order to account for death-related costs, we decompose the population into survivors and decedents, defined as persons who will die within one year. The age profile of per capita health care expenditure is also decomposed into two other age profiles: one for per capita health care expenditures of survivors and another for per capita health care expenditures of decedents (Van Ewijk et al. (2006) page 69 provide details on this decomposed age profile). Combining demographic projections with these two age profiles yields projections for the development of health expenditure of survivors and that of decedents through time. Upon aggregation, we then have the development of health expenditure through time. Note that this is quite similar to the standard approach. It just decomposes an age profile into two other age profiles and assumes that the latter is constant rather than the former.

Non-age-related expenditure

The second type of expenditure comprises the expenditure items that cannot be attributed to beneficiaries. This category consists of expenditure on defence, general government, transfers abroad and subsidies. For these expenditure items we assume a ‘flat’ age profile, entailing an equal benefit for each individual. The aggregate growth rate of these items are assumed to correspond to that of GDP (defined as the sum of production by the private and public sectors: $Y_g = Y_{gg} + Y_{ge}$). The rationale for this is that expenditure on these items is closely linked to the size of production in the economy, and GDP may be considered to be the best measure for this concept.[27] This leads to the following equation for projecting the aggregates for these items:

$$X_i(t) = X_i(t-1)\frac{Y_g(t)}{Y_g(t-1)} \tag{2.25}$$

The benefits from these expenditure items are assumed to be distributed equally over all age groups. This means that the attribution to individuals of age j in year t is arrived at by simply dividing the aggregate expenditure by the total population in that year:

$$X_i(j,t) = \frac{X_i(t)}{n_h^s(t-1)} \tag{2.26}$$

Interest payments are calculated by multiplying government debt in the previous year by the interest rate. Government debt increases each year by the sum of the EMU-deficit and the

[27] We use GDP at base prices rather than GDP at market prices. The reason is that the latter includes the revenues from indirect taxation, which cannot be considered as output.

purchases of financial assets (see hereafter).[28]

Revenues

Government revenues consist of direct taxes on households (including social security premiums), indirect and other taxes, corporate taxes and revenues from government assets (including natural gas). GAMMA distinguishes five sources, or tax bases, from which direct taxes on households are levied. These tax bases are: labour income exlusive of private pension and early retirement scheme (VUT) premiums ($Y_{wh}(j)-P_p(j)-P_{er}(j)$), pension income (public plus occupational $B_{gp}(j)+B_p(j)$), social security other than public pensions ($B_{sa}(j)+B_{da}(j)+B_{un}(j)$), third pillar pensions plus early retirement benefits and imputed income from private assets. The tax bases, which are treated in their respective sections of this paper, distinguish between age groups. This leads to the following equation for projecting direct taxes on households from source i for the different cohorts

$$T_y^i(j,t) = \tau^i(t)H_t^i(j,t), \tag{2.27}$$

where T_y^i is the volume of tax revenue, τ^i is the tax rate and H_t^i is the tax base. Aggregate tax receipts can be calculated using the aggregation rules explained in section 2.2.

The projection of indirect and other taxes[29] is split up into the part that stems from consumer spending and the part that is levied on investments. For the part that is based on consumption the projection into the future follows the projection for private consumption (see the section on households). The distribution of its burden across age groups simply follows the pattern of consumption. Indirect taxes on investment are derived by simply linking them to the growth of aggregate investments (see section 2.4).[30]

The growth rate of corporate tax revenues is assumed to be equal to that of corporate profits. In turn, corporate profits are modelled as the balance of private sector production and the sum of wages, depreciation and the cost of interest on corporate debt. The burden of this tax is distributed across age groups in line with asset holdings.

We assume that the value of government holdings of financial assets remains constant in real terms. This entails that their share in GDP will fall though time. For the part of government assets that yields a nominal return, such as bank deposits and bonds, this implies a need for the purchase of these assets because their real value would otherwise decline. As the EMU definition of the budget balance does not include expenditure on financial asset purchases, this implies that the growth of government debt in nominal terms is slightly larger than is indicated

[28] The latter is necessary because the costs of acquiring financial assets is not included in the concept of the EMU-deficit.

[29] This category consists of the value added tax, excises, environmental levies, taxes on the purchase and on the ownership of cars, taxes on the transfer of the ownership of a house, taxes on the ownership and occupation of a house, inheritance taxes and a number of taxes yielding small revenues.

[30] The ratio of these taxes to consumption and that of these taxes to investments are based on micro-data.

by the deficit according to the EMU definition, or that debt redemption by government is smaller than the surplus.

2.5.2 Sustainability and the sustainability gap

Government policies are unsustainable if, eventually, they lead to exploding budget deficits and debt levels. A sustainable policy is characterised by deficit and debt levels that are constant relative to GDP in the long run when all relevant economic ratios, and most specifically the old age dependency ratio, have reached a steady state value. By using this as a criterion, policies can be tested on the property of sustainability.

The sustainability gap is calculated as follows. If the government does not buy shares or bonds its debt changes with the EMU deficit. The EMU deficit can be split up into debt services and the primary deficit.[31] The debt to GDP ratio evolves over time according to:

$$q_{dg}^{s}(t) = \frac{1+r_g}{1+g} q_{dg}^{s}(t-1) + q_{dg}(t) \tag{2.28}$$

with g the growth rate of GDP, r_g the interest rate, q_{dg}^{s} the debt to GDP ratio, q_{dg} the primary deficit to GDP ratio. The debt-to-GDP ratio will in general explode in case of a bond rate larger than the growth rate. The debt to GDP ratio will stabilize in the long run if a sufficiently large adjustment in primary expenditure is made. We define the sustainability gap, q_s, as that change in the ratio of primary expenditure relative to GDP that is permanent and taken immediately and that stabilizes the debt to GDP ratio. For government finances to be sustainable, the discounted value of the debt to GDP ratio becomes zero (because $r_g > g$). Foreward solution of equation (2.28) after subtraction of the sustainability gap from the primary deficit, making use of its per definition fixed value and the fact that the debt ratio stabilizes, leads to an explicit expression:

$$(1+r_g)\, q_s(t) = (r_g - g)\left[q_{dg}^{s}(t-1) + \sum_{t=0}^{\infty} q_{dg}(t) \left(\frac{1+g}{1+r_g} \right)^{t} \right] \tag{2.29}$$

The sustainability gap is thus equal to the initial debt plus the discounted value of future primary shortages, expressed as an annuity.

2.5.3 Modelling how separate cohorts benefit from the government

The GAMMA model also translates the expenditure and revenue items of the government budget into the benefits and burdens they bring to individuals over their lifetimes. This takes three steps.

First step

Revenue and expenditure items must be distinguished between those that involve a benefit or burden to the private sector and those that do not. It is also necessary to include a number of items in the benefit and burden concept which do not form part of the government budget. Table

[31] The EMU deficit s_{vg} and the debt services are presented in the circular flow table A.1.

2.1 illustrates these adjustments for 2006. It shows how the composition of benefits differs from that of public expenditure, and how the composition of the burdens differs from that of public revenue. A significant difference is that flows between the government and the private sector can be counted as benefits and burdens only if there is no exchange in return. If there is an exchange in return, it is considered a 'normal' economic transaction and the flow can consequently not be classified as a benefit or burden. The fact that the public sector is the counterpart of the transaction is irrelevant. This issue is particularly important with regard to flows resulting from public assets and liabilities. Interest payments, for example, are not counted as a benefit since they are a return on an investment. Accordingly, revenues from financial assets and natural resources are not included. The same holds for the profit remitted by the central bank. However, although these expenditure and revenue items do not themselves constitute a benefit or burden, they do have an indirect effect by reducing, or respectively increasing, the budgetary room for other items that do involve a benefit or burden.

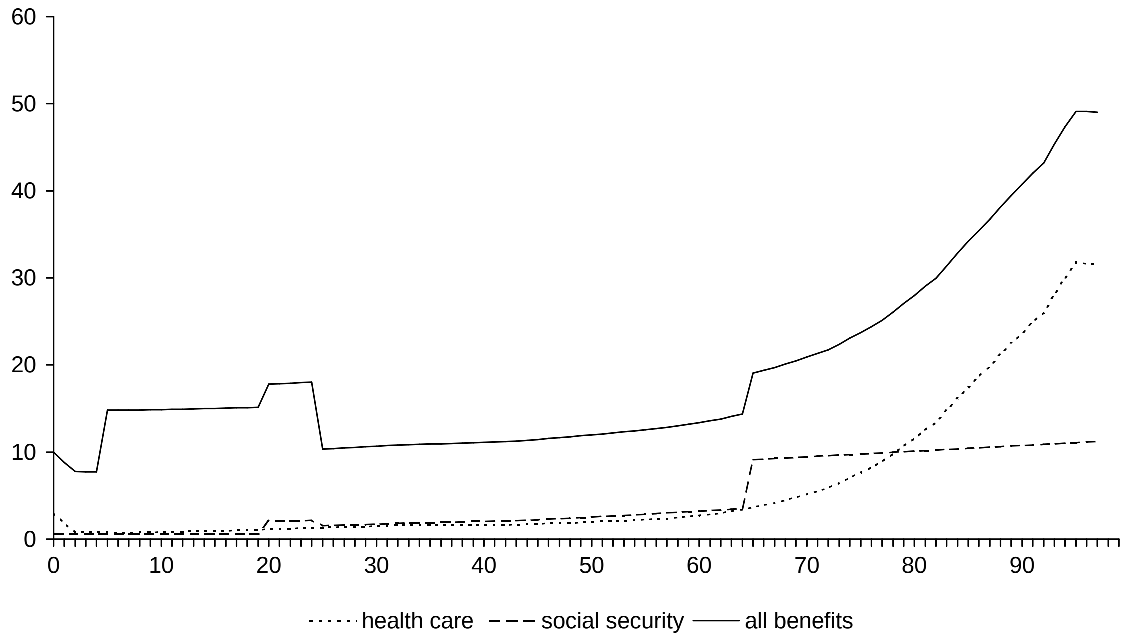
Table 2.1 Conversion of expenditure and revenues into benefits and burdens

	Expenditure	Adjustment (%GDP)	Benefits
Defence	1.3	0.0	1.3
General government	9.5	0.4	9.9
Infrastructure	1.5	0.7	2.2
Education	5.4	0.1	5.5
Subsidies	2.0	0.0	2 .0
Health	8.8	0.0	8.8
Social Security	12.0	0.0	12.0
Transfers abroad	2.2	0.0	2.2
Interest payments	2.5	− 2.5	0.0
Total	45.2	− 1.3	43.9
	Revenues	Adjustment	Burden
Taxes on households	21.8	0.0	21.8
Corporate taxes	2.6	0.0	2.6
Other taxes	14.9	0.0	14.9
Revenues from assets, incl. natural resources	2.8	− 2.8	0.0
Seigniorage	0.0	0.2	0.2
Other revenues	4.1	0.0	4.1
Total	46.2	− 2.6	43.6
	EMU-deficit	Adjustment	Net benefit
Balance	− 1.0	1.3	0.3

Another difference between the two classifications is that we do not include gross investments in the benefit concept. Nor do we include funds borrowed to finance the budget deficit as a burden.

These items have future effects on net benefits. However, the benefit concept does include benefits from the stock of public assets. These have to be estimated since there is no observable flow.

Figure 2.3 Age profile of benefits from the government for 2006 (thousands of euros)



A final difference is seigniorage. This is the cost to the private sector of holding notes and coins. This cost in a way counterbalances the remittance of the central bank's profit. The central bank earns its profit on assets (mainly on foreign exchange) that were deposited by the private sector to obtain money balances. Since interest is not earned on notes and coins, this exchange involves a loss of interest to the private sector, and thus forms a burden.

The conversion has a sizable effect of 1.3 procent of GDP. Whereas the EMU deficit equals −1.0% of GDP, aggregate net benefits from the government amount to 0.3% of GDP.

Second step

The second step assigns the benefits and burdens to individuals of each cohort. Figure 2.3 reveals the distribution of benefits across individuals (the age profile) in 2006 over the full range of age groups. The total benefit corresponds to the sum of the age profiles of age related and non-age related expenditure items (see equations (2.24) and (2.26)). Figure 2.3 shows that benefits generally rise with age. The two main components of this rise are social security and health care. Benefits from social security rise with age mainly due to public pensions (AOW), which are paid only to citizens over the age of 65, and disability benefits, which increase with age for those younger than 65 years. Health care costs rise with age because of growing costs of illness and of provisions for the elderly. Other benefits include those on education, among others. These are not shown separately.

The total benefit B of an individual of age j in a given year can be formalised by:

$$B(j) = \sum_{i=1}^{p} B_i(j) + \sum_{i=1}^{q} X_i(j) \tag{2.30}$$

for p age-related benefit categories and q non-age-related benefit categories.

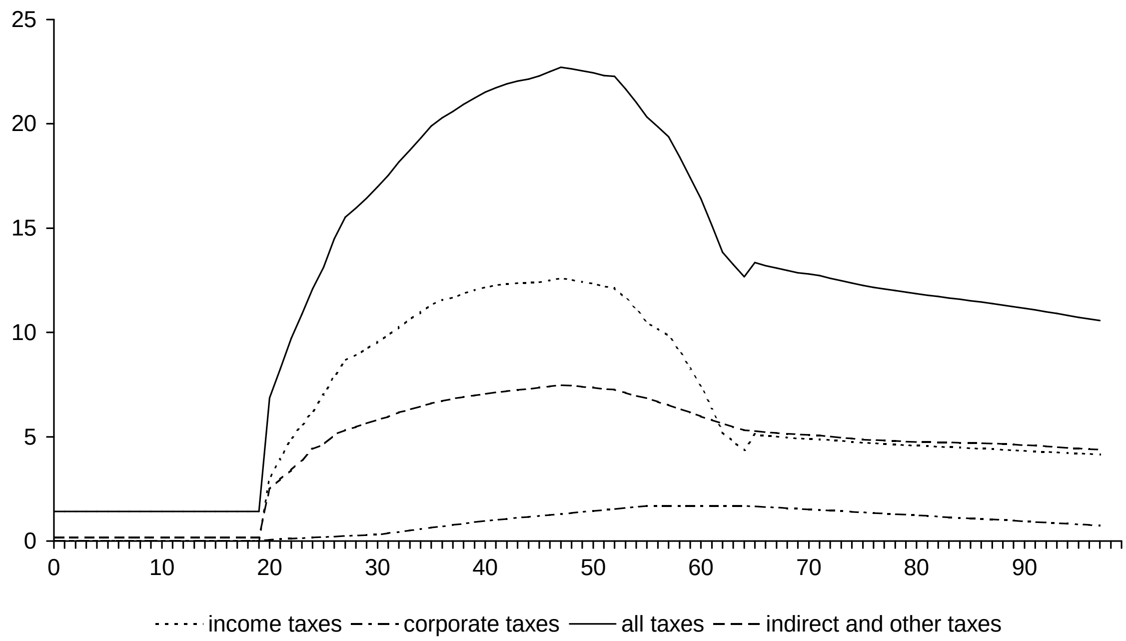
The age profiles of the burden from taxation also vary with age (see Figure 2.4). Until the age of about 50, labour incomes (and hence tax revenues from these incomes) rise with age. Beyond the age of 50, tax payments fall, due to the gradually decreasing labour force participation. The declining labour incomes are not fully offset by various forms of pension incomes, which are subject to income tax. Accordingly, income taxes fall with age.

The total burden for an average individual of age j in a given year can be expressed as:

$$T_{tb}(j) = T_y(j) + T_{in}(j) + T_p(j) + T_s(j) \tag{2.31}$$

In this equation T_y, T_{in}, T_p and T_s represent aggregate direct income taxes on households, aggregate indirect and other taxes, corporate taxes and seigniorage respectively.

Figure 2.4 Age profile of burdens from the government for 2006 (thousands of euros)



The average net benefit for individuals of age j in year t (B_n) can be calculated by:

$$B_n(j) = B(j) - T_{tb}(j) \tag{2.32}$$

The age profile of net benefits from the public sector is shown as Figure 3.3 in section 3.5.

Third step

The third step calculates the lifetime net benefits for average individuals of separate cohorts over their remaining lifetimes. Therefore, it uses the data on individual net benefits as presented in the first two steps. The net lifetime benefit for an average person of age j is derived in the following way:

$$B_n^s(j) = \sum_{i=j}^{j_e} d_n(i) B_n(i). \tag{2.33}$$

In this equation B_n represents the net benefits and $d_n(i)$ is the same as the objective discount factor as defined in equation 2.5 except that the discount rate on government debt r_g is used instead of the household discount rate r_h. The net lifetime benefit of future generations can be made comparable with current generations by discounting.[32]

2.6 Pension funds

The pension model discussed in this section describes the second pillar, the pillar consisting of supplementary pension arrangements. The pension model of GAMMA contains both a defined benefit (DB) and a defined contribution (DC) scheme. The default pension scheme is an average wage DB scheme which is described below in the main text. However, the opportunity exists to run the model with a final wage DB scheme or with a DC scheme. Descriptions of the last two pension schemes are presented in appendix (D). The rest of this section is organised as follows. We start with an overview of the main assumptions used in the pension block of the GAMMA model. Then we turn to the description of the pension model itself.

2.6.1 Overview

Modelling the second pillar of the Dutch pension system requires, first of all, a clear understanding of the key characteristics of this pillar. Therefore we start with a brief overview of the most important aspects of the second pillar:

- By law, second-pillar pensions must be fully funded. This means that during the working period assets are set aside. This stock of accumulated assets forms the basis of future pension benefits.
- In the second pillar the EET regime applies. According to this tax regime pension contributions and investment returns are tax exempt while pension benefits are taxed.
- A pension fund is obliged to charge a uniform contribution rate (‘doorsneepremie’). There is no premium differentiation because of age, sex or income. At the same time, every year a participant builds up the same percentage of her wage income as future pension benefits. The combination of a uniform premium rate and a time invariant accrual rate leads to

[32] In (Van Ewijk et al. (2006)) the total productivity growth factor was used as the discount factor.

intergenerational redistribution effects.

- The pensionable age is 65.

2.6.2 An average wage defined benefit system

We start the description of the pension fund with the contribution base. All households receive a public pension (AOW) benefit B_{gp} starting from the age of 65. Pensions in the second pillar are supplementary to the AOW benefit. This implies that households need not build up future pension benefits over their entire gross income. Instead, a franchise is deducted to correct for the AOW benefit. The contribution base of an individual of age j then equals the gross wage (p_{lg}) minus a franchise (f_p) if she works full time and participates fully in the pension system. We take account of the fact that many individuals work part-time and consequently, do not build up a full-time pension benefit, by correcting the full-time contribution base by the amount of leisure (v) consumed. We denote by ε the fraction of workers with a supplementary pension arrangement. The corrected contribution base (H_p) then equals

$$H_p(j) = \left(p_{lg}(j) - f_p\right)(1 - v(j))\,\varepsilon(j), \qquad j \in j_w, \ldots, j_r \tag{2.34}$$

In the DB scheme the pension benefit is guaranteed while the contribution rate is uncertain. The pension benefit is linked to gross labour income. As the name itself already reveals, in an average wage scheme the level of pension benefits depends on the average wage earned over the working life. Every year an individual builds up a fixed percentage (a_p^a) of his contribution base as (future) pension benefit, beginning from the first working year (j_w) until the year of retirement (j_r). After retirement the pension accrual only grows with the amount of indexation until the age of death (j_e). Formally, the pension rights of an agent of age j in an average wage scheme (W_{ph}^a) evolves according to:

$$W_{ph}^a(j) = \begin{cases} W_{ph}^a(j-1)\psi(j) + a_p^a H_p(j) & \text{if } j \in \{j_w, \ldots, j_r - 1\} \\ W_{ph}^a(j-1)\psi(j) & \text{if } j \in \{j_r, \ldots, j_e\} \end{cases} \tag{2.35}$$

with ψ the indexation parameter. So the pension rights $W_{ph}^a(j)$ gives the expected discounted value of the pension benefit at the retirement age accumulated up until year j. In the model any combination of wage indexation and inflation indexation is possible.

Pension funds must fund the expected value of the guaranteed pension liabilities. The expected nominal pension liabilities (Z_p^s) of the fund to a participant of age j then equal

$$Z_p^s(j) = W_{ph}^a(j) \sum_{i=\max\{j, j_r\}}^{j_e} d_p(i) \text{ and} \tag{2.36}$$

$$d_p(i) = [1 + r_b]^{-(i-j)} \prod_{l=j}^{i-1} \zeta(l).$$

In this equation d_p is the relevant objective discount factor for nominal pension liabilities and r_b the bond rate. The objective discount factor again consists of two elements. The first element makes all future expenditure streams comparable, by discounting them with the bond rate. The second element is the survival probability that also gives a lower weight to more distant years. This second element implies that expected nominal liabilities are calculated. Note that future price or wage indexation of the pension benefits are not included in the expected nominal liabilities.

The pension fund invests both in stocks and in bonds. In the model it is assumed that the portfolio choice is exogenous. As a consequence, the portfolio rate of return (r_p) is simply a weighted average of the rate of return on bonds and the rate of return on stocks. The assets of the fund (W^s_{fp}) evolve according to

$$W^s_{fp}(t) = (1+r_p)W^s_{fp}(t-1) + P_p - B_p(t) + W_{ip}(t) - W_{ep}(t) \tag{2.37}$$

Changes in these assets are due to five factors: investment returns ($r_p W^s_{fp}(t-1)$), pension contributions ($P_p = \tau_{pp} H_p$), pension benefits (B_p), immigration (W_{ip}) and emigration (W_{ep}). It is assumed that if people emigrate, they take with them all pension rights already built up. On the other hand, if people immigrate they buy into the pension system, acquiring the same pension rights as inhabitants of the same age.

Pension liabilities and assets together determine the funding ratio of the pension fund (q_f). The funding ratio is a solvency measure because it expresses the ratio between asset holding and liabilities of the pension fund.

$$q_f(t) = \frac{W^s_{fp}(t)}{Z^s_p(t)} \tag{2.38}$$

where Z^s_p is the aggregated stock of pension liabilities.

To determine the uniform contribution rate (τ_{uc}), the aggregated accrual of new pension liabilities is charged over the total contribution base[33]

$$\tau_{uc}(t) = \frac{\Delta Z^s_p(j) - W_{ip}(t) + W_{ep}(t)}{H_p(t)} \tag{2.39}$$

In a DB arrangement the pension benefit of a retiree of age j is simply equal to the pension build-up

$$B_p(j) = W^a_{ph}(j-1)\psi(j), \qquad j \in j_r, \ldots, j_e. \tag{2.40}$$

The pension benefits will be indexed to inflation or wage growth.

It may occur that the funding ratio is below the required level. A funding ratio below the required level must be restored to this level within a period of at most fifteen years. However, if

[33] Note, the actual model specification is a little bit different to prevent large immediate effects of interest rate shocks. This effect is distributed over time.

the funding ratio falls below 105 percent, the pension fund has to raise its funding ratio above this level within one year. This may be done by either implementing a 'catching-up' premium τ_{cu} or by applying cuts to the indexation of pension benefits.

The catching-up premium in GAMMA is chosen in such a way that it minimises the distorting labour market effects and at the same time restores the funding rate to the required level within the required time period. The catching-up premium is the solution to a dynamic optimization problem. Providing a detailed technical derivation of this problem is beyond the scope of this paper. For that we refer to Bonenkamp (2005). The catching-up premium together with the cost-effective contribution rate determine the total contribution rate a participant pays to the DB pension fund.

$$\tau_{pp}(t) = \tau_{uc}(t) + \tau_{cu}(t) \tag{2.41}$$

If a pension fund decides to finance its deficit with a temporary surcharge on the contribution rate, the burden of the deficit is completely absorbed by the working generations. One can imagine, for example in an ageing society, that this exerts too much pressure on the financial balances of working people. The instrument of indexation cuts provides a pension fund with a tool to shift a part of the burden to the retirees.

Indexation cuts mean that existing pension promises of workers and pension benefits of retirees will not be indexed fully to wages and/or prices. Especially in an average wage scheme, cutting down indexation is an effective instrument to restore the funding rate. In an average wage scheme, a pension fund not only cuts down indexation of the pension benefits (as in a final wage scheme), but also the indexation of the pension rights already accumulated by the working generations.

The pension fund in GAMMA sets indexation cuts as a linear function of the funding ratio. There exists a lower bound (mostly equal to 100 percent) below which participants get no indexation to wages and/or prices at all and an upper-bound (135 percent in the figure) above which pensioners get full indexation. Between the lower- and upper-bound the indexation linearly increases with the improvement of the funding ratio.

2.7 Equilibrium and the solution method

2.7.1 Equilibrium

The model has a dynamic general equilibrium solution[34] if:

- The allocations are feasible, and all markets clear: i.e. the rows and the columns in Table A.1 must sum up to zero;

[34] We use the same definition as Borsch-Supan et al. (2006).

- The goods market clears through volume adjustments on the export (import) market. The labour market clears through demand adjustment of firms: all labour supply is absorbed by the wages firms can pay. Exogenous factors, the goods market price and the user costs of capital determine the ability to pay (factor price frontier). Equity and bonds surpluses or shortages can be bought or sold on the international capital market at world market prices. The rate of return on equity and bonds is exogenously given;
- Capital is put into production such that the marginal productivities of both capital and labour equal the factor prices according to equation (2.18);
- Firms and households behave optimally and pension funds act according to the rules specified by the supervisory organ.
- We get a general equilibrium solution in case we enforce the no-Ponzi game condition. In particular, we require that the debt to GDP ratio stabilizes.

Unsustainable, or disequilibrium, solutions are obtained if we do not enforce the no-Ponzi game condition for government finances.

2.7.2 Solution method

Instead of solving the model for each period consecutively as in the Fair-Taylor method, GAMMA uses a Stacked-Time method to solve the model for all periods simultaneously. However, the traditional Stacked-Time method has been made more efficient. Van ’t Veer (2006) gives further details. Experiments with this solution method show faster convergence than the Fair-Taylor method.

3 Data and calibration

Projections and simulations using the GAMMA model are produced by combining the structure outlined in the previous chapter with real world data and economic parameters. This chapter elaborates on how these data and parameters are chosen and explains how they are used to calibrate the model. Section 3.1 shows the values of the circular flow variables for the calibration year 2006. The assumptions behind the extension of the demographic forecast are explained in section 3.2. Section 3.3 identifies the values for the exogenous parameters in the household utility function and presents the consumption and labour participation data for the Netherlands that are used to calibrate labour supply and Euler equations. Section 3.4 shows how the private sector labour and capital demand relations are calibrated. Government expenditure and revenue items are discussed in section 3.5, including the calculation of the age profile of health care expenditures and assumptions on tax rates. Finally section 3.6 explains the assumed parameter values of the pension sector.

3.1 Macro-economy

Table 3.1 presents the circular flow figures for the Dutch economy. These figures for the year 2006 correspond to the model variables presented in Table A.1. They are used to calibrate the macro-economic relations in the base path simulation. A cell in the table gives the transactions of agents on a market (+ receipt, − payment). The aggregated budget constraint of agents is obtained by adding up over the column: the sum of revenues minus expenditures equals savings. That is, the column totals are zero.

Table 3.1 Total accounts: circular flow for 2006

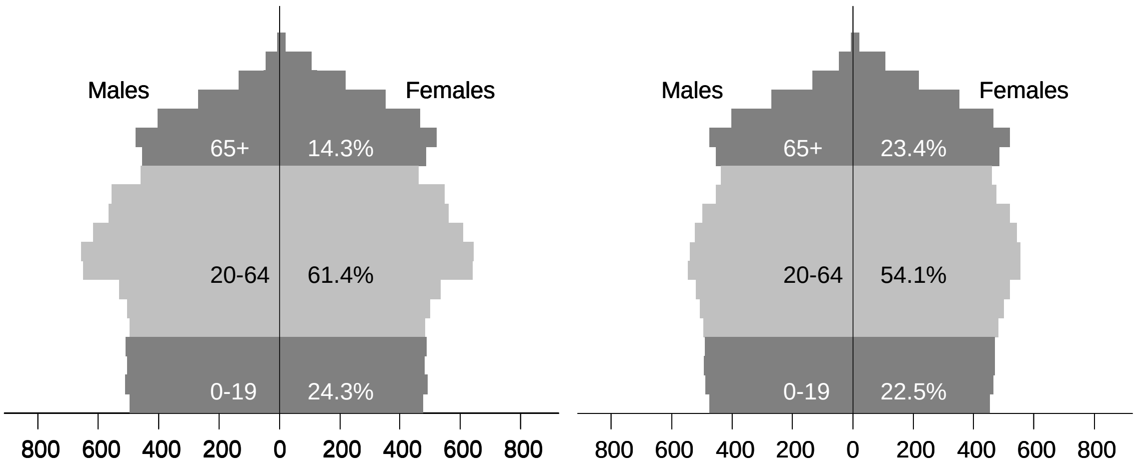
	House-holds	Pension funds	Capital	Government			Firms	Foreign sector	Row total
				Services	Taxes	Production			
Goods	− 240.0		− 103.3	− 134.9	59.5	62.9	390.0	− 34.3	0
Investments			103.3			− 15.1	− 88.2		0
Transfers	54.9			− 54.9					0
Labour income	257.0					− 50.4	− 206.6		0
Private pensions	25.7	− 25.7							0
Non-labour income	49.0	18.4		− 12.7	12.0		− 64.8	− 1.9	0
Income taxes	− 119.3				119.3			0	0
Profit tax					15.0		− 15.0		0
Private pension premiums	− 33.4	33.4							0
Transfers to foreigners				− 8.2				8.2	0
Savings(-)/shortage(+)	6.1	− 26.1			7.4		− 15.3	27.9	0
Column total	0	0	0		0		0	0	0

3.2 Demography

The demographic model reproduces the population forecast of Statistics Netherlands (See: De Jong (2005a), De Jong (2005b) and De Jong (2005c)) whose end year is 2050. The mortality and emigration probabilities and fertility rates are taken from this projection up to the year 2100 and afterwards fixed at their 2100 values. Up to 2100 immigration is set equal to the CBS projection, and afterwards is fixed at its 2100 value.

Figure 3.1 gives an overview of the age distribution of the Dutch population for the years 2006 and 2040. In 2006 the total population size is 16.4 million, of which 24% is younger than age 20, 61% is between 20 and 64 years old and 15% is older than age 64. The shape of the population distribution over the age cohorts and gender groups is like a house with an overhanging pitched roof. In the past the shape of this graph was more like a pyramid: older age cohorts were smaller than younger cohorts. The overhang is caused by a decline in fertility rates, during the late nineteen sixties and seventies. Since 1980 fertility rates have been stable. This has led to rather equal age cohort sizes for those born in recent decades.

Figure 3.1 Age structure of the population in 2006 (left) and 2040 (right)



The overhang in the distribution consists of the large cohorts between the ages 30 and 59. In the coming decades these large cohorts will rise in the age pyramid above the age of 65, as the right part of Figure 3.1 illustrates. The elderly dependency ratio, defined as the number of those aged 65 plus as a percentage of the 20 to 64-year olds (23.4% in 2006) will increase in the future due to these relatively large cohorts.

This baseline demographic scenario assumes that the total fertility rate remains at about 1.75 over the whole period, and that net immigration increases from its current negative value of around minus 2,000 annually to a structural level of plus 30,000. Mortality rates continue to decrease in the future, especially at older ages. As a result, life expectancy will also increase. Life expectancy at birth will increase from its present level of 76.7 years to 79.6 years for males in the period 2005-2050. Similarly, life expectancy at birth for females will increase from its present level of 81.2 years to 82.6 years in 2050. In the space of 45 years, average life expectancy overall will thus increase by a good two years. The gain is concentrated at higher ages: life expectancy at the age of 65 will increase by about 1.5 years.

Table 3.2 Population (millions) and its composition in the period 2006 up to 2100[a]

	2006	2020	2040	2060	2100
Age group					
0-19	4.0	3.8	3.8	3.8	3.9
20-64	10.0	9.8	9.2	9.5	9.7
65+	2.3	3.2	4.0	3.6	3.8
Total	16.4	16.8	17.0	16.9	17.5
Elderly dependency ratio	23.4%	33.0%	43.4%	37.4%	39.6%

[a] The data apply to the end of the year

Table 3.2 provides an overview of the changes in the age composition of the population that will be brought about by these developments. The elderly dependency ratio is projected to rise to 43.4% in 2040. After 2040, the ratio stabilises at a more-or-less constant level of around 39%. The total population will grow to just over 17 million in 2040, and after a dip around 2060, will rise further to 17.5 million in 2100.

3.3 Households

The GAMMA model specifies 99 as the maximum attainable age. All age cohorts between 0 and 99 are distinguished in the model. Consumption and leisure decisions are made from the age 20 onwards, so consumption of children is attributed to their parents.

The values of the parameters are based on evidence produced by national and international research. One of the crucial parameters is the substitution elasticity between leisure and consumption. This has a value of 0.25 ($\beta = 4$) and implies that on average the wage elasticity of labour supply equals 0.14.[35] Quite recently, CPB completed a meta-analysis on this parameter that was used to update the MIMIC model (Evers et al. (2005)). Our value of 0.14 is a little smaller than the corresponding value in the MIMIC model, but corresponds fairly well with the results from the meta-analysis.

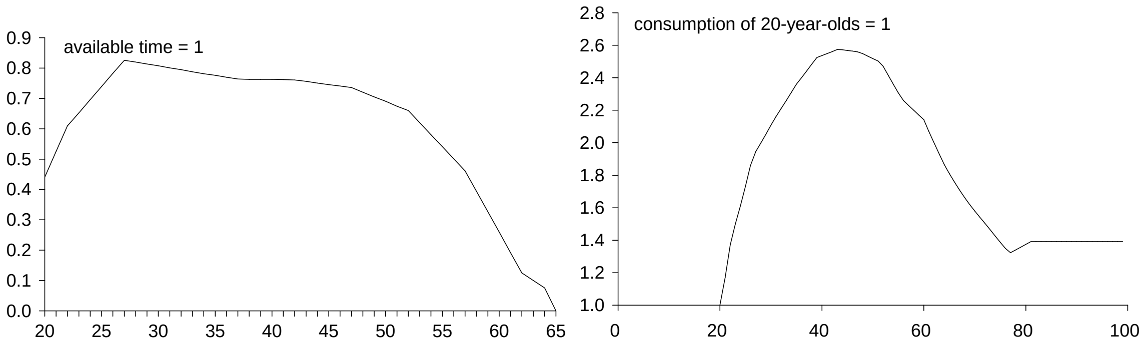
GAMMA's elasticity of intertemporal substitution equals 0.5 ($\gamma = 2$). Estimates of this elasticity typically vary widely in the range between zero and one. Research by Epstein and Zin (1991), which properly distinguishes between the aversion to risk and the aversion to

[35] In the labour supply function of GAMMA there is no income effect so the uncompensated and compensated labour supply elasticities are equivalent.

intertemporal substitution, confirms this result. Our value of 0.5 is well within their range of estimated values. The rate of time preference takes a value of 1.3%. This is somewhat higher than in Altig et al. (2001), and somewhat lower than in Bovenberg and Knaap (2005).

The left side of Figure 3.2 presents labour participation for the year 2006. Age is given on the horizontal axis of the figure, while the labour participation, i.e. employment in full-time equivalencies, l, in percentages of the cohort size, n, is depicted on the vertical axis. The figure reveals that labour participation grows between the ages twenty and thirty, stabilizes then up to age fifty, and decreases afterwards. Labour participation is expected to increase after 2006. This expected change is based on Euwals and van Vuuren (2005).

Figure 3.2 Labour participation (left) and consumption profile (right)



Leisure is defined as the complement of labour participation: $v = 1 - l/n$. This labour participation profile is calibrated by inverting equation (2.7) which gives the ratio of the parameters α_v and α_c as functions of leisure and the relative price of leisure.

The right hand side of figure 3.2 gives the consumption profile for 2006. This profile is based on estimates of Ree and Alessie (2007). Their estimates point to small differences between the life cycle profiles and cross section profiles in the period 1978-2001. This justifies our use of their (life cycle) estimates as a cross section profile. Above the age 75 the consumption level is assumed to be constant. Age is presented on the horizontal axis of the figure, while consumption is measured on the vertical axis. Consumption at the age 20 is scaled to one.

For the base run we assume that this consumption profile holds. This consumption profile together with total wealth imply a consumption level over the life cycle. The parameters α_c are used to calibrate this consumption level. More precisely: the Euler equation for total consumption can be obtained by dividing two equations from the equation set (2.8). This Euler equation can be written in such a form that the parameter α_c becomes a function of leisure and consumption.

3.4 Firms

The coefficients of the production function are calibrated by inverting the labour and capital demand relations

$$\theta = \left(\frac{p_{le}}{p}\right)^{\frac{\sigma}{1-\sigma}} \left(\frac{y_{ge}}{l_e}\right)^{-\frac{1}{1-\sigma}} \tag{3.1}$$

$$\kappa = \left(\frac{p_k}{p}\right)^{\frac{\sigma}{1-\sigma}} \left(\frac{y_{ge}}{k_e}\right)^{-\frac{1}{1-\sigma}}$$

The aggregated efficiency index is fixed at one in the calibration year. This assumption makes employment and wages in efficiency units equal to the actual employment and wage rate. The capital stock growth equals the employment growth in efficiency units in case of constant relative factor prices (see equation (2.18)). Constant relative factor prices hold in equilibrium. Given this assumption the capital stock follows by inverting the capital accumulation equation

$$k_e^s(t-1) = \frac{i_e(t)}{\phi + \frac{l_e(t+1)}{l_e(t)} - 1} \tag{3.2}$$

$$\phi = \frac{i_{de}(t)}{k_e^s(t-1)}$$

The technical rate of deterioration is set equal to depreciation i_{de} relative to the capital stock. Investment fluctuates a lot over the business cycle so we use steady state values of investment and the growth rate of employment to calibrate the capital stock and the rate of deterioration. This implies that the actual investment levels are not reproduced by our model.

The equity premium $r_{ep} = r_s - r_b$ is exogenous.[36] Mathur and Hassett (2006) have investigated the relationship between the profit tax and the wage rate for 72 countries over 22 years. They found that an increase of the profit tax leads to a drop in wages. According to our wage equation this implies that the tax system is not neutral in most countries. More specifically the tax system apparently increases the user costs of capital. This implies

$$1 > \frac{v}{v + r_s} + \rho_1 \frac{r_s}{\rho_0 + r_s} \tag{3.3}$$

This holds also in GAMMA because the fiscal depreciation rate (v) is calibrated at a value 0.048, the repayment fraction (ρ_0) at 0.038, the fraction (ρ_1) of new investment financed with new debt at 0.70, and the rate of return on shares (r_s) at 0.066.

In the model, private sector production includes an exogenous element in the base year in order to conform to data. Exogenous production includes, amongst other things, exploitation of natural gas reserves and a production surplus. The production surplus is defined as the observed production in factor costs minus wage and capital costs in the base year. Capital costs are based on the assumption for the market rate of return. An extended explanation can be found in Van Ewijk et al. (2006).

[36] The model does not take into account the disutility of risk. Since the portfolio decision of housholds is also exogenous, the model uses, in fact, one uniform market rate of return. This market rate of return is also used as the discount rate.

3.5 Government

3.5.1 Tax rates

As outlined in section 2.5 tax bases in GAMMA include: household income (including wages, public and private pensions, transfers, and income from asset wealth), private consumption, private investments and corporate profits as well as a number of miscellaneous tax sources. The tax rates for each of these bases is calibrated simply by dividing aggregate tax receipts for tax instrument *i* by the appropriate base in the calibration year:

$$\tau^i(t) = \frac{T_d^i(t)}{H_t^i(t)} \tag{3.4}$$

Apart from one exception we ignore progression in tax rates. Differences in average incomes across age groups therefore do not lead to differences in tax rates. This reflects the fact that, for these tax bases, the progression is relatively low. Moreover, as tax brackets are assumed to increase in line with productivity (rather than inflation only), the working of progression in the increase of tax revenues is fully eliminated. The taxation of pension incomes, however, does feature a sharp progression due to the fact that the tax rates in the first two tax brackets are very low for individuals over the age of 65. This is taken account of by imputing a progression factor of 1.8. As average pensions are projected to rise at a (slightly) higher pace than productivity this progression factor is effective.

3.5.2 Expenditure profiles

The aggregated age profile of health care expenditures is taken from The Institute for Medical Technology Assessment (*i*MTA). Important for the decomposition of the age profile of health expenditure into the age profiles of survivors and decedents is the age profile of decedents, *i.e.* the age profile of death-related costs. Here, we used data from Polder and Achterberg (2004)). These data pertain to the year 1999, which is quite recent for the purpose at hand. When disaggregated to acute and long-term care, the acute component is decreasing in age and the long-term care component is increasing in age. This pattern corresponds to what others have found for different countries.

We do not want to use the data in their raw form, however. The reason is that they measure costs in the last year of life. There is a great deal of evidence that indicates that death-related costs occur in a time period that is much longer than a year. Therefore, we multiply these figures by a blow-up factor. Note that this implicitly suggests that the ratio between costs in the last year of life and total death-related costs is the same for people of different age. To get a blow-up factor, we analysed four papers that presented data on death-related costs: WRR (1997), Seshamani and Gray (2004), Roos et al. (1987), and Jones (2002). The values for the blow-up factor that we calculated were 2.6, 2.2, 2.3 and 2.7 respectively. We took the average of these numbers: 2.45.

Given the age profile of death-related costs, the age profile of survivors follows from calibrating the age profile of total health expenditure (the weighted average of the age profiles of survivors and decedents) to statistics.

The age profiles of social security expenditures (public pensions, disability benefits, unemployment benefits, social assistance) are taken from SCP (1994).[37]

3.5.3 Aggregate expenditures and revenues

Table 3.3 presents the government budget for 2006 in percentages of GDP, with the left hand side of the table showing expenditures and the right hand side revenues. Most of these budgetary items are age related. In the case of expenditures, children receive education and child allowances while elderly people receive social security and higher health care benefits. Benefits from social security rise with age mainly due to public pensions (AOW) which are paid to citizens over the age of 65, and disability benefits, which increase with age for those younger than 65. Health care costs rise with age because of the growing costs of illness and provisions for the elderly.

Table 3.3 Public finances in 2006 in % GDP

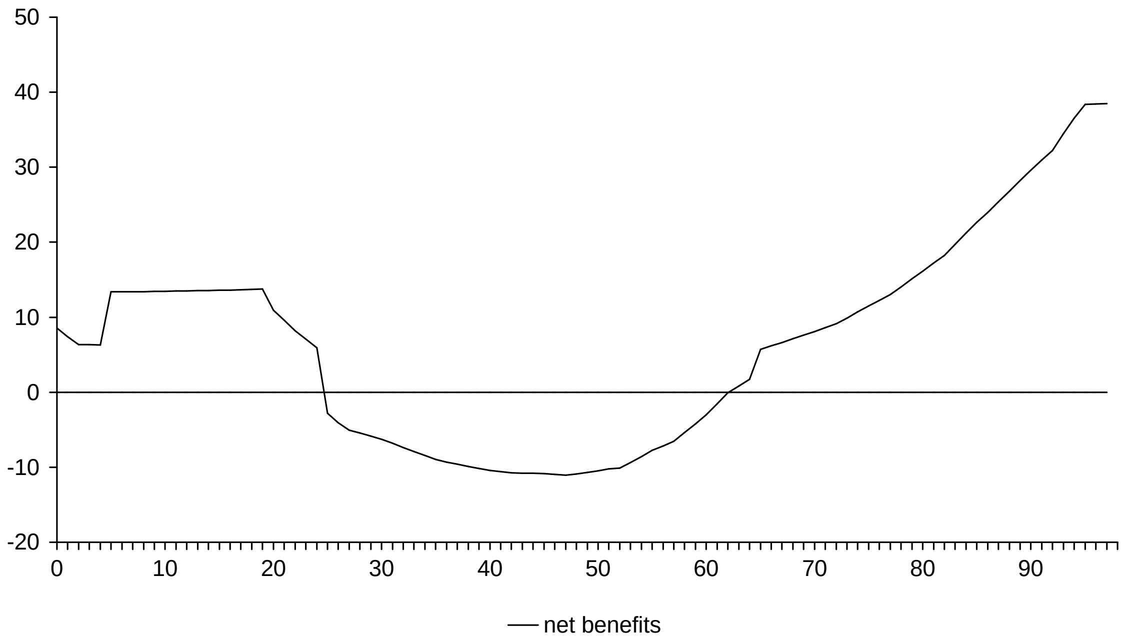
Expenditures		Revenues	
Social security	12.0	Income tax and social security contributions	21.8
- of which public pensions	4.7	Indirect and other commodity taxes	14.9
Health care	8.8	Corporate income tax	2.6
Education	5.4	Natural gas revenues	1.6
Other expenditure excluding interest payments	19.2	Other income	5.2
Interest payments	2.5	Total	46.1
Total	47.8		
EMU balance	− 1.7		
EMU debt [a]	54.4		

[a] Value at the end of the year

Items on the revenue side are also age related. Older workers pay more tax than retirees. Until the age of about 50, labour incomes (and hence tax revenues from these incomes) rise with age. Beyond the age of 50, tax payments fall due to gradually decreasing participation in the labour force. The declining labour incomes are not fully offset by various forms of pension income, which are also subject to income tax. Accordingly, income taxes fall with age.

[37] The distribution of average expenditure per individual over age groups is derived from data that usually apply to a past year. These data, which are provided by more specialized institutes as SCP and iMtA have to be updated in order to be consistent with the aggregate expenditure level in the base year. This updating procedure is carried out in a two stage process. In the first stage we calculate the aggregate level of the expenditure item that results from combining the original ('raw') data from the institutes with the age composition of the population in the base year. Then in the second stage, this result is compared with the actual forecast. The relative difference between these two aggregate figures is subsequently used to adjust the original data on the distribution over individuals.

Figure 3.3 Age profile of net benefits from the government for 2006 (thousands of euros)



The cumulative effect of the age profiles of revenues and burdens is shown in Figure 3.3 as the net benefits received by individuals from the government sector. It can be seen that, on the balance of benefits and burdens, it turns out that the young and the elderly are net beneficiaries from the government and the middle-aged are net contributors.

3.6 Pensions

We used as default an average-pay pension scheme. Because the build up of pension rights is linear, namely 2.25% (a_p^a) of the pension wage per year worked, our pension scheme aims at a replacement rate of 90% of average pay after 40 years of service (this includes the surviving relatives' pension). Not all workers build up occupational pensions in the Netherlands. Mostly self-employed workers do not participate in collective pension arrangements. We assume that 90% of the workers have occupational pensions (ε). The existence of the flat-rate public pension, the AOW, is taken into account by the pension fund through a franchise. Only workers with a wage above this franchise build up an occupational pension. The recent decline in the franchise in response to the disappearance of early retirement schemes is taken into account. In the base year (2006) the franchise (f_p) is set at 10,000 euro.

Most pension funds in the Netherlands aim at wage- or price indexation. This is not guaranteed, however, but is conditional on the financial position of the fund (funding ratio). Many pension funds have recently introduced more explicit indexation rules, providing for example no indexation at all if the funding ratio is below a certain lower bound, full indexation if the funding ratio is above an upper bound, and a linear cut in indexation for ratios in-between. Our average pension fund aims at a mixture of wage- (65%) and price indexation (35%), and gives full indexation at a funding ratio of 135% of the nominal liabilities (equivalent to about 95% of the indexed liabilities) or more. No indexation is given if the funding ratio is below 100% of nominal liabilities (70% of indexed liabilities).

Table 3.4 summarizes the parameter values in the GAMMA model for 2006.

Table 3.4 Exogenous variable and parameter values in the GAMMA model for 2006

	Symbol	Value
Growth and interest rates		
Productivity growth rate (%)	-	1.7
Inflation rate (%)	-	2.0
Nominal bond rate (%)	r_b	3.5
Equity premium (%)	r_{ep}	3.1
Households		
Intertemporal substitution elasticity	$1/\gamma$	0.5
Rate of time preference (%)	δ	1.3
Price elasticity of leisure demand	$1/\beta$	0.25
Fraction of financial wealth held in bonds (%)	-	50
Firms		
Substitution elasticity between labour and capital	σ	0.5
Fiscal depreciation rate of capital (%)	ν	4.8
Technical depreciation rate of capital (%)	ϕ	3.8
Fraction of debt repaid in each period (%)	ρ_0	3.8
Fraction of new investment financed with new debt (%)	ρ_1	70
Pension sector		
Target funding ratio	q_f	1.35
Pension rights accumulation rate (%)	a_p	2.3
Fraction of workers with occupational pensions (%)	ε	90
Tax rates		
Labour income (%) [a]	τ_{li}	34.1
Consumption (%)	τ_{in}	27.4
Transfer income (%)	τ_{sb}	10.4
Pension income (%)	τ_r	19.8
Firm profits (%)	τ_p	30.4
Household wealth (%) [b]	-	0.6

[a] includes public pension contribution
[b] imputed income from wealth is 4%

4 Baseline projection

The next two chapters of this document present and discuss the output for a variety of GAMMA simulations. This chapter deals with the baseline projection, assuming unchanged institutions. This is the same projection as that presented in Van Ewijk et al. (2006) that shows that without corrective budgetary reforms, Dutch public finances are on an unsustainable path.[38] Section 4.1 describes fiscal developments and section 4.2 describes the macroeconomic developments in the baseline.

4.1 Budgetary developments

Table 4.1 presents how public finances develop in our projection for the period 2006-2100. Until 2040 the demographic changes exert upward pressure on public expenditure by raising the costs of public pensions and health care by 4.1% and 4.3% of GDP respectively. In addition, natural gas revenues will decrease in this period by 1.5% of GDP as a result of the depletion of gas reserves. At the same time however, there are alleviating factors. First, tax revenues will increase through rising pension incomes that are subject to income taxation. First plus second pillar pension incomes will rise in the period until 2040 by 8.5% points of GDP (from 9.0% to 17.5% of GDP). This raises tax revenues from this source (both direct and indirect) by 4.1% of GDP. Second, expenditure on disability schemes is expected to fall considerably due to policy reforms that were implemented in recent years. On balance however, the burdening factors outweigh the alleviating factors. After an initial improvement due to cyclical factors, the primary balance deteriorates after 2011 as ageing and the decline of revenues from natural gas hit the budget. This eventually translates into declining EMU balances and eventually an explosion of public debt levels and interest payments. This in turn illustrates that government finances are currently on an unsustainable path and corrective measures are required to render public finances sustainable.

Total debt (the sum of the statutory debt and the implicit debt due to ageing and declining gas revenues) is about two times annual GDP. The corresponding sustainability gap amounts to 2.6% of GDP. This means that a permanent reduction in material government consumption by 2.6% of GDP starting in 2006 would suffice to fully restore fiscal sustainability. Pursuing this policy reform would lead to an improvement of government balances which in turn would reduce debt and interest payments and make it possible to cover the future costs of ageing. Table 4.2 shows how public finances are affected if a permanent reduction in material government consumption is adopted to achieve fiscal sustainability. Primary expenditure is reduced by 2.6% of GDP and the primary balance improves accordingly. As a result, the EMU balance improves. Next, the ratio

[38] Statistics Netherlands has updated the demographic forecast used in Van Ewijk et al. (2006) to allow for higher expected lifespans (see http://www.cpb.nl/nl/pub/cpbreeksen/notitie/05mrt2007_2/notitie.pdf). Here we retain the previous projection in order to reproduce the same baseline simulation.

Table 4.1 Public finances in the baseline projection

	2006	2020	2040	2060	2100
	% GDP				
Expenditure					
Social security	12.0	13.5	15.5	14.5	14.9
- public pensions	4.7	6.6	8.8	7.8	8.2
- disability benefits	2.0	1.9	1.6	1.6	1.6
- unemployment benefits	1.2	1.0	1.0	1.0	1.0
- other benefits	4.1	4.0	4.1	4.1	4.1
Health care	8.8	10.3	13.1	12.5	12.6
Education	5.4	5.4	5.8	5.7	5.8
Other expenditure excluding interest payments	19.2	18.4	18.2	18.3	18.3
Interest payments	2.5	1.5	2.5	4.2	7.2
Primary expenditure	45.3	47.8	52.5	51.0	51.5
Total	47.8	49.3	55.0	55.2	58.7
Revenues					
Income tax and social security contributions	21.8	23.7	25.3	24.9	25.2
- of which on pension income	1.8	2.5	3.6	3.4	3.6
Indirect and other taxation	14.9	15.9	17.3	16.7	16.8
- of which on consumption by population aged 65 and older	1.9	2.9	4.2	3.6	3.7
Corporate income tax	2.6	2.5	2.4	2.3	2.3
Natural gas revenues	1.6	0.8	0.1	0.0	0.0
Other income	5.2	5.2	4.9	4.7	4.4
Total	46.1	48.1	50.0	48.6	48.8
EMU balance	− 1.7	− 1.1	− 5.1	− 6.6	− 9.9
Primary EMU balance	0.7	0.4	− 2.6	− 2.4	− 2.7
EMU debt [a]	54.4	41.0	74.5	126.4	213.3

[a] Value at the end of the year

of debt to GDP shows a sharp decline and eventually becomes negative. The burden of interest payments develops accordingly. Eventually, government balances and debt levels stabilise at a constant ratio relative to GDP.[39]

4.2 Economic development in the baseline projection

Table 4.3 shows the development of a number of macroeconomic variables in the period 2006-2100. Notably, the share of consumption (private and public) in GDP increases strongly in this period. This share peaks around 2040, after which it decreases somewhat. This reflects the

[39] We use a discount rate equal to the average market rate of return for the determination of the sustainability gap.This does not imply that we also assume that the government actually finances its debt at this rate. Indeed, the bond tate is relevant for the rate at which the government finances its debt. (See Van Ewijk et al. (2006), page 54-56 for more information)

Table 4.2 Public finances in the sustainable projection (material public consumption reduced to achieve fiscal sustainability)

	2006	2020	2040	2060	2100
	% GDP				
Expenditure					
Primary expenditure	42.7	45.2	50.0	48.4	49.0
Interest payments	2.5	0.1	− 0.7	− 0.4	− 0.4
Total	45.2	45.3	49.3	48.0	48.6
Revenues	46.1	48.1	50.0	48.6	48.8
EMU balance	1.0	2.9	0.6	0.7	0.2
Primary EMU balance	3.4	3.0	− 0.0	0.2	− 0.2
EMU debt [a]	51.7	0.6	− 19.4	− 12.9	− 10.2

[a] Value at the end of the year

changing age structure of the population. Current saving levels are high and national wealth is built up through a surplus in the current account. Gradually, savings will decrease and consumption will increase relative to production when the share of elderly in terms of the working population starts to grow. The rapid increase in pensions that accompanies ageing results in an increase in the share of private consumption in GDP. Also, the share of government consumption in GDP increases in the projection. The increase occurs primarily after 2020 as a result of the comparatively strong growth in demand for health care. In 2040, the share of government consumption in GDP - especially as a result of rising expenditure on health care - is higher by 4.1% points in the baseline projection than in 2006. Finally, corporate investment also outpaces GDP in the projection in the period 2006-2040. The increase in corporate investment occurs entirely in the years 2020-2040. Up to 2020, there is an adverse effect on corporate investment from the slowdown in the growth of labour supply available for businesses. At a given capital intensity of the production process, a lower investment rate is required (in line with diminished employment growth). Overall, national spending increases more strongly than GDP. As a result, the surplus on the balance of trade deteriorates. Nonetheless, the Netherlands will realise very considerable surpluses in international trade in the coming decades. On the basis of sustainable policy, the trade balance surplus is expected to be 9.4% of GDP in 2006. That surplus is substantially greater than that of the baseline projection without budgetary measures, because the decrease in material government consumption implies a decline in national consumption. The international trade balance surplus gradually decreases in the projection and swings into a deficit as from around 2030. This eventually results in an international trade balance deficit of 4.9% of GDP in 2040, which gradually diminishes in subsequent years.

Table 4.3 Economic development on the basis of sustainable policies

	2006	2020	2040	2060	2100
	% GDP				
GDP components					
Wage income	50.1	52.8	52.6	53.0	52.9
Net other income	23.2	19.0	18.2	18.3	18.3
Depreciation	14.8	15.3	15.2	15.2	15.2
Indirect taxes less subsidies	11.6	12.6	13.8	13.4	13.4
Gross domestic product	100.0	100.0	100.0	100.0	100.0
Components of national consumption					
Private consumption	46.8	50.9	54.6	52.8	53.1
Government consumption	23.6	24.8	27.8	27.1	27.3
Corporate investment	17.5	17.2	20.0	19.5	19.7
Government investment	2.6	2.6	2.6	2.6	2.6
National consumption	90.6	95.5	104.9	102	102.6
Balance of trade surplus	9.4	4.5	− 4.9	− 2.0	− 2.6
Balance of primary revenues from abroad	0.4	3.2	4.5	3.9	4.6
Balance of secondary revenues from abroad	− 1.6	− 1.5	− 1.5	− 1.5	− 1.5
Balance of current foreign transactions	8.2	6.2	− 1.9	0.4	0.5
Net foreign assets [a,b]	0	76.1	103.1	86.3	104.9
Gross national product [c]	100.4	103.2	104.5	103.9	104.6

[a] Increase compared to 2006.
[b] Value at the end of the year.
[c] GDP plus balance of primary revenues from abroad.

5 Simulations

This chapter presents the results of three sets of simulations that demonstrate the workings of GAMMA. The first set (section 5.1) shows how the outcomes of the model are influenced by various alternative demographic assumptions. Since demographic developments are largely out of the control of the government, they are of particular interest for the projection of public finances. For these simulations we present the resulting unsustainable development and the change of the sustainability gap (see section 2.5.2 for a precise definition of the sustainability gap). The last two sets of simulations present the results of policy reforms, to the tax system (section 5.2) and the pension system (section 5.3) respectively. In these simulations, it is shown how household behavioural reactions are motivated by price changes caused by the reforms. In addition, the reforms will be assessed with respect to their efficiency and intergenerational consequences.[40]

Presenting these results as we do here, demonstrates the usefulness of the GAMMA model as a policy analysis tool: its results are intuitively understandable and straightforward to explain.

5.1 Demographic shocks

In this section, we present the results of three simulations that apply shocks to the baseline demographic projection. The shocks include: a permanent decrease in death rates, a permanent increase in the fertility rate and a permanent increase in immigration rates.[41] In order to draw comparisons between the scenarios, all three have been calibrated such that they all result in total population projections for the year 2041 that are 4.4% higher than in the baseline. (4.4% is half a standard deviation of the distribution in 2041 of population projections for the Netherlands produced by the PEP program. See Alho and Spencer (1997)). However, since each shock influences the demographic makeup of the population in a different way, they will have differing effects on the economy, the government's budget balance and the position of the pension sector.

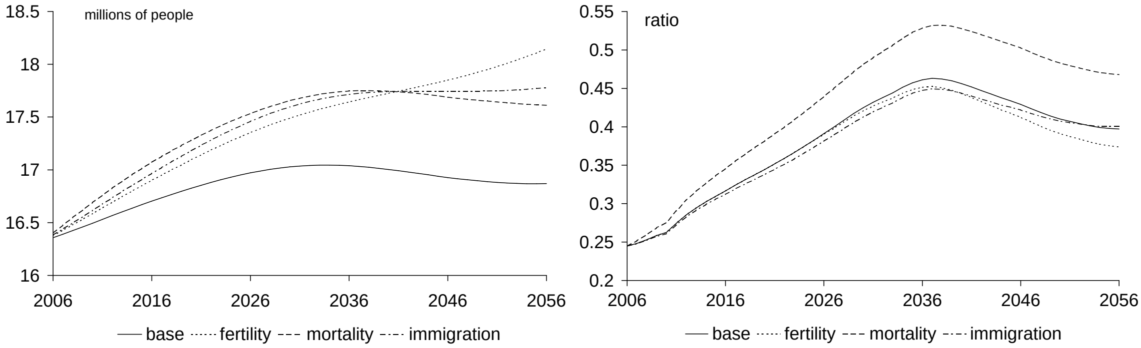
The left hand side of Figure 5.1 shows the time series projections of the total population from the three shock simulations as well as from the baseline projection. The calibration has enforced that the short-run consequences of each scenario are quite similar. However, after 2041 the population projections start to diverge once multiplier effects in the fertility and immigration shock scenarios kick in. In the mortality shock scenario, the population stabilizes by 2040 and begins to decline slightly thereafter.

The most relevant demographic statistic for government finance is the elderly dependency

[40] Appendix E investigates the homogeneity of the model for inflation, population growth and productivity growth.

[41] The immigration shock could also be interpreted as simulation of a decrease in emigration rates. Since the population model of GAMMA makes no relevant distinction between immigants and emigrants, decreasing emigration is equivalent to increasing immigration.

Figure 5.1 Total population (left) and old-age dependency ratio (right)



ratio, the number of pensioners divided by the number of potential workers. The right hand side of Figure 5.1 shows how each of the shocks affects the ratio differently. It can be seen that the largest divergence from the baseline occurs in the mortality shock scenario. Obviously, a decrease in the mortality rate would increase the number of retirees in the population while having very little effect on the number of workers. The positive fertility shock would have a slightly negative effect on the dependency ratio, although this would only occur after twenty years when the relatively larger cohorts reach working age. The immigration shock would also have a slightly negative effect, due to the specification of the age profile of immigrants. The majority of immigrants are of working age, so increasing the net immigration rate would increase the workforce.

The differing demographic implications of each of these scenarios will have consequences for the macroeconomy, the pension sector and government finances. In the following sections we address the consequences of each shock individually.

5.1.1 Mortality shock

Increased longevity will cause the elderly dependency ratio to stabilize at a much higher level than it is currently. In order to isolate the economic and budgetary effects of longer life spans, we present the outcomes of a mortality shock scenario in Table 5.1. The scenario simulates an increase of the expected lifespan by 3.4 years. The increase in lifespan has the effect of increasing the present value of future pension benefits to households, since the expected period of receiving benefits is now longer. This in turn increases the price of leisure, motivating less leisure demand (more labour supply) at the cohort level. The increase in the price of leisure occurs despite a rise in the average pension premium needed to cover the now higher liabilities of the pension system. The pension system subsidizes labour supply since pension benefits are taxed at a lower rate than labour income. Thus, the greater scope of the pension system due to the mortality shock increases the price of leisure. Despite this, the long-run aggregate demand for leisure increases due to a larger proportion of retirees in the population. This can be seen in Table 5.1: the aggregate leisure demand is a weighted average over the entire population

Table 5.1 Economic and fiscal effects of a decrease in mortality rates

		2006	2020	2040	2060	2100
Price of leisure	(%)	0.5	0.7	0.5	0.7	0.8
Tax wedge	(D)	− 0.3	− 0.4	− 0.3	− 0.4	− 0.5
Average pension premium		1.3	2.1	2.4	2.1	1.9
Leisure demand (weighted average)	(%)	− 0.1	0.1	0.2	0.2	0.1
- of twenty year-olds	(%)	− 0.1	0.0	0.0	0.0	− 0.1
- of fourty year-olds	(%)	− 0.1	− 0.1	− 0.1	− 0.1	− 0.2
- of sixty year-olds	(%)	− 0.2	− 0.2	− 0.2	− 0.3	− 0.3
Employment level	(%)	0.1	0.5	0.4	0.5	0.5
Household savings (% GDP)	(D)	1.5	0.8	− 0.1	− 0.1	0.1
Private consumption (% GDP)	(D)	− 1.6	− 0.4	1.1	1.4	1.3
Corporate investment (% GDP)	(D)	0.1	0.0	− 0.1	− 0.1	− 0.1
Government consumption (% GDP)	(D)	− 0.1	0.9	2.0	2.2	2.0
National consumption (% GDP)	(D)	− 1.6	0.6	3.1	3.5	3.2
Production (GDP in market prices)	(%)	− 0.4	0.4	0.8	0.9	0.9
Primary government expenditures (% GDP)	(D)	− 0.1	1.7	3.4	3.6	3.3
Government revenues (% GDP)	(D)	− 0.5	− 0.1	0.5	0.7	0.7
Sustainability gap	(D)	2.5				

including both retirees (whose leisure demand is scaled to the maximum by definition) and workers (whose leisure demand decreases). The long run increase in the proportion of retirees in the population causes this weighted average to increase despite the decease in leisure demand for workers at the cohort level. Longer lifespans also motivate an adjustment of goods consumption behaviour. Since they expect to live longer, households save a larger share of their working-age income for retirement.

At the aggregate level, the general employment level increases due to the increase in the price of leisure. The increase in employment is matched by an increase in private investment and the GDP is now larger due to the shock. Private consumption as a percentage of GDP falls in the short run because households initally must save because of the unexpected increase in their lifespans. Over the long run, however, longer lifespans mean that there is a permanently higher proportion of retirees in the population who consume more than they produce in any given year. In addition, the increase in the old-age dependency ratio drives up government spending on public pensions and health care. As a result, national consumption[42] as a percentage of GDP increases, meaning that longer lifespans would require a reversal of the current account surplus in the long run.

The fiscal effects of the shock are negative. Despite the fact that government revenues as a percentage of GDP increase due to higher tax receipts from labour income and indirect taxes,

[42] Defined as the sum of household and government consumption plus private and public investment.

greater spending on health care and public pension exacerbate the fiscal imbalance. The result of the shock is an increase of the sustainability gap of 2.5% of GDP.

5.1.2 Fertility shock

In this section we study the effects of a permanent increase in fertility rates from 2006 onward by 10%.[43] Table 5.2 shows the outcomes. In contrast to the mortality shock scenario there is

Table 5.2 Economic and fiscal effects of an increase in fertility rates

		2006	2020	2040	2060	2100
Price of leisure	(%)	0.0	0.0	− 0.8	− 0.5	− 0.7
Tax wedge	(D)	0.0	0.0	0.0	0.0	0.1
Average pension premium		0.0	0.0	− 0.2	− 0.1	0.1
Leisure demand (weighted average)	(%)	0.0	0.0	− 0.5	− 1.5	− 0.7
- of twenty year-olds	(%)	0.0	0.0	0.0	0.0	0.0
- of fourty year-olds	(%)	0.0	0.0	0.0	0.0	0.0
- of sixty year-olds	(%)	0.0	0.0	0.0	0.0	0.0
Employment level	(%)	0.0	0.0	3.2	8.3	18.5
Household savings (% GDP)	(D)	0.0	0.0	− 0.1	0.1	0.0
Private consumption (% GDP)	(D)	0.0	0.0	− 0.2	− 0.9	− 0.9
Corporate investment (% GDP)	(D)	0.0	0.0	0.8	1.1	0.7
Government consumption (% GDP)	(D)	0.0	0.3	0.3	0.0	− 0.1
National consumption (% GDP)	(D)	0.0	0.3	0.9	0.2	− 0.2
Production (GDP in market prices)	(%)	0.0	0.0	2.4	7.6	17.6
Primary government expenditures (% GDP)	(D)	0.0	0.3	0.2	− 0.5	− 0.6
Government revenues (% GDP)	(D)	0.0	0.0	− 0.2	− 0.6	− 0.6
Sustainability gap	(D)	0.2				

almost no influence from increased fertility on the price of leisure at the cohort level as is illustrated by the leisure demand effects at the cohort level. Thus, at the cohort level, the shock does not have a significant effect on household behaviour. However, after 2026, when the new relatively larger cohorts begin to join the workforce, there is an increase in the general employment level, accompanied by increases in private investment and gross domestic production. These effects are compounded over time because the larger cohorts begin to have proportionally more children themselves as they reach child-bearing ages.

There is also very little effect of higher fertility on public finances. In the medium run, there is an increase in public expenditures due to higher spending on education expenditures as expressed in percentages of GDP. Government revenue as a share of GDP changes temporarily as a result of the shock. The combined effect is a slight deterioration of the government's fiscal

[43] For example the fertility rate in 2006 for the 25-year-old cohort increases from .0672 to .0739.

position. However, sustainability could be restored with a budgetary adjustment of just 0.2% points of GDP.

That the increase in fertility actually makes the long run budgetary shortfall slightly worse can be explained by the fact that, on balance, individuals derive net benefits from the government over the course of their lifetimes. That is to say, viewed over the entire lifetime of a given individual, the discounted tax and premiums remitted to the government is lower than the discounted value of public expenditure from which that individual benefits. Therefore, a higher number of individuals in the population due to an increase in fertility will erode the fiscal position of the government, other things being equal.

5.1.3 Immigration shock

Table 5.3 gives the results of the immigration shock scenario that permanently decreases emigration rates in 2006 by 23%. As with the fertility shock scenario, higher net immigration

Table 5.3 Economic and fiscal effects of an increase in immigration rates

		2006	2020	2040	2060	2100
Price of leisure	(%)	− 0.1	0.0	0.1	0.3	0.3
Tax wedge	(D)	0.1	0.0	0.1	0.1	0.0
Average pension premium	(D)	0.0	0.0	0.3	0.2	0.1
Leisure demand (weighted average)	(%)	0.0	− 0.5	0.1	0.5	0.5
- of twenty year-olds	(%)	0.0	0.0	0.1	0.0	0.0
- of fourty year-olds	(%)	0.0	0.0	0.1	0.0	0.0
- of sixty year-olds	(%)	0.0	0.0	0.1	0.0	0.0
Employment level	(%)	0.0	2.9	5.9	6.0	5.6
Household savings (% GDP)	(D)	− 0.1	0.0	0.0	0.0	0.0
Private consumption (% GDP)	(D)	− 0.1	− 0.3	− 0.5	0.1	0.4
Corporate investment (% GDP)	(D)	0.6	0.8	0.2	0.0	0.0
Government consumption (% GDP)	(D)	0.0	− 0.2	− 0.5	− 0.2	0.0
National consumption (% GDP)	(D)	0.6	0.2	− 0.8	− 0.2	0.3
Production (GDP in market prices)	(%)	0.1	2.9	6.1	6.4	6.1
Primary government expenditures (% GDP)	(D)	0.0	− 0.4	− 0.8	− 0.2	0.2
Government revenues (% GDP)	(D)	0.0	− 0.2	− 0.3	0.1	0.2
Sustainability gap	(D)	− 0.2				

results in only very small variations in the price of leisure, and so behavioural reactions at the cohort level are insignificant. The aggregated economic impacts of the immigration shock mirror those of the fertility scenario except that the impacts are more immmediate. Employment increases due to the larger population, which in turn increases production.

It is apparent that the immigration shock has relatively little effect on public finances. In the beginning of the simulation period the main expenditure categories are slightly lower as percentages of GDP but only because production has increased by a greater magnitude due to a relatively larger workforce. Indeed, revenues as percentages of GDP also decrease in the short run for the same reason, but by not as much as expenditures. After 2040, tax revenues begin to increase. As a result, there is a small favourable development in the budget balance relative to the baseline. The sustainability gap as represented by the necessary budget adjustment is then negative (-0.2% of GDP), indicating that increased net immigration relieves the fiscal problem to some degree. However, compared to the positive necessary adjustment of 2.6% of GDP in the baseline scenario of chapter 4, decreasing emigration by 23% doesn't even come close to compensating for the demographic imbalance.

Note that this result must be qualified. By assumption, immigrants and emigrants in the GAMMA model are taken to be representative of the native cohort to which they belong. That is, there is no difference in terms of economic behaviour between immigrants and non-immigrants of the same age. In the real world, however, this seems not to be true. Immigrants, on average, have a lower level of labour force participation and wages than non-immigrants (see Ter Rele (2003)). Taking this fact into account would change the story considerably. In fact, encouraging immigration may make budgetary pressures worse if immigrants consume as much government expenditure as natives but provide less government revenue.

5.2 Tax reforms

5.2.1 Choice of tax base

The preferred choice of tax base has been the subject of investigation for applied general equilibrium models since their inception (see Auerbach and Kotlikoff (1987)). It is taken for granted that governments satisfy revenue requirements using distortionary taxes. By definition, distortionary taxes alter the economic behaviour of agents and erode the tax base. The relative efficiency of a particular tax depends not just on the the extent of the marginal economic distortion it creates but also the size of its base. For instance, a widely based tax spreads out the burden over more agents allowing the tax rate to be relatively low. Since tax distortions are judged to be convex in tax rates (Barro (1979)), a widely based tax will result in a lower aggregate distortion, all else being equal.

The GAMMA model is well-equipped to evaluate the efficiency of various tax instruments. Tax revenues are collected from a variety of sources including consumption, labour income, transfer income, asset income, pension income (public and private) and firms' profits. In addition, the effects of tax reforms can be simulated dynamically and not just at the instant the reform takes place.

In the simulation results presented here, we start from the sustainable projection presented in chapter 4 and roll back tax rates on total household income (exclusive of income from wealth) by ten percent (thus an equal relative change). Tax rates are then increased respectively on total pension income (public and private), labour income, and consumption in order to sustain the budget.[44] So this analysis is concerned with the effects of the trade-off involved in switching from one tax base to another. We look at both the efficiency and equity effects of the tax base change. As a measure of the efficiency effects of taxation, we use the long-run employment change as well as the aggregate utility change (measured as equivalent variations) resulting from the policy reform. The equity effects of taxation are measured by the distribution of utility changes over the age cohorts.

The switch to the tax solely on labour income results in a decrease in the price of leisure of just under 1% (Table 5.4). This occurs on the balance of two effects. First, the tax rate on pension income is reduced, resulting in an increase in the marginal benefit from supplying labour attributable to future pension benefits. Second, the tax on labour income is increased which decreases the marginal benefit of supplying labour attributable to the current wage. The latter effect outweighs the former. Due to the net decrease in the price of leisure, labour supply at both the cohort level and the macroeconomic level is reduced. Household goods consumption also decreases slightly over the long run because of the loss of net income caused by the reform for all cohorts except those who are presently retired or close to retirement. Those households benefit more from the reduction in pension taxes than they suffer from the increase in wage taxes.

Table 5.4 Economic effects of a switch from a tax on total income to a tax on labour income

		2006	2020	2040	2060	2100
Price of leisure	(%)	− 0.9	− 0.8	− 0.8	− 0.8	− 0.8
Goods price	(%)	0.0	0.0	0.0	0.0	0.0
Tax wedge	(D)	0.6	0.5	0.5	0.5	0.5
Average pension premium	(D)	0.0	0.0	0.0	0.0	0.0
Leisure demand	(%)	0.2	0.2	0.2	0.2	0.2
Employment level	(%)	− 0.1	− 0.1	− 0.1	− 0.1	− 0.1
Household savings (% GDP)	(D)	− 0.5	− 0.5	− 0.3	− 0.3	− 0.2
Private consumption (% GDP)	(D)	0.3	0.2	0.0	− 0.1	− 0.1
Corporate investment (% GDP)	(D)	− 0.4	0.0	0.0	0.0	0.0
Government consumption (% GDP)	(D)	0.0	0.0	0.0	0.0	0.0
National consumption (% GDP)	(D)	0.0	0.1	0.0	− 0.1	− 0.1
Production (GDP in market prices)	(%)	0.0	0.0	− 0.1	− 0.1	− 0.1

So a reform towards increased wage taxation is unambiguously distortionary, since the general employment level decreases by 0.1%. In this case, we have decreased taxes on two bases, transfers and pensions, that are relatively inelastic and increased taxes on a base that is directly

[44] These experiments are similar to those presented in chapter 5 of Auerbach and Kotlikoff (1987). In those simulations, the income tax was replaced entirely by taxes on wage income, consumption and capital income. Rather than imposing a long-run sustainability constraint as is done here, Auerbach and Kotlikoff impose a balanced budget constraint: that is, a constant stream of annual tax revenue.

affected by the tax rate through the wage elasticity of labour supply. Naturally the distortionary effect of the tax reform compounds itself: the distortion increases the aggregate tax burden which in turn increases the distortion.

Table 5.5 Economic effects of a switch from a tax on income to a tax on pension benefits

		2006	2020	2040	2060	2100
Price of leisure	(%)	2.7	2.3	2.5	2.4	2.4
Goods price	(%)	0.0	0.0	0.0	0.0	0.0
Tax wedge	(D)	− 1.8	− 1.6	− 1.7	− 1.6	− 1.7
Average pension premium	(D)	0.0	0.0	0.0	0.0	0.0
Leisure demand	(%)	− 0.6	− 0.5	− 0.6	− 0.5	− 0.5
Employment level	(%)	0.3	0.3	0.3	0.3	0.3
Household savings (% GDP)	(D)	2.0	1.9	1.0	1.0	0.9
Private consumption (% GDP)	(D)	− 1.2	− 0.6	0.1	0.3	0.3
Corporate investment (% GDP)	(D)	1.1	0.0	0.0	0.0	0.0
Government consumption (% GDP)	(D)	0.0	0.0	− 0.1	− 0.1	− 0.1
National consumption (% GDP)	(D)	− 0.1	− 0.5	0.1	0.2	0.3
Production (GDP in market prices)	(%)	0.0	0.1	0.3	0.4	0.4

In the case of replacing the total income tax with the tax on pension benefits (Table 5.5), the pension income tax has a large lump sum component: there is no labour supply reaction from presently retired cohorts resulting from the decrease in net pension receipts. For working-age cohorts the marginal benefit of working is reduced by lowering future net benefits, however this effect is more than offset by the increase in net wages resulting from the decrease in wage taxes. The reduction in the tax on labour income removes some of the distortion it caused in the base run scenario. So the shift of the burden to a tax base that is less affected by behavioural reactions decreases the distortions as measured by the long-run employment increase of 0.3%.

There is a short-run decrease in household goods consumption brought about by two factors. First, retired households see their net income decrease unexpectedly, and so they have less lifetime wealth from which to finance consumption. Second, working households must save more for retirement due to the increased tax on pension income. Over the long run, however, these effects subside and the efficiency of the reform is revealed by both by the long run increase in GDP and in household goods consumption as a percentage of GDP.

The consumption tax is also revealed to be more efficient than the tax on total income (Table 5.6). In this scenario there are influences on two prices which are relevant for household behaviour. First, the price of leisure is increased by over 4% due to a reduction of tax rates on labour and pension income. Second, the price of goods consumption is increased by 3.6% due to the higher indirect tax rate. As in the previous simulations the increase in the price of leisure stimulates household labour supply. In the short run, goods consumption decreases for those cohorts already in retirement or approaching retirement due to the unexpected loss of wealth in

Table 5.6 Economic effects of a switch from a tax on income to a tax on consumption

		2006	2020	2040	2060	2100
Price of leisure	(%)	4.2	4.1	4.2	4.2	4.2
Goods price	(%)	3.6	3.6	3.6	3.6	3.6
Tax wedge	(D)	− 0.4	− 0.4	− 0.4	− 0.4	− 0.4
Average pension premium	(D)	1.3	0.0	0.0	0.0	0.0
Leisure demand	(%)	− 0.1	− 0.1	− 0.2	− 0.1	− 0.2
Employment level	(%)	0.1	0.1	0.1	0.1	0.1
Household savings (% GDP)	(D)	0.0	0.2	0.1	0.1	0.1
Private consumption (% GDP)	(D)	0.7	0.9	1.0	1.1	1.1
Corporate investment (% GDP)	(D)	− 0.4	− 0.3	− 0.3	− 0.3	− 0.3
Government consumption (% GDP)	(D)	− 0.3	− 0.4	− 0.5	− 0.5	− 0.5
National consumption (% GDP)	(D)	− 0.1	0.2	0.2	0.2	0.2
Production (GDP in market prices)	(%)	1.4	1.6	1.8	1.7	1.8

real terms. However, for presently young and future cohorts, goods consumption increases as a result of the reform. At a glace, this seems counterintuitive since the price of goods consumption is now higher. However, counter-acting the price effect is an income effect: labour supply is increased which motivates and allows for increased goods consumption.

Why does the income effect dominate the price effect in this scenario? Because increasing indirect taxes shifts the burden in part to those who are presently retired, the reform is able to tax the existing wealth of those households. Since taxing wealth that has already been accumulated results in no behavioural feedback, the excess burden from taxation is decreased. So the increase in the consumption tax rate that must replace the decrease in the income rate can be less than it would be otherwise. The result of the reform is an increase in employment by 0.1% to increase the lifetime consumption possibilities of workers. So the shift to consumption taxation implies an efficiency increase.

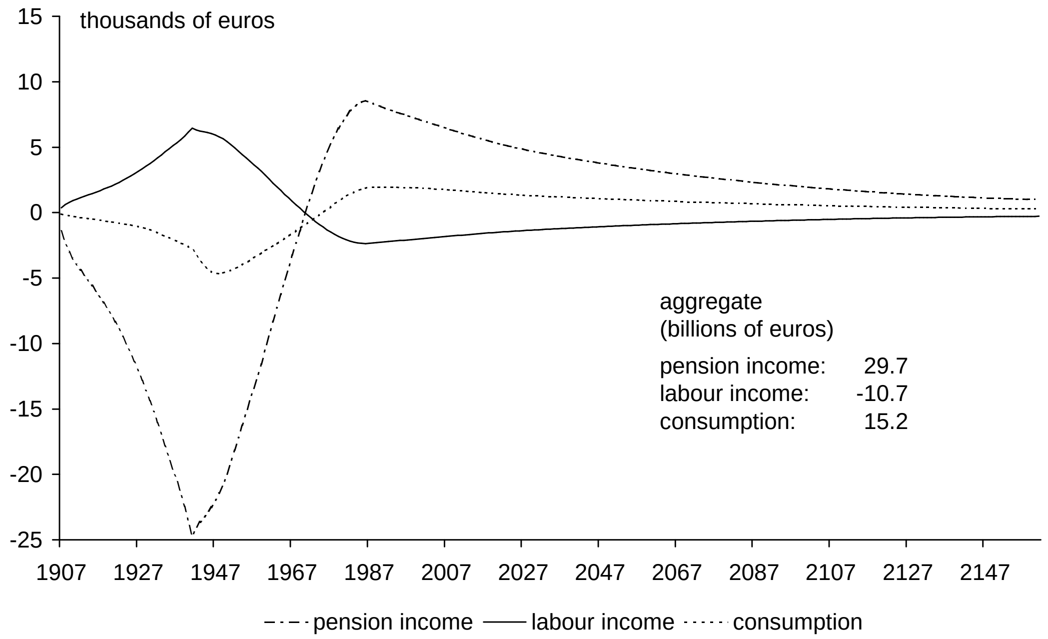
5.2.2 Welfare effects of tax reforms

In addition to the overall effects on efficiency of the tax reforms, switching between tax bases will have redistributional effects among generations. Figure 5.2 shows the equivalent variations for individuals by year of birth for the three tax policy simulations relative to the total income tax scenario.[45]

It can be seen that the switch to the tax on pension income has a very large negative impact on the welfare of those households who are presently retired. The cohort born in 1941 experiences

[45] The equivalent variation defined as the lump sum money transfer that would achieve the same change in lifetime utility as the policy reform. So a positive equivalent variation will indicate an efficiency over the baseline and vice versa. See appendix B, equations B.34 to B.36 for the derivation.

Figure 5.2 Equivalent variations of tax reforms for individuals by year of birth



the greatest utility loss since the households in that cohort turned 65 in 2006 and therefore retire the same year as the policy reform. As a result, they have the misfortune to be leaving the workforce the same year that the tax on labour income is reduced and the tax on pension income is increased. Older cohorts have shorter remaining expected lifespans and therefore require less compensation for the policy change. Younger cohorts benefit from a tax decrease every year until they retire and also have the opportunity to readjust their savings behaviour in expectation of lower pension income after retirement. The cohorts born in 1971 and after experience a net utility gain from the tax reform.

The switch to a tax solely on labour income has the opposite effect on per-cohort utility, although the welfare redistribution is less extreme. Presently retired and middle-aged cohorts experience a utility gain while working age cohorts born after 1970 experience a utility loss. Those cohorts that have yet to enter the workforce (born in 1987 and later) also experience a utility loss from the tax reform. As we saw above, the tax on labour income is inefficient relative to the tax on total income due to its smaller base and lack of non-distortionary components. As a result, future generations are subject to a higher aggregate tax burden.[46]

The consumption tax reform entails the smallest welfare redistribution of all the reforms. Since all income is eventually used up in consumption, switching from a tax on total income to a tax on consumption involves no change in the size of the base in the long run, only in the timing of the tax payment. The utility loss experienced by presently retired and middle-aged cohorts

[46] In the cases of the consumption tax and labour income tax, the welfare redistribution exhibits a similar pattern to that shown in Figure 5.4 of Auerbach and Kotlikoff (1987).

from the reform is due the fact that they will fund their consumption during retirement in part with existing financial wealth that was already taxed when it was earned as income. This double taxation will reduce the aggregate tax burden on following generations and as a result, they will experience a welfare gain from the reform. The 'break-even' cohort was born in 1975.

Figure 5.2 also shows the net effects on social welfare of the tax reforms. Not surprisingly, the switch to the labour income tax is the only scenario that entails an overall welfare loss. The reasons behind this should be clear from the discussion above. Both the tax on consumption and on pension income result in welfare gains. The pension income tax scenario is the more beneficial of the two because of its relatively large lump-sum component.

5.2.3 Delayed reforms

This section explores the intergenerational redistribution caused by postponing the tax reforms required to enforce fiscal sustainability for some time. In the simulations presented in this section, the rates for the labour income and consumption taxes, respectively, are raised from their present (i.e. unsustainable) level in particular years and smoothed over time in order to put government finances on a sustainable path. A later starting year of tax smoothing results in a larger debt in percentage of GDP at that date. A larger debt level in turn increases debt services which can only be raised through a larger future decrease of the primary deficit. Since the primary deficit must be covered by the tax increase, this implies that the later the sustainability policy starts, the larger the necessary tax rate change. Table 5.7 illustrates for both the consumption tax and the labour income tax.

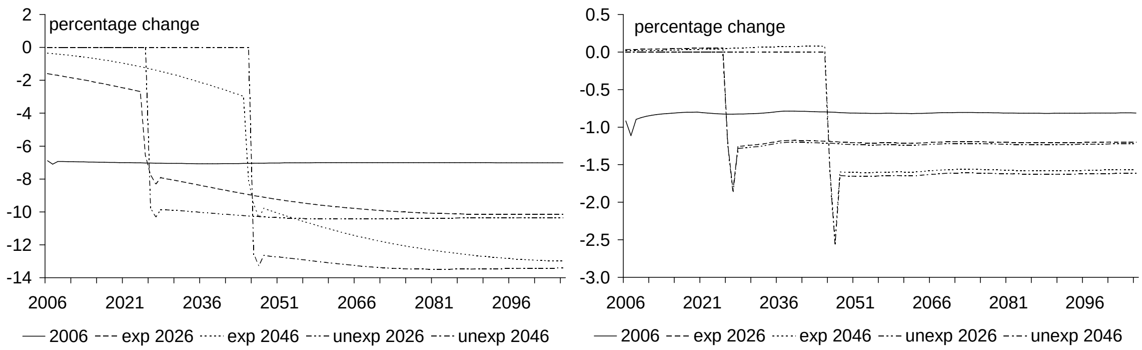
Table 5.7 Required tax rate increase according to the starting year (percentage points)

		2006	2026	2046
Consumption tax rate				
Anticipated	D	4.7	6.7	8.6
Unanticipated	D	4.7	6.8	8.8
Labour income tax rate				
Anticipated	D	7.9	10.1	13.0
Unanticipated	D	7.9	10.2	13.2

Table 5.7 also reveals that anticipating behaviour is favourable for the government budget. In order to smooth consumption over their lifecycles, households decrease their consumption in the period before the tax rate increases, but do not change their labour supply in this period (See Figure 5.3 in the case of the consumption tax increase). The boost in private saving increases financial wealth in the year that the policy reform takes place. Thus the total tax base (including that of asset taxes) increases because households anticipate the future tax rate increase. This explains why the tax rate increase can be slightly smaller if the policy adjustments are anticipated.

The left hand side of Figure 5.3 presents the development of private consumption relative to the baseline scenario according to the starting year of the consumption tax increase. For comparison, the effect of tax increases starting in 2006, 2026 and 2046 are shown for scenarios both when the policies are pre-announced by the government in 2006 and when the policy change is unexpected up until the year of implementation.

Figure 5.3 Change of consumption (left) and labour supply (right) according to the starting year of the tax increase; consumption taxation



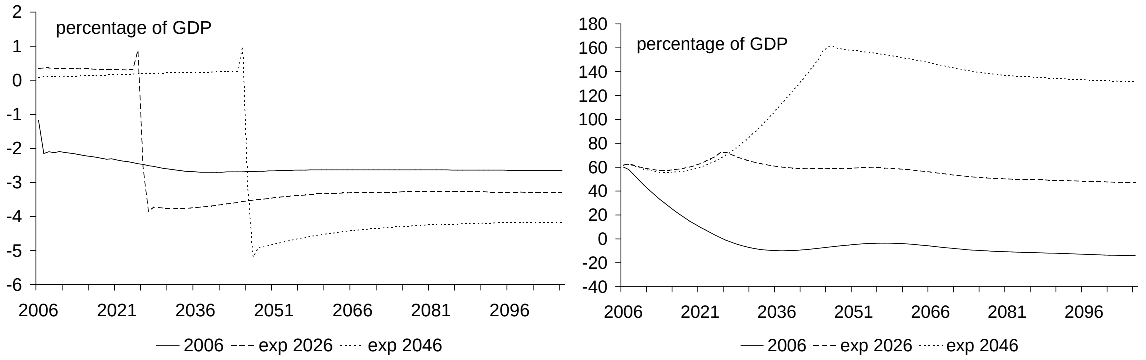
In the two unexpected scenarios, there are sudden and extreme reductions in consumption in the years when the policy changes takes place. Since the required tax increase in 2046 is higher than that in 2026, the decrease in consumption is also greater. When the future policy changes are known to individuals, the change in consumption behaviour is more gradual, as households accumulate more savings in anticipation of higher taxes in the future. The figure reveals that, despite the fact that pre-announcing is favourable relative to an unexpected policy measure, it does not fully compensate for the damage of postponing the tax increase.

The right hand side of Figure 5.3 presents the labour supply development under various consumption tax rate change scenarios and conveys a similar message as the consumption development. Moreover, it illustrates a characteristic of the model. There is no wealth effect on labour supply. Thus there is very little difference in labour supply movements between the expected and unexpected policy changes. In the case of an immediate tax increase (in 2006), the effect on labour supply is small and remains at a relatively constant level compared to the base case scenario.

The left hand side of Figure 5.4 presents the absolute change of the primary deficit as a percentage of GDP according to the starting year of the consumption tax increase. The figure illustrates an upwards spike in the primary deficit in the expected tax increase scenarios immediately before the policy measure becomes effective. This increase is linked to the decrease in labour supply. The year before the policy measure, investments decline sharply to make the capital stock consistent with the lower labour supply. This leads to a once-only decline in the indirect tax receipts on investments.

The development of the absolute level of the government debt to GDP ratio is shown in the

Figure 5.4 Absolute changes in the primary deficit (left) and debt (right) according to the starting year of the tax increase; consumption taxation



right hand side of Figure 5.4. It can be seen that postponing the necessary measure increases the long-run level of the ratio. In all cases the debt to GDP ratio is stabilized in the long run. However, only the immediate consumption tax increase reduces the debt to around zero. Fiscal sustainability requires that at any given date, the government's liabilities must be less than or equal to the present value of its future revenues. In each of the delayed tax increase scenarios, taxes are higher than in the immediate tax increase scenario and therefore so are the discounted future revenues summed indefinitely into the future. Thus the level of the government's debt relative to GDP can be higher. The longer the delay in implementation, the higher the required tax rate increase, the level of government debt and the associated debt service requirements.

Figure 5.5 Equivalent variations of delayed tax reforms for individuals by year of birth; left labour income tax, right consumption tax

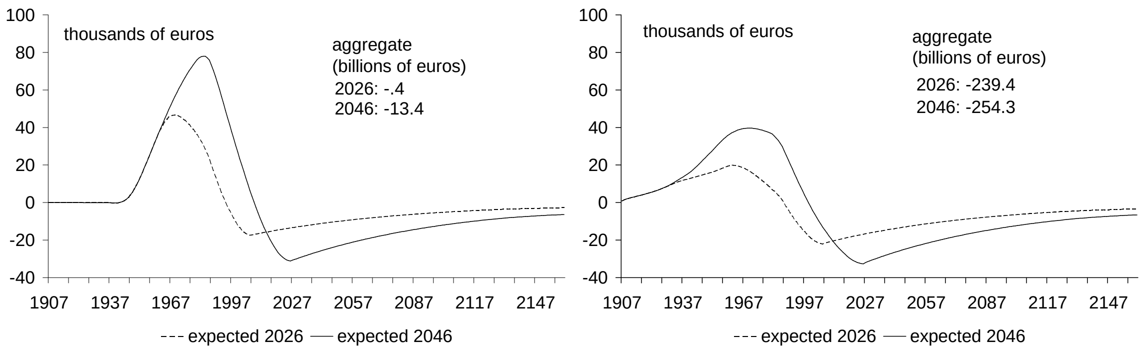


Figure 5.5 presents the welfare consequences of delaying the tax reforms for individuals by year of birth. These equivalent variation calculations are made relative to an immediate tax reform. In all cases, the cohort that suffers the most is that which is entering the workforce at the time the reform comes into effect. For example the cohort that was born in 2006 bears the highest loss if the reform is introduced in 2026.

In both the labour income and consumption tax cases, the longer the policy reform is delayed, the greater the welfare redistribution. This stands to reason because a longer delay implies that a higher tax rate increase is required (see Table 5.7). So the generation that experiences the effects of the reform over the greatest part of their working lives will suffer a larger welfare loss. Conversely a delay means that presently alive generations enjoy a larger welfare gain because the most escape the tax increase. On balance the aggregate welfare losses become more severe the more the reform is delayed.

It can be seen from the figure that a delay in the consumption tax reform entails a much larger aggregate welfare loss than a delay in the labour income tax reform. This result arises from the relative efficiency of introducing an immediate consumption tax increase rather than a labour income increase. The immediate introduction of a tax reform is unanticipated by households so the consumption tax increase can tax the existing financial wealth of households. A delay, by assumption, is preannounced. So presently living households have an incentive to save less (consume more now) and the future base of the consumption tax is eroded.

5.2.4 Public debt ratio smoothing

Postponement of policy reforms obviously benefits currently alive generations. However, the postponement scenarios feature highly discontinuous tax rate profiles. One way to avoid these discontinuities is to pursue a more gradual policy of keeping the public debt to GDP ratio at its initial level every year by adjusting tax rates accordingly. This is similar, although not the same as a balanced-budget policy in that accounting for the growth of the economy allows the government to run positive primary deficits. Like a tax smoothing policy, a public debt smoothing policy meets the requirement of fiscal sustainability. Indeed, policies are fiscally sustainable if they stabilize the long-run public debt to GDP ratio.

Here we present the results of two public debt smoothing simulations (for the consumption and labour income taxes) relative to tax smoothing simulations. Since the government stabilizes the debt to GDP ratio year-to-year at 2006 level, it is possible to postpone the necessary tax increase, which means a lower initial tax rate increase relative to the tax smoothing scenario (Figure 5.6). The tax rate development will then follow the dependency ratio, which is the main determinant of the debt ratio development. After about 25 years, tax rates become slightly higher than the tax smoothing rate. The fast increase of the tax rate in the first few decades slows down in line with the development of the dependency ratio. Following 60 years, both debt ratio smoothing tax rates remain at roughly constant (higher) levels relative to the tax smoothing rates. The difference is greatest in the case of the debt ratio smoothing with the labour income tax, again due to its smaller tax base compared to the consumption tax.

Figure 5.7 shows how this tax pattern is reflected in the private consumption and labour supply developments. Since taxes are initially relatively lower, consumption and labour supply are larger than in the tax smoothing scenario in the beginning of the simulation period. In the consumption tax case, private consumption becomes permanently lower by 2037, while in the

Figure 5.6 Change of the tax rate to smooth the public debt ratio; relative to a tax smoothing path

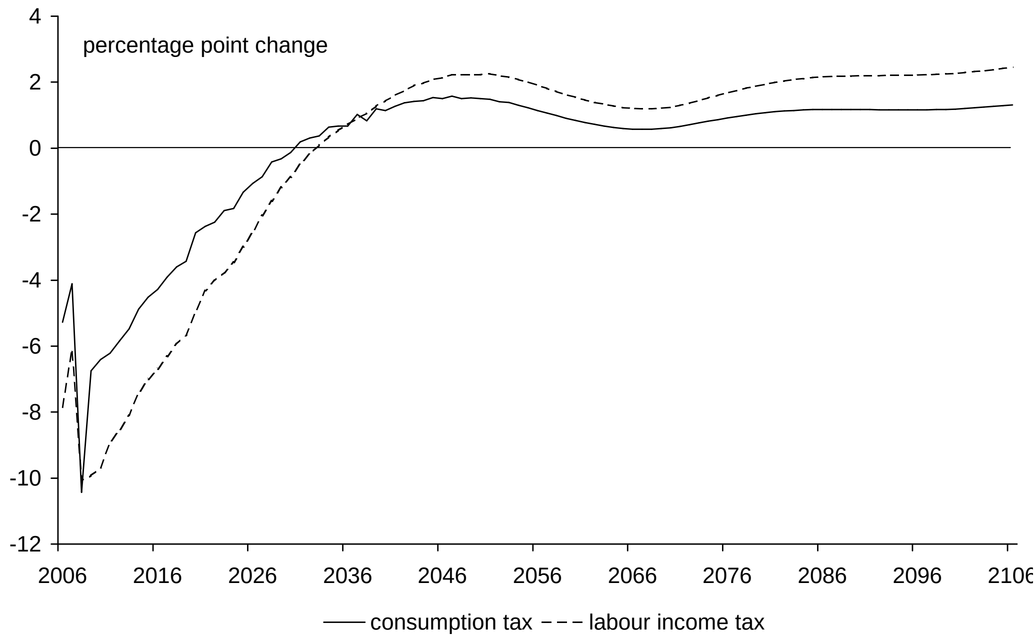
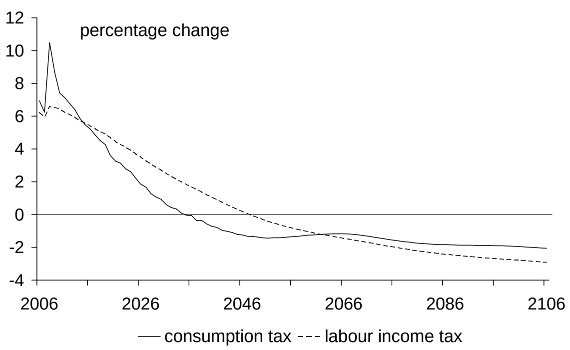
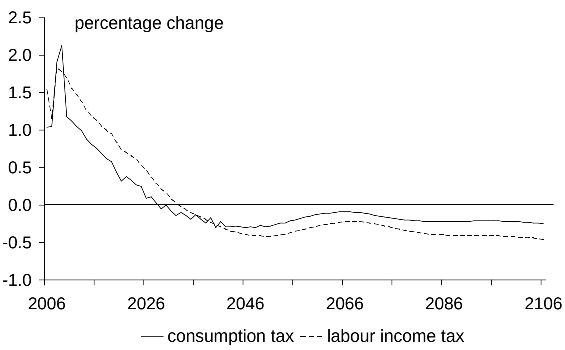


Figure 5.7 Public debt ratio smoothing relative to tax smoothing; left: percentage change of private consumption; right: percentage change of labour supply

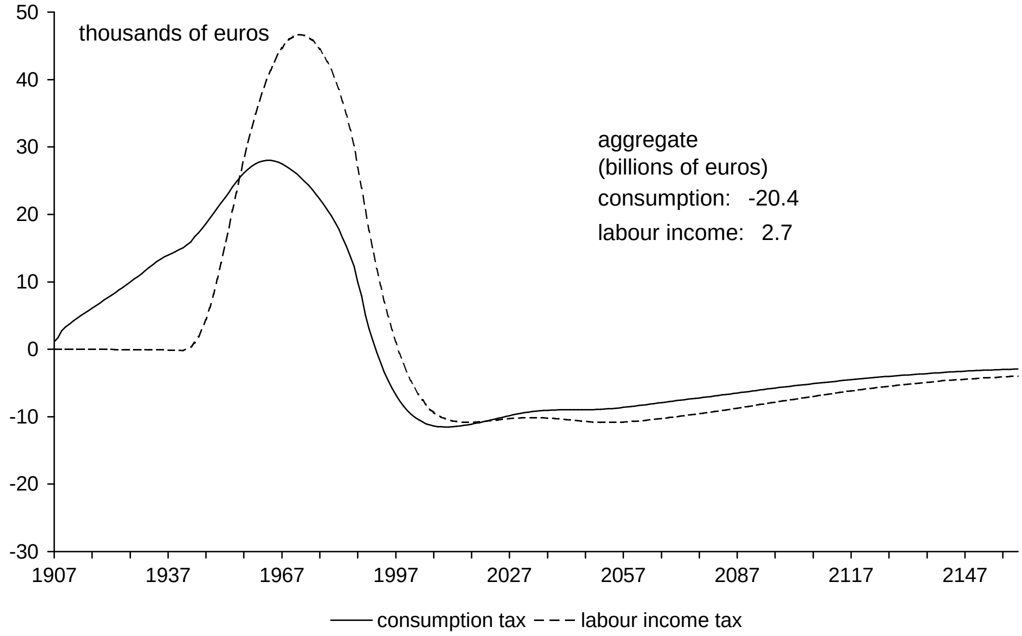




labour income tax case, consumption becomes lower about 20 years later. The long run effects on labour supply of the debt ratio smoothing policies are less severe, although there still is a decrease relative to the tax smoothing case after about 25 years. In the long run, the relative differences stabilize and the public debt ratio smoothing policy using the labour income tax has the most adverse effect on consumption and labour supply because of its higher tax burden.

Figure 5.8 illustrates the welfare effects for individuals by year of birth for each of the public debt ratio smoothing policies relative to the tax smoothing case. Comparing the two instruments, it can be seen that the utility gain from pursuing a gradual policy is distributed more widely among presently living cohorts in the case of the consumption tax. Presently retired cohorts are indifferent between a tax smoothing policy and a debt ratio smoothing policy using the labour income tax since they have no labour income. So in that case, all the welfare gains from the debt ratio smoothing policy are enjoyed by those cohorts of working age (and slightly younger) in 2006. For both tax instruments, future cohorts suffer from a gradual policy in approximately equal measures compared to tax smoothing.

Figure 5.8 Equivalent variations of public debt ratio smoothing relative to tax smoothing for individuals by year of birth



While the switch from tax smoothing to a public debt ratio smoothing policy in the consumption tax case entails an aggregated welfare loss valued at over 20 billion euros, in the labour income tax case debt ratio smoothing is welfare improving. At first glance this result is puzzling since the economic efficiency of tax smoothing is well established in the theoretical literature (see Barro (1979) and Kingston (1991)). However, keeping tax rates constant over time in GAMMA is not equivalent to tax smoothing as conceptualized by theoretical models. These models rely on the assumption of a direct relationship between tax rates and marginal distortions from taxation. In the case of labour income tax rates, tax smoothing in GAMMA (as well as in a real-world economy) does not necessarily imply constant marginal distortions due to a number of factors. These include a shift in the aggregate labour supply elasticity attributable to population ageing as well the decline of early retirement and catching up pension premiums in the coming years. Taking these factors into consideration implies that marginal distortion smoothing requires increasing labour income tax rates over time.

5.3 Pension reforms

5.3.1 A smaller pension scheme

In this section we investigate the effects on household behaviour, the macroeconomy and government finances of a reform that reduces the generosity of the second pillar pension system. The current tax regime for second pillar pension saving in the Netherlands is an "exempt-exempt-tax" (EET) scheme: pension contributions are subject to tax relief at the individual's then marginal rate (Exempt); investment returns are tax free (Exempt); and pension income is taxed when received at the individual's then marginal rate. Other private savings are made from taxed income (T), investment returns are rolled up 'after-tax' (T), and any money withdrawn from the saving account is exempt from tax (E). This results in a TTE regime. In this section, we investigate to what extent diminished pension savings are offset by private savings, i.e. what is the influence of the tax regime on savings? To answer this question, the accrual rate on supplementary pensions is decreased by ten percent starting from the year 2010.

Table 5.8 Economic and fiscal effects of a smaller pension system

		2006	2020	2040	2060	2100
Price of leisure	(%)	− 1.1	− 0.3	− 0.3	− 0.4	− 0.4
Tax wedge	(D)	0.7	0.2	0.2	0.3	0.3
Average pension premium	(D)	− 0.9	− 1.6	− 1.6	− 1.5	− 1.4
Leisure demand	(%)	0.3	0.1	0.1	0.1	0.1
Employment level	(%)	− 0.2	0.0	0.0	− 0.1	− 0.1
Household savings (% GDP)	(D)	0.4	0.8	0.6	0.5	0.4
Pension fund savings (% GDP)	(D)	− 0.3	− 0.9	− 0.7	− 0.6	− 0.5
Private consumption (% GDP)	(D)	− 0.1	− 0.1	− 0.1	− 0.2	− 0.2
Corporate investment (% GDP)	(D)	− 0.3	0.0	0.0	0.0	0.0
Government consumption (% GDP)	(D)	0.0	0.0	0.0	0.0	0.0
National consumption (% GDP)	(D)	− 0.4	− 0.1	− 0.1	− 0.1	− 0.1
Production (GDP in market prices)	(%)	− 0.2	− 0.1	− 0.1	− 0.1	− 0.1
Primary government expenditures (% GDP)	(D)	0.1	0.0	0.0	0.1	0.1
Government revenues (% GDP)	(D)	0.1	0.3	0.2	0.2	0.1
Sustainability gap	(D)	− 0.1				

Table 5.8 shows the economic and fiscal effects of the policy reform. Directly, there is a permanent decrease in the price of leisure. An uncompensated reduction in expected future retirement benefits results in a smaller marginal benefit from supplying labour. As a result, employment declines as does GDP. In addition, household goods consumption at the cohort level declines for three reasons. First, households are forced to save more for retirement in order to smooth consumption over their life-cycles. Second, because of the positive relationship between consumption and labour supply, goods consumption declines in step with the increase in leisure.

Third, household wealth, out of which consumption is financed, is diminished by the decline in both labour and pension income. At the aggregate level, private savings increase permanently as a percentage of GDP. It can be seen from the table that the increase in private savings almost (although not entirely) mirrors the decrease in pension fund savings. This illustrates the substitutability of private savings for pension wealth.

As illustrated in the table, there is little significant influence from the reform on public expenditures. Government revenues increase slightly due to the transition from EET taxed savings to TTE taxed savings. The EET regime is more generous because the labour income tax rate is larger than the pension income tax rate and due to the tax exemption of wealth increases. So the reform has a beneficial effect on public finances. However, the effect is relatively small: the necessary adjustment to restore sustainability is decreased by only .1% of GDP.

5.3.2 Increasing the retirement age

Increased - and still increasing - life expectancy has already prompted a number of countries to take pension reform measures, including an increase in retirement age. Here we introduce a variant to chart the effects of an increase in the statutory retirement age. This scenario assumes that the age at which peoples' entitlement to a public pension and supplementary pension commences is raised in two steps, by a total of two years, to the age of 67. These calculations assume a one-year step-up of the retirement age in both 2015 and 2025.[47] Table 5.9 reveals a

Table 5.9 Population effects of raising the statutory retirement age

		2006	2020	2040	2060	2100
Percentage of retirees in the population	(D)	0	− 1.2	− 2.1	− 2.3	− 2.3
Elderly dependency ratio	(D)	0	− 2.8	− 5.5	− 5.4	− 5.6

decline of the old age dependency ratio by 5.5 percentage points in 2040. However, the labour participation effects will likely be not as large. The computations assume that the productivity for the group aged 65 years (and 66 years, respectively) is the same as that for the group aged 64. This implies no wage differentials and the same leisure demands. So, the rate of labour participation after the increase in retirement age for the group aged 65 years (and 66 years, respectively) is the same as that for the group aged 64. On that assumption, the effects of the higher retirement age on effective labour supply in the alternative projection are limited, since the rate of participation for the 64-year-old group is rather low in the baseline projection. The baseline projection assumes a participation rate for this group of 10% and 11%, respectively, for 2014 and 2024. While this represents a slight increase in participation (as compared to the

[47] If a greater number of steps are assumed for bringing the retirement age up from 65 to 67, as is the case in the United States and Germany, the effects to be expected are rougly comparable.

present level of 8%), it is low compared to the labour market participation of the average cohort.[48]

Table 5.10 Economic and fiscal effects of raising the statutory retirement age

		2006	2020	2040	2060	2100
Price of leisure	(%)	− 0.6	− 0.1	− 0.3	− 0.4	− 0.5
Tax wedge	(D)	0.4	0.1	0.3	0.4	0.4
Average pension premium	(D)	− 0.9	− 2.4	− 2.4	− 2.3	− 2.1
Leisure demand	(%)	0.1	− 0.6	− 1.0	− 0.9	− 0.9
Employment level	(%)	0.0	0.3	0.5	0.5	0.5
Household savings (% GDP)	(D)	0.7	1.2	0.9	0.7	0.5
Pension fund savings (% GDP)	(D)	− 0.4	− 1.1	− 0.9	− 0.7	− 0.6
Private consumption (% GDP)	(D)	− 0.4	− 0.5	− 0.6	− 0.6	− 0.6
Corporate investment (% GDP)	(D)	0.2	0.0	0.0	0.0	0.0
Government consumption (% GDP)	(D)	0.0	0.0	− 0.1	− 0.1	− 0.1
National consumption (% GDP)	(D)	− 0.2	− 0.5	− 0.7	− 0.7	− 0.7
Production (GDP in market prices)	(%)	− 0.1	0.2	0.5	0.5	0.5
Primary government expenditures (% GDP)	(D)	0.1	− 0.3	− 0.6	− 0.6	− 0.6
Government revenues (% GDP)	(D)	0.1	0.2	0.1	0.0	0.0
Sustainability gap	(D)	− 0.6				

The increase of the retirement age makes pensions less generous and indirectly reduces the subsidy implicit in second-pillar pension schemes. This decreases the leisure price which in turn reduces labour supply for all age groups, except those between 65 and 67 years of age. At the aggregate level, employment increases as does GDP.

The increase in the retirement age means that the sustainability gap declines. Table 5.10 shows that it is 0.6%-points smaller than in the baseline projection. Fewer measures are now required for realising sustainable public finances since the higher retirement age results in a smaller increase in spending on public pensions in the coming decades, coupled with higher government revenues as employment and output are boosted. The increase in employment in this variant is smaller than the increase in the labour supply of the group aged 65 and over, since the higher retirement age results in a greater wedge.

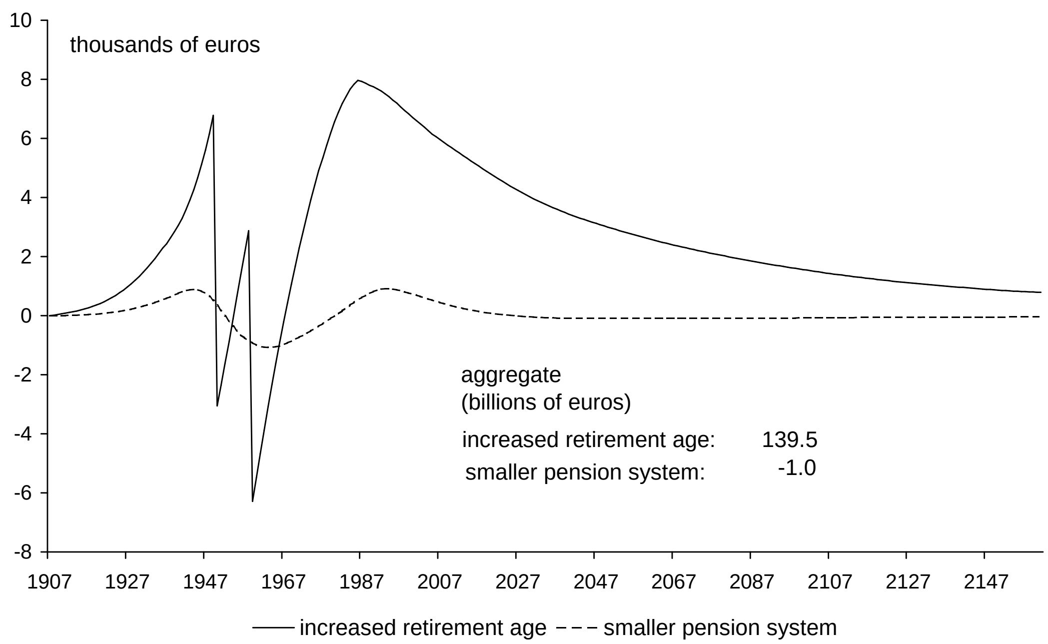
Table 5.10 reveals that even though employment and GDP increases household consumption declines both relative to GDP and in absolute terms. The reason for this decline is the greater wedge: the tax facilitated savings diminish. Households then have less lifetime wealth from which to consume goods.

[48] The employment effects are a little bit lower than those of Van Ewijk et al. (2006). The latter publication gives an autonomous impulse on the labour market participation change. In the present study we calculate welfare effects which needs a utility consist approach. This explans why the outcomes are slightly different.

5.3.3 Welfare effects of pension reforms

Figure 5.9 illustrates the welfare consequences over age groups of the two pension reforms described above. We have adjusted the labour income tax rate to sustain the government budget in order to account for the fiscal effects of the reforms. In this way, the simulations whose results are presented here are different from those presented in the previous two sub-sections.

Figure 5.9 Equivalent variations of pension reforms for individuals by year of birth



In the case of the switch to a smaller pension system, it can be seen that there is little welfare impact from the reform. Presently working and future cohorts experience both a reduction in future pension benefits and an increase in net (after-tax) wages. Those cohorts who are closer to retirement at the time of the reform suffer a net welfare loss. However, all other cohorts enjoy a welfare gain due to the fact that the benefit from reducing the relatively distortionary labour income tax outweighs the loss from decreasing the generosity of the pension system. Aggregated across all cohorts, the welfare effects of the reform are negative, although insignificant.

The reform that increases the retirment age has more extreme distributional effects. The two downward spikes in the figure correspond to the cohorts born in the years 1950 and 1959. Respectively, those households will turn 65 and 66 in the years 2015 and 2025, when the policy reforms come into place. As a result, those cohorts must work one more year than they would have otherwise before being able to receive pension benefits and, so, suffer a welfare loss. The reform is beneficial for government finances, however, and this enables a reduction in the labour income tax rate. As a result, future cohorts enjoy a welfare gain. It can be seen that presently retired cohorts also benefit from the policy reform. Since increasing the retirment age improves the financial position of private pension funds, benefits can increase due to greater indexation.

6 Final remarks

In chapters 4 and 5 it was shown how GAMMA can effectively be used to conduct long-term policy analysis. In addition to making projections about future economic and fiscal developments, the model is able to simulate convincing alternative scenarios that are highly relevant for policy makers. These scenarios may take the form of sensitivity analyses, such as changes to the input data or parameters of the model or the form of policy simulations which allow an evaluation of various reform options.

Two major publications of the CPB (Westerhout et al. (2004) and Van Ewijk et al. (2006)) have used GAMMA to demonstrate the consequences of population ageing on the sustainability of public finances and the private pension system in The Netherlands. Over time the model has been extended to allow it to be used for a wider range of purposes. Refinements made to the household block have enabled the production of more plausible feedback reactions in the simulations. In addition, the introduction of equivalent variations enabled consistent welfare analyses. This innovation is helpful in two respects. First, the equivalent variations of different cohorts from a policy simulation may be compared to assess the degree of intergenerational redistribution entailed by the reform. Second, aggregating equivalent variations over all cohorts is akin to creating a social welfare function since it indicates whether Pareto improvement is possible. While the GAMMA model will continue to be used to assess the sustainability of public finances and pensions, it will also be used increasingly to study the intergenerational consequences of specific reform measures. Examples of such measures are the introduction of life-cycle savings accounts, adjustments to the tax structure and alternative debt policy arrangements.

The GAMMA model is in a continual process of development. As with any economic model it relies on simplifying assumptions and, as such, suffers from a number of shortcomings that will be addressed in the future:

- A complete set of perfect markets exists. The neglect of capital constraints on households constitutes an important abstraction from reality that may have a bearing on the model's predictive power.
- There is one global market price for goods and capital. In doing so, there is no accomodation for terms-of-trade effects. The question whether such effects persist in the long run still has to be investigated.
- The risk premium applied to the return on equity is modelled in a rudimentary way. Portfolio shares are imposed exogenously so there is no correspondence between the premium and the risk aversion of households.
- Production occurs with only labour and capital as inputs. The influence of infrastructure and human capital on national income should otherwise be included in policy assessments.

- The form of the social welfare function does not account for changes in government consumption due to policy reforms. As a result, only welfare analyses of reforms to public revenue arrangements are possible.
- Agents face no uncertainty at the cohort level. This assumption has important implications for the behavioural underpinnings of the model.

GAMMA will continue to be improved in these aspects in the future.

Appendix A The circular flow

Table A.1 presents the total accounts of the Dutch economy as a circular flow, describing the macro-economic relations identified in the GAMMA model. The row entries of the total accounts are markets, or subdivisions of a market into transactions and the column entries are agents. The aggregated market transactions over agents (row total) sum to zero. The table distinguishes between markets for goods, capital and labour in addition to accounting for income transfers from the government to households. The capital market is subdivided into investment activities, income from capital and profit taxes. GAMMA identifies the following agents: households, pension funds, the government, firms and the foreign sector. Capital is included as an artificial agent, distinguished to make total investments and investments of the different sectors explicit. The model subdivides the household sector into one hundred age cohorts, however the table presents only the aggregated household sector. The government sector is subdivided into expenditures, income and production. A cell in the table gives the transactions of agents on a market (+ receipt, − payment). The aggregated budget constraint of agents is obtained by adding up over the column: the sum of revenues minus expenditures equals savings. That is, the column totals are zero.

Table A.1 Total accounts: circular flows

	Households	Pension funds	Capital	Government Expenditures	Income	Production	Firms	Foreign sector	Σ
Goods	$-C_h$		$-I$	$-C_g$	T_{in}	Y_{gg}	Y_{ge}	$-E_x$	0
Investments			I			$-I_g$	$-I_e$		0
Income transfers	Y_t			$-Y_t$					0
Labour income	Y_{wh}					$-Y_{wg}$	$-Y_{we}$		0
Private pensions	B_p	$-B_p$							0
Non-labour income	Y_{zh}	Y_{zp}		$-X_r$	Y_{zg}		$-Y_{ze}$	Y_{zf}	0
Income taxes	$-T_y$				T_y				0
Profit tax					T_p		$-T_p$		0
Private pension premiums	$-P_p$	P_p							0
Transfers to foreigners				$-X_{fo}$				X_{fo}	0
Savings(-)/shortage(+)	$-S_{vh}$	$-S_{vp}$			S_{vg}		W_{be}	$-S_{vf}$	0
Σ	0	0	0		0		0	0	

There is one good in GAMMA used for consumption, investment and exports. Imported goods are perfectly substitutable with domestic goods. The difference between domestically produced goods (by the government Y_{gg} and by firms Y_{ge}) and the domestic demand (private consumption C_h plus investments I plus government consumption C_g) minus indirect taxes plus subsidies T_{in} is sold abroad E_x. Households finance their consumption and pay tax and pension premiums (C_h, T_y and P_p respectively) with income from labour, transfers, pensions and asset wealth (Y_{wh}, Y_t, B_p and Y_{zh} respectively). In addition, they save privately for retirement S_{vh}.

Pension funds finance their payments B_p out of premium receipts P_p and income from wealth Y_{zp}. Wealth of pension funds grows with their savings S_{vp}. Government expenditures consist of government consumption C_g (defence, schooling, health care and public administration), income transfers Y_t (social assistance, children's assistance, public old-age-, disability-, unemployment-benefits and other social transfers), transfers to foreigners X_{fo} and debt services X_r. The government levies indirect taxes T_{in} (exclusive of subsidies), income taxes T_y, profit taxes T_p and other income taxes Y_{zg} (presented as non-labour income). The government produces services Y_{gg} using labour Y_{wg} and capital which is created with public investments I_g. The EMU deficit of the government is S_{vg}. Firms also produce Y_{ge} with labour Y_{we} and capital. Private capital formation takes place through investments I_e. The capital costs of firms consist of depreciation, profit taxes T_p and finance costs Y_{ze}.[49] The difference between income and expenditures of firms are the savings of firms W_{be}, which is the change of the stock of bonds issued by firms.

Table A.2 The total accounts: changes in total assets

	Households	Pension sector	Government	Firms	Foreign sector	Σ
Savings	S_{vh}	S_{vp}	$-S_{vg}$	$-W_{be}$	S_{vf}	0
Asset changes by emigration	$-W_{eh}$	$-W_{ep}$			W_{ef}	0
Asset changes by immigration	W_{ih}	W_{ip}			$-W_{if}$	0
Revaluation of assets	W_{rh}	W_{rp}	W_{rg}	$-W_{re}$	W_{rf}	0
Change of financial wealth	$-W_{fh}$	W_{fp}	W_{fg}	W_{fe}	$-W_{ff}$	0
Σ	0	0	0	0	0	0

The savings of agents is one of three determinants of the aggregated total asset change (W_{fh}, W_{fp}, W_{fg}, W_{fe}, W_{ff} for households, pension funds, government, firms and the foreign sector respectively). The other two determinants are net migration and revaluation of assets as Table A.2 reveals.[50] GAMMA assumes that migrants are representative agents: they have the same assets as natives of the same age. Migrants import (W_{ih}) or export (W_{eh}) those assets. Moreover, immigrants buy into the pension system (W_{ip}) with a purchase price equal to the value of pension rights while emigrants leave the pension system taking with them a money amount (W_{ep}) equal to the value of pension rights. Revaluation of bonds is not modelled: GAMMA implicitly assumes a one-year duration of bonds. That is why the revaluation of assets (W_{rh}, W_{rp}, W_{rg}, W_{rf} for households, pension funds, government and the foreign sector respectively) concerns the revaluation of shares only. The aggregate of these revaluations equals the change of the value of the firm (W_{re}).

[49] Y_{ze} consists of dividend payments, interest payments on debt, central bank profits paid to the government and income from leasing of land plus exogenous production Y_{go} in the base year (see section 3.4). So: $Y_{ze} = D_{iv} + r_b W^s_{be}(t-1) + G_{cb} + G_{pg} + Y_{go}$.

[50] Note, the government sector is not split up anymore, while the artificial capital sector is left out of the table.

Table A.3 The total accounts: portfolio changes

	Households	Pension sector	Government	Firms	Foreign sector	Σ
Change of financial wealth	W_{fh}	$-W_{fp}$	$-W_{vg}$	$-W_{fe}$	W_{ff}	0
Change of shares	$-W_{sh}$	$-W_{sp}$	$-W_{sg}$	W_{se}	$-W_{sf}$	0
Change of bonds	$-W_{bh}$	$-W_{bp}$	W_{bg}	W_{be}	$-W_{bf}$	0
Σ	0	0	0	0	0	

The total asset change is split up into the change of bonds and shares in Table A.3. The asset changes and investments determine the balances which are presented in Table A.4. The government has a claim W^s_{cg} on firms because it owns the central bank and land development companies. This is not the only reason for a difference between the value of the capital stock of firms (K^s_e) evaluated in production prices and the sum of the value of shares (W^s_{se}) and bonds (W^s_{be}). The effective price of capital deviates from the production price of investment goods through tax facilities (see equation C.23 in appendix C). This appraisal difference is variable W^s_{res}. The total capital stock (K^s) valued at current production prices deviates from the total capital stock evaluated at effective prices (W^s_{fc}). The total assets of households (W^s_{fh}), pension funds (W^s_{fp}), the government (W^s_{ng}) and the foreign sector (W^s_{ff}) equals the total capital stock evaluated at effective prices.

Table A.4 The total accounts: balances

	Households	Pension sector	Capital	Government	Firms	Foreign sector	Σ
Bonds	$-W^s_{bh}$	$-W^s_{bp}$		W^s_{bg}	W^s_{be}	$-W^s_{bf}$	0
Shares	$-W^s_{sh}$	$-W^s_{sp}$		$-W^s_{sg}$	W^s_{se}	$-W^s_{sf}$	0
Capital goods			K^s	$-K^s_g$	$-K^s_e$		0
Reserves government				$-W^s_{cg}$	W^s_{cg}		0
Reserves firms			W^s_{res}		$-W^s_{res}$		0
Total assets	W^s_{fh}	W^s_{fp}	$-W^s_{fc}$	W^s_{ng}		W^s_{ff}	0
Σ	0	0	0	0	0	0	

Appendix B Household model

B.1 Financial wealth; The Yaari assumption

The Yaari assumption is an easy way to close the household model. It describes how wealth of the deceased is distributed over the survivors. Assume homogeneity within an age cohort. Population size, n_h^s, of age cohort j develops over time according to

$$n_h^s(j,t) = n_h^s(j-1,t-1)\xi(j,t) \tag{B.1}$$
$$\text{with } \xi(j,t) = (1+q_i(j,t)-q_e(j,t))(1-q_d(j,t)),$$

q_i the immigration rate, q_e the emigration rate and q_d the death rate. Define households' nominal rate of return, r_h, financial wealth exclusive of pension rights, W_{fh}^s, annual after tax income income Y, consumption of goods C_h $(= c_h p_c)$, the change in wealth through immigration W_{ih}, the change in wealth through emigration W_{eh}. The relationship between the development of individual and aggregated financial wealth can now be stated as

$$W_{fh}^s(t) = (1+r_h)W_{fh}^s(t-1) + Y(t) - C_h(t) + W_{ih}(t) - W_{eh}(t) \tag{B.2}$$
$$\sum_j n_h^s(j,t)W_{fh}^s(j,t) = \sum_j n_h^s(j-1,t-1)\left[(1+r_h)W_{fh}^s(j-1,t-1) + \right.$$
$$\left. + Y(j,t) - C_h(j,t) + W_{ih}(j,t) - W_{eh}(j,t)\right]$$

Assume homogeneity of immigrants, emigrants and the non-migrant population. The change in wealth through emigration and immigration is then proportional to end of period wealth. Moreover, assume wealth from dying persons devolve upon their age cohort (Yaari assumption). This leads to the micro-economic budget equation[51]

$$W_{fh}^s(j,t) = \frac{1}{1-q_d(j,t)}\left[(1+r_h)W_{fh}^s(j-1,t-1) + Y(j,t) - C_h(j,t)\right]. \tag{B.3}$$

This micro-economic budget equation is consistent with the macro economic equation (B.2). So, we assume each individual receives an annuity from a life insurance company in return for bequeathing it its remaining assets upon death to close the model in an easy way.

In the next section we will describe how this budget equation is related to budget equation (2.5) in the main text. Because age and time are related on a one to one basis at the micro-level, the time indicator will be suppressed onwards.

Total wealth

In this appendix we attribute all income related to labour (wage income and pension benefits) directly to household labour supply. The starting point is the financial wealth equation (B.3).

[51] After aggregation of the individual variables their relation with the circular flow variables can be stated as: consumption C_h equals the circular flow variable; $r_h W_{fh}^s(-1) = Y_{zh} - T_k$ with T_k wealth taxes; $Y = Y_t + Y_{wh} - (T_y + T_k) + B_p - P_p$

Forward solution of the budget equation leads to:

$$(1+r_h)W^s_{fh}(j-1) = -\sum_{i=j}^{j_e} d_o(i)\,[Y(i)-C_h(i)] \tag{B.4}$$

$$d_o(i) = \left([1+r_h]^{-(i-j)}\prod_{l=j}^{i-1}\zeta(l)\right)$$

After tax income can be split up into after tax wage income Y_{wn}, after tax pension income Y_p and after tax transfer income Y_{tn}. After tax pension income can be split up into pension income linked to labour before the current year Y_{p1} and pension income linked to current and future labour Y_{p2}

$$Y(j) = Y_{wn}(j) + Y_{p1}(j) + Y_{p2}(j) + Y_{tn}(j) \tag{B.5}$$

The discounted value of labour income and pension benefits which can be attributed to current and future labour can be written as the discounted value of the product of labour time with a measure of total income per unit labour p_v

$$\sum_{i=j}^{j_r-1} d_o(i)Y_{wn}(i) + \sum_{i=j_r}^{j_e} d_o(i)Y_{p2}(i) = \sum_{i=j}^{j_r-1} d_o(i)\,[1-v(i)]\,p_v(i) \tag{B.6}$$

with leisure v and total available time per period scaled to 1. Define

$$(1+r_h)W^s_{ph}(j-1) = \sum_{i=j}^{j_e} d_o(i)Y_{p1}(i) \tag{B.7}$$

the pension rights which are built up at the end of period $j-1$. At the end of ones' life pension rights will be zero. Total financial wealth of households is defined as the sum of financial wealth actually in the hands of the households and pension rights $W^s_f(j) = W^s_{fh}(j) + W^s_{ph}(j)$. It is now easy to show that this can be written as equation (2.5) and (2.6) with $Y_f(j) = p_v(j) + Y_{tn}(j)$. We will now present details on the precise definitions of the different income sources.

B.2 Elaborating on the income sources

In this paragraph we will define after tax wage income Y_{wn}, after tax pension income Y_p and after tax transfer income Y_{tn}. The starting point is total annual after-tax income Y, which is the difference between non-capital income and taxes. Taxes consist of wage taxes T_l, transfer income taxes T_{sb}, taxes on private and public pensions T_r, other income taxes T_{er}, early retirement premiums P_{er}, and private pension premiums P_p, but exclude taxes on imputed income from wealth (included in the after tax rate of return). Labour and transfer income taxes include public pension premiums.

Income consists of transfers Y_t, private pensions benefits B_p, wage income Y_{wh} and early retirement benefits B_{er}. Transfer income Y_t includes social assistance B_{sa}, children's assistance B_{ca}, public old-age benefits B_{ea} (first pillar), disability B_{da} , unemployment benefits B_{un} and other social transfers B_{ot}

$$Y_t(j) = B_{sa}(j) + B_{ca}(j) + B_{ea}(j) + B_{da}(j) + B_{un}(j) + B_{ot}(j) \tag{B.8}$$

Wage income depends on the labour time and the wage rate

$$Y_{wh}(j) = (1 - v(j))p_l(j) \text{ and } j \in \{j_w, \dots, j_r - 1\} \tag{B.9}$$

Pension benefits are also a function of labour supply. We will distinguish between three pension systems: a final wage system, an average wage system and a defined contribution system. The relationship between work and pension benefits is different for these three systems. This will be elaborated in appendix B.3. Taxes and pension premiums are related to the tax and premium bases.

$$\begin{aligned}
T_l(j) &= (\tau_{li} + \tau_{gp})\,[Y_{wh}(j) - P_p(j) - P_{er}(j)] \text{ and } j \in \{j_w, \dots, j_r - 1\} \qquad \text{(B.10)}\\
T_{sb}(j) &= (\tau_{sb} + \tau_{gp})\,[B_{sa}(j) + B_{da}(j) + B_{un}(j)] \text{ and } j \in \{j_w, \dots, j_r - 1\}\\
T_r(j) &= \tau_r\,[B_{ea}(j) + B_p(j)] \text{ and } j \in \{j_r, \dots, j_e\}\\
P_p(j) &= \tau_{pp} H_p(j) \text{ and } j \in \{j_w, \dots, j_r - 1\}\\
P_{er}(j) &= \tau_{er} Y_{wh}(j) \text{ and } j \in \{j_w, \dots, j_r - 1\}
\end{aligned}$$

In these equations τ indicates a tax/premium rate and $H_p(j)$ the pension premium base. We can now define after tax wage income Y_{wn}, after tax pension income Y_p and after tax transfer income Y_{tn}

$$\begin{aligned}
Y_{wn}(j) &= [1 - (\tau_{li} + \tau_{gp})]\,[(1 - \tau_{er})p_l(j)(1 - v(j)) - P_p(j)] \qquad \text{(B.11)}\\
&\equiv p_{v1}(j)(1 - v(j))\\
Y_p(j) &= (1 - \tau_r)P_u(j)\\
Y_{tn}(j) &= (1 - \tau_r)B_{ea}(j) + B_{sa}(j) + B_{ca}(j) + B_{da}(j) + B_{un}(j) + B_{ot}(j) + B_{er}(j)\\
&\quad - (T_{sb}(j) + T_{pi}(j) + T_{er}(j))
\end{aligned}$$

This implies for the price of leisure

$$\begin{aligned}
p_v(j) &= p_{v1}(j) + p_{v2}(j) \qquad \text{(B.12)}\\
&= [1 - (\tau_{li} + \tau_{gp})]\left[(1 - \tau_{er})p_l(j) - \tau_{pp}\left(p_{lg}(j) - f_p\right)\varepsilon(j)\right] + p_{v2}
\end{aligned}$$

With p_{lg} the gross wage of households, f_p the pension franchise and $\varepsilon(j)$ the fraction of workers in the second pillar pension system as in equation (2.34) in section 2.6. We will now indicate how Y_{p2} can be rewritten to get an explicit expression for p_{v2} (see equation B.12 and B.11). This second part of the leisure price will depend on the pension system.

B.3 The price of leisure

Final wage system

In the final wage system private pensions are proportional to the premium base $(p_{lg}(j) - f_p)$ at the beginning of the pension period. The proportionality factor depends on the number of participation years and a built up percentage a_p^f. The pension benefits after the first pension year are indexed to prices, productivity and wages. Assume the pension benefits start in the year of participation. The after tax net benefit should then be $(1-\tau_r)a_p^f\left(p_{lg}(j)-f_p\right)\varepsilon(j)$. However, the pensions will start later on. Moreover, the benefits will be given several years. This delay and distribution effect can be represented in a discount value term d_{pv}^f. One more unit of labour supply leads to a discounted value

$$p_{v2}(j) \equiv (1-\tau_r)a_p^f\left(p_{lg}(j)-f_p\right)\varepsilon(j)d_{pv}^f(j) \tag{B.13}$$

of net, additional future pension benefits (p_{v2}).

Average wage system

In the average wage system private pensions (P_u) are proportional to pension rights. These pension rights increase over time for two reasons. First, the old rights are indexed. Second, through participation the rights increase proportionally to the premium base ($p_{lg}(j)-f_p$). The proportionality factor is a_p^a. The pension benefits after the first pension year are also indexed. This implies that we can write the net, additional future pension rights in the same way as in the case of the final wage system

$$p_{v2}(j) \equiv (1-\tau_r)a_p^a\left(p_{lg}(j)-f_p\right)\varepsilon(j)d_{pv}^a(j) \tag{B.14}$$

in which d_{pv}^a again represents the delay and distribution effect. The main difference with the final wage system is the difference of the discount factor, since in the final wage system old pension rights increase with the premium base, while in the average wage system they change with the general indexation factor. The difference is the incidental wage component.

Defined contribution system

In the defined contribution system private pensions P_u are an annuity dependent on wealth built up in the working ages. Wealth accumulates in the working ages. After the working ages wealth decumulates. One unit more of labour supply leads to a discounted value

$$p_{v2}(j) \equiv (1-\tau_r)\frac{\tau_{pp}^d\left(p_{lg}(j)-f_p\right)\varepsilon(j)}{1+m_p^d(j_r,j)}d_{pv}^d(j) \tag{B.15}$$

of net, additional future pension benefits (p_{v2}). In this equation d_{pv}^d again represents the delay and distribution term while m_p^d presents the annuity distribution of the premium over the pension years. The main diffrence with the final wage and the average wage system is the link of the benefits to the premium base.

B.4 The consumption problem

Households maximize the utility function (2.2) as a function of the consumption of goods and leisure given the budget equation (2.5) and given the restriction that leisure is less than or equal to one. The Lagrangian of this maximization problem is

$$L = \sum_{i=j}^{j_e} \frac{\alpha_c(i)^{1-\gamma}(c_h(i)-c_l(i))^{1-\gamma}}{1-\gamma} d_s(i) +$$
$$+\lambda\left[W^s(j-1) - \sum_{i=j}^{j_e} d_o(i)\left[c_h(i)p_c(i) + v(i))p_v(i)\right]\right]$$
$$-\lambda\sum_{i=j}^{j_e} d_o(i)\mu(i)\left[v(i)-1\right]. \tag{B.16}$$

First order conditions for an optimum are

$$L_c(i) = \alpha_c(i)^{1-\gamma}(c_h(i)-c_l)^{-\gamma} d_s(i) - \lambda d_o(i)p_c(i) = 0, \tag{B.17}$$

$$L_v(i) = \alpha_c(i)^{1-\gamma}(c_h(i)-c_l)^{-\gamma} d_s(i)v(i)^{-\beta}\frac{\alpha_v(i)}{\alpha_c(i)} - \lambda d_o(i)\widetilde{p}_v(i) = 0, \text{ and}$$
$$\widetilde{p}_v(i) = p_v(i) + \mu(i), \tag{B.18}$$

$$\mu(i)L_\mu(i) = \mu(i)\left[v(i)-1\right] = 0. \tag{B.19}$$

Division of the first two conditions gives for leisure for each period i

$$v(i) = \left(\frac{\alpha_v(i)}{\alpha_c(i)}\frac{p_c(i)}{\widetilde{p}_v(i)}\right)^{\frac{1}{\beta}}. \tag{B.20}$$

For pensioners $p_v = 0$, and $v = 1$ which implies

$$\mu(i) = p_c(i)\frac{\alpha_v(i)}{\alpha_c(i)}, \; i \geq j_r. \tag{B.21}$$

In case the leisure restriction binds for workers, the shadow price of leisure is:

$$\mu(i) = \left(\frac{p_c(i)}{p_v(i)}\frac{\alpha_v(i)}{\alpha_c(i)} - 1\right)p_v(i) \text{ if } v(i) = 1 \text{ and } i < j_r, \tag{B.22}$$

otherwise:

$$\mu(i) = 0 \text{ if } v(i) < 1 \text{ and } i < j_r.$$

Use the definition of X in (2.5), $\widetilde{p}_v(i)$ in (B.18) and leisure equation (B.20) to write consumption as

$$c_h(i) = \frac{X(i)}{p_c(i)} - \frac{\alpha_v(i)}{\alpha_c(i)}v(i)^{1-\beta} + \frac{\mu(i)}{p_c(i)}. \tag{B.23}$$

Define $x_f(i) = \alpha_c(i)(c(i)-c_l(i))$ and $p_x(i) = \alpha_c(i)^{-1}p_c(i)$ and substitute the consumption and leisure equation into $x_f(i)$

$$x_f(i) = \frac{X(i)}{p_x(i)} + \alpha_v(i)\frac{\beta}{1-\beta}v(i)^{1-\beta} + \frac{\mu(i)}{p_x(i)}. \tag{B.24}$$

Use the definition

$$x_l(i) = -\left[\alpha_v(i)\frac{\beta}{1-\beta}v(i)^{1-\beta} + \frac{\mu(i)}{p_x(i)}\right] \tag{B.25}$$

to obtain

$$x_f(i) + x_l(i) \equiv \frac{X(i)}{p_x(i)}. \tag{B.26}$$

Variable x_l attains its minimum value for pensioners for whom $v = 1$

$$x_l(i) = -\alpha_v(i)\frac{\beta}{1-\beta}\ . \tag{B.27}$$

For working-age households who supply labour ($v < 1$):[52]

$$x_l(i) = -\alpha_v(i)\frac{\beta}{1-\beta}v(i)^{1-\beta}. \tag{B.28}$$

So variable x_l decreases with an increase of v. Equivalently an increase of labour supply implies an increase of x_l. The most easy interpretation is an expenditure commitment to compensate for missing leisure.

Division of the first order conditions (B.17) for two subsequent periods leads to

$$x_f(i)p_x(i) = \left(\frac{p_x(i)}{p_x(i+1)}\right)^{1-\frac{1}{\gamma}}\left(\frac{1+r_h}{1+\delta}\right)^{-\frac{1}{\gamma}} x_f(i+1)p_x(i+1). \tag{B.29}$$

Define

$$\widetilde{W^s}(j-1) = W^s(j-1) - \sum_{i=j}^{j_e} d_o(i)x_l(i)p_x(i) = \sum_{i=j}^{j_e} d_o(i)x_f(i)p_x(i). \tag{B.30}$$

Substitution of the Euler equation (B.29) leads to

$$\widetilde{W^s}(j-1) = x_f(j)p_x(j)^{\frac{1}{\gamma}}p_W(j-1)^{1-\frac{1}{\gamma}}, \tag{B.31}$$

with p_W defined as in (2.9). This implies for total consumption over the rest of the life cycle

$$x_f(s) = \frac{\widetilde{W^s}(j-1)}{p_W(j-1)}\left(\frac{\frac{d_o(s)}{d_s(s)}p_x(s)}{p_W(j-1)}\right)^{-\frac{1}{\gamma}}. \tag{B.32}$$

This equation can be written as

$$c_h(s)p_c(s) = c_l(s)p_c(s) + \left[W^s(j-1) - \sum_{i=j}^{j_e} d_o(i)\left(\beta c_l(i) - \frac{\mu(i)}{p_c(i)}\right)p_c(i)\right]\times \left(\frac{d_o(s)}{d_s(s)}\right)^{-\frac{1}{\gamma}}\left(\frac{p_c(s)}{\alpha_c(s)p_W(j-1)}\right)^{1-\frac{1}{\gamma}}. \tag{B.33}$$

[52] The special case of working-aged households who supply no labour ($v = 1$ and $j < j_r$) gives: $x_l(i) = -\frac{\alpha_v(i)}{\alpha_c(i)}\frac{1}{1-\beta} + \frac{p_v(i)}{p_x(i)}$. In practise this result very rarely arises.

Equation (2.8) in the main text is obtained by setting $\mu(i) = 0$. Substitution into the utility function gives

$$U(j) = \sum_{i=j}^{j_e} \frac{x_f(i)^{1-\gamma}}{1-\gamma} d_s(i) \tag{B.34}$$
$$= \frac{1}{1-\gamma} \left(\frac{\widetilde{W^s}(j-1)}{p_W(j-1)} \right)^{1-\gamma}$$

The same utility order can also be presented by a monotonic transformation

$$U'(j) = \frac{\widetilde{W}^s(j-1)}{p_W(j-1)}. \tag{B.35}$$

The equivalent variations EV are defined as

$$0 = \frac{\widetilde{W}_0^s(j-1) + EV(j)}{p_{W0}(j-1)} - U_1'(j) \tag{B.36}$$

or

$$EV(j) = \frac{U_1'(j) - U_0'(j)}{U_0'(j)} \widetilde{W}_0^s(j-1) \tag{B.37}$$

Appendix C Firm model

The budget equation (2.12) in the main text can be written (after substitution of (2.13) and (2.14)) as

$$D_{iv}(t) = (1-\tau_p)\left[p(t)y_{ge}(t) - p_{le}(t)l_e(t)\right] - (1-\rho_1)p(t)i_g(t) + \tau_p A_f(t) - G_{nt}(t) \tag{C.1}$$

$$-\left[(1-\tau_p)r_b + \rho_0\right]W^s_{be}(t-1) \tag{C.2}$$

The variable G_{nt} includes payments to the government other than taxes and dividends.

$$G_{nt}(t) = G_{cb}(t) + (1-\tau_p)G_{pg}(t) \tag{C.3}$$

Forward expanding of the capital market arbitrage equation (2.15) results in

$$W^s_{se}(t) = \sum_{j=1}^{\infty} D_{iv}(t+j)\,(1+r_s)^{-j}\,. \tag{C.4}$$

The discounted value of the fiscal depreciation can be split up into depreciation on the existing capital stock at time t, AF_t, and the discounted value of depreciation on new investments:

$$\sum_{j=1}^{\infty}\left(\frac{1}{1+r_s}\right)^j A_f(t+j) = \sum_{j=1}^{\infty}\frac{v}{r_s+v}i_e(t+j)p(t+j)\,(1+r_s)^{-j} + AF_t \tag{C.5}$$

where $AF(t)$ equals the depreciation allowance on investments installed up to time t:

$$AF(t) = \sum_{j=0}^{\infty}(1-v)^j\frac{\upsilon}{r_s+v}i_e(t-j)p(t-j). \tag{C.6}$$

The value of $AF(t)$ is given and therefore does not affect the optimization problem.

The firm maximizes its own value (C.4) given the capital accumulation equation (2.17) and the production function (2.16). This leads to the Lagrangian

$$\begin{aligned} L = \sum_{j=1}^{\infty} &\left((1-\tau_p)p(t+j)\left[F\left\{k^s_e(t+j-1), l_e\right\} - p_{le}(t)l_e(t)\right]\right. \\ &- (1-\tau_p\frac{v}{r_s+v}-\rho_1)i_g(t+j)p(t+j) - G_{nt}(t+j) - \left[(1-\tau_p)r_b+\rho_0\right]W^s_{be}(t+j-1) \\ &- q(t+j)\left[k^s_e(t+j) - (1-\phi)k^s_e(t+j-1) - i_e(t+j)\right] \\ &+ \lambda(t+j)\left.\left[W^s_{be}(t+j) - (1-\rho_0)W^s_{be}(t+j-1) - \rho_1 p(t+j)i_e(t+j)\right]\right)(1+r_s)^{-j} \\ &+ \tau_p AF(t) \end{aligned} \tag{C.7}$$

with F the production function. First order conditions for an optimum are :

$$\imath.\ L_{l_e} = 0\,;\ \imath\imath.\ L_{k^s_e} = 0 \quad;\quad \imath\imath\imath.\ L_{i_e} = 0 \quad;\quad \imath\upsilon.\ L_{W^s_{be}} = 0 \tag{C.8}$$

$$\imath.\ F_{l_e}(t) = \frac{p_{le}(t)}{p(t)} \tag{C.9}$$

$$\iota\iota.\ q(t) = \frac{(1-\tau_p)\,p(t+1)F_{k_e^s}(t+1) + q(t+1)(1-\phi)}{1+r_s}, \tag{C.10}$$

$$\iota\iota\iota.\ q(t) = \left(1-\rho_1 - \frac{\tau_p \nu}{\nu + r_s}\right) p(t) + \lambda(t)\rho_1 p(t), \tag{C.11}$$

$$\iota\upsilon.\ \lambda(t) = \frac{[(1-\tau_p)r_b + \rho_0] + (1-\rho_0)\lambda(t+1)}{(1+r_s)} \tag{C.12}$$

This last condition can be simplified in case the interest rate and tax rate are constant.

$$\lambda(t) = \frac{(1-\tau_p)r_b + \rho_0}{r_s + \rho_0} \tag{C.13}$$

Substitution into the marginal cost of capital leads to

$$q(t) = \left[1 - \frac{\tau_p \nu}{\nu + r_s} - \rho_1 + \frac{(1-\tau_p)r_b + \rho_0}{r_s + \rho_0}\rho_1\right] p(t), \tag{C.14}$$

The first order condition for the marginal product of capital becomes

$$(1-\tau_p)p(t+1)F_{k_e^s}(t+1) = q(t)(1+r_s) - q(t+1)(1-\phi)$$

or

$$F_{k_e^s(t-1)}(t) = \frac{p_{k_e^s}(t)}{p(t)} \tag{C.15}$$

with

$$p_{k_e^s}(t) = \frac{1}{1-\tau_p}\left(\frac{p(t-1)}{p(t)}(1+r_s) - (1-\phi)\right) \times \left[1 - \frac{\tau_p \nu}{\nu + r_s} - \rho_1 + \frac{(1-\tau_p)r_b + \rho_0}{r_s + \rho_0}\rho_1\right] p(t) \tag{C.16}$$

The marginal product of capital is determined by exogenous variables only. So, we can use the marginal productivity equation to determine the capital stock. Labour supply is exogenous. The marginal productivity equation of labour and the production function determine the production level and the wage rate. Assume

$$y_{ge}(t) = \left(\kappa k_e^s(t-1)^{\frac{\sigma-1}{\sigma}} + \theta l_e(t)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{C.17}$$

Substitution of the marginal productivity relations into the first order conditions lead to the factor demand relations

$$l_e(t) = \theta^\sigma \left(\frac{p_{le}(t)}{p(t)}\right)^{-\sigma} y_{ge}(t) \tag{C.18}$$

$$k_e^s(t) = \kappa(t+1)^\sigma \left(\frac{p_{k_e^s}(t+1)}{p(t+1)}\right)^{-\sigma} y_{ge}(t+1) \tag{C.19}$$

Dividing both equations gives the capital equation (2.18) of the main text. Substitution of the factor demand relations into the production function gives

$$\begin{aligned} y_{ge}(t) &= \left(\kappa k_e^s(t-1)^{\frac{\sigma-1}{\sigma}} + \theta l_e(t)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \\ &= \left(\kappa^{\sigma}\left(\frac{p_{k_e^s}(t)}{p(t)}\right)^{-\sigma+1} y_{ge}(t)^{\frac{\sigma-1}{\sigma}} + \theta^{\sigma}\left(\frac{p_{le}(t)}{p(t)}\right)^{-\sigma+1} y_{ge}(t)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \end{aligned} \tag{C.20}$$

Dividing both sides by production and bringing the wage to the left side gives the wage equation (2.21) of the main text. We now derive now a relation between the capital stock and the value of the firm. We make use of the homogeneity of the production function. After some substitutions the first order conditions (C.10) and (C.12) lead to

$$\begin{aligned} &(1+r_s)\left(q(t)k_e^s(t) - \lambda(t)W_{be}^s(t)\right) = \\ &\quad = D_{iv}(t+1) - \tau_p\left(A_f(t+1) - \frac{v}{v+r_s}p(t+1)i_e(t+1)\right) + G_{nt}(t+1) \\ &\quad + q(t+1)k_e^s(t+1) - \lambda(t+1)W_{be}^s(t+1). \end{aligned} \tag{C.21}$$

Forward solution leads to the conclusion that the value of the capital stock minus the value of debt equals the discounted value of the dividend payments (the value of the firm) minus the depreciation allowance on investments installed up to time t and an arbitrary constant

$$q(t)k_e^s(t) - \lambda(t)W_{be}^s(t) = W_{se}^s(t) - \tau_p AF(t) + \sum_{j=1}^{\infty} G_{nt}(t+j)\,(1+r_s)^{-j} \tag{C.22}$$

So for the value of the firm we have the value of the capital stock valued at effective prices minus the value of debt plus the value of the depreciation allowance on investments installed up to time t minus the claims of the government on the firm other than taxes.

$$\begin{aligned} W_{se}^s(t) = &\left[1 - \tau_p\frac{v}{v+r_s} - \rho_1\frac{r_s-(1-\tau_p)r_b}{r_s+\rho_0}\right]p(t)k_e^s(t) - \frac{(1-\tau_p)r_b+\rho_0}{r_s+\rho_0}W_{be}^s(t) \\ &+ \tau_p AF(t) - W_{cg}^s(t) \end{aligned} \tag{C.23}$$

with $W_{cg}^s(t) = \sum_{j=1}^{\infty} G_{nt}(t+j)\,(1+r_s)^{-j}$ the net present value of non-tax government claims on firms.

Appendix D Pension model

We distinguish three private pension schemes. The main text described the average wage defined benefit system, which is representative of most current pension arrangements. This appendix describes the two other schemes that may be implemented in GAMMA, the final wage defined benefit system and the defined contribution system.

D.1 Final wage scheme

In a final wage scheme the level of a pension benefit is determined by the final wage and the number of past working years (l_h). The pension rights build-up for this scheme (W^f_{ph}) equals:

$$W^f_{ph}(j) = \begin{cases} a^f_p l_h(j) H_p(j) & \text{if } j \in \{j_w, \dots, j_r - 1\} \\ W^f_{ph}(j-1)\psi(j) & \text{if } j \in \{j_r, \dots, j_e\} \end{cases} \tag{D.1}$$

with a^f_p the accrual rate of the final wage scheme. Like in the average wage scheme, after retirement the pension build-up grows with the indexation parameter ψ. The nominal pension liabilities (Z^s_p) of the fund to a participant of age j then equal,

$$Z^s_p(j) = W^f_{ph}(j) \sum_{i=\max\{j,j_r\}}^{j_e} d_p(i) \text{ and}$$
$$d_p(i) = [1 + r_b]^{-(i-j)} \prod_{l=j}^{i-1} \zeta(l). \tag{D.2}$$

Otherwise the final wage scheme is the same as the average wage system.

D.2 Defined Contribution (DC) scheme

Now we turn to a DC arrangement. In its most pure form a DC arrangement is characterized by a fixed premium rate and a variable pension benefit. At this moment only four percent of the workers participating in a supplementary pension arrangement have a DC scheme in the Netherlands.

Pension liabilities

By definition in a DC scheme the pension liabilities of the pension fund are equal to the assets. Accordingly, the pension rights of an individual of age j, are equivalent to his accumulated pension contributions. These pension rights (W^d_{ph}) evolve according to the following equation:

$$W^d_{ph}(j) = \frac{(1 + r_p(j))W^d_{ph}(j-1) - B^d_p(j) + \tau^d_p(j)H_p(j)}{\zeta(j)} \qquad j \in \{j_w, \dots, j_e\} \tag{D.3}$$

In GAMMA it is imposed that the assets of deceased individuals belonging to generation j is distributed equally between the survivors of that generation. As a consequence, individuals who survive have more assets than they would if nobody passed away. This explains why the right-hand side of equation (D.3) is divided by the survival probability (ζ).[53]

Contribution rate and pension benefit

The DC scheme of GAMMA is characterized by a fixed premium rate and a variable pension benefit. The contribution rate is determined once, namely at the age an individual enters the labour market (j_w), and from then on this individual always pays this contribution rate until the retirement age (j_r). The determination of the contribution rate is based on a target level for the future pension benefits. This pension target (P_{rt}) is set according to the pension rights build-up of the last working generation (with age $j_r - 1$). That is,

$$P_{rt}(j) = a_p^d H_p^s(j_r - 1) l_h(j_r) m_p^d(j_w) \tag{D.4}$$

with a_p^d the accrual rate in the DC scheme and,

$$m_p^d(j) = \begin{cases} \frac{\zeta(j)\psi(j)}{1+r_p(j+1)} m_p^d(j+1) & \text{if } j \in \{j_w, \dots, j_{r-2}\} \\ \frac{\zeta(j)\psi(j)}{1+r_p(j+1)} \left(1 + m_p^d(j+1)\right) & \text{if } j \in \{j_{r-1}, .., j_{e-1}\} \\ 0 & \text{if } j = e \end{cases} \tag{D.5}$$

the present value of a real pension benefit of one euro.

The total contribution base at the time an individual starts working is equal to his stock of human capital. Human capital (H_p^s) is defined as the present value of the sum of all yearly contribution bases that this individual will earn during her career. That is,

$$H_p^s(j) = \begin{cases} H_p(j) + \frac{\zeta(j)}{1+r_p(j+1)} H_p^s(j+1) & \text{if } j \in \{j_w, \dots, j_{r-1}\} \\ 0 & \text{if } j \in \{j_r, .., j_e\} \end{cases} \tag{D.6}$$

The age-specific contribution rate can now be stated as follows:

$$\tau_{pp}^d(j) = \begin{cases} \frac{P_{rt}(j)}{H_p^s(j)} & \text{if } j = j_w \\ \tau_{pp}^d(j-1) & \text{if } j \in \{j_r, \dots, j_e\} \end{cases} \tag{D.7}$$

At the time an individual retires, the pension fund calculates the actual pension benefit as an indexed annuity. Every year the individual receives this pension benefit (B_p^d) according to the following rule:

$$B_p^d(j) = \frac{W_{ph}^d(j-1)(1 + r_p(j))}{1 + m_p^d(j)}, \qquad j \in \{j_r, \dots, j_e\} \tag{D.8}$$

[53] For the interested reader, Bonenkamp (2006, forthcoming) provides a derivation of equation (D.3).

Appendix E Growth characteristics of the GAMMA model

In this appendix we present three simulations that demonstrate the growth characteristics of the model. The simulations apply shocks to the inflation rate, the productivity growth rate and the population growth rate. We investigate the homogeneity of the model by showing how these rate changes affect the model outcomes in terms of GDP.

E.1 Inflation

Table E.1 shows the effects of an increase in the inflation rate by half a percentage point in 2006. It can be seen that there is no influence from the shock on public expenditures beyond a small decrease in debt service payments arising from a net increase in tax revenues.

Table E.1 Fiscal effects of 0.5 percent more inflation

		2006	2020	2040	2060	2100
		% GDP				
Expenditures						
Social security	(D)	0.0	0.0	0.0	0.0	0.0
Health care	(D)	0.0	0.0	0.0	0.0	0.0
Education	(D)	0.0	0.0	0.0	0.0	0.0
Other expenditures exclusive debt services	(D)	0.1	0.1	0.1	0.1	0.1
Debt services	(D)	0.0	− 0.1	− 0.1	0.0	0.0
Total	(D)	0.1	− 0.1	− 0.1	0.0	0.0
Revenues						
Income taxes and social premiums	(D)	0.2	0.1	− 0.1	− 0.1	− 0.1
- on pension income	(D)	0.0	0.0	− 0.1	0.0	0.0
Indirect taxes	(D)	− 0.4	− 0.1	0.0	0.0	0.0
- on consumption by 65+	(D)	0.0	0.0	0.0	0.0	0.0
Profit taxes	(D)	− 0.1	0.1	0.1	0.1	0.1
Other income	(D)	0.0	0.0	0.0	0.0	0.0
Total	(D)	− 0.3	0.1	0.0	0.0	0.0
Sustainability gap	(D)	− 0.1				
Primary EMU balance	(D)	− 0.4	0.0	− 0.1	0.0	0.0
EMU balance	(D)	− 0.4	0.2	0.0	0.0	0.0
EMU-debt	(D)	− 0.9	− 3.1	− 0.5	1.1	1.8

The small changes in government revenues as percentages of GDP follow two opposing paths. The long run developments in labour income and pension income taxation are explained by the influence of inflation on the user cost of capital.[54] Inflation affects the user cost of capital

[54] Defined in equation 2.19 in section 2.4.2.

through the historical cost price depreciation allowance as well as the structure of debt service deductability. An increase in the inflation rate increases the user cost of capital which, in the long-run, will decrease private sector investment and the capital stock. As a result, real wages decline and so there is a negative effect on wage and pension income tax receipts. This result also explains the negative development of consumption tax receipts due to the positive correlation between consumption and labour income. The movement in profit tax revenue is also explained by the changes in the fiscal depreciation allowance. The real value of private sector profits falls due to the increase in the user cost of capital. However, since the depreciation allowance is based on the price of capital at the time it is purchased, the deduction decreases at a faster rate than profits and the base for corporate taxation grows. The net effect of the changes in government revenue is negligible: the required adjustment decreases by .1% point of GDP as a result of the shock.

Table E.2 Macroeconomic effects of 0.5 percent more inflation

		2006	2020	2040	2060	2100
		% GDP				
Expenditures						
Wage income	(D)	− 0.1	− 0.1	− 0.1	− 0.1	− 0.1
Net other income	(D)	0.4	0.2	0.2	0.2	0.2
Depreciation	(D)	0.1	− 0.1	− 0.1	− 0.1	− 0.1
Indirect taxes less subsidies	(D)	− 0.4	0.0	0.0	0.0	0.0
Gross domestic product	(D)	0.0	0.0	0.0	0.0	0.0
Components of national consumption						
Private consumption	(D)	− 0.2	− 0.2	− 0.1	− 0.1	− 0.1
Government consumption	(D)	0.1	0.1	0.1	0.1	0.0
Corporate investment	(D)	− 3.2	− 0.1	− 0.1	− 0.1	− 0.1
Government investment	(D)	0.0	0.0	0.0	0.0	0.0
National consumption	(D)	− 3.3	− 0.2	− 0.2	− 0.2	− 0.2
Balance of current foreign transactions	(D)	3.0	0.4	0.5	0.5	0.5
Net foreign assets [a]	(D)	1.8	5.5	6.5	6.8	7.2
Gross national product [b]	(D)	− 0.2	0.2	0.3	0.3	0.3
		(level)				
Production (GDP)	(%)	− 0.6	− 0.5	− 0.5	− 0.5	− 0.5
Employment	(%)	− 0.2	0.0	− 0.1	− 0.1	− 0.1

[a] Value at the end of the year
[b] GDP plus balance of primary revenues from abroad

In the short run the temporary increase in revenue from wage taxes is notable. The wage decline leads to lower pension fund liabilities and, therefore, lower premiums. On balance the wage tax base increases temporarily.

These effects are also apparent in the development of macroeconomic variables. Table E.2

shows a change in the composition of GDP. The decline in the absolute levels of wage income and depreciation arising from lower capital stock levels is compensated by the residual category ‘net other income’. Indeed, net other income increases on balance of the user cost of capital increase and the capital stock decline due to a subsitution elasticity between capital and labour lower than one The combined effects of lower household consumption and corporate investment due to lower wages and a higher user cost of capital result in a decrease in national consumption and, hence, an increase in the balance of trade surplus.

E.2 Productivity growth

The second variant in Table E.3 shows the effects of growth in labour productivity pegged at 0.5% points higher than in the baseline projection. The structural growth of labour productivity will not affect labour supply because the utility parameters will adjust in line with the increase.[55] This variant pushes up the extent of the corrective budgetary measures required in 2006 by about

Table E.3 Fiscal effects of a 0.5 percent larger productivity growth

		2006	2020	2040	2060	2100
		% GDP				
Expenditures						
Social security	(D)	0.0	0.0	0.0	0.0	0.0
Health care	(D)	0.0	0.0	0.0	0.0	0.0
Education	(D)	0.0	0.0	0.0	0.0	0.0
Other expenditures exclusive debt services	(D)	− 1.1	− 1.0	− 0.9	− 0.9	− 0.9
Debt services	(D)	0.0	− 0.6	− 0.5	− 0.3	− 0.2
Total	(D)	− 1.2	− 1.5	− 1.3	− 1.2	− 1.0
Revenues						
Income taxes and social premiums	(D)	− 0.1	− 0.3	− 0.6	− 0.6	− 0.5
- on pension income	(D)	0.0	− 0.1	− 0.2	− 0.1	− 0.1
Indirect taxes	(D)	0.6	0.1	− 0.1	− 0.1	− 0.1
- on consumption by 65+	(D)	0.0	− 0.1	− 0.3	− 0.3	− 0.3
Profit taxes	(D)	0.0	− 0.1	− 0.1	− 0.1	− 0.1
Other income	(D)	0.0	− 0.1	− 0.1	− 0.1	− 0.1
Total	(D)	0.4	− 0.5	− 0.9	− 0.9	− 0.8
Sustainability gap	(D)	1.0				
Primary EMU balance	(D)	1.6	0.5	− 0.1	− 0.1	0.0
EMU balance	(D)	1.6	1.0	0.4	0.3	0.1
EMU-debt	(D)	− 2.1	− 15.6	− 14.5	− 9.6	− 4.7

1% of GDP. It can be seen that there is no long-run influence from the shock on the relative development of public expenditures beyond an increase in debt service payments arising from a

[55] Note 15 details on this assumption.

net decrease in tax revenues. Indeed, all government expenditures rise with productivity and so will stay in constant proportion to GDP. However, several revenue sources lag behind GDP development, in particular income taxes.

Within income taxes, the tax on income from financial wealth explains about half of the decrease. Financial wealth declines relatively because, without liquidity constraints, consumption takes place out of total wealth, the sum of financial wealth and human wealth. A rise of human wealth due to increased productivity growth leads to more consumption when households are young and so as a result, there is a relative decline in financial wealth. The labour income tax in percentages of GDP also declines. The labour income tax deductibility of the pension premiums is the origin of this development. The pension premium rate has to rise because second pillar pensions are funded. In case of a permanent productivity growth rise, future pensions rise more than the wages out of which the pensions have to be financed. This pension premium rise would not occur in a pay-as-you-go pension system but is characteristic for a funded pension system.

Table E.4 Macroeconomic effects of a 0.5 percent larger productivity growth

		2006	2020	2040	2060	2100
		% GDP				
Expenditures						
Wage income	(D)	− 0.2	0.0	0.1	0.1	0.1
Net other income	(D)	− 0.2	0.0	0.0	0.1	0.1
Depreciation	(D)	− 0.1	− 0.1	− 0.1	− 0.1	− 0.1
Indirect taxes less subsidies	(D)	0.5	0.1	− 0.1	− 0.1	− 0.1
Gross domestic product	(D)	0.0	0.0	0.0	0.0	0.0
Components of national consumption						
Private consumption	(D)	0.6	− 0.4	− 1.1	− 1.1	− 1.1
Government consumption	(D)	− 1.1	− 1.0	− 0.9	− 0.9	− 0.9
Corporate investment	(D)	3.4	1.7	1.8	1.8	1.8
Government investment	(D)	0.0	0.0	0.0	0.0	0.0
National consumption	(D)	2.9	0.4	− 0.1	− 0.1	− 0.1
Balance of current foreign transactions	(D)	− 2.8	− 0.6	− 0.1	0.0	0.1
Net foreign assets [a]	(D)	− 1.1	− 9.6	− 14.3	− 11.8	− 8.7
Gross national product [b]	(D)	0.1	− 0.1	− 0.2	− 0.2	− 0.1
		(level)				
Production (GDP)	(%)	0.9	7.6	18.4	30.6	58.8
Employment	(%)	0.0	0.1	0.0	0.0	− 0.1

[a] Value at the end of the year
[b] GDP plus balance of primary revenues from abroad

The other revenues stay more in line with the development of GDP. The indirect tax and profit tax decreases are rather small. However, in the case of indirect taxes this is the result of two counterbalancing effects: indirect taxes on investment increase, while indirect taxes on consumption decrease. More precisely, indirect taxes on consumption decrease relative to GDP in the long run, which is relevant for the sustainability, but rise in the short run. This is caused by consumption smoothing which leads to frontloading: households anticipate future income rises by increasing their consumption earlier. The investment tax rise and profit tax decline are caused by an investment ratio rise. Note that the tax base for the profit tax is profits net of depreciation allowances and net of debt service payments, both of which are linked to new investments.

Table E.4 shows the macroeconomic effects of a 0.5% point higher productivity growth. With regard to the long-run, the larger investment ratio is notable. Higher labour augmenting technological progress encourages investment. Indeed, the investment ratio is proportional to the growth rate plus the depreciation rate. The development of the consumption ratio is explained above.

So the labour productivity increases have a negative impact on sustainability, due to the life cycle behaviour of agents and due to the funded second pillar pension system.

E.3 Population growth

The third variant in Table E.5 shows the effects of a 0.5 percentage point higher population growth. The population growth is established by net migration such that the dependency ratio is not affected.

This variant pushes up the extent of the measures required in 2006 by about 0.3% of GDP. It can be seen that there is no long-run influence from the shock on the relative development of public expenditures beyond an increase in debt service arising from a net decrease in tax revenues. Indeed, all government expenditures rise with the population growth and stay in constant proportion to GDP.

A larger population implies more employment, which induces more investments which increase indirect tax revenues. However, this favourable development for government finances is counterbalanced by the profit tax development. The tax base for profit taxes in percentage of GDP shrinks through the fiscal depreciation allowance and the deductibility of the increased debt, which is used to finance investments. The labour income tax in percentage of GDP also declines. The labour income tax deductibility of the pension premiums is the origin of this development. Pension premiums increase because migrants buy into the pension fund according to the nominal pension rights while they receive real pension rights. This leads to a decline in the coverage rate and thus to larger pension premiums.

Table E.6 shows the main macro economic effects: a relative increase of the investments and a relative decrease of consumption. More growth means relative more investments. Consumption declines relative to GDP due to the larger pension premiums.

Table E.5 Fiscal effects of 0.5 percent larger population growth

		2006	2020	2040	2060	2100
		% GDP				
Expenditures						
Social security	(D)	0.0	0.0	0.0	0.0	0.0
Health care	(D)	0.0	0.0	0.0	0.0	0.0
Education	(D)	0.0	0.0	0.0	0.0	0.0
Other expenditures exclusive debt services	(D)	− 0.3	− 0.3	− 0.3	− 0.3	− 0.3
Debt services	(D)	0.0	− 0.3	− 0.2	− 0.2	− 0.1
Total	(D)	− 0.2	− 0.6	− 0.5	− 0.5	− 0.4
Revenues						
Income taxes and social premiums	(D)	0.0	0.0	− 0.2	− 0.1	− 0.1
- on pension income	(D)	0.0	0.0	0.0	0.0	0.0
Indirect taxes	(D)	− 0.1	0.1	0.2	0.2	0.1
- on consumption by 65+	(D)	0.0	0.0	0.0	0.0	0.0
Profit taxes	(D)	0.0	− 0.1	− 0.1	− 0.1	− 0.1
Other income	(D)	0.0	− 0.1	− 0.1	− 0.1	− 0.1
Total	(D)	0.0	0.0	− 0.3	− 0.2	− 0.2
Sustainability gap	(D)	0.3				
Primary EMU balance	(D)	0.2	0.3	0.0	0.0	0.1
EMU balance	(D)	0.2	0.6	0.2	0.2	0.2
EMU-debt	(D)	− 0.2	− 7.5	− 7.1	− 5.6	− 4.2

E.4 Homogeneity of the Model

Figure E.1 illustrates the homogeneity of the GAMMA model. For each of the three simulations presented above, the difference from the baseline scenario in the stock of net foreign assets as percentages GDP are plotted.[56] Homogeneity requires that the growth in macroeconomic variables as percentages of GDP stabilize at constant levels in the long run. As the figure demonstrates, this condition is satisfied. While there are initial deviations in the asset levels with respect to GDP relative to the baseline, after 2106 the differences stabilize in all three scenarios. Thus the long-run growth rates are unaffected.

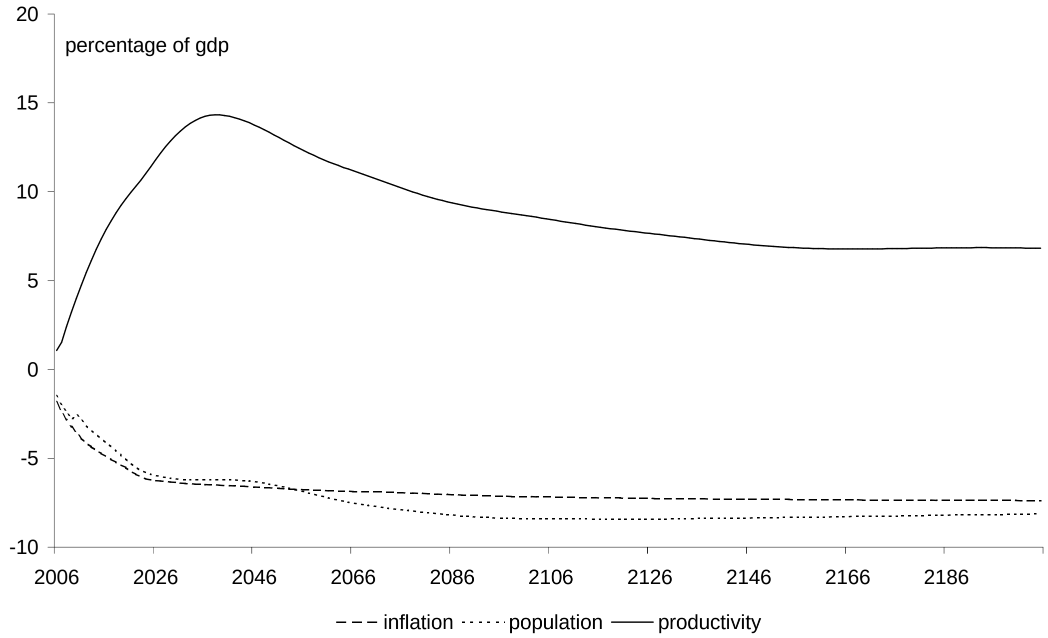
[56] Any one of several macroeconomic variables would make this point. We chose 'net foreign asset holdings' because its development is not directly affected by the fiscal sustainability mechanism that is applied to the simulations.

Table E.6 Macroeconomic effects of a 0.5 percent larger population growth

		2006	2020	2040	2060	2100
		% GDP				
Expenditures						
Wage income	(D)	0.0	0.0	0.0	0.0	0.0
Net other income	(D)	0.0	0.0	0.0	0.0	0.0
Depreciation	(D)	0.0	− 0.1	− 0.1	− 0.1	− 0.1
Indirect taxes less subsidies	(D)	0.0	0.1	0.1	0.1	0.1
Gross domestic product	(D)	0.0	0.0	0.0	0.0	0.0
Components of national consumption						
Private consumption	(D)	− 0.1	− 0.2	− 0.2	− 0.2	− 0.2
Government consumption	(D)	− 0.3	− 0.3	− 0.3	− 0.3	− 0.3
Corporate investment	(D)	− 0.1	1.7	1.8	1.8	1.8
Government investment	(D)	0.0	0.0	0.0	0.0	0.0
National consumption	(D)	− 0.5	1.2	1.3	1.3	1.3
Balance of current foreign transactions	(D)	0.6	− 1.0	− 1.1	− 1.0	− 1.0
Net foreign assets [a]	(D)	1.5	5.0	6.2	7.2	8.4
Gross national product [b]	(D)	0.0	0.2	0.2	0.3	0.3
		(level)				
Production (GDP)	(%)	− 0.1	5.2	16.0	28.2	56.5
Employment	(%)	0.0	5.1	16.1	28.2	56.5

[a] Value at the end of the year
[b] GDP plus balance of primary revenues from abroad

Figure E.1 Difference in net foreign assets holdings in the growth rate shock scenarios relative to the baseline



Appendix F Symbols

In the text we used upper case letters for values and lower case letters for volumes. We use suffixes as indicators for variables that refer to specific time periods or ages. For individual variables we use only the age suffix j, for intergenerational variables we use both the age suffix j and the time suffix t, for aggregated (macro) variables we use only time suffix t. At the individual level time and age are related on a one-to-one basis. So using the age indicator j is sufficient. Stocks are measured at the end of the period and are indicated with a superscript s.

a_p^i pension accrual rate $i = a$ (average wage DB), f (final wage DB), d (DC)
A_f fiscal depreciation allowance
AF value of fiscal depreciation on existing capital stock

b total benefits received from the government
b_{ca} children's assistance transfers
b_{da} disability transfers
b_e public outlays on education
b_{er} early retirment benefits
b_{gp} public pension benefits
b_h public outlays on health care
b_i age-related benefits from the government
b_n net benefits of households from the public sector
b_n^s net present value of lifetime household benefits from the public sector
b_{ot} transfers other than pensions, disability, unemployment and social assistance
b_p^i private pension benefits, $i = a, f, d$
b_{sa} social assistance
b_{un} public unemployment transfer

c_h household goods consumption
c_g aggregate public consumption
c_l labour induced commodity consumption$d$:

d_{iv} dividends
d_n discount factor for net benefits
d_o objective discount factor household optimaization problem
d_p objective discount factor on pension liabilities
d_{pv}^i discount factor on pensions in the price of leisure $i = a, f, d$
d_s subjective discount factor

e_f labour efficiency index
EV equivalent variation
e_x net exports

f_p franchise for pension premium payments

g growth rate of GDP
g_{as} public revenues from natural gas
g_{cb} income of the Central Bank
g_{nt} payments of firms to the government other than taxes and dividends
g_{pg} government income from the lease of public grounds

h_p premium base for private pension premiums
h_p^s the discounted value of the premium base - DC system
h_t tax base

i total gross domestic investments, exclusive clearing of land for building
i_e gross investments of the private sector
i_g gross public investments

k total domestic capital stock
k_e^s capital stock of enterprises
k_g^s aggregate public capital stock

l_d labour demand enterprises
l_e employment in efficiency units enterprises
l_h labour history (passed working years)

m_p^d weighted number of pensionable years

n_g^s cohort population size $g = f$ (females), m (males), h (females and males)
n_i immigrants

p composite good price
p_c goods consumption price
p_{er} early retirement premiums
p_k user cost of capital
p_l wage rate per worker of a certain age

p_{le} macro wage per worker in efficiency units

p_{lg} gross wage of households

p_p^i pension premiums, $i = a, f, d$

p_{rt} pension rights target defined contribution scheme

$\widetilde{p}_v$ unrestricted price of leisure

p_v price of leisure

p_{v1} price of leisure attributable to net wages

p_{v2} price of leisure attributable to future pension rights

p_W price of wealth

p_x total consumption price

q Tobin's q

q_d mortality rate

q_{dg}^s debt to gdp ratio

q_{dg} primary deficit ratio

q_e emigration rate

q_f funding ratio of the pension fund

q_i immigration rate

q_s sustainability gap

r_b rate of return on bonds

r_{ep} equity premium

r_g interest rate on government debt

r_h interest rate on household wealth

r_p rate of return pension funds

r_s rate of return on shares

s_{vf} foreign sector savings

s_{vg} government savings

s_{vh} household's savings

s_{vp} private pension fund savings

t_{er} taxes paid on early retirement benefits

t_{in} indirect taxes minus subsidies

t_k capital income tax

t_l labour income tax

t_p profit tax

t_{pi} tax on private pension benefits

t_r pension income tax (private plus public)
t_s seigniorage
t_{sb} taxes on social benefits
t_{tb} total burden from the government
t_y direct income tax on households

u per-period direct utility
U expected lifetime utility
U' lifetime household utility, transformed

v household leisure
$\widetilde{v}$ unrestricted demand for leisure

W^s total lifetime wealth households
$\widetilde{W^s}$ total lifetime wealth less the present value of future labour induced total consumption
w_{be} bonds issued by firms
w_{bf} bonds owned by foreigners
w_{bg} bonds issued by the government net of bonds owned by the government
w_{bh} bonds owned by households
w_{bp} bonds owned by pension funds
w_{cg} claims of the government on firms
w_{ef} change in assets through emigration
w_{eh} change in household assets through emigration
w_{ep} change in pension sector's assets through emigration
w_{fc} total assets
w_{fe} financial wealth of firms
w_{ff} aggregate foreign asset holdings
w_{fg} aggregate government asset holdings
w_{fh} aggregate household asset holdings
w_{fp} aggregate pension fund assets holdings
w_{fph} household lifetime financial wealth (including pension rights)
w_{if} change in assets through immigration
w_{ih} change household assets through immigration
w_{ip} change in pension sector assets through immigration
w_{ng} net wealth of the government
w^i_{ph} pension rights $i = a, f, d$
w_{re} revaluation of assets owned by firms
w_{res} firms' reserves

w_{rf} revaluation of assets owned by foreigners
w_{rg} revaluation of government assets
w_{rh} revaluation of assets owned by households
w_{rp} revaluation of assets owned by pension funds
w_{se} total shares issued by domestic private firms
w_{sf} shares owned by foreigners
w_{sg} shares owned by the government
w_{sh} shares owned by households
w_{sp} shares owned by pension funds

x_a public outlays on public administration
x_d public outlays on defence
x_f total consumption above labour induced total consumption
x_{fo} public transfers to foreigners
x_h total household consumption
x_i non-age related benefits from the government
x_l labour induced total consumption
x_r public interest payments
x_{su} government subsidies

y total after tax income households
y_f full, after-tax non-capital income of households
y_g gross domestic product in factor prices
y_{ge} gross domestic product of the private sector in factor prices
y_{gg} gross domestic product of the government
y_{go} exogenous production of firms in the base year
y_p after-tax pension income households
y_{p1} pension income linked to labour before current year
y_{p2} pension income linked to current and future labour
y_{tn} after-tax transfer income
y_t individual transfer income
y_{we} wage sum of enterprises
y_{wh} gross wage income households
y_{wg} wage sum of the government
y_{wn} after-tax wage income households
y_{ze} dividend payments, debt interest payments, central bank profits and land lease payments of firms
y_{zf} foreign capital income

y_{zg} government capital income
y_{zh} household capital income
y_{zp} pension fund capital income

z_p nominal liabilities of the pension fund

Greek symbols :
α_c taste shift parameter - goods consumption
α_v taste shift parameter - leisure consumption
β inverse of the price elasticity of leisure demand
γ inverse of the elasticity of intertemporal substitution
δ pure rate of time preference
ε fraction of workers with supplementary pension arrangement
ζ survival probability
θ share of labour in production
κ share of capital in production
μ shadow price of leisure
ν fiscal depreciation rate of firms
ξ survival rates corrected for immigration and emigration
ρ_0 fraction of firm debt repaid in each period
ρ_1 fraction of new investment financed with new debt
σ substitution elasticity between capital and labour
τ tax or premium rate
τ_{cu}^i catching-up premium rate $i = a, f$
τ_{er} early retirement (VUT) premium rate
τ_{gp} public pension premium rate
τ_{li} tax rate on labour income
τ_p corporate tax rate
τ_{pp}^i pension premium rate $i = a, f, d$
τ_r tax rate on public and private pension benefits
τ_{sb} tax rate on social benefits
τ_{uc}^i uniform contribution rate $i = a, f$
τ_y average tax rate on total income
ϕ technical rate of capital deterioration
ψ indexation factor for DB pension rights

References

Alho, J. M. and B. Spencer, 1997, Practical specification of the expected error of population forecasts, *Journal of Official Statistics*, 13:203–225.

Altig, D., A. J. Auerbach, L. J. Kotlikoff, K. A. Smetters, and J. Walliser, 2001, Simulating fundamental tax reform in the United States, *American Economic Review*, 91(3):574–595.

Armstrong, A., D. Draper, and E. Westerhout, 2007, *Uncertain Demographics and Fiscal Sustainability*, chapter Ageing, Demographic Uncertainty and Optimal Tax Policy, Cambridge University Press.

Auerbach, A. and L. Kotlikoff, 1987, *Dynamic fiscal policy*, Cambridge University Press.

Auerbach, J. G., A.J. and L. Kotlikoff, 1991, *Tax Policy and the Economy 5*, chapter Generational Accounts: A Meaningful Alternative to Deficit Accounting, 55–110, Cambridge, MA, MIT Press.

Auerbach, L. K., A.J. and W. Leibfritz, 1999, *Generational Accounting Around the World*, The University of Chicago Press.

Barro, R. J., 1979, On determination of the public debt, *Journal of Political Economy*, 87(5):940–971.

Blundell, R. and T. Macurdy, 1999, *Handbook of Labor Economics,*, volume 3, chapter Labor Supply: a Review of Alternative Approaches, North Holland.

Bonenkamp, J., 2005, A comparison of catching-up premium rate models, CPB Memorandum 127.

Borsch-Supan, A., A. Ludwig, and J. Winter, 2006, Ageing, pension reform and capital flows: A multi-country simulation model, *Economica*, 73(292):625–659.

Bovenberg, A. and T. Knaap, 2005, Ageing, funded pensions and the Dutch economy, Working Paper 1403, CESifo.

Bovenberg, A. and H. ter Rele, 2000, Generational accounts for the Netherlands: An update, *International Tax and Public Finance, Special Issue on Public Finance and Transitions in Social*

Security, 411–430.

Broer, D., 2001, Growth and welfare distribution in an ageing society: an applied general equilibrium analysis for the Netherlands, *De Economist*, 149:53–79.

Buiter, W., 1995, Do generational accounts reveal the effect of the budget on savings and intergenerational redistribution?, NBER working paper 5087.

De Jong, A., 2005a, Bevolkingsprognose 2004-2050: maximaal 17 miljoen inwoners, *Bevolkingstrends, Statistisch kwartaalblad over de demografie van Nederland*, 53(1):12–18.

De Jong, A., 2005b, Bevolkingsprognose 2004-2050: Veronderstellingen, *Bevolkingstrends, Statistisch kwartaalblad over de demografie van Nederland*, 53(1):19–23.

De Jong, A., 2005c, Bevolkingsprognose 2004-2050: Veronderstellingen over immigratie, *Bevolkingstrends, Statistisch kwartaalblad over de demografie van Nederland*, 53(2):33–38.

Draper, D. and F. Huizinga, 2001, The effect of corporate taxes on investment and the capital stock, CPB Memorandum 13.

Epstein, L. G. and S. E. Zin, 1991, Substitution, risk aversion, and the temporal behavior of consumption and asset returns: An empirical analysis, *Journal of Political Economy*, 99(2):263–286.

European Commission, 2006, *The Long Term Sustainability of Public Finances in the European Union*, Brussels.

Euwals, R. and D. van Vuuren, 2005, Arbeidsaanbod tot 2050: een beleidsneutraal scenario, CPB Memorandum 134.

Evers, M., R. de Mooij, and D. van Vuuren, 2005, What explains the variation in estimates of labour supply elasticities?, CPB Discussion Paper 51.

Fehr, H. and L. J. Kotlikoff, 1997, Generational accounting in general equilibrium, NBER working paper W5090.

Flavin, M. A., 1981, The adjustment of consumption to changing expectations about future income, *Journal of Political Economy*, 89(5):974–1009.

Geary, R. C., 1950, A note on "a constant-utility index of the cost of living", *Review of Economic Studies*, 18(1):65–66.

Gelauff, G. and J. Graafland, 1994, *Modelling Welfare State Reform*, North-Holland.

Greenwood, J., Z. Hercowitz, and G. W. Huffman, 1988, Investment, capacity utilization, and the real business cycle, *American Economic Review*, 78(3):402–417.

Guest, R., 2006, Population ageing, fiscal pressure and tax smoothing: A CGE application to Australia, *Fiscal Studies*, 27:183–203.

Haveman, R., 1994, Should generational accounts replace public budgets and deficits?, *Journal of Economic Perspectives*, 8(1):95–111.

Heijdra, B. J., 1998, Fiscal policy multipliers: The role of monopolistic competition, scale economies,and intertemporal substitution in labour supply, *International Economic Review*, 39(3):659–696.

INGENUE, 2005, Scenarios for global ageing: An investigation with the INGENUE 2 model, Technical report, ENEPRI Research Report No. 9.

Jensen, S. E. H., U. Nødgaard, and L. H. Pedersen, 2002, Fiscal sustainability and generational burden sharing in denmark, *Nordic Journal of Political Economy*, 28:43–60.

Jones, C., 2002, Why have health expenditures as a share of GDP risen so much?, NBER Working Paper 9325.

King, R. G., C. I. Plosser, and S. T. Rebelo, 1988a, Production, growth and business cycles : I. the basic neoclassical model, *Journal of Monetary Economics*, 21(2-3):195–232.

King, R. G., C. I. Plosser, and S. T. Rebelo, 1988b, Production, growth and business cycles : Ii. new directions, *Journal of Monetary Economics*, 21:309–341.

Kingston, G., 1991, Should marginal tax rates be equalized through time?, *Quarterly Journal of Economics*, 106(3):911–924.

Lassila, J., 1999, An overlapping-generations simulation model for the Lithuanian economy, ETLA discussion papers.

Lassila, J. and T. Valkonen, 2001, Pension prefunding, ageing, and demographic uncertainty, *International Tax and Public Finance*, 8:573–593.

Lumsdaine, R. and O. Mitchell, 1999, *Handbook of Labor Economics*, volume 3, chapter New Developments in the Economic Analysis of Retirement, Elsevier Science.

Mathur, A. and K. A. Hassett, 2006, Taxes and wages, Technical report, AEI, American Enterprise Institue for Public Policy Research.

Polder, J. and P. Achterberg, 2004, Cost of illness in the Netherlands; highlights, Technical report, RIVM, Bilthoven.

Raffelhueschen, B., 1999a, Generational accounting in Europe, *American Economic Review - AEA Papers and Proceedings*, 89:167–170.

Raffelhueschen, B., 1999b, Generational accounting: Method, data and limitations, in *Generational Accounting in Europe, Brussels*, European Commission.

Ree, J. d. and R. Alessie, 2007, Estimating a life cycle model of female and no-durable consumption, Technical report, Utrecht University, forthcoming.

Roos, N., P. Montgomery, and L. L. Roos, 1987, Health care utilisation in the years prior to death, *The Milbank Quarterly*, 65:231–254.

SCP, 1994, *Sociaal Cultureel Rapport 1994*, SCP.

Seshamani, M. and A. Gray, 2004, A longitudinal study of the effects of age and time to death on hospitalcosts, *Journal of Health Economics*, 23:217–235.

Stone, R., 1954, Linear expenditure systems and demand analysis: An application to the pattern of British demand, *The Economic Journal*, 64(255):511–527.

Ter Rele, H., 1998, Generational accounts for the Netherlands, *De Economist*, 146:555–584.

Ter Rele, H., 2003, The fiscal impact of immigration, *CPB Report*, 2.

Van Ewijk, C., D. Draper, H. ter Rele, and E. Westerhout, 2006, *Ageing and the sustainability of Dutch public finances*, CPB, Netherlands Bureau for Economic Policy Analysis, The Hague, CPB Special Publication 61.

Van 't Veer, O., 2006, Solving large scale normalised rational expectations models, CPB Discussion Paper 54, CPB.

Westerhout, E., M. van de Ven, C. van Ewijk, and D. Draper, 2004, Naar een schokbestendig pensioenstelsel; verkenning van enkele beleidsopties op pensioengebied, CPB Document 67.

WRR, 1997, Volksgezondheidszorg, Rapporten aan de Regering 52, Wetenschappelijke Raad voor het Regeringsbeleid, The Hague.

Yaari, M. E., 1965, Uncertain lifetime, life insurance, and the theory of the consumer, *The Review of Economic Studies*, 32(2):137–150.