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A small stochastic model of a pension fund with endogenous saving[1]

In this paper we investigate whether uncertainty on the real rate of return on capital and productivity growth (labelled as economic uncertainty) is more or less important than mortality and fertility uncertainty (labelled as demographic uncertainty) for a consumer facing a decision how much to save. Furthermore we look at the errors that are made when uncertainty is neglected in consumer behaviour. The results indicate that economic uncertainty is far more important than demographic uncertainty. The welfare costs of neglecting uncertainty in consumer behaviour seem to be small.

Keywords: economic uncertainty, demographic uncertainty, certainty equivalent
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Contents

1 Introduction

We live in an uncertain world, yet a lot of research into the sustainability of welfare states is done in the context of certainty. There are good reasons why the analysis is mostly confined to a model of a certain world. A full analysis of the sustainability of welfare states which includes all relevant economic interactions is already intricate in a certain world because it requires the use of complex dynamic general equilibrium models. Even without stochastics, understanding all the mechanisms and its results is sometimes difficult. In addition, when building stochastics into these type of models one may run into the limitations of computer capacity.

This paper explores, at a very general level, the consequences of a stochastic environment for sustainability and economic behaviour. We will use of rather simple setting of only two overlapping generations where consumers only work the first period of their lives and are retired in the second period. Labour supply is exogenous. Hence, consumer behaviour here is restricted to the choice of private savings in the first period. When choosing how much to save, consumers take into account the contributions they have to pay to a mandatory pension fund and the benefits they expect to receive from this pension fund. And, of course, they take into account the economic and demographic uncertainty they face. Even in this simple setting, we have to rely on (stochastic) simulations to derive results.

This research has been done as part of a large project called DEMWEL where economic analysis of population ageing is combined with statistical analysis of demographic uncertainty. One of the parts of the project is to explore on a general level the value added of the tool of stochastic simulations. Two main questions lay the foundation of this paper. First, which source of uncertainty is more important for the savings decision of an optimising consumer, economic uncertainty or demographic uncertainty? Second, what is the impact of both sources of uncertainty on consumer behaviour? Or put it differently, what is the error economic agents make when uncertainty is neglected in consumer behaviour.

We distinguish four sources of economic uncertainty. First, there is uncertain productivity growth. In our model, productivity growth affects consumer behaviour via the pension fund. The other three sources of economic uncertainty are bond returns, equity returns and inflation rates. Together they determine the real portfolio return of private savings and mandatory pension savings. For simplicity we impose that there is no endogenous portfolio selection for the pension fund and consumers.

Demographic uncertainty comes from two sources: aggregate fertility risks and aggregate mortality risks. Cross-sectional mortality risks are assumed to be fully insurable. Examples of possible aggregate mortality risks are the discovery of a cure against cancer which implies a higher life expectancy (which is good news for individuals but bad news for pension funds and life insurance companies). An aggregate mortality risk in the other direction is the mutation of the bird flu into a human variant. In so far the decline in fertility rates in the previous century was unforeseen, it is an example of an aggregate fertility risk.

We will show that the exact behavioural effects of all these sources of uncertainty depend on the type of pension scheme. We distinguish four types of pension schemes that can labelled along two dimensions. The first dimension is the distinction between defined benefit (DB) and defined contribution (DC). The second dimension deals with the way pension benefits are financed; that is, by funding or at a pay-as-you-go (PAYG) basis.

Assessing the the importance of economic uncertainty vis-à-vis demographic uncertainty, the results indicate that for a consumer facing both types of uncertainty, it is economic uncertainty that matters the most. As to the second question, the consequences for behaviour of including uncertainty, the results suggest that the average welfare costs of not taking uncertainty into account are fairly low.

The rest of the paper is organised as follows. Section 2 gives an introductory discussion about the potential advantages and limitations of stochastic simulations. Also some related literature is discussed. Section 3 presents the model used which is a simple two-period overlapping generations model of a small open economy with consumers and a mandatory pension fund. Section 4 presents some analytical results regarding consumption and saving. In section 5 we discuss the characteristics of the random variables and analyse the impact of deterministic shocks in these variables on consumer behaviour. In section 6 we turn to stochastic simulations and try to answer the main questions raised above. Finally, section 7 concludes with a discussion of the results and some suggestions for future research.

2 The advantages and limitations of stochastic simulations

Uncertainties are all around. We can use a number of classifications. First, uncertainties can be demographic or non-demographic, like economic or financial types of uncertainty. Second, uncertainties can apply to different future periods. Indeed, uncertainties on financial markets are more short term, whereas demographic uncertainties typically apply to longer terms. A third distinction is the level at which uncertainties are relevant. Some uncertainties are especially relevant at the microeconomic level and play less of a role at a macroeconomic level; others are relevant at both levels. Uncertainties are even not confined to the values of variables. Indeed, the values of model parameters may be uncertain as well and the same applies even to the type of model that describes the economy. In applied work, one often faces different types of uncertainty simultaneously. For example, in making an assessment of the viability of welfare state programs with an economic model, one uses a specific model, combined with specific parameter values and specific time paths for demographic and non-demographic variables.

The scope of the research in this paper is much more moderate. It takes as given the model that is used to describe the economy and also the specific values of model parameters. It focuses on aggregate shocks. It models a variety of shocks and basically asks two questions. First, what is the role of demographic uncertainties relative to non-demographic uncertainties? Second, do predictions on the impact of demographic and non-demographic shocks change when the analysis accounts for an impact of uncertainty on consumers saving behaviour?

Traditionally, we adopt scenario analysis or sensitivity analysis to demonstrate the impact of uncertainties. In sensitivity analysis, we change the time path of one exogenous variable at a time in order to see the robustness of outcomes with respect to that particular variable. In scenario analysis, we do something similar, but change two or more exogenous variables simultaneously. These two types of uncertainty analysis have in common that they tend to use a small number of variants: an analysis may contain not more than five or ten sensitivity variants, whereas scenarios usually come in much smaller numbers.

Stochastic simulation analysis differs from the other two types of uncertainty analysis in the number of simulations or variants. Indeed, stochastic simulations use hundreds or thousands of simulations. Therefore, stochastic simulation analysis allows one to depict the full distribution of the endogenous variables, which may be more informative than one or two points of that distribution.

Obviously, stochastic simulations take more time. Hence, it is good to ask beforehand what precisely are the benefits one can expect to reap from the tool of stochastic simulation analysis? We argue that there are two types of benefits. The first relates to a better exploitation of available information. We propose to call this descriptive benefits. The second refers to the possibility to make model adjustments on the basis of the information provided by stochastic simulation analysis. We propose to call the corresponding benefits analytical benefits. Let us start with the descriptive benefits. We think of four types of descriptive benefits:

1. The average values of endogenous variables that follow from a stochastic simulation analysis may differ from the values that correspond with the average simulation, i.e. the simulation that uses the average values taken by the exogenous variables in the model. The means of a stochastic simulation exercise and the outcome of the path that corresponds to the means of exogenous variables will always be different in case there are non-linearities in the model. The interesting question is whether these non-linearities are such that a simple simulation of the average path gives a completely false picture.
2. Stochastic simulations give an idea about the whole distribution, i.e. its location, its variance, its higher moments. They allow for calculating quintiles, quartiles etc. and are therefore more informative than traditional projection analysis. Note that the distribution of the variables itself may be relevant to policymakers. There are examples of policy reforms that are driven by the aim to change the variance rather than the mean of some variables. Think of policies that aim to reduce the vulnerability of pensions to shocks in life expectancy, like in Finland, or policies that aim to reduce the impact of inflation shocks on the government budget (indexed bonds). In the pension sphere, one may think of portfolio-allocation policies (asset-liability matching). Traditional projection analysis does not assess the spread of possible outcomes and cannot be informative about this aspect of government policies.
3. Stochastic simulation analysis allows an assessment of the plausibility of typical scenarios and typical projections (Lee and Edwards (2002)). This may be helpful in defining scenarios. Indeed, given the information that is provided by stochastic simulation analysis, it is possible to select scenarios on the condition that they are more or less equally plausible. The same applies to sensitivity analysis. Stochastic simulation analysis makes it possible to make shocks in different exogenous variables more comparable by quantifying their standard deviations. This is helpful in choosing the size of shocks in exogenous variables.
4. Stochastic simulation analysis also allows to account for correlations between innovations in different variables which may be fundamental in assessing the distribution of those endogenous variables that are driven by many correlated exogenous variables. In a different dimension, stochastic simulation analysis can account for serial correlations in variables and therefore study a variety of shocks that range from temporary to permanent. Both types of correlations may be relevant in projection studies (Lee and Tuljapurkar (2001)).

On the downside, as said, stochastic simulations take more time. If the model at hand is rather simple, this argument may have little relevance. But if the model used consists of thousands of equations and a number of simultaneities, then it may become hard to do stochastic simulation analysis. One may then economize on the number of runs, but that could imply a severe loss of quality.

More fundamentally, stochastic simulation analysis fails to indicate the role of fundamental uncertainties. This relates to the Knightian distinction between risk and uncertainty (Knight (1921)). If uncertainties are so overwhelmingly large that even the distribution of outcomes cannot be meaningfully defined, then stochastic simulation analysis is out of place. A difficult issue is that it is hard to tell a priori which variables should be classified as uncertain and risky. Probably, one can say something about that afterwards, but this conclusion obviously does not help that much.

Sometimes it is possible to make a compromise. Indeed, exogenous variables may behave in the future differently than in the past, but not in every aspect. For example, the distribution may exhibit a change of mean, leaving the rest of the distribution intact. In this case, it remains possible to perform stochastic simulation analysis, be it that one has to add one or more sensibility or plausibility checks.

What are the analytical benefits of stochastic simulation analysis? These benefits arise when a case can be made that the distribution of variables affects decision-making. Examples are consumers who engage in precautionary saving in order to mitigate future income uncertainty. This mechanism has been known in the literature for several decades (Leland (1968), Sandmo (1970)). A number of papers investigate the saving effects of income uncertainty on an empirical basis (Hubbard et al. (1995), Engen and Gruber (2001)). Another way for consumers to absorb income uncertainty is to adjust portfolios. In particular, by reducing portfolio investments in risky assets, in particular in those the returns of which correlate positively with labour income, consumers can make their consumption less volatile (Bodie et al. (1992), Viceira (2001)). However, it is not only consumers who can anticipate future uncertainty. A case can also be made for precautionary savings by the government (Auerbach and Hasset (2001), Steigum (2001)) and for government portfolio strategies (Lucas and Stokey (1983), Bohn (1990)). A third example is funded pension schemes, which will be explored in this paper. It is difficult to see how one can account for these types of behaviour without stochastic simulation analysis.

3 The model

We use a Diamond model of a small open economy. The model contains consumers and a pension fund. There is uncertainty on four economic variables: the return on nominal bonds (r_t^b), equity returns (r_t^e), productivity growth (g_t) (which equals wage growth) and, finally, inflation (i_t). There is also uncertainty on two demographic variables: the fertility rate (n_t) and the survival rate (ε_t). We assume that economic uncertainty and demographic uncertainty are uncorrelated.

The sequence of events is as follows. Shocks occur at the beginning of a period. After these shocks have occurred, consumers and funds decide on savings and contribution rates respectively. Consequently, when people make their choices, they know the interest rate, the equity return, the productivity growth (and, hence, their wages) and the population size of that period. However, when deciding on the level of private saving, the consumers face uncertainty with respect to the return on their private savings in the next period and the pension fund benefit they will receive. Furthermore, they are uncertain about their life expectancy.

3.1 Consumers

The model is populated by a large number of identical consumers who live for two periods. So in each period both a young and old generation are alive. The young generation works and the old generation is retired. The size of a generation born at (the beginning of) period t is denoted by L_{1t} and it grows at rate n_t, thus $L_{1t} = (1+n_t)L_{1t-1}$. A decrease in n_t can be interpreted as a decrease in the fertility rate. Furthermore, we assume that a consumer born at t lives throughout old age with probability ε_{t+1}. Therefore, at time t there are $L_{2t} = \varepsilon_t L_{1t-1}$ old consumers. We can interpret ε_t as an average life expectancy: when ε_t rises, people expect to live longer. Note that both the growth rate n_t as the longevity rate ε_t are random variables.

As mentioned, consumers work in the first period and are retired in the second period. In the first period they earn labour income (y_t). Labour supply is exogenous. The consumers participate in a mandatory pension fund. The first period they pay a contribution rate (π_t), the second period they receive a benefit (b_{t+1}). In the second period agents consume the pension benefit received from the pension fund and the proceeds from their private savings (s_t). Thus, first period consumption (c_{1t}) and second period consumption (c_{2t+1}) equal:

$$c_{1t} = y_t(1-\pi_t) - s_t \tag{3.1}$$

$$c_{2t+1} = \frac{(1+r_{t+1})}{\varepsilon_{t+1}} s_t + b_{t+1} \tag{3.2}$$

with r_{t+1} the random real return to be defined later. We assume that private savings are invested in a perfect annuity market so that savings of the deceased will be distributed among the survivors. The return on savings for those who survive is therefore $(1+r_{t+1})/\varepsilon_{t+1}$. Note that the ex ante expected return on savings is equal to $(1+\mathrm{E}_t r_{t+1})$, where E_t denotes the expectations operator of time t. Also note that ε_{t+1} is a random variable which implies that there is aggregate mortality risk. Contrary to cross-sectional mortality risk, aggregate mortality risk cannot be insured away. Hence, consumers bear the aggregate mortality risk themselves.

Combining equations (3.1) and (3.2), the lifetime budget constraint of the consumers reads:

$$c_{1t} + \frac{\varepsilon_{t+1}}{1+r_{t+1}} c_{2t+1} = y_t^{LF} \tag{3.3}$$

where y_t^{LF} denotes lifetime income which is given by:

$$y_t^{LF} = y_t(1-\pi_t) + \frac{\varepsilon_{t+1}}{1+r_{t+1}} b_{t+1} \tag{3.4}$$

The budget constraint has to hold for each state of nature.

Consumers maximize expected utility with a CRRA instantaneous utility function. That is, at time t they maximize:

$$\begin{aligned} \mathrm{E}_t U(c_{1t}, c_{2t+1}) &= \mathrm{E}_t \left[\frac{c_{1t}^{1-\theta} - 1}{1-\theta} + \frac{1}{1+\rho} \varepsilon_{t+1} \frac{c_{2t+1}^{1-\theta} - 1}{1-\theta} \right] \\ &= \frac{c_{1t}^{1-\theta} - 1}{1-\theta} + \frac{1}{1+\rho} \mathrm{E}_t \left[\varepsilon_{t+1} \frac{c_{2t+1}^{1-\theta} - 1}{1-\theta} \right] \end{aligned} \tag{3.5}$$

subject to equation (3.3). In equation (3.5) ρ is the subjective discount rate and θ is the coefficient of relative risk aversion. The second line follows by noticing that uncertainty only appears in the second period.

3.2 Capital market

It is assumed that the portfolio of consumers and pension funds are identical and fixed to contain a share β equity and a share $1-\beta$ (nominal) bonds. Thus, there is no endogenous portfolio selection. Because of the fixed portfolio there is only one risky asset in this economy whose nominal return equals

$$r_t^n = (1-\beta) r_t^b + \beta r_t^e \tag{3.6}$$

With inflation denoted by i_t, the real return equals

$$r_t = \frac{1+r_t^n}{1+i_t} - 1 \tag{3.7}$$

3.3 Pension fund

Our pension fund aims at a wage-indexed pension with a replacement rate α of the previous wage income, i.e. the target benefit level is $b_{t+1} = (1+g_{t+1})\alpha y_t$.[1] Since g_{t+1} is random, the target benefit level is random as well. Hence, the expected target benefit equals $\mathrm{E}_t b_{t+1} = (1+\mathrm{E}_t g_{t+1})\alpha y_t$.

The budget constraint of the pension fund reads:

$$A_t = (1+r_t)A_{t-1} + \pi_t Y_t - B_t \tag{3.8}$$

where A_t are the accumulated assets of the pension fund at the end of period t. Throughout we use a capital letter as the aggregate in cohort terms, hence in equation (3.8) $Y_t = y_t L_{1t}$ denotes total income earned by workers in period t and $B_t = b_t L_{2t}$ denotes total pension benefits received by the retirees.

Pensions can either be funded or be financed on a PAYG basis. In case pensions are funded, the contribution rate π_t, charged by the pension fund, is invested to cover the benefit paid out the next period. The deficit (D_t) of a pension fund is the difference between the assets accumulated by a generation and the actual benefits paid to this generation. That is,

$$D_{t+1} = (1+r_{t+1})A_t - B_{t+1} \tag{3.9}$$

When determining the contribution rates for the funded schemes, we require the (expected) deficit to be zero.[2] In case PAYG financing is used there is no capital accumulation and equation (3.8) simplifies to:

$$\pi_t Y_t = B_t \tag{3.10}$$

We distinguish four types of pension schemes that can labelled along two dimensions. One dimension is the distinction between Defined Contribution (DC) schemes where the contribution rate is fixed and the benefit depends on the uncertain return on the invested contribution, and Defined Benefit (DB) schemes where the benefit is fixed and the contribution rate is uncertain because it depends on possible shortfalls or surpluses of the funding of the benefits. The other dimension is the distinction between methods of financing the pension benefits: PAYG versus funding.

[1] Remember that we assumed wage growth to be equal to productivity growth.

[2] Alternatively, one could also look at the funding ratio (F_t) of the pension fund, defined as $F_t = A_t/(B_{t+1}/(1+r_{t+1}))$. In a deterministic world, requiring the deficit to be zero is equal to requiring the funding ratio to be one. In a stochastic world, requiring a zero expected deficit implies $A_t \cdot \mathrm{E}_t[1+r_{t+1}]/\mathrm{E}_t[B_{t+1}] = 1$; eequiring the expected funding ratio to be one implies $A_t \cdot \mathrm{E}_t[(1+r_{t+1})/B_{t+1}] = 1$. In general $\mathrm{E}_t[1+r_{t+1}]/\mathrm{E}_t[B_{t+1}] \neq \mathrm{E}_t[(1+r_{t+1})/B_{t+1}]$. Hence, a zero expected deficit does not imply a funding ratio of one.

Actuarial fairness, actuarial neutrality and net benefits

Lindbeck and Persson (2003) have used three dimensions to classify pension schemes. In addition to the two dimensions applied in this paper, they also deployed actuarial versus non-actuarial as a third dimension. They define actuarial fairness as a scheme where the marginal return on a consumers contribution is equal to the market rate of interest. This differs from Börsch-Supan (1992) who defines actuarial fairness as zero net benefits, i.e. benefits minus contributions, for all (retirement) ages. Furthermore, Börsch-Supan (1992) makes a distinction with actuarial neutrality which is defined as unchanged net benefits in case retirement is postponed or advanced.

Actuarial fairness as defined by Börsch-Supan (1992) is also of interest for new entrants into a pension fund because it tells them whether participation in this pension scheme is a good deal. Therefore, we will look at this net benefit for each of the four pension schemes. Hence, in our model, the (expected) net benefit (NB) is defined as:

$$\mathrm{E}_t[NB] = \mathrm{E}_t\left[\frac{\varepsilon_{t+1} b_{t+1}}{1+r_{t+1}}\right] - \pi_t y_t \tag{3.11}$$

The sign of this net benefit indicates whether it is a good or bad deal to participate in a particular pension scheme.

3.3.1 Funded DC pensions

In case of a funded DC scheme, the pension fund fixes the contribution rate at a level such that, ex ante, the expected target benefit level can be paid. This is equivalent to a zero expected pension fund deficit. This implies that the fixed contribution rate is equal to:[3]

$$\pi_t^{F,DC} = \frac{\mathrm{E}_t \varepsilon_{t+1}}{1+\mathrm{E}_t r_{t+1}}(1+\mathrm{E}_t g_{t+1})\alpha \tag{3.12}$$

The realized benefit depends on the realized return on the invested contribution:

$$b_{t+1}^{F,DC} = \frac{1+r_{t+1}}{\varepsilon_{t+1}}\pi_t^{F,DC} y_t \tag{3.13}$$

Note that the realized return on the invested contribution is higher than $(1+r_{t+1})$ because, as with private savings, the assets of those who do not make it into the second period are equally divided between the survivors. From combining equation (3.12) and (3.13) it follows immediately that the benefit is not necessarily equal to the target benefit level. The realizations of r_{t+1}, g_{t+1} and ε_{t+1} can differ from their expected values. Furthermore, the net benefit of a funded DC scheme is always zero.

3.3.2 Funded DB pensions

In case of a DB scheme, the benefit is fixed conditional on the uncertain wage indexation g_{t+1} and equals:

$$b_{t+1}^{F,DB} = (1+g_{t+1})\alpha y_t \tag{3.14}$$

[3] Throughout this paper superscripts F and P denote respectively a Funded scheme and a Pay-As-You-Go scheme, while superscripts DC and DB denote respectively a Defined Contribution scheme and Defined Benefit scheme.

The contribution rate, $\pi_t^{F,DB}$, equals:

$$\pi_t^{F,DB} = \overbrace{\frac{\mathrm{E}_t \varepsilon_{t+1}}{1+\mathrm{E}_t r_{t+1}}(1+\mathrm{E}_t g_{t+1})\alpha}^{\pi_t^1} + \overbrace{\frac{\varepsilon_t(1+g_t)\alpha y_{t-1} - (1+r_t)\pi_{t-1}^1 y_{t-1}}{y_t(1+n_t)}}^{\pi_t^2} \tag{3.15}$$

Equation (3.15) can be split in two parts. The first part, π_t^1, is the actuarial cost price of the retirement benefit. This contribution rate is set so that, in expectation, there is no surplus or shortfall in the fund, i.e. $\mathrm{E}_t[D_{t+1}] = 0$. Note that this part of the contribution rate equals the DC contribution rate in equation (3.12). The second part, π_t^2, is a possible surcharge on the actuarial contribution rate that compensates any losses or gains in the funding of the benefits of the previous generation. In latter case the surcharge is negative, hence the total contribution rate is below actuarial cost price. The net benefit is given by:

$$\mathrm{E}_t[NB] = \mathrm{E}_t\left[\frac{\varepsilon_{t+1}(1+g_{t+1})}{1+r_{t+1}}\right]\alpha y_t - \frac{\mathrm{E}_t \varepsilon_{t+1}}{1+\mathrm{E}_t r_{t+1}}(1+\mathrm{E}_t g_{t+1})\alpha y_t - \pi_t^2 y_t \tag{3.16}$$

Note that even if the fund starts with zero deficit (thus with a zero surcharge π_t^2), the net benefit is not equal to zero since $\mathrm{E}_t\left[\varepsilon_{t+1}(1+g_{t+1})/(1+r_{t+1})\right] \neq (1+\mathrm{E}_t g_{t+1}) \cdot \mathrm{E}_t \varepsilon_{t+1}/(1+\mathrm{E}_t r_{t+1})$.[4]

3.3.3 PAYG DC pensions

Instead of funding the DC and DB schemes can also be financed on a PAYG basis. When we have a DC scheme, the contribution rate is fixed and based on the expected implicit return in a PAYG scheme which equals the expected growth of the contribution base.

$$\pi_t^{P,DC} = \frac{\mathrm{E}_t \varepsilon_{t+1}}{1+\mathrm{E}_t n_{t+1}}\alpha \tag{3.17}$$

The benefit then equals,

$$b_{t+1}^{P,DC} = \frac{(1+n_{t+1})(1+g_{t+1})}{\varepsilon_{t+1}}\pi_t^{P,DC} y_t \tag{3.18}$$

A PAYG financed DC pension scheme is also known as a notional defined contribution (NDC) scheme. The net benefit equals:

$$\mathrm{E}_t[NB] = \mathrm{E}_t\left[\frac{(1+n_{t+1})(1+g_{t+1})}{1+r_{t+1}}\right]\pi_t y_t - \pi_t y_t \tag{3.19}$$

In general, whether the net benefit is positive or negative depends on whether $(1+n_{t+1})(1+g_{t+1})$ is larger or smaller than $(1+r_{t+1})$.[5]

[4] See also footnote 2.

[5] This condition has already been derived in Aaron (1966).

3.3.4 PAYG DB pensions

Again, as in the funded DB scheme, the benefit is fixed:

$$b_{t+1}^{P,DB} = (1+g_{t+1})\alpha y_t \tag{3.20}$$

A PAYG financed DB scheme leads to the following expression for the contribution rate:

$$\pi_t^{P,DB} = \frac{\varepsilon_t}{1+n_t}\alpha \tag{3.21}$$

The net benefit equals:

$$\mathrm{E}_t[NB] = \mathrm{E}_t\left[\frac{\varepsilon_{t+1}(1+g_{t+1})}{1+r_{t+1}}\right]\alpha y_t - \frac{\varepsilon_t}{1+n_t}\alpha y_t \tag{3.22}$$

3.3.5 Summary

Table 3.1 summarizes the preceding sections. From this table, one can easily see where the uncertainty comes in. In case of DC schemes, the contribution rates are fixed and based on the expected values of the random variables. The uncertainty shows up in the pension benefits. With DB schemes, the pension benefits are only uncertain with respect to the indexation to productivity growth, all the other uncertainty is borne by the working generation.

Table 3.1 Summary of pension schemes

	Funded	PAYG
DC	$\pi_t = \frac{\mathrm{E}_t\varepsilon_{t+1}}{1+\mathrm{E}_t r_{t+1}}(1+\mathrm{E}_t g_{t+1})\alpha$ $b_{t+1} = \frac{1+r_{t+1}}{\varepsilon_{t+1}}\pi_t y_t$ $A_t = \pi_t Y_t$ $\mathrm{E}_t[NB] = 0$	$\pi_t = \frac{\mathrm{E}_t\varepsilon_{t+1}}{1+\mathrm{E}_t n_{t+1}}\alpha$ $b_{t+1} = \frac{(1+n_{t+1})(1+g_{t+1})}{\varepsilon_{t+1}}\pi_t y_t$ $A_t = 0$ $\mathrm{E}_t[NB] \lesseqgtr 0$
DB	$\pi_t = \frac{\mathrm{E}_t\varepsilon_{t+1}}{1+\mathrm{E}_t r_{t+1}}(1+\mathrm{E}_t g_{t+1})\alpha + \frac{\varepsilon_t(1+g_t)\alpha y_{t-1} - (1+r_t)\frac{\mathrm{E}_{t-1}\varepsilon_t}{1+\mathrm{E}_{t-1}r_t}(1+\mathrm{E}_{t-1}g_t)\alpha y_{t-1}}{y_t(1+n_t)}$ $b_{t+1} = (1+g_{t+1})\alpha y_t$ $A_t = \pi_t Y_t$ $\mathrm{E}_t[NB] \lesseqgtr 0$	$\pi_t = \frac{\varepsilon_t}{1+n_t}\alpha$ $b_{t+1} = (1+g_{t+1})\alpha y_t$ $A_t = 0$ $\mathrm{E}_t[NB] \lesseqgtr 0$

4 Analytical results: effects on savings and consumption...

Solving the first-order condition of the optimisation of equation (3.5) subject to (3.3) yields the following Euler equation:

$$c_{1t}^{-\theta} = \frac{1}{1+\rho}\mathrm{E}_t\left[c_{2t+1}^{-\theta}(1+r_{t+1})\right] \tag{4.1}$$

This is a standard result which can be found in e.g. Romer (2001). It states that the marginal contribution of consumption to (expected) lifetime utility in the first period of life must be equal to the marginal contribution in the second period of life. Solving the budget constraint (3.3) for c_{2t+1} and inserting the resulting expression in equation (4.1) gives the (implicit) consumption function:

$$c_{1t}^{-\theta} = \frac{1}{1+\rho}\mathrm{E}_t\left[\varepsilon_{t+1}\left(y_t(1-\pi_t)+\frac{\varepsilon_{t+1}}{1+r_{t+1}}b_{t+1}-c_{1t}\right)^{-\theta}\left(\frac{1+r_{t+1}}{\varepsilon_{t+1}}\right)^{1-\theta}\right] \tag{4.2}$$

In general this equation cannot be solved analytically for c_{1t} and we have to use stochastic simulation techniques to derive further results in this more general setup. However, before turning to the simulation results, we look at some special and standard cases where we are able to derive analytical solutions.

4.1 ...when there is no uncertainty

When there is no uncertainty we can omit the expectations operator. Using this, the first-order condition becomes:

$$c_{1t}^{-\theta} = \frac{\varepsilon_{t+1}}{1+\rho}\left(y_t(1-\pi_t)+\frac{\varepsilon_{t+1}}{1+r_{t+1}}b_{t+1}-c_{1t}\right)^{-\theta}\left(\frac{1+r_{t+1}}{\varepsilon_{t+1}}\right)^{1-\theta} \tag{4.3}$$

Solving this using equations (3.1) and (3.2) gives for consumption in the first and second period respectively:

$$c_{1t} = (1-s(r_{t+1},\varepsilon_{t+1}))y_t^{LF} \tag{4.4}$$

$$c_{2t+1} = \frac{1+r_{t+1}}{\varepsilon_{t+1}}s(r_{t+1},\varepsilon_{t+1})y_t^{LF} \tag{4.5}$$

where

$$s(r,\varepsilon) = \frac{\varepsilon^{\frac{1}{\theta}}\left(\frac{1+r}{\varepsilon}\right)^{\frac{1}{\theta}-1}}{(1+\rho)^{\frac{1}{\theta}}+\varepsilon^{\frac{1}{\theta}}\left(\frac{1+r}{\varepsilon}\right)^{\frac{1}{\theta}-1}} \tag{4.6}$$

Intuitively, a rise in r_t has both an income and a substitution effect. For $\theta < 1$ ($\theta > 1$) the substitution (income) effect dominates and equation (4.6) is increasing (decreasing) in r_t. In the special case of $\theta = 1$ (logarithmic utility), the two effects balance, and equation (4.6) is independent of r_t. Recall that $y_t^{LF} = y_t(1-\pi_t) + \varepsilon_{t+1}b_{t+1}/(1+r_{t+1})$ denotes lifetime income.

Private savings (s^*) equal:

$$s_t^* = y_t(1-\pi_t) - c_{1t}$$
$$= s(r_{t+1}, \varepsilon_{t+1})y_t^{LF} - \frac{\varepsilon_{t+1}}{1+r_{t+1}}b_{t+1} \tag{4.7}$$

Private savings are lowered by an amount equal to the present value of the future pension benefit. Hence, private savings plus pension contributions, denoted as total savings (s^T), are then equal to:

$$s_t^T = s_t^* + \pi_t y_t$$
$$= s(r_{t+1}, \varepsilon_{t+1})y_t^{LF} - \frac{\varepsilon_{t+1}}{1+r_{t+1}}b_{t+1} + \pi_t y_t \tag{4.8}$$

Several remarks can be made. First, in a certain world, the difference between DB and DC becomes irrelevant. Because DB and DC aim for the same pension benefit ex ante, the absence of uncertainty implies that, ex post, the benefits will be equal as well. Furthermore, when the pensions are funded, pension savings are replacing private savings one-to-one since both forms of savings are invested with the same portfolio composition. Hence, first period consumption and second period consumption are identical to the values that would have resulted if there would not have been a pension fund. This can easily be seen in equation (4.3) from noticing that in the case of funded pensions $\pi_t y_t = \varepsilon_{t+1}b_{t+1}/(1+r_{t+1})$ for any value of π and b. By looking at equations (4.7) and (4.8) it can immediately be seen that private savings are reduced by an amount equal to the contribution to the pension fund, whereas total savings are unaffected.

In case the mandatory pension benefits are PAYG financed, the replacement need not to be one-to-one since the implicit rate of return on the PAYG pension contributions need not to be equal to the explicit rate of return on private savings. That means, when the implicit rate of return on a PAYG contribution is lower than the rate of return on the private savings, consumers are forced to ‘save’ part of their resources in a low yielding ‘asset’ which implies a lower lifetime income. This lower lifetime income is then divided over first period and second period consumption such that marginal utility of consumption in both periods is equal. Hence, in this case, under a PAYG scheme, consumption will be lower in both periods.

This difference between funding and PAYG financing also shows up in the net benefits. In case of funding, the net benefits are zero. In case of PAYG financing, the net benefits equal:

$$NB_t^{P,DC} = \left(\frac{(1+n_{t+1})(1+g_{t+1})}{1+r_{t+1}} - 1 \right) d_{t+1}\alpha y_t \tag{4.9}$$

$$NB_t^{P,DB} = \left(\frac{(1+n_{t+1})(1+g_{t+1})}{1+r_{t+1}} - \frac{d_t}{d_{t+1}} \right) d_{t+1}\alpha y_t \tag{4.10}$$

where d_t is the dependency ratio which is the number of retirees over the number of active people i.e. $d_t = \varepsilon_t/(1+n_t)$. In both equations the Aaron-rule shows up. Under a PAYG DC scheme the net benefit is negative if the implicit return of the PAYG scheme is lower than the explicit market return, . Under a PAYG DB scheme the same holds if ε and n are constant over time and, thus, the dependency ratio is constant over time. Otherwise, it depends on the development of the dependency ratio.

4.2 ...when there is uncertainty but no pension fund

In case there is uncertainty but no pension fund, consumers only face uncertainty with respect to the returns on their private savings and their life expectancy. Since there are no PAYG financed transfers, consumers are not affected by uncertainty with respect to generation growth n_t. Furthermore, wages are determined before they choose their level of savings. Thus, uncertainty on productivity growth has also no effect.

Straightforward calculation shows that the optimal level of saving (s^{**}) equals

$$s_t^{**} = \mathrm{E}_t s(r_{t+1}, \varepsilon_{t+1}) y_t^{LF} \tag{4.11}$$

where

$$\mathrm{E}_t s(r_{t+1}, \varepsilon_{t+1}) = \frac{\left(\mathrm{E}_t \left[\varepsilon_{t+1} \left(\frac{1+r_{t+1}}{\varepsilon_{t+1}}\right)^{1-\theta}\right]\right)^{\frac{1}{\theta}}}{(1+\rho)^{\frac{1}{\theta}} + \left(\mathrm{E}_t \left[\varepsilon_{t+1} \left(\frac{1+r_{t+1}}{\varepsilon_{t+1}}\right)^{1-\theta}\right]\right)^{\frac{1}{\theta}}} \tag{4.12}$$

Note that, because there is no pension fund, $y_t^{LF} = y_t$. It can easily be shown that if $\theta > 1$ savings are higher than in the case with no uncertainty (and no pension fund or funded pensions which is irrelevant as shown before), i.e. $\mathrm{E}_t s(r_{t+1}, \varepsilon_{t+1}) > s(r_{t+1}, \varepsilon_{t+1})$. These extra savings are precautionary savings. Uncertainty on the rate of return and on aggregate mortality both imply capital market uncertainty. For the latter case, recall that the return on saving equals $(1+r_{t+1})/\varepsilon_{t+1}$. Thus, more uncertainty on aggregate mortality implies more uncertainty on the return on private savings. As shown in Sandmo (1970), capital market uncertainty induces a substitution effect and an income effect. In case $\theta > 1$, the income effect dominates and the uncertainty leads to an increase in (private) savings.

First and second period consumption equal:

$$c_{1t} = (1 - \mathrm{E}_t s(r_{t+1}, \varepsilon_{t+1})) y_t^{LF} \tag{4.13}$$

$$c_{2t+1} = \frac{1+r_{t+1}}{\varepsilon_{t+1}} \mathrm{E}_t s(r_{t+1}, \varepsilon_{t+1}) y_t^{LF} \tag{4.14}$$

where $\mathrm{E}_t s(r_{t+1}, \varepsilon_{t+1})$ is given in equation (4.12). Because of the extra savings, first period consumption is lower than in the case with no uncertainty, second period consumption is ex ante expected to be higher but, of course, ex post it can be either higher or lower.

4.3 ...when there is uncertainty and a pension fund

When there is uncertainty and a pension fund, we can only derive analytical solutions in case of funded DC pensions. For solutions of the other cases we have to use stochastic simulation techniques.

In a DC scheme the contribution rate is fixed. Hence, we can insert equation (3.12) into the first-order condition (4.2). Because in a DC scheme ex ante as well as ex post it holds that $\pi_t y_t = \varepsilon_{t+1} b_{t+1}/(1+r_{t+1})$, the first-order condition simplifies to:

$$c_{1t}^{-\theta} = \frac{\mathrm{E}_t \varepsilon_{t+1}}{1+\rho} (y_t - c_{1t})^{-\theta} \, \mathrm{E}_t \left[\left(\frac{1+r_{t+1}}{\varepsilon_{t+1}} \right)^{1-\theta} \right] \tag{4.15}$$

In this case private savings equal:

$$\begin{aligned} s_t^{F,DC} &= \mathrm{E}_t s(r_{t+1}, \varepsilon_{t+1}) y_t - \pi_t^{F,DC} y_t \\ &= s_t^{**} - \pi_t^{F,DC} y_t \end{aligned} \tag{4.16}$$

Given that the portfolio of the pension fund is equal to the portfolio of the consumer, there is no essential difference between private saving and saving through the pension fund except for the fact that pension saving is compulsory and private saving voluntary. Hence, the consumer lowers his private savings exactly by the amount he is forced to save through the pension fund. Logically it follows that first period and second period consumption equals consumption in the case without a pension fund, i.e. equation (4.13) and (4.14).

For the other three pension schemes, analytical solutions cannot be derived. Following the analysis in Sandmo (1970) we can conclude that in each case precautionary savings show up if $\theta > 1$. In all cases, the uncertainty on the rate of return and on aggregate mortality risk has a positive effect on private savings because the income effect dominates the substitution effect. In all schemes consumers are exposed to uncertain productivity growth. Uncertain productivity growth has no effect on the return on savings but, through indexation in DB pension schemes, it does have an effect on future income. Sandmo (1970) labels this as 'income risk' and shows that an increase in income risk always leads to higher private savings. Uncertain fertility only plays a role in case of a PAYG DC scheme where it induces an income effect only and, thus, increase private savings.

4.4 Concluding remarks

In this section, we looked at some analytical results from the model developed in the previous section. However, analytical solutions could only be derived in very special, but less interesting cases. To get answers to the questions raised in this paper, we have to use stochastic simulations. Nevertheless, the results derived in this section are worth noticing because they are useful in the stochastic simulations analysis.

If we look at the model in a world without uncertainty, we noticed that the difference between DB and DC becomes irrelevant. However, the difference between PAYG and funding does matter. In the latter case, pension savings are replacing private savings one-to-one. But the lower implicit rate of return on the PAYG contributions implies a lower lifetime income and, hence, lower consumption in both periods. This result is also seen in the net benefits. In case of the funded schemes these are zero. However for the PAYG schemes these are negative (with an exception for one generation: the retirees when the PAYG scheme is introduced).

If we bring uncertainty into the model but leave the pension funds aside, precautionary savings shows up in case the coefficient of risk aversion (θ) is high enough. Consumers face uncertainty on the real rate of return on their private savings and on aggregate mortality risk. As already shown in Sandmo (1970) this leads to precautionary savings if $\theta > 1$.

Introducing pension funds into the model leads only to analytical solutions in case the pension fund offers a funded DC scheme. Then, however, pensions savings are a perfect substitute for private savings. For the other three types of pension schemes one can deduce that precautionary savings will show up if consumers are sufficiently risk averse.

5 Data and shock analysis

In this section we provide a description of the data. The calibration of the parameters as well as the random variables are treated. Second, we analyse the impact of economic shocks and demographic shocks under the four pension schemes introduced in the preceding sections.

5.1 Calibration and data

5.1.1 Calibration of model parameters

Our model contains four parameters which have to be calibrated, the rate of time preference (ρ), the coefficient of relative risk aversion (θ), the ambition level of pension benefits (α) and, finally, the share of stocks in the asset holdings of a pension fund (β).

The value of the rate of time preference ρ is set at 25% per thirty years, which corresponds to an annual rate of around 0.75%. We impose $\theta = 4$ as our benchmark value. Further, the ambition level of pensions is 50% of first period income, so $\alpha = 0.5$. Finally, we assume that the pension fund and consumers invest an equal share in bonds and stocks, i.e. $\beta = 0.5$.

5.1.2 Characteristics of the random variables

Our model contains six exogenous variables which are all random. The economic random variables are: inflation (i_t), productivity growth (g), return on bonds (r^b) and return on stocks (r^e). The inflation rate does not directly enter into the model, because all variables are expressed in real terms. The two demographic random variables are the fertility rate (n) and longevity rate (ε). Throughout the analysis we impose that the economic and demographic random variables are mutually independent, while the economic random variables themselves are mutually dependent.

The random draws of the economic variables are obtained from ORTEC.[6] They first estimate a VAR model for the economic variables based on annual data of the Dutch economy over the period 1971-2002. The covariance matrix of the residuals is then used to simulate random scenarios that all contain one value for each of the economic variables. An annual scenario x is converted into a thirty-years scenario X by the following formula:

$$X = \prod_{t=1}^{30} (1 + x_t) - 1 \tag{5.1}$$

We received 500 scenarios from Ortec. All these scenarios have been used in our simulation

[6] ORTEC is an independent international consultancy firm that, among other activities, conducts Asset Liability Management (ALM) studies to address potential risk factors for pension funds.

analysis. As a consequence, we do not have to make arbitrary assumptions regarding the distribution of the economic random variables. The correlations of the economic variables that can be derived from the 500 scenarios are shown in table 5.1.

Table 5.1 Correlation matrix[a]

inflation	1.00	0.95	− 0.27	0.73
productivity growth	0.95	1.00	− 0.13	0.74
return on equity (nominal)	− 0.27	− 0.13	1.00	− 0.07
return on bonds (nominal)	0.73	0.74	− 0.07	1.00

[a] Source: ORTEC

We also need random draws for the fertility rate (n) and the longevity rate (ε). These variables are computed using a computer program PEP (Program for Error Propagation). This program, written at the Department of Statistics, University of Joensuu, generates long-term stochastic population forecasts for many European countries, including the Netherlands.[7]

Several remarks can be made at this point. First, recall that $n_t = L_{1t}/L_{1t-1}$ and $\varepsilon_t = L_{2t}/L_{1t-1}$, with L_1 the working generation and L_2 the retired generation. We assume that the working generation consists of people whose age ranges from 31 to 60 while the retired generation consists of people with an age between 61 and 90 years old.[8] Second, n_t and ε_t reflect thirty-years figures.

The random draws of the fertility rate are directly taken from PEP. However, the computation of the longevity draws involves a two-step procedure. In the first step we derive the mean and variance of the longevity rate from the first and second moments of the fertility rate and the dependency ratio. It makes sense to use the uncertainty in the dependency ratio, because this variable captures both key demographic trends in the Netherlands, low fertility and increasing life expectancy. Recall that the dependency ratio is defined as $d = \varepsilon/(1+n)$. Hence, once we have obtained the first and second moments of n and d from PEP, the mean and variance of ε can be calculated by:

$$\begin{aligned} \mu_\varepsilon &= \mu_d(1+\mu_n) + \text{cov}(d,n) \\ \sigma_\varepsilon^2 &= \left(\sigma_d^2+\mu_d^2\right)\left(\sigma_n^2+(1+\mu_n)^2\right) + \text{cov}(d^2,n^2) - \left(\mu_d\,(1+\mu_n)+\text{cov}(d,n)\right)^2 \end{aligned} \tag{5.2}$$

where μ_x and σ_x^2 denote, respectively, the mean and variance of x. In the second step, we postulate that the longevity rate is normally distributed. That is, we draw 500 times from the normal distribution with mean μ_ε and variance σ_ε^2.

[7] See Alho and Spencer (1997) for a description of the PEP program.

[8] We have checked for other definitions of L_1 and L_2 as well. We found that the most important conclusions from the model simulations (see section 6) are not sensitive for this definition.

Table 5.2 presents the mean, median and standard deviation of the random variables. Since the economic variables are cumulated annual figures, we also report the corresponding one-year numbers in parentheses. In absolute terms the standard deviation of the demographic variables is rather low compared to that of the economic variables. However, relative to its corresponding mean value, the standard deviation of the fertility rate is the highest one, while the standard deviation of the longevity rate is the smallest. The standard deviation of the return on equity is high, also in relative terms.

Table 5.2 Characteristics random variables, cumulated thirty-years figures[a]

	productivity growth	return on equity	return on bonds	inflation	fertility rate	longevity rate
mean	0.73	15.37	7.16	2.24	− 0.05	0.73
	(0.018)	(0.098)	(0.072)	(0.040)	-	-
median	0.73	12.19	6.99	2.09	− 0.04	0.73
standard deviation	0.28	10.89	1.64	0.96	0.05	0.08

[a] Numbers in parentheses reflect the corresponding one-year means.

5.2 Shock analysis

In this section we investigate the role of the economic and demographic random variables in more detail. For all four pension schemes distinguished in this paper, we will analyse the welfare effects of both a positive and a negative one standard deviation shock in one of the random variables. We are interested how the sensitivity of individual welfare for economic and demographic shocks differs between the pension schemes. This sensitivity gives us a first insight which risk factors may be important. To simplify the analysis at this point, we leave out any form of uncertainty. We assume, however, that shocks are temporary and unexpected. It is further imposed that consumers know that *if* a shock takes place, it will last just one period. As a consequence, consumers and the pension fund will not change their expectations after a shock.

5.2.1 Welfare measure

The notion of compensating variation provides a natural way to address the welfare implications of shocks or policy measures in a deterministic context. Compensating variation is the compensating payment that leaves the consumer as well off as before the economic change. The payment is positive for a welfare loss and negative for a welfare gain. For technical details, we refer to appendix A in which we derive explicit expressions for the compensating variations of the working generation and the retired generation.

Social welfare (SW) can be defined as the sum of the compensating variations of the currently working and retired consumers plus the discounted value of the compensating variations of future generations. That is,

$$SW_t = L_{2t}CV_{2t} + \sum_{i=0}^{\infty} \frac{CV_{1t+i}L_{1t+i}}{(1+r_{t+i})^i} \tag{5.3}$$

where CV_1 and CV_2 denote, respectively, the consumer compensating variation of a working consumer and a retired consumer. Again, equation (5.3) is positive for a welfare loss and negative for a welfare gain. Social welfare must be interpreted as society's maximum willingness to pay for a utility increasing shock and its minimum acceptable compensation for a utility loss.

With regard to a fertility and longevity shock, equation (5.3) can in principle be computed with the baseline population or with the new (after-shock) population. Due to the definition of a compensation variation, we have chosen for the baseline population.[9] Obviously, one can only compensate consumers in response to a shock if they were already alive in the starting situation.

5.2.2 Base projection

Table 5.3 shows some important ratios for the baseline calibration of the model. In all pension schemes agents consume 82% of their lifetime income in the first period and 18% in the second period. There is a remarkable difference between the contribution rate in a funded scheme (14%) and in a PAYG scheme (38%). This is due to the fact that in the model a funded scheme is more efficient than a PAYG scheme, i.e., $(1+n)(1+g) < (1+r)$.[10] Given this relatively high contribution rate, consumption smoothing induces consumers in the PAYG schemes to borrow in their first period of life. This debt must be repaid during the second period. Because of this reason second period consumption is lower than the level of pension benefits and hence, pension benefits as percentage of second period consumption exceeds the hundred percent.

Table 5.3 Baseline scenario

		DB funded	DC funded	DB PAYG	DC PAYG
consumption young	(% y^{LF})	81.55	81.55	81.55	81.55
consumption old	(% y^{LF})	18.45	18.45	18.45	18.45
contribution rate	(% y)	14.26	14.26	38.21	38.21
pension benefits	(% c_2)	77.32	77.32	101.66	101.66
private savings	(% y)	4.18	4.18	− 0.23	− 0.23

Note: y^{LF} is lifetime income, y is individual income and c_2 is old-age consumption.

[9] As a sensitivity analysis we have also calculated equation (5.3) using the after-shock population. We found that the results are not senstive to the choice of population.

[10] Assuming certainty one can check that if $(1+n)(1+g) = (1+r)$ the cost-effective contribution rates in all pension schemes are equal. However if $(1+n)(1+g) > (<) (1+r)$ the contribution rate in PAYG schemes is lower (higher) than in funded schemes.

5.2.3 Productivity shock

Let us first consider the welfare implications of a positive and negative shock in productivity growth (g). The shock is unexpected and hence, does not change the cost-effective contribution rate in funded schemes, see table 3.1. Recall from the same table that the contribution rate in PAYG schemes does not depend on productivity growth at all. In addition, since consumers do not alter their expectations, it follows from equation (4.6) that they will not change behaviour. The upper panel of table 5.4 presents the welfare effects, both for a positive (+) and a negative (-) productivity shock. Recall that a positive number implies a welfare loss, a negative number a welfare gain. As a general remark, observe that the welfare effects of a negative shock are larger than the effects of a positive shock. This is direct consequence of the risk aversion of consumers.

If we focus on the differences between the pension schemes we see, first, that for working consumers the welfare effects are the highest in the funded DC scheme and the lowest in the funded DB scheme. The effects in the PAYG scheme fall somewhere in between. Higher (lower) productivity growth increases (decreases) the income of the working consumers and hence, raises (declines) welfare in all pension schemes. In the funded DB scheme however these welfare effects are reduced by the catching up premium rate. Recall that in this scheme the pension benefits are wage-indexed. Therefore, if productivity growth is higher than expected, the pension savings of the retired generation are not sufficient to pay for the higher indexation. As a consequence, the working generation has to pay for the higher pension benefits of the old which lowers welfare. For a negative shock it is just the other way around. Second, with respect to retirees, we observe that in the funded DC scheme welfare is not sensitive to productivity shocks. For the other schemes, in which pension benefits are all wage-indexed, the welfare effects are equal.

To conclude, the differences in welfare indicate that for working consumers productivity risk is most relevant in the funded DC scheme, followed by the PAYG schemes and the funded DB scheme. For the retired generation productivity risk does not play a role in the funded DC scheme as opposed to the other schemes in which productivity growth determines the indexation of pension benefits.

5.2.4 Portfolio return shock

A shock in the portfolio rate of return (r) leads to different welfare effects. As for the productivity shock, an unexpected one-shot portfolio return shock does not influence consumer behaviour and the cost-effective contribution rates (see table 3.1). Therefore, the welfare of working consumers is not sensitive to this shock in the funded DC scheme and in the PAYG schemes. However, from equation (3.15) it follows that this shock affects the catching-up premium in case of a funded DB scheme. That is, if r increases (decreases) the pension savings

Table 5.4 Welfare effects economic shocks

		DB funded		DC funded		DB PAYG		DC PAYG	
Productivity shock		+	-	+	-	+	-	+	-
compensating variation young	(% y)	− 1.95	2.70	− 3.15	4.36	− 2.40	3.32	− 2.40	3.32
compensating variation old	(% b)	− 2.30	3.18	0.00	0.00	− 2.30	3.18	− 2.30	3.18
social welfare	(% Y)	− 4.70	6.51	− 5.03	6.96	− 4.70	6.51	− 4.70	6.51
Portfolio return shock		+	-	+	-	+	-	+	-
compensating variation young	(% y)	− 3.28	12.59	− 0.00	0.00	− 0.00	0.00	0.00	0.00
compensating variation old	(% b)	− 1.84	7.05	− 5.03	76.46	0.10	− 0.39	0.10	− 0.39
social welfare	(% Y)	− 3.99	15.29	− 3.10	11.88	0.04	− 0.15	0.04	− 0.15

Note: y is individual income, b is pension benefit and Y is aggregated income.

of the currently retired generation are too much (few) to cover the guaranteed pension benefits. Consequently, the catching-up premium is negative (positive) and hence welfare increases (decreases). Note again that the welfare effects of a negative shock are much larger than those of a positive shock.

For the retired generation we observe a large difference between the funded schemes and the PAYG schemes. Not only are the welfare effects for funded schemes much larger, also the direction of the effects differs. The welfare effects are the highest in the funded DC scheme, followed by the funded DB scheme. In the first one both income sources (pension benefit and private savings) depend on the portfolio return, while in the second one only private savings are sensitive to this rate of return. Due to the higher contribution rate, in PAYG schemes private savings are very small and even negative (see table 5.3). Consequently, retirees suffer (benefit) from a higher (lower) interest rate, because it increases (decreases) the interest payments on their debt.

We conclude that for working consumers the risk associated with an unexpected portfolio return shock is important in the funded DB scheme. In other schemes this risk does not play a role since the shock does not affect welfare. For retired consumers portfolio return risk is most relevant in the funded DC scheme, followed by the funded DB scheme. Since the amount of private savings is very small in PAYG schemes, consumers are hardly confronted with portfolio return risk.

5.2.5 Fertility shock

From the upper panel of table 5.5 we see that a shock in the fertility rate (n) has no welfare effects in funded schemes. In these schemes the actuarial contribution rate does not depend on the fertility rate, see equation (3.15). We therefore concentrate the analysis to the PAYG schemes.

A fertility shock affects the contribution rate in a PAYG DB scheme (see table 3.1). An increase (decrease) in the fertility rate implies that there are more (less) working people to pay for the pension benefits of the currently retired generation. This lowers (rises) the contribution rate in a DB scheme and improves (deteriorates) welfare of the young. The pension benefit of the old does not change and hence, there is no effect on welfare of the old. For the DC scheme the picture is reversed. The contribution rate does not change after a positive (negative) fertility shock, but the pension benefits will increase (decrease).

To summarize, in funded schemes a fertility shock has no individual welfare consequences. This indicates that fertility risk will not be important in these schemes. For PAYG schemes fertility risk matters because a fertility shock affects welfare of working consumers in a DB scheme and that of retired consumers in a DC scheme.

Table 5.5 Welfare effects demographic shocks

		DB funded		DC funded		DB PAYG		DC PAYG	
Fertility shock		+	-	+	-	+	-	+	-
compensating variation young	(% y)	0.00	0.00	0.00	0.00	− 0.39	0.53	0.00	0.00
compensating variation old	(% b)	0.00	0.00	0.00	0.00	0.00	0.00	− 0.74	1.01
social welfare	(% Y)	0.00	0.00	0.00	0.00	− 0.37	0.56	− 0.28	0.39
Longevity shock		+	-	+	-	+	-	+	-
compensating variation young	(% y)	0.95	− 0.94	0.00	0.00	0.95	− 0.94	0.00	0.00
compensating variation old	(% b)	0.53	− 0.53	2.60	− 2.08	− 0.03	0.03	1.98	− 1.58
social welfare	(% Y)	1.15	− 1.15	0.90	− 0.89	0.94	− 0.93	0.68	− 0.68

Note: y is individual income, b is pension benefit and Y is aggregated income.

5.2.6 Longevity shock

The last shock we consider, is a one standard deviation increase and decrease in the longevity rate (ε). See the lower panel of table 5.5 for the results. A longevity shock does not affect the cost-effective contribution rate except for the PAYG DB scheme. If longevity increases, in this scheme consumers has to pay a higher contribution rate. This lowers welfare of the young. For a negative shock it is just the other way around. In a funded DB scheme the longevity shock generates a mismatch between the pension benefits and the amount of funding. Consequently, the catching-up premium increases (decreases) if the longevity rate goes up (down). This leads to the same welfare implications as in the PAYG DB scheme.

Longevity influences welfare of retired consumers in all pension schemes. As for the portfolio return shock, the funded DC scheme is most sensitive to longevity, followed by the PAYG DC scheme. To understand this result, note that longevity determines the ex-post return on private savings, i.e. $(1+r)/\varepsilon$. Since in the funded DC scheme pension savings and private savings both face this return, the welfare effects are relatively large. In addition, in DC schemes the pension benefit itself depends negatively on the longevity rate which reinforces the welfare effects in the funded DC scheme. Note the direction of the welfare effects in the PAYG DB scheme differs from that in the other schemes. In this scheme we observe a welfare gain (loss) if longevity increases (decreases). In the PAYG DB scheme consumers borrow in the first period and pay back the debt in the second period. An increase in longevity lowers the effective interest rate on their debt and, hence boosts welfare.

The welfare effects suggest that working consumers only face longevity risk in the two DB schemes. While in all pension schemes retired consumers are subject to longeviy risk, the extent of the risk exposure is most pronounced in the funded DC scheme, followed by the PAYG DC scheme.

6 Simulation results

The main objectives of this paper are, first, the analysis of the impact of uncertainty on behaviour and, second, the assessment what source of uncertainty, economic or demographic, is more important. In this section we turn to these questions. For each of the four pension schemes we simulate and compare the cases of full certainty, full uncertainty, economic uncertainty only and demographic uncertainty only. To assess the importance of economic and demographic uncertainty we calculate the certainty equivalents which can be interpreted as the insurance premium people are willing to pay to avoid (part of) the uncertainty. The impact of uncertainty on behaviour is analysed by comparing behaviour based on rational expectations with behaviour based only on the expected values of the random variables (point forecasting).

6.1 Full uncertainty

Looking at the effects of uncertainty on behaviour implies, in this model, looking at the effects on savings. Because θ, the parameter of risk aversion, is larger than 1, one expects precautionary savings. Figure 6.1 shows the extra, precautionary private savings under uncertainty as a percentage of labour income for each of the four pension schemes. These extra private savings are largest in a funded DC scheme and lowest in a funded DB scheme.

Figure 6.1 Extra private savings under uncertainty (% labour income)

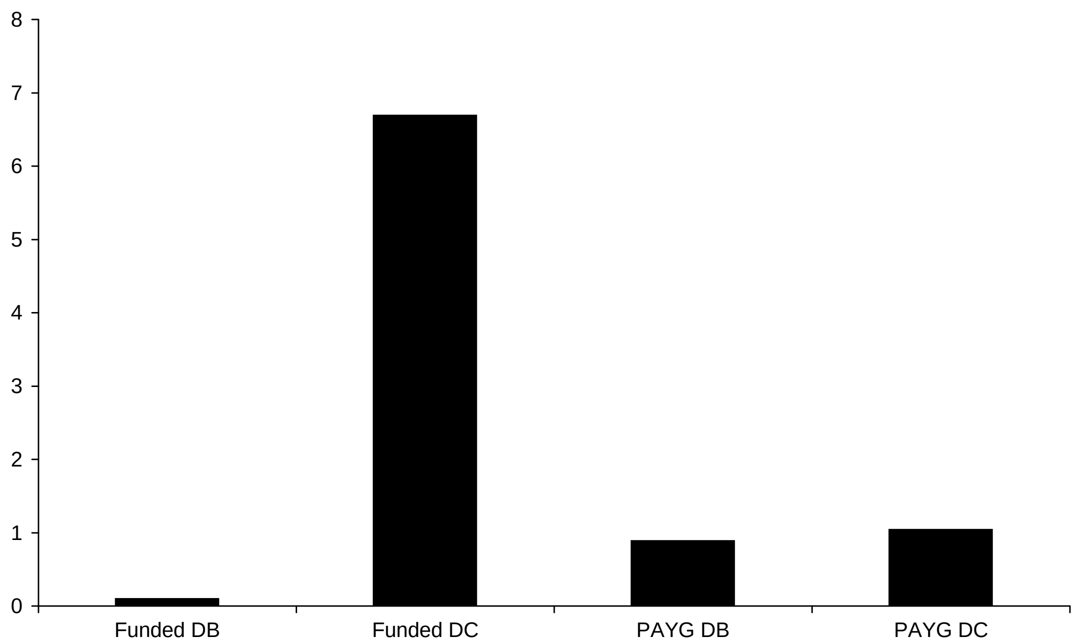

Table 6.1 Certainty and uncertainty: Utility, Consumption, Savings

	$E_t y_t^{LF}$	c_{1t} (% $E_t y^{LF}$)	$E_t c_{2t+1}$ (% $E_t y^{LF}$)	s_t (% y)	$s_t + \pi_t y_t$ (% y)	$E_t U$
Certainty						
DB, Funded	17.30	81.55	18.45	4.18	18.45	52.778
DC, Funded	17.30	81.55	18.45	4.18	18.45	52.778
DB, PAYG	13.16	81.55	18.45	− 0.23	37.97	52.759
DC, PAYG	13.16	81.55	18.45	− 0.23	37.97	52.759
Full Uncertainty						
DB, Funded	18.25	77.20	22.80	4.28	18.54	52.777
DC, Funded	17.30	74.86	25.14	10.88	25.14	52.772
DB, PAYG	14.11	74.96	25.05	0.66	38.86	52.758
DC, PAYG	14.10	74.84	25.16	0.81	39.01	52.758
Economic uncertainty						
DB, Funded	18.24	77.23	22.77	4.31	18.58	52.777
DC, Funded	17.30	75.27	24.73	10.47	24.73	52.772
DB, PAYG	13.16	75.02	24.98	0.67	38.87	52.758
DC, PAYG	14.10	75.02	24.98	0.67	38.87	52.758
Demographic uncertainty						
DB, Funded	17.30	81.57	18.43	4.16	18.43	52.778
DC, Funded	17.30	81.29	18.71	4.44	18.71	52.778
DB, PAYG	13.16	81.55	18.45	− 0.23	37.98	52.759
DC, PAYG	13.16	81.20	18.80	0.03	38.24	52.759

The top half of table 6.1 shows the optimal levels of first period and second period consumption (as percentage of expected lifetime income), private savings and private savings plus pension contributions (as percentage of labour income) as well as expected lifetime income and expected utility under both full certainty and full uncertainty.

What can we conclude from this table? First, whether there is certainty or uncertainty, funded schemes give a higher utility than PAYG schemes if, as is the case here, either scheme is already operative.[11] This is no surprise as under a PAYG scheme people are forced to 'save' for retirement through a pension fund with a lower (implicit) rate of return than under a funded scheme. This lower rate of return implies a lower (expected) lifetime income. Furthermore, utility is lower under uncertainty compared to certainty. Note also that expected lifetime income under a funded DB scheme and under both PAYG schemes is higher than the certain lifetime income under the same pension schemes. For the funded DC scheme expected lifetime income is

[11] This does not mean that it is always preferable to introduce a funded scheme or switch from a PAYG scheme to a funded scheme. There is long list of literature comparing PAYG and funded schemes (e.g. Lindbeck and Persson (2003), Sinn (2000) or Barr (2000)).

equal to the certain lifetime income.[12]

Table 6.2 Certainty equivalents (CE)

	Total CE (% y)	Economic CE (% y)	Demographic CE (% y)
Full uncertainty			
DB, Funded	0.71	0.68	0.02
DC, Funded	10.79	10.41	0.72
DB, PAYG	0.84	0.84	− 0.01
DC, PAYG	1.24	0.79	0.44
Economic uncertainty			
DB, Funded		0.69	
DC, Funded		10.14	
DB, PAYG		0.86	
DC, PAYG		0.86	
Demographic uncertainty			
DB, Funded			0.02
DC, Funded			0.42
DB, PAYG			0.00
DC, PAYG			0.51

Table 6.2, under the label 'Full uncertainty', gives the certainty equivalents for total uncertainty (economic and demographic uncertainty) and for economic and demographic uncertainty separately. The latter two show how much people want to pay to avoid economic (demographic) uncertainty and keep demographic (economic) uncertainty. From this table we can make the following observations. First, the funded DB scheme is very close to full certainty. People want to pay only a very small amount of money, less than 1% of income, to avoid uncertainty. This confirms the observation in table 6.1 where we saw that utility of a funded DB scheme under uncertainty was almost identical to utility of a funded DB scheme under certainty.

Second, both DB schemes, whether PAYG financed or funded, show low certainty equivalents. The DC schemes are regarded more risky as people are willing to pay a larger amount of money to keep off the uncertainty. However, one may not conclude that the PAYG DB scheme is preferred to the funded DC scheme because the certainty equivalent is higher for the latter. What matters is expected lifetime utility which is higher for a funded DC scheme than for a PAYG DB scheme (see table 6.1).

Finally, comparing the second and third column of the full uncertainty panel, reveals that economic uncertainty is much more important than demographic uncertainty. For all pension

[12] See appendix C for a formal proof.

schemes the economic certainty equivalents are higher than the demographic counterparts, indicating that consumers are willing to give up more income to avoid economic risks than to avoid demographic risks.

6.2 Demographic or economic uncertainty only

This section considers the situation where consumers only face either economic uncertainty or demographic uncertainty. The bottom half of table 6.1 shows the results for these cases. Table 6.2 gives the corresponding certainty equivalents. These figures confirm the finding of the previous section, that economic uncertainty is much more important for consumers than demographic uncertainty.

Using table 6.2 we can decompose the total certainty equivalent into the economic certainty equivalent and the demographic certainty equivalent. To illustrate, combining the first two columns under ‘Full uncertainty’ with the column under ‘Demographic uncertainty’ gives a decomposition where, first, economic uncertainty is separated and then the remaining demographic uncertainty is valued. It follows for example that for the funded DC scheme the total certainty equivalent of 10.8% of income can be split up in a economic certainty equivalent of 10.4% and a demographic certainty equivalent of 0.4%. Doing it the other way around by first separating the demographic uncertainty, the values for economic and demographic uncertainty become respectively 10.1% and 0.7%.

Demographic uncertainty only plays a significant role in the PAYG DC scheme where the total certainty equivalent of 1.2% can be divided into almost 0.8% for economic uncertainty and 0.5% for demographic uncertainty. So in this case approximately one third of the value of the uncertainty is explained by demographic uncertainty. For the other three schemes this is less than 10%.

6.3 Sensitivity analysis

To investigate the robustness of our results, we perform a sensitivity analysis for alternative parameter values. Table 6.3 gives an overview of the parameters analysed and the results for the certainty equivalents. We observe from this table that the conclusion that economic uncertainty matters more than demographic uncertainty is robust.

If θ is higher (lower) one would expect that the certainty equivalent will be higher (lower) as well since this corresponds to more (less) risk aversion. This holds for the funded DC scheme and for both PAYG schemes. However, it does not hold for the funded DB scheme. There, both a higher and a lower θ imply a higher certainty equivalent compared to the benchmark case. This

Table 6.3 Sensitivity analysis for certainty equivalents

	Baseline	θ		ρ		α		β		y_t	
		2	6	0.1	0.4	0.3	0.7	0.05	0.95	$0.5y_t$	$1.5y_t$
Full uncertainty											
DB, Funded	0.71	0.92	0.86	0.77	0.66	2.46	1.10	0.83	1.29	0.71	0.71
DC, Funded	10.79	7.20	14.01	11.05	10.57	10.79	10.79	3.34	24.54	10.79	10.79
DB, PAYG	0.84	0.42	1.56	0.79	0.91	1.80	6.85	1.53	0.58	0.84	0.84
DC, PAYG	1.29	0.74	2.08	1.26	1.33	2.29	6.70	2.09	0.99	1.29	1.29
Economic uncertainty											
DB, Funded	0.68	0.87	0.85	0.75	0.64	2.34	1.10	0.83	1.26	0.68	0.68
DC, Funded	10.41	6.95	13.52	10.66	10.19	10.41	10.41	2.81	24.28	10.41	10.41
DB, PAYG	0.84	0.40	1.55	0.78	0.90	1.64	6.68	1.46	0.57	0.84	0.84
DC, PAYG	0.79	0.43	1.38	0.75	0.84	1.83	6.09	1.44	0.57	0.79	0.79
Demographic uncertainty											
DB, Funded	0.02	0.04	0.02	0.03	0.02	0.16	− 0.01	− 0.01	0.04	0.02	0.02
DC, Funded	0.72	0.48	0.66	0.73	0.70	0.72	0.72	0.76	− 2.13	0.72	0.72
DB, PAYG	− 0.01	− 0.00	− 0.01	− 0.01	− 0.01	0.10	0.25	− 0.01	− 0.01	− 0.01	− 0.01
DC, PAYG	0.44	0.33	0.52	0.46	0.42	0.58	0.09	0.57	0.40	0.44	0.44

could be explained by noting that in our CRRA utility function a higher θ implies higher risk aversion as well as a lower intertemporal elasticity of substitution. Hence, a higher θ implies, first, that the consumer becomes more risk averse and will be prepared to pay a higher certainty equivalent for the same amount of risk. At the same time, a lower intertemporal elasticity of substitution implies that the consumer is less willing to substitute consumption between periods and thus implies lower private savings. But lower private savings imply less risk and, thus, a lower certainty equivalent. These two opposing effects on the certainty equivalent apparently lead to a non-monotonic relationship between θ and the certainty equivalents.

In case of funded schemes, a higher subjective discount rate ρ implies lower certainty equivalents. For PAYG schemes the opposite holds. If ρ is higher less weight will be attached to the future and, thus, to future uncertainty. Hence, one would expect certainty equivalents to drop as is indeed the case with funded pension schemes. The opposite results for the PAYG pension schemes needs further investigation.

A higher or lower replacement rate α for the pension fund has no effect in a funded DC scheme which is obvious since funded DC savings and private savings are perfect substitutes in our model. For the other schemes certainty equivalents are higher. In a funded DB scheme the pension savings of consumers are protected against the risks in the rate of return on these savings. If the replacement rate is lower, consumers have relatively more private savings on which they run rate of return risks. This explains a higher certainty equivalent in this case. In case the replacement rate is lower for PAYG pension schemes, a similar effect is at work.

More risky assets in the portfolio of consumers and pension funds (a higher β) obviously leads to higher certainty equivalents for the funded DC scheme. When there are PAYG financed pension schemes and there is a switch to more equity, the certainty equivalents drop. In case of a funded DB scheme or PAYG schemes, an increase in equity has two opposing effects on the certainty equivalent. First, a switch to more equity provides a hedge against productivity growth risks. Table 5.1 shows that the correlation between productivity growth and equity returns is negative. Thus, for the DB schemes, low pension benefits due to low productivity growth is (partly) compensated by higher returns on private savings. The same holds for the PAYG DC scheme since the 'return' on the PAYG contribution is linked to productivity growth as well. Second, the risk on private savings increases with an increase of equity in the portfolio. This second effect, however, does not undo the hedge effect in PAYG schemes because in these schemes private savings are low. This does not hold in case of funded DB pensions.

The simulations reveal a non-monotonic relation between the share of equity in the portfolio and the value of the certainty equivalent in the funded DB scheme. Very low shares of equity as well as very high shares of equity give higher values for the certainty equivalents than the baseline case where the portfolio contains 50% equity and 50% bonds. Apparently, when equity is increased if the share of equity is still low, the hedge effect dominates the increased risk on private savings (which are low in case of a low equity share) and the certainty equivalent declines. If the share of equity is increased further, private savings grow and the increased risk on these private savings starts to dominate the hedge effect of the pension benefit. As a result, the certainty equivalent increases.

Finally, the initial income has no effect on the certainty equivalents. It only affects (expected) lifetime utility because lifetime income is higher or lower (not shown here).

6.4 The impact of uncertainty on consumer behaviour

In reality consumers are confronted with a lot of uncertainties. Undoubtedly, these uncertainties have a significant influence on economic behaviour. Yet, most research into the sustainability of welfare states has been done in the context of certainty, or rather, in a world in which consumers do not respond to the uncertainties they face. A natural question is what do we miss, in terms of welfare costs, if we assume that consumers are not aware of uncertainty.

We compare two different informational assumptions of an optimising consumer with respect to the economic and demogaphic random variables. The benchmark case is the rational expectations assumption that is already used throughout this chapter. In this case, consumers are well-informed in that they know the complete distribution of the random variables. As a consequence, the consumer actually recognizes that he is confronted with uncertainty and takes this uncertainty explicitly into account when he decides upon his amount of savings (precautionary savings motive).

We compare the benchmark with the case where consumers only know a point forecast. We impose that this point forecast is equal to the sample mean. Since consumers behave as if there is no uncertainty, the optimising consumption rules for the benchmark are sub-optimal. Instead, the consumer applies the rules derived in section 4.1 under certainty. Hence, the consumer does not have any precautionary savings motive in this case.

How to measure the welfare gain of a consumer that has rational expectations versus a naive consumer that only has a point forecast? As in section 5.2 we will use the concept of compensating variation. That is, we compute the minimal amount of income that must be given to the less-informed consumer (point forecast assumption) to give him the same utility level he would get in the benchmark situation (rational expectations assumption). See appendix B.3 for the technical details.

Table 6.4 Compensating variations (% income)

	Baseline ($\theta = 4$)	$\theta = 2$	$\theta = 6$
DB, Funded	0.00	0.02	0.06
DC, Funded	6.71	0.43	24.79
DB, PAYG	0.36	0.00	14.59
DC, PAYG	0.47	0.00	7.00

Table 6.4 displays the compensating variations as percentage of wage income for each pension scheme and for different degrees of risk aversion. In general, the compensating variations increase with the degree of risk aversion. This makes intuitively sense, because a more risk averse consumer will engage in more precautionary savings if he takes uncertainty explicitly into account. As a consequence, the sub-optimality of the point-forecast decision rule, increases and hence, a consumer is more willing to pay for additional information.[13]

In case of a funded DB scheme the compensating variations are very low, indicating that in this scheme a consumers willingness to pay for additional information regarding the distribution of the random variables (rational expectations) is rather minimal. This result is robust for the degree of risk aversion.

In case of PAYG schemes the compensating variation is also rather small for the baseline. In a DB scheme, for example, consumers are willing to pay at most 0.36% for additional information. For a DC scheme this percentage is somewhat higher but still small, 0.47%.

[13] Note that the funded DB results are not completely monotone in θ. In the sensitivity analysis of section 6.3 we observed the same discontinuity for the funded DB scheme. There we explained that this probably has to deal with the fact that for CRRA utility θ not only determines risk aversion but also intertemporal substitution in consumption.

Interestingly, for $\theta = 6$, the compensating variation is much higher for a DB than for a DC scheme. This contra-intuitive result needs further investigation.

The sub-optimality of point-forecasting is most severe for a funded DC scheme. Note that consumers are willing to pay at most 6.7% of their income for more information. This result is not surprising, because the amount of uncertainty consumers face is the largest in this scheme. Note further that the compensating variation heavily depends on the degree of risk aversion. For $\theta = 2$ the compensating variation declines to 0.43%, while for $\theta = 6$ it increases to 25%.

Overall, our analysis indicates that in most cases the welfare costs of neglecting uncertainty are small.[14] However, this result depends on the type of pension scheme and the degree of risk aversion. For a funded DC scheme, for example, the welfare costs of the point-forecast decision rule can be really large.

[14] Although their analysis only considers mortality risk, Alho and Määttänen (2006) come to the same preliminary conclusion.

7 Concluding remarks

Two questions were at the centre of this paper. First, what type uncertainty is more important: economic or demographic? Second, what are the consequences for consumer behaviour of including uncertainty? We analysed these questions in the context of a two-period overlapping generations model. The overall conclusion of the assessment of the importance of economic uncertainty vis-à-vis demographic uncertainty is that for a consumer facing both types of uncertainty, it is economic uncertainty that matters the most. As to the second question, the consequences for behaviour of including uncertainty, the results suggest that the average welfare costs of not taking uncertainty into account are fairly low.

The sensitivity analysis revealed several non-monotonic relations between exogenous parameters and the certainty equivalents that warrant a further investigation. One obvious extension therefore is to replace the standard CRRA utility function with a utility function where the risk aversion and the intertemporal elasticity of substitution can be separated. Not only can we analyse in more detail the double role of θ in the current model, we can also take account of the equity premium puzzle that we passed over in this paper by our choice for θ.

Another extension is to endogenize portfolio selection. In the present version, consumers and pension funds have a fixed portfolio of nominal bonds and equity. Endogenizing portfolio choice for consumers is an obvious and straightforward extension because it gives consumers an extra instrument to diversify risk. Since the risks imposed on consumers by the mandatory pension funds differ between the four pension schemes, consumers may adapt the choice between stocks and bonds for their private portfolio accordingly. Doing this for pension funds is less straightforward because it requires the formulation of an objective function for these funds. Including only the utility of present generations may lead to excessive investment in equity in funded DB schemes since the risks on these investments are (partly) transferred to future, unrepresented generations.

Appendix A Welfare analysis

Technically, the compensating variation can be derived from the value function which expresses maximal attainable utility as function of total wealth (financial wealth plus human wealth). For this purpose, the utility-maximisation problem of equation (3.5) can be written in a more general form by the following Bellman equation:

$$V_t(a_{t-1}) = \max_{c_t} \frac{c_t^{1-\theta}-1}{1-\theta} + \frac{\varepsilon_{t+1}}{1+\rho} V_{t+1}(a_t) \tag{A.1}$$

Financial wealth, a_t, evolves according to:

$$a_t = \frac{1+r_t}{\varepsilon_t} a_{t-1} + w_t - c_t \tag{A.2}$$

where w_t is age-specific (disposable) income. Assume that the value function has the following solution:[15]

$$V_t(a_{t-1}) = H_t^{\theta} \left[\frac{\left(a_{t-1} + \frac{\varepsilon_t K_t}{1+r_t}\right)^{1-\theta} - \phi}{1-\theta} \right] \tag{A.3}$$

where H_t is an arbitrary constant and K_t is human wealth. The constant ϕ is a shift parameter that ensures that the value function exactly gives the same outcome as equation (A.1).[16] Substituting the value function in the Bellman equation and solving the resulting maximisation problem, we obtain the following expressions for H_t and K_t:

$$H_t = \left(\frac{1+r_t}{\varepsilon_t}\right)^{\frac{1}{\theta}-1} (1+\eta_t H_{t+1}) \tag{A.4}$$

$$K_t = w_t + \frac{K_{t+1}}{1+r_{t+1}} \tag{A.5}$$

Denoting the original utility level by superscript ’o’ and the new utility level by superscript ’n’, the compensating variation (CV) is defined as:

$$\begin{aligned} 0 &= V_t^{\mathrm{o}}(a_{t-1}) - V_t^{\mathrm{n}}(a_{t-1} + CV_t) \\ 0 &= H_t^{\mathrm{o}\theta} \left[\frac{\left(a_{t-1}^{\mathrm{o}} + \frac{\varepsilon_t^{\mathrm{o}} K_t^{\mathrm{o}}}{1+r_t^{\mathrm{o}}}\right)^{1-\theta} - \phi_t^{\mathrm{o}}}{1-\theta} \right] - H_t^{\mathrm{n}\theta} \left[\frac{\left(a_{t-1}^{\mathrm{n}} + CV_t + \frac{\varepsilon_t^{\mathrm{n}} K_t^{\mathrm{n}}}{1+r_t^{\mathrm{n}}}\right)^{1-\theta} - \phi_t^{\mathrm{n}}}{1-\theta} \right] \end{aligned} \tag{A.6}$$

Solving CV_t from equation (A.6) gives:

$$CV_t = \left\{ \left(\frac{H_t^{\mathrm{o}}}{H_t^{\mathrm{n}}}\right)^{\theta} \left[\left(a_{t-1}^{\mathrm{o}} + \frac{\varepsilon_t^{\mathrm{o}} K_t^{\mathrm{o}}}{1+r_t^{\mathrm{o}}}\right)^{1-\theta} - \phi^{\mathrm{o}} \right] + \phi^{\mathrm{n}} \right\}^{\frac{1}{1-\theta}} - a_{t-1}^{\mathrm{n}} - \frac{\varepsilon_t^{\mathrm{n}} K_t^{\mathrm{n}}}{1+r_t^{\mathrm{n}}} \tag{A.7}$$

[15] For HARA class utility (as CRRA utility), the value function has the same form as the utility function (Merton (1990)).

[16] It is easy to show that $\phi = 1$ for retired consumers in the two-period OLG model.

Equations (A.4) and (A.5) can be worked out for our two-period OLG model. Since $w_{1t} \equiv (1-\pi_t)y_t$ and $w_{2t} \equiv b_t$ and it is assumed that there are no deaths in the first period of life ($\varepsilon_{1t} = 1$), we have:

$$
\begin{aligned}
K_{1t} &= (1-\pi_t)y_t + \frac{\varepsilon_{t+1}b_{t+1}}{1+r_{t+1}} \equiv y_t^{LF} \\
K_{2t} &= b_t
\end{aligned}
\tag{A.8}
$$

and

$$
\begin{aligned}
H_{1t} &= (1+r_t)^{\frac{1}{\theta}-1}\left[1+\left(\frac{\varepsilon_{t+1}}{1+\rho}\right)^{\frac{1}{\theta}}\left(\frac{1+r_{t+1}}{\varepsilon_{t+1}}\right)^{\frac{1}{\theta}-1}\right] \\
H_{2t} &= \left(\frac{1+r_t}{\varepsilon_t}\right)^{\frac{1}{\theta}-1}
\end{aligned}
\tag{A.9}
$$

Substituting equations (A.8) and (A.9), together with $a_{1t} = s_t$, in (A.7), we ultimately obtain:

$$
CV_{1t} = \left[\left(\frac{H_{1t}^{\mathrm{o}}}{H_{1t}^{\mathrm{n}}}\right)^{\theta}\left(\left(\frac{y_t^{LF\mathrm{o}}}{1+r_t^{\mathrm{o}}}\right)^{1-\theta} - \phi^{\mathrm{o}}\right) + \phi^{\mathrm{n}}\right]^{\frac{1}{1-\theta}} - \frac{y_t^{LF\mathrm{n}}}{1+r_t^{\mathrm{n}}} \tag{A.10}
$$

$$
CV_{2t} = \left(\frac{H_{2t}^{\mathrm{o}}}{H_{2t}^{\mathrm{n}}}\right)^{\frac{\theta}{1-\theta}}\left(s_{t-1}^{\mathrm{o}} + \frac{\varepsilon_t^{\mathrm{o}}b_t^{\mathrm{o}}}{1+r_t^{\mathrm{o}}}\right) - s_{t-1}^{\mathrm{n}} - \frac{\varepsilon_t^{\mathrm{n}}b_t^{\mathrm{n}}}{1+r_t^{\mathrm{n}}} \tag{A.11}
$$

Appendix B The computations of the certainty equivalents

From the stochastic simulations we know the maximal expected utility:

$$\overline{U} = \mathrm{E}_t U(\overline{c}_{1t}, \overline{c}_{2t+1}) = \frac{\overline{c}_{1t}^{1-\theta} - 1}{1-\theta} + \frac{1}{1+\rho} \mathrm{E}_t \left[\varepsilon_{t+1} \frac{\overline{c}_{2t+1}^{1-\theta} - 1}{1-\theta} \right] \tag{B.1}$$

What we want to know is how much income y_t (or, equivalently, lifetime income y_t^{LF}) people are willing to forgo to avoid the economic and/or demographic uncertainty. That is, what certain income level $\widetilde{y}_t$ (with matching consumption levels $\widetilde{c}_{1t}$ and $\widetilde{c}_{2t+1}$) gives just the same utility level as under uncertainty, $\overline{U}$:

$$\frac{\widetilde{c}_{1t}^{1-\theta} - 1}{1-\theta} + \frac{1}{1+\rho} \left(\varepsilon_{t+1} \frac{\widetilde{c}_{2t+1}^{1-\theta} - 1}{1-\theta} \right) = \overline{U} \tag{B.2}$$

We are interested in three certainty equivalents:

- avoiding all uncertainty;
- avoiding only economic uncertainty;
- avoiding only demographic uncertainty.

B.1 Avoiding all uncertainty

For a given certain level of income $\widetilde{y}_t$, consumption in both periods equals:

$$\begin{aligned} \widetilde{c}_{1t} &= (1 - s(r_{t+1}, \varepsilon_{t+1}))F\widetilde{y}_t \\ \widetilde{c}_{2t+1} &= \frac{1+r_{t+1}}{\varepsilon_{t+1}} s(r_{t+1}, \varepsilon_{t+1}) F \widetilde{y}_t \end{aligned} \tag{B.3}$$

where

$$F = 1 - \pi_t + \frac{\varepsilon_{t+1}}{1+r_{t+1}} \frac{b_{t+1}}{y_t} \tag{B.4}$$

Note that, in case of certainty, for all pension schemes we have $b_{t+1}/y_t = (1+g_{t+1})\alpha$. Inserting the consumption levels into equation (B.2) and solving for $\widetilde{y}_t$ gives:

$$\widetilde{y}_t = \left(\frac{\overline{U} + \frac{1+\rho+\varepsilon_{t+1}}{(1-\theta)(1+\rho)}}{D} \right)^{\frac{1}{1-\theta}} \tag{B.5}$$

with

$$D = \frac{[(1-s(r,\varepsilon))F]^{1-\theta}}{1-\theta} + \frac{\varepsilon_{t+1}}{1+\rho} \frac{\left[\frac{1+r_{t+1}}{\varepsilon_{t+1}} s(r,\varepsilon) F\right]^{1-\theta}}{1-\theta} \tag{B.6}$$

Since uncertainty has a different impact in each of the four pension schemes, we have to compute the certainty equivalent income for each of the pension schemes.

B.2 Avoiding economic or demographic uncertainty

To compute the certainty equivalent for economic or demographic uncertainty only is more involved because it implies comparing full uncertainty with partial uncertainty and, in general, we cannot derive explicit solution for either situation. The question we want answer is comparable to the question in the previous section of this appendix: what income level $\hat{y}_t$ (under only economic or demographic uncertainty and with matching consumption levels $\hat{c}_{1t}$ and $\hat{c}_{2t+1}$) gives just the same utility level as under full uncertainty, $\overline{U}$:

$$\frac{\hat{c}_{1t}^{1-\theta}-1}{1-\theta}+\frac{1}{1+\rho}\mathrm{E}_t\left[\varepsilon_{t+1}\frac{\hat{c}_{2t+1}^{1-\theta}-1}{1-\theta}\right]=\overline{U} \tag{B.7}$$

Furthermore, the first-order condition has to hold:

$$\hat{c}_{1t}^{-\theta}=\frac{1}{1+\rho}\mathrm{E}_t\left[\hat{c}_{2t+1}^{-\theta}(1+r_{t+1})\right] \tag{B.8}$$

Finally, for each state of nature i it must hold that:

$$\hat{c}_{2t+1}^{i}=\frac{1+r_{t+1}^{i}}{\varepsilon_{t+1}^{i}}\left[\hat{F}^{i}\hat{y}_t-\hat{c}_{1t}\right] \tag{B.9}$$

with $\hat{F}^i=(1-\pi_t)+\frac{\varepsilon_{t+1}^{i}}{1+r_{t+1}^{i}}\frac{b_{t+1}^{i}}{\hat{y}_t}$.

By inserting the last equation for $\hat{c}_{2t+1}$ in the other two, we end up with the following system of two non-linear equations in $\hat{c}_{1t}$ and $\hat{y}_t$:

$$f^1(\hat{c}_{1t},\hat{y}_t)=\frac{\hat{c}_{1t}^{1-\theta}-1}{1-\theta}+\frac{1}{1+\rho}\frac{\mathrm{E}_t\left[\varepsilon_{t+1}(\hat{F}\hat{y}_t-\hat{c}_{1t})^{1-\theta}\left(\frac{1+r_{t+1}}{\varepsilon_{t+1}}\right)^{1-\theta}\right]-1}{1-\theta}-\overline{U}=0 \tag{B.10}$$

$$f^2(\hat{c}_{1t},\hat{y}_t)=\hat{c}_{1t}^{-\theta}-\frac{1}{1+\rho}\mathrm{E}_t\left[\varepsilon_{t+1}\left(F\hat{y}_t-\hat{c}_{1t}\right)^{-\theta}\left(\frac{1+r_{t+1}}{\varepsilon_{t+1}}\right)^{1-\theta}\right]=0 \tag{B.11}$$

To solve this system of equations we have to approach the two expectations terms by:

$$\mathrm{E}_t\left[\varepsilon_{t+1}\left(\hat{F}\hat{y}_t-\hat{c}_{1t}\right)^{-\theta}\left(\frac{1+r_{t+1}}{\varepsilon_{t+1}}\right)^{1-\theta}\right]=\frac{1}{N}\sum_{i=1}^{N}\varepsilon_{t+1}^{i}\left(\hat{F}^{i}\hat{y}_t-\hat{c}_{1t}\right)^{-\theta}\left(\frac{1+r_{t+1}^{i}}{\varepsilon_{t+1}^{i}}\right)^{1-\theta} \tag{B.12}$$

and

$$\mathrm{E}_t\left[\varepsilon_{t+1}\left(\hat{F}\hat{y}_t-\hat{c}_{1t}\right)^{1-\theta}\left(\frac{1+r_{t+1}}{\varepsilon_{t+1}}\right)^{1-\theta}\right]=\frac{1}{N}\sum_{i=1}^{N}\varepsilon_{t+1}^{i}\left(\hat{F}^{i}\hat{y}_t-\hat{c}_{1t}\right)^{1-\theta}\left(\frac{1+r_{t+1}^{i}}{\varepsilon_{t+1}^{i}}\right)^{1-\theta} \tag{B.13}$$

B.3 Welfare cost of neglecting uncertainty

Let the optimising consumer only have a point forecast (PFC) regarding the random variables. Solving the maximisation problem of equation (3.5) then gives for first period consumption:

$$c_{1t} = y_t M_t \tag{B.14}$$

$$M_t = \mathrm{E}_t\Big|_{PFC}\Big[(1 - s(r_{t+1}, \varepsilon_{t+1})F_t\Big] \tag{B.15}$$

with F_t defined in equation (B.4). Substituting equation (B.14) in the intertemporal budget constraint, we derive for second period consumption:

$$c_{2t+1} = \frac{1 + r_{t+1}}{\varepsilon_{t+1}} y_t N_t \tag{B.16}$$

$$N_t = F_t - M_t \tag{B.17}$$

Note that c_{1t} is deterministic, while c_{2t+1} is random. In addition, since F_t depends on the type of pension scheme, the same holds for M_t and N_t.

Now we ask the question what income level, say $\check{y}_t$ (with corresponding consumption levels $\check{c}_{1t}$ and $\check{c}_{2t+1}$), gives a consumer that is endowed with only a point forecast the same utility as he would get under rational expectations (REX). That is,

$$U^{REX} = \frac{\check{c}_{1t}^{1-\theta} - 1}{1 - \theta} + \frac{1}{1 + \rho}\mathrm{E}_t\left[\varepsilon_{t+1}\frac{\check{c}_{2t+1}^{1-\theta} - 1}{1 - \theta}\right] \tag{B.18}$$

Substituting equations (B.14) together with (B.16) in (B.18) and solving for $\check{y}_t$ gives:

$$\check{y}_t = \left(\frac{U^{REX} + \frac{1+\rho+\varepsilon_{t+1}}{(1-\theta)(1+\rho)}}{D}\right)^{\frac{1}{1-\theta}} \tag{B.19}$$

$$D = \frac{1}{1 - \theta} M_t^{1-\theta} + \frac{1}{(1 + \rho)(1 - \theta)}\mathrm{E}_t\left[\varepsilon_{t+1}\left(\frac{1 + r_{t+1}}{\varepsilon_{t+1}} N_t\right)^{1-\theta}\right] \tag{B.20}$$

Finally, the compensating variation (CV) is the compensating payment that leaves the less-informed consumer as well off as a well-informed consumer. Thus,

$$CV_t = \check{y}_t - y_t \tag{B.21}$$

Appendix C Lifetime income under certainty and uncertainty

From the definition of lifetime income, equation 3.4, it follows that expected lifetime income equals:

$$\mathrm{E}_t y_t^{LF} = y_t(1-\pi_t) + \mathrm{E}_t\left[\frac{\varepsilon_{t+1}}{1+r_{t+1}} b_{t+1}\right] \tag{C.1}$$

When there is no uncertainty the random variables always equal their expected values and the certain lifetime income is given by:

$$y_t^{LF,C} = y_t(1-\pi_t) + \left(\frac{\mathrm{E}_t\varepsilon_{t+1}}{1+\mathrm{E}_t r_{t+1}} \mathrm{E}_t b_{t+1}\right) \tag{C.2}$$

Combining these two equations we get:

$$\mathrm{E}_t y_t^{LF} = y_t^{LF,C} + \mathrm{E}_t\left[\frac{\varepsilon_{t+1}}{1+r_{t+1}} b_{t+1}\right] - \left(\frac{\mathrm{E}_t\varepsilon_{t+1}}{1+\mathrm{E}_t r_{t+1}} \mathrm{E}_t b_{t+1}\right) \tag{C.3}$$

By inserting the appropriate equation for b_{t+1} it follows that for the funded DC scheme expected lifetime income equals certain lifetime income. For the two DB pension schemes the following equation can be derived:

$$\mathrm{E}_t y_t^{LF} = y_t^{LF,C} + \mathrm{E}_t\varepsilon_{t+1}\left(\mathrm{E}_t\left[\frac{1+g_{t+1}}{1+r_{t+1}}\right] - \frac{1+\mathrm{E}_t g_{t+1}}{1+\mathrm{E}_t r_{t+1}}\right)\alpha y_t \tag{C.4}$$

For the PAYG DC scheme the following equation can be derived:

$$\mathrm{E}_t y_t^{LF} = y_t^{LF,C} + (1+\mathrm{E}_t n_{t+1})\left(\mathrm{E}_t\left[\frac{1+g_{t+1}}{1+r_{t+1}}\right] - \frac{1+\mathrm{E}_t g_{t+1}}{1+\mathrm{E}_t r_{t+1}}\right)\alpha y_t \tag{C.5}$$

Using that $\mathrm{E}[A/B] = \mathrm{E}[A]\cdot\mathrm{E}[1/B] + \mathrm{cov}(A, 1/B)$ and applying Jensen's inequality that $\mathrm{E}[1/B] \geq 1/\mathrm{E}[B]$, we get for the two DB schemes:

$$\mathrm{E}_t y_t^{LF} \geq y_t^{LF,C} + \mathrm{E}_t\varepsilon_{t+1}\mathrm{cov}(1+g_{t+1}, \frac{1}{1+r_{t+1}})\alpha y_t \tag{C.6}$$

For the PAYG DC scheme we get

$$\mathrm{E}_t y_t^{LF} \geq y_t^{LF,C} + (1+\mathrm{E}_t n_{t+1})\mathrm{cov}(1+g_{t+1}, \frac{1}{1+r_{t+1}})\alpha y_t \tag{C.7}$$

Hence, if $\mathrm{cov}(1+g_{t+1}, \frac{1}{1+r_{t+1}}) > 0$ expected lifetime income under uncertainty is higher than lifetime income under certainty.

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