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Varieties and the terms of trade

Frederik Huizinga and Sjak Smulders[a]

[a] Frederik Huizinga is affiliated with the CPB Netherlands Bureau for Economic Policy Analysis and Sjak Smulders with Tilburg University. Corresponding author: Free Huizinga, P.O. Box 80510, 2508 GM The Hague, the Netherlands. Email: Huizinga@cpb.nl. The authors thank George Gelauff, Coen Teulings, Ruud Okker and participants at various seminars.

CPB Netherlands Bureau for Economic Policy Analysis
Van Stolkweg 14
P.O. Box 80510
2508 GM The Hague, the Netherlands

Telephone +31 70 338 33 80
Telefax +31 70 338 33 50
Internet www.cpb.nl

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Abstract in English

This paper analyzes the dynamic adjustment of the terms of trade in an intertemporal, two country model with endogenous product variety. In the base model all workers are identical. In an extended version the development of new varieties requires skilled labor while manufacturing uses skilled and unskilled labor. In the model without skill, a population increase in one of the countries has no effect on its terms of trade, not even in the short run. In the model with skill, the terms of trade initially worsen, but eventually return to their original level. The terms of trade immediately and permanently worsen in response to a productivity increase in manufacturing. However, they gradually improve if the productivity in variety research rises. If productivity in both activities rises equiproportionally, the terms of trade respond in the same manner as after a population shock.

Key words: terms of trade, product variety, scale effect, productivity

JEL codes: F12, F41, O31, O41

Abstract in Dutch

Dit artikel bestudeert de aanpassing van de ruilvoet in een intertemporeel tweelandenmodel met een endogeen aantal gedifferentieerde producten. In het basismodel zijn alle arbeiders in dezelfde mate geschoold. In een uitgebreide versie vereist de ontwikkeling van nieuwe variëteiten geschoolde arbeid terwijl de productiesector geschoolde en ongeschoolde arbeid gebruikt. In het model zonder geschoolde arbeid heeft een verandering in de bevolkingsomvang van een land geen enkel effect op diens ruilvoet, zelfs niet op de korte termijn. In het model met geschoolde arbeid verslechtert de ruilvoet op de korte termijn, maar keert deze op de lange termijn terug naar zijn oorspronkelijke waarde. De ruilvoet verslechtert onmiddellijk en permanent na een stijging van de productiviteit in de productiesector. De ruilvoet verbetert geleidelijk na een stijging van de productiviteit van onderzoekswerk. Als de productiviteit in beide activiteiten stijgt, reageert de ruilvoet net zo als na een schok in de bevolkingsomvang.

Steekwoorden: ruilvoet, groei, product differentiatie, schaaleffecten, productiviteit

Contents

1 Introduction and summary

Expansion of the labor force participation and raising productivity are two important policy objectives in many countries. In both cases domestic output rises. However, the welfare effect of an increase in output is not clear. A common view in trade and growth theory is that increased domestic output causes a deterioration of the terms of trade. The reason is that the higher level of domestic output can only be sold in the world market if its relative price falls. As a result, welfare rises less than output, and in extreme cases, may even fall. Many applied policy models, such as the Federal Reserve Board's multi-country dynamics general equilibrium model SIGMA (see e g. Gagnon (2008)) and the macro-econometric model of CPB, SAFFIER (see Kranendonk and Verbruggen (2007)), incorporate this effect.

Recently however, models such as Peretto and Smulders (2002) and Young (1998), have been developed that allow for an expansion of the domestic labor supply without suffering a loss in the terms of trade, at least not in the long run. The key ingredient of these models is an endogenous number of varieties that may be produced in a country. This opens up the possibility that a positive labor supply shock does not lead to increased output per variety (with an accompanying worsening of the terms of trade) but to an expansion of the number of varieties. In these models, the number of varieties typically rises proportionally with the domestic labor supply, so that employment per variety remains constant. Since new and existing varieties are fully symmetric, new varieties command the same price as existing ones, and the terms of trade are unaffected by the expansion of domestic supply. The negative relationship between output and price still holds at the level of individual firms, but not necessarily at the level of a country.[1]

Almost all variety models, however, only analyze steady states. In particular, we know of no growth model that analyses the dynamic adjustment of the terms of trade after a shock to domestic output. Empirical evidence suggests that there is a loss in the terms of trade in the short run, even if there is none in the long run. Intuitively, one would also expect such a result from the variety literature point of view, simply because it takes time to develop new varieties. In this case, the short run welfare effect of an economic expansion is overstated by a steady state analysis.

This paper focuses on the dynamic adjustment of the terms of trade after several population and productivity shocks. We build an intertemporal two country general equilibrium model with optimizing consumers and producers, a perfect labor market within each country, free trade and perfect international capital markets. All consumers have the same Dixit-Stiglitz utility function.

[1] In an interesting twist, Corsetti et al. (2007) show that a expansion of the domestic labor supply may actually *improve* the terms of trade if there is a bias for domestic products. They also show that transport cost effectively create such a home bias. The reason is that with home bias, an expansion of the domestic labor supply shifts the world market for goods in favor of domestic goods so that the number of domestic varieties rises more than proportionally with the rise in the domestic population

Each variety is produced by a single firm that has a worldwide monopoly right to produce this variety.

We start out with a benchmark model in which the number of varieties, or firms, in each country is exogenous. We confirm the standard result that a country with a rising domestic output (caused by a rise in manufacturing productivity or in the labor supply) experiences a loss in its terms of trade. However, a rise in manufacturing productivity and labor supply are not the same in all respects. After a rise in the labor supply, the lower terms of trade translate into a lower wage and lower profits per capita. However, profits per firm and the value of a firm rise. Hence, the ratio of the value of a firm to the wage rises as well, leading to an economic imbalance. The incentive for new firms to be created has risen. Standard trade models would allow for entry of firms producing the same goods as existing firms until excess profits are zero. This still would expand output of existing products and worsen the terms of trade. After a rise in manufacturing productivity, however, the wage rises, despite the loss in the terms of trade. Now, wages and profits per firm rise in tandem, and the incentives to create new firms do not change.

The distinction between an increase in manufacturing productivity and labor supply comes to the fore in the subsequent section, where we allow new firms to be created by performing research to develop a new variety. We initially assume that all workers within a country have the same skill and therefore earn the same wage. We show that, also in this case, an increase in manufacturing productivity leads to an immediate deterioration of the terms of trade. This makes sense, since an expansion of manufacturing productivity does not lead to an incentive to create additional firms in the first place. Profits rise, but at the same time the opportunity cost of R&D rises as well, so that R&D does not become more profitable. With a constant number of firms, higher manufacturing productivity simply raises output per firm and reduces the terms of trade without dynamic adjustment.

As argued above, an expanding labor supply does lead to an incentive to start new firms, and the number of firms expands. Rather surprisingly, and contrary to our intuition, we find that the terms of trade do not change at all after a shock to a country’s labor supply, not even in the short run. This result may be understood as follows. Both the cost and the benefit of a unit of research are independent of the scale of research. As a result, research can absorb a flexible number of workers without affecting relative prices. Existing firms, however, can only absorb more workers at the cost of a reduction in relative prices. The market outcome avoids this loss in relative prices by initially letting (most of) [2] the additional workers perform research. As a result, the number of varieties in the country expands. Over time the expanding number of firms absorb more and more of the expansion in the labor supply, until the ratio of workers in research and

[2] We say "most of" because an increase in the population of a country also raises the interest rate, which raises employment per firm in both countries. So, some of the additional workers in the expanding country start out in manufacturing. However, since employment per firm rises in both countries, the terms of trade are not affected. Moreover, if the expanding country is relatively small, the rise in interest rate may be ignored.

manufacturing (the existing firms) has returned to its original level. At that point the country has expanded completely proportionally, with all individual firms producing the same amount of output as before the shock.

The result that the terms of trade do not change might sound counterintuitive. Empirical evidence suggests that, in the short run, there is a terms of trade effect. And one does not usually think of research as the residual absorber of population shocks. One objection, for instance, is that research is a specialized activity that not all workers can perform equally well.

To explore the consequences of this objection, we introduce skill differences among workers. Individuals exogenously have either high or low skill. Research can only be performed by high-skilled labor, while manufacturing uses both, according to a Cobb-Douglas production function. We now find that after a population shock, the terms of trade initially fall, but eventually rise again to their former level. In other words, there is a short term loss in the terms of trade.

The intuition for this result is as follows. It cannot be the case that all additional workers start out in manufacturing. Then we would have the outcome of the benchmark model, with no additional research being performed and no additional varieties being created. The resulting loss in the terms of trade and the relative wage, combined with the rising profits per firm would raise the rate of return to entry, and entry would jump up. Therefore, some of the additional workers must start out in research, increasing the growth rate of the number of firms in their country. However, the research sector can no longer absorb (almost) all of the additional workers either. Since research is skill-intensive, an expanding research sector raises the relative demand for skilled labor. This raises its relative wage and thus the cost of research. This puts a break on the ability of research to be the residual absorber of a population shock.

So, employment in manufacturing and research both rise. The increase in manufacturing employment initially raises output per firm, resulting in a loss in the terms of trade. Over time, the additional research raises the number of firms. Output per firm falls and the terms of trade rise. The expanding number of firms absorbs more and more research workers until, in equilibrium, the ratio of research workers to manufacturing workers is the same as in the old equilibrium. In the new equilibrium, output per firm, wages and the terms of trade all have gone back to their old levels.

In addition, this model allows us to analyze the effects of an increase in the ratio of skilled to unskilled workers. We find that an increase in this ratio raises the terms of trade in the long-run equilibrium. The reason is that the increased relative supply of skilled workers reduces their relative wage, and thus the relative cost of research. Research increases, and the number of varieties rises. Output per variety falls and the terms of trade improve.

On impact, however, the terms of trade may either rise or fall, depending on the initial fraction of skilled workers. If this fraction is very low, the increase in the availability of skilled workers raises productivity in manufacturing so much that output per firm initially rises and the terms of trade worsen. Over time, however, the increase in the number of firms unambiguously reduces output per firm and raises the terms of trade.

Finally, we analyze the effect of raising productivity. A positive shock in the productivity in manufacturing still has no effect on the number of firms. Thus, it again simply raises output per firm and reduces the terms of trade without dynamic adjustment. A positive shock in the productivity of research raises the number of firms in equilibrium, reduces output per firm and raises the terms of trade. Because the increase in the number of firms is gradual, so is the improvement in the terms of trade. A positive proportional shock in research and manufacturing has the same effect on domestic output and the terms of trade as a rise in the population. The only difference is that now the rise in output is not spread out over more individuals. So, output per capita is higher after a productivity shock than after a population shock.

In an appendix we show that all versions of the model are globally stable and have a unique equilibrium.

The model closest to ours in the literature is Wälde (1966). He analyses a model that is the same as our model with homogeneous skill. However, his focus is different, namely the stability properties of the model. He shows that his model is globally stable. In terms of his focus, our paper extends his result to a model with different productivity and skill levels. However, our main contribution lies in the dynamics of the terms of trade. The model of Wälde, like our model with homogeneous labor, has the property that the terms of trade are constant throughout the adjustment path, but Waelde does not address this issue. He also does not analyze productivity shocks.

Arnold (2007) also studies the transitional dynamics in trade models. He, however, restricts the analysis to growth models in which international trade can lead to factor price equalization and studies the dynamics of equilibria with factor price equalization. This rules out terms of trade effects, since with all prices and factor prices the same across countries at any point in time, relative prices obviously stay constant.

Models of trade dynamics and skills are scarce in the literature. Grossman and Helpman (1991), chapters 7, 8, and 9, do allow for skilled and unskilled labor in their models of growth and trade, but analyze specialization patterns in the long-run only. Vandenbussche et al. (2006) model different skill intensities in imitation and innovation to explain how the skill requirements for growth change as a country catches up to the international production frontier. Apart from a different focus, their model differs from ours in that it is a two period model, has no goods production sector and does not allow for entry. The idea that skilled labor may be a bottleneck in the dynamics adjustment of the terms of trade is, to the best of our knowledge, new.

The empirical literature on the effects of an expansion of domestic output on the terms of trade is mixed. Acemoglu and Ventura (2002) take the conventional view that a domestic expansion reduces the terms of trade. They show that this effectively introduces decreasing returns to capital within any country, and as a result, a stable world income distribution.

However, their empirical analysis also points to a positive correlation between human capital and the terms of trade. If we interpret an increase in the capital stock as an increase in labor productivity in manufacturing and an increase in human capital as an increase in the skill ratio, then their findings are consistent with our model. Corsetti et al. (2007) report that in a panel regression for 20 OECD countries over the period 1980-2004, the effect of a country's GDP growth on its terms of trade is basically zero. However, if R&D is added to the regression, the effect of GDP growth becomes significantly negative, while the effect of R&D is significantly positive. This is in line with our model, if we assume that R&D is helpful to create new varieties, while GDP growth that is not associated with R&D simply consists of raising output per variety.

The paper is organized as follows. The next section derives the consumer demand functions, including export and import functions from a Dixit-Stiglitz type utility function. Section 3 presents the benchmark model in which labor is homogeneous and the number of varieties in each country is exogenous. In section 4 we endogenize the number of varieties and in section 5 we introduce different skill levels. In the appendix we prove that all models used are globally stable.

2 Consumption

The world consists of two countries, A and B. The superscript $m \in \{A, B\}$ refers to a country in general. The superscripts A, B, W and R refer to the specific countries A and B, the whole world and the ratio of the specific variable in countries A and B. So, for any variable x, $x^R = \frac{x^A}{x^B}$. The countries engage in free trade of goods and financial assets. There are no transportation costs. Country m has a continuum of individuals with mass L^m, called the population of country m. The populations of both countries are constant, except for a possible exogenous unexpected shock. All individuals inelastically supply one unit of labor. Countries A and B produce a continuum of (consumer) goods located along the intervals $[-N^A, 0)$ and $[0, N^B]$. Define $N^W = N^A + N^B$. We will refer to N^m as the number of goods produced by the countries.

The subscripts i, j and t refer to a good, an individual and time. Depending on the context, individual j is called worker j, consumer j, or just generically, individual j. The variables p, d, c and q denote the price, spending, consumption and production of goods. Lower case letters refer to single agents (individuals or firms), and upper case letter to macro aggregates. For example, c^m_{ijt} is the consumption of good i by consumer j in country m at time t, q^A_{it} the production of good i in country A at time t, and Q^B_t total output of goods in country B at time t.

All consumers have identical preferences. The utility function of consumer j in country m is

$$U^m_j = \int_0^\infty \log \left[\int_{-N^A}^{N^B} \left(c^m_{ijt}\right)^\eta di \right]^{\frac{1}{\eta}} e^{-\rho t}\, \mathrm{d}t \tag{2.1}$$

where η and ρ are parameters, with $0 < \eta < 1$ and $\rho > 0$. Define $\sigma = \frac{1}{1-\eta}$, so that $\sigma \in (1, \infty)$. σ is the elasticity of substitution between any two products. The consumer faces the intertemporal budget constraint

$$\int_0^\infty d^m_{jt} e^{-\int_0^t r_\omega d\omega} dt = \int_0^\infty w^m_{jt} e^{-\int_0^t r_\omega d\omega} dt + a^m_{j0} \tag{2.2}$$

where w^m_{jt} is the wage rate and a^m_{jt} the non-wage wealth of consumer j, both in country m at time t. r_t is the rate of interest on financial assets at time t. Free trade in financial assets ensures that the rate of interest is the same in both countries. d^m_{jt} is the consumer's spending on goods at time t, given by

$$d^m_{jt} = \int_{-N^A}^{N^B} p_{it} c^m_{ijt} di \tag{2.3}$$

where p_{it} is the price of good i at time t. Because of free trade, this price is the same in both countries.

For any variable x, $\dot{x}$ denotes its time derivative: $\dot{x} = \frac{\mathrm{d}x}{\mathrm{d}t}$ and $\hat{x}$ denotes its growth rate: $\hat{x} = \frac{\mathrm{d}x}{\mathrm{d}t} \frac{1}{x}$. The first order conditions for optimal consumption by consumer j in country m imply

$$d^m_{j0} = \rho \left[\int_0^\infty w^m_{jt} e^{-\int_0^t r_\omega \mathrm{d}\omega} \mathrm{d}t + a^m_{j0} \right] \tag{2.4}$$

$$d^m_{jt} = d^m_{j0} e^{-\rho t + \int_0^t r_\omega \,\mathrm{d}\omega} \tag{2.5}$$

$$\hat{d}^m_{jt} = r_t - \rho \tag{2.6}$$

$$c^m_{ijt} = \frac{p_{it}^{-\sigma}}{P_t^W} d^m_{jt} \tag{2.7}$$

$$P_t^W = \int_{-N^A}^{N^B} p_{it}^{1-\sigma} di \tag{2.8}$$

In the remainder of the paper the subscript t is dropped unless doing so is confusing.

Let D^m denote country m's total spending on goods, c_i^m its total consumption of good i, C^m its total consumption, and A^m its total non-wage wealth. Then $D^m = \int_0^{L^m} d_j^m \,\mathrm{d}j$, $c_i^m = \int_0^{L^m} c_{ij}^m \,\mathrm{d}j$, $C^m = \int_{-N^A}^{N^B} c_i \,\mathrm{d}i$, and $A^m = \int_0^{L^m} a_j^m \,\mathrm{d}j$. It follows that:

$$D_0^m = \rho \left[\int_0^{L^m} \int_0^\infty w^m_{jt} e^{-\int_0^t r_\omega \,\mathrm{d}\omega} \,\mathrm{d}t \,\mathrm{d}j + A_0^m \right] \tag{2.9}$$

$$\hat{D}^m = r - \rho \tag{2.10}$$

$$c_i^m = \frac{D^m}{P^W} p_i^{-\sigma} \tag{2.11}$$

$$C^m = \frac{D^m}{P^W} \int_{-N^A}^{N^B} p_i^{-\sigma} \,\mathrm{d}i \tag{2.12}$$

Equation 2.9 shows that the non-wage wealth effects on macro spending on goods by country m only depend on the total non-wage wealth of the country, not on how it is distributed across its individuals. Moreover, an unexpected redistribution of this wealth does not affect macro spending either. This means that after an unexpected shock to a country's population, it does not matter how the existing total wealth is divided up over the expanded population, as long as any redistribution is also unexpected. We assume that all individuals start out with the same level of initial wealth, and that after a population shock, the existing non-labor wealth is unexpectedly divided equally over all individuals. So, all individuals in a country always are alike, also in terms of non-wage wealth.

Let D^W denote worldwide spending on goods, $D^W = D^A + D^B$. Let c_i^w denote worldwide consumption of good i, $c_i^W = c_i^A + c_i^B$. Then

$$\hat{D}^W = r - \rho \tag{2.13}$$

$$c_i^W = \frac{D^W}{P^W} p_i^{-\sigma} \tag{2.14}$$

Let X^m and M^m denote country m's total exports and imports of goods. Country A's exports, and thus country B's imports, equal country B's spending on goods produced by country A:

$$X^A = M^B = \int_{-N^A}^0 c_i^B \,\mathrm{d}i \tag{2.15}$$

$$= \frac{D^B}{P^W} \int_{-N^A}^0 p_i^{-\sigma} \,\mathrm{d}i \tag{2.16}$$

Similarly,

$$X^B = M^A = \int_0^{N^B} c_i^A \,\mathrm{d}i \tag{2.17}$$

$$= \frac{D^A}{P^W} \int_0^{N^B} p_i^{-\sigma} \, \mathrm{d}i \tag{2.18}$$

Compared to standard trade models the innovation here is that, in addition to foreign spending and prices, exports and imports also depend on the number of products a country produces. This allows for the possibility that exports expand in equilibrium, without a loss in the terms of trade.

3 Production with an exogenous number of varieties

Each good i is produced by a profit maximizing firm who has a monopoly right on the production of that good. The labor market in each country is perfectly competitive. The goods on the interval $[-N^A, 0)$ are produced in country A and those on the interval $[0, N^B]$ in country B. All firms within a country are completely symmetric. In particular, as we will show shortly, they all charge the same price for their output. We will set the price of goods produced in country B equal to 1. The terms of trade of country A are defined as $p^R = \frac{p^A}{p^B} = p^A$. However, to preserve the symmetry in the presentation of the equations, we will generally keep writing p^B instead of 1. The symmetry between firms in a country and across countries allows us to describe the actions of a generic firm in this world, and omit the superscript m and the subscript i referring to a specific firm. Sometimes we still use these identifiers if we want to be more specific.

Production takes place with labor only, according to the following production function

$$q = H_q L_q \tag{3.1}$$

where H_q is the exogenous country specific labor productivity, which is the same for all firms in a country. L_q is an individual firm's employment level. Define $L_Q = NL_q$. L_Q is the total employment in all firms. Since in this version of the model, these firms are the only source of employment, and there is no unemployment, it must be that $L_Q = L$. However, to make the transition to the next section easier, we will use L_Q for total employment, instead of L. Similarly, we will refer to the firms collectively as the manufacturing sector and to L_Q as manufacturing employment. For each firm i, $q_i = c_i^W$, so that the firm's demand function is given by equation 2.14:

$$q = p^{-\sigma} \frac{D^W}{P^W} \tag{3.2}$$

Combining the above two equations yields

$$p = \left(\frac{ND^W}{H_q L_Q P^W} \right)^{1-\eta} \tag{3.3}$$

It follows that the terms of trade of country A, p^R, are given by

$$p^R = \left(\frac{N^R}{L_Q^R H_q^R} \right)^{1-\eta} \tag{3.4}$$

So, the terms of trade of country A fall with the relative total output (employment times productivity).This result is the standard outcome of most macro-econometric models. The higher level of output can only be sold in the world market - that is, to the domestic and foreign consumers - if the relative price of that output falls.

However, equations 3.2 and 3.4 show that the negative relationship between output and price holds at the level of individual firms, not necessarily at the level of a country. A rise in the number of varieties in a country causes total manufacturing employment to be spread out over more varieties, and thus output per variety falls. This reduction in output leads to a higher price for these varieties, and the country's terms of trade rise. If an expansion of manufacturing output were for some reason to be accompanied by a proportional rise in the number of varieties, the terms of trade of the expanding country would not fall. Since, in this section, as in most macro-econometric models, this possibility is not modeled, an expanding country always suffers a loss in its terms of trade.

The deterioration in the terms of trade of the expanding country affects other variables as well. Let v_{it}^m denote the expected value at time t of the discounted profits of the firm producing good i in country m (called firm i in country m). Then

$$v_{it}^m = \int_t^\infty \pi_{i\tau}^m e^{-\int_t^\tau r_\omega d\omega} d\tau \tag{3.5}$$

where π_{it}^m denotes the profits of firm i in country m time t. It follows from equations 3.1 and 3.2 that revenue, pq equals

$$pq = (H_q L_q)^\eta \left(\frac{D^W}{P^W}\right)^{1-\eta} \tag{3.6}$$

A perfect labor market ensures that workers all earn the same wage w. A firm's profits, π, are given by $\pi = pq - wL_q$, which equals

$$\pi = (H_q L_q)^\eta \left(\frac{D^W}{P^W}\right)^{1-\eta} - wL_q \tag{3.7}$$

Maximizing a firm's value amounts to maximizing its profits every period. The first order condition for maximizing π with respect to L_q is

$$wL_q = \eta pq \tag{3.8}$$

Substituting equation 3.1 yields

$$w = \eta p H_q \tag{3.9}$$

Since all firms within a country face the same wage, this equation implies, as claimed above, that they all charge the same price. The relative wage is given by

$$w^R = \left(\frac{N^R}{L_Q^R}\right)^{1-\eta} \left(H_q^R\right)^\eta \tag{3.10}$$

So, the relative wage of a country whose employment expands falls. If a country is able to raise its labor productivity, the relative wage rises, but less than proportionally with the rise in productivity. Both results follow directly from the fall in the terms of trade of the expanding country.

Equation 3.8 implies that $\pi = (1-\eta)pq$. Substituting equation 3.1, 3.6 and using the fact that, in equilibrium, $L_Q = NL_q$, yields

$$\pi = (1-\eta)\left(\frac{H_q L_Q}{N}\right)^\eta \left(\frac{D^W}{P^W}\right)^{1-\eta} \tag{3.11}$$

so that relative profits are

$$\pi^R = \left(\frac{H_q^R L_Q^R}{N^R} \right)^\eta \tag{3.12}$$

The relative profits of a firm rise with its labor productivity, although less that proportionally. It also rises with the relative size of the firm in terms of employment, $\frac{L_Q^R}{N^R}$. Even though the price falls with output, revenue and profits still rise with the level of output. Profits per capita, denoted π_{cap}, are equal to $\frac{\pi N}{L_Q}$, and thus equal

$$\pi_{cap} = (1-\eta) H_q^\eta \left(\frac{N}{L_Q} \right)^{1-\eta} \left(\frac{D^W}{P^W} \right)^{1-\eta} \tag{3.13}$$

and relative profits per capita are

$$\pi_{cap}^R = \left(H_q^R \right)^\eta \left(\frac{N^R}{L_Q^R} \right)^{1-\eta} \tag{3.14}$$

So, relative profits per capita of country whose employment expands fall as well.

In this version of the model, there is no dynamic adjustment, and after a shock, adjustment to the new equilibrium is immediate. In equilibrium all variables are constant, and thus, by equation 2.13, the interest r always equals the utility discount rate ρ. It follows that the value of a firm equals its instantaneous profits π divided by this discount rate. The relative value of a firm in country A, v^R therefore equals the relative profit rate, π^R.

The central conclusion of this section is that with a fixed number of varieties, a country with expanding total employment will experience a loss in its terms of trade, a lower wage and lower profits per capita. However, profits per firm and thus the value of a firm rise. The ratio of the value of a firm to the wage therefore rises, leading to an economic imbalance and a rising incentive for new firms to be created. Standard trade models ignore this tension. We explore the consequences of this tension in the next section.

A final result is that while an increase in total employment and in labor productivity H_q both lead to a loss in the terms of trade, the effect on the ratio of profits to the wage, and thus on the incentives for entry differ. A rise in H_q leaves this ratio unchanged, and therefore does not lead to a rising incentive for new firms to be created. This also will be further explored in the next section.

4 Entry

Now, assume that in both countries, new firms may enter at any time by creating a new variety of goods, and thus raising the number of goods produced in their country. After creating a new variety, the new firm gains a monopoly right to exploit it forever. New varieties are created by R&D which again only involves labor. The perfect labor market ensures that workers earn the same wage regardless of where they work.

When R&D takes place, it is no longer the case that all workers work in the manufacturing firms of the previous section. However, within the manufacturing sector, all equations of the previous section still hold. The only modification is that macro manufacturing employment, L_Q, no longer equals total employment L.

We assume that the productivity of research rises with the current total stock of knowledge in the world. This stock has been accumulated by the N^W research projects undertaken so far, resulting in the N^W current varieties. We assume that the worldwide stock of knowledge simply equals the total amount of research performed so far, that is, N^W. However, knowledge gained from new research projects is understood best by the original researchers. The benefit to other researchers is a spillover effect. We assume that these spillover are to some extent local in the sense that the spillover effects fall with the distance between researchers. We assume that the average distance between researchers rises with the size of the world, which we measure by the size of the world population. So, the spillover effects of knowledge rise with the total stock of knowledge N^W and fall with the size of the world population, L^W. Specifically, we assume that the rate at which new varieties are created in county m is

$$\dot{N}^m = H_n^m L_N^m \left(\frac{N^W}{L^W}\right)^{\phi} \tag{4.1}$$

where L_N^m is total employment used for developing new varieties in country m, and H_n^m is a country specific research productivity variable for country m. $\left(\frac{N^W}{L^W}\right)^{\phi}$ is the spillover effect from existing knowledge, and ϕ is a scale parameter common to the whole world. Note that $L_Q^m + L_N^m = L^m$.

There is free entry so that new firms will enter until doing so yields zero profits. Employing one worker for a short period of time $\mathrm{d}t$ costs $w\,\mathrm{d}t$ and yields $H_n\left(\frac{N^W}{L^W}\right)^{\phi}\mathrm{d}t$ new firms with total value $vH_n\left(\frac{N^W}{L^W}\right)^{\phi}\mathrm{d}t$. Free entry implies that

$$v = \frac{w}{H_n\left(\frac{N^W}{L^W}\right)^{\phi}} \tag{4.2}$$

$$\hat{v} = \hat{w} - \phi\hat{N}^W \tag{4.3}$$

Perfect capital markets and profit maximization of firms imply the standard no-arbitrage condition

$$r = \frac{\pi}{v} + \hat{v} \tag{4.4}$$

Remember that profits π are given by $\pi = (1-\eta)pq$. Substituting the above equations for v and $\hat{v}$ and using equations 3.2, 3.9 and the fact that, by equation 3.9, $\hat{w} = \hat{p}$, we find for country m

$$r = \frac{(p^m)^{-\sigma} H_n^m D^W}{(\sigma-1) H_q^m P^W} \left(\frac{N^W}{L^W}\right)^{\phi} + \hat{p}^m - \phi \hat{N}^W, \qquad m \in \{A, B\} \tag{4.5}$$

The interest rate r is the same in both countries because of the perfect capital markets. Equating the right hand sides of the above equation for $m = A$ and $m = B$, and using $\hat{p}^R = \hat{p}^A - \hat{p}$ yields

$$\hat{p}^R = \frac{\left(p^A\right)^{-\sigma} H_n^A D^W}{(\sigma-1) H_q^A P^W} \left(\frac{N^W}{L^W}\right)^{\phi} \left[\frac{H_q^R \left(p^R\right)^{\sigma}}{H_n^R} - 1\right] \tag{4.6}$$

Since the term in front of the square brackets is always positive, this equation is an unstable differential equation in p^R. For any variable x, x^* denotes its equilibrium value. The unique equilibrium of the above equation is given by

$$p^{*R} = \left(\frac{H_n^R}{H_q^R}\right)^{1-\eta} \tag{4.7}$$

On the unique stable trajectory towards this equilibrium, $p^R = p^{*R}$ always holds, that is, p^R immediately jumps to its steady state. Equation 4.7 shows that the terms of trade no longer depend on the relative population. Therefore, an economy with a growing population no longer suffers from a deterioration of the terms of trade, not even in the short run.

To develop an intuition for why the terms of trade are not affected by relative population, we first need an intermediate result. Substituting equation 4.7 into equation 3.9 implies that

$$w^R = \left(H_n^R\right)^{1-\eta} \left(H_q^R\right)^{\eta} \tag{4.8}$$

which, together with equation 4.2 implies

$$v^R = \left(\frac{H_q^R}{H_n^R}\right)^{\eta} \tag{4.9}$$

which is constant. It follows that $\hat{v}^A = \hat{v}^B$, and thus, by equation 4.4 that $\pi^R = v^R$. Substituting 4.2 yields

$$\frac{w^R}{H_n^R} = \pi^R \tag{4.10}$$

The left hand side of the equation is the relative cost of entry and the right hand side the relative benefit of entry. The equation thus says that the rate of return (defined as benefit divided by cost) of entry has to be equal in both countries, that is, the relative rate of return always equals 1. This result follows immediately from the free entry condition. The free entry condition is actually stronger, requiring not only that the relative rate of return has to equal 1, but also that the actual rates of return in both countries equal 1, so that entry always yields zero profits in both countries.

The intuition that the terms of trade are not affected by the relative size of the population may now be presented as follows. First, by equation 3.2, the terms of trade, p^R, are inversely related to the relative output per firm, q^R. This result follows directly from the CES utility function. So, all we need is an intuition about why the relative output per firm is not affected by the relative population. We do this by contradiction. Suppose that the population of country A suddenly rises and that, contrary to the result above, its relative output per firm rises in response, so that its terms of trade fall. The relative wage in country A falls (see equation 3.9), and thus the relative cost of entry. On the other hand, equation 3.12 showed that, with given relative productivity, the rise in output per firm implies a rise in relative profits per firm, despite the loss in the terms of trade. Thus the relative rate of return of entry rises. However, this result violates the assumption of free entry, which requires that the relative rate of return always equals 1.To summarize, the relative rate of return of entry is inversely related to the terms of trade. Free entry requires a constant relative rate of return and thus also a constant terms of trade,

So, output per firm and the terms of trade adjust to ensure that the rates of return to entry in both countries are equal. However, what happens if entry in both countries yields equal but non-zero profits? Movement in the relative output per firm (and the terms of trade) by itself would not help, since that would just create unequal rates of return. In this case, the interest rate adjusts. A rise in the interest rate reduces the discounted value of profits, and thus the profitability of entry in both countries. So, the world average rate of return of entry is inverse related to the interest rate, and the relative rate of return is inverse related to the terms of trade. The free entry condition of zero and thus equal profits of entry in both countries thus pins down both the terms of trade and the interest rate.

The terms of trade are not completely fixed, however. They are affected by the relative productivity in manufacturing and research. The intuition here is as follows. Suppose that starting from equilibrium, the relative labor productivity of research in country A, H_n^R rises. Then ceteris paribus, the relative rate of return of entry in country A rises. To bring the relative rate of return back to equality, the terms of trade immediately rise.

Similarly, suppose that the relative labor productivity in manufacturing in country A, H_q^R rises. The previous section showed that with fixed employment per firm, relative output per firm rises and the terms of trade fall. In addition, the relative wage and relative profits per firm both rise, and by the same percentage. It follows that the relative rate of return of entry is not affected. Hence no further movement of the terms of trade is needed, and the result of the previous section still hold if entry is allowed.

4.1 The number of firms

The counterpart of the result that the terms of trade are not affected by the relative size of the population is that the number of firms change, and in such a way that output per firm remains constant. To investigate the evolution of the number of firms, we first use equations 3.2, 3.8 and

3.9 to write

$$\frac{L_Q^m}{M^m} = \frac{(p^m)^{-\sigma} D^W}{H_q^m P^W} \tag{4.11}$$

Substituting this and the result that $\hat{p}^m = 0$ into equation 4.5 yields

$$r = \frac{H_n^m L_Q^m}{(\sigma - 1) N^m} \left(\frac{N^W}{L^W} \right)^{\phi} - \phi \hat{N}^W \tag{4.12}$$

Since this equations applies to both countries, it follows that

$$N^R = H_n^R L_Q^R \tag{4.13}$$

which implies directly that for given relative productivity in research, relative employment per firm in manufacturing is constant. Using this result, and noting that $\hat{N}^R = \hat{N}^A - \hat{N}^B$, equation 4.1 implies

$$\hat{N}^R = \frac{H_n^B L^B}{N^B} \left(\frac{N^W}{L^W} \right)^{\phi} \left[\frac{H_n^R L^R}{N^R} - 1 \right] \tag{4.14}$$

Since the term in front of the square brackets is always positive, this is a stable differential equation in N^R. The unique equilibrium is given by

$$N^{*R} = H_n^R L^R \tag{4.15}$$

The equilibrium relative number of varieties is proportional to the relative population. This explains why, in equilibrium, there is no terms of trade effect after an expansion of the population. The relative number of firms rises proportionally to the increase in the relative population, and relative employment per firm and relative prices do not change. Moreover, substituting the above equation into equation 4.13 yields

$$L_Q^{*R} = L^R \tag{4.16}$$

Let l_Q^m and l_N^m denote the fraction of total employment of country m working in manufacturing and research: $l_Q^m = \frac{L_Q^m}{L^m}$ and $l_N^m = \frac{L_N^m}{L^m}$. The above equation says that, in equilibrium, these fractions are the same in both counties, and thus equal to the corresponding fractions at the world level, $l_Q^{*m} = l_Q^{*W}$ and $l_N^{*m} = l_N^{*W}$.

So far, we only analyzed the evolution of relative variables of the two countries, such as the terms of trade. This is also the focus of the paper. In the appendix, we investigate the properties of the full model, including the levels of the variables. We prove that the full model has a unique equilibrium and that for $\phi < 1$, the only possible steady state value for $\hat{N}^W$ is zero, so that the number of varieties in the world is constant in equilibrium. Furthermore, in equilibrium,

$$l_Q^{*m} = 1 \tag{4.17}$$

$$l_N^{*m} = 0 \tag{4.18}$$

$$N^{*m} = \frac{H_n^m}{\rho(\sigma-1)}\left[\frac{H_n^W}{\rho(\sigma-1)}\right]^{\frac{\phi}{1-\phi}} L^m \tag{4.19}$$

$$N^{*W} = \left(\frac{H_n^W}{(\sigma-1)\rho}\right)^{\frac{1}{1-\phi}} L^W \tag{4.20}$$

$$r = \rho \tag{4.21}$$

where $H_n^W = \frac{H_N^A L^A + H_n^B L^B}{L^W}$. H_n^W. H_n^W is the average world labor productivity in research. These equations are equations A.23 through A.27 in the appendix.

In equilibrium, all workers work in manufacturing and no research takes place. The number of varieties in both countries is proportional to the population, and the world number of varieties is proportional to the world population. Total output in each country and thus in the world is constant. World spending is thus also constant, which by equation 2.13 implies that the interest rate r equals the utility discount rate ρ.

For $\phi = 1$, the appendix shows that the model still has a unique equilibrium, which is balanced growth path with

$$l_Q^{*m} = \eta\left(1 + \frac{\rho}{H_n^W}\right) \tag{4.22}$$

$$l_N^{*m} = 1 - l_Q^{*m} \tag{4.23}$$

$$\hat{N}^{*m} = \hat{N}^{*W} \tag{4.24}$$

$$\hat{N}^{*W} = (1-\eta)H_n^W - \eta\rho \tag{4.25}$$

$$r = \rho \tag{4.26}$$

These equations are equations A.28 through A.32 in the appendix. In this case, research takes place in equilibrium, so that the number of varieties grows. The growth rate is the same in both countries, and thus equal to the world growth rate. Total output is again constant in the steady state, so that $r = \rho$. This implies that employment in manufacturing is spread out over an ever increasing number of firms, each of which produces less and less. However, because consumer like variety, instantaneous utility does keep growing.

4.2 A population shock

We can now complete the description of the response to an increase in the population of country A. We already know that the terms of trade and the relative employment per firm remain constant throughout the adjustment process. What happens to the interest rate? In both the old and the new equilibrium, $r = \rho$. However, in the new equilibrium, total output has risen because of the higher output in country A. So, on the transition path, output rises, which by equation 2.13 implies that the interest rate is higher than its steady state value ρ. This reduces the value of the firm in both countries. Since nothing happens to the wage in either country, the higher interest rate would render entry unprofitable in both countries. To compensate, profits per firm have to rise during the transition. This implies by equation 3.11 that employment per firm rises in both countries.

This interest rate effect presents a problem if $\phi < 0$. For then there is no research in either country in equilibrium. So, there are no research workers who can move into manufacturing to increase the number of workers per firm, and a boundary solution would result. Note, however, that equation 2.13 shows that the rise in the interest rate depends on the growth rate of worldwide spending, not on the growth rate of spending in country A. If country A is relatively very small, the rise in its population would not affect world spending much, and in the limit not at all.

If $\phi < 1$ and country A is small, the constant terms of trade and interest rate imply that employment per firm in both countries also remains constant throughout the adjustment path. This implies that all of the additional workers in country A start out in research. This leads to a gradual increase in the number of firms in this country. As soon as these additional firms are created, they start employing the same number of workers as existing firms. In equilibrium, the number of firms in country A has expanded proportionally to the increase in population, and all of the additional labor has found work in the new firms. In country B, total employment in manufacturing remains constant and equal to total employment. No research takes place throughout the adjustment path and the number of firms is constant.

If $\phi = 1$, both countries start out with some research, and so the model can handle a general relative size of country A. The rise in the interest rate leads to an expansion of employment per firm in both counties. Since on impact the number of firms is constant in both countries, total manufacturing employment and output in both countries jumps up when employment in country A expands. So, on impact some of the additional workers in country A do find work in manufacturing. There is no loss in the terms of trade, because the same thing happens in country B. In that country, the increase in manufacturing employment leads to a fall in research and in the growth rate of varieties. In country A, the expansion in total employment causes both manufacturing and research in that country to rise, and the growth rate of varieties rises. The relative number of varieties in country A increases, until in the new equilibrium it has risen as much as the relative population of the country. Equation 4.24 and 4.25 show that the growth rate of varieties in the old and new equilibrium are the same in both countries.

In both cases, the additional workers in research in country A do not produce any consumer goods, but do consume based on their lifetime income. This leads to a trade deficit of country A. In the new equilibrium, the individuals in country A have accumulated a debt to country B. Equation 2.9 shows that this debt reduces consumption by ρ times this debt. This debt results in interest payments of r times the debt. In the new equilibrium, $r = \rho$, and the reduction in consumption is just enough to pay the interest payment. As a result the debt is never paid off, and the interest payment last forever. The counterpart of this debt service of country A is a trade surplus, which also lasts forever.

In addition, overall trade increases. In country B, the same number of firms produce the same amount of total output as before, However, the fraction of their (worldwide) customers living in the other country increased, and their exports have risen. The consumers of country *B* still spend an equal fraction of their total spending on all available varieties, but now a larger fraction of those varieties is produced abroad. In addition, their overall consumption rises because of the accumulated bond holdings. So, the imports of country *B* rise as well.

A final comment on this model is that the results that the terms of trade do not move at all and that an increase in the population of a (small) country is initially fully absorbed by the research sector seem a bit counterintuitive. One would expect that at least in the short run there is a terms of trade effect, and one does not think of research as the absorber of population shocks. One obvious real world objection is that research is a specialized activity that not all workers can perform equally well. In the next section, we explore the consequences of this objection.

5 Skill

In this section, individuals exogenously have either high or low skill. We have two reasons for this extension. First, research is a skill-intensive activity and modeling this makes the model more realistic. Second, and closer to the focus of the paper, if research is skill-intensive and the research sector expands, the relative demand for skilled labor rises. This raises the relative wage of skilled labor and the relative cost of research versus manufacturing. This puts a break on the ability of research to be the absorber of population shocks. We would expect that with the rising cost of research, more workers start working in manufacturing. This would raise employment per firm in that sector and reduce the terms of trade. This section formalizes this intuition.

First, we set up some notation. L_s^m and L_u^m are the number of high- and low-skilled workers in country m. s^m and u^m are the fractions of high- and low-skilled workers in country m, so that $s^m = \frac{L_s^m}{L^m}$ and $u^m = \frac{L_u^m}{L^m}$. Low skilled workers can only do manufacturing work, so that all of them are employed in that sector. High skilled workers may work in manufacturing and research. L_{uq}^m and L_{sq}^m denote an individual manufacturing firm's employment of low and high skilled workers in country m. L_{uQ}^m and L_{sQ}^m are the total employment of low- and high-skilled workers in country m, so that $L_{sQ}^m = N^m L_{sq}^m$ and $L_{uQ}^m = N^m L_{uq}^m$. L_{sN}^m is the total employment of skilled workers in research in country m. Clearly, $L_s^m = L_{sQ}^m + L_{sN}^m$. l_{sQ} and l_{sN} denote the fractions of high- and low-skilled workers in manufacturing and research, that is, $l_{sQ} = \frac{L_{sQ}^m}{L_s^m}$ and $l_{sN} = \frac{L_{sN}^m}{L_s^m}$. Since all low-skilled workers are employed in manufacturing, we also have $L_u = L_{uQ}$. Remember that, for any variable x, $x^R = \frac{x^A}{x^B}$. As before, the symmetry between firms in a country and across countries allows us to describe the actions of a generic firm in this world, and omit the superscript m and the subscript i referring to a specific firm. Sometimes we still use these identifiers if we want to be more specific.

We now quickly rewalk the derivation in the previous two section and adjust the derivations for the inclusion of skill. The production function for manufacturing is Cobb-Douglas in low- and high-skilled labor:

$$q = H_q L_{uq}^{1-\alpha} L_{sq}^{\alpha} \tag{5.1}$$

where H_q. as before, is the country specific labor productivity in manufacturing, which is the same for all firms in a country, and α is the Cobb-Douglas parameter common to all firms in the whole world. The demand function is still given by equation 3.2. A firm's profits are given by

$$\pi = H_q^{\eta} L_{uq}^{(1-\alpha)\eta} L_{sq}^{\alpha\eta} \left(\frac{D^W}{P^W}\right)^{\frac{1}{\sigma}} - w_u L_{uq} - w_s L_{sq} \tag{5.2}$$

The firm maximizes profits with respect to the high- and low-skilled labor input. The first order conditions are

$$w_u L_{uq} = (1-\alpha)\eta p q \tag{5.3}$$

$$w_s L_{sq} = \alpha \eta p q \tag{5.4}$$

which again implies $\pi = (1-\eta)pq$. Define

$$\omega = \left(\frac{w_u}{1-\alpha}\right)^{1-\alpha}\left(\frac{w_s}{\alpha}\right)^{\alpha} \tag{5.5}$$

Substituting this equations into equation 5.1 yields

$$p = \frac{\omega}{\eta H_q} \tag{5.6}$$

The rate at which new varieties are created in country m is

$$\dot{N}^m = H_n^m L_{sN}^m \left(\frac{N^W}{L^W}\right)^{\phi} \tag{5.7}$$

This is a modification of equation 4.1 to account for the fact that only skilled workers can perform research. Free entry now implies that

$$v = \frac{w_s}{H_n\left(\frac{N^W}{L^W}\right)^{\phi}} \tag{5.8}$$

$$\hat{v} = \hat{w}_s - \phi \hat{N}^w \tag{5.9}$$

Remember that profits π are given by $\pi = (1-\eta)pq$. Substituting this and equations 5.4, 5.8 and 5.9 into the no-arbitrage equation $r = \frac{\pi}{v} + \hat{v}$ yields

$$r = \frac{H_n L_{sQ}}{N\alpha(\sigma-1)}\left(\frac{N^W}{L^W}\right)^{\phi} + \hat{w}_s - \phi\hat{N}^w \tag{5.10}$$

5.1 The terms of trade

The main focus of the paper is the dynamic adjustment of the terms of trade, to which we now turn. The analysis below is fully general, and in particular, it holds if $\phi < 1$ and if $\phi = 1$. First, we define the following two variables:

$$R_n = \frac{N^R}{H_n^R s^R L^R} \tag{5.11}$$

$$R_w = \frac{w_s^R}{\left(H_q^R\right)^{\eta}\left(H_n^R\right)^{1-\eta}}\left(\frac{s^R}{u^R}\right)^{\eta(1-\alpha)} \tag{5.12}$$

R_n and R_w are the relative number of varieties and the relative wage in the two countries multiplied be some constants, These constants are chosen so that R_n and R_w equal 1 in equilibrium, as we will see shortly. To study the dynamics of the terms of trade, we build a phase diagram in these two variables.

Equations 3.2 and 5.4 imply

$$w_s^m L_{sQ}^m = N^m \alpha\eta\left(p^m\right)^{1-\sigma}\frac{D^W}{P^W}, \quad m \in A, B \tag{5.13}$$

Dividing these equations for countries A and B gives

$$\left(p^R\right)^{1-\sigma} = \frac{w_s^R L_{sQ}^R}{N^R} \tag{5.14}$$

Let Q^m denote the total demand for goods produced by country m: $Q^m = N^m q^m$. Equation 3.2 implies that $Q^R = N^R \left(p^R\right)^{-\sigma}$, while equation 5.1 yields $Q^R = H_q^R \left(L_u^R\right)^{1-\alpha} \left(L_{sQ}^R\right)^{\alpha}$ It follows that

$$\left(p^R\right)^{\sigma} = \frac{N^R}{H_q^R \left(L_u^R\right)^{1-\alpha} \left(L_{sQ}^R\right)^{\alpha}} \tag{5.15}$$

which is the equivalent of equation 3.4. Multiplying this equation and equation 5.14 yields

$$p^R = \frac{w_s^R}{H_q^R \left(\frac{L_u^R}{L_{sQ}^R}\right)^{1-\alpha}} \tag{5.16}$$

Raising this equation to the power σ and equating the result to the previous equation yields the equation for the relative wage of high-skilled workers

$$w_s^R = \left(N^R\right)^{1-\eta} \left(H_q^R\right)^{\eta} \left(L_u^R\right)^{\eta(1-\alpha)} \left(L_{sQ}^R\right)^{\eta\alpha-1} \tag{5.17}$$

For any variable x, $\log(x)$ denotes the natural logarithm of x. Using $l_{sQ} = \frac{L_{sQ}^m}{L_s^m}$, we write equation 5.17 as

$$\log(R_w) = (1-\eta)\log(R_n) - (1-\eta\alpha)\log\left(l_{sQ}^R\right) \tag{5.18}$$

Define the *ll* line as the set of points for which $l_{sQ}^A = l_{sQ}^B$. Its equation is

$$\log(R_w) = (1-\eta)\log(R_n) \tag{5.19}$$

The line is labeled *ll* in figure 5.1 . Above this line $l_{sQ}^A < l_{sQ}^B$, and below it, $l_{sQ}^A > l_{sQ}^B$.

Since $\hat{R}_n = \hat{n}^A - \hat{n}^B$, equations 5.7 yields

$$\hat{R}_n = \left(\frac{N^W}{L^W}\right)^{\phi} \left[\frac{H_n^A L_{sN}^A}{N^A} - \frac{H_n^B L_{sN}^B}{N^B}\right] \tag{5.20}$$

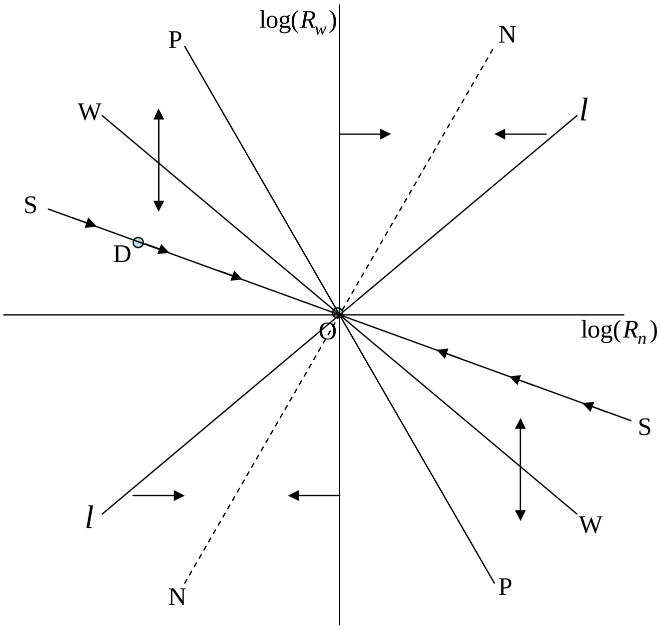
$$= \frac{H_n^B L_{sN}^B}{N^B} \left(\frac{N^W}{L^W}\right)^{\phi} \left[\frac{1}{R_n}\frac{l_{sN}^A}{l_{sN}^B} - 1\right] \tag{5.21}$$

For a given value of R_w, this is a stable differential equation in R_n. Define the NN line as the set of points for which $\hat{R}_n = 0$. Its equation is

$$\log(R_n) = \log(1 - l_{sQ}^A) - \log(1 - l_{sQ}^B) \tag{5.22}$$

Since the location of this line depends on l_{sQ}^A and l_{sQ}^B, it cannot be drawn directly in figure 5.1. Moreover, the line may move over time as l_{sQ}^A and l_{sQ}^B adjust to their equilibrium. However, we can establish that the line must lie within a certain region, and that is enough for our purposes. Equation 5.22 shows that if $\log(R_n) > 0$, $l_{sQ}^A < l_{sQ}^B$. Therefore, if $\log(R_n) > 0$, the NN line must lie above the *ll* line, and thus lie in the first quadrant between the *ll* line and the vertical axis. Similarly, if $\log(R_n) < 0$, the line must lie in the third quadrant, again between the *ll* line and the vertical axis. Finally, if $\log(R_n) = 0$, $l_{sQ}^A = l_{sQ}^B$, and the line must coincide with the *ll* line, that is,

Figure 5.1 Phase diagram for $\log(R_n)$ and $\log(R_w)$



lie at the origin. This information is conveyed in figure 5.1 by drawing the NN line as a dotted line. The horizontal arrows give the direction of movement in the phase diagram implied by equation 5.21. No claim is made about the direction of the arrows between the *ll* line and vertical axis.

The perfect capital market ensures that the interest rate r is the same in both countries. Applying equation 5.10 to both countries yields

$$\hat{w}_s^A - \hat{w}_s^B = \frac{H_n^A L_{sQ}^A}{\alpha(\sigma-1)N^A}\left(\frac{N^W}{L^W}\right)^{\phi}\left[R_n \frac{l_{sQ}^B}{l_{sQ}^A} - 1\right] \tag{5.23}$$

Noting that $\hat{R}_w = \hat{w}_s^A - \hat{w}_s^B$, yields

$$\hat{R}_w = \frac{1}{\alpha(\sigma-1)}\left(\frac{N^W}{L^W}\right)^{\phi}\left[\frac{H_n^B L_{sQ}^B}{N^B} - \frac{H_n^A L_{sQ}^A}{N^A}\right] \tag{5.24}$$

$$= \frac{H_n^A L_{sQ}^A}{\alpha(\sigma-1)N^A}\left(\frac{N^W}{L^W}\right)^{\phi}\left[\frac{R_n}{l_{sQ}^R} - 1\right] \tag{5.25}$$

Using equation 5.18 to substitute out l_{sQ}^R yields

$$\hat{R}_w = \frac{H_n^A L_{sQ}^A}{\alpha(\sigma-1)N^A}\left(\frac{N^W}{L^W}\right)^{\phi}\left[(R_w)^{\frac{1}{1-\eta\alpha}}(R_n)^{\frac{\eta(1-\alpha)}{1-\eta\alpha}} - 1\right] \tag{5.26}$$

For a given value of R_n, this is an unstable differential equation in R_w. Define the WW line as the set of points for which $\hat{R}_w = 0$. Its equation is

$$\log(R_w) = -\eta(1-\alpha)\log(R_n) \tag{5.27}$$

The line is labeled WW in figure 5.1, with the vertical arrows indicating the movement in the phase diagram implied by equation 5.26. The phase diagram in the figure implies that there is a unique equilibrium, namely the origin O, with $R_n = R_w = 1$ as postulated above. Moreover, in equilibrium, $l^A_{sQ} = l^B_{sQ}$. There is a unique saddle path leading to this equilibrium, labeled SS. The saddle path lies between the WW line and the horizontal axis.

The economy moves towards its equilibrium along the saddle path. For instance, if the world economy starts at point D, it moves along the SS line towards the origin O. At D, $R_n < 0$, and on the adjustment path R_n rises to 0, which involves a rise in the ratio $\frac{N^A}{N^B}$. Similarly, at D, $R_w > 0$, and on the adjustment path R_n falls to 0, which involves a fall in the ratio $\frac{w^A}{B}$.

Substituting the equilibrium conditions $R_w = 1$ and $l^A_{sQ} = l^B_{sQ}$ into equation 5.16 yields the equilibrium level of the terms of trade:

$$p^{*R} = \left[\frac{H^R_n}{H^R_q} \left(\frac{s^R}{u^R} \right)^{1-\alpha} \right]^{1-\eta} \tag{5.28}$$

This is a modification of equation 4.7. Now the terms of trade also rise with the relative ratio of skilled workers. A higher ratio of skilled workers reduces the relative wage of skilled workers by virtue of the Cobb-Douglas production function in manufacturing. As a result, the relative cost of research falls. Research becomes profitable, and the number of firms increase. Output per firm falls, and the terms of trade improve.

To study the dynamics of the terms of trade, use equation 5.14 to substitute L^R_{sQ} out of equation 5.16:

$$\left(p^R\right)^{\alpha+\sigma(1-\alpha)} = \left(w^R_s\right)^{\alpha} \left(\frac{n^R}{u^R L^R} \right)^{1-\alpha} \frac{1}{H^R_q} \tag{5.29}$$

Combining this equation and equation 5.28 yields

$$\log(R_w) = -\frac{1-\alpha}{\alpha} \log(R_n) + \left(1 + \sigma \frac{1-\alpha}{\alpha}\right) \log\left(\frac{p^R}{p^{*R}}\right) \tag{5.30}$$

Define the PP line as the set of points for which $p^R = p^{*R}$. Its equation is

$$\log(R_w) = -\frac{1-\alpha}{\alpha} \log(R_n) \tag{5.31}$$

The line is labeled PP in figure 5.1. It is steeper than the WW line. Points above the PP line imply $p^R > p^{*R}$ and vice versa. So, Along the saddle path, $p^A > p^{*A}$ if $\log(R_n) > 0$ and vice versa. For instance, at D, $p^A < p^{*A}$, and on the adjustment path p^A rises to p^{*A}.

The intuition behind the model is best explored by studying some simulations. This we do in the remainder of this section. In each simulation, we assume that the world economy is in equilibrium, when country A is hit by an unexpected shock. As in the previous section, to avoid a violation of the boundary conditions, we assume that country A is small for the case $\phi < 1$. For $\phi = 1$, we just need to assume that the shock is not so large that boundary conditions are violated. To compare this model with the previous one, we start with the same population shock as analyzed in the previous model.

5.2 A population shock

Suppose that, starting from a situation of general equilibrium, the population of country A suddenly and unexpectedly jumps up with both skilled and unskilled labor rising proportionally. The change in equilibrium is the same as before: country A expands proportionally. We refer to the previous section for the intuition on this and as a point of reference for this shock and its intuition.

The dynamics are very different though. On impact $\log(R_n)$ falls by $\Delta \log(L^A)$ and thus becomes negative. Let figure 5.1 represent the situation right after the shock. The economy has jumped to a point D, on the saddle path with $\log(R_n) < 0$. From D the economy gradually moves back to the origin. $\log(R_n)$ gradually rises back to zero, that is $\frac{N^A}{N^B}$ rises proportionally with the increase in the population of country A. As D lies below the PP line, $p^A < p^{*A}$ on impact. Since p^{*A} never changes, it must be that p^A on impact jumps down and then gradually moves back up to its old value.

figures 5.2 through 5.7 illustrate some of the results of for the case of an increase in the population of country A by 20% and $\phi = 1$. No attempt has been made to calibrate the model in an empirically realistic way. All effects are shows in terms of deviations from the original balanced growth path.

The intuition for these results is as follows. It cannot be the case that all additional workers start out in manufacturing. Then we would have the outcome of the benchmark model, with no additional research being performed and no additional varieties being created. The resulting loss in the terms of trade and the relative wage, combined with the rising profits per firm would raise the rate of return to entry, and entry would jump up. Therefore, some of the additional workers must start out in research, increasing the growth rate of the number of firms in their country. both absolutely and relative to country B. This is illustrated in figure 5.2 However, the research sector in country A can no longer absorb all of the additional workers either. Since research is skill-intensive, an expanding research sector raises the relative demand for skilled labor. This raises its relative wage and thus the cost of research, see figure 5.3. This puts a brake on the ability of research to be the residual absorber of a population shock. The increase in manufacturing employment initially raises output per firm, resulting in a loss in the terms of trade, see figure 5.4.

In addition to these effects, the increased research investment in country A raises the worldwide interest rate. Ceteris paribus, a rise in the interest rate reduces research in both countries. In country A, the rise in interest rate puts a second brake on the expansion of research employment. The net result is an unambiguous increase in both manufacturing and research employment in country A, see figure 5.5. In country B, employment in research initially actually falls, as employment shifts to manufacturing, see figure 5.6. As a result, the growth rate of new varieties rises in country A and initially falls in country B. This is illustrated in figure 5.7.

Over time, the additional research raises the number of firms. Output per firm falls and the terms of trade rise. The expanding number of firms absorbs more and more research workers until, in equilibrium, the ratio of research workers to manufacturing workers is the same as in the old equilibrium. In the new equilibrium, output per firm, wages, the rate of interest and the terms of trade all have gone back to their old levels.

Figure 5.2 **The ratio of the number of varieties in countries A and B in deviation from the original balanced growth path**

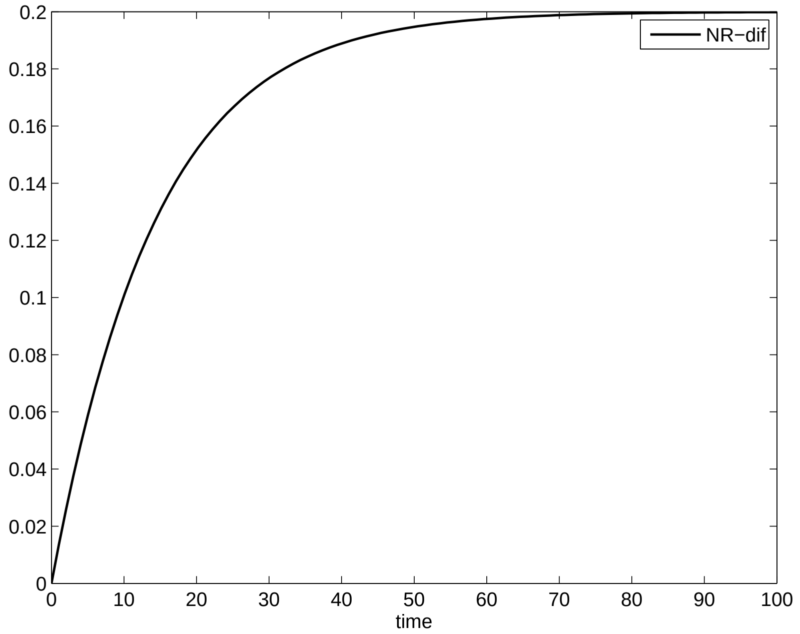


Figure 5.3 **The ratio of high- to low skilled wages in country A in deviation from the original balanced growth path**

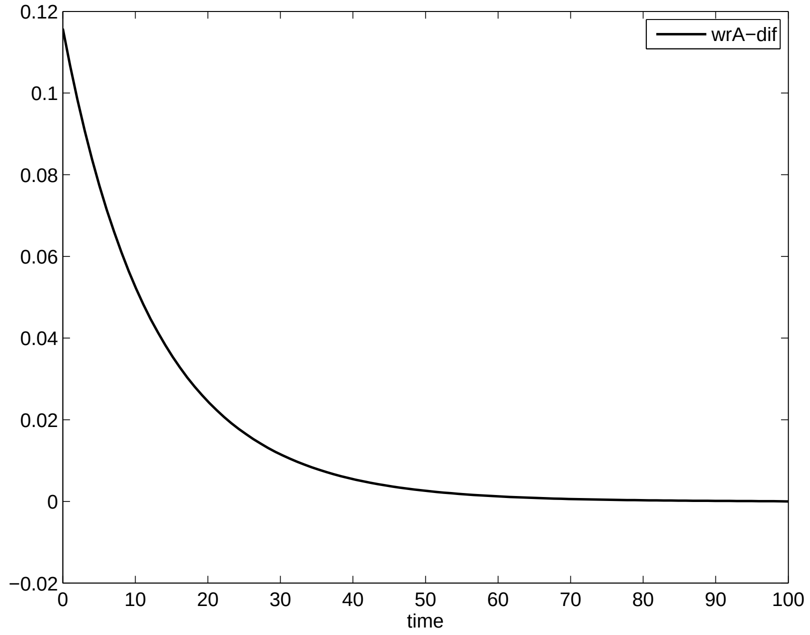


Figure 5.4 The terms of trade in deviation from the original balanced growth path

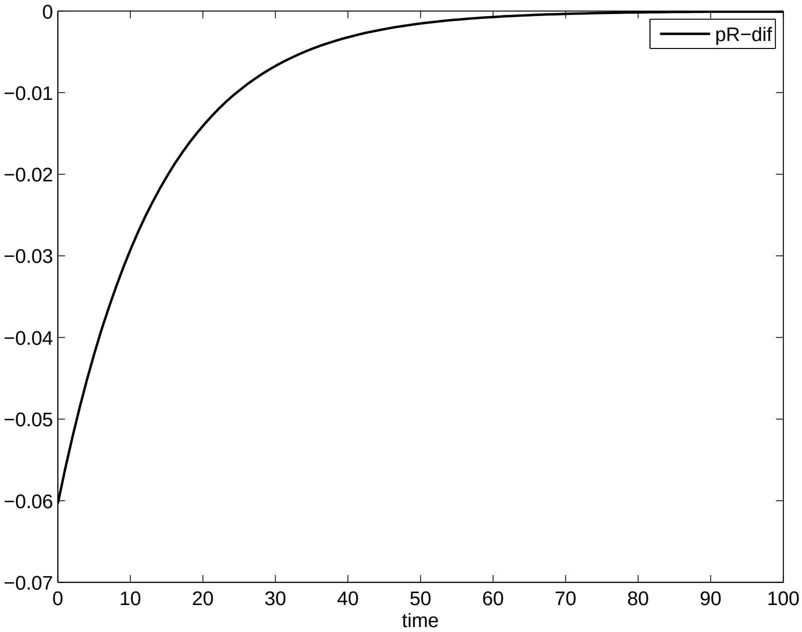


Figure 5.5 Employment in research, indicated by ——, and in production, indicated by - - - , in country A, in deviation from the original balanced growth path

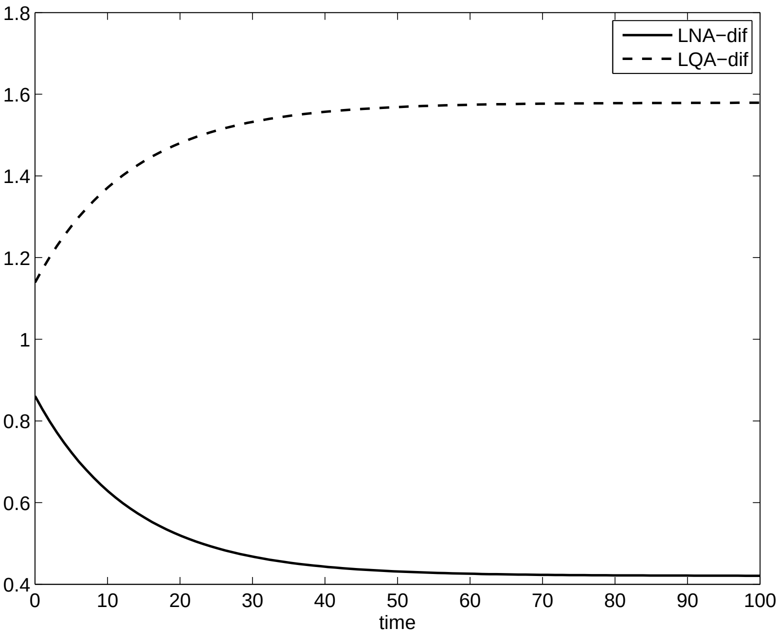


Figure 5.6 Employment in research, indicated by ——, and in production, indicated by - - - , in country B, in deviation from the original balanced growth path

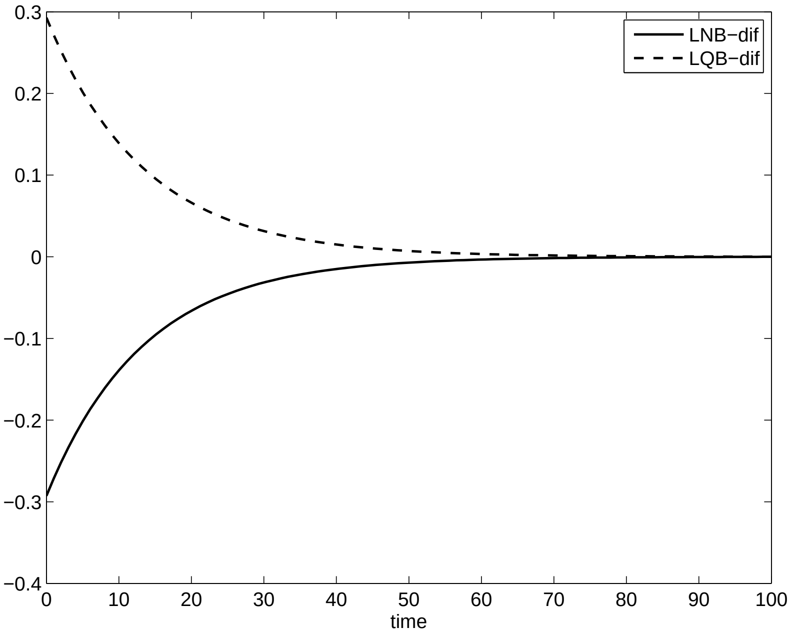
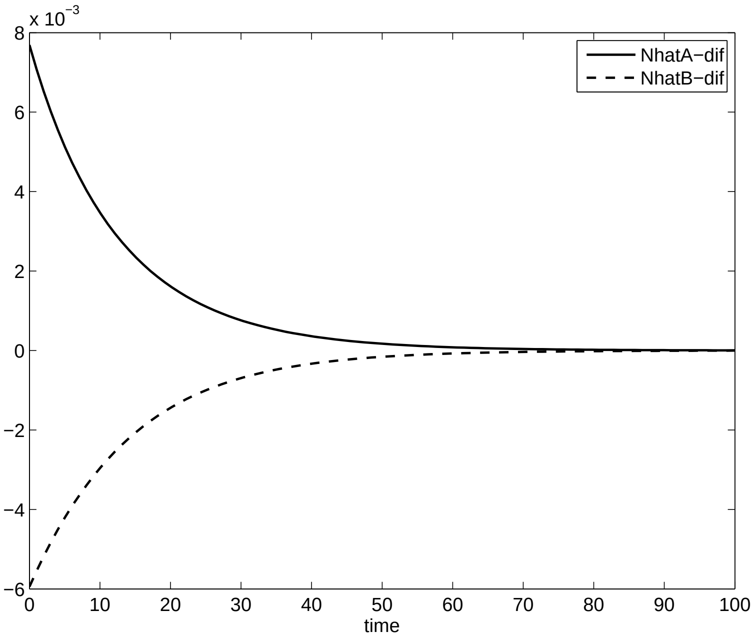


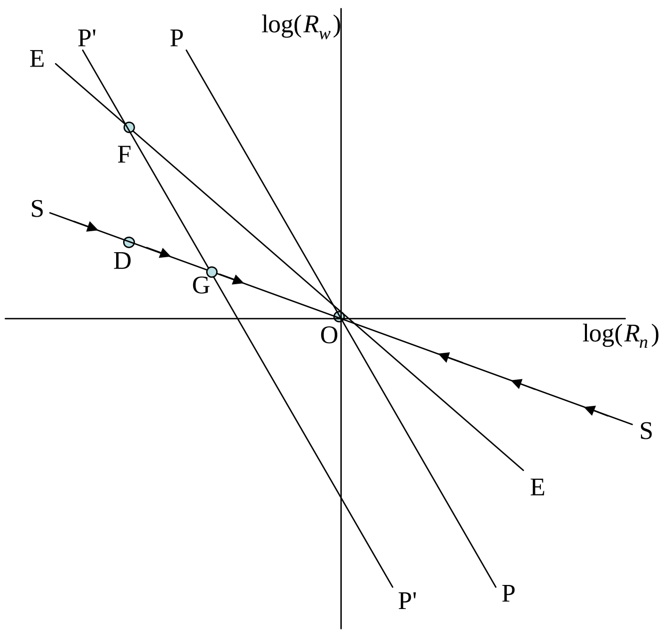
Figure 5.7 The growth rates of the number of varieties in country A, indicated by —— and in country B, indicated by - - - , in deviation from the original balanced growth path



5.3 A shock in the relative supply of skilled labor

Now suppose that the ratio of skilled to unskilled workers rises in country A, keeping total employment constant. s^A rises, and u^A falls. Figure 5.8 shows the situation right after the shock. On impact, $\log(R_n)$ falls by $\Delta\log(s^A)$, and $\log\left(p^{*A}\right)$ rises by $\frac{1-\alpha}{\sigma}\Delta\log\left(\frac{s^A}{1-s^A}\right)$. The economy

Figure 5.8 A shock in skilled labor



jumps to point D and gradually moves back to the origin. At D, p^A lies below its new equilibrium value and gradually rises to it.

What happens to p^A on impact? To answer this question, define the P'P' line as the set of point for which p^A equals its old equilibrium value, denoted p^{*A}_{old}. Substituting $p^A = p^{*A}_{old}$ into equation 5.30 yields the equation for the P'P' line:

$$\log(R_w) = -\frac{1-\alpha}{\alpha}\log(R_n) + \left(1+\sigma\frac{1-\alpha}{\alpha}\right)\log\left(\frac{p^{*A}_{old}}{p^{*A}_{new}}\right) \tag{5.32}$$

$$= -\frac{1-\alpha}{\alpha}\log(R_n) - \left(1+\sigma\frac{1-\alpha}{\alpha}\right)\Delta\log\left(p^{*A}\right) \tag{5.33}$$

At points below the P'P' line, $p^A < p^{*A}_{old}$, and vice versa. So, p^A jumps up on impact if point D in figure 5.8 lies above the P'P' line and vice versa.

For this particular shock, we have

$$\Delta\log\left(p^{*A}\right) = \frac{1-\alpha}{\sigma}\Delta\log\left(\frac{s^A}{1-s^A}\right) \tag{5.34}$$

so that the equation for the P'P' line becomes

$$\log(R_w) = -\frac{1-\alpha}{\alpha}\log(R_n) - \left(1+\sigma\frac{1-\alpha}{\alpha}\right)\frac{1-\alpha}{\sigma}\Delta\log\left(\frac{s^A}{1-s^A}\right) \tag{5.35}$$

We want to know whether the vertical line through point D crosses the P'P' line above or below point D. At point D, $\log(R_n) = -\Delta\log(s^A)$. Substituting this yields

$$\log(R_w) = +\frac{1-\alpha}{\alpha}\Delta\log(s^A) - \left(1+\sigma\frac{1-\alpha}{\alpha}\right)\frac{1-\alpha}{\sigma}\Delta\log\left(\frac{s^A}{1-s^A}\right) \tag{5.36}$$

$$= \eta(1-\alpha)\Delta\log(s^A) + \left(1+\sigma\frac{1-\alpha}{\alpha}\right)\frac{1-\alpha}{\sigma}\Delta\log(1-s^A) \tag{5.37}$$

The point F in figure 5.8 lies straight above D on the WW line. Its coordinates are $(-\Delta\log(s^A), \eta(1-\alpha)\Delta\log(s^A))$. Since the second term in the above equation is negative, the P'P' line goes underneath point F. Since both point D and the P'P' line are below point F, we cannot say in general whether point D is above or below the P'P' line. However, a given increase in s^A affects the first term in equation 5.37 more and the second term less if the initial level of s^A is lower, and vice versa. If the initial level of s^A is very close to zero and the change in s^A is very small, the second term becomes arbitrarily close to zero and the P'P' line gets arbitrarily close to point F. This case is shown in figure 5.8. Now D lies below the P'P' line, and thus p^A initially drops down. As the economy moves to the origin, it crosses the P'P' line at G, where p^A has recovered to its old equilibrium value. From then p^A rises further until it reaches its new equilibrium value in the new steady state (at the origin).

If on the other hand, If the initial level of s^A is very close to one, and the change in s^A is very small, the first term in equation 5.8 may be ignored. Then the P'P' line crosses the vertical line through point D below the horizontal axis and point D lies above the P'P' line. In this case, p^A jumps up on impact and then gradually rises further towards its new equilibrium.

The intuition for this result is as follows. On impact, the increase in the number of high-skilled workers in country A reduces their relative wage. This has two effects on output per firm. First, employment in research rises as the relative cost of research falls. So, total employment in production falls. Since the number of firms is given on impact, employment per firm falls. Second, firms will use relatively more skilled labor in production, which affects average labor productivity. The sign and size of this effect depends on the initial relative share of high-skilled labor in production. Labor productivity rises more if the initial level of s^A was low, that is, if skilled labor was initially relatively scarce. If the initial level of s^A goes to zero, the marginal product of high-skilled workers in production goes to infinity. Then the marginal effect of the increase in the share of high-skilled workers on productivity becomes very large, and outweighs the negative effect of the drop in employment. In that case, output per firm must rise and the initial effect on the terms of trade must be negative. After the shock, the increase in the number of firms gradually reduces output per firm again and the terms of trade rise. Eventually the terms of trade even rise above their original level.

If the initial level of s^A is high, raising the share of skilled workers in production does not raise average productivity much, and at some point will even hurt it (as low skilled labor becomes the scarce factor), and so the negative effect of the reduction in total employment in production dominates. On impact, output per firms drops and the terms of trade rise. Over time, the terms of trade keep rising as the increase in the number of firms reduces output per firm even further, until the new equilibrium is reached.

5.4 A shock in the productivity of research of country A

Next, we consider a positive shock in H_n^A, the productivity of research in new varieties in country A. On impact, $\log(R_n)$ falls by $\Delta\log(H_n^A)$, and $\log\left(p^{*A}\right)$ rises by $\frac{1}{\sigma}\Delta\log(H_n^A)$. The situation right after the shock may be represented by figure 5.1. The economy jumps to point D and gradually moves back to the origin. At D, p^A lies below its new equilibrium and gradually rises to it.

What happens to p^A on impact? The P'P' line (equation 5.33) becomes

$$\log(R_w) = -\frac{1-\alpha}{\alpha}\log(R_n) - \left(1-\sigma\frac{1-\alpha}{\alpha}\right)\frac{1}{\sigma}\Delta\log(H_n^A) \tag{5.38}$$

Substituting $R_n = -\Delta\log(H_n^A)$ yields

$$\log(R_w) = -\frac{1}{\sigma}\Delta\log(H_n^A) \tag{5.39}$$

So, the P'P' line crosses the vertical line through point D below the horizontal axis. Point D, therefore, lies above that line and, on impact, p^A jumps up and then rises further to its new equilibrium value.

The intuition is that a rise in research productivity makes it more attractive to invest in new varieties. As a result, output per firm will fall and the terms of trade will improve.

5.5 A shock in the productivity of production of country A

A positive shock in H_q^A, the productivity of production in country A, does not affect $\log(R_n)$ on impact. $\log\left(p^{*A}\right)$ falls by $\frac{1-\alpha}{\sigma}\Delta\log(H_q^A)$. In terms of figure 5.1, the world economy remains at the origin which is also the new equilibrium. So, there are no dynamics in this case. The only thing that happens is a sudden and permanent jump in output of country A and a sudden and permanent fall in its terms of trade. The intuition is that the increase in productivity in country A raises the relative output of the firms of that country and thus reduces its terms of trade.

5.6 A shock in the overall productivity of country A

Now consider a general shock in the productivity of country A, that is, both H_n^A and H_q^A rise proportionally by a certain percentage. This is a combination of the previous two shocks. So, on impact, $\log(R_n)$ falls by $\Delta\log(H_n^A)$, and $\log\left(p^{*A}\right)$ remains unchanged. The analysis is identical to the first shock, the rise in the population of country A. On impact, output per firm in country A rises, and its terms of trade fall. Over time the relative number of firms in country A rises, and the terms of trade fall back to their original level. The only difference between the two shocks is that in the case of a population shock output per capita and wages do not change in either country, while in the second case output per capita and wages in country *A* rise proportionally with the rise in productivity.

Appendix A The full model and global stability

The previous section showed that the model is globally stable in its relative variables. We now use this result to show that the complete model is globally stable as well. Our strategy is as follows. From the previous section, we know that N^R and p^R always converge to their equilibrium values and that $l^A_{sQ} - l^B_{sQ}$ converges to zero. We use these facts to show that the full model converges to a model that is globally stable, and therefore, is itself globally stable as well. We do this by imposing the equilibrium conditions $N^R = N^{*R}, p^R = p^{*R}$ and $l^A_{sQ} = l^B_{sQ}$ onto the full model and then analyze the result.

First, we need some more notation. Y^m is country m's total manufacturing revenue, that is, $Y^m = N^m p^m q^m$. Define f^m_Y as country m's share in world manufacturing revenue, that is, $f^m_Y = \frac{Y^m}{Y^A+Y^B}$. Since world revenue equals world spending[3], $Y^A + Y^B = D^W$, and we can write

$$p^m q^m = \frac{f^m_y D^W}{N^m}, \qquad m \in \{A, B\} \tag{A.1}$$

Substituting this into 5.4 and using $L^m_{sQ} = N^m L^m_{sq}$ yields

$$L^m_{sQ} = \frac{\alpha\eta f^m_Y D^W}{w^m_s} \tag{A.2}$$

so that

$$\hat{L}^m_{sQ} = \hat{f}^m_Y + \hat{D}^W - \hat{w}^m_s \tag{A.3}$$

Substituting equations 2.13 and 5.10 yields

$$\hat{L}^m_{sQ} = \frac{H^m_n L^m_{sQ}}{(\sigma-1)\alpha N^m}\left(\frac{N^W}{L^W}\right)^{\phi} - \phi\hat{N}^W - \rho + \hat{f}^m_Y \tag{A.4}$$

This equation and equations 5.7, 5.21 and 5.26 form a dynamic system in L^m_{sQ}, N^m, R_n and R_w. From these variables, all other variables in the model can be derived.

Using the definition of f^m_Y and noting that $Y^R = \frac{Y^A}{Y^B}$, we write

$$\frac{1}{f^A_Y} = 1 + \frac{1}{Y^R} \tag{A.5}$$

$$\frac{1}{f^B_Y} = 1 + Y^R \tag{A.6}$$

Equation 3.2 implies that

$$Y^m = N^m \left(p^m\right)^{1-\sigma} \frac{D^W}{P^W} \tag{A.7}$$

so that

$$Y^R = N^R \left(p^R\right)^{1-\sigma} \tag{A.8}$$

[3] This can be shown formally by using equation 2.3, the definitions above equation 2.9 and $q_i = c^W_i$.

Imposing $N^R = N^{*R}$ and $p^R = p^{*R}$ yields a constant for Y^R and thus constants for f_Y^A and f_Y^B as well, so that $\hat{f}_Y^A = \hat{f}_Y^B = 0$.

Next, define L_{sN}^W and L_{sQ}^W as the world employment of skilled labor in research and manufacturing. Define $L_s^W = L_{sN}^W + L_{sQ}^W$,and $l_{sN}^W = \frac{L_{sN}^W}{L_s^W}$. Then $l_{sN}^A = l_{sN}^B$ implies $l_{sN}^A = l_{sN}^B = l_{sN}^W$, $l_{sQ}^A = l_{sQ}^B = l_{sQ}^W$ and $l_{sN}^W + l_{sQ}^W = 1$. Also define H_n^W as

$$H_n^W = \frac{H_n^A s^A L^A + H_n^B s^B L^B}{L^W} \tag{A.9}$$

H_n^W is an population weighted average of the product of the skill ratio and the research productivity of the two countries. Using these definitions, and the facts that$\hat{L}_{sQ}^m = \hat{l}_{sQ}^m = \hat{l}_{sQ}^W$ and $\hat{f}_Y^m = 0$, equation A.4 can be written as

$$\hat{l}_{sQ}^W = \frac{H_n^m L_{sQ}^m}{(\sigma-1)\alpha N^m}\left(\frac{N^W}{L^W}\right)^{\phi} - \phi \hat{N}^W - \rho \tag{A.10}$$

Since this equation holds for $m \in \{A, B\}$, it follows that

$$\frac{H_n^A L_{sQ}^A}{N^A} = \frac{H_n^B L_{sQ}^B}{N^B} \tag{A.11}$$

which implies that

$$\frac{H_n^m L_{sQ}^m}{N^m} = l_{sQ}^W H_n^W \frac{L^W}{N^W} \tag{A.12}$$

Adding equation 5.7 for both counties yields

$$\hat{N}^W = \frac{H_n^A L_{sN}^A + H_n^B L_{sN}^B}{L^W}\left(\frac{N^W}{L^W}\right)^{\phi-1} \tag{A.13}$$

$$= (1 - l_{sQ}^W) H_n^W \left(\frac{N^W}{L^W}\right)^{\phi-1} \tag{A.14}$$

Substituting this equation and the equation above it into equation A.10 yields

$$\hat{l}_{sQ}^W = \left(\frac{1}{(\sigma-1)\alpha} + \phi\right) l_{sQ}^W H_n^W \left(\frac{N^W}{L^W}\right)^{\phi-1} - \phi H_n^W \left(\frac{N^W}{L^W}\right)^{\phi-1} - \rho \tag{A.15}$$

So, equations 5.7 and A.4 converge to equations A.14 and A.15. We now analyze the system of these last two equations. Define the LL line as the set of points for which $\hat{l}_{sQ}^W = 0$. Its equation is

$$l_{sQ}^W = \frac{\phi + \frac{\rho}{H_n^W}\left(\frac{N^W}{L^W}\right)^{1-\phi}}{\phi + \frac{1}{(\sigma-1)\alpha}} \tag{A.16}$$

Equation A.14 shows that for $\phi < 1$, the only possible state value for $\hat{N}^W$ is zero, that is, the number of varieties in the world must be constant in equilibrium. For $\phi = 1$, a non-zero steady state value for $\hat{N}^W$ is possible. We will treat these two cases separately.

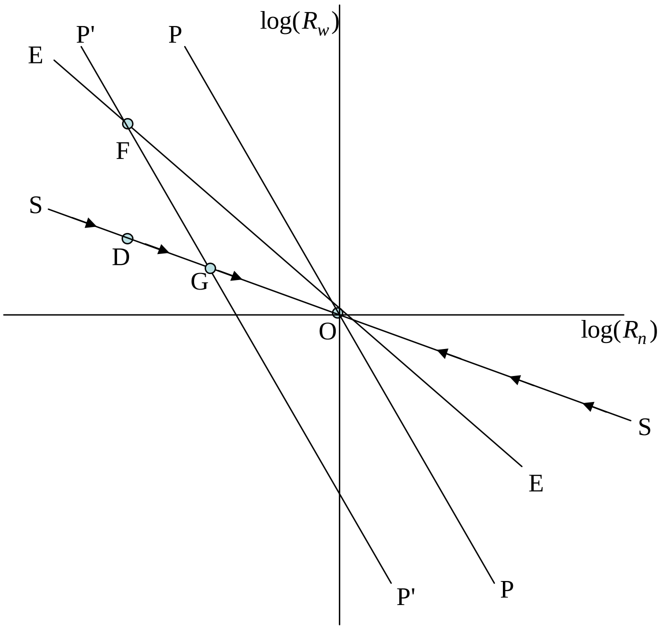
A.1 ϕ<1

For the case $\phi < 1$, define the NN line as the set of points for which $\hat{N}^W = 0$. Its equation is given by

$$l_{sQ}^W = 1 \tag{A.17}$$

The line is labeled NN in figure A.1. The horizontal arrow below the line gives the direction of

Figure A.1 Phase diagram for $\phi = 1$



movement in the phase diagram implied by equation A.14. Points above the line imply negative employment in research and are not allowed. The LL line is labeled LL in the figure. The vertical arrows give the direction of movement implied by equation A.15. The line stops when it hits the NN line. The phase diagram implies that there is a unique equilibrium at point E, and a unique saddle path leading to it, labeled SS. At E, $l_{sQ}^W = 1$, $l_{sN}^W = 0$ and by equation A.16,

$$N^{*W} = \left[\frac{H_n^W}{\alpha\rho(\sigma-1)}\right]^{\frac{1}{1=\phi}} L^W \tag{A.18}$$

The logic of the model implies that an increase in ϕ (an increase in knowledge spillovers in research) does not lead to a decrease in the number of varieties. This implies that $H_n^W \geq \alpha\rho(\sigma-1)$, which we assume to be the case. Substituting equation A.18 and $l_{sQ}^W = 1$ into equation A.12 yields

$$N^{*m} = \frac{s^m H_n^m L^m}{\alpha\rho(\sigma-1)} \left[\frac{H_n^W}{\alpha\rho(\sigma-1)}\right]^{\frac{\phi}{1-\phi}} \tag{A.19}$$

In equilibrium, all workers work in manufacturing and no research takes place. The number of varieties in both countries is proportional to the population, and the world number of varieties is proportional to the world population. Total output in each country and thus in the world is constant. World spending is thus also constant, which by equation 2.13 implies that the interest rate r equals the utility discount rate ρ. Equation A.19 shows that a rise in the skill level of country m raises the number of firm in both countries (through the rise in H_n^m), but relatively more in country m itself (through a rise in s^m). The rise of the number of firm abroad is due to the spillover effects of knowledge.

A.2 ϕ = 1

For the case $\phi = 1$, define the N_1N_1 line as the set of points for which equation A.14 holds. We rewrite it as

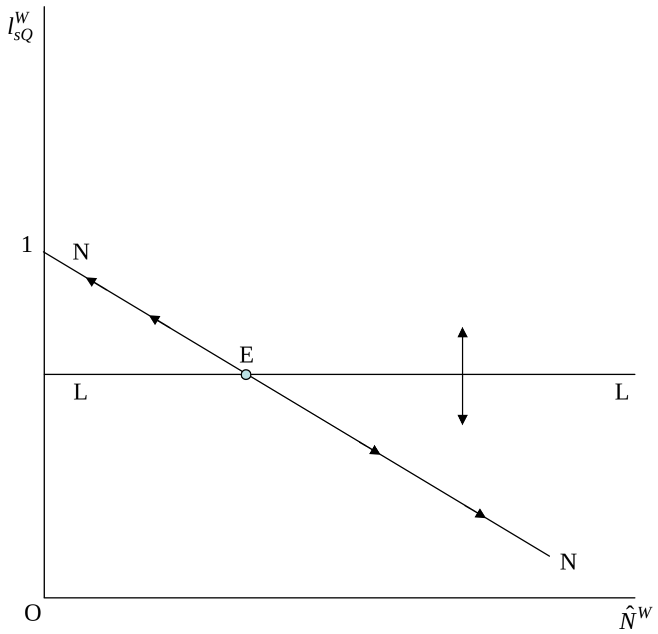
$$l_{sQ}^{W} = 1 - \frac{\hat{N}^{W}}{H_n^{W}} \tag{A.20}$$

Since this equation always holds, the world economy must always be on this line.The line is labeled N_1N_1 in figure A.2. The equation for the LL in this case becomes

$$l_{sQ}^{W} = \frac{1 + \frac{\rho}{H_n^W}}{1 + \frac{1}{(\sigma-1)\alpha}} \tag{A.21}$$

The line is again horizontal and labeled LL in the figure. The vertical arrows again give the

Figure A.2 Phase diagram for $\phi = 1$



direction of motion implied by equation A.15. These arrows and the fact that the economy must always move along the NN line imply that if the economy is ever not at point E, it will always move further away from that point, as indicated by the arrows on the NN line. So, E is a unique, unstable equilibrium, and the only way the economy reaches it is by immediately jumping to it. Substituting equation A.21 into equation A.14 yields

$$\hat{N}^{*W} = \frac{H_n^W - \alpha\rho(\sigma-1)}{1 + \alpha(\sigma-1)} \tag{A.22}$$

The requirement that the number of varieties does not implode to zero implies, just as above, that $H_n^W \geq \alpha\rho(\sigma-1)$, which we assumed to be the case.

In this case, research takes place in equilibrium, so that the number of varieties grows. The growth rate is the same in both countries, and thus equal to the world growth rate. Total output is again constant in the steady state, so that $r = \rho$. This implies that employment in manufacturing is spread out over an ever increasing number of firms, each of which produces less and less. Because consumer like variety, instantaneous utility does keep growing. A rise in the skill level of country m raises the equilibrium growth rate of firm in both countries equally (through the spillover effects), but the relative level of the number of firms in country m rises. So, on the transition path, the growth rate of the number of firms in country m is higher than in the other country.

Note that figures A.1 and A.2 are show the phase diagrams for the simplified model of equations A.14 and A.15, not the original full model of equations 5.7 and A.4. However, since the latter system converges to the former, the simplified model is enough the analyze the equilibrium properties of the original model. The conclusion is that the original model has a unique equilibrium, given by the equations above.

A.3 Homogeneous labor

The model reduces to the version with only one type of skill if we set $\alpha = 1$ and $s^m = 1$. Substituting these values into the equations above yields for the case of $\phi \leq 1$,

$$l_Q^{*m} = 1 \tag{A.23}$$

$$l_N^{*m} = 0 \tag{A.24}$$

$$N^{*m} = \frac{H_n^m}{\rho(\sigma-1)} \left[\frac{H_n^W}{\rho(\sigma-1)}\right]^{\frac{\phi}{1-\phi}} L^m \tag{A.25}$$

$$N^{*W} = \left(\frac{H_n^W}{(\sigma-1)\rho}\right)^{\frac{1}{1-\phi}} L^W \tag{A.26}$$

$$r = \rho \tag{A.27}$$

where $H_n^W = \frac{H_N^A L^A + H_n^B L^B}{L^W}$. H_n^W. H_n^W is the average world labor productivity in research. For $\phi = 1$, the solution for the balanced growth path has

$$l_Q^{*m} = \eta \left(1 + \frac{\rho}{H_n^W}\right) \tag{A.28}$$

$$l_N^{*m} = 1 - l_Q^{*m} \tag{A.29}$$

$$\hat{N}^{*m} = \hat{N}^{*W} \tag{A.30}$$

$$\hat{N}^{*W} = (1-\eta)H_n^W - \eta\rho \tag{A.31}$$

$$r = \rho \tag{A.32}$$

These equations are copied as equations 4.17 through 4.26 in the main text. .

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