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Competition and access price regulation in the broadband market[a]

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Abstract in English

In most European broadband Internet markets local loop unbundling is mandated under a cost-based regulated access price. We construct a model for differentiated Cournot competition between service-based and infrastructure-based firms, out of which one infrastructure-based firm (the incumbent) supplies to the service-based firms. We seek for and compare the socially optimal and the incumbent's profit maximizing access price in two scenarios: (i) service-based firms and incumbent supply homogeneous services (partial differentiation), and (ii) all services are horizontally differentiated (uniform differentiation). We show that in both cases the incumbent never forecloses service-based firms if infrastructure-based competition is present or if services are somewhat differentiated. Under uniform differentiation the welfare optimizing access price is below marginal cost, hence the incumbent subsidizes the production of service-based firms and makes zero profit. In the case of partial differentiation, the same result obtains when both markets are concentrated. However, if markets are not concentrated, the socially optimal access fee exceeds the marginal cost.

Key words: broadband Internet market, imperfect competition, product differentiation, access regulation

JEL code: L13, L51, L86, L96

Abstract in Dutch

In de meeste Europese landen valt toegang tot de local loop van netwerken voor breedband internet onder een regime van kostengeoriënteerde regulering. Dit paper analyseert een model waarin bedrijven zonder en met een eigen netwerk met elkaar concurreren. We nemen aan dat één bedrijf met eigen netwerk (de incumbent) toegang verleent aan bedrijven zonder eigen netwerk. We vergelijken de welvaartsoptimale toegangsprijs met de toegangsprijs die de winst van de incumbent maximaliseert in twee scenario's: (i) de producten van alle bedrijven zijn even sterk gedifferentieerd en (ii) de producten van de incumbent en de bedrijven zonder eigen netwerk zijn homogeen. We laten zien dat in beide gevallen de incumbent bedrijven zonder eigen netwerk niet uitsluit als er concurrerende netwerken bestaan of als de producten van de incumbent en de bedrijven zonder eigen netwerk gedifferentieerd zijn. In het eerste scenario ligt de welvaartsoptimale prijs onder de marginale kosten. De incumbent maakt dan zelf geen winst en subsidieert de productie van bedrijven zonder eigen netwerk. In het tweede scenario ligt de welvaartsoptimale toegangsprijs boven de marginale kosten als de markt niet geconcentreerd is.

Sleutelwoorden: broadband Internet market, imperfect competition, product differentiation,

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Contents

Summary

In most European broadband Internet markets local loop unbundling is mandated under a cost-based regulated access price. We construct a model for differentiated Cournot competition between service-based and infrastructure-based firms, out of which one infrastructure-based firm (the incumbent) supplies to the service-based firms. We seek for and compare the socially optimal and the incumbent's profit maximizing access price in two scenarios: (i) service-based firms and incumbent supply homogeneous services (partial differentiation), and (ii) all services are horizontally differentiated (uniform differentiation). We show that in both cases the incumbent never forecloses service-based firms if infrastructure-based competition is present or if services are somewhat differentiated. Under uniform differentiation the welfare optimizing access price is below marginal cost, hence the incumbent subsidizes the production of service-based firms and makes zero profit. In the case of partial differentiation, the same result obtains when both markets are concentrated. Nonetheless, if markets are not concentrated, the socially optimal access fee exceeds the marginal cost.

1 Introduction

Since the emergence of broadband Internet access services in the 1990s, telecommunication networks have been regulated. In most EU countries regulatory authorities require the dominant fixed telecommunication firm to provide entrants with wholesale access to its network on a cost oriented basis.[1] This requirement is in compliance with the guideline of the Access Directive of the European Parliament and of the Council passed in 2002 (European Commission (2002)). Access regulation aims to increase competition at the services level by enabling entry without the need for high infrastructure investments.

Since then, discussion has centered on whether this policy has been successful and whether regulation should take a different direction. So far, consensus on the effects of unbundling and a higher number of service-based firms has not been reached. On the contrary, concerns have been raised that mandatory unbundling and access price regulation have dampened firms' incentives to engage in infrastructure-based competition.

Several empirical studies (Howell (2002), Aron and Burnstein (2003), Höffler (2005), and Wallsten (2006)) suggest that wholesale access policies aimed at increasing broadband diffusion through service-based competition have not been successful. Using data of Western European countries, Höffler (2005) finds service-based competition to do poorly in stimulating broadband diffusion while infrastructure-based competition has a significant positive effect. Based on OECD and ITU country data, Wallsten (2006) concludes that local loop unbundling has no significant effect on broadband penetration and, furthermore, sub-loop unbundling is negatively related to broadband diffusion. He finds only on-site collocation to be positively correlated with broadband penetration.

From the theoretical point of view, Distaso et al. (2006) study the relationship between service-based and infrastructure-based competition and broadband diffusion, and find somewhat different results. They assume one incumbent and a variable number of service-based and infrastructure-based firms compete in quantities. Broadband diffusion is interpreted as the total demand for broadband access. Distaso et al. find that infrastructure-based competition unambiguously stimulates broadband adoption. Also service-based competition is found to spur broadband diffusion given that the ratio of service-based firms to infrastructure-based firms is not remarkably high. If the ratio is high, however, the positive effect of a further increase in the number of firms within the dominant technology is partially or fully neutralized by the negative effect of increased infrastructure-based concentration. Distaso et al. also find that broadband diffusion decreases in the access price but they do not define the socially optimal access fee.

The risk of exclusion is an important motivation for access regulation. Such regulation

[1] Access may take place at various levels of the infrastructure: through resale, bit-stream access, shared access lines or fully unbundled lines.

prevents the incumbent from foreclosing service-based firms through high access prices. In markets with multiple competing infrastructures whose owners are also present in the downstream market, competition between networks may reduce the incentive for the incumbent to exclude its downstream rivals. Several papers (Bourreau et al. (2007), Brito and Pereira (2007) and Ordover and Shaffer (2006)) analyze the effects of unregulated access pricing in the case of such a bilateral oligopoly. They conclude that in many cases an upstream network has an incentive to supply its downstream rivals. After all, entry often reduces the market shares of the incumbent's upstream competitors. Competition between the upstream suppliers may then result in low access prices. Ordover and Shaffer (2006) find an exception to this result. If a downstream retailer cannibalizes its supplier's downstream market disproportionately, it may lead downstream rivals to be foreclosed. In addition, Bourreau et al. (2007) find that, depending on the level of downstream product differentiation, upstream firms do not always compete to be the downstream rival's supplier which leads to access at higher prices.

In this paper we examine optimal access price regulation in a market with both service-based and infrastructure-based competition. We assume that service-based firms have access to the network of one incumbent. This is the case in most EU countries. In a model of Cournot competition in differentiated services offered by service-based and infrastructure-based firms, we compare the socially optimal access price with the incumbent's privately chosen optimal access price. We consider two scenarios. In the first scenario service-based firms and the incumbent offer homogeneous services, but infrastructure-based firms are differentiated. In the second scenario, all firms are differentiated. In contrast to related papers, we have multiple service-based firms, as well as multiple infrastructure-based firms. However, we do not analyze the optimal number of firms.

Our first finding is that when the market is unregulated and service-based firms supply homogenous services, the incumbent forecloses service-based competition only if no other infrastructure-based firm is present in the market. When service-based firms supply heterogeneous services, the incumbent never forecloses them from the market. These results are in line with the general literature on foreclosure (see for instance Bijlsma et al. (2008)). This suggests that under network competition, access prices do not necessarily have to be regulated.

Our second finding is that irrespective of how the services are differentiated, the incumbent always chooses a higher access price than the socially optimal one. The socially optimal access fee is however not necessarily cost-based. Under uniform service differentiation the socially optimal access price falls below cost. When services are partially differentiated, the socially optimal access fee can be above or below cost, that is, contain a positive or a negative price mark-up. The mark-up depends on the degree of market power of service-based and infrastructure-based firms. This is in line with Distaso et al. who find that it is not always desirable to promote the output of service-based firms, i.e. to set a low access price. If competition is sufficient between and over the networks, a positive access mark-up may subsidize the output of the incumbent and other infrastructure-based firms. Our findings therefore support the notion that regulated access prices do not have to be cost-based, as is the current practise in many European countries.

This paper proceeds as follows. Chapter 2 introduces the model. In chapter 3 we analyze the first scenario where only the infrastructure-based firms differentiate their services. In chapter 4 we study the second scenario where services of all firms are differentiated. Chapter 5 concludes. Proofs of the existence of equilibria and propositions are in the appendices.

2 Model Description

The model describes Cournot competition with horizontally differentiated goods, between n service-based and $m+1$ infrastructure-based firms ($n \geq 1, m \geq 0$). Cournot competition is often used in the industrial organization literature to model competition in the telecommunications market even in the absence of capacity constraints,[2] like for instance in our motivating paper Distaso et al. (2006). Cournot competition enables us to model market power even when services are stronger substitutes, which characteristics can be observed in the broadband Internet access market. As evidence of that, one can think of the price mark-ups realized in the market. The currently observed end-user prices are considerably higher than the cost of providing the service to the customer (the marginal costs of providing broadband services are almost zero).

Service-based firms are indexed by $i = 1, ..., n$, the incumbent by $i = n+1$, and the other infrastructure-based firms by $i = n+2, ..., n+1+m$. Infrastructure-based firms supply vertically integrated services, that is, they own the networks over which they serve consumers. Service-based firms, on the contrary, access the network of an infrastructure-based firm to supply retail services. We assume that only one of the infrastructure-based firms, referred to as the incumbent, supplies network capacity in the upstream market. The incumbent is not allowed to discriminate in the access prices.

To keep the analysis tractable, we assume that a representative consumer has a linear inverse demand function given by

$$\mathbf{p} = \alpha - \mathbf{Bq}$$

where $\mathbf{p}$ denotes the vector of prices that a consumer pays for the services, $\mathbf{q}$ is the vector of quantities consumed at these prices, and α and $\mathbf{B}$ parameterize the demand for given $\mathbf{q}$. The vector α and the symmetric matrix $\mathbf{B}$ are of dimension $1 \times (n+1+m)$ and $(n+1+m) \times (n+1+m)$, the components of which are denoted by α_i and $\beta_{i,j}$, respectively. The diagonal element $\beta_{i,i}$ of $\mathbf{B}$ is the own price effect of firm i's output on the price set by firm i, and $\beta_{i,j}$ is the cross price effect of firm j's output on the price set by firm i. The relative magnitudes of $\beta_{i,i}$ and $\beta_{i,j}$ thus define the level of differentiation between the services provided by firms i and j. When $\beta_{i,i}$ equals $\beta_{i,j}$, firms i and j supply homogenous services. The consumer surplus

[2] Service-based firms might face capacity constraint in the backbones, however this case we assume away.

associated with this demand function reads[3]

$$CS = (\alpha^T - \frac{1}{2}\mathbf{q}^T\mathbf{B})\mathbf{q} - \mathbf{p}^T\mathbf{q}$$

The numbers of service-based and infrastructure-based firms and the degree of service differentiation between the firms can be seen as proxies for the degree of service-based and infrastructure-based competition. We assume that firms have no fixed costs.[4] Service-based firms are charged symmetric access prices for providing access services to end-users over the incumbent's network. For later use we define **a** as a vector of access prices a_i with the following characteristics

$$a_i = a \text{ for all } i \leq n \text{ and } a_i = 0 \text{ otherwise.}$$

The access prices paid by service-based firms constitute the wholesale access revenue to the incumbent. We assume the upstream marginal costs to be equal for all networks, and it is set to zero for simplicity. The access price charged by the incumbent can therefore be interpreted as a price mark-up over its upstream marginal cost. Consequently, the firms' profits are given by

$$\pi_i = q_i(p_i - a) \text{ for all } i \leq n$$
$$\pi_i = q_i p_i + \sum_{j \leq n} q_j a \text{ for } i = n+1$$
$$\pi_i = q_i p_i \text{ for } i > n+1$$

Total welfare, equal to the sum of consumer surplus and producer surplus, is given by

$$W = CS + \sum_i \pi_i = \left(\alpha^T - \frac{1}{2}\mathbf{q}^T\mathbf{B}\right)\mathbf{q} \qquad (2.1)$$

The model consists of two stages. In the first stage, the wholesale access price is set either by the incumbent or the regulator. In the second stage, firms set their profit maximizing outputs, given the output of their competitors under differentiated Cournot competition. We find the equilibrium of the model by backward induction. We solve the model in two different scenarios. In the first scenario, referred to as partial differentiation, all services carried over a particular network are homogenous, irrespective of whether the services are provided by service-based

[3] Our definition of consumer surplus is in line with the literature on horizontal product differentiation, e.g. Singh and Vives (1984). This definition is supported by any utility function yielding the above demand system; for instance the following simple form satisfies

$$u(\mathbf{q}) = \left(\alpha^T - \frac{1}{2}\mathbf{q}^T\mathbf{B}\right)\mathbf{q}$$

From this utility function exactly the same demand system arises when the representative consumer maximizes his utility given his budget constraint.

[4] Since the analysis is *ex post* with respect to investment and entry, all the infrastructure-based firms' investments in networks are sunk costs. Moreover, fixed costs would be relevant only for the socially optimal access price. When networks are not tradable, the opportunity cost of a network is zero. If networks were yet tradable, the regulator could set the access price such that it yields zero profit to the incumbent and thus zero opportunity cost to its network. In both cases fixed costs remain irrelevant in the analysis.

firms or the incumbent. All services carried over different networks are horizontally differentiated, that is, infrastructure-based firms supply differentiated downstream services. In the second scenario, referred to as uniform differentiation, all downstream services are uniformly differentiated, irrespective of the network through which the services are supplied.

3 Partial Differentiation

In this scenario service-based firms and the incumbent supply homogeneous services, while infrastructure-based firms supply heterogenous services. The demand parameters are hence defined as

$$\alpha_i = \alpha > 0 \text{ for all } i$$
$$\beta_{i,j} = \beta > 0 \text{ for all } i = j \text{ or } i, j \leq n+1$$
$$\beta_{i,j} = \gamma \geq 0 \text{ for all } i \neq j \text{ and } i > n+1 \text{ or } j > n+1$$

To compare the access prices that prevail in the presence and absence of regulation, the socially optimal access fee and the incumbent's choice of access fee are solved for. The regulated access price is set such that the sum of consumer and producer surpluses is maximized, provided that the incumbent and the service-based firms are active in the market at that access price.[5] When the access price is not regulated, the incumbent sets an access price that maximizes its profit.

3.1 Differentiated Cournot Equilibrium

In the second stage of the game, firms compete à la Cournot and set their profit maximizing output, given the output of their competitors and the access price which is determined in the first stage. The first order conditions can be written as

$$\mathbf{p} - \mathbf{a} - \mathbf{D}\mathbf{q} = \mathbf{0}$$

where $\mathbf{D}$ is a diagonal matrix of the own price parameters β. The equilibrium outputs at a given access price a are

$$\mathbf{q}(a) = (\mathbf{B} + \mathbf{D})^{-1}(\alpha - \mathbf{a})$$

Since all firms of a certain type are symmetric, they produce the same output quantity. Due to the symmetry in the equilibrium, from now on let S denote a service-based, A an infrastructure-based firm, and I the incumbent. The outputs of each types of firms are

[5] This definition of the social optimal access price follows the general literature on one-way access, such as Laffont and Tirole (2000) and Vogelsang (2003). The Ramsey or second-best approach for social optimum is suitable since no additional entry takes place in the markets.

$$q_S(a)=\frac{\alpha\beta\,(2\beta-\gamma)-(2\beta-\gamma)\,(2\beta+\gamma m)\,a}{\beta\,[\beta(2\beta+\gamma(m-1))(n+2)-\gamma^2(n+1)m]}=q_I(a)-\frac{a}{\beta} \tag{3.1}$$

$$q_I(a)=\frac{\alpha\beta\,(2\beta-\gamma)+(\beta\,(2\beta-\gamma)+(\beta-\gamma)\,\gamma m)\,na}{\beta\,[\beta(2\beta+\gamma(m-1))(n+2)-\gamma^2(n+1)m]}$$

$$q_A(a)=\frac{\alpha\beta\,(2\beta-\gamma)+n\alpha\beta\,(\beta-\gamma)+\beta\gamma na}{\beta\,[\beta(2\beta+\gamma(m-1))(n+2)-\gamma^2(n+1)m]}$$

The equilibrium $q_i(a)$s are always non-negative if $0\leq\gamma\leq\beta$ (or for later notation $0\leq\delta\leq 1$ where $\delta=\gamma/\beta$). The equilibrium prices are

$$\begin{aligned} P_S(a) &= \beta q_S(a)+a \\ P_I(a) &= P_S(a)=\beta q_I(a) \\ P_A(a) &= \beta q_A(a) \end{aligned} \tag{3.2}$$

Service-based firms and the incumbent charge equal prices since they provide homogenous products. The equilibrium quantities and prices solved in the second stage of the game yield the following profit functions

$$\begin{aligned} \pi_A\,(a) &= \beta q_A^2\,(a) \\ \pi_I\,(a) &= \beta q_I^2\,(a)+naq_S \\ \pi_S\,(a) &= \beta q_S^2\,(a) \end{aligned}$$

The relationship between access pricing, outputs (3.1) and end-user prices (3.2) is described in the following proposition.

Proposition 1. *In the equilibrium*

(i) the output of a service-based firm decreases, whereas the price set by a service-based firm increases in the access price,

(ii) the output of and the price set by the incumbent increase in the access price,

(iii) the output of and the price set by an infrastructure-based firm increase in the access price if $\gamma>0$; the access price has no effect on an infrastructure-based firm's output and price if $\gamma=0$.

An increasing access price raises the marginal cost of a service-based firm, thus reducing its output. The incumbent's output increases in the access price because the outputs of service-based firms and the incumbent are strategic substitutes. Since the aggregate decrease in the service-based firms' output is larger than the increase in the incumbent's output, their total output decreases. Therefore, the price for the homogeneous services provided by the service-based firms and the incumbent increases in the access price. Because the services provided over different networks are strategic substitutes, the output of an infrastructure-based firm will also increase in the access price. However the increase in the aggregate output of infrastructure-based firms is less than the decrease in aggregate output provided over the incumbent's network, shifting the total demand for services of infrastructure-based firms upwards. This increased demand leads to a higher price set by the infrastructure-based firms.

3.2 The Incumbent's Choice of Access price

In the first stage of the game, the incumbent sets the access price to maximize its profit. Given the equilibrium quantities (3.1) and (3.2) of the second stage, the incumbent's profit can be written as

$$\pi_I(a) = \beta q_I^2(a) + anq_S(a)$$

The first order condition for the incumbent is

$$\frac{\partial \pi_I}{\partial a} = 2\beta q_I \frac{\partial q_I}{\partial a} + nq_S + an\frac{\partial q_S}{\partial a} = 0$$

from which the profit maximizing access price follows

$$a_I = -\frac{q_I(0)\left(\frac{\partial q_I}{\partial a} + \frac{n}{2\beta}\right)}{\left(\frac{\partial q_I}{\partial a} + \frac{n}{2\beta}\left(1+\sqrt{1+\frac{4}{n}}\right)\right)\left(\frac{\partial q_I}{\partial a} + \frac{n}{2\beta}\left(1-\sqrt{1+\frac{4}{n}}\right)\right)} \tag{3.3}$$

where $q_I(0)$ refers to the incumbent's output when $a = 0$, and $q_I(0) > 0$. This access price is positive if the second order condition holds which is

$$\frac{\partial \pi_I^2}{\partial^2 a} = 2\beta\left(\frac{\partial q_I}{\partial a}\right)^2 + 2n\frac{\partial q_S}{\partial a} < 0 \Leftrightarrow \frac{\partial q_I}{\partial a} < -\frac{n}{2\beta}\left(1-\sqrt{1+\frac{4}{n}}\right)$$

This condition is satisfied for all $n \geq 1$ and m if $0 \leq \delta \leq 1$.

In order to see whether the incumbent has an incentive to set an excessively high access price at which it forecloses service-based firms, we compare the incumbent's optimal choice with the exclusionary access price. At the exclusionary access fee a service-based firm earns zero profit, that is

$$\pi_S(a_S^E) = \beta q_S^2(a_S^E) = 0$$

which here is equivalent to $q_S(a_S^E) = 0$ and results in

$$a_S^E = \frac{\alpha}{2+\delta m} \tag{3.4}$$

The relationship between (3.3) and (3.4) is described in the following proposition.

Proposition 2. *In equilibrium, for $n \geq 1$, the following holds for the incumbent's profit maximizing access price (3.3) and the service-based firm's exclusionary access price (3.4):*

(i) $a_I = a_S^E$ if $m = 0$ or $\delta = 0$, that is, foreclosure takes place in the case of network monopoly or independent network services,

(ii) $a_I < a_S^E$ if $m \geq 1$ and $\delta > 0$, that is, the incumbent allows for service-based competition if it competes against any infrastructure-based firm and the services are at least somewhat substitutes.

If services provided over different networks are maximally differentiated from each other ($\delta = 0$), service-based firms can only gain consumers at the expense of the incumbent. In this case it is more profitable for the incumbent not to supply service-based firms, hence the latter are foreclosed from the market. The same argument holds in the absence of infrastructure-based competitors ($m = 0$): it is more profitable for the incumbent to operate as a monopolist in the market. Note, however, that in this scenario, a (monopolist) incumbent and a service-based firm always offer homogeneous services. If this assumption is relaxed it will be profitable for the incumbent to supply service-based firms.

If infrastructure-based competition is present ($m \geq 1$) and a certain level of substitution between networks' services ($\delta > 0$), the incumbent sets the access price below the exclusionary level. This strategy is profitable for the incumbent because service-based firms supply not only some of the end-users the incumbent would supply if it foreclosed service-based competition, but also some of the end-users the other infrastructure-based firms would serve. The increases in wholesale revenues including from the market that the service-based firms gain from the infrastructure-based firms outweigh the downstream market revenues foregone due to the loss of consumers to the service-based companies.

3.3 The Socially Optimal Access Price

The socially optimal access price maximizes the social welfare as measured by the sum of consumer and producer surpluses, subject to the participation constraints, that is, all firms should make non-negative profits. Because the infrastructure-based firms' profits are nonnegative independent of the level of the access price, we only consider the participation constraints of the incumbent and the service-based firms. As a first step, we calculate the unconstrained access price that maximizes welfare (2.1). The first order condition is

$$\frac{\partial W}{\partial a} = \left(\alpha^T - \mathbf{q}^T \mathbf{B}\right) \frac{\partial \mathbf{q}(a)}{\partial a} = 0$$

from which

$$a_W = \frac{\left(\alpha^T - \mathbf{q}^T(0)\mathbf{B}\right) \frac{\partial \mathbf{q}(a)}{\partial a}}{\frac{\partial \mathbf{q}(a)}{\partial a}^T \mathbf{B} \frac{\partial \mathbf{q}(a)}{\partial a}} \tag{3.5}$$

$$= -\frac{\alpha\left((2-\delta)^2 - mn\delta(1-\delta)\right)}{n\left((2-\delta)^2 + m\delta(1-\delta)(4-(m-1)\delta)\right)}$$

The sign of (3.5) depends on the numbers of service-based and infrastructure-based firms and the degree of product differentiation, as is concluded below.

Proposition 3. *In the equilibrium, when* $0 \leq \delta \leq 1$, *the following holds for the unconstrained welfare maximizing access price* a_W*:*

(i) if $mn \geq 8$ *and* $\delta \varepsilon \left[\frac{4+mn-mn\sqrt{1-\frac{8}{mn}}}{2(1+mn)}, \frac{4+mn+mn\sqrt{1-\frac{8}{mn}}}{2(1+mn)}\right]$, *then* $a_W \geq 0$,

(ii) otherwise, that is, if $mn < 8$ *or* δ *is very low,* $a_W < 0$,

(iii) $a_W < a_I$, *that is, the unconstrained welfare maximizing access price is always lower than the incumbent's choice of access price.*

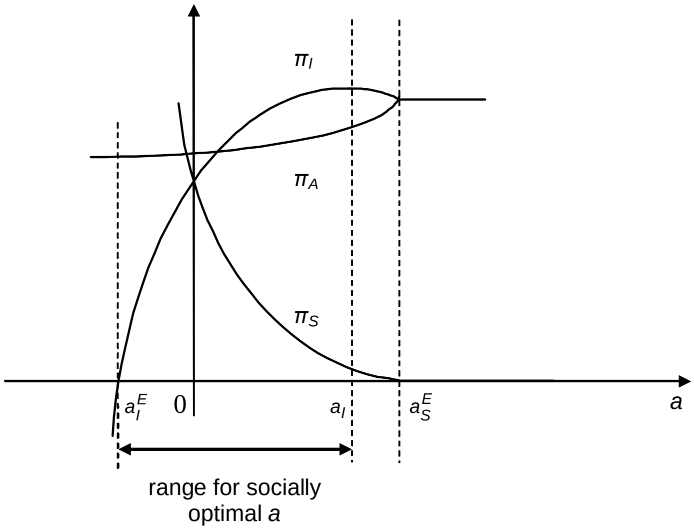
The welfare maximizing access price can therefore be positive or negative. It is positive when there is sufficient competition either at service-based level or at infrastructure-based level and when the services are relatively close substitutes. A positive access mark-up means that the output of the incumbent and the other infrastructure-based firms is subsidized. If both markets are concentrated ($mn < 8$, that is, the number of service-based and infrastructure-based firms is low), then the welfare maximizing access price is negative and subsidizes the output of the service-based firms.

However the welfare maximizing access price may yield negative profit to the incumbent or to the service-based firms, in which case the equilibrium is not feasible. Taking this into consideration, the socially optimal access price cannot be lower than the exclusionary access price of the incumbent and higher than the one of a service-based firm. The potential range of the socially access fee is depicted in Figure 3.1 where each firm's profit is displayed as a function of the access price.

We first assess whether the exclusionary access price for a service-based firm will ever bind. According to Proposition 2, the incumbent's profit maximizing access price is never higher than the exclusionary access price for a service-based firm. As it is shown in Figure 3.1, the profit of a service-based firm (see π_S) increases when the access price decreases. Moreover Proposition 3 guarantees that $a_W < a_I$, therefore the social welfare maximizing access price never exceeds the exclusionary access price of a service-based firm, that is, $a_W < a_S^E$. Hence for $\delta < 1$ the service-based firm's profit constraint will never bind.

The lower the access price, the lower the profit of the incumbent is (see π_I in Figure 3.1). Therefore there exists an access price a_I^E at which the incumbent's profit equals zero ($\pi_I\left(a_I^E\right) = 0$).

Figure 3.1 Profits as functions of access price, partial differentiation



It then determines a lower limit for the socially optimal access fee and equals

$$a_I^E = -\frac{q_I(0)}{\frac{\partial q_I}{\partial a} + \frac{n}{2\beta}\left(1 - \sqrt{1+\frac{4}{n}}\right)} < 0$$

which is always negative. If the unconstrained welfare maximizing access price is below this lower limit, the participation constraint of the incumbent is binding, and the socially *optimal* access price equals a_I^E. However if the unconstrained welfare maximizing access price is higher than the incumbent's exclusionary fee, the participation constraints are not binding, and the former is the socially optimal one. Whether the participation constraints are binding or not depends on the intensity of competition and the degree of service differentiation which are the two measures of market power. According to Proposition 3, in highly concentrated markets or when services between networks are weak substitutes, the welfare maximizing access fee is negative. Since the regulator intends to intensify downstream competition, it can obtain it via an access price as low as possible: that is the exclusionary fee which yields zero profit for the incumbent. The higher the number of service-based (n) or infrastructure-based firms (m), the lower the market power downstream. As a consequence, it is more likely that the unconstrained welfare maximizing access fee, which increases in n and m, exceeds the exclusionary fee. This result is intuitive. Due to more intensive competition the regulator will let the incumbent earn more (or lose less) in the wholesale market by setting a higher access fee than its exclusionary one. This result holds true even for more differentiated services (small δ). However when the number of service-based firms is small, the regulator will allow for a higher access price than the exclusionary one only when competition can be still kept sufficient, that is if services provided over different networks are strong substitutes (large δ).

This result is summarized in the following proposition.

Proposition 4. *In the equilibrium, where* $0 \leq \delta \leq 1$,

(i) if $a_I^E < a_W$*, then the socially optimal access price equals* a_W *as defined in (3.5) and it is an internal equilibrium of the welfare optimization problem,*

(ii) if $a_W \leq a_I^E$*, then the socially optimal access price equals* $a_I^E < 0$*, yielding a cornering solution of the welfare maximization problem.*

To summarize these findings, the socially optimal access price depends on the numbers of service-based and infrastructure-based firms as well as on the own and cross price effects. Generally speaking, the socially optimal access price is positive when the numbers of infrastructure-based and service-based firms are high. Similarly, the access price is negative when the firms are few in number. Setting the socially optimal access fee negative alleviates the detrimental effects of market power by subsidizing the output of service-based firms. When the access price is positive, on the other hand, it subsidizes the output of infrastructure-based firms, including the incumbent. If the service-based firms and infrastructure-based firms are equal in number, the socially optimal access price increases with the number of firms from negative to positive.

4 Uniform Service Differentiation

In this section we assume that all firms' services are uniformly horizontally differentiated. The demand parameters are therefore defined as follows

$$\begin{aligned}
\alpha_i &= \alpha > 0 \text{ for all } i \\
\beta_{i,j} &= \beta > 0 \text{ for all } i = j \\
\beta_{i,j} &= \gamma \geq 0 \text{ for all } i \neq j
\end{aligned}$$

Again we intend to find the incumbent's profit maximizing access price in the absence of regulation and the socially optimal access price maximizing the sum of consumer and producer surpluses. We compare the equilibria with the ones of the first scenario where only infrastructure-based firms differentiate their services.

4.1 Differentiated Cournot Equilibrium

We consider the same game as described in section 2. Solving for the second stage, the outputs of a service-based firm q_S, the incumbent q_I and an infrastructure-based firm q_A as a function of the access price a are given by

$$q_S(a) = \frac{\alpha(2\beta-\gamma)-(2\beta+\gamma m)a}{(2\beta-\gamma)(2\beta+\gamma(n+m))} = q_I(a) - \frac{a}{2\beta-\gamma} \tag{4.1}$$

$$q_I(a) = q_A(a) = \frac{\alpha(2\beta-\gamma)+\gamma n a}{(2\beta-\gamma)(2\beta+\gamma(n+m))}$$

In these expressions we explicitly denoted the dependency of the equilibrium quantities on the access price, and these quantities are always non-negative if $0 \leq \gamma \leq \beta$ (or $0 \leq \delta \leq 1$ where $\delta = \gamma/\beta$). The corresponding second stage equilibrium prices are

$$\begin{aligned}
p_S(a) &= \beta q_S(a) + a \\
p_I(a) &= \beta q_I(a) = \beta q_A(a) = p_A(a)
\end{aligned} \tag{4.2}$$

whereas the firms' profits satisfy

$$\begin{aligned}
\pi_S(a) &= \beta q_S^2(a) \\
\pi_I(a) &= \beta q_I^2(a) + naq_S \\
\pi_A(a) &= \beta q_A^2(a)
\end{aligned}$$

In contrast to the first scenario, in this case where all services are uniformly differentiated, the incumbent and the infrastructure-based firms charge equal prices and supply the same quantities. This result obtains because these firms face now symmetric competitors and have symmetric costs. As before, the output produced by a service-based firm equals the output of the incumbent minus a term linear in the access price. Thus, for a positive access price, the output of a service-based firm is always lower than the output of an infrastructure-based firm. From (4.1) and (4.2), the next proposition follows.

Proposition 5. *In the equilibrium*

(i) the output of a service-based firm decreases, whereas the price set by a service-based firm increases in the access price,

(ii) the outputs of and the prices set by an infrastructure-based firm and the incumbent increase in the access price if $\gamma > 0$*; the access price has no effect on the output of and the price set by any infrastructure-based firm if* $\gamma = 0$.

This proposition shows that our qualitative findings on the relationship between access pricing, output and end-user prices are the same in both models.

4.2 The Incumbent's Choice of Access Price

The incumbent's profit maximizing access price and the exclusionary access price for a service-based firm can be determined in the same way as in chapter 3. The exclusionary access price for a service-based firm a_S^E, i.e. the access price at which the service-based firm's profit maximizing strategy is zero output, is

$$a_S^E = \frac{\alpha\,(2-\delta)}{2+\delta m} \tag{4.3}$$

The incumbent's profit maximizing access price a_I equals

$$a_I = a_S^E \left(1 - \frac{1}{2}\frac{(2+(m+n)\,\delta)\,(4\,(1-\delta)+m\,(2-\delta)\,\delta)}{(2-\delta)\,(2+(m+n)\,\delta)\,(2+m\delta)-n\delta^2}\right) \tag{4.4}$$

Note that (4.4) is positive if the second order condition (SOC) is satisfied. The SOC states that the incumbent's profit is convex in a when

$$\frac{\partial \pi_I^2}{\partial^2 a} = 2\beta\left(\frac{\partial q_I}{\partial a}\right)^2 + 2n\frac{\partial q_S}{\partial a} < 0 \Leftrightarrow \frac{\partial q_I}{\partial a} < -\frac{n}{2\beta}\left(1-\sqrt{1+\frac{4}{(2-\delta)\,n}}\right)$$

For $0 \leq \delta \leq 1$ and any n and m, this condition holds true. Given (4.3) and (4.4), the incumbent's access pricing behavior and its incentive to foreclose service-based competition can be characterized as follows.

Proposition 6. *In the equilibrium, for* $n \geq 1$*, the following holds for the incumbent's profit maximizing access price (4.4) and a service-based firm's exclusionary access price (4.3):*

(i) $a_I < a_S^E$ *if* $m > 0$ *or* $0 \leq \delta < 1$*, that is, the incumbent allows for service-based competition if it competes against any infrastructure-based firm or if the services are at least somewhat differentiated,*

(ii) $a_I = a_S^E$ if $m = 0$ and $\delta = 1$, that is, foreclosure takes place in the case of network monopoly and if services are homogenous.

Proposition 6 implies that the incumbent refrains from foreclosing service-based firms if there is any other infrastructure-based firm or if the services are differentiated. Only when the incumbent is a network monopolist and downstream services are homogenous it is optimal for the incumbent to foreclose service-based competitors. Note the difference with the previous case of partial service differentiation where the incumbent always forecloses service-based competitors if $\delta = 0$, whereas in this scenario when $\delta = 0$ (that is maximum service differentiation), the incumbent never forecloses service-based firms. This difference arises because here not only the services provided in different networks, but also the services of service-based firms and the incumbent are differentiated. It means that even at $\delta = 0$ service-based firms generate extra revenue for the incumbent, whereas in the previous case they cannibalize exclusively on the incumbent's market share. Recall for a moment Proposition 2. It implies that in the first scenario where service-based firms and the incumbent provide homogenous products, the incumbent forecloses the service-based firms from the market only if no infrastructure-based competition prevails. Proposition 6 is thus in line with Proposition 2. However, it also states that if service-based firms differentiate their services, the incumbent has no incentive to foreclose them, irrespective of the degree of infrastructure-based competition.

4.3 The Socially Optimal Access Price

The socially optimal access price is subject to the constraints that service-based firm and the incumbent earn non-negative profits,[6] otherwise these firms will leave the market. As in chapter 3 and disregarding the constraints, the access price that maximizes the sum of consumer and producer surpluses is

$$a_W = -\frac{\alpha(2-\delta)^2}{\delta(2-\delta)n + (1-\delta)(2+\delta m)(2+\delta(n+m))} \tag{4.5}$$

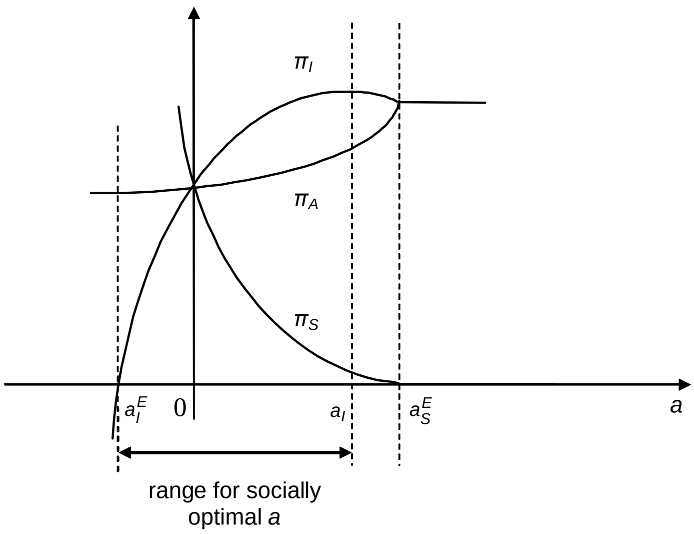
This access price is always negative if $0 \leq \delta \leq 1$. From (4.4) and (4.5) the following proposition is straightforward.

Proposition 7. *In the equilibrium, $a_W < 0 < a_I$, that is, the unconstrained welfare maximizing access price a_W is negative and always lower than the incumbent's choice of access price a_I which is positive.*

[6] As in the previous case, infrastructure-based firms always earn non-negative profits.

Now the question is whether any of the participation constraints is binding. As in the preceding scenario, there are different possibilities: either the non-negative profit condition of a service-based firm or the incumbent binds, or potentially none of the constraints bind (see Figure 4.1).

Figure 4.1 Profits as functions of access price, uniform differentiation



According to Proposition 6, at the incumbent's profit maximizing access price and for $\delta < 1$, none of the constraints is binding, implying that a_I is the upper bound for the socially optimal access price. Furthermore, the unconstrained welfare maximizing access price is smaller than the incumbent's profit maximizing access price (Proposition 7), which in turn is smaller than the exclusionary access fee for a service-based firm, therefore the constraint for a service-based firm output (4.3) never binds at the welfare maximizing access price. As for the lower bound, we are left with the possibility that at the welfare maximizing access price the non-zero profit condition for the incumbent does not satisfy. The exclusionary access price for the incumbent is defined by $\pi_I(a_I^E) = 0$ and is given by

$$a_I^E = -\frac{q_I(0)}{\frac{\partial q_I}{\partial a} + \frac{n}{2\beta}\left(1 - \sqrt{1 + \frac{4}{(2-\delta)n}}\right)}$$

The welfare maximizing access price as defined by (4.5), i.e. ignoring the participation constraints, is lower than the exclusionary access price for the incumbent. Thus, for this access price the incumbent's participation constraint always binds. Based on the previous argument, the following proposition can be stated.

Proposition 8. *For* $0 \leq \delta \leq 1$ *and for any* $n \geq 1$ *and* m, $a_W < a_I^E < 0$. *It implies that in the* *social optimum the participation constraint for the incumbent is binding and therefore the socially optimal access price* a_I^E *is.*

Like in the previous scenario, in this case the access price that maximizes the consumer and producer surpluses is lower than the access price that is optimal for the incumbent. Moreover, the socially optimal access price always equals the incumbent's exclusionary access price, yielding zero profit to the incumbent and subsidizing the output of service-based firms. This is due to the fact that all services are differentiated and therefore each firm carries some market power over its services.

5 Discussion and Conclusion

In this paper, we have studied wholesale access pricing in the absence of regulation for an arbitrary number of m infrastructure-based and n service-based firms competing à la Cournot. We compared these outcomes with the socially optimal access price. In the case of uniform differentiation we find that the socially optimal access price is always below cost, which subsidizes the output of service-based firms, allowing the incumbent to earn zero profit only. In the case of partial differentiation the socially optimal access price can be above or below costs, subsidizing in the first case the output of the infrastructure-based firms, including the incumbent, and in the second case the output of service-based firms. The socially optimal access fee only in concentrated markets can yield zero profit for the incumbent.

We also find that in the presence of infrastructure-based competition, exclusion by means of access pricing is never an equilibrium if products are differentiated. However there is an important difference between the cases of partial and uniform differentiation if the services are maximally differentiated (i.e. product are sold in separate markets). In the case of partial differentiation, exclusion is an equilibrium, whereas in the case of uniform differentiation it is not. This relates to the following effect identified by Ordover and Shaffer (2006). If entry cannibalizes the incumbents' products proportionally, which happens in the case of uniform differentiation, competition to supply the entrant ensures that the entrant always obtains access. When entry predominantly cannibalizes the incumbent's products, which happens in the case of partial differentiation, the entrant is not supplied in the equilibrium.

One might wonder what the equilibrium will be if other infrastructure-based firms than the incumbent can also offer access to their networks. In that case the analysis of whether exclusion is an equilibrium remains valid. Exclusion can only be an equilibrium if for any infrastructure-based firm i given that no other firm offers access to its network, it is optimal to charge the exclusive access price to a service-based firm. To analyze what the equilibrium will be, we turn to Figures 3.1 and 4.1. At the access price which is optimal given that no other firm provides access, all infrastructure-based firms want to be the incumbent: the profits of the firm providing access are higher than the profits of a firm not providing access. Therefore the latter will undercut the former. This will continue until there is no more to gain from undercutting, i.e. until profit π_I equals π_A. In the partially differentiated case the competitive access price will therefore be larger than zero, whereas in the uniformly differentiated case the competitive access fee will be zero. Under uniform product differentiation, the competitive access price will be larger than the socially optimal access fee. However, in the partially differentiated case the competitive access price can be either larger, smaller or equal to the socially optimal fee.

Several possible extensions come to mind that might change these results: (1) introducing two-part wholesale tariffs, (2) allowing the infrastructure-based firm to discriminate between different retailers, and (3) assuming Bertrand competition instead of Cournot. In addition, from a regulatory point of view it would be interesting to analyze the optimal number of service-based and infrastructure-based firms in our model. This requires that we introduce fixed costs and determine welfare as a function of the access price and the number of firms. Of course, in the absence of fixed costs, increasing the number of firms increases welfare.

6 Appendix: Partial Service Differentiation

Proof. *Existence of the equilibrium, stage 2* The equilibrium exists if the second order conditions (SOCs) are negative. The SOCs are $-\mathbf{D}$, which satisfies when the diagonal values of matrix $\mathbf{D}$, i.e. βs are positive. This always holds due to our assumption for β. ■

Proof. *Proposition 1* A sufficient condition for the equilibrium quantities to be non-negative is if $0 \leq \delta \leq 1$ (where $\delta = \gamma/\beta$).

(i) From the (3.1) equilibrium output of a service-based firm:

$$\frac{\partial q_S}{\partial a} = -\frac{(2\beta-\gamma)(2\beta+\gamma m)}{\beta\left(\beta(2\beta+\gamma(m-1))(n+2)-\gamma^2(n+1)m\right)}$$

For $0 \leq \delta \leq 1$ and any $n \geq 1$ and m, the nominator and the denominator of this expression are positive. Therefore $\frac{\partial q_S}{\partial a} < 0$. For price change see (ii).

(ii) From the (3.1) equilibrium output of the incumbent:

$$\frac{\partial q_I}{\partial a} = \frac{(\beta\left(2\beta-\gamma\right)+\left(\beta-\gamma\right)\gamma m)n}{\beta\left(\beta(2\beta+\gamma(m-1))(n+2)-\gamma^2(n+1)m\right)} \tag{6.1}$$

For the same reason as (i), $\frac{\partial q_I}{\partial a} > 0$. From (3.2) and $\frac{\partial q_I}{\partial a} > 0:$

$$\frac{\partial P_S}{\partial a} = \frac{\partial P_I}{\partial a} = \beta\frac{\partial q_I}{\partial a} > 0$$

(iii) From the (3.1) equilibrium output of the infrastructure-based firm:

$$\frac{\partial q_A}{\partial a} = \frac{\gamma n}{\beta(2\beta+\gamma(m-1))(n+2)-\gamma^2(n+1)m}$$

which is always positive if $\gamma > 0$. Then

$$\frac{\partial P_S}{\partial a} = \beta\frac{\partial q_A}{\partial a} > 0$$

If $\gamma = 0$, i.e. independent services, then $\frac{\partial q_A}{\partial a} = 0$, so neither the quantity, nor the price of the infrastructure-based firm changes in a. ■

Proof. *Existence of equilibrium. Stage 1, incumbent* From the incumbent's profit function the first order condition and the optimal access price are:

$$\frac{\partial \pi_I}{\partial a} = 2\beta q_I\frac{\partial q_I}{\partial a} + n\left(q_S + \frac{\partial q_S}{\partial a}a\right) = 0$$

$$a_I = -\frac{2\beta q_I(0)\frac{\partial q_I}{\partial a} + nq_S(0)}{2\beta\left(\frac{\partial q_I}{\partial a}\right)^2 + 2n\frac{\partial q_S}{\partial a}} \tag{6.2}$$

The first-order condition yields a maximum, if the second-order condition holds, i.e. $\frac{\partial^2 \pi_I}{\partial a^2} < 0$.

$$\frac{\partial^2 \pi_I}{\partial a^2} = 2\left(\beta\left(\frac{\partial q_I}{\partial a}\right)^2 + n\frac{\partial q_S}{\partial a}\right) = 2\left(\beta\left(\frac{\partial q_I}{\partial a}\right)^2 + n\left(\frac{\partial q_I}{\partial a} - \frac{1}{\beta}\right)\right)$$

which is negative if

$$0 < \frac{\partial q_I}{\partial a} < -\frac{n}{2\beta}\left(1-\sqrt{1+\frac{4}{n}}\right) \tag{6.3}$$

By unfolding (6.3)

$$\frac{(\beta\,(2\beta-\gamma)+(\beta-\gamma)\,\gamma m)n}{\beta\,(\beta(2\beta+\gamma(m-1))(n+2)-\gamma^2(n+1)m)} < \tfrac{n}{2\beta}\left(-1+\sqrt{1+\tfrac{4}{n}}\right)$$

$$\left(1+\frac{4+2\,(m-1)\,\delta-2m\delta^2}{2\,(n+2)+(m-1)\,(n+2)\,\delta-m\,(n+1)\,\delta^2}\right)^2 \qquad < 1+\tfrac{4}{n}$$

For the sufficient equilibrium condition $0 \leq \delta \leq 1$ the second term in brackets is positive, therefore the left-hand side of the previous expression is smaller than the right-hand side if

$$2\,(n+4)+(m-1)\,(n+4)\,\delta-m\,(n+2)\,\delta^2 > 0$$

This expression draws an inverted parabola in δ. Since at $\delta = 0$ it takes $2\,(n+4) > 0$ and at $\delta = 1$ it takes $2m+n+4 > 0$, for $0 \leq \delta \leq 1$ it has only positive values (for any $n \geq 1$ and m). Therefore for any $0 \leq \delta \leq 1$ SOC holds true. ■

Proof. *Proposition 2* In the expression for the incumbent's optimal access price (6.2) $q_i\,(0)$ is the quantity served by firm i at $a = 0$, and from (3.1) $q_I(0) = q_S(0)$ and $\frac{\partial q_S}{\partial a} = \frac{\partial q_I}{\partial a} - \frac{1}{\beta}$. Using $\delta = \frac{\gamma}{\beta}$ and (3.1), (6.2) can be rewritten as

$$a_I = -\frac{q_I(0)\left(2\beta\frac{\partial q_I}{\partial a}+n\right)}{2\left(\beta\left(\frac{\partial q_I}{\partial a}\right)^2+n\frac{\partial q_I}{\partial a}-\frac{n}{\beta}\right)}$$

$$= \frac{\alpha\,(2-\delta)\left((n+4)\,(2-\delta+\delta m\,(1-\delta))+m\delta^2\right)}{2\,(n+4)\,(4+4\delta\,(m-1)+\delta^2\,(m^2-4m+1)-m\delta^3\,(m-1))+2m^2\delta^4}$$

$$= a_S^E\,\frac{(2+\delta m)\,(2-\delta)\left((n+4)\,(2-\delta+\delta m\,(1-\delta))+m\delta^2\right)}{2\,(n+4)\,(4+4\delta\,(m-1)+\delta^2\,(m^2-4m+1)-m\delta^3\,(m-1))+2m^2\delta^4}$$

To prove that $a_I < a_S^E$, it is sufficient to show that the nominator of the RHS is smaller than its denominator. The difference between the two is

$$\begin{aligned}&(2+\delta m)\,(2-\delta)\left((n+4)\,(2-\delta+\delta m\,(1-\delta))+m\delta^2\right)\\ &-2\,(n+4)\left(4+4\delta\,(m-1)+\delta^2\left(m^2-4m+1\right)-m\delta^3\,(m-1)\right)-2m^2\delta^4\\ &\qquad = m\delta^2\,(((n+1)\,\delta-(n+2))\,m\delta-(2-\delta)\,(n+2))\end{aligned}$$

The first term in the difference is positive for $0 < \delta \leq 1$ and $m > 0$, and equals zero for $\delta = 0$ or $m = 0$. This latter result implies that if $\delta = 0$ or $m = 0$ then the fraction on the RHS equals 1, so $a_I = a_S^E$, that is the incumbent forecloses service-based firms (proof of (i)). The second term is always negative, since both parts of that expression are negative for $0 \leq \delta \leq 1$ and $m > 0, n > 0$. Therefore for $0 < \delta \leq 1$ and $m > 0$, $a_I < a_S^E$. ■

Proof. *Existence of social welfare maximizing fee* From the welfare function (2.1), the first-order condition with respect to access price is

$$\frac{\partial W}{\partial a} = \left(\alpha^T - \mathbf{q}^T(a)\mathbf{B}\right)\frac{\partial \mathbf{q}(a)}{\partial a} =$$

$$= \left(\alpha^T - \left(\mathbf{q}^T(0) + \frac{\partial \mathbf{q}(a)}{\partial a}^T a\right)\mathbf{B}\right)\frac{\partial \mathbf{q}(a)}{\partial a} = 0$$

and the access price that maximizes the total welfare is

$$a_W = \frac{\left(\alpha^T - \mathbf{q}^T(0)\mathbf{B}\right)\frac{\partial \mathbf{q}(a)}{\partial a}}{\frac{\partial \mathbf{q}(a)}{\partial a}^T \mathbf{B}\frac{\partial \mathbf{q}(a)}{\partial a}} = -\frac{\alpha\left((2-\delta)^2 - mn(1-\delta)\delta\right)}{n\left((2-\delta)^2 - m(m-1)(1-\delta)\delta^2\right)}$$

This access price yields a maximum when the SOC holds, that is

$$\frac{\partial^2 W}{\partial a^2} = -\frac{\partial \mathbf{q}(a)}{\partial a}^T \mathbf{B}\frac{\partial \mathbf{q}(a)}{\partial a}$$

$$= (\gamma - \beta)\left(n\left(\frac{\partial q_A}{\partial a}\right)^2 + \left(\frac{\partial q_I}{\partial a} + m\frac{\partial q_S}{\partial a}\right)^2\right) - \gamma\left(\frac{\partial q_I}{\partial a} + n\frac{\partial q_A}{\partial a} + m\frac{\partial q_S}{\partial a}\right)^2 < 0$$

This expression is strictly negative for $\gamma \leq \beta$ (i.e. $\delta \leq 1$). ■

Proof. *Proposition 4* The incumbent is an active player if it earns non-negative profit, hence it is excluded from the market if

$$\pi_I < 0 \iff \beta q_I^2 + n q_S a < 0$$

In the equilibrium we require that profit has a maximum which satisfies if SOC satisfies (see (6.3)). Therefore the profit function in a is a parabola which opens downward and has two intersections:

$$a_1 = -\frac{q_I(0)}{\frac{\partial q_I}{\partial a} + \frac{n - \sqrt{n(n+4)}}{2\beta}}$$

$$a_2 = -\frac{q_I(0)}{\frac{\partial q_I}{\partial a} + \frac{n + \sqrt{n(n+4)}}{2\beta}}$$

The access fee providing maximum profit for the incumbent lays in between there intersection and it is positive. Therefore for non-negative profit only the lower a has to be considered. The nominator of both a_1 and a_2 is always positive. According to (6.3), the denominator of a_1 is negative, and of case a_2 is positive. Hence the lower value for the exclusionary access fee, is

$$a_I^E = -\frac{q_I(0)}{\frac{\partial q_I}{\partial a} + \frac{n - \sqrt{n(n+4)}}{2\beta}} < 0$$

which is negative, and according to the proposition may bind. ■

7 Appendix: Uniform Service Differentiation

Proof. *Existence of the 2nd stage equilibrium* The second order conditions are satisfied when the diagonal values of matrix **D** are positive, that is $\beta > 0$. ■

Proof. *Proposition 5* A sufficient condition for the equilibrium quantities to be non-negative is if $0 \le \delta \le 1$ (where $\delta = \gamma/\beta$). Point (*i*) and (*ii*) is immediate for q_I, p_I, q_A, p_A and q_S. For p_S we get that

$$\frac{\partial p_S}{\partial a} = \frac{(\beta-\gamma)(2\beta+m\gamma+n\gamma)+n\beta\gamma}{(2\beta-\gamma)(2\beta+m\gamma+n\gamma)}$$

It follows that for the previous sufficient condition $\frac{\partial p_S}{\partial a} > 0$ holds. ■

Proof. *Existence of 1st stage equilibrium, incumbent* Similarly to the partial differentiation case, a_I yields a maximum if $\frac{\partial^2 \pi_I}{\partial a^2} < 0$:

$$\frac{\partial^2 \pi_I}{\partial a^2} = 2\left(\beta\left(\frac{\partial q_I}{\partial a}\right)^2 + n\left(\frac{\partial q_I}{\partial a} - \frac{1}{2\beta-\gamma}\right)\right)$$

which is negative if

$$\frac{\partial q_I}{\partial a} < \frac{-n+\sqrt{n\left(n+\frac{4\beta}{2\beta-\gamma}\right)}}{2\beta}$$

Inserting the explicit expression for $\frac{\partial q_I}{\partial a}$, we find that

$$\frac{n\gamma}{(2\beta-\gamma)(2\beta+m\gamma+n\gamma)} < \frac{-n+\sqrt{n\left(n+\frac{4\beta}{2\beta-\gamma}\right)}}{2\beta} \Leftrightarrow$$

$$\left(1+\frac{2\delta}{(2-\delta)(2+m\delta+n\delta)}\right)^2 < \left(1+\frac{4}{(2-\delta)n}\right) \Leftrightarrow$$

$$\frac{2\delta}{(2-\delta)(2+m\delta+n\delta)} < \frac{4}{(2-\delta)n} \Leftrightarrow$$

$$2(\delta-2)(2m\delta+n\delta+4) < 0$$

where $\delta = \gamma/\beta$. This shows that the second order condition is always satisfied for $0 \le \delta \le 1$. ■

Proof. *Proposition 6* For $0 \le \delta \le 1$ the exclusive access fee a_S^E is positive. Therefore, it follows from expression (4.4) that $a_I < a_S^E$ if $4\delta - 2m\delta + m\delta^2 - 4 < 0$. Define $f(m,\delta) = 4\delta - 2m\delta + m\delta^2 - 4 = -4(1-\delta) - m\delta(2-\delta)$. If $m > 0$, this expression is strictly negative for $0 \le \delta \le 1$. If $m = 0$,

$$a_I = a_S^E \frac{2\delta+(2-\delta)(2+n\delta)}{(2-\delta)(2+n\delta)2-n\delta^2}$$

By taking the difference between the nominator and the denominator:

$2\delta+(2-\delta)(2+n\delta)-(2-\delta)(2+n\delta)2+n\delta^2=2(\delta-1)(n\delta+2)$. This expression is negative for any $\delta<1$ and zero for $\delta=1$, therefore $a_I>a_S^E$ for any $\delta<1$ and $a_I=a_S^E$ if for $\delta=1$. ■

Proof. *Existence of social welfare maximizing fee* From the welfare function 2.1 the second-order condition with respect to access price is

$$\begin{aligned}\frac{\partial^2 W}{\partial a^2} &= -\frac{\partial \mathbf{q}(a)}{\partial a}^T \mathbf{B}\frac{\partial \mathbf{q}(a)}{\partial a}\\ &= (\gamma-\beta)\left[\left(\frac{\partial q_I}{\partial a}\right)^2+n\left(\frac{\partial q_A}{\partial a}\right)^2+m\left(\frac{\partial q_S}{\partial a}\right)^2\right]-\gamma\left(\frac{\partial q_I}{\partial a}+n\frac{\partial q_A}{\partial a}+m\frac{\partial q_S}{\partial a}\right)^2\end{aligned}$$

This expression is strictly negative for $\gamma\leq\beta$ (i.e. $\delta\leq 1$).. ■

Proof. *Proposition 8* The exclusionary access fee for the incumbent is given by

$$a_I^E = -\frac{q_I(0)}{\frac{\partial q_I}{\partial a}+\frac{n}{2\beta}\left(1-\sqrt{1+\frac{4}{(2-\delta)n}}\right)}$$

$$= -\frac{2\alpha(2-\delta)}{n\left(2\delta+(2-\delta)(2+(n+m)\delta)\left(1-\sqrt{1+\frac{4}{(2-\delta)n}}\right)\right)}$$

This value is always negative for $0\leq\delta\leq 1$.

To determine the relationship between a_I^E and a_W, we calculate the following:

$$\left(\frac{1}{a_W}-\frac{1}{a_I^E}\right)=\frac{2+(n+m)\delta}{2\alpha}\left(n\left(1-\sqrt{1+\frac{4}{(2-\delta)n}}\right)-\frac{2(1-\delta)(2+m\delta)}{(2-\delta)^2}\right)$$

This expression is always negative if $\delta\leq 1$, which is therefore also a sufficient condition for $a_W<a_I^E$ to be satisfied. ■

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