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Intergenerational risk sharing and labour supply in collective funded pension schemes with defined benefits[a]

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Abstract in English

In many countries, collective funded pension schemes with defined benefits (DB) are being replaced by individual schemes with defined contributions. Collective funded DB pensions may indeed reduce social welfare. This will be the case when the schemes feature income-related contributions that distort the labour-leisure decision. However, these schemes also share risks between generations. This adds to welfare if these risks cannot be traded on capital markets. This paper compares the welfare gains from intergenerational risk sharing with the welfare losses that are due to labour market distortions. We adopt a two-period overlapping-generations model for a small open economy with risky returns to equity holdings. We derive analytically that the gains dominate the losses for the case of Cobb-Douglas preferences between labour and leisure. Numerical simulations for the more general CES case confirm these findings which also withstand a number of other model modifications, like the introduction of a short-sale constraint for households and the inclusion of a labour income tax. These results suggest that collective funded schemes with well-organized risk sharing are preferable over individual schemes, even if labour market distortions are taken into account.

Key words: risk sharing, labour market distortion, funded pensions, defined benefits
JEL Code: E21, G11, H55

Abstract in Dutch

Het is algemeen bekend dat collectieve kapitaalgedekte pensioenstelsels die risico's spreiden over generaties, bijdragen aan een hogere welvaart. Het is evenwel minder bekend dat dergelijke pensioenstelsels ook een verstorend effect hebben op de arbeidsaanbodbeslissing en langs die route de welvaart juist kunnen verlagen. De reden hiervoor is dat pensioenpremies in veel gevallen impliciete belastingen (of subsidies) bevatten die samenhangen met een discrepantie tussen de risico's waaraan pensioenvermogens en -verplichtingen blootgesteld zijn. Dit paper weegt de voordelen van collectieve kapitaalgedekte pensioenstelsels in termen van risicodeling af tegen de nadelen in termen van arbeidsmarktverstoring. Daartoe wordt een stochastisch model ontwikkeld met twee overlappende generaties dat een kleine open economie (zoals de Nederlandse) representeert. We vinden dat de welvaartswinsten van risicodeling veel groter zijn dan de welvaartsverliezen van arbeidsmarktverstoringen. Dit resultaat is robuust voor een breed scala aan alternatieve modelspecificaties, zoals de specificatie van de nutsfunctie, de implementatie van een verbod om met geleend geld in aandelen te beleggen en de introductie van een initiële belasting op arbeid.

Contents

Summary

It is widely recognized that risk sharing across generations can be welfare improving. Competitive financial markets often fail to fully exploit the merits of intergenerational risk sharing because current generations cannot sign insurance contracts with those who are not born yet. This market incompleteness leaves a role for collective institutions like pension funds. By offering collective contracts, pension funds are able to commit future generations to an intergenerational risk-sharing scheme.

The key feature of these collective schemes is that they smooth shocks over and beyond the lifetime of any single generation. A collective pension scheme makes intergenerational risk sharing possible by disconnecting individual contributions to individual benefits. Because in most real-world pension plans contributions are related to labour income, this disconnection between contributions and benefits distorts the labour supply decision. The aim of this paper is to trade the advantages of funded collective pensions in terms of intergenerational risk sharing against its drawbacks that are due to labour market distortions.

This trade-off is analysed with a stochastic overlapping-generations model that represents a small open economy. The economy is subject to macroeconomic capital-market risk. There are two overlapping generations, a young generation that is active in the labour market and an old generation which is retired. The young generation decides upon the amount of private savings, labour supply and the portfolio allocation in order to maximize expected lifetime utility. The pension fund offers risk-free benefits and raises state-contingent contributions proportional to labour income. Given the optimal behavioural responses of the consumer, the pension fund acts as a benevolent Stackelberg leader by optimally choosing its portfolio allocation in order to maximize (ex ante) social welfare.

The source of market incompleteness that justifies the existence of the collective pension fund in the model is the inability of generations to trade risks before they are born. By defining benefits independently from realized financial returns, the pension fund facilitates opportunities for intergenerational risk sharing that agents cannot undo through transactions in financial markets. In this way, the pension fund creates an opportunity for the young generation to exchange financial risk with the old generation ex ante, which reduces market incompleteness. This young generation is better equipped to bear capital-market risk than the old generation because they can use their human capital (which is assumed to be risk free in our model) to absorb financial shocks and they still have the flexibility to adjust their labour supply. When there is no collective pension arrangement, the old generation only has financial wealth as consumption source which makes them vulnerable for financial shocks.

For a specific utility function we analytically show that the introduction of a collective funded scheme with defined benefits and state-contingent contributions involves an ex ante Pareto improvement. The benefit of this risk-sharing pension scheme is not to reduce risk but

rather to increase the expected pay-off from risky investments by generations who have not entered the labour market yet. We find that the welfare gains from intergenerational risk sharing outweigh the losses from labour market distortions. Using numerical simulations, we show that this result also holds for more general utility functions. However, we also find that proportional transfers reduce the risk-bearing capacity of the working generation because they introduce a positive correlation between returns on financial assets and labour income.

The results of this paper have an important implication related to the current pension reforms worldwide. Due to increasing demographic pressure, in various countries collective defined benefit pension schemes are being replaced by individual defined contribution systems, in which benefits are subject to various types of risk. Our result emphasizes that individual pension schemes that do not share risks among generations are not optimal, even if we take care of the potential labour market distortions that are related to collective funded pension schemes.

1 Introduction

In the industrialized world, population ageing and the current financial crisis jeopardize the sustainability of public finances. Consequently, many countries are reforming or planning to reform their social security systems. Various countries are gradually reducing their pay-as-you-go (PAYG) systems in favour of more funded-based systems. In other countries with traditionally large funded systems, like Australia, Switzerland, the United States and the Netherlands, we observe a clear trend away from collective defined-benefit (DB) systems towards individual defined-contribution (DC) systems. Currently DC schemes represent 42% of total pension assets, compared to 40% in 2004 and 32% in 1999 (Towers Watson (2010)). This move to DC schemes is a global trend, driven largely by financial considerations, as sponsors seek to take control of both the volatility and the overall cost of their DB plans.

A priori it is not immediately clear that a movement from collective funded DB pensions towards individual funded DC pensions will improve social welfare. On the one hand, collective funded DB pension schemes allow for welfare-improving intergenerational risk sharing wich better protect workers for sizeable downturns of financial markets. As demonstrated in e.g. Gordon and Varian (1988), Shiller (1999) and Gottardi and Kubler (2006), in competitive financial markets currently living generations are not able to share risks with those who are not born yet. Mandatory participation in a collective funded DB pension scheme can (at least partly) solve this market incompleteness. The main feature of such a pension scheme is that it smoothes shocks over and beyond the lifetime of a single generation by disconnecting individual contributions from individual benefits.

On the other hand, collective funded DB pensions often involve distortions on labour markets, an aspect that certainly decreases welfare. In most real-world pension plans, pension contributions are related to labour income.[1] A disconnection between individual contributions and benefits then implies that the contribution rate contains an implicit tax or subsidy which distorts the labour supply decision. The aim of this paper is to trade the advantage of collective funded DB pensions in terms of intergenerational risk sharing against the drawback of a labour market distortion.

The risk-sharing characteristics of alternative pension and social security systems have recently gained increasing attention in the literature. Much of these papers focus on PAYG schemes (see, e.g., Krueger and Kubler (2006), Sánchez-Marcos and Sánchez-Martín (2006), Miles and Černý (2006), Nishiyama and Smetters (2007) and Fehr and Habermann (2008)). Some recent papers also look at the role of funded pension schemes in facilitating

[1] Since individual abilities are unobservable, policy makers (or pension funds) necessarily use observable wages to distribute shocks. Wage-related contributions can also be justified from constant relative risk aversion. In that case, optimal risk sharing implies that shocks should be distributed proportionally over pension members, based on total wealth (Bovenberg et al. (2007)). One way to implement this is to use income-related contributions.

intergenerational risk sharing (see, e.g., Beetsma et al. (2008), Bovenberg et al. (2007), Cui et al. (2006), Gollier (2008), Matsen and Thøgersen (2004) and Teulings and de Vries (2006)). Although these papers differ in the way risk sharing is designed, they all conclude that funded pension schemes allow for substantial welfare gains. However, none of these studies compares these gains with the losses due to labour market distortions.

To analyse this trade-off, we consider a model that represents a small open economy populated with two overlapping generations and a collective pension fund. The economy is subject to macroeconomic capital-market risk. The old generation is retired, while the young generation is active in the labour market. The two overlapping generations cannot trade risks because the young is not able to participate in the capital market before shocks occur. The young generation decides upon the amount of private saving, labour supply and the portfolio allocation in order to maximize expected lifetime utility. The pension fund provides risk-free benefits and raises state-contingent contributions proportional to individual labour income. Hence, the young generation bears the full mismatch risk between the benefit guarantees provided to the old and the accumulated pension assets. Labour is assumed to be perfectly immobile so that agents are not able to avoid implicit taxes by moving abroad. Taking into account the behavioural response of the consumer to its actions, the pension fund optimally chooses the portfolio allocation in order to maximize an (ex ante) social welfare function.

This paper provides some interesting results. For a specific utility function we analytically show that the introduction of a collective funded scheme with defined benefits and state-contingent contributions involves an ex ante Pareto improvement. Using numerical simulations, we show that this result also holds for more general utility functions. As demonstrated by Gollier (2008), the benefits of risk sharing do not only imply a lower level of risk, but also show up in a different guise. Indeed, in his and our paper, households react to the risk reduction that is due to the risk-sharing scheme by shifting their portfolios towards equity. This increases the average rate of return that households earn on their portfolios and increases the welfare gain from the pension scheme. The present paper adds that households may also choose a different combination of labour and leisure, thereby even further increasing the welfare gain from risk sharing.

In addition, we find that labour supply flexibility decreases the risk appetite of consumers if pension contributions are distortionary. This result contrasts with existing studies on the interaction between labour supply and portfolio choice (see e.g. Bodie et al. (1992), Choi and Shim (2006) and Farhi and Panageas (2007)). These studies show that labour supply flexibility offers insurance against adverse shocks which justifies more risky asset portfolios. The idea is that income effects in labour supply behaviour cause a negative correlation between asset returns and labour income allowing individuals to take more risk. This paper, however, shows that income-related intergenerational transfers also introduce substitution effects. These substitution effects work in the opposite direction and generate a positive correlation between labour income and asset returns. Hence, labour supply is subject to pro-cyclical pressure which reduces the risk-bearing capacity of consumers.

The results of this paper are relevant for the worldwide pension reform towards the establishment of individual DC-type schemes. Indeed, our results emphasize that individual pension schemes that do not share risks among generations may not be optimal. Collective funded schemes with well-structured intergenerational risk sharing are preferable from a welfare point of view, even if the losses from labour market distortions are taken into account.

A few other studies are related to the present study. Mehlkopf (2009) assesses also the labour market distortions from collective funded schemes, but relates them to the gains of a different kind of market incompleteness. Indeed, he focuses on the inability of individual agents to borrow against their human capital. He shows that the welfare costs of recovery policies (due to labour market distortions) are smaller than the associated welfare gains (due to the alleviation of borrowing constraints). Also related is Draper and Westerhout (2009) who, like us, conclude that the gains from intergenerational risk sharing dominate the losses from labour market distortions. Their model is multi-period and allows for a more detailed modelling of household behaviour. However, they do not present analytical solutions and they do not optimize on the investment policy of the pension fund, as we do here.

The remainder of this paper is structured as follows. In Section 2 we discuss the model. Section 3 derives an analytical solution for the ex ante welfare gain of the pension scheme in case of a Cobb-Douglas felicity function in consumption and leisure. Section 4 decomposes the derived welfare gain into the gain due to risk sharing and the loss due to the labour supply distortion. Section 5 presents numerical simulation results for the more general version of the model that allows the intratemporal substitution elasticity to be lower or higher than one. This section also explores the welfare consequences of a short-sale constraint for households and of the inclusion of a labour income tax. Finally, Section 6 concludes.

2 Model

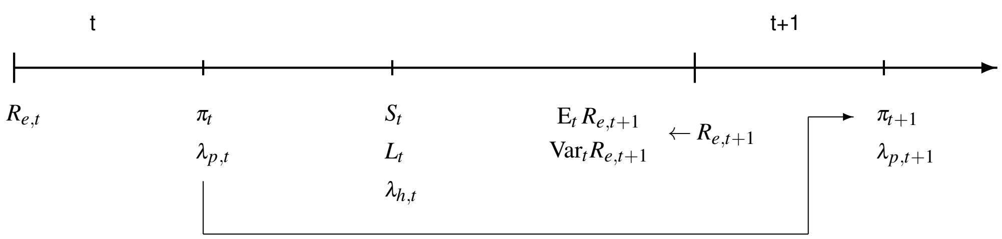
We consider an economy populated with two overlapping generations of households and a collective pension fund. Each generation is modelled as a representative household who consumes in the two periods of his life and can supply labour when young. We abstract from demographic risk: all generations are equally large (normalized at unity). There are two financial assets in the economy, a risk-free bond and risky equity. We will jointly optimize on the investment policy of the pension fund and on household behaviour in terms of consumption, labour supply and portfolio allocation. The model represents a small open economy for which factor prices (the wage rate, the risk-free interest rate and the rate of return on equity) can be taken as given. As usual, capital is assumed to be perfectly mobile and labour is perfectly immobile.

The added value of the pension is that it alleviates the distortion of a missing market, i.e. the market for risks that occur before a generation is born. In particular, by providing safe benefits to the elderly, investing savings (partly) in equity and imposing the mismatch risk between assets and liabilities upon young generations, the pension fund lets young generations share in risks in which they cannot trade on the private capital market. The creation of a new asset will, if supplied in the right amount, increase the welfare of all generations that participate in the scheme.

2.1 Timing

The sequence of events is graphically shown in Figure 2.1. At the beginning of period t, a shock occurs in the equity rate of return ($R_{e,t}$). After this shock has revealed, first the pension fund decides on the contribution rate (π_t) and the portfolio share ($\lambda_{p,t}$) to be invested in equity. The pension fund acts as a benevolent Stackelberg leader, taking into account the future reactions of households to its decisions. An important property of the model is that the portfolio choice of the pension fund at time t only affects lifetime utility of the next generation, born in $t+1$. As

Figure 2.1 Timing of events



visually emphasized with an arrow in Figure 2.1, this dependency is driven by the direct impact of the pension fund's equity investment on next period's contribution rate. After the actions of the pension fund, the consumers decide upon their private savings (S_t), the amount of leisure (L_t) and the portfolio share ($\lambda_{h,t}$) to be invested in equity, taking the pension contribution rate as given. The decisions of the consumers are based on the distribution of the future asset return. Since future pension benefits are safe, consumers only face uncertainty about the return on their private savings.

2.2 Households

Agents derive utility from consumption and leisure. The preference structure is represented by a time-separable, nested constant-elasticity-of-substitution (CES) utility function that separates the aversion to risk and to intertemporal variation (Epstein and Zin (1991)). This separation is important for the analysis of pension contracts that alter both the risk properties and the timing of individual consumption flows. The utility function of an agent born at time t is defined as:

$$U_t = \left\{ u\left(C_{1,t}, L_t\right)^{1-\gamma} + \beta \left[\mathrm{E}_t\, u\left(C_{2,t+1}, 1\right)^{1-\theta} \right]^{\frac{1-\gamma}{1-\theta}} \right\}^{\frac{1}{1-\gamma}}, \quad \gamma > 0, \theta > 0 \tag{2.1}$$

where $C_{1,t}$ and L_t denote consumption and leisure when young at time t, $C_{2,t+1}$ denotes consumption when old at time $t+1$ and β is the time discount factor. The parameters θ and γ define the coefficient of relative risk aversion and the inverse of the intertemporal substitution elasticity.[2] When $\gamma = \theta$, equation (2.1) reduces to a standard expected utility formation where no distinction is made between risk aversion and intertemporal substitution. The felicity function $u(\cdot)$ is defined over commodities and leisure consumption, assuming a CES specification:

$$u(C,L) = \begin{cases} \left[(1-\eta)C^{1-\rho} + \eta L^{1-\rho}\right]^{\frac{1}{1-\rho}} & \text{for} \quad \rho > 0, \rho \neq 1 \\ C^{1-\eta} L^{\eta} & \text{for} \quad \rho = 1 \end{cases} \tag{2.2}$$

with $0 < \eta < 1$. The inverse of the intratemporal substitution elasticity is given by ρ; the utility parameter η governs the relative preference for leisure. In the following, we will use the following shorthand notation: $u_{1,t} \equiv u(C_{1,t}, L_t)$ and $u_{2,t+1} \equiv u(C_{2,t+1}, 1)$.

There are two assets in the economy, a risk-free asset with return R_f and a risky asset with return R_e. When young, agents start to save out of their labour income (they enter the economy with zero endowment of assets). They determine the portfolio allocation by choosing a share $\lambda_{h,t}$ of private savings to invest in the risky asset and a share $1 - \lambda_{h,t}$ to invest in the risk-free asset. The rate of return on the private portfolio is thus equal to:

$$R_{h,t+1} = \left(1 - \lambda_{h,t}\right) R_f + \lambda_{h,t} R_{e,t+1} \tag{2.3}$$

[2] The parameters θ and γ define risk aversion and the aversion to intertemporal substitution with respect to total consumption, i.e. consumption of goods *and* leisure. Risk aversion and aversion to intertemporal substitution for the two goods separately will be defined below.

We impose that the log return on the risky asset in excess of the log risk-free return, i.e. $\log(1+R_e)-\log(1+R_f)$, is an independently and identically distributed normal variable with mean μ and variance σ^2.

When young, an agent spends a fraction L_t of his time endowment on leisure. We normalize the time endowment at unity, so that $0 \leq L_t \leq 1$. A fraction π_t of labour income is contributed to the pension fund; the rest is devoted to consumption and private saving S_t. During the second period, the agent is retired. Consumption in this period consists of a labour-related pension benefit $(1-L_t)B$, where B denotes the maximum attainable level and the factor $1-L_t$ reflects the accumulation of pension benefits. The consolidated lifetime budget constraint reads as:

$$C_{1,t}+\frac{C_{2,t+1}}{1+R_{h,t+1}}=(1-L_t)(1-\pi_t)Y+\frac{(1-L_t)B}{1+R_{h,t+1}} \tag{2.4}$$

where Y denotes the wage rate, which is assumed to be constant over time.[3]

Maximizing the objective function, equation (2.1), subject to the intertemporal budget constraint, equation (2.4), gives the following set of first-order conditions with respect to $C_{1,t}$, L_t and $\lambda_{h,t}$:

$$u_{1,t}^{\rho-\gamma}C_{1,t}^{-\rho}=\beta\left(\mathrm{E}_t\, u_{2,t+1}^{1-\theta}\right)^{\frac{\theta-\gamma}{1-\theta}}\mathrm{E}_t\left[\left(1+R_{h,t+1}\right)u_{2,t+1}^{\rho-\theta}C_{2,t+1}^{-\rho}\right] \tag{2.5a}$$

$$\eta u_{1,t}^{\rho-\gamma}L_t^{-\rho}=(1-\eta)\beta\left(\mathrm{E}_t\, u_{2,t+1}^{1-\theta}\right)^{\frac{\theta-\gamma}{1-\theta}}\mathrm{E}_t\left\{\left[\left(1+R_{h,t+1}\right)(1-\pi_t)Y+B\right]u_{2,t+1}^{\rho-\theta}C_{2,t+1}^{-\rho}\right\} \tag{2.5b}$$

$$0=\mathrm{E}_t\left[\left(R_{e,t+1}-R_f\right)u_{2,t+1}^{\rho-\theta}C_{2,t+1}^{-\rho}\right] \tag{2.5c}$$

Equation (2.5a) is the Euler equation which equalizes the marginal utility of first-period consumption to the discounted expected marginal utility of second-period consumption. Equation (2.5b) is the first-order condition with respect to leisure, while equation (2.5c) is the condition for optimal portfolio allocation.

The Euler equation specifies a relation between the marginal utility of consumption and the rate of return on assets. This relation becomes more clear by rewriting equation (2.5a) in the form $\mathrm{E}_t\left[m_{t+1}\left(1+R_{h,t+1}\right)\right]=1$, where

$$m_{t+1}=\left[\frac{\left(\mathrm{E}_t\, u_{2,t+1}^{1-\theta}\right)^{\frac{1}{1-\theta}}}{u_{2,t+1}}\right]^{\theta-\gamma}\left(\frac{C_{2,t+1}}{C_{1,t}}\right)^{-\rho}\left(\frac{u_{2,t+1}}{u_{1,t}}\right)^{\rho-\gamma}\beta \tag{2.6}$$

defines the stochastic discount factor (SDF). The SDF measures the marginal value of a unit of consumption next period per unit of current consumption. The term in square brackets enters because of non-expected utility and compares next-period utility with its certainty-equivalent counterpart. A consumer that is relatively risk averse ($\theta > \gamma$) has a certainty-equivalent utility that is lower than expected utility.[4] That is, the consumer applies a correction factor to next

[3] Non-stochastic wages can go together with stochastic equity returns if i) there is depreciation risk and no productivity risk and ii) the production function is linear in capital and labour implying an infinite elasticity of substitution.

[4] By Jensen's inequality, we have that $\left(\mathrm{E}_t\, u_{2,t+1}^{1-\theta}\right)^{\frac{1}{1-\theta}} < \mathrm{E}_t\, u_{2,t+1}$.

period's marginal utility which is less than one on average, implying that he discounts the future more heavily on average than an expected-utility consumer.

The equation for the SDF can be used to derive a few more relations, which we will apply further on in the paper. From equation (2.5c), using (2.3), it follows that:

$$\mathrm{E}_t\left[u_{2,t+1}^{\rho-\theta}C_{2,t+1}^{-\rho}(1+R_{h,t+1})\right]=\mathrm{E}_t\left[u_{2,t+1}^{\rho-\theta}C_{2,t+1}^{-\rho}(1+R_{i,t+1})\right],\quad i=f,e \tag{2.7}$$

Using equation (2.5a), we then have:

$$\mathrm{E}_t\left[m_{t+1}\left(1+R_{i,t+1}\right)\right]=1,\qquad i=f,h,e \tag{2.8}$$

and, hence,

$$\frac{1}{1+R_f}=\mathrm{E}_t\, m_{t+1} \tag{2.9}$$

2.3 Pension fund

We consider a collective pension fund scheme in which households are obliged to participate. Switching from obligatory participation to voluntary participation could give rise to discontinuity problems. These problems will be discussed later on.

The pension fund collects contributions from the young generation, invests these contributions in the capital market and pays out benefits to the same generation in the second period of life. The maximal attainable pension benefit B is risk free and defined as:

$$B=\alpha Y \tag{2.10}$$

with α the replacement rate. Recall that the pension contract is related to labour history so that the actual pension benefit paid out to the old generation at time $t+1$ equals $(1-L_t)B$.

The pension fund invests the collected pension contributions in the risk-free asset and the risky asset. It invests a share $\lambda_{p,t}$ of the contributions in the risky asset and the remaining part $1-\lambda_{p,t}$ in the risk-free asset. The portfolio return of the pension fund R_p is thus equal to:

$$R_{p,t+1}=(1-\lambda_{p,t})R_f+\lambda_{p,t}R_{e,t+1} \tag{2.11}$$

If the pension fund chooses a risky investment strategy (i.e. $\lambda_{p,t}\neq 0$), the contribution rate expressed as percentage of the wage rate (π_t) consists of two parts: a cost-effective component ($\pi_{b,t}$) and a recovery component ($\pi_{c,t}$) reflecting the mismatch risk between liabilities and assets, i.e. $\pi_t=\pi_{b,t}+\pi_{c,t}$. From an ex ante point of view, the pension scheme is a fair deal if the cost-effective component of the contribution rate is equal to the value the participant attaches to the future pension benefit. That is,

$$\pi_{b,t}(1-L_t)Y=\mathrm{E}_t\left[m_{t+1}(1-L_t)B\right] \tag{2.12}$$

Equation (2.12) is the funding condition which ensures that the pension contract does not contain ex ante redistribution. Solving for the cost-effective component of the contribution rate, using (2.9), then gives:

$$\pi_{b,t} = \frac{\alpha}{1+R_f} \tag{2.13}$$

Let us now focus upon the recovery component, $\pi_{c,t}$. This component reflects the transfers that the young generation will make to or receive from the old generation. In case of a funding deficit, the recovery rate is positive and the young generation effectively makes a payment to the retired generation. In case of a funding surplus, this rate is negative and it is the old generation who makes a transfer to the young generation. Hence, risk sharing is restricted to only two overlapping generations. The solvency constraint for the pension fund thus equals:

$$(1+R_{p,t+1})\,\pi_{b,t}(1-L_t)Y + \pi_{c,t+1}(1-L_{t+1})Y = B(1-L_t)$$

This equation says that the pension fund finances pension benefits in period $t+1$ (that reflects the rights accumulated in period t) with cost-effective premiums levied in period t, the portfolio return earned on this in period $t+1$ and an intergenerational transfer levied in period $t+1$ on the basis of period $t+1$ labour supply. Hence, risk sharing is confined to two overlapping generations. As one model period represents roughly twenty years, the potential for risk sharing is maximized at forty years. This is not unrealistic if we look at risk-sharing mechanisms in actual pension schemes, which are often restricted by rigid solvency regimes.

Using equations (2.10), (2.11) and (2.13), we can convert the solvency constraint into an expression for the recovery rate $\pi_{c,t+1}$:

$$\pi_{c,t+1} = -\frac{N_t\left(R_{e,t+1} - R_f\right)}{(1-L_{t+1})\,Y} \tag{2.14}$$

where $N_t \equiv \pi_{b,t}\lambda_{p,t}\,(1-L_t)\,Y$ is the absolute amount of collected contributions invested in equity. For this pension scheme to be sustainable, we must have that $\pi_{c,t+1} < 1$. Otherwise the young generation is not always able to guarantee a safe benefit to the old generation. Later on, when we solve the model, we will derive the necessary and sufficient condition such that $\pi_{c,t+1} < 1$. Note from equation (2.14) that $\pi_{c,t+1} = 0$ if the pension fund does not invest in equity. Further, note that the average transfer is negative: even if the rate of return on equity happens to be equal to its mean, there is a non-zero transfer which reflects the risk premium on equity. Since the intergenerational transfer is related to income of the young, it serves on average as an implicit subsidy to labour supply.

The pension fund uses its investment policy (N_t) to maximize social welfare. Due to the simple model structure, this boils down to a static optimization problem. The only intergenerational link in the model is the recovery rate. As already shown in Figure 2.1, the portfolio decision of the pension fund at time t only affects lifetime utility of the generation born at $t+1$ – through its direct impact on the intergenerational transfer. This property is a consequence of the utility-based valuation of the cost-effective contribution rate which is based on the risk-free return (and not the portfolio return!). Hence, both the benefit of a risky investment strategy at time t (i.e. a higher expected portfolio return) and the mismatch risk between assets and liabilities only shows up in the transfer of a single generation which is young at time $t+1$.

For the benevolent pension fund who aims to maximize expected utility of all currently living and future generations, it it therefore sufficient to maximize ex ante lifetime utility of one representative generation, i.e. lifetime utility evaluated before the occurrence of the shock in the first period of the life of the household and based upon the distribution of shocks in the two periods of his life. Consequently, if the investment strategy of the pension fund improves welfare for this single generation, the policy is automatically a Pareto improvement. The pension fund thus maximizes the following social welfare function:

$$W_t = \left(\mathrm{E}_t\, U_{t+1}^{1-\theta}\right)^{\frac{1}{1-\theta}} \tag{2.15}$$

subject to the solvency constraint (2.14) and the first-order conditions of the household, equations (2.5a)-(2.5c).

3 Solution

For Cobb-Douglas felicity utility ($\rho = 1$) the model can be solved analytically. In the simulations later on, we will present results for the general model in which we can play around with the intratemporal substitution elasticity. In case of Cobb-Douglas utility, the first-order conditions (2.5a)-(2.5c) simplify to:

$$C_{1,t}^{-\psi} L_t^{1-\omega} = \beta \left(1 + R_f\right) \left(\mathrm{E}_t\, C_{2,t+1}^{1-\zeta}\right)^{\nu} \mathrm{E}_t\, C_{2,t+1}^{-\zeta} \tag{3.1a}$$

$$\eta C_{1,t}^{1-\psi} L_t^{-\omega} = (1-\eta)\beta \left(1 + R_f\right) (1 - \pi_{c,t})\, Y \left(\mathrm{E}_t\, C_{2,t+1}^{1-\zeta}\right)^{\nu} \mathrm{E}_t\, C_{2,t+1}^{-\zeta} \tag{3.1b}$$

$$0 = \mathrm{E}_t \left[\left(R_{e,t+1} - R_f\right) C_{2,t+1}^{-\zeta}\right] \tag{3.1c}$$

where we used equation (2.7) to substitute out the stochastic portfolio return $R_{h,t+1}$ for the risk-free return R_f.

Equations (3.1a) to (3.1c) include the parameters $\psi \equiv 1-(1-\eta)(1-\gamma) > 0$, $\omega \equiv 1-\eta(1-\gamma) > 0$, $\zeta \equiv 1-(1-\eta)(1-\theta) > 0$ and $\nu \equiv (\theta-\gamma)/(1-\theta)$. The parameters ψ and ω are the inverse of the intertemporal substitution elasticity with respect to consumption and leisure, respectively; ζ denotes the coefficient of relative risk aversion with respect to consumption and ν reflects the importance of non-expected utility.

3.1 Consumer problem

We start to solve for the individual portfolio allocation. We first rewrite the intertemporal budget constraint (2.4) into:

$$C_{2,t+1} = \left(1 + R_{T,t+1}\right) \left[(1-L_t)(1-\pi_{c,t})Y - C_{1,t}\right] \tag{3.2}$$

with,

$$R_{T,t+1} \equiv (1-a_t) R_f + a_t R_{e,t+1} \tag{3.3}$$

$$a_t \equiv \frac{\lambda_{h,t} S_t}{S_t + \frac{(1-L_t)B}{1+R_f}} \tag{3.4}$$

Note that the portfolio share a_t relates the household's investment in equity to its total wealth, which is defined as the sum of financial wealth and pension wealth. As a result, $R_{T,t+1}$ is the effective return on the individual's total portfolio. Substituting equation (3.2) in first-order condition (3.1c) gives:

$$\mathrm{E}_t \left[\left(1 + R_{T,t+1}\right)^{-\zeta} \left(R_{e,t+1} - R_f\right)\right] = 0 \tag{3.5}$$

Since shocks in $R_{e,t+1}$ are independently and identically distributed, there is a unique solution to equation (3.5). In Appendix A.1 we show that a_t is approximately equal to:

$$a = \frac{\bar{\mu}}{\zeta \sigma^2} \tag{3.6}$$

where $\bar{\mu} = \mu + \frac{1}{2}\sigma^2$ is the expectation of the excess return on the risky asset. Equation (3.6) is similar to the result obtained by Merton (1969) and Samuelson (1969). The portfolio fraction invested in the risky asset is increasing in the expected excess return of the risky asset and decreasing in the variance of the excess return and the preference from consumption smoothing as measured by relative risk aversion ζ.[5] In the limit of continuous time with continuous paths for asset prices, equation (3.6) is exact (Campbell and Viceira (2002)).

To solve for consumption and leisure demand, we first substitute the budget constraint (3.2) in first-order condition (3.1a) to obtain:

$$C_{1,t} = \frac{1}{Z_t + 1}(1 - L_t)(1 - \pi_{c,t})Y$$

with Z_t and the certainty-equivalent (CE) rate of return $\bar{R}_t$ defined as:

$$Z_t \equiv \left[\beta L_t^{\omega - 1}(1 + \bar{R})^{1-\psi}\right]^{\frac{1}{\psi}} \tag{3.7}$$

$$1 + \bar{R}_t \equiv \left[\mathrm{E}_t\left(1 + R_{T,t+1}\right)^{1-\zeta}\right]^{\frac{1+\nu}{1-\psi}} \tag{3.8}$$

The CE rate of return is the return on a hypothetical risk-free investment strategy that provides individuals the same expected utility level as they receive from optimally investing their wealth into the tradable risk-free and risky asset.[6] Since it is assumed that the equity return is independently and identically distributed, the CE rate of return can be treated as an unconditional expectation. In Appendix A.2 we show that this return is approximately equal to:

$$\bar{R} = R_f + \frac{1}{2}\frac{\bar{\mu}^2}{\zeta\sigma^2} \tag{3.9}$$

If the risky asset offers no excess return ($\bar{\mu} = 0$), agents will not invest in the risky asset so that the CE rate of return is equal to the risk-free return. In the more interesting case in which $\bar{\mu} > 0$, the CE rate of return exceeds the risk-free rate of return. The second term on the right-hand side of equation (3.9) is the risk premium of the market portfolio. It follows that $d\bar{R}/d\bar{\mu} > 0$ and $d\bar{R}/d\sigma^2 < 0$. Hence, if the expected excess return increases, the CE rate of return also increases. When uncertainty increases, however, the CE return decreases.

Dividing (3.1b) by equation (3.1a) shows that the marginal rate of substitution between leisure and consumption is equal to the price of leisure,

$$\frac{\eta}{1-\eta}\frac{C_{1,t}}{L_t} = (1 - \pi_{c,t})Y \tag{3.10}$$

[5] In Appendix A.1 we show that $\bar{\mu} \approx \mathrm{E}(R_e - R_f)$ and $\sigma^2 \approx \mathrm{Var}(R_e - R_f)$.

[6] To derive the definition of $\bar{R}$ we use:

$$\begin{aligned}
\mathrm{E}_t\left(1 + R_{T,t+1}\right)^{1-\zeta} &= \mathrm{E}_t\left[(1 + R_{T,t+1})^{-\zeta}(1 + R_{T,t+1})\right] \\
&= \mathrm{E}_t(1 + R_{T,t+1})^{-\zeta}\left(1 + R_f\right) + a\,\mathrm{E}_t\left[(1 + R_{T,t+1})^{-\zeta}\left(R_{e,t+1} - R_f\right)\right] \\
&= \mathrm{E}_t(1 + R_{T,t+1})^{-\zeta}\left(1 + R_f\right)
\end{aligned}$$

where the last step follows from first-order condition (3.5).

Only the recovery rate shows up in the price of leisure, because, by construction, the cost-effective contribution rate is equal to the utility-based value of the accrued pension entitlement (that is, the cost-effective contribution rate is actuarially fair). Using equation (3.10), we can solve for consumption and leisure demand:

$$C_{1,t} = \frac{1-\eta}{(1-\eta)Z_t+1}(1-\pi_{c,t})Y \tag{3.11}$$

$$C_{2,t+1} = \frac{(1-\eta)Z_t\,(1+R_{T,t+1})}{(1-\eta)Z_t+1}\,(1-\pi_{c,t})\,Y \tag{3.12}$$

$$L_t = \frac{\eta}{(1-\eta)Z_t+1} \tag{3.13}$$

We can define Z_t as an implicit equation that only depends on exogenous variables and structural parameters. Inserting equation (3.13) in (3.7), yields:

$$Z_t = \beta^{\frac{1}{\psi}}\left[\frac{(1-\eta)Z_t+1}{\eta}\right]^{\frac{1-\omega}{\psi}}(1+\bar{R})^{\frac{1-\psi}{\psi}} \tag{3.14}$$

We assume that this equation has a unique and positive solution for Z. Note that this solution is constant over time.

The solution for consumption enables us to solve for the fraction of private savings invested in the risky asset. This fraction is equal to:

$$\lambda_{h,t} = \frac{aZC_{1,t}}{S_t} = \frac{a\,(1-\pi_{c,t})\,Z}{(1-\pi_{c,t})\,Z-(1+Z)\,\pi_{b,t}} \tag{3.15}$$

If the pension fund increases the cost-effective contribution rate it levies upon workers, workers respond by increasing the fraction of private savings that they invest in the risky asset. Since the pension benefit is risk free, the cost-effective contribution is equivalent to an investment in the risk-free asset. Agents counteract the actions of the pension fund with their private savings in such a way that − in terms of total wealth − the investment in the risky asset is constant. This offsetting response will be reinforced when agents are confronted with a positive surcharge ($\pi_{c,t} > 0$), because in that case the share of financial wealth in total wealth declines. To ensure that the stock of risky asset holdings in total wealth does not change, agents invest a larger share of financial wealth in the risky asset. Note that $\lambda_{h,t}$ can be larger than unity. In that case, the worker goes short in bonds to buy risky equity.

Equation (3.14) can be used to derive the effect of uncertainty on the consumption and leisure decision. Taking the total differential of this equation and rearranging terms gives:

$$\frac{dZ}{d\bar{R}} = \frac{\frac{1-\psi}{\psi}\frac{Z}{1+\bar{R}}}{1-\frac{1-\omega}{\psi}\frac{(1-\eta)Z}{(1-\eta)Z+1}} \tag{3.16}$$

Using the definition of ψ, it can be shown that equation (3.16) is positive for $\gamma < 1$, negative for $\gamma > 1$ and zero for $\gamma = 1$. Recall from equation (3.9) that $d\bar{R}/d\bar{\mu} > 0$ and $d\bar{R}/d\sigma^2 < 0$. Hence, if $\gamma > 1$, Z increases if σ^2 increases, implying that first-period consumption and leisure both decrease. This corresponds to the case that the negative income effect on consumption dominates the positive substitution effect (Sandmo (1970)). The additional savings can then be viewed as a self-insurance on capital markets against future income risk (i.e. precautionary savings). Similarly, Z decreases if $\bar{\mu}$ increases, leading to higher first-period consumption and leisure. If $\gamma < 1$, the substitution effect of a change in the CE rate of return dominates the corresponding income effect, thereby reversing the signs of $dZ/d\bar{\mu}$ and $dZ/d\sigma^2$.

The constancy of Z has some important implications. First, note from equation (3.13) that labour supply is a constant. This is due to the Cobb-Douglas specification that features a zero uncompensated labour supply elasticity. Hence, we omit the time index of labour supply in the rest of the section. Second, it implies that first-period consumption is a constant fraction of lifetime income $(1-\pi_{c,t})Y$.

3.2 Pension fund problem

To derive the optimal policy of the pension fund we need to specify the indirect utility function for young agents. Substitution of the solution for consumption, equation (3.11) and equation (3.12), and leisure, equation (3.13), in equation (2.1), gives the following indirect utility function:

$$V_t = \frac{(1+Z)^{\frac{1}{1-\gamma}} \eta^{\eta} (1-\eta)^{1-\eta}}{(1-\eta)Z+1} \left[(1-\pi_{c,t})Y\right]^{1-\eta} \tag{3.17}$$

Inserting this function together with the recovery rate, equation (2.14), in the objective function of the pension fund, equation (2.15), and taking the derivative with respect to N_t, we obtain the following first-order condition:

$$\mathrm{E}_t \left\{ \left[1+R_f+\alpha\lambda_{p,t}\left(R_{e,t+1}-R_f\right)\right]^{-\zeta} \left(R_{e,t+1}-R_f\right) \right\} = 0 \tag{3.18}$$

Note that condition (3.18) has exactly the same structure as the optimality condition of the portfolio choice of households, see equation (3.5). This implies that $\alpha\lambda_{p,t} = a_t$ so that equation (3.19) describes the share of equity in the pension fund's portfolio:

$$\lambda_p = \frac{\bar{\mu}}{\alpha\zeta\sigma^2} \tag{3.19}$$

$$N = \frac{\bar{\mu}\,(1-L)\,Y}{\zeta\sigma^2\left(1+R_f\right)} \tag{3.20}$$

Equation (3.20) describes the pension fund's absolute investment in equity. Each period the pension fund invests a fixed amount in the risky asset. Like households, the pension fund invests a smaller amount in the risky asset if risk aversion increases (higher ζ), reflecting the higher preference for consumption smoothing across states of nature. Note that the absolute amount of equity exposure in the pension fund portfolio does not depend on the accrual rate α. For lower (higher) values of α, the fund collects relatively less (much) pension contributions. In these cases, the pension fund will invest a larger (smaller) share λ^p of the contributions in the risky asset so that its risk exposure in absolute terms is left unchanged.

The maximum loss young agents can be confronted with occurs when $R_{e,t} = -1$. Then the implicit tax equals:

$$\pi_c^{MAX} = \frac{\bar{\mu}}{\zeta\sigma^2}$$

Hence, as long as $\bar{\mu} < \zeta\sigma^2$, the pension scheme is always sustainable. That is, the young generation will in any case be able to provide safe benefits to the old generation.

3.3 Welfare measure

In this section, we show that the collective DB pension scheme involves an ex ante Pareto improvement compared to the situation without a collective pension scheme. We denote this benchmark situation as the individual DC system. We measure the welfare change as an income-equivalent variation: we ask with how much the labour income of the representative agent (x) should be increased in the situation without a pension scheme in order to make him indifferent between participating in the funded pension scheme or not. We answer this question from an ex ante perspective, i.e., before an agent knows the state of nature in the first period of life. Positive numbers thus indicate welfare gains and negative numbers welfare losses. Let $W^0(\cdot)$ denote ex ante indirect utility in case there is no collective pension scheme and $W^1(\cdot)$ ex ante indirect utility in case there is a collective scheme. Then we have to solve for x that satisfies the following equality:[7]

$$W^1(Y) = W^0\left[(1+x)Y\right] \tag{3.21}$$

In Appendix A.3 we show that x is approximately equal to:

$$x = \frac{1}{2}\frac{\bar{\mu}^2}{\zeta\sigma^2} \tag{3.22}$$

If $\bar{\mu} = 0$, then it follows from equation (3.22) that $x = 0$: if the expected excess return of the risky asset is zero, the pension fund will not invest in this asset, thereby eliminating the scope for intergenerational risk sharing. As long as the expected excess return is positive ($\bar{\mu} > 0$), the pension will invest in the risky asset so that households can capture the equity premium which is welfare enhancing ($x > 0$).

To show that this pension scheme is Pareto-improving, suppose that the pension scheme is introduced at time t^*. Then the generations born before time $t^* + 1$ are obviously indifferent between the case with and without a pension fund. The young generation at time t^* only contributes the cost-effective rate π_{b,t^*} of disposable income to the pension fund. Since there are no intergenerational transfers in t^* (i.e. $\pi_{c,t^*} = 0$), we have that the utility of this generation also

[7] For brevity, we suppress the time index t: the welfare change from the introduction of the pension scheme is independent of the initial state.

remains unaffected. The generations born at the beginning of time $t^* + 1$ and beyond, benefit from the introduction of the pension scheme. The reform is thus Pareto-improving.

The welfare gain is increasing with the reward for risk taking $(\bar{\mu}/\sigma)$ and decreasing with the coefficient of relative risk aversion (ζ). From equation (3.20) it follows that a higher reward for risk taking increases the intergenerational payments and hence, raises the scope for intergenerational risk sharing. For higher degrees of risk aversion, in turn, it follows from equation (3.20) that the pension fund invests a smaller amount in the risky asset. As a consequence, the scope for intergenerational risk sharing decreases, resulting in lower welfare gains.

Interestingly, equation (3.22) does not depend on the pension fund size α. In the most extreme case, in which $\alpha = 0$, the pension fund does not collect cost-effective contributions from the young at all but still provides a welfare gain of x. In this situation, the pension fund explicitly takes short positions in safe assets to buy stocks on behalf of future generations (Teulings and de Vries (2006)). However, if we introduce short-sale constraints on investment behaviour of consumers, equation (3.15), or the pension fund, equation (3.19), this independency between the welfare gain and the pension fund size breaks down, as will be discussed in Section 5.6.

4 Lump sum transfers

To disentangle the welfare gains from intergenerational risk sharing from the labour market distortions associated with income-related intergenerational transfers, we now solve the model in case of lump sum transfers. We continue to assume Cobb-Douglas utility ($\rho = 1$). The budget constraint now becomes:

$$C_{2,t+1} = (1 + R_{T,t+1}) \left[(1 - L_t)Y + T_t - C_{1,t} \right]$$

with $T_t \equiv N_{t-1} \left(R_{e,t} - R_f \right)$. Defined in this way, a positive transfer ($T_t > 0$) implies that an agent *receives* a transfer from the pension fund, a negative transfer ($T_t < 0$) means that an agent *pays* a transfer to the pension fund. With lump sum transfers, the first-order condition with respect to consumption, equation (3.1a), and the first-order condition with respect to optimal portfolio allocation, equation (3.5), do not change, implying that $a = \bar{\mu}/\zeta\sigma^2$ continues to hold true. The first-order conditions with respect to leisure changes, however, into:

$$\eta C_{1,t}^{1-\psi} L_t^{-\omega} = (1-\eta)\beta \left(1 + R_f\right) Y \left(\mathrm{E}_t \, C_{2,t+1}^{1-\zeta} \right)^{\nu} \mathrm{E}_t \, C_{2,t+1}^{-\zeta} \tag{4.1}$$

Combining (4.1) with (3.1a) gives the result that the marginal rate of substitution between leisure and consumption equals the gross wage rate rather than the wage rate after pension contributions as in the previous section (see equation (3.10)),

$$\frac{\eta}{1-\eta} \frac{C_{1,t}}{L_t} = Y \tag{4.2}$$

First-period consumption, second-period consumption and leisure then satisfy:

$$C_{1,t} = \frac{1-\eta}{(1-\eta)Z_t + 1} (Y + T_t) \tag{4.3}$$

$$C_{2,t+1} = \frac{(1-\eta)Z_t (1 + R_{T,t+1})}{(1-\eta)Z_t + 1} (Y + T_t) \tag{4.4}$$

$$L_t = \frac{\eta}{(1-\eta)Z_t + 1} \frac{Y + T_t}{Y} \tag{4.5}$$

with Z_t already defined in equation (3.7).

Equations (4.3)-(4.5) are no closed-form solutions in the sense that their right-hand sides contain endogenous variables. Indeed, Z_t is a function of leisure which in case of lump sum transfers is not a constant. As a consequence, it is not possible in general to solve for the optimal investment policy of the pension fund analytically. Only for a particular case, when lifetime utility is log-linear in first-period and second-period consumption and leisure (i.e. $\gamma = \theta = \rho = 1$), is it possible to get closed-form expressions and to derive the optimal pension fund policy. In the case of log-linear lifetime utility, Z_t is constant and does not depend on leisure. Appendix A.4 shows that for log-linear lifetime utility the optimal pension fund policy is given by equation (4.6):

$$\lambda_{p,t} = \frac{\bar{\mu}}{\alpha\left(1-L_t\right)\sigma^2} \tag{4.6}$$

$$N = \frac{\bar{\mu}Y}{\sigma^2\left(1+R_f\right)} \tag{4.7}$$

Comparing solution (4.7) with the solution in case of distortionary transfers, see equation (3.20) with $\gamma = \theta = \rho = 1$, it follows that in the latter case the pension fund invests a smaller amount in the risky asset. In the numerical simulations, later on, it will be shown that this result also holds if $\gamma = \theta = \rho = 1$ does not apply.

If contributions relate to labour income, the intergenerational payments introduce a substitution effect in the labour supply decision. This substitution effect creates a positive correlation between labour income and asset returns, because if equity returns drop down, the pension fund has to increase the contribution rate which reduces the price of leisure, depresses labour supply and hence, reduces labour income. This procyclical pressure on labour supply behaviour reduces the risk-bearing capacity of consumers leading to lower equity investments of the pension fund in case of income-dependent transfers. As discussed in the introduction, this result differs from the existing literature on the interaction between labour supply and portfolio selection (see e.g. Bodie et al. (1992)).

For the case of log-linear lifetime utility, the welfare gain can be decomposed into the welfare gain from risk sharing and the welfare loss from the labour market distortion associated with the recovery rate. Let us denote x^L as the income-equivalent variation for lump sum transfers expressed as percentage of labour income: it measures the additional amount of income that should be given to an agent in the situation without a pension scheme in order to make him indifferent between participating in the funded pension scheme or not. Then we have:[8]

$$x^L = \frac{1}{2}\frac{\bar{\mu}^2}{\zeta\sigma^2}\left[1+\frac{\eta}{(1-\eta)(1+\beta)}\right] \tag{4.8}$$

We define the labour market distortion (x^D) as the difference between the income-equivalent variation in the presence of proportional transfers and that in the presence of lump sum transfers, i.e. $x^D \equiv x - x^L$. Hence,

$$x^D = -\frac{1}{2}\frac{\bar{\mu}^2}{\sigma^2}\frac{\eta}{(1-\eta)(1+\beta)} \tag{4.9}$$

Note that $x^D < 0$ is negative, implying that the welfare gain from risk sharing is larger in case of lump sum transfers than in case of proportional transfers. From equation (4.9), it follows that if the share of leisure expenditures in total expenditures increases (higher η), the welfare loss associated with the labour market distortion increases. In addition, if more weight is given to future consumption (higher β), the welfare loss decreases. The reason is that the labour market distortion only affects first-period consumption since people are retired in the second period.

[8] See Appendix A.4 for the derivation.

5 Simulations

5.1 Numerical procedure

This section reports on numerical simulations with the model. There are two reasons for switching to numerical simulation. First, the model version with CES preferences ($\rho \neq 1$) cannot be solved explicitly for optimal consumption, asset accumulation, labour supply and the optimal pension fund policy. Second, in the case of Cobb-Douglas preferences ($\rho = 1$), the computation of an analytical solution involves some approximation of the portfolio choice. Indeed, the derivations of the portfolio choices and the welfare gain are based on an approximation of the log portfolio return which exactly holds in continuous time, but becomes somewhat less accurate over longer time intervals (Barberis (2000)). To illustrate, based on the numerical approach we calculate an equity share in the individual portfolio of 25%, whereas a calculation based on the approximate analytical solution (3.6) gives an equity share of 27%.[9]

We use Monte Carlo simulation methods to solve the first-order conditions (2.5a)-(2.5c) of the consumer and first-order condition (3.18) of the pension fund. Recall that equation (3.18) only solves the pension fund problem for Cobb-Douglas utility. In case of CES utility, we will use grid search to compute to what extent the Cobb-Douglas solution must be adapted to obtain the CES optimum.

In solving the individual optimization problem, individuals form rational expectations based on the true distribution of variables. So in each period a young agent makes a decision, based on the state variable at that time (which is the net wage rate) in such a way that the first-order conditions are satisfied and expectations are based on the assumed distributions of next-period random variables. We will use 2,000 realized paths of the equity return to compute the distribution of the endogenous stochastic variables. This seems a natural choice in the trade-off between sufficient accuracy of the results and very long computer running time.[10]

5.2 Model parameters

The key parameters in the model are the utility parameters, i.e., the time discount factor (β), the (inverse of the) intertemporal substitution elasticity (γ), the coefficient of relative risk aversion (θ), the parameter governing the relative preference for leisure (η) and the (inverse of the) intratemporal substitution elasticity (ρ). The rest of the parameters involve the replacement rate (α), the risk-free rate of return (R_f), the wage rate (Y) and, finally, the mean and standard deviation of the stochastic equity return. Table 5.1 provides the values for these parameters used

[9] To derive equation (3.6), we assume that $1+R_T$ is lognormally distributed (see appendix A). This is not completely true because $1+R_T$ is a linear combination of the risk-free return $1+R_f$ and the lognormally distributed equity return $1+R_e$.

[10] See Appendix B for more details.

Table 5.1 Benchmark parameterization

Parameter	θ	γ	η	ρ	β	α	R_f	$\mathrm{E}(R_e)$	$\mathrm{Stdev}(R_e)$	Y
Value	5	2	0.5	1	0.8	0.4	0.02	0.05	0.20	1

The risk-free return and the mean and standard deviation of the equity return are annual figures.

in the baseline scenario.

In our model economy, agents live for two periods. Therefore, we interpret one model period to last twenty years. Like in van Hemert (2005), we set the time discount factor at $\beta = 0.8$ in the baseline calculation, which corresponds to an annual time discount rate of 1.1%. We choose as benchmark an intertemporal elasticity of substitution of 0.5 (i.e. $\gamma = 2$) and an intratemporal substitution of $\rho = 1$. An intertemporal substitution elasticity of one half is commonly used in the macro and public finance literature and it lies well within the range of available estimates (e.g., Attanasio and Weber (1995) or Blundell et al. (1994)). We set the coefficient of relative risk aversion at $\theta = 5$. The calibrated share parameter η is set at 0.5 and the risk-free return (R_f) is set at 2% per year, implying that the the twenty-year return is 48.5%. The replacement rate α is set at 40%, which is quite realistic for Dutch occupational pension schemes.[11] The wage rate Y is normalized at unity so that consumption and leisure have the same order of dimension in the utility function.

We assume that the annual mean of the equity return is 5%, implying a risk premium of 3% per year. The standard deviation of the annual equity return is set at 20%. To construct twenty-year shocks, we transform the annual mean and variance of the lognormal distribution (of equity returns) to the corresponding moments of the normal distribution (of log equity returns).[12] Then we calculate twenty-year log returns as the sum of twenty randomly chosen yearly log returns. To reduce the sample variation, these twenty-year log returns are scaled in a uniform way to ensure that their mean and standard deviation are equal to the theoretical values.

5.3 Baseline results

Since the wage rate is normalized at unity, variables like consumption (C_1 and C_2) and the stock of risky assets of the pension fund (N) can be interpreted as percentages of the wage rate.

[11] In the Netherlands, an individual with an average income level and an unbroken career typically has a (gross) replacement rate of 70%, which consists for about 30%-points of (first pillar) PAYG public pensions and for 40%-points of (second pillar) funded occupational pensions.

[12] If $\mathrm{E}(1+R_e)$ and $\mathrm{Var}(1+R_e)$ are the mean and variance of the lognormally distributed gross equity return, then μ and σ^2 satisfy:

$$\mu = 2\log \mathrm{E}(1+R_e) - \log(1+R_f) - \frac{1}{2}\log\left\{\mathrm{Var}(1+R_e) + [\mathrm{E}(1+R_e)]^2\right\}$$

$$\sigma^2 = \log\left\{\frac{\mathrm{Var}(1+R_e)}{[\mathrm{E}(1+R_e)]^2} + 1\right\}$$

Table 5.2 Simulation results in the benchmark economy

	C_1	C_2	L	a	$\lambda_h S$	N	x
Individual DC							
expectation	39.4	37.8	39.4	24.9	5.3		
10% quantile	39.4	27.2	39.4	24.9	5.3		
90% quantile	39.4	52.4	39.4	24.9	5.3		
Collective DB: proportional							
expectation	47.0	45.5	39.4	24.9	6.3	10.2	7.0
10% quantile	33.9	26.0	39.4	24.9	4.6		
90% quantile	65.2	69.9	39.4	24.9	8.8		
Collective DB: lump sum							
expectation	44.1	44.7	43.9	24.9	6.2	11.2	7.8
10% quantile	35.9	26.5	35.9	24.9	4.7		
90% quantile	55.2	67.5	55.2	24.9	8.3		

Results are based on 2,000 simulations and expressed in percentages. The wage rate Y is normalized at unity.

Table 5.2 shows the simulation results for the benchmark parameterization. The table shows results for the private economy (which we denote as an individual DC system) and the economy with a collective DB scheme, whereby it distinguishes between proportional and lump sum intergenerational transfers.[13] The table reports expected outcomes and 10% and 90% quantiles.

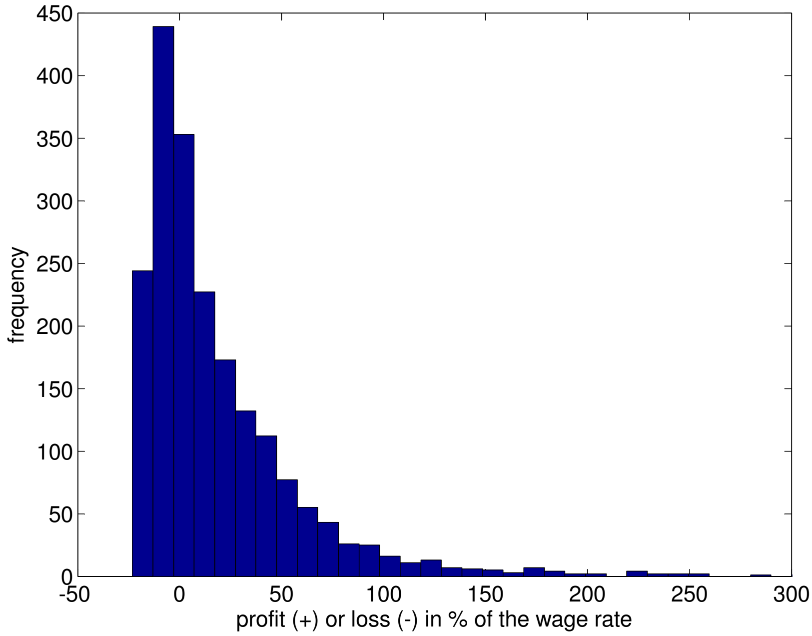
Note first that in all situations, regardless of whether there is a funded DB scheme or not, agents invest the same percentage (24.9%) of lifetime income in the risky asset. Because the pension benefit is safe, the asset span of households in the presence of collective pensions is the same as that in the private economy resulting in equivalent lifetime portfolio choices.

In the private economy and the DB pension economy with lump sum transfers, in which labour market distortions are absent, the marginal rate of substitution between first-period consumption and leisure equals the gross wage rate. Since we assume that agents spend the same fraction of total expenditures on both goods ($\eta = 0.5$), first-period consumption (expressed as percentage of the wage rate) and leisure should then be equal to each other. This is only approximately true in the case of lump sum transfers. This is due to the fact that leisure cannot exceed the value of one. This boundary constraint is binding in case of an extremely large shock in the excess return on the risky asset. The numerical effect of this boundary constraint is, as Table 5.2 shows, very small however.

The introduction of the collective pension scheme does not change labour supply due to our assumption of a unitary elasticity of intratemporal substitution. However, compared to the private economy, the introduction of the pension scheme does increase the expected consumption levels in the first and second period. At the same time, it also raises the risk born by each generation as reflected by the wider 80%-confidence intervals of both first-period and

[13] In the following, if we do not explicitly mention how transfers are financed, we refer to the pension scheme with proportional premiums on labour income.

Figure 5.1 Distribution of the intergenerational transfer



second-period consumption. The intuition is that the risk sharing provided by the pension scheme increases lifetime wealth of consumers which leads them, in turn, to raise their demand for risky assets ($\lambda_h S$) from 5.3% to an average level of 6.3%. Hence, in our small open economy setting, the benefit of risk sharing is not to reduce risk but rather to increase the expected payoff from risky investments by generations who have not entered the labour market yet. Cui et al. (2006), Bovenberg et al. (2007) and Gollier (2008) also point to this effect of intergenerational risk sharing.

Consumers can only capture the equity premium in the first period if they choose to work in this period. In this way, the equity premium acts as a subsidy on labour supply. Indeed, leisure is about 4.5%-points lower than if transfers were financed lump sum, while average first-period consumption is about 3%-points higher. The distortionary effect of the intergenerational payments leads to a slightly lower investment in the risky asset by the pension fund: in case of lump sum transfer, risk taking by the pension fund is 10.2%, while in case of proportional transfers it is 11.2%. We have already shown that this result holds analytically for a specific version of the model. It also holds for a more general case however: labour market distortions reduce the risk-bearing capacity of consumers.[14]

The pension scheme involves a significant ex ante Pareto improvement. The income-equivalent variation amounts 7%.[15] The income-equivalent variation in case of lump

[14] See Mehlkopf (2009) for a similar result.

[15] If we compute the income-equivalent variation from the analytical solution, equation (3.22), we obtain a welfare gain of 7.9%.

sum transfers is 7.8%. This means that the distortionary effect of income-related transfers is 0.8%-point, roughly 10% of the pure welfare gain from risk sharing.

Within our intergenerational risk-sharing contract, any transfer between generations is positive in expectation, but it can be negative for some bad states of the world. In principle, consumers are only willing to participate in the contract if the initial loss is not larger than the expected gains from risk sharing. We have avoided this discontinuity issue by imposing mandatory participation, but it is still useful to analyse the sustainability of the contract under voluntary participation. Figure 5.3 show the histogram of the intergenerational transfer as percentage of the wage rate. The expected transfer is positive and equal to 19% of the wage rate, but there is a probability of 40% that agents enter the pension contract with a negative transfer. The profits are generally much larger than the losses: the average loss agents incur is 10% of the wage rate, while the average profit amounts to 39% of the wage rate. Note that the minimum transfer is bounded at a value of about 25%.

5.4 Sensitivity analysis

In this section, we check the robustness of the baseline result by solving the model for alternative parameter values. We will solve for the income-equivalent variation in the presence of proportional and lump sum transfers in order to be able to calculate the size of the labour market distortion. We consider alternative values for the time discount factor (β), the inverse of the intertemporal substitution elasticity (γ), the preference parameter for leisure (η), the equity premium ($\bar{\mu}$), the standard deviation of the excess return (σ) and the coefficient of relative risk aversion (θ). The findings are shown in Table 5.3. To make a comparison of the results easier, we consider for each parameter reported a value change of $+50\%$ and -50% compared to the baseline value.

Consistent with equation (3.22), the computed income-equivalent variations do not depend on the time discount factor or the intertemporal substitution elasticity in case of proportional transfers.[16] In case of lump sum transfers, however, the income-equivalent variation increases (decreases) if the time discount factor decreases (increases). Recall that the labour market distortion only affects the intratemporal decision between consumption and leisure in the first period because in the second period people do not work at all. As a result, the distortion increases for lower values of β and decreases for higher values of β, although quantitatively this effect is very small.

The welfare effects of changes in the leisure parameter and in the degree of risk aversion are similar in size. Note first that the welfare gain is decreasing with the coefficient of relative risk

[16] Given that the welfare gain is not very sensitive to changes in the intertemporal substitution elasticity, we lose not much generality if we would confine the analysis to the standard expected utility formation instead of non-expected utility. At this stage, however, we prefer to use the more general non-expected utility framework.

Table 5.3	Sensitivity analysis: welfare gains			
Parameter	Value	Proportional	Lump sum	Distortion
β	0.4	7.0	7.9	– 0.9
	0.8*	7.0	7.8	– 0.8
	1.2	7.0	7.7	– 0.7
γ	1.1	7.0	7.8	– 0.8
	2.0*	7.0	7.8	– 0.8
	3.0	7.0	7.8	– 0.8
η	0.25	5.1	5.3	– 0.2
	0.50*	7.0	7.8	– 0.8
	0.75	11.3	15.3	– 4.1
θ	2.5	13.2	16.5	– 3.2
	5.0*	7.0	7.8	– 0.8
	7.5	4.8	5.1	– 0.3
$\bar{\mu}$	0.015	1.5	1.7	– 0.2
	0.030*	7.0	7.8	– 0.8
	0.045	17.8	19.8	– 2.0
σ	0.1	36.3	42.3	– 5.9
	0.2*	7.0	7.8	– 0.8
	0.3	2.2	2.5	– 0.2

Results are based on 2,000 simulations. All equivalent variations are expressed as percentages (of the wage rate), with benchmark parameters marked as *.

aversion, a result already derived in the analytical exposition. For lower degrees of risk aversion, the pension fund takes more risk which raises the scope for risk sharing (and hence, welfare). On the contrary, the income-equivalent variation is increasing in the leisure parameter η. If η increases (decreases), the welfare gain increases (decreases) because the average subsidy the pension scheme provides increases (decreases) as a percentage of labour income. At the same time, an increase in η also increases the volatility of the intergenerational payments, which widens the gap between the income-equivalent variation in case of proportional transfers and lump sum transfers.

The income-equivalent variation is very sensitive to changes in the equity premium and the standard deviation of the excess return. The increase (decrease) of the equity premium with 1.5 %-point raises the welfare gain by almost 11%-points (5.5%-points). A 50%-increase of the standard deviation of the yearly excess return from 20% to 30%, in turn, reduces the income-equivalent variation from 7% to 2.2%. A 50%-decrease of the standard deviation, however, increases the welfare gain to more than 36%.

Calculated for a wide range of realistic parameter values, the welfare gain has a large spread of possible values; it varies somewhere between 2% and 36% of the wage rate. Most importantly, the figures in Table 5.3 show that the welfare gains from risk sharing in a funded

DB scheme are large compared to the labour market distortion associated with the intergenerational payments. Still, the welfare costs of labour market distortions are not negligible. For some cases, the fraction of surplus that is eroded by distortions is almost 30%.

5.5 Simulations with CES utility

So far, we have assumed that the intratemporal substitution elasticity between consumption and leisure ($1/\rho$) is unity, implying an uncompensated labour supply elasticity (ε) of zero (see Appendix A.5 for the relationship between the two). Actually, there is a lot of evidence suggesting a non-zero labour supply elasticity (see Blundell and MaCurdy (1999) for an overview). Evers et al. (2008) use 30 different studies to construct a data set of empirical estimates of the uncompensated labour supply elasticity. They show that the mean elasticity of men equals 0.07, while for women it equals 0.34. Mean elasticities for men range between -0.08 and 0.18. For women, mean elasticities range between 0.03 and 2.79. To capture the empirical evidence regaring the labour supply elasticity, we present simulation results for different values for the intratemporal substitution elasticity.

Table 5.4 Results with CES utility: individual DC scheme

$1/\rho$	C_1	L	a
0.5	36.8	36.8	17.7
1.0	39.4	39.4	24.9
2.0	41.9	41.9	37.9

Results are based on 2,000 simulations.

Table 5.4 reports the simulation results for the private economy and Table 5.5 shows the simulation results for the economy with DB pensions. In Appendix A.5 we formally show that the labour supply elasticity is positive if the intratemporal labour supply elasticity is larger than unity, zero if it is equal to unity and negative if the intratemporal substitution elasticity is smaller than unity. Tables 5.4 and 5.5 show simulation results for three different values for the intratemporal substitution elasticity, corresponding to an interval for ε that ranges from -0.2 to 0.3.

If the labour supply elasticity is unequal to zero, the income and substitution effect on leisure are different so that labour supply and portfolio allocation become state-dependent. For example, suppose $\varepsilon > 0$, then agents will work less hours if they are confronted with a positive recovery rate ($\pi_c > 0$) implying that they build up less pension rights to finance future consumption. To compensate for this, agents take more risk by investing a larger share of total wealth in the risky asset (higher a).

Compared to the private economy, consumption is higher in the DB pension economy, reflecting the welfare gain from risk sharing. The average level of labour supply, instead, is

Table 5.5 Results with CES utility: collective DB scheme

			Proportional transfer				Lump sum transfer			
$1/\rho$		ε	C_1	L	a	N	C_1	L	a	N
0.5	expectation	– 0.2	42.3	38.4	18.1	9.7	41.2	41.1	18.0	10.2
	10% quantile	– 0.2	33.0	35.3	17.4	9.7	33.7	33.7	17.4	10.2
	90% quantile	– 0.2	55.3	42.9	19.1	9.7	51.4	51.4	18.8	10.2
1.0	expectation	0.0	47.0	39.4	24.9	10.2	44.1	43.9	24.9	11.2
	10% quantile	0.0	33.9	39.4	24.9	10.2	35.9	35.9	24.9	11.2
	90% quantile	0.0	65.2	39.4	24.9	10.2	55.2	55.2	24.9	11.2
2.0	expectation	0.3	51.8	39.5	36.6	9.7	46.9	46.7	36.5	12.2
	10% quantile	0.3	33.2	31.8	32.4	9.7	38.2	38.2	32.0	12.2
	90% quantile	0.4	76.8	45.7	40.2	9.7	58.6	58.6	39.9	12.2

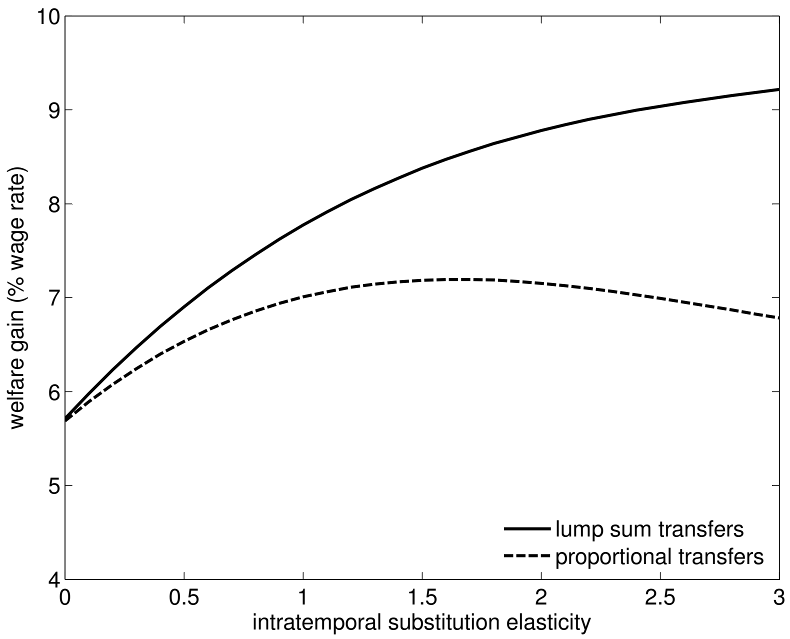
Results are based on 2,000 simulations.

higher if the labour supply elasticity is positive and lower if this elasticity is negative. Recall that the pension scheme enables agents to capture the equity premium already in the first period. In this way, the pension scheme stimulates labour supply if $\varepsilon > 0$ (the substitution effect dominates the income effect) and depresses labour supply if $\varepsilon < 0$ (vice versa).

The fourth and fifth column of Table 5.5 reveal that the levels of consumption and leisure are diverging in the DB pension economy for higher levels of the intratemporal substitution elasticity, reflecting the fact that the two goods become closer substitutes. In addition, since the substitution effect on leisure becomes more dominant relative to the income effect, consumption and leisure get also further away from the corresponding lump sum levels. As a consequence, the distortionary effect of the intergenerational transfers becomes larger for higher levels of the intratemporal substitution elasticity which drives away the optimal equity investment of the pension fund from the corresponding lump sum levels. For an intratemporal substitution elasticity of 0.5, the equity investment of the pension fund is only 0.5%-points lower than in case of lump sum transfers. When the intratemporal substitution elasticity is 2, however, this difference has been increased to 2.5%-points.

But even if intratemporal substitution is high, the labour market distortion is quite modest compared to the pure welfare gains of risk sharing. Figure 5.2 shows the welfare gain of risk sharing as function of the intratemporal substitution elasticity. The displayed values of substitution elasticities from (approximately) 0 to 3 correspond to an interval for the labour supply elasticity that ranges between -0.4 and 0.7. The difference between the solid line (corresponding to lump sum transfers) and the dashed line (corresponding to proportional transfers) is the labour market distortion. The labour market distortion increases from 0% of the wage rate to about 2.5% of the wage rate for a substitution elasticity of 3. The pure welfare gain belonging to this high level of intratemporal substitution is 9.3%, hence, the labour market

Figure 5.2 Welfare gain in case of CES utility



distortion amounts more than one quarter of this percentage.[17]

5.6 Short-sale constraint consumers

Until now, we have assumed that agents do not face any borrowing or liquidity constraint. That means, agents can take short positions in either the risky asset or the risk-free asset. In addition, if that would be optimal, agents can choose to borrow in the first period (negative private savings) to optimally smooth consumption over the first and second period. In practice, though, it is often difficult or even impossible for young people to take short positions in an asset, because human capital alone does not collateralize major loans in modern economies for reasons of moral hazard and adverse selection problems. To overcome this objection, this section solves the model with a short-sale constraint for households, i.e. $0 \leq \lambda_{h,t} \leq 1$. We do not have to consider a non-negativity constraint on private savings because in the baseline scenario the young's optimal savings turn out to be always positive in the simulations.

In case there is no short-sale constraint, the size of the collective pension scheme in terms of the exogenous accrual rate (α) does not play a role in the model. Any actions by the pension scheme can be undone by the household. However, if there is a short-sale constraint, the size of the pension sector will matter because there is a possibility that agents cannot offset the decision

[17] As an additional check, we have also calculated the sensitivity of the labour market distortion for changes in the intratemporal substitution elasticity keeping constant the ratio between first-period and second-period consumption at the baseline level. To that end, we considered several model parameters to recalibrate this consumption ratio. In all cases, these modifications hardly changed the welfare effects as represented in Figure 5.2.

Table 5.6 **Short selling versus no short selling**

	λ_h	λ_p	x
Baseline			
expectation	136.9	62.2	7.0
10% quantile	46.3	62.2	
90% quantile	227.8	62.2	
No short selling			
expectation	80.5	59.2	6.7
10% quantile	47.1	59.1	
90% quantile	100.0	59.2	

Results are based on 2,000 simulations and expressed in percentages.

of the pension fund. In our DB pension scheme, whose size is based on Dutch evidence, it is indeed true that the implementation of a short-sale constraint restricts household behaviour in some states. We already discussed that for the benchmark parameterization agents invest 25% of *total* wealth in the risky asset. As a percentage of *financial* wealth, however, this equity investment is much larger in the economy with a funded DB scheme. Table 5.6 shows that the equity investment is 137% of financial wealth in the expected path, which means that agents take short positions in the risk-free asset on average.

The introduction of the short-sale constraint reduces the risk exposure considerably: the expected value of λ_h declines from 136.9% to 80.5% and the 90% quantile decreases from 227.8% to the threshold value of 100%. The pension fund does not take short positions. In the baseline scenario the pension fund invests 62.2% of the collected contributions in the risky asset. In case of a short-sale constraint, it is optimal to decrease this share to 59.2%. As discussed in Section 3.1, agents are inclined to take short positions if there are funding deficits ($\pi_c > 0$) to ensure that the fraction of their risky asset holdings in total wealth will not change. However, when agents are not allowed to do this, a welfare-maximizing pension fund takes care of this by investing a smaller amount in stocks. This investment strategy decreases the probability of funding surpluses and deficits, and hence, the probability that the short-sale constraint will bind the household. The welfare consequences of the inability to take short positions are modest: the income equivalence decreases with only 0.3%-points.

5.7 Labour income tax

In general, the labour supply decision is determined by the total marginal tax burden which is not only affected by implicit taxes or subsidies in collective pension schemes but also by explicit labour income taxes. In the model analysed so far, we have ignored the role of labour income taxation. This section investigates how the introduction of a funded DB pension scheme affects individual welfare if there is already an initial labour income tax in the economy. We consider

Table 5.7 **Welfare effects in case of labour income taxation**

		Without lump sum transfer			With lump sum transfer		
$1/\rho$	$\tau = 0$	$\tau = 0.2$	$\tau = 0.3$	$\tau = 0.4$	$\tau = 0.2$	$\tau = 0.3$	$\tau = 0.4$
0.5	6.5	6.3	6.1	5.9	6.4	6.3	6.1
1.0	7.0	7.0	7.0	7.0	6.7	6.4	6.0
2.0	7.1	7.6	7.9	8.3	6.1	5.5	4.9

Results are based on 2,000 simulations and expressed in percentages.

two different cases: in the first case the government spends the tax revenues on services from which the consumer does not derive utility while in the second case tax revenues are redistributed back in the form of lump sum income transfers. Hence, in the first case the labour income tax has a substitution and income effect, in the second case it only has a substitution effect. We analyse the welfare implications for a labour income tax rate (τ) of 20%, 30% and 40% and for different values of the intratemporal substitution elasticity.

Table 5.7 shows the results. Consider first the case without lump sum redistribution of the collected tax revenues, the left panel of the table. Note that in the benchmark parameterization with a substitution elasticity of unity the welfare gain is independent of labour income taxation.[18] If $\rho = 1$ the income and substitution effects cancel against each other so that the labour supply decision is not affected by labour income taxation. For substitution elasticities below unity, however, the welfare gain of the funded pension scheme is lower in case there is an initial tax distortion. In this case, the income effect dominates the substitution effect implying that labour supply in the economy is higher if $\tau > 0$ compared to $\tau = 0$. Consequently, if there is an initial tax distortion, the welfare-improving intergenerational transfers associated with the risk-sharing contract decrease as percentage of net disposable labour income. This reduces the scope for risk sharing, resulting in lower welfare gains. If the substitution elasticity is higher than unity, instead, the opposite holds. In this case income taxation reduces labour supply incentives which relatively increases the welfare-improving intergenerational transfers.

Concentrating on the case in which the government redistributes the collected tax receipts using lump sum payments, the right panel of Table 5.7, it follows that the welfare gain of the pension scheme is unambiguously lower if $\tau > 0$ compared to $\tau = 0$. Labour income taxation introduces an additional (negative) substitution effect on labour supply which reduces the risk-bearing capacity of consumers. The optimizing pension fund responds to this by therefore reducing its equity exposure, resulting in lower welfare gains.

[18] This can also be proved formally. If $\rho = 1$, we have that $\tilde{\lambda}_{p,t} = (1-\tau)\lambda_{p,t}$, where $\tilde{\lambda}_{p,t}$ denotes the optimal equity investment of the pension fund in case of an initial income tax rate τ. Substituting this expression in the condition for x_t, i.e. $(1+x_t)^{1-\zeta} = \mathrm{E}_t\left[1+\pi_{b,t}\tilde{\lambda}_{p,t}(1-\tau)^{-1}\left(R_{e,t+1}-R_f\right)\right]^{1-\zeta}$, we obtain equation (A.13), which does not depend on τ anymore.

6 Concluding remarks

In this paper, we have developed a stylized two-period overlapping-generations model to investigate the welfare gains from intergenerational risk sharing in collective funded pension schemes. We have analysed the welfare implications of collective funded pensions both in terms of risk sharing and in terms of labour market distortions. To that end, we have contructed a pension scheme with defined benefits related to previous working life and with contributions contingent on the capital-market shock. This funded DB scheme creates opportunities for intergenerational risk sharing that agents cannot offset through transactions in financial markets. In our model, the pension fund facilitates intergenerational risk sharing by allowing the young generation to trade in equity before it is born. As the contingent intergenerational transfers in this pension scheme relate to labour income, they also distort the labour-leisure decision.

We have analytically shown for a Cobb-Douglas felicity function that a funded DB scheme involves an ex ante Pareto improvement, implying that the welfare gains from risk sharing outweigh the negative effect regarding labour market distortions. Using numerical simulations, we showed that this result also holds under a more general CES felicity function. In terms of income-equivalent variation, the welfare gain of risk sharing can be quite substantial, varying somewhere between 2% and 36%. We have also shown that a funded DB scheme may change labour supply, because the pension contract changes the price of leisure. To the earlier literature that emphasizes that risk sharing may be reflected in a higher average portfolio rate of return, our paper thus adds that risk sharing may also affect labour supply.

Our findings are relevant for the debate on worldwide pension reforms. Due to increasing demographic pressure, various countries have moved or are considering to move away from collective funded DB schemes towards individual funded DC schemes. Our result emphasizes that the loss this implies in terms of reduced intergenerational risk sharing may be far more important than the gain that is due to reduced labour market distortions.

While our model allows studying the welfare effects of collective pensions, it has its limitations. It restricts intergenerational risk sharing to two generations only. Because one model period represents twenty years, shocks can maximally be smoothed out over a period of fourthy years. One might argue that this time span is relatively short. Extending risk-sharing possibilities to more generations will increase the welfare gains from risk sharing.[19] In this respect, our calculated welfare gain can be viewed as a lower bound of the potential gains of risk sharing.

We have abstracted from stochastic labour productivity. Accounting for the stochastic nature

[19] Increasing the risk-sharing possibilities could be implemented in the model by spreading risks over an infinite number of generations. However, extending the model along this line is numerically much more demanding. For a particular version of the model (i.e. when per-period utility has a Cobb-Douglas form and transfers are income-related) this problem is analytically well-defined. It gives a welfare gain of 8.7%, which is only 1.7%-points higher than if risk sharing is restricted to two generations. Hence, alleviation of this restriction does not seem to add much potential for welfare improvement.

of labour productivity would enhance the role of the pension scheme, which then would not only share equity return risks across generations, but also labour productivity risks. This would increase the scope for risk sharing. This also suggests that our analysis underestimates the welfare gain from funded DB schemes. Furthermore, as riskless assets are rarely seen in practice, a model with two types of risky assets to describe equity and bonds could be more realistic. In such a world, pension schemes would be even more attractive, as they provide nonstochastic benefits which households cannot obtain if pension funds are absent. This adds to our claim that our result on the welfare gain of collective pension schemes should not be interpreted literally.

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Appendix A Derivations

In this appendix we derive equation (3.6), equation (3.9), equation (3.22) and equations (4.7)-(4.9). We also derive the labour supply elasticity (ε) reported in Table 5.5.

A.1 Portfolio allocation households

Following Campbell and Viceira (2002), we assume that both $1+R_{T,t+1}$ and $(1+R_{e,t+1})/(1+R_f)$ are lognormally distributed. If a variable X is lognormally distributed, then there holds:

$$\log \mathrm{E}_t X_{t+1} = \mathrm{E}_t \log X_{t+1} + \frac{1}{2}\mathrm{Var}_t \log X_{t+1} \tag{A.1}$$

Taking logs of equation (3.5) and using equation (A.1), we obtain:

$$\begin{aligned} &\mathrm{E}_t\left(-\zeta r_{T,t+1} + r_{e,t+1}\right) + \frac{1}{2}\mathrm{Var}_t\left(-\zeta r_{T,t+1} + r_{e,t+1}\right) = \\ &\qquad \mathrm{E}_t\left(-\zeta r_{T,t+1} + r_f\right) + \frac{1}{2}\mathrm{Var}_t\left(-\zeta r_{T,t+1} + r_f\right) \end{aligned}$$

where $r_i \equiv \log(1+R_i)$ and $i = e, f, T$. Simplifying this expression gives:

$$\mathrm{E}_t\, r_{e,t+1} + \frac{1}{2}\mathrm{Var}_t r_{e,t+1} - r_f = \zeta \mathrm{Cov}_t\left(r_{T,t+1}, r_{e,t+1}\right) \tag{A.2}$$

As the return on the portfolio is a linear combination of the return on stocks and the return on bonds, see equation (2.3), and the log of a linear combination is not the same as a linear combination of logs, we follow Campbell and Viceira (2002) and use a Taylor approximation of the nonlinear function relating log individual-asset returns to log portfolio returns. First note that equation (3.3) can be written as:

$$1 + R_{T,t+1} = 1 + R_f + a_t\left[(1+R_{e,t+1}) - \left(1 - R_f\right)\right] \tag{A.3}$$

Dividing this expression by $1+R_f$ and then taking logs gives:

$$r_{T,t+1} - r_f = \underbrace{\log\left\{1 + a_t\left[\exp\left(r_{e,t+1} - r_f\right) - 1\right]\right\}}_{f\left(r_{e,t+1} - r_f\right)}$$

Now we take a second-order Taylor expansion of $f(\cdot)$ around $r_{e,t+1} - r_f = 0$, which gives:

$$r_{T,t+1} \approx r_f + a_t\left(r_{e,t+1} - r_f\right) + \frac{1}{2}a_t(1-a_t)\mathrm{Var}_t r_{e,t+1} \tag{A.4}$$

From equation (A.4) it follows:

$$\mathrm{Cov}_t\left(r_{T,t+1}, r_{e,t+1}\right) = a_t \mathrm{Var}_t r_{e,t+1} \tag{A.5}$$

Substituting equation (A.5) into (A.2) then gives:

$$a_t = \frac{\mathrm{E}_t\, r_{e,t+1} - r_f + \frac{1}{2}\mathrm{Var}_t r_{e,t+1}}{\zeta \mathrm{Var}_t r_{e,t+1}} \tag{A.6}$$

Recall our statistical assumptions:

$$\mathrm{E}_t\left(r_{e,t+1} - r_f\right) = \mu \tag{A.7}$$

$$\mathrm{Var}_t\, r_{e,t+1} = \sigma^2 \tag{A.8}$$

Note from equation (A.1) that:

$$\bar{\mu} \equiv \log \mathrm{E}_t\left(\frac{1+R_{e,t+1}}{1+R_f}\right) = \mu + \frac{1}{2}\sigma^2 \tag{A.9}$$

Inserting equations (A.7)-(A.9) in equation (A.6) gives equation (3.6).

Note that for small x we have that $\log(1+x) \approx x$. Then it follows from (A.9) that $\bar{\mu}$ is approximately equal to the excess return:

$$\bar{\mu} \approx \mathrm{E}\left(R_e - R_f\right)$$

We can also derive that:

$$\mathrm{Var}\left(R_e - R_f\right) = \mathrm{Var}\left(R_e\right) = \left[\mathrm{E}\left(1+R_e\right)\right]^2\left[\exp\left(\sigma^2\right) - 1\right] \approx \left[\mathrm{E}\left(1+R_e\right)\right]^2\sigma^2 \approx \sigma^2$$

where the two $\approx$ signs are associated with small σ^2 and small $\mathrm{E}(R_e)$. Hence, σ^2 is approximately equal to the variance of the (excess) equity return.

A.2 Certainty-equivalent rate of return

Taking logs of equation (3.8) and using (A.1), we obtain:

$$\bar{r}_t = \frac{1+\nu}{1-\psi}\underbrace{\left[(1-\zeta)\,\mathrm{E}_t\, r_{T,t+1} + \frac{1}{2}(1-\zeta)^2 \mathrm{Var}_t\, r_{T,t+1}\right]}_{k_t} \tag{A.10}$$

with $\bar{r} \equiv \log\left(1+\bar{R}\right)$. Using equation (A.4), the term k_t can be rewritten to:

$$\begin{aligned} k_t = {} & (1-\zeta)r_f + (1-\zeta)a_t\left(\mathrm{E}_t\, r_{e,t+1} - r_f\right) + \frac{1}{2}(1-\zeta)a_t(1-a_t)\mathrm{Var}_t\, r_{e,t+1} \\ & + \frac{1}{2}(1-\zeta)^2 a_t^2 \mathrm{Var}_t\, r_{e,t+1} \end{aligned} \tag{A.11}$$

Inserting equation (A.6) into (A.11) and rearranging gives:

$$k_t = (1-\zeta)r_f + \frac{1-\zeta}{2\zeta}\frac{\left(\mathrm{E}_t\, r_{e,t+1} - r_f + \frac{1}{2}\mathrm{Var}_t\, r_{e,t+1}\right)^2}{\mathrm{Var}_t\, r_{e,t+1}}$$

Using equations (A.7)-(A.8) together with equation (A.9), we obtain:

$$k_t = (1-\zeta)\left(r_f + \frac{1}{2}\frac{\bar{\mu}^2}{\zeta\sigma^2}\right) \tag{A.12}$$

Inserting equation (A.12) in equation (A.10), and again using the fact that $\log(1+x) \approx x$, we obtain equation (3.9).

A.3 Welfare gain

Substituting the indirect utility function (3.17) in the objective function of the pension fund, equation (2.15), gives the following expression for $W_t^1(\cdot)$:

$$W_t^1(Y) = \frac{(1+Z)^{\frac{1}{1-\gamma}}\eta^\eta\left[(1-\eta)Y\right]^{1-\eta}}{(1-\eta)Z+1}\left[\mathrm{E}_t\left(1-\pi_{c,t+1}\right)^{1-\zeta}\right]^{\frac{1}{1-\theta}}$$

Note that $W_t^0(\cdot)$ is simply equal to:

$$W_t^0(Y) = \frac{(1+Z)^{\frac{1}{1-\gamma}}\eta^\eta\left[(1-\eta)Y\right]^{1-\eta}}{(1-\eta)Z+1}$$

Then equation (3.21) implies:

$$(1+x_t)^{1-\zeta} = \mathrm{E}_t\left[1+\pi_{b,t}\lambda_{p,t}\left(R_{e,t+1}-R_f\right)\right]^{1-\zeta} \tag{A.13}$$

Multiplying both sides of equation (A.13) with $\left(1+R_f\right)^{1-\zeta}$ gives:

$$\left[(1+x_t)\left(1+R_f\right)\right]^{1-\zeta} = \mathrm{E}_t\left[1+R_f+\alpha\lambda_{p,t}\left(R_{e,t+1}-R_f\right)\right]^{1-\zeta}$$

Recall that $a_t = \alpha\lambda_{p,t}$. Hence, this equation can rewritten into:

$$\left[(1+x_t)\left(1+R_f\right)\right]^{1-\zeta} = \mathrm{E}_t\left(1+R_{T,t+1}\right)^{1-\zeta}$$

Taking logs on both sides gives:

$$(1-\zeta)\log(1+x_t)+(1-\zeta)r_f = k_t \tag{A.14}$$

Substituting equation (A.12) in (A.14) and (again) using the approximation $\log(1+x)\approx x$, we obtain equation (3.22).

A.4 Labour market distortion

For log-linear lifetime utility (i.e. $\gamma=\theta=\rho=1$) we are able to solve the model with lump sum transfers analytically. In that case, it follows from equation (3.7) that $Z=\beta$ and, hence, does not depend on leisure anymore. Lifetime utility equals,

$$U_t = (1-\eta)\log C_{1,t}+\eta\log L_t+\beta(1-\eta)\,\mathrm{E}_t\log C_{2,t+1}$$

Substituting equation (4.3)-(4.5) in this expression gives the following indirect utility function:

$$V_t = F-\eta\log Y+\left[1+\beta(1-\eta)\right]\log(Y+T_t) \tag{A.15}$$

$$\begin{aligned} F \equiv\ & (1-\eta)(1+\beta)\log(1-\eta)+\eta\log\eta+\beta(1-\eta)\log\beta+\beta(1-\eta)\,\mathrm{E}_t\,r_{T,t+1} \\ & -\left[1+\beta(1-\eta)\right]\log\left[1+\beta(1-\eta)\right] \end{aligned}$$

Note that F is a constant term. Social welfare is maximized if,

$$\frac{\partial W_t}{\partial N_t} = \frac{\partial \, \mathrm{E}_t V_{t+1}}{\partial N_t} = 0$$

leading to first-order condition:

$$\mathrm{E}_t \left\{ \left[1 + R_f + \frac{\left(1 + R_f\right) N_t}{Y} \left(R_{e,t+1} - R_f\right) \right]^{-1} \left(R_{e,t+1} - R_f\right) \right\} = 0 \tag{A.16}$$

Comparing equation (3.5) and (A.16), there must hold:

$$\frac{\left(1 + R_f\right) N_t}{Y} = a_t \tag{A.17}$$

This implies:

$$N = \frac{\bar{\mu} Y}{\sigma^2 \left(1 + R_f\right)} \tag{A.18}$$

$$\lambda_{p,t} = \frac{\bar{\mu}}{\alpha \left(1 - L_t\right) \sigma^2} \tag{A.19}$$

To derive the welfare gain of the pension scheme in the presence of lump sum transfers, we need to solve for the additional amount of income (x^L) an agent in the situation without a pension scheme needs in order to make him indifferent between participating in the pension scheme or not:

$$\mathrm{E}_t V_{t+1}^1(Y) = \mathrm{E}_t V_{t+1}^0 \left[(1 + x_t^L) Y\right] \tag{A.20}$$

From equation (A.15) it follows:

$$V_t^0(Y) = (1 - \eta)(1 + \beta) \log Y + F \tag{A.21}$$

$$V_t^1(Y) = F - \eta \log Y + [1 + \beta(1 - \eta)] \log(Y + T_t) \tag{A.22}$$

Inserting equation (A.21) and (A.22) in equation (A.20) then gives:

$$\begin{aligned} (1 - \eta)(1 + \beta) \log(1 + x_t^L) = & - [1 + \beta(1 - \eta)] \, r_f + [1 + \beta(1 - \eta)] \\ & \times \mathrm{E}_t \log \left[1 + R_f + \frac{\left(1 + R_f\right) N_t}{Y} \left(R_{e,t+1} - R_f\right) \right] \end{aligned} \tag{A.23}$$

Using equation (A.3) and (A.17), this expression can be simplified to:

$$(1 - \eta)(1 + \beta) \log(1 + x_t^L) = [1 + \beta(1 - \eta)] \left(\mathrm{E}_t \, r_{T,t+1} - r_f\right) \tag{A.24}$$

Using equations (A.4), (A.6) and (A.7)-(A.9), it follows that:

$$\mathrm{E}_t \, r_{T,t+1} = r_f + \frac{1}{2} \frac{\bar{\mu}^2}{\sigma^2}$$

Inserting this expression in equation (A.24), we find:

$$x^L = \frac{1}{2} \frac{\bar{\mu}^2}{\sigma^2} \left[1 + \frac{\eta}{(1 - \eta)(1 + \beta)} \right] \tag{A.25}$$

Equation (A.25) and (3.22) determine the labour market distortion $x^D \equiv x - x^L$ mentioned in equation (4.9).

A.5 Labour supply elasticity

This section derives the uncompensated labour supply elasticity, as shown in Table 5.5. To that end, we first need to approximate $u_{1,t}$ and $u_{2,t+1}$. Taking logs of equation (2.2) for the first period gives:

$$\log u_{1,t} = \frac{1}{1-\rho} \underbrace{\log\Big\{\exp\left[\log(1-\eta)+(1-\rho)\log C_{1,t}\right]+\exp\left[\log\eta+(1-\rho)\log L_t\right]\Big\}}_{f\left(\log C_{1,t},\log L_t\right)}$$

Approximating $f(\cdot)$ with a first-order Taylor expansion around the point $(\log C_{1,t}, \log L_t) = (0,0)$ gives:

$$f\left(\log C_{1,t}, \log L_t\right) \approx (1-\rho)\left[(1-\eta)\log C_{1,t}+\eta\log L_t\right]$$

So, we have:

$$u_{1,t} \approx C_{1,t}^{1-\eta} L_t^{\eta} \tag{A.26}$$

Along the same lines, we can approximate equation (2.2) for the second period, resulting in:

$$u_{2,t+1} \approx C_{2,t+1}^{1-\eta} \tag{A.27}$$

Substituting equation (A.26) and (A.27) in (2.5a), gives:

$$C_{1,t} = \frac{1}{Z_t+1}\left(1-L_t\right)\left(1-\pi_{c,t}\right)Y$$

with:

$$Z_t = \left\{\beta\left(1+R_f\right)L_t^{\omega-1}\,\mathrm{E}_t\left(1+R_{T,t+1}\right)^{-\zeta}\left[\mathrm{E}_t\left(1+R_{T,t+1}\right)^{(1-\eta)(1-\theta)}\right]^{\nu}\right\}^{\frac{1}{\psi}}$$

and $\psi \equiv \rho-(1-\eta)(\rho-\gamma)$, $\zeta \equiv \rho-(1-\eta)(\rho-\theta)$, $\omega \equiv 1-\eta(\rho-\gamma)$ and ν defined as in the text. Dividing equation (2.5b) by equation (2.5a), we obtain:

$$\frac{C_{1,t}}{L_t} = \left(\frac{1-\eta}{\eta}I_t\right)^{\frac{1}{\rho}}$$

with $I_t \equiv \left(1-\pi_{c,t}\right)Y$ the price of leisure. Using this condition, we have:

$$\begin{aligned}
C_{1,t} &= \frac{\left(\frac{1-\eta}{\eta}I_t\right)^{\frac{1}{\rho}}}{\left(1+Z_t\right)\left(\frac{1-\eta}{\eta}\right)^{\frac{1}{\rho}}I_t^{\frac{1}{\rho}-1}+1}\\
L_t &= \frac{1}{\left(1+Z_t\right)\left(\frac{1-\eta}{\eta}\right)^{\frac{1}{\rho}}I_t^{\frac{1}{\rho}-1}+1}
\end{aligned} \tag{A.28}$$

From equation (A.28) we derive the labour supply elasticity ε, which is equal to:

$$\varepsilon \equiv \frac{d\left(1-L_t\right)}{dI_t}\frac{I_t}{1-L_t} = \frac{L_t}{1-L_t}\frac{1-\rho}{\rho}\frac{1+Z_t}{1+\frac{\gamma}{\psi}Z_t+\left(\frac{\eta}{1-\eta}\right)^{\frac{1}{\rho}}I_t^{1-\frac{1}{\rho}}}$$

so we have that $\varepsilon > 0$ if $\rho < 1$, $\varepsilon < 0$ if $\rho > 1$ and $\varepsilon = 0$ if $\rho = 1$.

Appendix B Accuracy of the simulations

Table B.1 shows the distribution of the income-equivalent variation for various sets of drawings. As the number of drawings increase, the standard deviation of the computed welfare gains declines from 0.21 (in case of 250 drawings) to 0.07 (in case of 2,000 drawings). The difference between the maximum and minimum value decreases from 0.67%-points to 0.24%-points.

Running more simulations increases the accuracy of the results, but it also increases the solvency time exponentially. Hence, we are forced to make a trade-off between accuracy on the one hand and a feasible solvency time on the other hand. Our results in the text are based on 2,000 simulations. For this number of drawings the difference between the maximum and minimum value of the income-equivalent variation will not exceed the 0.25%-points, which we consider as a reasonable margin of error.

Table B.1 Accuracy simulations

	Number of drawings			
	250	500	1000	2000
Mean	6.94	7.05	7.02	7.03
Median	6.89	7.00	7.04	7.03
Stdev	0.21	0.16	0.11	0.07
Min	6.61	6.75	6.78	6.92
Max	7.28	7.37	7.18	7.15

Reported statistics are based on 25 observations.