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Endogenous Growth and International Trade

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ENDOGENOUS GROWTH AND INTERNATIONAL TRADE

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1. Introduction

Endogenous or Schumpeterian growth models have been presented by Paul Romer (1986; 1990), Robert Lucas (1988), Paul Segerstrom, et. al. (1990), Philippe Aghion and Peter Howitt (1990), and Gene Grossman and Elhanan Helpman (1991). These models have great potential for understanding economic growth because they incorporate three stylized facts into the theory of economic growth: (1) the limits economies of scale impose on product differentiation; (2) innovation is endogenous; and (3) innovation has spillover effects.

The motivation for endogenous growth theory is that it gives a tentative explanation for three stylized facts about economic growth. The first is the point by Angus Maddison (1991) and Romer (1986) that per capita growth rates in the industrialized countries have increased over the past several centuries. The second is that free international trade appears to have a much larger impact on economic growth rates than can be accounted for by neoclassical trade theory (De Long and Summers, 1991; Gould and Ruffin, 1992). Third, government policies have a substantial impact on economic growth rates (Barro, 1991).

The wonderful new book by Grossman and Helpman (GH) synthesizes most of the contributions under one roof. It explains the new models simply and without excessive mathematical formalism. The economics, rather than the mathematics, is stressed. While the reader is treated to simple explanations, he is also confronted with a rather flat landscape without the signposts to distinguish the trivial from the profound.

The purpose of this paper is to clarify and slightly generalize the basic endogenous growth model. Romer (1990) and Grossman and Helpman (1991) assume that the distribution of knowledge around the world is irrelevant; I prove the basic theorems without this assumption. I shall try to state the basic results of the model in terms of lemmas, theorems, and corollaries in order to bring out as clearly as possible the assumptions on which each result is based. A single factor of production is assumed so that I ignore a major theme of Grossman and Helpman (1991) dealing with certain factors being well suited for research and development in a Heckscher-Ohlin model.

2. Monopolistic Competition and International Trade

The Grossman-Helpman model is really just a dynamic version of the Krugman model of monopolistic competition (Krugman, 1979; 1981). Accordingly, this section begins with a simple explanation of trade with monopolistic competition. The Krugman model was inspired by the seminal work of Avinash Dixit and Joseph Stiglitz (1977).

Suppose there are L people and workers. Each consumes n varieties of some differentiated product. There are n monopolistically competitive firms. The production of each variety is denoted by x_k. There are economies of scale in the production of each variety. The production function for each firm is

$$l_k = a + bx_k, \tag{1}$$

where l_k is the input needed to produce x_k units of the kth variety. Economies of scale exist because the parameter a is a positive constant; and marginal costs are constant because b is a positive constant. Demand and costs are perfectly symmetrical so that the production of each variety is the same. Thus, we can let $l_k = l$ and $x_k = x$. Since there are n varieties, full employment requires that $nl = L$ or

$$n = L/(a + bx). \tag{2}$$

The elasticity of demand facing each variety is exactly the same so that each of the n firms charge the same price,

$$p = bw/\alpha, \tag{3}$$

where w is the wage rate, bw is marginal costs, and $1/\alpha$ is the markup $(0 < \alpha < 1)$. Free entry implies that price drops to the level of average costs, so that

$$p = w(a + bx)/x = wa/x + wb. \tag{4}$$

Combining (3) and (4) we get

$$x = a\alpha/b(1-\alpha). \tag{5}$$

Substituting into (2) yields the equilibrium number of products

$$n = L(1-\alpha)/a. \tag{6}$$

The number of products increases with the size of the population, the higher the markup (i.e. the lower α) on products, and the lower fixed costs.

The utility function popularized by Dixit and Stiglitz (1977) can be used to justify the markup pricing rule:

$$U = (\Sigma_{k=0}^{n}(x_k)^{\alpha})^{1/\alpha}. \tag{7}$$

We can also interpret (7) as a production function in which the U is a final product and the x_k's are intermediate inputs (Ethier, 1982). It can be shown that the demand for good k is

$$x_k = E(p_k)^{-\epsilon}/[\Sigma_f(p_f)^{1-\epsilon}], \tag{8}$$

where $\epsilon = 1/(1-\alpha)$, E is expenditure, and p_k is the price of variety k. If n is very large, then ϵ is the price elasticity of demand.[1] Indeed, in the sequel we will assume that n defined on a continuum. The parameter α will be in the range $(0,1)$ when $\epsilon > 1$; it also measures the elastic values of the elasticity. The value of this parameter is that the famous equation, $MR = P(1-1/\epsilon)$ can be written as $MR = P\alpha$.

Suppose countries A and B share the same preferences and technology. If they engage in free trade in goods, the number of products in the world will be:

$$n^A + n^B = (L^A + L^B)(1-\alpha)/a. \tag{9}$$

Which varieties each country produces is indeterminate: only the total number produced in a country is known. Consumers in both countries purchase equal quantities of all varieties so that there is trade between the two countries: the basis for trade is economies of scale. The gain from trade consists of having more variety.

With the utility function in equation (7), the gain from trade is easily seen. Since all the x_k's are the same, the utility function can be written as

$$U = (n^A + n^B)^{1/\alpha}x. \tag{10}$$

Clearly, as the number of products increases, the utility of the representative consumer rises. We see from equation (5) that the size of firm, x, remains constant.

3. Endogenous Growth

The step between the Krugman model and the GH model is a short one. Basically, GH make two changes in the Krugman model. First, the fixed cost a in the production function in (2), $l = a + bx$, should reflect technological spillovers. Second, to take account of economic growth they consider a representative consumer maximizing an intertemporal utility function over an intertemporal budget constraint.

Technological Spillovers

Romer (1990) introduced two basic ideas, borrowed by Grossman and Helpman. First, there are technological spillovers so that innovations often lower the cost of making future innovations. Second, there are so many possible combinations of physical elements that the acquisition of new knowledge is not subject to diminishing returns to scale.

To capture these ideas, we can think of the number of products, n, as a measure of the stock of human knowledge. From now on we will treat n as a continuous variable. Let a^j be the amount of labor required in country j per unit of the flow of product development in country j, $\dot{n}^j$. Grossman and Helpman use the Romer (1990) assumptions:

$$a^j = c/n^j, \tag{11}$$

$$a^j = c/(n^A + n^B), \tag{12}$$

where c is a positive constant. In the case of (11), technological spillovers are limited to domestic innovators; in the case of (12), technological spillovers are additively global.

A key feature of (12) is that the distribution of world knowledge is irrelevant. Assumption (12) is the obvious extension of specification (11) for an autarkic economy. Thus, it is not surprising that under this assumption the global economy works like a single, but larger, integrated economy. However, there is nothing natural about (11). It is just as plausible to assume diminishing returns to knowledge in a single, isolated country. For example, China's self-imposed cultural isolation from the rest of the world between 1000 and 1500 A.D. appeared to even limit the usefulness of the spectacular innovations, such as printing and gunpowder (McNeill, 1963, Chapter X). From this perspective when the rest of the world is introduced it might well have a multiplicative effect on the productivity of knowledge in innovation. A more general assumption would be

$$a^j = a^j(n^A,\ n^B). \qquad (13)$$

We can now preserve the common property of both (11) and (12), namely, constant returns to all the knowledge inputs.[2] In other words, if knowledge doubles everywhere in the world, the labor required to produce a new good ("knowledge") would fall by one-half. This captures Romer's idea (Romer, 1990) that there are infinite discovery possibilities and yet allows for the possibility of a distinctive role for international knowledge as opposed to domestic knowledge. An increase in knowledge in one country, holding knowledge elsewhere constant, could still lead to diminishing returns because of, say, the costs of coordinating or comprehending knowledge in different countries. I show that assumption (12) is not required for any basic theorem, though it is used in the special case studied by Romer (1990) and Grossman and Helpman (1991).

Intertemporal Maximization

To dynamize the Krugman-Dixit-Stiglitz model of monopolistic competition, consider a pure consumption loan model in which a representative agent spends $E(t)$ and earns $Y(t)$ at time t. The agent maximizes

$$\mathcal{L} = \int_0^\infty e^{-\rho t}(\ln C(t))dt - \lambda\int_0^\infty e^{-R(t)}[E(t) - Y(t)]dt \tag{14}$$

where $C(t)$ is an index of consumption at time t, ρ is the rate of time preference, λ is the present value of the marginal utility of expenditure, $R(t) = \int r(t)dt$ (the cumulative discount factor), and $r(t)$ is the instantaneous (nominal) interest rate at time t. The index $C(t)$ might be utility or the production of some final product. Let $P(t) \equiv E(t)/C(t)$. At each t we can maximize (14) by simply differentiating inside the integrals, obtaining

$$\partial\mathcal{L}/\partial C(t) = e^{-\rho t}[1/C(t)] - \lambda e^{-R(t)}P(t) = 0,$$

implying

$$e^{R(t)-\rho t} = \lambda E. \tag{15}$$

Since λ is a constant, the time derivative of (15) implies

$$\dot{E}/E = r(t) - \rho. \tag{16}$$

This is intuitive because if $r(t) > \rho$, people will save for the future causing expenditures to rise over time.

This is a general equilibrium model: only relative prices matter. To achieve this a numeraire is required. GH make the assumption that $E(t) = 1$. This implies, from (16), that at all points in time, $r(t) = \rho$. Thus, ρ is the nominal rate of interest.

The Basic Model

To recapitulate: there is one productive factor--labor or human capital--that can be used to not only produce the differentiated product, but also the blueprint for a new variety. The a^j variable is the cost of a blueprint. Since new differentiated products require devoting labor to creating a blueprint ("R and D"), we shall sometimes refer to them as "high-tech" goods. It is assumed that it always takes exactly *1* unit of labor to produce *1* unit of any high-tech good for which there is a blueprint. Knowledge of this blueprint is proprietary.

There is one firm producing each variety of the good. Accordingly, the profit-maximizing condition in monopoly is simply $p = MC/\alpha$. Since a unit of each variety uses one unit of labor and labor earns the wage, w, the price of each variety is the same:

$$p = w/\alpha. \tag{17}$$

Due to symmetry the output of each firm is the same so that $x_k = x$ for all k. There are n firms, so each firm sells E/pn units. The profit of each firm is [using (17)]:

$$\pi = E(p-w)/pn = E(1-\alpha)/n \tag{18}$$

Since $\alpha = w/p$ is the cost of \$1 worth of product, the fraction $(1-\alpha)$ may be interpreted as the profit on one dollar of sales.

The rest of the model consists of conditions describing free entry, full employment, and one guaranteeing that there are no arbitrage profits from buying and selling firms. A two country world does not substantially complicate the model. A complete description follows.

The World Economy

Countries A and B have the same tastes (α and ρ) and it costs 1 unit of labor in either country to produce a unit of the high-tech good. As in the Krugman model, we abstract from comparative advantage and focus only on scale effects. World commodity markets are integrated, with country j producing n^j varieties of the differentiated good. Again, world expenditure E is normalized to unity. Expenditure on country j's goods is given by s^j, where $\Sigma s^j = 1 = E$. If there is international capital mobility, $r^A = r^B$. It follows from (16) that $r = \rho$ in each country at each point in time. If there is no international capital mobility, in the steady state the nominal interest rate is ρ in each country.

As before every good in country j sells for the same price because wages and the markup are the same. Thus, in country j, each variety sells for

$$p^j = w^j/\alpha. \tag{19}$$

Multiplying both sides of equation (8) by p^j, we can see that the share of world expenditure on country j's goods will be:

$$s^j = n^j(p^j)^{1-\epsilon}/[\Sigma_i n^i(p^i)^{1-\epsilon}]. \tag{20}$$

A closed economy can be examined just by letting $s^i = 1$.

The profit of each firm in country j is now

$$\pi^j = (s^j/p^j n^j)(p^j - w^j) = s^j(1-\alpha)/n^j, \tag{21}$$

which is obvious because s^j/n^j is the dollar sales of each firm in country j and $(1-\alpha)$ is the profit per dollar of sales.

Firms will engage in R&D if the value of a monopoly exceeds the cost of research and development. If v^j is the present value of the future stream of profits from a monopoly in country j, the free entry condition is described by:

$$w^j a^j \geq v^j, \tag{22}$$

where

$$\dot{\underline{n}}^j (w^j a^j - v^j) = 0, \tag{23}$$

implying the formation of new products or firms, $\dot{\underline{n}}^j > 0$, whenever

$$w^j a^j = v^j \tag{24}$$

Grossman and Helpman call (24) the **free entry condition.** This is appropriate because new products and entry are the same. Thus, if new products or new firms are prohibited, then it would be possible for $v^j > w^j a^j$.

There can be no arbitrage profits from an entrepreneur keeping the firm or selling the firm. The **no-arbitrage** condition is:

$$\pi^j + \dot{\underline{v}}^j = \rho v^j, \tag{25}$$

where the left-hand-side is the gain from keeping the firm and the right-hand-side is the gain from selling the firm each period.

Finally, there must be full employment. The supply of labor, L^j, can produce blueprints or goods. The full employment condition is:

$$a^j \dot{\underline{n}}^j + s^j / p^j = L^j. \tag{26}$$

This can be seen by remembering each blueprint requires a^j units of labor and $\dot{\underline{n}}^j$ blueprints are created at each moment; and each unit of the high-tech product produced requires 1 unit of labor and there are s^j / p^j units of these.

It should be noted that in the interpretation of this model L^j can be interpreted as the stock of human capital (Romer, 1990). When there is only one factor, which can be used to produce either new knowledge or old goods, then we are dealing with the allocation of human capital between different activities.

<u>Workings of the Model</u>

This is a model in which we have trade in differentiated products. There are economies of scale because to invent a new product requires $w^j a^j$ worth of fixed costs. Each country specializes in some non-overlapping subset of the $n = n^A + n^B$ available goods.

If we assume that there are technological spillovers, economies will grow over time because fixed costs fall with the number of products, increasing the entry of new firms. We seek to examine the steady-state properties of the model.

It is convenient to summarize the basic model:

MARKUP PRICING	$p^j = w^j/\alpha$	**(19)**
EXPENDITURE IN COUNTRY J	$s^j = n^j (p^j)^{1-\epsilon} / [\Sigma_i n^i (p^i)^{1-\epsilon}]$	**(20)**
FIRM PROFITS IN COUNTRY J	$\pi^j = s^j (1-\alpha)/n^j$	**(21)**
FREE ENTRY	$w^j a^j = v^j$	**(24)**
NO ARBITRAGE PROFITS	$\pi^j + \dot{v}^j = \rho v^j$	**(25)**
FULL EMPLOYMENT	$a^j \dot{n}^j + s^j/p^j = L^j$	**(26)**

We begin with a set of lemmas that hold when the two countries do not face the same technology for innovation. It is instructive--but not crucial to the remaining argument--to begin with the fairly obvious lemma that the high wage country has lower profits.

Lemma 1. $\pi^A > (<)\ \pi^B$ *iff* $w^A < (>)\ w^B$.

Proof. Substitute (19) into (20) and (20) into (21). So

$\pi^j = (w^j/\alpha)^{1-\epsilon}(1-\alpha)/[n^A(w^A/\alpha)^{1-\epsilon} + n^B(w^B/\alpha)^{1-\epsilon}]$

Since $\epsilon > 1$, a rise in w^j lowers π^j.

Remark. This result is useful in dealing with models in which the innovation technology a^j differs between countries and holds on or off any steady-state growth path (see Grossman and Helpman, p. 224).

Definition. A steady-state is where the rate of growth $g^j = \dot{n}^j/n^j$, the allocation of labor to research and production, and the market share s^j are constant.

Lemma 2. In a steady-state, the wage rate, w^j is constant.

Proof. From the full employment equation and the pricing equation we can write

$$a^j\dot{n}^j + s^j\alpha/w^j = L^j. \qquad (27)$$

Since in a steady-state s^j and the allocation of labor to production must be constant, w^j must also be constant.

Lemma 3. In a steady-state, n^i/n^j is constant.

Proof. From the expenditure share equation, (20), and the pricing equation it follows that

$$s^A/s^B = (n^A/n^B)(w^A/w^B)^{1-\epsilon}.$$

Therefore, since the s^j's and the w^j's are constant in a steady-state, so is the ratio n^A/n^B.

Remark. The preceding three lemmas hold without any assumption concerning the nature of technological spillovers. These are very weak results. For example, lemma 3 is consistent with equal growth rates in both countries or unequal growth rates when the shares are either zero or one. For deeper results we introduce the assumption that the productivity of innovation is subject to constant returns to scale.

Lemma 4. In a steady-state, if $a^j = a^j(n^A, n^B)$ is homogeneous of degree -1, then $n^j a^j$ is constant.

<u>Proof</u>. Homogeneity of degree -1 implies that

$$n^j a^j(n^A, n^B) = a^j(1, n^i/n^j). \tag{28}$$

In the steady-state, n^i/n^j (lemma 3) is constant; thus, (28) implies that $n^j a^j$ is constant.

Lemma 5. In a steady-state, if $a^j(.)$ is homogeneous of degree -1, the aggregate value of the stock market of all firms, $v^j n^j$, is constant and equals $s^j(1-\alpha)/(g^j + \rho)$.

<u>Proof</u>. By free-entry, $w^j a^j n^j = v^j n^j$. Since w^j and $n^j a^j$ are constant in the steady-state (lemmas 2 and 4), so is $v^j n^j$. Now divide both sides of (25) by v^j and add $\dot{n}^j/n^j$ to both sides and using the definition of profit in (21) we have

$$\dot{v}^j/v^j + \dot{n}^j/n^j = \rho + g^j - s^j(1-\alpha)/v^j n^j. \tag{29}$$

The market share, s^j, is constant in a steady-state. The left-hand-side of (29) is the proportionate change in $v^j n^j$; therefore, the left-hand-side is zero in the steady-state. Thus,

$$v^j n^j = s^j(1-\alpha)/(\rho + g^j). \tag{30}$$

Remark. This is a useful result. It is intuitive because $s^j(1-\alpha)$ is the total profit of firms in country j and $(\rho + g^j)$ is the rate of interest plus rate of decline in profits. In effect, we can think of g^j as the rate of obsolescence because the profits of old firms decline as new firms enter. The stock market value of all firms is just a present value of profits when there is perfect foresight about the obsolescence rate of decline in profits.

We will now consider the implications of an additional assumption: that $a^j(n^A, n^B) = a(n^A, n^B)$ for $j = A, B$, that is, the knowledge production function is the same across countries.

Theorem 1. If $a^j(.) = a(.)$ for $j = A, B$ and is subject to constant returns (homogenous of degree minus one), the steady-state growth rates are the same, that is, $g^A = g^B = g$.

<u>Proof.</u> Follows immediately from lemma 4. Since $n^j a^j$ is constant, it follows that $(\hat{\underline{x}} = (dx/dt)/x)$

$$\hat{\underline{n}}^j + \hat{\underline{a}}^j = 0$$

If $a^A = a^B$, it follows that $g^A = g^B$.

Corollary 1. If $a^j(.) = a(.)$ for $j = A, B$ and is subject to constant returns , then $s^A/v^A n^B = s^B/v^B n^B$.

<u>Proof.</u> Follows from lemma 5 and Theorem 1 by simply noting that $v^j n^j/s^j = (1-\alpha)/(g + \rho)$ for $j = A, B$.

Remark. This results simply says is that if all production functions are everywhere the same and subject to constant returns to scale, the stock market value of each country is proportional to the dollar value of sales in each country.

Corollary 2. If $a^j(.) = a(.)$ for $j = A, B$ and is subject to constant returns , then in the steady-state, $n^A/n^B = L^A/L^B$.

<u>Proof.</u> The full employment condition (26) may be written as, using the pricing equation (19),

$$a\dot{\underline{n}}^j + s^j\alpha/w^j = L^j.$$

Using theorem 1 that under identical innovation technologies $g^A = g^B = g$, and the free entry condition (24), we can rewrite the above equation as

$$n^j ag + \alpha s^j a/v^j = L^j. \tag{31}$$

Dividing (31) by n^j we get

$$ag + a\alpha s^j/v^j n^j = L^j/n^j. \tag{32}$$

By the fundamental corollary 1 the left-hand-sides of (32) are the same for $j = A, B$.

Remark. Corollary 2 gives a soothing message. If production and innovation technologies are the same, and doubling the numbers of goods in each country cuts innovation costs by one-half, the number of varieties produced by each economy will be exactly proportional to the size of each economy. It is interesting because it states that in the steady-state the distribution of human knowledge around the world is the same as the distribution of human capital.

Theorem 2. If $a^j(.) = a(.)$ for $j = A, B$ and is subject to constant returns , then in the steady-state there is factor price equalization, $w^A = w^B$.

Proof. By the fundamental corollary 1 $s^A/s^B = v^A n^A/v^B n^B$. Using the free entry condition we can write this as

$$s^A/s^B = w^A n^A/w^B n^B. \tag{33}$$

By inserting (19) into (20), taking the ratio we also get

$$s^A/s^B = n^A (w^A)^{\underline{1-\epsilon}}/n^B (w^B)^{\underline{1-\epsilon}} \tag{34}$$

Equating (33) and (34) we get

$$w^A/w^B = (w^A/w^B)^{\underline{1-\epsilon}}. \tag{35}$$

For $\epsilon > 1$, as it must be, (35) holds if and only if $w^A = w^B$.

Remark. As in the standard factor price equalization theorem, it is necessary to assume not only identical production functions but constant returns to inputs and knowledge. In the case where $a^j = c/n^j$, if $n^A > n^B$, then it is cheaper to carry out innovations in economy A. There will be uneven rates of innovation in this case and factor price equalization will break down. But, evidently, factor price equalization will also break down if $a^j(.) = a(.)$ for $j = A, B$ but is not homogeneous of degree minus one.

Theorem 3. If $a^j(.) = a(.)$ for $j = A, B$ and is subject to constant returns, then the steady state growth rate

$$g = \dot{\underline{n}}/n = (1-\alpha)\bar{\underline{L}}/a(\theta^A, \theta^B) - \alpha\rho,$$

where $\bar{\underline{L}} = L^A + L^B$ and $\theta^j = L^j/\bar{\underline{L}}$.

<u>Proof.</u> Writing down the full employment equation (26) and using the markup (19), free entry condition (24), and theorem 1 that $g^A = g^B = g$ yields:

$$g = L^j/a(n^A, n^B)n^j - s^j\alpha/v^j n^j. \tag{36}$$

Substituting the aggregate stock market value from (30) into (36) and rearranging yields:

$$g = (1-\alpha)L^j/a(n^A, n^B)n^j - \alpha\rho. \tag{37}$$

From corollary 2 it follows that $L^j/\bar{\underline{L}} = n^j/n$. Thus, (37) becomes

$$g = (1-\alpha)\bar{\underline{L}}/na(n^A, n^B) - \alpha\rho. \tag{38}$$

Since $a(.)$ is homogeneous, we can let $\theta^j = n^j/n = L^j/\bar{\underline{L}}$; thus

$$g = (1-\alpha)\bar{\underline{L}}/a(\theta^A, \theta^B) - \alpha\rho, \tag{39}$$

where, of course, $\theta^A + \theta^B = 1$.

Remark. Equation (39) is the basic endogenous growth equation when innovation costs are the same the world over and subject to constant returns. It is obvious that the growth rate, g, must be unique. If $a(n^A, n^B) = c/(n^A + n^B)$, as in Grossman and Helpman, so that $a(\theta^A, \theta^B) = c/1 = c$, then we obtain the special case derived by Romer (1990) and Grossman and Helpman (1991):

$$g = (1-\alpha)\tilde{\underline{L}}/c - \alpha\rho. \tag{40}$$

In a single isolated economy,

$$g^j = (1-\alpha)L^j/c - \alpha\rho. \tag{41}$$

Clearly, mostly because of the broadening of knowledge spillovers, an open economy substantially increases a country's growth rate, which is not true in neoclassical trade theory.

A study of equation (39) or (40) shows that the growth rate is higher the higher the profit per dollar of sales, since the expected reward from a monopoly will be higher; the larger the population capable of engaging in R&D, since more innovation lowers innovation costs due to spillover effects; the smaller the rate of time preference, since a thrifty population faces smaller interest rates and future profits have a higher present value.

The difference between (39) and (40) is in the role of the distribution of human knowledge or (in the steady-state) human capital (corollary 2). In the Grossman-Helpman-Romer equation, (40), the distribution of human capital between countries A and B is irrelevant to economic growth. But in the more general model, equation (39), the distribution of human knowledge and, therefore, human capital, matters. If there is a globally-known production function for new knowledge, it may well be that redistributing human capital from a large country to a small country would raise the world rate of growth. For example, if $1/a(\theta^A, \theta^B) = (\theta^A\theta^B)^{1/2}$, then the growth rate would be maximized by letting endowments of the two countries be equal. This is clearly an empirical question.

The theory implies that if two separate economies are not closely linked they will have quite different growth rates and, therefore, their levels of income might diverge through time. But if two separate economies become suddenly integrated, their growth rates will both increase to a common level.

Finally, we inquire into the condition for the economy to stagnate.

Corollary 3. The economy will stagnate ($g = 0$) if the real interest rate is less than the rate of time preference.

Proof. We know that $P(t)C(t) = 1$, where $C(t)$ is utility or some index of final consumption. From equation (10) we can write $C = n^{1/\alpha}x$, where x is a constant, so that

$$\hat{P} + \hat{C} = \hat{P} + g/\alpha = 0.$$

The real interest rate is $\delta = \rho - \hat{P} = \rho + g/\alpha$. Using (39) we have

$$\delta = (1-\alpha)\bar{L}/\alpha a(\theta^A, \theta^B). \tag{42}$$

Equation (39) can be rewritten as

$$g/\alpha = [(1-\alpha)\bar{L}/\alpha a(\theta^A, \theta^B) - \rho] = [\delta - \rho]. \tag{43}$$

Clearly, if $\delta \leq \rho$, the economy will stagnate at the initial stock of knowledge, n.

Remark. Equation (43) is not surprising. In a Solow model with the same intertemporal utility function as (14), the growth path for consumption is simply equal to the real interest rate minus the discount rate (Grossman and Helpman, 1991, p. 33). This suggests that if the stock of human capital is growing over time, so should the real interest rate! This is equivalent to the proposition, discussed by Romer (1986), that over time the rate of growth of per capita income should be accelerating.

4. Schumpeterian Economic Growth

We have seen that when new firms enter, the profits of old firms are reduced. This is a mild version of the creative destruction models of Aghion and Howitt (1990), Segerstrom, et. al. (1990), and Grossman and Helpman (1991, Ch. 12).

Imagine that instead of horizontal product differentiation, in which consumers buy all varieties, consumers only buy the state-of-the art product from each industry. Each industry is capable of producing different qualities at the same marginal cost of w. But consumers are indifferent between different qualities when they sell for the same quality-adjusted price. It is assumed that firms engage in Bertrand competition, so that the highest-quality firm simply charges epsilon less than the price

$$p = \lambda w, \tag{44}$$

where $\lambda > 1$ measures the quality advantage. The state-of-the-art firm drives all other competitors out of business; accordingly, its profits would be constant as long as it holds its monopoly power. Thus, the first equation that is affected is the profit equation, (21).

But the firm faces the risk that an innovator will discover the next generation of the state-of-the-art quality. The only other equation that is affected in the basic model is the no-arbitrage condition, which becomes

$$\pi^j + \dot{v}^j = (\iota + \rho)v^j, \tag{45}$$

where ι is the risk of losing one's monopoly position.

With sufficiently careful assumptions, equations (44) and (45) result in the same basic form of the Romer-Grossman-Helpman endogenous growth equation, (40). For example, ι is the probability that a researcher will discover the next generation of each good. If we assume that ι also measures research intensity, so that $a\iota$ measures the labor devoted to R&D, the free entry condition simply becomes $wa\iota = \iota v$ because $wa\iota$ is the cost of invention and ιv is the expected value of a new product. Since the ι cancels, the free entry condition is the same as before. Except for notational differences, the full employment equation is also the same as before.

In terms of the final growth equation, the model is equivalent to the horizontal product differential model because in that model the growth of new products measures the extent to which profit falls. In the rising product quality model, the firm faces the risk that profits will remain constant and then, at some time in the future, drop to zero with probability ι. The ι and the g play the same role in the two theories: both discount future profits. In the Schumpeterian model, the aggregate stock market value = (total industry profit)/$(\rho + \iota)$. This is the same form as (30); accordingly, the Schumpeterian model has the same final equation.

The Schumpeterian model is useful in analyzing the product cycle (Segerstrom, et. al, 1990; Grossman and Helpman, Ch. 12). At different times the lead product goes back and forth between countries.

IV. Conclusions

International trade theorists have for many years loosely discussed how trade is advantageous for the new ideas and cultural contacts that accompany international business. The gains from these contacts may be of a different order of magnitude than the traditional gains from specialization (comparative advantage or economies of scale). Endogenous growth theory appears to offer to trade theorists the possibility of formalizing the gains from new ideas (Rivera-Batiz and Romer, 1991). So far the mechanism is only that knowledge in the rest of the world may cheapen the cost of creating new knowledge. It remains to study the microeconomic details of this mechanism.

This paper has assumed that there is only one factor. But as Grossman and Helpman (1991) stress it is theoretically possible for factor endowments to play a role in determining economic growth in multi-factor, multi-sector models. If trade causes specialization in goods that do not promote R&D, then with different knowledge functions different countries may have different growth rates. But the empirical importance of this Heckscher-Ohlin effect remains to be seen.

Footnotes

1. To derive (8), first calculate the marginal utility from (7):

$$\partial U/\partial x_{\underline{h}} = U_{\underline{h}} = (\Sigma x_{\underline{h}}^{\alpha})^{(1/\alpha - 1)} x_{\underline{h}}^{(\alpha-1)}. \tag{i}$$

Thus, setting

$$U_{\underline{h}}/U_{\underline{k}} = (x_{\underline{h}}/x_{\underline{k}})^{(\alpha-1)} = p_{\underline{h}}/p_{\underline{k}}. \tag{ii}$$

Let $\epsilon = 1/(1-\alpha)$. From (ii) we can then write

$$x_{\underline{h}} = (p_{\underline{h}}/p_{\underline{k}})^{\epsilon} x_{\underline{k}}. \tag{iii}$$

Substitute this in the budget constant $E = \Sigma_{\underline{h}} p_{\underline{h}} x_{\underline{h}}$, yielding

$$E = p_{\underline{k}}^{\epsilon} x_{\underline{k}} \Sigma p_{\underline{h}}^{(1-\epsilon)}. \tag{iv}$$

2. Grossman and Helpman (1991, p. 307) use a similar assumption when they examine the implications of imitation when the South can produce knowledge under constant returns.

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