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Sticky Information and Determinacy

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Alexander Meyer-Gohde*†

Abstract

The infinite-dimensional sticky-information Phillips curve is cast as a finite-dimensional time-varying system of difference equations in order to directly assess determinacy in the model with demand given by the forward-looking IS equation and monetary policy by an interest rate rule. An equivalence to the model without lagged expectations holds (albeit tenuously) for the particular specification and a common truncation method produces spurious determinacy.

JEL classification: C62; E31; E43; E52
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1 Introduction

The sticky-information model of Mankiw and Reis (2002),[1] with an infinite regress of lagged expectations, cannot be brought into the canonical form of Blanchard and Kahn (1980) to assess determinacy (existence of a unique, bounded equilibrium). I analytically derive the determinacy properties for a standard New Keynesian model with sticky information by recasting the system as a time-varying system of difference equations. I show that for standard dynamic IS demand and the interest rate rule examined here, the parameter restriction to ensure determinacy is the same as would be obtained by examining the model without lagged expectations. Such an equivalence need not hold in general, however, as the non-singularity constraints and finite variational bounds satisfied by the particular model analyzed here need not be satisfied by other models with lagged expectations. With analytical results in hand, I conclude by demonstrating that a standard truncation method produces spurious determinacy.

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[1]For a recent overview of applications and extensions of sticky information, see Mankiw and Reis (Forthcoming).

2 A Sticky-Information Model

A basic sticky-information New Keynesian model can be written as[2]

(1) $$y_t = E_t[y_{t+1}] - a_1 R_t + a_1 E_t[\pi_{t+1}]$$

(2) $$\pi_t = \frac{1-\lambda}{\lambda}\xi y_t + (1-\lambda)\sum_{i=0}^{\infty}\lambda^i E_{t-i-1}[\pi_t + \xi(y_t - y_{t-1})]$$

where y_t is the output gap, π_t inflation , and R_t the nominal interest rate. Equation (1) is a dynamic IS-curve and (2) is Mankiw and Reis's (2002) sticky-information Phillips curve. Here, a_1 and ξ are assumed positive,[3] and $0 < 1-\lambda < 1$ is a firm's probability of receiving an information update.

Monetary policy will be described by the following rule for the interest rate to close the model

(3) $$R_t = \phi_R R_{t-1} + \phi_\pi[(1-\psi_\pi)E_t[\pi_{t+1}] + \psi_\pi \pi_t] + \phi_y y_t$$

where $0 \leq \phi_R < 1$ describes the degree of interest-rate smoothing, $0 \leq \phi_\pi < \infty$ of inflation targeting, and $0 \leq \phi_y < \infty$ of output-gap targeting. The coefficient $0 \leq \psi_\pi \leq 1$ nests contemporaneous inflation targeting ($\psi_\pi = 1$) and inflation forecast targeting ($\psi_\pi = 0$) into the rule.

3 Endogenous Fluctuations and Determinacy

Without loss of generality, I abstract from exogenous driving forces.[4] By examining the infinite moving average representation of the model in response to endogenous fluctuations (i.e., to sunspot shocks), the system of difference equations originating from the model of sticky information will yield a non-autonomous or time-varying system of homogenous non-stochastic difference equations.

Consider a sunspot shock that occurs at time 0 and denote with x_t the response of the variable x in period t to the sunspot. The response of the model, defined by (1), (2), and (3), is given by

[2]See, e.g., Trabandt (2007) for a first-principles derivation analogous to Woodford (2003, Ch. 4).

[3]See, e.g., Woodford (2003, pp. 160–164 & 243–245)

[4]With bounded exogenous driving forces, the boundedness of the particular solution will rest on that of the homogenous solution. See Taylor (1986), Woodford (2003, pp. 252, & 636) and Pituk (2002).

the system of deterministic time-varying difference equations

(4) $$y_t = y_{t+1} - a_1 R_t + a_1 \pi_{t+1}$$

(5) $$\lambda^{t+1}\pi_t = \left(1-\lambda^{t+1}\right)\xi y_t - \xi\lambda\left(1-\lambda^t\right)y_{t-1}$$

(6) $$R_t = \phi_R R^R_{t-1} + \phi_\pi\left[(1-\psi_\pi)\pi_{t+1} + \psi_\pi\pi_t\right] + \phi_y y_t$$

with $R_{-1} = 0$, where (4) and (6) correspond straightforwardly to (1) and (3), and where (5) follows from (2) after noting that both the response of variables and expectations dated before 0 are zero.[5]

Equation (5) gives the time-varying difference equation described by the sticky-information Phillips curve (2). As $t \to \infty$, the foregoing converges to the "unrestricted" perfect-foresight version of the model, given by $y_t = \lambda y_{t-1}$ as all outdated information sets are updated. The lagged expectations serve to transition the Phillips curve from having a positive trade-off at time 0, given by $\lambda\pi_0 = (1-\lambda))\xi y_0$, to being vertical with no trade-off in the limit. This contrasts with the sticky-price Phillips curve, which always posits the same dynamic trade-off between inflation and output: $\pi_t - \beta\pi_{t+1} = \kappa y_t$ under perfect foresight.[6] Although the model itself is time invariant, the response of a variable under sticky information is time varying: the equilibrium relationships between the responses of endogenous variables to a shock change as the shock becomes more outdated. The model will be determinate (sunspots can be ruled out), if the only sequence of impulse responses to a sunspot shock that remains bounded is the trivial sequence of zeros for all variables at all horizons; i.e., if the only bounded response of endogenous variables to sunspots is no response at all.

Lagging (5) forward and noting the additional initial condition yields the following system

(7) $$\begin{bmatrix} \lambda^{t+2} & -\xi\left(1-\lambda^{t+2}\right) & 0 \\ a_1 & 1 & -a_1 \\ -\phi_\pi(1-\psi_\pi) & 0 & 1 \end{bmatrix}\begin{bmatrix}\pi_{t+1}\\ y_{t+1}\\ R_t\end{bmatrix} = \begin{bmatrix} 0-\xi\lambda\left(1-\lambda^{t+1}\right) & 0 & \\ 0 & 1 & 0 \\ \phi_\pi\psi_\pi & \phi_y & \phi_R \end{bmatrix}\begin{bmatrix}\pi_t\\ y_t\\ R_{t-1}\end{bmatrix}$$

for $i = 0, 1, 2, \ldots$, with $R_{-1} = 0$ and $\lambda\pi_0 = (1-\lambda)\xi y_0$.

The foregoing system has two initial conditions but three variables. As the system is homogenous, one solution is $\pi_t, y_t, R_t = 0$, $t = 0, 1, \ldots$, but it may not be the only bounded solution. Different potential solutions can be indexed by different values for the "missing" initial condition—i.e., a value for y_0 or π_0; if the system (7) is stable, then it will remain bounded for

[5]See, likewise, Mankiw and Reis's (2002) Appendix.

[6]The notation follows Woodford (2003).

any such finite initial condition and, thus, the sunspots cannot be ruled out. If the system, however, has a one-dimensional unstable manifold that can be associated with this condition, then the boundedness requirement will provide the missing initial condition and sunspots can be ruled out.

Proposition 3.1. *The model given by (1), (2), and (3) is determinate iff* $\left|\frac{\phi_R+\phi_\pi\psi_\pi}{1-\phi_\pi(1-\psi_\pi)}\right| > 1$.[7]

Proof. The system of difference equations in (7) can be inverted to yield

$$\begin{bmatrix}\pi_{t+1} & y_{t+1} & R_t\end{bmatrix}' = (C+D(i))\begin{bmatrix}\pi_t & y_t & R_{t-1}\end{bmatrix}' \tag{8}$$

so long as $\phi_\pi(1-\psi_\pi) \neq 1 + \frac{\lambda^{t+2}}{(1-\lambda^{t+2})\xi a_1}$, $\forall t \geq 0$. Where

$$C = \begin{bmatrix} \frac{\phi_\pi\psi_\pi}{1-\phi_\pi(1-\psi_\pi)} & \frac{a_1\phi_y+1-\lambda}{a_1(1-\phi_\pi(1-\psi_\pi))} & \frac{\phi_R}{1-\phi_\pi(1-\psi_\pi)} \\ 0 & \lambda & 0 \\ \frac{\phi_\pi\psi_\pi}{1-\phi_\pi(1-\psi_\pi)} & \frac{a_1\phi_y+(1-\lambda)\phi_\pi-\phi_\pi\psi_\pi}{a_1(1-\phi_\pi(1-\psi_\pi))} & \frac{\phi_R}{1-\phi_\pi(1-\psi_\pi)} \end{bmatrix}$$

$$D(i) = \alpha(i)D$$

$$\alpha(i) = \frac{\lambda^{i+2}}{(1-\lambda^{i+2})a_1\xi(1-\phi_\pi(1-\psi_\pi))+\lambda^{i+2}}$$

$$D = \begin{bmatrix}\frac{-1}{1-\phi_\pi(1-\psi_\pi)} & a_1 & \frac{-\phi_\pi(1-\psi_\pi)}{1-\phi_\pi(1-\psi_\pi)}\end{bmatrix}' \begin{bmatrix}\phi_\pi\psi_\pi & 1-\lambda+a_1[\phi_y-\xi(1-\lambda)(1-\phi_\pi(1-\psi_\pi))] & \phi_r\end{bmatrix}$$

Using the ratio test, $\sum_{i=0}^{\infty}|\alpha(i)| < \infty$, and thus

$$\sum_{i=0}^{\infty}||D(i)|| \leq ||D||\sum_{i=0}^{\infty}|\alpha(i)| < \infty \tag{9}$$

Noting (9) and following Ludyk's (1985) Theorem 3-29, the system in (8) is stable if C is stable and, from Ludyk's (1985) Theorem 3-12, will remain bounded for any bounded initial conditions.

Examining the eigenvalues of C, $z_1 = 0$, $z_2 = \lambda$, $z_3 = \frac{\phi_R+\phi_\pi\psi_\pi}{1-\phi_\pi(1-\psi_\pi)}$, the first two of which are necessarily inside the unit circle. If $|z_3| < 1$, then (8) is stable for any set of bounded initial conditions. In this case, the boundedness condition will be insufficient to pin down the missing initial condition and one cannot rule out sunspot equilibria (i.e., the model is indeterminate).

Should $|z_3| > 1$, then z_3 is a simple dominant eigenvalue. Noting (9) and following Pituk's (2002) Theorem 1, solutions of (8) are related asymptotically to solutions of the system $x_{t+1} = Cx_t$ via $\lim_{t\to\infty}\left(z_3^{-t}\begin{bmatrix}\pi_t & y_t & R_{t-1}\end{bmatrix}'\right) = \gamma\Xi$, where γ is a constant and Ξ is the eigenvector of C corresponding to z_3. The eigenvector is $\begin{bmatrix}1 & 0 & 1\end{bmatrix}'$ and as $|z_3| > 1$, π_t and R_t will be unbounded

[7] The analysis will abstract from cases with eigenvalues on the unit circle, following Woodford (2003, p. 254).

unless $\gamma = 0$, which, following Pituk's (2002) Theorem 3, would require $\begin{bmatrix}\pi_0 & y_0 & R_{-1}\end{bmatrix}' = \begin{bmatrix}0 & 0 & 0\end{bmatrix}'$. Hence the requirement of boundedness provides the system with an additional restriction that rules out the sunspot equilibria (i.e., the model is determinate).

Should $\phi_\pi(1-\psi_\pi) = 1 + \frac{\lambda^{t+2}}{(1-\lambda^{t+2})\xi a_1}$ for some t (say τ), then (7) cannot be brought into the form of (8) for all t. The singularity of the coefficient matrix at τ provides one linear restriction, which, when combined with the two original restrictions, implies that $R_{t-1} = y_t = \pi_t = 0$, $t \leq \tau$. The recursion then delivers two new initial conditions, $\left(1-\lambda^{\tau+2}\right)\xi y_{\tau+1} = \frac{\lambda^{\tau+2}}{\phi_\pi} R_\tau$ and $\pi_{\tau+1} = \frac{\xi(1-\lambda)}{\lambda^{\tau+2}(1-\xi a_1)+\lambda a_1 \xi} y_{t_\tau+1} + \frac{a_1\xi\left(\lambda-\lambda^{\tau+2}\right)}{\lambda^{\tau+2}(1-\xi a_1)+\lambda a_1\xi} R_{t_\tau}$, which result in a non-singular recursion for $i = i_\tau + 1, i_\tau + 2, \ldots$, with the same stability characteristics as in the recursion without the singularity. □

4 Equivalence and Specious Determinacy

It is conspicuous that the parameter bound for determinacy is independent of the parameters outside of the interest rate rule. This independence is related to the equivalence of the determinacy bounds to those in a frictionless version of the model. To see this, note that in the absence of lagged expectations, (5) reduces to $y_t = \lambda y_{t-1}$, which is necessarily stable. Thus, determinacy of the system without lagged expectations can be ascertained by means of the following system

$$R_t = E_t\left[\pi_{t+1}\right]$$

$$R_t = \phi_R R_{t-1}^R + \phi_\pi\left[(1-\psi_\pi)E_t\left[\pi_{t+1}\right] + \psi_\pi \pi_t\right]$$

Following Blanchard and Kahn (1980), $\left|\frac{\phi_R+\phi_\pi\psi_\pi}{1-\phi_\pi(1-\psi_\pi)}\right| > 1$ is required for determinacy. This is, of course, the same bound as in (3.1). This equivalence is, however, more tenuous than one might infer from Wang and Wen (2006). As can be seen in the proof of (3.1), both singular coefficient matrices and infinite variation—if (9) does not hold—can cause this equivalence to break down: neither of which can be *a priori* ruled out.

Generally,[8] the sticky-information model needs to be truncated when a particular solution is sought. The truncation used by Trabandt (2007) and Andrés, López-Salido, and Nelson (2005), which eliminates the tail of the distribution of lagged expectations, leads to a specious determinacy region for an otherwise indeterminate monetary policy rule.

[8] See Meyer-Gohde (2010) for an overview.

Equation (2) is truncated at some $I < \infty$ as

$$\pi_t = \frac{1-\lambda}{\lambda}\xi y_t + (1-\lambda)\sum_{i=0}^{I-1}\lambda^i E_{t-i-1}\left[\pi_t + \xi\left(y_t - y_{t-1}\right)\right]$$

To simplify the calculations, consider pure inflation forecast targeting by the monetary authority: the special case of $\psi_\pi = \phi_R = \phi_y = 0$ in (3). The system can now be written in matrix form as

$$0 = \sum_{i=0}^{I} A_i E_{t-i}\left[Y_{t+1}\right] + \sum_{i=0}^{I} B_i E_{t-i}\left[Y_t\right] + \sum_{i=0}^{I} C_i E_{t-i}\left[Y_{t-1}\right]$$

where $Y_t = \begin{bmatrix}\pi_t & y_t & R_t\end{bmatrix}'$. This is the canonical form of Meyer-Gohde (2010) and determinacy can be ascertained by examining the eigenvalues (Γ) of the matrix pencil[9]

$$\begin{bmatrix}\sum_{i=0}^{I} C_i & \sum_{i=0}^{I} B_i \\ 0 & I\end{bmatrix} - \Gamma\begin{bmatrix}0 & -\sum_{i=0}^{I} A_i \\ I & 0\end{bmatrix}$$

the determinate of which yields

$$\left[\left(\lambda^{I+1} - \left(\lambda - \lambda^{I+1}\right) a_1\xi\left(\phi_\pi - 1\right)\right) - \Gamma\left(\lambda^{I+1} - \left(1 - \lambda^{I+1}\right) a_1\xi\left(\phi_\pi - 1\right)\right)\right]\Gamma^3 = 0$$

The two "missing" eigenvalues are called "infinite." Of the remaining four eigenvalues, it is trivial to see that three are equal to zero. Thus, determinacy will rest upon the final eigenvalue

$$\Gamma = \frac{\lambda^{I+1} - \left(\lambda - \lambda^{I+1}\right) a_1\xi\left(\phi_\pi - 1\right)}{\lambda^{I+1} - \left(1 - \lambda^{I+1}\right) a_1\xi\left(\phi_\pi - 1\right)}$$

being outside the unit circle. This holds if

$$1 < \phi_\pi < 1 + \frac{2\lambda^{I+1}}{a_1\xi\left(1 + \lambda - 2\lambda^{I+1}\right)}$$

This requires the interest-rate rule to satisfy the Taylor Principle and to not react "too strongly" to expected inflation. Proposition 3.1, with $\psi_\pi = \phi_R = \phi_y = 0$, states that the true, non-truncated model is necessarily indeterminate. As the tail of the distribution of lagged expectations *never* adjust, the truncation scheme causes the long-run Phillips curve to become non-vertical like in the standard sticky-price model and leads to the emergence of a specious determinacy region.

[9]The lagged expectations also have to be resolvable—i.e., Meyer-Gohde's (2010) Equation (12) has to be invertible. This held with the non-truncated model and thus follows here as both models are identical up to the truncation.

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