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**Incentives for Anticompetitive Behavior
by Public Enterprises**

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Executive Summary

We examine the incentives that public enterprises may have to undertake anticompetitive activities. These activities include setting prices below marginal cost, raising the operating costs of existing rivals, erecting entry barriers to preclude the operation of new competitors, and circumventing regulations designed to foster competition. We find that public enterprises often have stronger incentives to pursue these activities than do their private, profit-maximizing counterparts.

Incentives for Anticompetitive Behavior by Public Enterprises

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1. Introduction.

Most formal analyses of competition among firms assume that firms act to maximize their profit. This is a reasonable approximation in many settings. But public enterprises do not typically seek to maximize profit, and public enterprises compete directly with private, profit-maximizing enterprises in many important markets. In the United States, for example, the U.S. Postal Service provides overnight mail and package shipping services in direct competition with private delivery companies. Many public hospitals and educational institutions also compete directly with private suppliers of similar services. Production by public enterprises is even more widespread in many other countries. To illustrate, during the 1980s public enterprises accounted for approximately 14 percent of gross domestic product (GDP) in African nations, and for approximately 11 percent of GDP in developing countries as a whole (World Bank , 1995, p. 30).¹

Because public enterprises are often charged with objectives other than profit maximization, one might suspect that public enterprises would act less aggressively toward their competitors than would their profit-maximizing counterparts.² This is generally not the case, though. We identify a variety of plausible settings in which public enterprises have stronger incentives than profit-maximizing firms to pursue activities that disadvantage competitors. Quite often, the less concerned is the public enterprise with profit, the stronger are its incentives to undertake activities that disadvantage competitors. These activities include setting prices below cost, misstating costs and choosing

¹These statistics are consistent with Short's (1984, p. 118) earlier findings that, on average, public enterprises accounted for 8.6 percent of GDP and 27.0 percent of capital formation in the late 1970s. The corresponding percentages for Africa were 17.5 and 32.4, respectively.

²The U.S. Postal Service, for example, is required by statute to consider the fairness, equity, and simplicity of its rate structure as well as the relationships among prices, production costs, and the value of the service provided (39 U.S.C. § 3622).

inefficient technologies in order to circumvent restrictions on below-cost pricing, raising the operating costs of existing rivals, and erecting entry barriers to preclude the operation of new competitors.

Our analysis differs from other analyses in the literature because we focus on the strategic actions that a public enterprise might undertake to disadvantage competitors and evade regulations designed to foster competition. Although others have shown that profit-maximizing firms may pursue some of these actions, we demonstrate that public enterprises will often have the incentive to pursue these actions even more aggressively.³ Since these actions can reduce welfare, our findings complement those of other researchers who have shown that the operation of a public enterprise can be detrimental even when the public enterprise seeks to maximize social welfare (Cremer et al., 1991; De Fraja and Delbono, 1989).⁴ Our analysis also extends Lott's (1990) important observation that public enterprises may set prices below marginal production costs and thereby harm competition and reduce welfare.⁵ We extend Lott's analysis by specifying precisely when a public enterprise will price below marginal cost and how the prices that a public enterprise sets vary as its concern with profit varies.

We do not provide a comprehensive assessment of the benefits and costs of public enterprises. In particular, we do not analyze why the operation of public enterprises may be preferred to operation by private, profit-maximizing firms.⁶ We also abstract from any innate cost differences

³Salop (1979), Salop and Scheffman (1983, 1987), Brock (1983), Salop et al. (1984), Krattenmaker and Salop (1986), Ordover and Saloner (1989), and Economides (1998), among others, analyze the incentives for profit-maximizing firms to raise their rivals' operating costs.

⁴The welfare loss in Cremer et al.'s (1991) model arises because the presence of a public enterprises induces private firms to choose more similar qualities, which is detrimental to consumers. The loss in De Fraja and Delbono's (1989) model occurs because the public enterprise produces a disproportionate share of industry output, thereby raising total production costs.

⁵Lott (1999) reiterates this observation and provides some supporting empirical evidence.

⁶See Baumol (1984), Ruys (1988), Cremer et al. (1989), Delbono and Rossini (1992), Delbono and Denicolo (1993), MacAvoy and McIsaac (1995), Hansmann (1996), Cremer et al. (1997), Hart et al. (1997), and Shleifer (1998), among others, for some analyses of this issue.

between public and private enterprises.⁷ Therefore, our research is not designed to deliver broad policy prescriptions regarding the proper scope of public enterprises. Our analysis does suggest, however, that the incentives that public enterprises have to engage in various forms of anticompetitive behavior deserve careful consideration in any comprehensive assessment of the benefits and costs of public enterprises.⁸

Our formal analysis begins in section 2, where we examine the prices that a public enterprise would set in the absence of any pricing restrictions. The analysis provides sufficient conditions for a public enterprise to set prices below marginal production costs. In section 3, we investigate some of the methods that a public enterprise might employ to relax a binding prohibition against below-cost pricing. We show that a public enterprise will typically have stronger incentives than a profit-maximizing firm to manipulate accounting data in order to understate marginal costs and to overinvest in capital in order to reduce marginal production costs.

Section 4 examines the incentives of a public enterprise to raise the costs of existing rivals and to erect barriers to keep potential rivals from entering the market. We identify plausible conditions under which a public enterprise will have stronger incentives to undertake these activities than will a private, profit-maximizing firm. Conclusions and directions for future research are discussed in section 5. The proofs of all formal results are provided in the Appendix.

2. Public Enterprise Pricing.

In this section, we examine the prices that a public enterprise will set for its products. We first show how the prices preferred by a public enterprise can be characterized by a modified inverse-

⁷Boardman and Vining (1989) provide a review of the empirical literature that addresses this issue.

⁸Our focus throughout is on public enterprises. However, to the extent that private, nonprofit firms share similar objectives with public enterprises, some of our conclusions may pertain to nonprofit firms. See Hansmann (1996), Salamon and Anheier (1996), Rose-Ackerman (1996), and Weisbrod (1997) for recent analyses of nonprofit organizations.

elasticity rule. Then we demonstrate how a public enterprise will alter the prices it charges as it becomes less concerned with the profit it generates. Finally, we examine the conditions under which a public enterprise will set prices below marginal production costs.

In contrast to the typical private firm in a capitalist society, a public enterprise seldom seeks solely to maximize the profit it generates. The profit that public enterprises are permitted to earn is often explicitly limited, and public enterprises are commonly instructed to pursue goals that are distinct from, if not fundamentally incompatible with, profit maximization.⁹ In addition, the managers of public enterprises often have considerable discretion to pursue their own objectives. This discretion stems in part from the fact that public enterprises are not subject to takeover threats and are generally less subject to the discipline of capital markets than are private enterprises (Geddes, 1994, 1999; Oster, 1995). In practice, managers of public enterprises often have considerable interest in expanding the scale or scope of their activities (Niskanen, 1971, 1975), in part because a manager's abilities are often inferred from the size of the operations that he or she oversees.

The revenue that an operation generates often serves as a proxy for the size and scope of the operation.¹⁰ Therefore, as one representation of a public enterprise's reduced focus on profit and its expanded focus on operational scale, we assume that the public enterprise seeks to maximize a weighted average of revenue and profit. The parameter $w \in [0, 1]$ will denote the weight that the public enterprise places on revenue and $1 - w$ will denote the corresponding weight on profit. Although we focus on this objective in the ensuing analysis, most of our qualitative conclusions hold more generally (for example, when the public enterprise seeks to maximize a weighted average of

⁹ Universal service -- providing high-quality service that is affordable to all citizens -- is one such common goal.

¹⁰As Baumol (1959, pp. 32, 45) points out, "In ordinary business parlance the term 'sales' refers not to the number of physical units ... but, rather, to the *total revenue* obtained by the firm from the purchases of its customers". Furthermore, "In the near universal multi-product firm any measure of overall physical volume must involve index number problems, and the adoption of a value measure is doubtless to be expected".

output and profit). The critical assumption is that the public enterprise values expanded output more highly than does its profit-maximizing counterpart.¹¹

We assume that the public enterprise supplies $n \geq 1$ products. Let $p_i \geq 0$ denote the price of the i^{th} product, and $p \equiv (p_1, \dots, p_n)$ the vector of prices for all n products. Also let $Q_i(\cdot)$ denote the demand for the public enterprise's i^{th} product and $Q \equiv (Q_1(\cdot), \dots, Q_n(\cdot))$ the vector of demands for the public enterprise's n products. $C(Q)$ will denote the public enterprise's cost of producing output Q . The public enterprise is assumed to choose prices to maximize:

$$w \left[\sum_{i=1}^n p_i Q_i(p) \right] + [1 - w] \left[\sum_{i=1}^n p_i Q_i(p) - C(Q(\cdot)) \right]. \quad (2.1)$$

The first term in square brackets in expression (2.1) is the public enterprise's total revenue. The last term in square brackets is the public enterprise's profit. Thus, expression (2.1) is simply the aforementioned weighted average of revenue and profit.

For analytic simplicity, we will focus throughout on the case of independent demands (so $\partial Q_i(\cdot) / \partial p_j = 0$ for all $j \neq i$) and separable production costs. In this case, demand for the public enterprise's i^{th} product can be written as $Q_i(p_i)$, and the public enterprise's cost function can be expressed as $C(Q) = \sum_{i=1}^n C_i(Q_i(\cdot))$. We will also assume that the public enterprise's problem is concave.¹²

Lemma 1 characterizes the public enterprise's preferred prices. The lemma refers to $\epsilon_i = \left| \frac{\partial Q_i}{\partial p_i} \frac{p_i}{Q_i} \right|$, which is the price elasticity of demand for the public enterprise's i^{th} product.

Lemma 1. The public enterprise's preferred prices are determined by the following modified inverse-

¹¹Expanded output can also promote expanded employment, which can be a goal of public enterprises (Geddes, 1999).

¹²This assumption allows us to focus on the necessary conditions for a solution to the public enterprise's problem. Concavity in prices is facilitated by two assumptions that are maintained throughout: (1) demand is a concave function of price ($Q_i''(p_i) \leq 0$, where primes denote derivatives); and (2) either marginal production costs increase with output ($C_i''(Q_i) \geq 0$) or they decline with output less rapidly than price declines with output along the inverse demand curve ($C_i''(Q_i(\cdot)) Q_i'(p_i) < 1$).

elasticity rule:

$$\frac{p_i - [1 - w] \frac{\partial C_i(\cdot)}{\partial Q_i}}{p_i} = \frac{1}{\epsilon_i} . \quad (2.2)$$

Expression (2.2) can be viewed as a modified inverse-elasticity rule (Ramsey, 1927; Baumol and Bradford, 1970). The public enterprise implements proportional mark-ups of price over modified marginal cost ($[1 - w] \partial C_i(\cdot) / \partial Q_i$) that vary inversely with the price elasticity of demand. Prices are set further above cost the more inelastic is the demand for the product. This pricing rule is the same rule that a profit-maximizing firm follows, except that marginal costs are scaled down by the factor $1 - w$ to reflect the public enterprise's reduced focus on profit. The greater is its focus on revenue rather than profit (that is, the larger is w), the more the public enterprise discounts marginal costs in the modified inverse-elasticity rule.

Expression (2.2) provides the following conclusion:

Lemma 2. The less profit-oriented is the public enterprise, the lower the price it will set for each of its products (that is, $\frac{dp_i}{dw} < 0$ for all $i = 1, \dots, n$).

The magnitudes of the price reductions that a public enterprise implements as it becomes less profit-oriented generally vary with the shapes of the relevant demand and cost curves. Lemma 3 specifies conditions under which a reduced focus on profit leads the public enterprise to increase the extent to which it implements relatively high proportional mark-ups of price above cost on products with inelastic demand.

Lemma 3. Suppose the public enterprise faces constant elasticity demand functions.¹³ Then the less

profit-oriented is the public enterprise, the greater is the difference in proportional price-cost mark-ups it will implement for products with less elastic versus more elastic demand (that is, if

$Q_i(p_i) = \alpha_i p_i^{-\epsilon_i}$ where $\epsilon_i > 1$ for all $i = 1, \dots, n$, then $\frac{d}{dw} \left[\frac{p_i - c_i}{p_i} - \frac{p_j - c_j}{p_j} \right] > 0$ for $\epsilon_i < \epsilon_j$).¹⁴

Lemma 3 reflects the fact that as the public enterprise becomes more concerned with revenue relative to profit, it becomes less averse to the higher costs that arise from increased output. Consequently, the public enterprise favors more highly the expanded output and revenue that result when the prices of products with more elastic demand are reduced. In practice, a public enterprise often faces the most elastic demand on those products for which competition from alternative suppliers is most pronounced. Lemma 3 suggests that when this is the case, a reduced focus on profit may lead the public enterprise to allocate price reductions disproportionately toward those products for which it faces the most intense competition.

This conclusion supports Lott's (1990) observation that a public enterprise might set the price of a product below its marginal cost of production. Equation (2.1) and Lemma 3 suggest that below-cost pricing is most likely when the public enterprise's focus on profit is limited and when the demand for the public enterprise's product is elastic. Observations 1A and 1B confirm this intuition and extend Lott's (1990) observation by providing sufficient conditions for a public enterprise to price

¹³We assume all constant elasticity demand functions represent elastic demands ($\epsilon_i > 1$). This assumption rules out unrealistic cases in which the public enterprise can generate unbounded profit by setting an arbitrarily high price for a product.

¹⁴The conclusion in Lemma 3 also holds if the public enterprise faces linear demands and constant marginal production costs. Simulations reveal that the conclusion also holds more generally.

below marginal cost.

Observation 1A. Suppose the public enterprise faces constant elasticity demand functions ($Q_i(p_i) = \alpha p_i^{-\epsilon_i}$). Then the public enterprise will set price below marginal cost on those products for which the price elasticity of demand exceeds $1/w$.

Observation 1B. Suppose the public enterprise faces linear demand functions ($Q_i(p_i) = a_i - b_i p_i$) and quadratic production costs ($C_i(Q_i) = F_i + c_i^0 Q_i + 0.5 c_i^1 Q_i^2$). Then the public enterprise will set price below marginal cost on those products for which $1 > w > [a_i - b_i c_i^0] / [b_i (a_i c_i^1 + c_i^0)]$.

Corollary 1. Suppose the conditions of Observation 1B hold and the public enterprise's marginal cost is zero when output is zero (that is, $c_i^0 = 0$). Then the public enterprise will set the price of its i^{th} product below marginal cost if $c_i^1 > \frac{1}{b_i}$ and $w > \frac{1}{b_i c_i^1}$.

Observations 1A and 1B reflect the fact that even though profit declines as price is reduced below marginal cost, revenue can increase as price declines. Therefore, if the public enterprise's relative valuation of revenue is sufficiently pronounced and/or if demand is sufficiently elastic, the public enterprise may choose to set prices below marginal production costs. To illustrate, Observation 1A reports that if the public enterprise faces constant-elasticity demand functions and values profit and revenue equally, then it will set prices below marginal cost on all products for which the price elasticity of demand exceeds 2.

Observation 1B and Corollary 1 consider the case of linear demands and quadratic production costs. In this case, the public enterprise is more likely to price below marginal cost the less profit-oriented it is, the more rapidly marginal costs rise with output,¹⁵ and the more sensitive is demand to price (that is, the larger are w , c_i^1 , and b_i). These conclusions emerge because a reduced focus on profit renders output expansion more attractive to the public enterprise. Furthermore, a more steeply sloped marginal cost curve and a flatter inverse demand curve increase the likelihood that the marginal cost curve will lie above the inverse demand curve as output expands beyond the profit-maximizing level.

Observations 1A and 1B reveal that even in the absence of any predatory intent, a public enterprise may set prices so low that they do not cover marginal production costs.¹⁶ In doing so, the public enterprise may drive a more efficient profit-maximizing firm from the market. It will do so, for example, if the competitor operates with a constant marginal cost that lies above the public enterprise's preferred price and below the public enterprise's marginal cost of production.

3. Avoiding Restrictions on Below-Cost Pricing.

The analysis to this point has focused on the prices that a public enterprise will set when its pricing flexibility is unrestricted. In practice, a public enterprise may face restrictions on feasible prices. For example, a public enterprise may be prohibited from pricing below marginal cost, as

¹⁵The public enterprise will not price below marginal cost if its marginal cost does not vary with output. This conclusion is sensitive to the presumed objective of the public enterprise, however. If the public enterprise seeks to maximize a weighted average of output and profit (rather than revenue and profit), the public enterprise that operates with linear demand ($Q(p) = a - bp$) and constant marginal cost (c) will price below marginal cost if $w > [a - bc] / [a - b(c - 1)]$.

¹⁶This fact underlies Lott's (1990, 1999) observation that a public enterprise's threat to price below marginal cost may be more credible than the identical threat of a profit-maximizing firm.

private, profit-maximizing firms typically are.¹⁷ The purpose of this section is two-fold. First, we illustrate how a public enterprise might attempt to relax a binding prohibition against below-cost pricing. Second, and more importantly, we show that a public enterprise typically has stronger incentives than a profit-maximizing firm to devote resources to relaxing this prohibition.

A. Manipulating Accounting Data.

One obvious way in which a firm might attempt to relax a binding constraint against pricing below marginal cost is to manipulate accounting data so as to understate realized marginal cost.¹⁸ Such understatement might be achieved by classifying as overhead (fixed) production costs some or all of the costs that truly vary as output varies. For example, the firm might count some of the personnel hired to supply the product in question as central management. An alternate way for the firm to understate its realized marginal cost is to record as variable costs incurred in the provision of a different product costs that are truly incurred in producing the product whose price the firm would like to set below marginal cost. For example, the firm might claim that materials and supplies employed to produce the product in question were employed to produce a different product.

Intentional understatement of marginal production costs is likely to entail personal risk. Laws against fraud can carry severe financial penalties, and career prospects can be dimmed for managers who are even suspected of knowingly reporting false information. We capture these and other costs of understating marginal production costs in the function $D(u)$, which denotes the firm's expected

¹⁷In American law, the doctrine of sovereign immunity may shield public enterprises of the federal or state governments from application of the antitrust laws. In addition, public enterprises of state or municipal governments may be exempt from the antitrust laws under the state action immunity. See Areeda and Hovenkamp (1999, ¶ 2.12). If neither immunity applies, the public enterprise will be subject to general antitrust constraints, including those on below-cost pricing.

¹⁸See Sidak and Spulber (1996, pp. 105-126).

disutility or cost of understating marginal cost by u dollars. This disutility is assumed to increase at an increasing rate with the degree of understatement.¹⁹ So as not to bias our analysis against the public enterprise, we analyze the case in which the public enterprise views the costs of manipulating accounting data exactly as a profit-maximizing firm does. In particular, the public enterprise bears the full costs ($D(\cdot)$) of the manipulation, and does not discount these costs by the factor $1 - w$, as it implicitly discounts production costs.

The public enterprise's problem in this setting with possible cost understatement, labeled $[P - u]$, is:

$$\underset{p, u}{\text{Maximize}} \quad w [p Q(p)] + [1 - w][p Q(P) - C(Q(p))] - D(u) \quad (3.1)$$

$$\text{subject to :} \quad p \geq C'(Q(p)) - u. \quad (3.2)$$

Expression (3.1) reflects the public enterprise's desire to maximize a weighted average of revenue and profit less the disutility associated with understating marginal cost. Expression (3.2) captures the prohibition against pricing below measured marginal cost, which is true marginal cost ($C'(\cdot)$) less any understatement (u) of marginal cost. For simplicity, we assume that the public enterprise produces only one product, but the conclusion reported in Observation 2 holds more generally.²⁰

Observation 2. In the setting with possible cost understatement, the public enterprise will understate its marginal cost of production in order to relax a binding prohibition against pricing below cost.

¹⁹Formally, $D'(u) > 0$ and $D''(u) > 0$ for all $u > 0$. It is also convenient to assume that the costs of understatement initially increase slowly but eventually increase very rapidly with u , that is, $\lim_{u \rightarrow 0} D'(u) = 0$ and $\lim_{u \rightarrow \infty} D'(u) = \infty$.

²⁰The presumed concavity of $[P - u]$ is ensured if, for example, demand is linear and marginal cost increases with output at an increasing rate or if demand is concave and marginal cost is constant.

The less profit-oriented is the public enterprise, the more it will understate its marginal cost (that is, $u > 0$ and $\frac{du}{dw} > 0$ when constraint (3.2) binds at the solution to $[P - u]$).

Observation 2 reveals that when they face the same risks from understating costs, a public enterprise will typically understate its marginal cost more than will a profit-maximizing firm. The public enterprise is willing to bear the higher costs that accompany more pronounced understatement because it values more highly the expanded output and revenue that result from the lower price that the understatement facilitates.

B. Strategic Choice of Technology.

Now consider a more subtle strategy that the public enterprise might pursue to relax a binding prohibition against pricing below cost. Suppose that instead of misstating realized marginal cost, the firm chooses an inefficient operating technology that secures a relatively low marginal cost at the expense of a particularly high overhead (fixed) cost of production. In practice, a firm might do so by installing general-purpose equipment on a large scale and thereby reduce the need for project-specific equipment, or by retaining a large on-site staff with broad legal, engineering, computing, and/or marketing expertise that can substitute for specific expertise on individual products.

To capture this tradeoff formally, suppose the public enterprise has a choice among production technologies and suppose this choice is indexed by the amount of overhead (fixed) productive resources the firm employs. Let $F \geq 0$ denote the level of overhead resources, which we call capital for expositional convenience.²¹ We will denote by $k > 0$ the unit cost of capital. The more capital the firm installs, the lower are its variable and marginal costs of production. Formally, $V_F(Q, F) < 0$

²¹The overhead cost could include labor. The critical feature of overhead cost is that it does not vary with the level of output produced by the firm.

and $V_{QF}(Q, F) < 0$, where $V(Q, F)$ is the variable cost of producing output Q when F units of capital are installed, and where subscripts denote partial derivatives.^{22, 23}

The public enterprise's problem in this setting with strategic choice of technology, labeled $[P - F]$, is:

$$\underset{F, p}{\text{Maximize}} \quad w [p Q(p)] + [1 - w] [p Q(p) - V(Q(p), F) - kF] \quad (3.3)$$

$$\text{subject to:} \quad p \geq V_Q(Q(p), F) \quad (3.4)$$

Expression (3.3) reflects the public enterprise's desire to maximize a weighted average of revenue and profit (where profit is the difference between revenue and the sum of variable and capital costs).

Expression (3.4) restricts the price chosen by the public enterprise to exceed its marginal cost of production ($V_Q(\cdot)$).

Of central interest is whether a public enterprise might be particularly inclined to choose an inefficient technology in order to relax a binding prohibition on pricing below cost.²⁴ Observation 3 reports that this is the case. The Observation refers to $F^*(Q)$, which is the level of capital that minimizes the cost of producing Q units of output.²⁵

Observation 3. In the setting with strategic choice of technology, the public enterprise will overinvest in capital to relax a binding prohibition on pricing below cost. The less profit-oriented is

²²To ensure an interior choice of F , it is convenient to assume $\lim_{F \rightarrow 0} V_F(Q, F) = -\infty$ and $\lim_{F \rightarrow \infty} V_F(Q, F) = 0$.

²³Diminishing returns to increasing F are also assumed. In particular, increases in F decrease variable costs and marginal costs at a decreasing rate (so $V_{FF}(\cdot) \geq 0$ and $V_{QFF}(\cdot) \geq 0$).

²⁴Baseman (1971) and Spence (1977) illustrate how a profit-maximizing firm might employ an inefficient technology to deter entry. Brennan (1990) and Crew and Crocker (1991) explain how a regulated, profit-maximizing firm might choose an inefficient technology in order to arbitrage cost allocation rules.

²⁵Formally, $F^*(Q) \equiv \underset{F}{\text{argmin}} \{V(Q, F) + kF\}$.

the public enterprise, the more it will over-invest in capital (that is, $F > F^*(Q)$ and $\frac{dF}{dw} > 0$ when constraint (3.4) binds at the solution to $[P - F]$).

The more highly the public enterprise values revenue relative to profit, the more it benefits from the expanded output and revenue that a lower price provides, and thus the greater the technological inefficiency it will endure to secure a lower price. Observation 3 reports that the public enterprise will install an inefficiently large level of capital in order to reduce its marginal cost even if it faces the same market cost of capital that private enterprises face. If the public enterprise's capital purchases are subsidized (as they often are in practice, since public enterprises are commonly afforded privileged access to government funds)²⁶, then inefficient over-capitalization becomes even more pronounced, as Corollary 2 reports.

Corollary 2. The public enterprise's over-investment in capital to relax a binding restriction on pricing below cost will be more pronounced the more heavily its capital purchases are subsidized (that is, $\frac{dF}{dk} < 0$ when constraint (3.4) binds at the solution to $[P - F]$).

In sum, it is apparent that by strategically relaxing a binding prohibition against below-cost pricing, a public enterprise may disadvantage its competitors. Section 4 next considers alternative methods that a public enterprise might employ to disadvantage its competitors.

4. Raising Rivals' Costs.

Other activities that firms might undertake that can serve to disadvantage their rivals include

²⁶See Sidak and Spulber (1996), for example.

lobbying for regulations that increase rivals' operating costs, restricting rivals' access to essential productive inputs, and buying excessive amounts of inputs in order to raise their market price (Salop, 1979; Brock, 1983; Salop et al., 1984; Salop and Scheffman, 1983, 1987; Krattenmaker and Salop, 1986). In this section, we show that public enterprises often have stronger incentives than their profit-maximizing counterparts to engage in such activities. We examine three representative settings: (1) where the public enterprise and a fringe of competitive firms produce a homogeneous product; (2) where the public enterprise and a rival produce differentiated products; and (3) where the public enterprise enjoys a monopoly position and can undertake actions that promote the continued exclusion of competitors.

A. Dominant Firm Setting.

Consider, first, the setting where the public enterprise is a dominant firm that faces a fringe of competitive suppliers of an identical product. The public enterprise chooses a price, recognizing that the fringe will take this price as given and deliver the output that maximizes the fringe's profit. The public enterprise then supplies the residual demand, $Q^R(\cdot)$, which is the difference between market demand ($Q(p)$) and fringe supply ($Q^F(\cdot)$) at the chosen price.

Fringe supply is determined by its cost function. Denote by $C^F(Q, \mathbf{r})$ the fringe's cost of producing output Q when the public enterprise invests resources $\mathbf{r} \geq \mathbf{0}$ to raise its rivals' costs. Cost-raising expenditures by the public enterprise increase both the total and the marginal cost of the fringe (so, $C_r^F(\cdot) > \mathbf{0}$ and $C_{Qr}^F(\cdot) > \mathbf{0}$, where subscripts denote partial derivatives). They also raise the public enterprise's production costs (so $C_r(\cdot) > \mathbf{0}$). The fringe produces with increasing marginal cost (so $C_{QQ}^F(\cdot) > \mathbf{0}$).

The public enterprise's problem is to choose a price (p) and cost-raising expenditures (r) to

maximize its objective function. This problem, labeled $[P - r]$, is the following:

$$\text{Maximize}_{p, r \geq 0} w[pQ^R(p, r)] + [1 - w][pQ^R(p, r) - C(Q^R(p, r), r)] \quad (4.1)$$

$$\text{subject to: } Q^R(p, r) = Q(p) - Q^F(p, r); \text{ and} \quad (4.2)$$

$$Q^F(p, r) = \underset{q}{\text{argmax}} [pq - C^F(q, r)]. \quad (4.3)$$

Expression (4.1) reflects the public enterprise's objective of maximizing a weighted average of revenue and profit. Equation (4.2) defines the residual demand facing the public enterprise as the difference between market demand and the supply of the competitive fringe. Equation (4.3) identifies the output of the competitive fringe as the level of output that maximizes the fringe's profit, given the market price (p) and the cost-raising activities (r) of the public enterprise.

The following conditions help to determine the extent to which the public enterprise will attempt to raise its rivals' costs.

$$(C1) \quad \left. \frac{\partial p}{\partial r} \right|_{Q^0, r=0} > [1 - w] \frac{C_r(Q^0, 0)}{Q^0}, \quad \text{where } Q^0 \text{ is the public enterprise's output at the solution to } [P - r] \text{ when } r = 0.$$

$$(C2) \quad \frac{Q_r^F(p, r)}{Q_p^R(p, r)} \geq [1 - w] \left[C_{Q^R}(\cdot) - C_r(\cdot) \frac{Q_{pr}^F(\cdot)}{Q_p^R(\cdot) Q_r^F(\cdot)} \right] \text{ at the solution to } [P - r].$$

Condition (C1) says that the incremental benefit from initial amounts of the cost-raising activity outweigh the associated incremental cost. The incremental benefit ($\left. \frac{\partial p}{\partial r} \right|_{Q^0, r=0} = Q_r^F(\cdot) / Q_p^R(\cdot)$) is the vertical shift in the public enterprise's residual demand curve that arises from the reduction in the fringe's output as its costs increase. The incremental cost is the increase in the public enterprise's average cost, discounted by the factor $1 - w$, which reflects the public enterprise's diminished focus

on profit.²⁷ Notice that the value of the public enterprise's objective function when it supplies output Q is $[p - (1 - w)C(\cdot)/Q]Q$. Therefore, the public enterprise's objective function increases whenever p rises more rapidly than $[1 - w]C(\cdot)/Q$, holding Q constant. This explains why the public enterprise optimally undertakes some cost-raising activity whenever condition (C1) holds.

Condition (C2) ensures that a public enterprise will be more aggressive in raising its rivals' costs than an otherwise identical profit-maximizing firm. The condition simply requires that the predominant effect of the cost-raising activity be to increase the public enterprise's residual demand by decreasing the fringe's supply. This effect ($Q_r^F(\cdot)/Q_p^R(\cdot)$) must outweigh any adverse impact on the public enterprise of higher marginal costs ($C_{Qr}(\cdot)$), higher total costs ($C_r(\cdot)$), or decreased slope of the fringe's supply curve ($-Q_{pr}^F(\cdot)$).²⁸ Condition (C2) will be satisfied, for example, if: (1) the cost-raising activity does not affect the public enterprise's marginal cost of production (that is, $C_{Qr}(\cdot) = 0$ for all Q and r); and (2) the impact of higher r on the rival's marginal cost does not vary with the level of output (as when $C^F(Q, r) = F(r) + c_0(r)Q + 0.5 c_1 Q^2$, where $F'(r) \geq 0$ and $c_0'(r) > 0$).

Observation 4. Suppose conditions (C1) and (C2) hold. Then the public enterprise will act to raise

its rivals' costs, and will do so more extensively the less profit-oriented it is (that is, $r > 0$ and

$$\frac{dr}{dw} > 0 \text{ at the solution to } [P - r].$$

Observation 4 indicates that as long as the public enterprise's activities raise the fringe's cost

²⁷Condition (C1) is analogous to expression (5) in Salop and Scheffman (1987, p. 23), with the exception of the discount factor, $1 - w$.

²⁸As the slope of the fringe's supply curve declines, residual demand declines more rapidly as the market price increases. The reduced residual demand is disadvantageous for the public enterprise.

more than they raise its own cost, the public enterprise will raise its rivals' costs more aggressively than will a profit-maximizing firm. The additional aggression by the public enterprise is motivated by its reduced focus on profit. The reduced focus on profit effectively renders the cost of expanded output less onerous for the public enterprise. The public enterprise secures the expanded output that it values highly by reducing the output of its rivals via raising their costs.

B. Duopoly Interaction.

It is important to determine whether a public enterprise's incentive to raise its rivals' costs persists in settings other than those considered in Observation 4. In practice, public enterprises are not always dominant firms facing a fringe of competitive suppliers, and their products often differ from those of their competitors. Therefore, it is instructive to consider the following simple setting where the public enterprise is one of two firms producing differentiated products. The two firms establish prices for their products simultaneously after learning the amount (r) by which the public enterprise has raised its rival's constant marginal cost of production (c^r). For simplicity, the public enterprise is assumed to incur a separable cost, $L(r)$, that increases at an increasing rate with its cost-raising activity (that is, $L'(r) > 0$ and $L''(r) > 0$ for all $r > 0$). To illustrate, this cost might constitute expected penalties for anticompetitive behavior or the costs of lobbying for regulations that restrict its rival's access to key inputs (for example, transmission or delivery media). The public enterprise's production cost is c per unit.²⁹

The higher is the price that one firm sets for its product, the greater is the demand for the other firm's product. This is why the public enterprise may act to raise its rival's marginal cost of

²⁹For expositional simplicity, we abstract from fixed costs of production.

production, even though doing so is personally costly. As its marginal cost increases, the rival increases the price it charges for its product, thereby increasing the demand for the public enterprise's product. For analytic simplicity, demand curves are assumed to be linear in prices. The public enterprise's demand curve is:

$$Q(p, p^r) = a - b_0 p + b_1 p^r ; \quad (4.4)$$

and the rival's demand curve is:

$$Q^r(p^r, p) = a^r - b_0^r p^r + b_1^r p , \quad (4.5)$$

where $p \geq 0$ is the price the public enterprise sets for its product, $p^r \geq 0$ is the price of the rival's product, and $a, a^r, b_0, b_0^r, b_1,$ and $b_1^r,$ are all strictly positive constants. Each firm's demand is assumed to be more responsive to changes in its own price than to changes in its competitor's price (that is, $b_0 > b_1$ and $b_0^r > b_1^r$). In addition, demand for the public enterprise's product is substantial in the sense that the intercept of the public enterprise's demand curve exceeds the public enterprise's marginal cost of production (c) even when the rival's price (p^r) is zero.

The public enterprise's problem in this duopoly setting, labeled $[P - d]$, is the following:

$$\text{Maximize}_{p, r \geq 0} \quad w [p Q(p, p^r)] + [1 - w] [(p - c) Q(p, p^r)] - L(r) \quad (4.6)$$

subject to: (4.4); (4.5);

$$p = \underset{\tilde{p}}{\text{argmax}} \{ w [\tilde{p} Q(\tilde{p}, p^r)] + [1 - w] [(\tilde{p} - c) Q(\tilde{p}, p^r)] \}; \quad \text{and} \quad (4.7)$$

$$p^r = \underset{\tilde{p}}{\text{argmax}} \{ [\tilde{p} - (c^r + r)] Q^r(\tilde{p}, p) \} . \quad (4.8)$$

Expression (4.6) reflect's the public enterprise's desire to maximize a weighted average of

revenue and profit, less the cost of raising its rival's cost.³⁰ Expressions (4.7) and (4.8) reflect the fact

³⁰Notice that in this formulation, the public enterprise bears the full costs ($L(\cdot)$) of r , and does not discount these cost by $1 - w$, as it implicitly discounts production costs. It is apparent that if $L(\cdot)$ is sufficiently large for all r , no

that the public enterprise and its rival choose prices simultaneously to maximize their objectives, after observing the extent of the public enterprise's cost-raising activities, r . The key features of the solution to $[P - d]$ are recorded in Observation 5.

Observation 5. In the duopoly setting, the public enterprise will raise its rival's cost, and will do so to a greater extent the less profit-oriented it is (that is, $r > 0$ and $\frac{dr}{dw} > 0$ at the solution to $[P - d]$).

A public enterprise will raise its rival's cost more extensively than will a profit-maximizing firm *ceteris paribus* because the public enterprise is more eager than its profit-maximizing counterpart to expand output. Consequently, the public enterprise raises its rival's cost more dramatically in order to restrict its rival's supply more severely, and thereby increase the demand for its own product more extensively.

C. Excluding Potential Competitors.

In addition to raising the operating costs of an existing rival, a public enterprise may undertake activities designed to preclude the operation of potential rivals. For example, the public enterprise may lobby key policymakers to erect impenetrable entry barriers, such as outright prohibitions on entry. To determine whether a public enterprise has more or less incentive than a private, profit-maximizing enterprise to undertake such exclusionary activities, consider the following simple model.

firm will act to raise its rival's cost. We abstract from this possibility by assuming $L(0) = 0$ and $L'(r)|_{r=0} = 0$. We also avoid the situation in which the public enterprise raises its rival's cost so much that the rival exits the market. We do so by assuming $L'(r)$ and $L''(r)$ are sufficiently large for all $r > 0$. Sufficient conditions are $L'(r^*) > b_0^r b_1 [a - b_0(1-w)c + b_1 p^r] / A$ and $L''(r^*) > 2b_0(b_0^r)^2 (b_1)^2 / A$, where r^* is the optimal r for the public enterprise at the solution to $[P - d]$, and where $A \equiv 4b_0 b_0^r - b_1 b_1^r$.

Let $\phi(\cdot) \in [0, 1]$ denote the probability that potential competitors are excluded from the market in which the public enterprise operates. This probability increases at a decreasing rate with the effort, $e \geq 0$, that the public enterprise devotes to securing exclusion.³¹ The unit cost of effort is normalized to 1. If competitors are excluded from the market, the public enterprise faces the demand curve $Q^m(p)$, where p is the price that the public enterprise charges for its product. If competitors are not excluded, the public enterprise faces demand curve $Q^c(p)$. Competition reduces demand for the public enterprise's product, so $Q^m(p) > Q^c(p)$ for all $p \geq 0$.³²

Let p^m denote the price that the public enterprise will set for its product if its efforts to exclude competitors are successful. Let p^c denote the corresponding price if its efforts are unsuccessful. Then the public enterprise's problem in this setting with potential exclusion (denoted $[P - e]$) is the following.³³

$$\begin{aligned} \text{Maximize}_{p^m, p^c, e} \quad & \phi(e) \{w [p^m Q^m(p^m)] + [1 - w][p^m Q^m(p^m) - C(Q^m(p^m))]\} \\ & + [1 - \phi(e)] \{w [p^c Q^c(p^c)] + [1 - w][p^c Q^c(p^c) - C(Q^c(p^c))]\} - e. \end{aligned} \quad (4.9)$$

Observation 6. In the setting with potential exclusion, the public enterprise will undertake exclusionary effort. The level of exclusionary effort increases as the public enterprise becomes less profit-oriented whenever competition reduces the public enterprise's output (that is, $e >$

³¹Formally, $\phi'(e) > 0$ and $\phi''(e) < 0$ for all $e \geq 0$. To ensure an interior value for e , we assume $\lim_{e \rightarrow 0} \phi'(e) = \infty$ and $\lim_{e \rightarrow 1} \phi'(e) = 0$.

³²Because competition reduces the demand for the public enterprise's product, the equilibrium value of the public enterprise's objective function declines when competition is admitted.

³³ $[P - e]$ will be concave in p^m, p^c and e if, for example, the demand curves facing the public enterprise are concave and if its cost function is convex.

0 and, if $Q^m(p^m) > Q^c(p^c)$, $\frac{de}{dw} > 0$ at the solution to $[P - e]$.

As the public enterprise becomes less profit-oriented (so w increases), it implicitly discounts its production costs more highly, and therefore finds the extra cost of higher output less onerous. Consequently, when exclusion of rivals leads to more output by the public enterprise, it will find exclusion to be particularly valuable as w increases, and so it optimally increases its exclusionary activities. There are many settings in which the public enterprise will sell more output when competition is precluded than when it is admitted. One important setting is when potential competitors have lower costs than the public enterprise and pricing below marginal cost is prohibited. In this setting, if the firms engage in price competition and produce a homogenous product with constant marginal cost, the public enterprise will be driven from the market when more efficient suppliers are authorized to produce. Consequently, as Observation 6 reveals, the public enterprise has particularly strong incentives in this setting to act aggressively to exclude rivals.³⁴

5. Conclusions.

We have shown how the diverse goals that a public enterprise faces may provide it with particularly strong incentives to act as an aggressive competitor. A reduced focus on profit was shown to lead the public enterprise to price certain products below cost, to raise the costs of existing rivals, to erect entry barriers to preclude entry by potential rivals, and to understate costs and adopt

³⁴A public enterprise can have even greater incentive to exclude rivals when its production technology exhibits cost complementarities. To illustrate this point, suppose that a public enterprise produces two products, A and B , and that product B is also supplied by competitors. Further suppose that the firm's marginal cost of producing product B declines as its output of product A increases. In this setting, if the public enterprise successfully precludes competition on product A and thereby increases its output of product A , the public enterprise reduces its marginal cost of delivering product B . By doing so, the public enterprise is likely to strengthen its competitive position and so increase its output in the market for product B . Therefore, in the presence of cost complementarities, the public enterprise can secure benefits in multiple markets by limiting competition in one market.

inefficient production technologies in order to circumvent regulations designed to foster competition. Each of these activities can preclude the operation of more efficient competitors, and thereby reduce social welfare.

We have analyzed selected anticompetitive activities that public enterprises might undertake. We have not undertaken a comprehensive benefit-cost analysis of public enterprises. Therefore, our analysis alone cannot provide broad policy prescriptions regarding the proper scope of public enterprises. However, the fact that public enterprises may pursue anticompetitive actions particularly aggressively suggests that the costs of public enterprises need to be weighed carefully against any benefits that such firms may provide.

A comprehensive benefit-cost analysis of public enterprises is beyond the scope of this research. Such an analysis would need to consider other possible objectives of the public enterprise, including national security and income redistribution. The analysis would also need to consider market failures that a public enterprise might help to correct, and contrast the internal operations of public and private enterprises. The analysis should also endow the public enterprise with a richer set of policy instruments, including non-linear and discriminatory prices, products of varying quality, and different intensities of product and process innovation.

A comprehensive assessment of the merits of public enterprises would also need to account for the fact that public enterprises often face some regulations, even though the regulations can be less stringent than those faced by private firms that operate in regulated industries (Sidak and Spulber, 1996, pp. 83-100). The optimal design of regulatory policy for public enterprises has received little attention in the literature, and deserves careful study. It is important to determine, for example, whether the benefits that price-cap regulation can provide when applied to profit-maximizing firms persist when price-cap regulation is applied to public enterprises. It is conceivable, for example, that

a public enterprise might have greater incentive than its private counterpart to set prices strategically in order to relax a binding price-cap constraint (Sappington and Sibley, 1992; Law, 1997; and Foreman, 1995), or to employ the expanded freedoms of price-cap regulation to price below marginal cost (Armstrong and Vickers, 1993).

The optimal design of antitrust law as applied to public enterprises also merits extensive study. We have shown that a public enterprise may have greater incentive to engage in anticompetitive practices and circumvent antitrust laws than its private counterpart. Therefore, more stringent antitrust laws and harsher penalties for violating these laws may be appropriate for public enterprises. Such legislation or enforcement policy would necessarily raise the question of the proper scope of sovereign immunity for the proprietary, as opposed to political, actions of governments. Of course, financial penalties may have little force if the public enterprise is able to pass financial penalties on to taxpayers.

In short, the incentives for anticompetitive behavior by public enterprises invite further theoretical and empirical research on a wide range of issues. In turn, that research will have the opportunity to inform an emerging body of public policy having great practical significance in many nations.

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APPENDIX

Proof of Lemma 1.

Setting the derivative of (2.1) with respect to p_i equal to zero provides:

$$L_{p_i} \equiv p_i Q_i'(p_i) + Q_i(p_i) - [1 - w] \frac{\partial C_i(\cdot)}{\partial Q_i} Q_i'(p_i) = 0. \quad (\text{A L1.1})$$

Rearranging (A L1.1) provides:

$$p_i - [1 - w] \frac{\partial C_i(\cdot)}{\partial Q_i} = - \frac{Q_i(\cdot)}{Q_i'(p_i)}. \quad (\text{A L1.2})$$

Dividing both sides of equation (A L1.2) by p_i and substituting for ϵ_i provides equation (2.2). ■

Proof of Lemma 2.

$$L_{p_i w} = \frac{\partial C_i(\cdot)}{\partial Q_i} Q_i'(p_i) < 0, \text{ from (A L1.1)}. \quad (\text{A L2.1})$$

Therefore, since concavity of (2.1) ensures $L_{p_i p_i} < 0$, it follows from (A L1.1) and (A L2.1) that

$$\frac{dp_i}{dw} = - \frac{L_{p_i w}}{L_{p_i p_i}} < 0. \quad \blacksquare \quad (\text{A L2.2})$$

Proof of Lemma 3.

Let ϵ_i, ϵ_j denote the (constant) price elasticities of demand for products i and j , respectively.

Equation (2.2) implies:

$$z \equiv \frac{p_i - \partial C_i(\cdot)/\partial Q_i}{p_i} - \frac{p_j - \partial C_j(\cdot)/\partial Q_j}{p_j} = \frac{\partial C_j(\cdot)/\partial Q_j}{p_j} - \frac{\partial C_i(\cdot)/\partial Q_i}{p_i}. \quad (\text{A L3.1})$$

Equation (2.2) reveals that in the present setting:

$$p_i - [1 - w] \partial C_i(\cdot)/\partial Q_i = \frac{p_i}{\epsilon_i}. \quad (\text{A L3.2})$$

Rearranging equation (A L3.2) provides:

$$\frac{\partial C_i(\cdot)/\partial Q_i}{p_i} = \frac{1 - 1/\epsilon_i}{1 - w}. \quad (\text{A L3.3})$$

Substituting equation (A L3.3) and its counterpart for product j into equation (A L3.1) and rearranging terms provides:

$$z = \frac{1}{1-w} \left[\frac{1}{\epsilon_i} - \frac{1}{\epsilon_j} \right]. \quad (\text{A L3.4})$$

Differentiating (A L3.4) with respect to w provides:

$$\frac{dz}{dw} = \frac{1}{[1-w]^2} \left[\frac{1}{\epsilon_i} - \frac{1}{\epsilon_j} \right] \begin{matrix} > \\ \equiv \\ < \end{matrix} 0 \text{ as } \epsilon_i \begin{matrix} < \\ \equiv \\ > \end{matrix} \epsilon_j. \blacksquare \quad (\text{A L3.5})$$

Proof of Observation 1A.

Since $Q_i'(p_i) = -\alpha_i \epsilon_i p_i^{-(\epsilon_i+1)}$ in the present setting, it follows from equation (A L1.1) that the public enterprise's preferred price for product i is given by:

$$-\alpha_i \epsilon_i p_i^{-\epsilon_i} + \alpha_i p_i^{-\epsilon_i} + [1-w] \frac{\partial C_i(\cdot)}{\partial Q_i} \alpha_i \epsilon_i p_i^{-(\epsilon_i+1)} = 0. \quad (\text{A1.1})$$

Rearranging the terms in (A1.1) and simplifying provides:

$$p_i = [1-w] \frac{\epsilon_i}{\epsilon_i - 1} \frac{\partial C_i(\cdot)}{\partial Q_i}. \quad (\text{A1.2})$$

Subtracting $\frac{\partial C_i(\cdot)}{\partial Q_i}$ from both sides of the equality in (A1.2) provides:

$$p_i - \frac{\partial C_i(\cdot)}{\partial Q_i} = [(1-w) \frac{\epsilon_i}{\epsilon_i - 1} - 1] \frac{\partial C_i(\cdot)}{\partial Q_i} < 0 \text{ if and only if } \epsilon_i > \frac{1}{w}. \blacksquare \quad (\text{A1.3})$$

Proof of Observation 1B.

Since $Q_i(p_i) = a_i - b_i p_i$ and $\frac{\partial C_i(\cdot)}{\partial Q_i} = c_i^0 + c_i^1 Q_i$ in the present setting, it follows from equation (A L1.1) that the public enterprise's preferred price for product i is given by:

$$-b_i p_i + a_i - b_i p_i + b_i [1-w] [c_i^0 + c_i^1 (a_i - b_i p_i)] = 0. \quad (\text{A1.4})$$

Rearranging the terms in (A1.4) and simplifying provides:

$$p_i = \frac{a_i + [1-w] [a_i b_i c_i^1 + b_i c_i^0]}{b_i [2 + b_i [1-w] c_i^1]}. \quad (\text{A1.5})$$

It follows from (A1.5) that $p_i < \frac{\partial C_i(\cdot)}{\partial Q_i}$ if and only if:

$$\frac{\alpha_i + [1 - w][\alpha_i b_i c_i^1 + b_i c_i^0]}{b_i[2 + b_i[1 - w]c_i^1]} [1 + b_i c_i^1] < c_i^0 + c_i^1 \alpha_i. \quad (\text{A1.6})$$

Straightforward manipulation of terms reveals that inequality (A1.6) holds if and only if:

$$w > \frac{\alpha_i - b_i c_i^0}{b_i[\alpha_i c_i^1 + c_i^0]}. \quad \blacksquare \quad (\text{A1.7})$$

Proof of Corollary 1.

The corollary follows immediately from equation (A1.7), once c_i^0 is set to 0. \blacksquare

Proof of Observation 2.

Let $\lambda^u \geq 0$ denote the Lagrange multiplier associated with constraint (3.2). Then the Lagrangean function associated with problem $[P - u]$ is :

$$L^u \equiv pQ(p) + [1 - w]C(Q(p)) - D(u) + \lambda^u[p - C'(Q(p)) + u]. \quad (\text{A2.1})$$

The necessary conditions for a solution to $[P - u]$ are:

$$L_p^u \equiv [p - (1 - w)C'(Q(p))]Q'(p) + Q(p) + \lambda^u[1 - C''(\cdot)Q'(p)] = 0; \quad (\text{A2.2})$$

$$L_u^u = -D'(u) + \lambda^u \leq 0; \quad u[-D'(u) + \lambda^u] = 0; \quad (\text{A2.3})$$

$$L_{\lambda^u} = p - C'(Q(p)) + u \geq 0; \quad \lambda^u[p - C'(\cdot) + u] = 0. \quad (\text{A2.4})$$

Since $\lambda^u > 0$ and $\lim_{u \rightarrow 0} D'(u) = 0$ by assumption, $u > 0$ from (A2.3).

Let H^u denote the matrix of second order partial derivatives $\begin{bmatrix} L_{pp}^u & L_{pu}^u & L_{p\lambda^u}^u \\ L_{up}^u & L_{uu}^u & L_{u\lambda^u}^u \\ L_{\lambda^u p}^u & L_{\lambda^u u}^u & L_{\lambda^u \lambda^u}^u \end{bmatrix}$. It

follows

from (A2.2) - (A2.4) that $|H^u| = D''(u)(L_{p\lambda^u}^u)^2 - L_{pp}^u > 0$. Cramer's rule implies:

$$\frac{du}{dw} \stackrel{s}{=} \begin{vmatrix} L_{pp}^u - L_{pw}^u & L_{p\lambda^u}^u \\ 0 & 0 & 1 \\ L_{p\lambda^u}^u & 0 & 0 \end{vmatrix} = -L_{p\lambda^u}^u L_{pw}^u. \quad (\text{A2.5})$$

From (A2.2), $L_{pw}^u = C'(Q(\cdot))Q'(p) < 0$ and $L_{p\lambda^u}^u = 1 - C''(\cdot)Q'(p) > 0$. Therefore, from

$$(A2.5), \quad \frac{du}{dw} > 0. \quad \blacksquare$$

Proof of Observation 3.

Let $\lambda^F \geq 0$ denote the Lagrange multiplier associated with constraint (3.4). Then the Lagrangean function associated with problem $[P - F]$ is:

$$L^F = pQ(p) - [1 - w][V(Q(p), F) + kF] + \lambda^F[p - V_Q(Q(p), F)]. \quad (A3.1)$$

The necessary conditions for a solution to $[P - F]$ are:

$$L_p^F \equiv [p - (1 - w)V_Q(\cdot)]Q'(p) + Q(p) + \lambda^F[1 - V_{QQ}(\cdot)Q'(p)] = 0; \quad (A3.2)$$

$$L_F^F \equiv -[1 - w][V_F(\cdot) + k] - \lambda^F V_{QF}(\cdot) = 0; \quad (A3.3)$$

$$L_{\lambda^F}^F \equiv p - V_Q(\cdot) \geq 0; \quad \lambda^F[p - V_Q(\cdot)] = 0. \quad (A3.4)$$

Since $\lambda^F > 0$ and $V_{QF}(\cdot) < 0$ by assumption, (A3.3) implies:

$$\frac{\partial [V(\cdot) + kF]}{\partial F} = V_F(\cdot) + k > 0. \quad (A3.5)$$

Since $V_{FF} > 0$, (A3.5) implies $F > F^*(Q)$.

Define $H^F \equiv \begin{bmatrix} L_{pp}^F & L_{pF}^F & L_{p\lambda^F}^F \\ L_{Fp}^F & L_{FF}^F & L_{F\lambda^F}^F \\ L_{\lambda^F p}^F & L_{\lambda^F F}^F & L_{\lambda^F \lambda^F}^F \end{bmatrix}$. The second order conditions for an interior maximum require:

$$|H^F| = 2L_{pF}^F L_{F\lambda^F}^F L_{p\lambda^F}^F - L_{FF}^F (L_{p\lambda^F}^F)^2 - L_{pp}^F (L_{F\lambda^F}^F)^2 > 0. \quad (A3.6)$$

Furthermore, Cramer's Rule implies:

$$\frac{dF}{dw} \stackrel{s}{=} \begin{vmatrix} L_{pp}^F & -L_{pw}^F & L_{p\lambda^F}^F \\ L_{pF}^F & -L_{Fw}^F & L_{F\lambda^F}^F \\ L_{p\lambda^F}^F & -L_{\lambda^F w}^F & 0 \end{vmatrix} = L_{p\lambda^F}^F [L_{Fw}^F L_{p\lambda^F}^F - L_{F\lambda^F}^F L_{pw}^F]. \quad (A3.7)$$

It follows from (A3.2) - (A3.4) that $L_{p\lambda^F}^F = 1 - V_{QQ}(\cdot)Q'(p) > 0$, $L_{F\lambda^F}^F = -V_{QF}(\cdot) > 0$,

$L_{pw}^F = V_Q(\cdot)Q'(p) < 0$, and $L_{Fw}^F = V_F(\cdot) + k > 0$ from (A3.5). Therefore, from (A3.7),

$$\frac{dF}{dw} > 0. \blacksquare$$

Proof of Corollary 2.

From (A3.2) - (A3.4) and (A3.6), Cramer's rule implies:

$$\frac{dF}{dk} \stackrel{s}{=} \begin{vmatrix} L_{pp}^F & -L_{pk}^F & L_{p\lambda^F}^F \\ L_{Fp}^F & -L_{Fk}^F & L_{F\lambda^F}^F \\ L_{\lambda^F F}^F & -L_{\lambda^F k}^F & L_{\lambda^F \lambda^F}^F \end{vmatrix} = \begin{vmatrix} L_{pp}^F & 0 & L_{p\lambda^F}^F \\ L_{Fp}^F & 1-w & L_{F\lambda^F}^F \\ L_{p\lambda^F}^F & 0 & 0 \end{vmatrix} = - (L_{p\lambda^F}^F)^2 [1-w] < 0. \blacksquare$$

Proof of Observation 4.

The necessary conditions for a solution to $[P - r]$ are:

$$L_p^r = [p - [1 - w]C_{Q^R}(\cdot)] Q_p^R(p, r) + Q^R(p) = 0; \text{ and} \quad (A4.1)$$

$$L_r^r \equiv - [p - [1 - w]C_{Q^R}(\cdot)] Q_r^F(p, r) - [1 - w]C_r(\cdot) \leq 0; \quad r[L_r^r] = 0. \quad (A4.2)$$

Solving (A4.1) for $p - [1 - w]C_{Q^R}(\cdot)$, substituting into (A4.2), and dividing by $Q^R(p, r)$ provides:

$$m \equiv \frac{Q_r^F(p, r)}{Q_p^R(p, r)} - [1 - w] \frac{C_r(Q^R(\cdot), r)}{Q^R(p, r)} \leq 0; \quad r[m] = 0. \quad (A4.3)$$

Since $\frac{Q_r^F(\cdot)}{Q_p^R(\cdot)} = \frac{-Q_r^R(\cdot)}{\partial Q^R(\cdot)/\partial p} = - \frac{\partial p}{\partial Q^R(\cdot)} \frac{\partial Q^R(\cdot)}{\partial r} = \frac{\partial p}{\partial r}$, it follows from (A4.3) that $r > 0$ if condition (C1) holds.

Define $H^r \equiv \begin{bmatrix} L_{pp}^r & L_{pr}^r \\ L_{pr}^r & L_{rr}^r \end{bmatrix}$. The second order conditions for an interior solution to $[P - r]$

require $L_{pp}^r < 0$, $L_{rr}^r < 0$, and $|H^r| = L_{pp}^r L_{rr}^r - (L_{pr}^r)^2 > 0$. Cramer's rule implies:

$$\frac{dr}{dw} \stackrel{s}{=} \begin{vmatrix} L_{pp}^r & -L_{pw}^r \\ L_{pr}^r & -L_{rw}^r \end{vmatrix} = L_{pw}^r L_{pr}^r - L_{pp}^r L_{rw}^r. \quad (A4.4)$$

From (A4.1) and (A4.2):

$$L_{pr}^r = - [p - [1 - w] C_{Q^R(\cdot)}] Q_{pr}^F(\cdot) - Q_p^R(\cdot) [1 - w] [C_{Q^R r}(\cdot) - C_{Q^R Q^R(\cdot)}] Q_r^F(\cdot) - Q_r^F(\cdot); \quad (A4.5)$$

$$L_{pw}^r = C_{Q^R(\cdot)} Q_p^R(\cdot) < 0; \text{ and} \quad (A4.6)$$

$$L_{rw}^r = C_r(\cdot) - C_{Q^R(\cdot)} Q_r^F(\cdot) > 0. \quad (A4.7)$$

After some simplification, it follows from (A4.5) - (A4.7) that

$$\begin{aligned} L_{pw}^r L_{pr}^r - L_{pp}^r L_{rw}^r &= [p - [1 - w] C_{Q^R(\cdot)}] Q_{pp}^R(\cdot) [C_{Q^R(\cdot)} Q_r^F(\cdot) - C_r(\cdot)] \\ &\quad - Q_p^R(\cdot) C_r(\cdot) [2 - (1 - w) Q_p^R(\cdot) C_{Q^R Q^R(\cdot)}] \\ &\quad + Q_p^R(\cdot) C_{Q^R(\cdot)} [Q_r^F(\cdot) - (p - [1 - w] C_{Q^R(\cdot)}) Q_{pr}^F(\cdot) - [1 - w] Q_p^R(\cdot) C_{Q^R r}(\cdot)] \end{aligned} \quad (A4.8)$$

Notice that $p > (1 - w) C_{Q^R(\cdot)}$ from (A4.1). Therefore, the expression in (A4.8) will be strictly positive if $Q_{pp}^R(\cdot) \leq 0$, $C_{Q^R Q^R(\cdot)} \geq 0$, and:

$$Q_p^R(\cdot) C_{Q^R(\cdot)} [Q_r^F(\cdot) - (p - [1 - w] C_{Q^R(\cdot)}) Q_{pr}^F(\cdot) - Q_p^R(\cdot) [1 - w] C_{Q^R r}(\cdot)] \geq 0. \quad (A4.9)$$

Substituting for $p - [1 - w] C_{Q^R(\cdot)}$ from (A4.2), it is apparent that the inequality in (A4.9) will hold if and only if:

$$Q_r^F(\cdot) \leq [1 - w] \left[Q_p^R(\cdot) C_{Q^R r}(\cdot) - C_r(\cdot) \frac{Q_{pr}^F(\cdot)}{Q_r^F(\cdot)} \right]. \quad (A4.10)$$

Dividing both sides of inequality (A4.10) by $Q_p^R(\cdot)$ reveals that the inequality will hold if (C2) holds.

Therefore, from (A4.4) - (A4.10), $\frac{dr}{dw} > 0$ if condition (C2) holds. ■

Proof of Observation 5.

From (4.4) - (4.6), the objective of the public enterprise is to:

$$\text{Maximize}_p [p - (1 - w)c] [a - b_0 p + b_1 p^r]. \quad (A5.1)$$

Setting the partial derivative of (A5.1) with respect to p equal to zero and solving for p provides:

$$p = \frac{1}{2b_0} \left[a + b_0 [1 - w]c + b_1 p^r \right]. \quad (A5.2)$$

The corresponding analysis for the rival provides:

$$p^r = \frac{1}{2b_0^r} [a^r + b_0^r [c^r + r] + b_1^r p]. \quad (\text{A5.3})$$

Solving (A5.2) and (A5.3) simultaneously provides:

$$p = \frac{1}{A} [2ab_0^r + b_1a^r + 2b_0b_0^r[1-w]c + b_0^rb_1[c^r + r]]; \text{ and} \quad (\text{A5.4})$$

$$p^r = \frac{1}{A} [2a^rb_0 + b_1^ra + 2b_0b_0^r[c^r + r] + b_0b_1^r[1-w]c], \quad (\text{A5.5})$$

where $A \equiv 4b_0b_0^r - b_1b_1^r > 0$.

$[P - d]$ can now be rewritten as:

$$\text{Maximize } U \equiv [p - (1-w)c][a - b_0p + b_1p^r] - L(r) \quad (\text{A5.6})$$

$$r \geq 0$$

subject to (A5.4) and (A5.5).

Differentiating (A5.6) with respect to r provides:

$$U_r = \{[a - b_0p + b_1p^r] - b_0[p - (1-w)c]\} \frac{dp}{dr} + [p - (1-w)c]b_1 \frac{dp^r}{dr} - L'(r). \quad (\text{A5.7})$$

From (A5.4) and (A5.5),

$$\frac{dp}{dr} = \frac{b_0^rb_1}{A} \quad \text{and} \quad \frac{dp^r}{dr} = \frac{2b_0b_0^r}{A}. \quad (\text{A5.8})$$

Substituting (A5.8) into (A5.7), simplifying, and rearranging terms provides:

$$U_r = \frac{b_0^rb_1}{A} [a - b_0[1-w]c + b_1p^r] - L'(r). \quad (\text{A5.9})$$

Since $L'(0) = 0$ and $a > b_0c$ by assumption, (A5.9) implies $U_r|_{r=0} > 0$, which ensures $r > 0$.

Under the maintained assumptions, $U_{rr} < 0$. Furthermore, from (A5.8) and (A5.9):

$$U_{rw} = \frac{b_0^rb_1}{A} [b_0c - \frac{1}{A} b_1b_0b_1^rc] = \frac{2b_0b_0^rb_1c}{A^2} [2b_0b_0^r - b_1b_1^r] > 0. \quad (\text{A5.10})$$

The inequality in (A5.10) holds because $b_0 > b_1$ and $b_0^r > b_1^r$. Therefore, $\frac{dr}{dw} = -\frac{U_{rw}}{U_{rr}} > 0$. ■

Proof of Observation 6.

Let U^e denote the objective function of the public enterprise, as defined in expression (4.9). The necessary conditions for a solution to $[P - e]$ are:

$$U_e^e = \phi'(e)G - 1 \leq 0 \quad \text{and} \quad e \left[U_e^e \right] = 0, \quad (\text{A6.1})$$

where $G \equiv p^m Q^m(p^m) - [1 - w] C(Q^m(p^m)) - [p^c Q^c(p^c) - [1 - w] C(Q^c(p^c))] > 0$; (A6.2)

$$U_{p^m}^e = \phi(e) \left[[p^m - (1 - w) C'(Q^m(p^m))] Q^{m'}(p^m) + Q^m(p^m) \right] = 0; \quad \text{and} \quad (\text{A6.3})$$

$$U_{p^c}^e = [1 - \phi(e)] \left[[p^c - (1 - w) C'(Q^c(p^c))] Q^{c'}(p^c) + Q^c(p^c) \right] = 0. \quad (\text{A6.4})$$

Since $G > 0$ and $\lim_{e \rightarrow 0} \phi'(e) = \infty$, (A6.1) implies that $e > 0$.

The second order conditions for an interior maximum require $U_{ee}^e < 0$, $U_{p^m p^m}^e < 0$, and

$$|H^e| < 0, \quad \text{where} \quad H^e \equiv \begin{bmatrix} U_{ee}^e & U_{ep^m}^e & U_{ep^c}^e \\ U_{p^m e}^e & U_{p^m p^m}^e & U_{p^m p^c}^e \\ U_{p^c e}^e & U_{p^c p^m}^e & U_{p^c p^c}^e \end{bmatrix}. \quad \text{Since} \quad U_{p^m p^c}^e = U_{ep^m}^e = U_{ep^c}^e = 0, \quad |H^e| < 0$$

if and only if $U_{p^c p^c}^e < 0$.

From (A6.1) - (A6.4), Cramer's rule implies:

$$\frac{de}{dw} \stackrel{s}{=} - \begin{vmatrix} -U_{ew}^e & 0 & 0 \\ -U_{p^m w}^e & U_{p^m p^m}^e & 0 \\ -U_{p^c w}^e & 0 & U_{p^c p^c}^e \end{vmatrix} = U_{ew}^e U_{p^m p^m}^e U_{p^c p^c}^e \stackrel{s}{=} U_{ew}^e. \quad (\text{A6.5})$$

From (A6.1) and (A6.2),

$$U_{ew}^e = \phi'(e) [C(Q^m(p^m)) - C(Q^c(p^c))] \stackrel{>}{\leq} 0 \quad \text{as} \quad Q^m(p^m) \stackrel{>}{\leq} Q^c(p^c). \quad (\text{A6.6})$$

(A6.5) and (A6.6) imply $\frac{de}{dw} > 0$ if $Q^m(p^m) > Q^c(p^c)$. ■