J O I N T   C E N T E R

AEI-BROOKINGS JOINT CENTER FOR REGULATORY STUDIES

Does Liquidity Affect Securities Market Efficiency?

Paul C. Tetlock[*]

Working Paper 06-24

November 2006

J O I N T   C E N T E R

AEI-BROOKINGS JOINT CENTER FOR REGULATORY STUDIES

In order to promote public understanding of the impact of regulations on consumers, business, and government, the American Enterprise Institute and the Brookings Institution established the AEI-Brookings Joint Center for Regulatory Studies. The Joint Center's primary purpose is to hold lawmakers and regulators more accountable by providing thoughtful, objective analysis of relevant laws and regulations. Over the past three decades, AEI and Brookings have generated an impressive body of research on regulation. The Joint Center builds on this solid foundation, evaluating the economic impact of laws and regulations and offering constructive suggestions for reforms to enhance productivity and welfare. The views expressed in Joint Center publications are those of the authors and do not necessarily reflect the views of the Joint Center.

All AEI-Brookings Joint Center publications can be found at www.aei-brookings.org

Executive Summary

I investigate the impact of liquidity on market efficiency using data from short-horizon binary outcome securities traded on an online exchange. I show that the most liquid securities markets exhibit significant pricing anomalies, such as overpricing low probability events and underpricing high probability events, whereas less liquid markets do not exhibit these anomalies. I also find that the prices of illiquid securities converge more quickly toward their terminal cash flows. These results are consistent with the idea that liquidity is a proxy for non-informational or noise trading, which can impede market efficiency; but they are inconsistent with models in which increases in liquidity have no impact or a favorable impact on efficiency.

I. Introduction

Numerous theoretical arguments and mounting empirical evidence suggest that securities market liquidity is related to informational efficiency. One view is that illiquidity represents a transaction cost for informed arbitrageurs whose trades make prices more efficient. For example, when liquidity increases in Kyle's (1985) model, informed traders bet more aggressively based on their existing information because their trades have a smaller impact on prices. In addition, informed traders have greater incentives to acquire more precise information in liquid markets. If informed arbitrageurs are less active in illiquid markets where trading is expensive, securities' prices in these markets may deviate by large amounts from their fundamental values. Many recent papers provide indirect empirical support for the view that securities mispricing is greater in illiquid markets—*e.g.*, Wurgler and Zhuravskaya (2002), Kumar and Lee (2006), Sadka and Scherbina (2006), and Chordia, Roll, and Subrahmanyam (2006).

An alternative view is that liquidity is a proxy for non-informational trading, referred to as noise trading hereafter, which may harm informational efficiency.[1] In behavioral finance models, various limits to arbitrage prevent rational agents from making aggressive bets against noise traders. For example, in DeLong, Shleifer, Summers, and Waldmann (1990b), rational arbitrageurs may reinforce demand shocks from noise traders because they anticipate mispricing will worsen in the short-run.[2] If

---

[1] Variations in liquidity correspond to variations in noise trading in Kyle (1985) and Glosten and Milgrom (1985), but they may also result from variations in the search costs that buyers and sellers incur in their efforts to transact—*e.g.*, Duffie, Garleanu, and Pedersen (2005).

[2] Another notable example is DeLong, Shleifer, Summers, and Waldmann (1990a). In that model, risk-averse arbitrageurs attenuate their demands because they must liquidate their positions at uncertain prices that are set, in part, by noise traders.

liquid markets have more noise trading than illiquid markets and rational agents do not fully offset noise traders' demands, then securities prices in liquid markets may be inefficient relative to prices in illiquid markets.

I test these competing predictions in a real-world setting that allows particularly clean tests of whether liquidity affects market efficiency. Specifically, I use data from Arrow-Debreu securities that represent bets on one-day sports and financial events traded on an online exchange, TradeSports.com. These securities possess many advantages for measuring absolute pricing efficiency relative to wagering markets, experimental markets, and conventional financial markets.

Unlike trading in many wagering markets, trading on the TradeSports exchange takes place in standard continuous double auctions, comparable to the mechanisms used in the world's major stock, currency, commodity and derivatives exchanges.[3] Unlike the participants in most experimental markets, many professional traders from Chicago, London, and New York routinely wager thousands of dollars in sports and financial markets on TradeSports. However, unlike securities in conventional financial markets, the TradeSports securities pay a single terminal cash flow at the end of their extremely short horizons, allowing me to directly observe and measure securities' fundamentals (Thaler and Ziemba (1988)).[4] Equally important, most TradeSports securities are exposed to little or no systematic risk, making it straightforward to price these securities. For these

---

[3] In both theory and practice, double auctions appear to be particularly robust mechanisms that promote rapid adjustment towards market equilibrium even in the presence of market frictions and trader irrationality. For theoretical models, see Gjerstad and Dickhaut (1998) and Satterthwaite and Williams (2002). For empirical evidence, see Gode and Sunder (1993), Friedman and Ostroy (1995), Cason and Friedman (1996), and Noussair *et al*. (1998).

[4] Accordingly, this paper's tests of absolute pricing efficiency are not directly comparable to efficiency tests in traditional equity markets. For example, Chordia, Roll, and Subrahmanyam (2006) argue that liquidity increases market efficiency based on evidence from high-frequency return predictability tests. Because their efficiency tests do not employ measures of stocks' fundamentals, one cannot infer whether liquidity increases or decreases absolute pricing efficiency.

reasons, tests of efficiency using TradeSports data nicely complement the evidence from wagering markets, experimental markets, and conventional financial markets.

To measure the liquidity of securities markets on the TradeSports exchange, I rely on two indicators designed to capture O'Hara's (1995) definition of liquid markets: "those that accommodate trading with the least effect on price" (page 216). She suggests two measures that exemplify the cross-sectional dimension and time series dimension of liquidity from the perspective of an individual trader: low bid-ask spreads and low price movements after trading, respectively. The idea is that, in a liquid market, traders can cheaply conduct round-trip trades at a given time, and can do so almost continuously.

Using both the cross-sectional and time series measures of liquidity, I show that more liquid securities markets on TradeSports exhibit significant pricing anomalies, whereas less liquid securities markets do not exhibit these anomalies. In liquid markets, low probability events are overpriced and high probability events are underpriced. This specific mispricing pattern is consistent with Kahneman and Tversky's (1979) theory of individual probability misperception, suggesting that their theory applies to traders who are active in liquid markets.

To interpret this result further, I test two additional implications of the reasoning that liquidity could impede efficiency. First, if liquidity does represent noise trading, then the degree of mispricing in liquid markets will depend on arbitrageurs' incentives to offset noise traders' demands. A key factor in whether arbitrageurs will bet on fundamental information is whether they expect to be able to liquidate their positions before a security expires. In markets that are persistently liquid, arbitrageurs may destabilize prices because they can liquidate their positions before expiration at prices

that may differ from fundamentals. By contrast, in sporadically liquid markets, arbitrageurs cannot reliably liquidate their positions until securities yield their terminal cash flows and prices equal fundamentals. Consistent with these incentives for arbitrageurs, I show that markets with persistently high liquidity in consecutive time periods exhibit substantially greater mispricing than those with currently high, but sporadic, liquidity.

Second, I explore whether liquidity (illiquidity) serves as a proxy for noise (informed) trading, as it does in Kyle (1985) and Glosten and Milgrom (1985). As I explain later, if periods of illiquidity tend to precede the release of information about securities' fundamental values and its incorporation into prices, then illiquidity (liquidity) could represent informed (noise) trading. Indeed, I find strong empirical support for the hypothesis that the prices of illiquid securities converge more quickly toward their terminal cash flows.

Collectively, these results suggest that liquidity is a proxy for noise trading, which can harm informational efficiency, particularly in persistently liquid markets.[5] The findings are consistent with theoretical models in which rational agents face limits to arbitrage, but inconsistent with models in which increases in liquidity have no impact or a favorable impact on efficiency. Although related work identifies similar pricing patterns in wagering markets (*e.g.*, Jullien and Selanie (2000), Wolfers and Zitzewitz (2004), and Zitzewitz (2006)) and financial markets (Rubinstein (1985), Brav and Heaton (1996), and Barberis and Huang (2005)), none of these studies draws a link between securities mispricing and market liquidity. The main contribution of this paper is to show that the

---

[5] These findings are consistent with evidence from experimental markets in Bloomfield, O'Hara, and Saar (2006). The authors show that laboratory markets are less efficient when noise traders are more active.

Kahneman and Tversky (1979) pattern in securities mispricing is largely confined to liquid markets, and does not apply to illiquid markets.

The layout of the paper is as follows. In Section II, I describe the structure of the securities data from the TradeSports exchange and the measures of liquidity used throughout the paper. In Section III, I perform the initial efficiency tests for liquid and illiquid securities markets. In Section IV.A, I examine whether mispricing is greater for persistently liquid securities; in Section IV.B, I explore whether illiquidity is related to the incorporation of information about securities' terminal cash flows into prices. In Section V, I conclude and suggest directions for further research on liquidity and securities market efficiency.

II. Securities Data and Measures of Liquidity

I construct an automatic data retrieval program to collect comprehensive statistics about each security traded on the TradeSports exchange.[6] The program runs at 30-minute intervals almost continuously from March 17, 2003 to October 23, 2006.[7] All empirical tests in this paper include only data from the one-day sports and financial securities recorded by the program. The vast majority of TradeSports' securities are based on one-day sports or financial events, such whether the Yankees will win a particular baseball game or whether the Dow Jones Index will close 50 or more points above the previous

---

[6] I am grateful to TradeSports Exchange Limited for granting me permission to run this program.

[7] The program's 30-minute interval is approximate because it records securities sequentially, implying that the exact time interval depends on whether new securities have been added or subtracted and precise download speeds. In practice, these factors rarely affect the time interval by more than a few minutes. The program stops running only for random author-specific events, such as software installations, operating system updates, power failures, office relocation—and technical TradeSports issues, such as daily server maintenance and occasional changes in the web site's HTML code.

day's close. I focus on these securities to limit the number of factors needed as controls in the statistical analysis that follows.[8] Roughly 70% of TradeSports' securities are based on sports events, 25% are based on financial events and fewer than 5% are based on events in all other categories combined—*e.g.*, political, entertainment, legal, weather, and miscellaneous.

The TradeSports exchange solely facilitates the trading of binary outcome securities by its members, and does not conduct transactions for its own account.[9] Securities owners receive $10 if a pre-specified, verifiable event occurs and $0 otherwise—*e.g.*, the owner of the Dow Jones security mentioned above receives $10 if and only if the index goes up by 50 points or more. For ease of interpretation, the exchange divides its security prices into 100 points, worth $0.10 per point. The minimum price increment, or tick size, ranges between one point for thinly traded securities and 0.1 point for heavily traded securities.

TradeSports levies a commission equal to 0.4% of the maximum securities price ($10) on a per security basis whenever a security is bought or sold.[10] At the time of security expiration, traders must liquidate all outstanding securities positions and incur commissions.[11] Note that the $0.08 round-trip transaction fee is smaller than the value of

---

[8] For sports events, I consider only securities with an official TradeSports categorization that includes either the text "game," "bout," or "match"; for financial events, I consider only securities that do not include "weekly," "monthly," or "yearly." Over 96% and 99.6% of qualifying financial securities expire on the day of or the day after listing, respectively. Over 97% of all the eligible financial securities are based on the level of daily financial indices. Even though virtually all of the uncertainty for qualifying sports and financial events is resolved on the day in which the security expires, some securities are listed and traded on the exchange before the expiration day. I keep all observations within one week of the expiration day.
[9] TradeSports limits the risk that the counterparty in a security transaction will default by imposing stringent margin requirements for each sale or purchase of a security by one of its members. In most cases, members must retain sufficient funds in their TradeSports account to guard against the maximum possible loss on a transaction. TradeSports also settles and clears all transactions conducted on its exchange.
[10] The exchange has recently eliminated commissions for non-marketable limit orders.
[11] Expiration is the time at which the payoff event is verifiable and the owner receives payment.

one point ($0.10) for most securities. This implies arbitrageurs have an incentive to push prices back towards fundamental values if they stray by even one point.

Following conventions in other studies of financial markets, I eliminate observations from the least active markets on the TradeSports exchange, where few traders participate and price data are of poor quality. Specifically, I exclude observations on securities with a cumulative trading volume below 50 securities ($500), market depth below 10 securities ($100), or bid-ask spreads exceeding 10% ($1.00).[12] I compute market depth as the sum of all outstanding buy and sell limit orders within the maximum 10% bid-ask spread. These restrictions are designed to exclude securities without well-established market prices for which tests of efficiency are unlikely to be meaningful.

I use two measures of securities market liquidity to capture its cross-sectional and time series dimensions. I define cross-sectional liquidity as the ability to cheaply conduct a round-trip trade at a given time, and time series liquidity as the ability to trade almost continuously at prices that do not change very much. To maximize the power of the statistical tests that follow, I partition all securities into two equal-sized halves based on the amount of liquidity according to each of these two measures.[13]

The cross-sectional indicator for the amount of market liquidity is a low bid-ask spread. Following convention, I define the spread as the difference between the inside (lowest) ask and (highest) bid quotations. I consider all securities markets with spreads below the median, usually around $0.20, to have "low spreads" and all other securities to

---

[12] The observations excluded by the combination of all three restrictions represent only 10% of volume on the exchange. Using more stringent market activity criteria tends to strengthen the statistical results below.
[13] Other partitions, such as quartiles based on the amount of liquidity, produce similar results.

have "high spreads."[14] To avoid any look-ahead bias in the partition cutoff value, I use the median spread from the distribution of all qualifying observations on TradeSports securities over the prior six calendar months.

The time series indicator of market liquidity is based on the absolute deviation between the bid-ask midpoint and the previous transaction price, which I call post-trade price movement. I infer that securities markets with large post-trade price movements are costly to trade, or illiquid. In these markets, a buyer or seller cannot repeatedly trade at the same price, which represents a transaction cost. For example, suppose that the last trade price is 66 points and the current bid and ask quotations are 67 and 69 points, implying the midpoint is 68 points. If the last trade was buyer-initiated and the next trade will execute at the bid-ask midpoint, the same buyer would have to pay two points more (68 minus 66) for the security. Thus, high post-trade price movement can be viewed as a proxy for high price impact and illiquidity. I categorize all trades with below-median post-trade price movement, usually around $0.10, as "low-impact" securities and all others as "high-impact" securities. I use the median post-trade price movement from all TradeSports securities over the past six months as the cutoff value.

One would hope that two different measures designed to capture the same liquidity phenomenon would exhibit some degree of similarity. Indeed, I find that the correlation between the logarithms of bid-ask spread and post-trade price movement is over 60%, which is strongly statistically significant at any level.[15] This is comforting in

---

[14] In the first six months of data, I use the median of all spreads in the sample to date. In general, using the median spread from the previous six months as the cutoff does not divide securities into two groups of identical size because of lumpiness in the distribution of spreads. Using *ad hoc* spread cutoffs of $0.10, $0.20 and $0.30 produces qualitatively similar results.
[15] I compute the logarithms of post-trade price movement and spreads to reduce the substantial skewness in these measures before computing their correlation. Before taking logs, I add 0.1 to post-trade price movements. This allows me to use observations with zero post-trade price movement, and makes the

that the two measures, one cross-sectional and one time series, probably describe some common aspect of market activity.[16]

III. Tests of Market Efficiency

Now I analyze the absolute pricing efficiency of securities on the TradeSports exchange. I conduct these tests separately for the liquid securities, the illiquid securities, and all securities. I also analyze the efficiency of sports and financial markets separately based on the results in Tetlock (2004), which reports evidence that pricing in these markets differs significantly. Fortunately, the key results in this study apply to both sports securities and financial securities, regardless of their exposure to market risk.[17]

Microstructure theory measures absolute pricing efficiency as the expected mean squared error of prices minus cash flows, which can be decomposed into two components. First, I focus on measuring the squared bias component of absolute pricing efficiency, as opposed to the conditional variance component. This emphasis is standard practice in the literature on market efficiency in binary prediction markets because expected cash flows are not directly observable for each security. Thus, researchers cannot measure securities' conditional variances without making assumptions about biases, but they can estimate average biases by comparing securities' average cash flows to their prices. In this section, I estimate biases in prices following conventions from prior

variable's minimum value equal to the minimum spread of 0.1 point. The correlation between raw spreads and raw post-trade price movements is still greater than 45%.

[16] I find similar results using alternative measures based on the common component of cross-sectional and time series liquidity—*e.g.*, linear combinations of bid-ask spreads and post-trade price movement.

[17] In unreported tests, I allow for the possibility that financial securities with positive exposure to market risk have different expected returns from those with negative risk. I find a positive, but insignificant, risk premium of less than 1% for the typical financial security with positive exposure to the market. This is not surprising because three years of data is usually insufficient for estimating market risk premiums.

work—*e.g.*, Wolfers and Zitzewitz (2004), Tetlock (2004), and Zitzewitz (2006). In the subsequent section, I make additional assumptions in an effort to indirectly estimate the conditional variance component of mean squared error.

I employ a straightforward regression methodology to test the null hypothesis that securities prices are unbiased predictors of securities' cash flows. A single observation consists of a security's current price and its returns until expiration. I measure all current prices using the midpoints of the inside bid and ask quotations to avoid the problem of bid-ask bounce that could affect transaction prices.[18] I calculate a security's percentage returns to expiration by subtracting its current price from its expiration value, which is either 0 or 100 points, then dividing by 100 points.[19] The null hypothesis is that securities' expected returns to expiration are zero, regardless of the current securities price. The alternative hypothesis is that Kahneman and Tversky's (1979) theory of probability perception describes the pattern of expected returns across securities with different current prices.

The S-shaped form of the probability weighting function hypothesized in Kahneman and Tversky (1979) and formalized in Prelec (1998) informs my choice of pricing categories and statistical tests. The S-shape refers to a graph of subjective versus objective probabilities, as shown first in Kahneman and Tversky (1979). Prelec (1998) derives their theory of probability misperception from axiomatic foundations. He predicts that agents overestimate the likelihood of events with objective probabilities less than $1/e$ = 36.79% and underestimate the likelihood of events with objective probabilities greater than $1/e$. There is also an ample body of empirical evidence that is consistent with a

---

[18] All results are robust to using the most recent transaction price instead.
[19] I exclude the very small fraction of TradeSports contracts that do not expire at 0 or 100 points. I divide by 100 points to represent the combined amount of capital that buyers and sellers invest in the security.

probability weighting function having a fixed point in the neighborhood of $1/e$ (Tversky and Kahneman (1992), Camerer and Ho (1994), and Wu and Gonzalez (1996)).

Based on this evidence, I construct dummy variables, Price1 through Price5, for five equally-spaced pricing intervals: (0,20), [20,40), [40,60), [60,80), and [80,100) points. I then measure the returns until expiration for securities in each pricing category. I test the null hypothesis that all returns to expiration are equal to zero against the alternative that securities in the first two categories—Price1 and Price2—based on small probability events ($p < 40\%$), are overpriced and securities in the last three categories— Price3, Price4, and Price5—based on large probability events ($p > 40\%$), are underpriced.

I report the results from three Wald (1943) tests based on this simple idea. The first Wald test measures whether small probability events are overpriced on average:[20]

(1)   (Price1 + Price2) / 2 = 0

The second Wald test assesses whether large probability events are underpriced:

(2)   (Price3 + Price4 + Price5) / 3 = 0

The third Wald test measures whether large probability events are more underpriced than small probability events—*i.e.*, whether the mispricing function is S-shaped:

(3)   (Price3 + Price4 + Price5) / 3 − (Price1 + Price2) / 2 = 0

Of the three, this is the most powerful test of the null hypothesis against the Kahneman and Tversky (1979) alternative because it accounts for other factors that could influence the level of mispricing of both small and large probability events.[21]

---

[20] Despite the specific nature of over- and underpricing predicted by Kahneman and Tversky (1979), I use two-tailed Wald tests to be conservative.

[21] The choice of how to partition the pricing categories has little effect on the Wald tests because, regardless of the partitioning, these tests assess whether the returns to expiration of securities priced below 40 points differ from the returns of securities priced above 40 points.

I use standard ordinary least squares to estimate the coefficients of the five pricing categories. For all regression coefficients, I compute robust standard errors to account for the repeated sampling of the same security over multiple time periods and the sampling of different securities based on related events. I employ the clustering methodology developed by Froot (1989) to allow for correlations in the error terms of all securities expiring on the same calendar day, which simultaneously corrects for the repeated sampling of the same security and the sampling of related events.[22] This clustering procedure exploits the fact that all event uncertainty is resolved on the day of expiration (see footnote 8).

To illustrate the efficiency tests and give an overview of the data, I first examine the returns to expiration for all sports securities, all financial securities, and both groups together. Table I displays the regression coefficient estimates for Price1 through Price5 along with the three Wald tests described above. The main result is that neither sports nor financial securities exhibit substantial mispricing, which is consistent with Wolfers and Zitzewitz (2004) and Tetlock (2004).

[Insert Table I around here.]

The qualitative patterns in the pricing of both sets of securities and in their aggregate suggest, however, that the probability weighting function could play a role in any mispricing that does exist. To aid the reader in identifying the S-shaped pattern, Figure 1 provides a visual representation of mispricing in each pricing category for sports, financial, and both types of securities.

[Insert Figure 1 around here.]

---

[22] Using a finer clustering unit based on the expiration day and type of security does not affect the results.

The securities based on small probability sports events in pricing categories 1 and 2 appear to be overpriced by 1.70 points (p-value = 0.063) and financial securities based on large probability events in categories 3, 4, and 5 are underpriced by 2.28 points (p-value = 0.024). The Wald test for the S-shaped pattern rejects the null hypothesis that returns do not differ across pricing categories at the 5% level for both sports and financial securities. Interestingly, the magnitude of mispricing decreases from an average of 2.55 points across the sports and financial groups to just 1.69 points in the aggregate group, which is barely statistically significant at the 5% level. This reduction occurs because of differences in the pricing patterns of sports and financial securities and the changing relative composition of sports and financial securities within pricing categories.[23]

Having established that both sports and financial securities show a limited degree of inefficiency, I now turn to the key test of whether the S-shaped Kahneman and Tversky (1979) pattern is more pronounced in the illiquid or liquid securities. Table II and Table III each report the results from nine regressions that attempt to address this question using the cross-sectional and time series measures of liquidity. Each table includes separate regression results for sports, financial and both types of securities sorted by their degree of liquidity. The tests for differences in the coefficients (Columns Three, Six, and Nine) come from joint regressions in which I estimate coefficients on Price1 through Price5 for both liquid and illiquid securities simultaneously by adding five interaction terms—between liquidity and Price1 through Price5.

[Insert Table II around here.]

---

[23] The disparity between the average of the individual estimates and the aggregate estimate illustrates the importance of estimating the effects on sports and financial securities separately.

Table II reports the returns to expiration of various securities sorted by whether their bid-ask spreads fall below the median spread on the TradeSports exchange during the previous six-month period. For both sports and financial securities, the S-shaped probability weighting function pattern is strongly statistically and economically significant only in the securities with high cross-sectional liquidity. For example, the S-shaped pattern is virtually nonexistent in the illiquid sports securities (0.75 points), but is quite pronounced in the liquid sports securities (4.60 points, p-value = 0.004). Similarly, the S-shaped pattern is small in the illiquid financial securities (2.27 points), but strongly significant and large in the liquid financial securities (6.11 points, p-value = 0.001). Moreover, both of the two tests for whether the S-shaped patterns are more pronounced in the liquid securities than in the illiquid securities reject the null hypothesis at the 5% level. Again, the rejection of the null hypothesis is slightly weaker for the aggregate of sports and financial securities, but still significant at the 5% level.

Note that the signs of 17 of the 20 individual coefficients on the liquid securities and interaction terms in the sports and financial security regressions agree with the predictions of the S-shaped probability weighting function. This precise pattern in mispricing is highly unlikely to occur by chance (p-value = 0.001). Indeed, the qualitative pattern in the coefficients explains why, even though only a few of the individual coefficients on the pricing category dummy variables are statistically significant, the Wald tests for the S-shape easily reject the null hypothesis of zero expected returns.

The effect of liquidity on the magnitude of the S-shaped pattern is similar for sports and financial securities (3.85 points and 3.84 points). Using the same bid-ask spread or price impact cutoff values for both types of securities, however, I find that a

much greater fraction of the sports securities fall into the high liquidity classification (more than 50%) relative to the financial securities (fewer than 20%), regardless of the liquidity measure.[24] One interpretation is that liquidity is a proxy for non-informational or noise trading, which is more widespread in sports securities. Nevertheless, the relationship between liquidity and market efficiency is similar for both security types.

Table III shows the same analysis in Table II for securities sorted by their degree of time series liquidity. Specifically, Table III reports the returns to expiration for each of the five pricing categories for securities sorted by whether their post-trade price movement falls below the median post-trade price movement on the exchange during the previous six-month period. Columns One and Four establish that the liquid (low post-trade price movement) sports and financial securities have returns to expiration that exhibit a strong S-shaped pattern across pricing categories of roughly five points. By contrast, Columns Two and Five show that the illiquid (high post-trade price movement) securities exhibit at most weak evidence of an S-shaped pattern. Again, for both sports and financial securities, the S-shaped pattern is much larger in liquid securities—by magnitudes of 4.61 points and 2.98 points, respectively, both significant at the 1% level.

[Insert Table III around here.]

Figure 2 graphically represents the difference in returns to expiration for the illiquid and liquid sports and financial securities in each of the five pricing categories from Columns Three and Six of Table II and Table III. The figure shows the interaction terms from the regressions in these tables, which separately measure the effect of the two liquidity measures on mispricing. The vertical axis shows the returns to expiration for each security grouping while the horizontal axis shows the pricing categories. The visual

---

[24] Some disparity in these fractions is inevitable because I use the same cutoff value for both security types.

impression from the figure confirms the statistical results in Table II and Table III: the differences in returns to expiration for both security types show distinct S-shaped patterns.

[Insert Figure 2 around here.]

In the liquid securities, the overpricing of low probability events is much larger, especially in sports securities, and the underpricing of high probability events is far more severe, especially in financial securities. The differences between the pricing category coefficients have the sign predicted by the S-shaped pattern in 18 out of 20 cases, which is virtually impossible to occur by chance (p-value < 0.001). This visual and intuitive evidence is consistent with the numerical impression from the tables. If one accepts market liquidity as an adequate proxy for noise trading, these results suggest that noise traders cause significant harm to pricing efficiency in exactly the manner predicted by the classic S-shaped probability weighting function.

It is important to recognize that the findings summarized in Figure 2 are distinct from the well-known empirical relationships between liquidity risk and expected returns in traditional financial markets (Pastor and Stambaugh (2003)). On the TradeSports exchange, there is little room for interpreting the expected returns on liquid securities as compensation for risk because the sports securities exhibit the same pattern as the financial securities even though they are not susceptible to systematic risks. In fact, the evidence here suggests a logical alternative to risk-based explanations: the expected returns in liquid securities could be attributable to the probability misperceptions of the agents who are most active in liquid markets.

17

Next, I evaluate the possibility that other security characteristics, such as return volatility, trading volume, order imbalance, and time horizon, could explain the relationship between mispricing and liquidity. I measure these effects using 20 (4x5) interaction terms between each of the four control variables and the five pricing category dummy variables. This estimation procedure is directly analogous to the procedure used to measure the effect of liquidity on efficiency. It also allows me to use the same Wald tests described in Equations (1) through (3) to assess whether volatility, volume, order imbalance, or time horizon effects could explain the S-shaped mispricing pattern.

To implement the Wald tests for the control variables, I convert volatility, volume, order imbalances, and time horizon into dummy variables before generating the necessary interaction terms. Analogous to the procedure used to generate the liquidity dummies, I partition all observations into two equal-sized halves based on market depth to generate dummies for above-median values of each control variable, using rolling six-month windows to compute the medians. The full-sample median value cutoffs for the first three control variables are: 0.5 points ($0.05) per 30 minutes for the volatility dummy, 213 securities ($2,130) for the volume dummy, and 4.47% of market depth for the order imbalance dummy.[25] To capture the effect of information arrival, I create a time horizon dummy that is equal to one if the observation occurs before the event begins.[26]

The regression coefficients in Columns One, Two, and Three in Table IV reveal that the effect of liquidity on pricing efficiency is not particularly sensitive to the

---

[25] Imposing *ad hoc* full-sample cutoffs for volume, volatility, and order imbalance produces similar results.
[26] Because I do not know precisely when each event begins, I assume that the event is in progress once the securities price has moved by a "significant" amount, five or more points, relative to the first trading price. I use this definition for the time horizon dummy to avoid any potential look-ahead bias in knowing when the event will end. I find similar results for alternative definitions of the time horizon dummy.

inclusion of additional interaction terms.[27] Columns Two and Three report regression results using bid-ask spreads and post-trade price movement as the liquidity measures. Column One uses a liquidity dummy equal to one for securities with both cross-sectional and time series liquidity above their respective median values. The estimates from all three columns show that the S-shaped mispricing pattern changes very little when I allow for the possibility that volatility, volume, order imbalances, and time horizon affect mispricing. The magnitude of the S-shape remains roughly three or four points and is statistically significant at the 5% or 1% level, depending on the liquidity measure used.

[Insert Table IV around here.]

Rather than report the individual coefficients, I summarize the results from the Wald tests based on the 20 coefficient estimates of the control variable interaction terms in Table IV. I cannot reject the null hypothesis for the joint significance of the 20 control variables at even the 10% level. None of the four sets of control variable interactions exerts a significant effect on mispricing at the 5% level. None of the four sets of coefficients shows the same S-shaped pattern as the liquidity coefficients—*i.e.*, no estimate is statistically significant and the largest absolute value is less than two points. Overall, the inclusion of the controls based on volatility, volume, order imbalance, and time horizon appears to introduce estimation error without increasing the regression's explanatory power. Proponents of market efficiency can take comfort in the fact that few variables, other than liquidity and price, are useful predictors of returns to expiration.

---

[27] Rather than run separate regressions for sports and financial securities, to conserve space, I include five interaction terms that account for differences in mispricing across the two types.

IV. Interpreting Mispricing and Liquidity

In this section, I examine whether certain types of liquid securities exhibit greater mispricing, and explore the nature of liquidity on the TradeSports exchange. Both sets of tests shed light on the hypothesis that liquidity is a proxy for noise trading, which adversely affects market efficiency.

IV.A Variation in Mispricing within Liquid Securities

In the first set of tests, I analyze how mispricing varies within the liquid securities identified in Section III. If liquidity does represent noise trading, then the extent to which informed arbitrageurs' bet against noise traders will determine the degree of mispricing in liquid markets. I use two variables to measure informed traders' ability or willingness to offset the demands of noise traders.

First, I construct an indicator of persistent liquidity in consecutive time periods, which could reduce arbitrageurs' incentives to trade on fundamental information. For example, theoretical models such as DeLong, Shleifer, Summers, and Waldmann (1990a and 1990b) suggest that informed traders are more reluctant to offset the demands of noise traders, and may even reinforce their demands, when there is a risk that mispricing will worsen in the short-run. Such increases in mispricing can only occur in securities with persistent liquidity, which is necessary for arbitrageurs to liquidate their positions at prices that may differ from securities' fundamental values. I consider securities that qualify for the high liquidity classification in three consecutive data recording periods,

over one hour, to exhibit "persistent liquidity," and all others to exhibit "sporadic liquidity."[28] Because the high liquidity classification shows strong positive serial correlation, 43.9% of liquid securities meet the criterion for persistent liquidity.[29]

Second, I test whether arbitrageurs are more or less effective in counteracting mispricing in liquid securities with greater trading volume. One view is that arbitrageurs have greater incentives to offset mispricing in securities with greater transaction volumes. A competing view is that securities with high trading volume are likely to have more noise traders, whose demands are not fully offset by increased informed trading. To discriminate among these two hypotheses, I assess whether the degree of mispricing in liquid securities is related to trading volume.

The regressions in Table V include estimates of the effect of persistent liquidity and high trading volume on mispricing in liquid securities. The most notable result is that securities with persistent high liquidity show significantly greater mispricing than securities with sporadically high liquidity. The test for whether the S-shaped pattern is equally pronounced in these two groups rejects the null hypothesis at the 1% level. This result suggests that persistent liquidity does correspond to the classic theoretical risk that mispricing could worsen in the short-run. This interpretation is that arbitrageurs do not offset, and may even reinforce, noise traders' probability misperceptions in securities that exhibit persistent liquidity. However, arbitrage is quite effective in the securities with sporadically high liquidity, where mispricing is 73% smaller (2.59 points vs. 9.62 points).

---

[28] The results are similar for alternative definitions of persistent liquidity, such as high liquidity in two or five periods in a row. Using cutoff values higher than five eliminates many events from the sample. In these events, trading activity lasts for only a few hours because the event itself lasts for only a few hours.
[29] Less than 0.2% of the persistently liquid securities are financial securities because high levels of liquidity are not persistent or common in financial securities. Using a less stringent criterion, I confirm that the qualitative results are similar for financial securities that exhibit relatively persistent liquidity.

In these securities, arbitrageurs effectively offset noise traders' demands because they expect the security to yield a payoff equal to its fundamental value at expiration.

[Insert Table V around here.]

Figure 3 visually compares the returns to expiration of securities with low liquidity, those with high but sporadic liquidity, and those with persistently high liquidity. The immediate impression from the figure is that the dark gray bars representing the mispricing of persistently liquid securities are very large relative to the mispricing bars for other securities. Comparing the white and the light gray mispricing bars, one sees that the S-shaped pattern is only slightly larger in securities with high but sporadic liquidity relative to securities with low liquidity (2.59 points vs. 1.09 points). That is, there is only slight mispricing in securities without persistent liquidity even when there is currently high liquidity.

[Insert Figure 3 around here.]

The second set of tests in Table V provides further insight into how mispricing varies within liquid securities. Columns Four, Five, and Six show that the liquid securities with greater trading volume are less efficiently priced, not more, than the liquid securities with lower trading volume. Although this effect is not as large or robust as the effect of persistent liquidity, it is surprising that arbitrage is less effective in high-volume, liquid securities where arbitrageurs would seem to have larger incentives to counteract mispricing. One possible resolution to this puzzle is that heavy trading in liquid securities is a proxy for large demands from noise traders, which arbitrageurs only partially offset.[30]

---

[30] In Table IV, I use trading volume as a control variable in regressions that include liquid and illiquid securities, finding that the overall effect of trading volume on mispricing is not large. Thus, the results in Table V show that trading volume primarily affects mispricing in liquid securities.

22

As a final check on the magnitude and relevance of the mispricing in securities with high cross-sectional and time series liquidity, I explore the profitability of a simple trading strategy based on the pattern of mispricing identified in Column One of Table IV.[31] A natural trading strategy would be to sell all of the overpriced liquid securities in pricing categories 1 and 2, and buy all of the underpriced liquid securities in pricing categories 3, 4, and 5. I augment this simple rule to reflect the stylized fact that the overpricing of low probability events is more severe in sports securities. I also disregard the financial securities because these are few in number and could be susceptible to systematic risk. Thus, I analyze the trading strategy that sells liquid sports securities in pricing categories 1 and 2.[32]

To make this trading strategy implementable, I assume that a trader submits a market order to TradeSports as soon as the automated data retrieval program records a price on the exchange. This means that all buy orders execute at the lowest asking price and all sell orders execute at the highest bid price at the time of retrieval. Unfortunately, the trader must bear a substantial liquidity cost, which he or she could possibly avoid by using a limit order.[33] In addition, I assume that the trader must incur the maximum round-trip commission on TradeSports, which is 0.8% per round-trip. This set of assumptions leads to a conservative estimate of realizable trading returns.

Despite the substantial liquidity and commission costs of implementing the strategy, Table VI reports that selling the liquid sports securities in pricing categories 1 and 2 yields realizable expected returns of 3.42% and 2.94% over the time span of a day

---

[31] Obviously, one can improve upon this trading strategy using the information in Table V.
[32] A number of closely related trading strategies yield profits that are comparable in magnitude.
[33] Strategies using complex limit order rules may be profitable, but this is extremely difficult to evaluate because of the adverse selection problem associated with the execution of standing limit orders.

or so.[34] Table VI also shows that buying the liquid financial securities in pricing

categories 3, 4, and 5 produces expected returns of 10.17%, 9.38% and -0.08%. Figure 4

visually represents this evidence on returns to expiration for buyers and sellers of sports

and financial securities. The expected returns in Table VI and Figure 4 are not weighted

by the amount of capital that could be invested in each security and do not account for the

cost of obtaining this capital.

[Insert Table VI around here.]

[Insert Figure 4 around here.]

I make three additional assumptions to address these issues and obtain a

conservative estimate of the total dollar profits from the strategy that sells liquid sports

securities in pricing categories 1 and 2. First, I assume that a retail investor could

establish a line of credit allowing her to access capital at an annualized interest rate of

10%, or 0.027% per day.[35] Second, to simplify strategy implementation, I consider a

strategy based on short-selling only the liquid sports securities in pricing categories 1 and

2 that expire on the same calendar day in which the data retrieval program records their

prices.[36] Third, I assume that the investor submits a market sell order for the quantity of

securities exactly equal to the size of the current inside bid quote.

I find that this trading strategy earns an average of only $390 per day over the

course of the 471 days in which it is implementable. Restated in more familiar terms, the

strategy yields a total of $184,000 during the 1,109-day data sample, or $60,000 per year.

Based on this analysis, there appears to be sufficient competition among arbitrageurs on

---

[34] The vast majority of these securities expire within the same day.

[35] I assume that the investor pays a full day's worth of interest for funds used less than 24 hours.

[36] To be conservative, I assume traders must post margin equal to the maximum possible loss in order to sell a security—*e.g.*, a trader must post $8 to sell a security priced at $2 because it could expire at $10.

the TradeSports exchange to ensure that realizable trading profits do not become too large, which is consistent with endogenous information acquisition models—*e.g.*, Grossman and Stiglitz (1980). A wage of $60,000 per year is a reasonable estimate of the equilibrium compensation for actively monitoring the securities on the exchange.

IV.B Is Liquidity a Proxy for Noise Trading?

Although the previous tests provide indirect evidence that liquidity could serve as a proxy for non-informational or noise trading, I have not yet established a direct relationship between liquidity and informed or non-informational trading. In this section, I attempt to infer the relative amount of informed trading by examining the behavior of prices following periods of liquidity and periods of illiquidity. If periods of illiquidity tend to precede the release of information about securities' fundamental values and its incorporation into prices, I infer that illiquidity (liquidity) could represent informed (noise) trading.

I measure the extent of information release during an event as the convergence of prices toward securities' terminal payoffs at expiration—*i.e.*, the reduction in the squared difference between prices and terminal payoffs. This measurement of information release requires two key assumptions. First, the degree of informed trading prior to an event must be a monotonic function of the amount of information released during and after the event. Second, I assume that reductions in mean squared error occur only from reduction in the conditional variance component—*i.e.*, the squared bias component does not change. This may be a reasonable approximation for short time horizons, but the previous results

suggest this assumption will fail for long time horizons as squared bias approaches zero. Because liquid markets exhibit greater mispricing that is corrected over time, periods of liquidity will appear to precede greater information release even if there is no true relationship. This reasoning suggests my second assumption is conservative in the sense that it will be difficult to interpret liquidity as a proxy for non-informational or noise trading.

Under the two assumptions above, a comparison of the reduction in mean squared error following periods of liquidity and periods of illiquidity reveals whether illiquidity (liquidity) is a plausible proxy for informed (noise) trading. If prices converge faster toward fundamentals after periods of illiquidity (liquidity), I infer that these periods are characterized by relatively more informed (noise) trading. I use a regression methodology to explore whether illiquidity precedes information release in the next data recording period, and a visual representation to show the evolution of information release in the ten data recording periods surrounding the time of liquidity measurement.

Table VII displays two regression estimates of the effect of liquidity on the reduction in mean squared error over the next data recording period (roughly 30 minutes). There is one regression for each of the two measures of liquidity: the logarithm of bid-ask spread (*LnSpread*) and the logarithm of post-trade price movement (*LnImpact*). Each regression includes control variables for expected information release through expiration (*ExpInfo*), whether the event is in progress (*InEvent*), two measures for the time elapsed since initial security listing (*LnTime* and *LnMoves*), and the trading volume (*LnVolHr*)

26

and volatility (*LnAbsRet*) during the prior period.[37] To eliminate the influence of omitted

factors that could affect information release and market-wide liquidity, all regressions

also include time dummy variables for each data recording period. I report the coefficient

estimates and standard errors based on the equal-weighted averages from monthly cross-

sectional regressions because these estimates are the most conservative (Fama and

MacBeth (1973)).[38] The coefficient standard errors are robust to heteroskedasticity and

autocorrelation up to three lags in the monthly time series of coefficients (Newey and

West (1987)).

[Insert Table VII around here.]

For both liquidity measures, periods of illiquidity precede the release of

information about securities' terminal payoffs over the next 30 minutes or so. The

coefficient estimates on the *LnSpread* and *LnImpact* illiquidity variables are extremely

statistically robust—both have p-values less than 0.001 and t-statistics comparable to

even the expected information release (*ExpInfo*) variable. The magnitudes of the

illiquidity coefficients are also comparable to the strongest predictors of information

release, such as expected information release (*ExpInfo*) and the two event timing

---

[37] *ExpInfo* is defined as (100 – Price) * Price, which is the expected value of squared returns to expiration
under the assumption of market efficiency. *InEvent* is a dummy variable indicating whether the securities'
price has moved by five points or more during the expiration day. *LnTime* is the logarithm of one plus the
number of hours elapsed since the first time the security was recorded. *LnMoves* is the logarithm of one
plus the number of five-point or more price movements since initial listing. *LnAbsRet* is the logarithm of
one plus the absolute value of returns over the prior 30-minute period. *LnVolHr* is the logarithm of one plus
hourly volume, extrapolated from the preceding 30-minute period. In robustness checks, I find that the
liquidity coefficient estimates are not particularly sensitive to the use of alternative sets of control variables.
[38] I do not include regressions from months with fewer than 200 observations. Including all months and
attributing greater weights to months with more observations produces smaller standard errors. Performing
pooled OLS estimates also produces smaller standard errors.

variables (*LnTime* and *LnMoves*).[39] These results suggest that liquidity could proxy for

non-informational or noise trading, as it does in Kyle (1985) and Glosten and Milgrom

(1985). In the 30 minutes following liquidity measurement, the prices of liquid securities

converge much more slowly toward their terminal cash flows.

To show how this convergence takes place over longer time periods, Figure 5

presents the cumulative reductions in mean squared error for liquid and illiquid securities

in the ten data recording periods surrounding the time of liquidity measurement. For ease

of interpretation, the figure also depicts the difference between the information release

during a liquidity event and an illiquidity event. To reduce the impact of event timing on

the analysis, I include only observations that occur after the underlying sports or financial

event has begun, as defined by the *InEvent* dummy variable described earlier.[40]

[Insert Figure 5 around here.]

As expected, the prices of both liquid and illiquid securities monotonically

converge toward fundamentals as time passes. However, this convergence does not take

place at the same rate. In the two hours after an illiquidity event, prices incorporate

information much faster than they do after a liquidity event. This result holds for both

sports and financial securities. A notable aspect of Figure 5 is that liquidity events tend to

follow periods in which prices incorporate a lot of information, whereas illiquidity events

precede these informative periods. A simple interpretation is that informed trading

precedes informative events, which subsequently attract the interest of noise traders. In

---

[39] To interpret the coefficient magnitudes, I standardize each coefficient using each independent variable's full-sample standard deviation. For example, a one-standard-deviation change in *LnSpread* would predict the movement of a security priced at 89.7 to its eventual expiration value of 100—*i.e.*, $(100 – 89.7)^2 =$ 106.3 squared percentage points is the coefficient magnitude.

[40] I find similar results comparing information release in liquid and illiquid securities before the underlying event has begun. The only qualitative difference is that prices incorporate almost no information prior to the liquidity or illiquidity events that occur before the event has begun.

summary, Table VII and Figure 5 provide strong statistical and visual support for the interpretation that liquidity is a proxy for non-informational or noise trading.

V. Conclusions

Using data from conventional financial markets to uncover the complex relationships between liquidity and market efficiency is a daunting task. Rather than confront this task directly, I conduct tests of absolute pricing efficiency using TradeSports data on simple short-horizon securities with observable fundamentals and negligible exposure to systematic risk. The hope is that identifying empirical regularities in these simple sports and financial securities can inform future theoretical and empirical studies of more complex environments.

In this setting, tests of market efficiency consistently reject theoretical models in which liquidity either does not affect or enhances informational efficiency. In particular, securities markets with persistently high liquidity show significant pricing anomalies, such as overpricing low probability events and underpricing high probability events. These pricing patterns correspond closely to the predictions of Kahneman and Tversky's (1979) probability weighting function, suggesting that their theory applies to noise traders, who dominate pricing in persistently liquid markets. Conversely, the sporadically liquid and illiquid securities markets are remarkably efficient. A leading explanation is that illiquid markets have fewer noise traders, and periods of illiquidity prevent arbitrageurs from profiting on short-term trades that would destabilize prices.

Additional statistical tests provide support for the idea that liquidity serves as a proxy for non-informational or noise trading. The key finding is that the prices of illiquid securities converge toward terminal cash flows much more rapidly than the prices of liquid securities. This implies that non-informational or noise trading is prevalent during periods of liquidity, which may help explain the observed mispricing in liquid securities.

Although these results are unlikely to generalize without modification to conventional financial markets with long-horizon securities and larger stakes, they do suggest three interesting directions for future research. First, liquidity may only appear to be a priced risk factor because it captures some systematic element of mispricing. Second, different types and sources of liquidity may have opposing effects on the costs of arbitrage and equilibrium mispricing—*e.g.*, liquidity from noise trading may harm efficiency, whereas liquidity from low search costs may enhance efficiency. Third, because there appear to be significant limits to arbitrage on an online exchange with few capital constraints and securities that expire within a single day, the limits to arbitrage in conventional markets may be more severe than previously thought.

References

Barberis, Nicholas and Ming Huang, 2005, Stocks as Lotteries: The Implications of Probability Weighting for Security Prices, Yale University Working Paper.

Bloomfield, Robert, Maureen O'Hara, and Gideon Saar, 2006, The Limits of Noise Trading: An Experimental Analysis, Cornell University Working Paper.

Brav, Alon, and John B. Heaton, 1996, Explaining the Underperformance of Initial Public Offerings: A Cumulative Prospect Utility Approach, Duke University Working Paper.

Camerer, Colin and Teck H. Ho, 1994, Violations of the Betweenness Axiom and Nonlinearity in Probability, *Journal of Risk and Uncertainty* 8, 167-196.

Cason, Timothy and Daniel Friedman, 1996, Price Formation in Double Auction Markets, *Journal of Economic Dynamics and Control* 20, 1307-1337.

Chordia, Tarun, Richard Roll, and Avanidhar Subrahmanyam, 2006, Liquidity and Market Efficiency, Emory University Working Paper.

DeLong, J. Bradford, Andrei Shleifer, Lawrence H. Summers, and Robert J. Waldmann, 1990a, Noise Trader Risk in Financial Markets, *Journal of Political Economy* 98, 703-738.

DeLong, J. Bradford, Andrei Shleifer, Lawrence H. Summers, and Robert J. Waldmann, 1990b, Positive Feedback Investment Strategies and Destabilizing Rational Speculation, *Journal of Finance* 45, 379-395.

Duffie, Darrell, Garleanu, Nicolae, and Lasse Heje Pedersen, 2005, Over-the-Counter Markets, *Econometrica* 73, 1815-1847.

Fama, Eugene F., 1970, Efficient Capital Markets: A Review of Theory and Empirical Work, *Journal of Finance* 25, 383-417.

Fama, Eugene F., 1991, Efficient Capital Markets: II, *Journal of Finance* 46, 1575-1617.

Fama, Eugene F., and James D. MacBeth, 1973, Risk, Return, and Equilibrium: Empirical Tests, *Journal of Political Economy* 81, 607-636.

Friedman, Daniel, and Joseph Ostroy, 1995, Competitivity in Auction Markets: An Experimental and Theoretical Investigation, *Economic Journal* 105, 22-53.

Froot, Kenneth A., 1989, Consistent Covariance Matrix Estimation with Cross-Sectional Dependence and Heteroskedasticity in Financial Data, *Journal of Financial and Quantitative Analysis* 24, 333-355.

Gjerstad, Steven, and John Dickhaut, 1998, Price Formation in Double Auctions, *Games and Economic Behavior* 22, 1-29.

Glosten, Lawrence, and Paul Milgrom, 1985, Bid, Ask and Transaction Prices in a Specialist Market with Heterogeneously Informed Traders, *Journal of Financial Economics* 14, 71-100.

Gode, Dhananjay K., and Shyam Sunder, 1993, Allocative Efficiency of Markets with Zero-Intelligence Traders: Markets as a Partial Substitute for Individual Rationality, *Journal of Political Economy* 101, 119-137.

Grossman, Sanford J., and Joseph Stiglitz, 1980, On the Impossibility of Informationally Efficient Markets, *American Economic Review* 70, 393-408.

Jullien, Bruno, and Bernard Selanie, 2000, Estimating Preferences under Risk: The Case of Racetrack Bettors, *Journal of Political Economy* 108, 503-530.

Kahneman, Daniel, and Amos Tversky, 1979, Prospect Theory: An Analysis of Decision under Risk, *Econometrica* 47, 263-291.

Kumar, Alok, and Charles Lee, 2006, Retail Investor Sentiment and Return Comovements, *Journal of Finance* 61, 2451-2486.

Kyle, Albert S., 1985, Continuous Auctions and Insider Trading, *Econometrica* 53, 1315-1336.

Newey, Whitney K., and Kenneth D. West, 1987, A Simple, Positive-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix, *Econometrica* 55, 703-708.

Noussair, Charles, Stephane Robin, and Bernard Ruffieux, 1998, The Effect of Transactions Costs on Double Auction Markets, *Journal of Economic Behavior and Organization* 36, 221-233.

O'Hara, Maureen, 1995, *Market Microstructure Theory*, Blackwell Publishing.

Pastor, Lubos, and Robert Stambaugh, 2003, Liquidity and Expected Returns, *Journal of Political Economy* 111, 642-685.

Prelec, Drazen, 1998, The Probability Weighting Function, *Econometrica* 66, 497-527.

Rubinstein, Mark, 1985, Nonparametric Tests of Alternative Option Pricing Models Using All Reported Trades and Quotes on the 30 Most Active CBOE Option Classes from August 23, 1976 through August 31, 1978," *Journal of Finance* 40, 455-480.

Sadka, Ronnie, and Anna Scherbina, 2006, Analyst Disagreement, Mispricing, and Liquidity, *Journal of Finance*, forthcoming.

Satterthwaite, Mark A., and Steven R. Williams, 2002, The Optimality of a Simple Market Mechanism, *Econometrica* 70, 1841-1863.

Tetlock, Paul C., 2004, How Efficient Are Information Markets? Evidence from an Online Exchange, University of Texas Working Paper.

Thaler, Richard, and William Ziemba, 1988, Anomalies: Parimutuel Betting Markets: Racetracks and Lotteries, *Journal of Economic Perspectives* 2, 161-174.

Tversky, Amos, and Daniel Kahneman, 1992, Advances in Prospect Theory: Cumulative Representation of Uncertainty, *Journal of Risk and Uncertainty* 5, 297-323.

Wald, Abraham, 1943, Tests of Statistical Hypotheses Concerning Several Parameters When the Number of Observations is Large, *Transactions of the American Mathematical Society*, 54, 426-482.

Wolfers, Justin, and Eric Zitzewitz, 2004, Prediction Markets, *Journal of Economic Perspectives* 18, 107-126.

Wu, George, and Richard Gonzalez, 1996, Curvature of the Probability Weighting Function, *Management Science* 42, 1676-1690.

Wurgler, Jeffrey, and Ekaterina Zhuravskaya, 2002, Does Arbitrage Flatten Demand Curves for Stocks? *Journal of Business* 75, 583-608.

Zitzewitz, Eric, 2006, Price Discovery among the Punters: Using New Financial Betting Markets to Predict Intraday Volatility, Stanford University Working Paper.

Table I: Returns to Expiration for Sports and Financial Securities

This table reports the results from three ordinary least squares (OLS) regressions of securities' returns to expiration on five dummy variables, Price1 through Price5. The three regressions include observations of the returns and prices of securities based on one-day sports, financial, and both types of events recorded at 30-minute intervals in which there is active trading (see text for details). I compute returns to expiration as the payoff at expiration, 0 or 100 points, minus the bid-ask midpoint divided by 100 points. I construct dummy variables (Price1 through Price5) for five equally-spaced pricing intervals: (0,20), [20,40), [40,60), [60,80), and [80,100) points. The small probabilities row displays the magnitude and significance of the average coefficient on Price1 and Price2. The large probabilities row displays the magnitude and significance of the average coefficient on Price3, Price4, and Price5. The large minus small row reports the magnitude and significance of the difference in these two averages. I compute clustered standard errors to account for correlations within and across securities that expire on the same calendar day (Froot (1989)).

	Sports	Financial	Both Types
$0 < \text{Price} < 20$	-1.45**	-0.45	-0.63
	(0.69)	(0.65)	(0.55)
$20 \leq \text{Price} < 40$	-1.95	-0.29	-1.04
	(1.60)	(1.15)	(0.96)
$40 \leq \text{Price} < 60$	-0.76	2.81**	-0.47
	(0.67)	(1.42)	(0.63)
$60 \leq \text{Price} < 80$	1.10	2.82**	1.41
	(1.06)	(1.40)	(0.90)
$80 \leq \text{Price} < 100$	1.90**	1.20	1.63***
	(0.89)	(0.84)	(0.62)
Small Probabilities	-1.70*	-0.37	-0.83
	(0.91)	(0.80)	(0.64)
Large Probabilities	0.75	2.28**	0.86*
	(0.61)	(1.01)	(0.51)
Large – Small	2.45**	2.65***	1.69**
	(1.01)	(1.03)	(0.76)
R-squared	0.0005	0.0019	0.0004
Expiration Days	1109	776	1123
Observations	132583	41798	174381

Robust standard errors are in parentheses.
* significant at 10%; ** significant at 5%; *** significant at 1%.

Table II: Returns to Expiration for Sports and Financial Securities Sorted by Bid-Ask Spread

This table reports the results from nine (3x3) ordinary least squares (OLS) regressions of securities' returns to expiration on five dummy variables, Price1 through Price5 (see text or Table I for details). The nine regressions include three sets of regressions for securities based on one-day sports, financial and all events. Each set includes regressions for high bid-ask spread, low bid-ask spread, and both groups of securities (the Low – High columns). See text for further details.

	Sports			Financial			Both Types		
	High Spread	Low Spread	Low – High	High Spread	Low Spread	Low – High	High Spread	Low Spread	Low – High
0 < Price < 20	-0.22	-3.46***	-3.24***	-0.34	-0.68	-0.34	-0.33	-1.98***	-1.66*
	(0.85)	(0.89)	(1.13)	(0.68)	(1.09)	(1.10)	(0.61)	(0.75)	(0.85)
20 ≤ Price < 40	-0.82	-3.46	-2.65	0.00	-3.86*	-3.86*	-0.27	-3.52	-3.25
	(1.47)	(2.67)	(2.64)	(1.15)	(2.33)	(2.12)	(0.92)	(2.32)	(2.31)
40 ≤ Price < 60	-1.19*	-0.46	0.73	2.53*	5.46*	2.93	-0.54	-0.39	0.15
	(0.61)	(0.87)	(0.77)	(1.42)	(2.90)	(2.57)	(0.56)	(0.86)	(0.78)
60 ≤ Price < 80	0.27	1.84	1.57	2.43*	6.11***	3.69*	1.00	1.96	0.97
	(1.07)	(1.29)	(1.12)	(1.43)	(2.36)	(2.14)	(0.86)	(1.25)	(1.15)
80 ≤ Price < 100	1.61*	2.04	0.43	1.35	-0.04	-1.38	1.48**	1.66	0.18
	(0.85)	(1.38)	(1.34)	(0.84)	(1.30)	(1.06)	(0.59)	(1.14)	(1.10)
Small Probabilities	-0.52	-3.46**	-2.94***	-0.17	-2.27*	-2.10*	-0.30	-2.75**	-2.45**
	(0.87)	(1.43)	(1.43)	(0.82)	(1.37)	(1.26)	(0.66)	(1.23)	(1.26)
Large Probabilities	0.23	1.14	0.91	2.10**	3.85**	1.74	0.64	1.07	0.43
	(0.57)	(0.80)	(0.66)	(1.01)	(1.57)	(1.26)	(0.48)	(0.74)	(0.65)
Large – Small	0.75	4.60***	3.85**	2.27**	6.11***	3.84**	0.94	3.83***	2.88**
	(0.93)	(1.33)	(1.60)	(1.05)	(1.88)	(1.73)	(0.73)	(1.41)	(1.39)
R-squared	0.0005	0.0008	0.0007	0.0015	0.0094	0.0021	0.0003	0.0008	0.0005
Expiration Days	1088	1067	1094	764	636	765	1105	1085	1108
Observations	55174	75279	130453	37764	3699	41463	92938	78978	171916

Robust standard errors are in parentheses.
* significant at 10%; ** significant at 5%; *** significant at 1%.

Table III: Returns to Expiration for Different Securities Sorted by Price Impact

This table reports the results from nine (3x3) ordinary least squares (OLS) regressions of securities' returns to expiration on five dummy variables, Price1 through Price5 (see text or Table I for details). The nine regressions include three sets of regressions for securities based on one-day sports, financial and all events. Each set includes regressions for high post-trade price movement (High Impact), low post-trade price movement (Low Impact), and both groups of securities (Low – High). See text for further details.

	Sports			Financial			Both Types		
	High Impact	Low Impact	Low – High	High Impact	Low Impact	Low – High	High Impact	Low Impact	Low – High
0 < Price < 20	0.03	-3.72***	-3.74***	-0.28	-0.79	-0.52	-0.24	-1.74***	-1.50***
	(0.92)	(0.82)	(1.16)	(0.66)	(0.83)	(0.64)	(0.59)	(0.63)	(0.58)
20 ≤ Price < 40	-0.38	-3.98	-3.59	-0.25	-0.02	0.22	-0.30	-2.82	-2.52
	(1.33)	(2.55)	(2.28)	(1.16)	(1.71)	(1.42)	(0.89)	(1.88)	(1.76)
40 ≤ Price < 60	-0.99*	-0.55	0.44	2.17	5.95***	3.78**	-0.53	-0.40	0.13
	(0.58)	(0.90)	(0.75)	(1.42)	(2.09)	(1.67)	(0.54)	(0.88)	(0.75)
60 ≤ Price < 80	0.76	1.56	0.80	2.21	5.34***	3.14**	1.19	1.79	0.60
	(0.95)	(1.39)	(1.15)	(1.45)	(1.85)	(1.58)	(0.79)	(1.31)	(1.15)
80 ≤ Price < 100	1.08	2.67**	1.59	0.78	2.39***	1.61**	0.93	2.60***	1.67*
	(0.88)	(1.29)	(1.24)	(0.92)	(0.81)	(0.75)	(0.63)	(0.99)	(0.97)
Small Probabilities	-0.18	-3.85***	-3.67***	-0.26	-0.41	-0.15	-0.27	-2.28**	-2.01**
	(0.84)	(1.33)	(1.19)	(0.81)	(1.04)	(0.78)	(0.64)	(1.02)	(0.94)
Large Probabilities	0.28	1.22	0.94	1.72*	4.56***	2.84***	0.53	1.33*	0.80
	(0.53)	(0.82)	(0.65)	(1.04)	(1.14)	(0.82)	(0.46)	(0.73)	(0.63)
Large – Small	0.46	5.07***	4.61***	1.98*	4.97***	2.98***	0.80	3.61***	2.81***
	(0.90)	(1.49)	(1.34)	(1.06)	(1.36)	(1.08)	(0.71)	(1.20)	(1.07)
R-squared	0.0004	0.0009	0.0007	0.0011	0.0080	0.0022	0.0002	0.0008	0.0005
Expiration Days	1087	1066	1094	761	721	765	1105	1092	1108
Observations	60368	70085	130453	34321	7142	41463	94689	77227	171916

Robust standard errors are in parentheses.
* significant at 10%; ** significant at 5%; *** significant at 1%.

Table IV: Returns to Expiration for Securities Sorted by Liquidity and Controls

This table reports the results from three ordinary least squares (OLS) regressions of securities' returns to expiration on five dummy variables, Price1 through Price5, and six sets of interaction terms with these five dummies (see text for details). The table displays only the coefficients from the set of five liquidity interaction terms. The three regressions use three different liquidity measures as the basis for these five interaction terms (see text for construction). All regressions also include four sets of five control interaction terms based on return volatility, trading volume, order imbalance, and time horizon. I report the p-values for these 20 interaction terms below. Because each regression includes all securities based on one-day sports and financial events, I include a sixth set of five interactions to allow for variation in mispricing across sports and financial securities.

Liquidity Measure	Low Spread and Low Impact	Low Bid-Ask Spread	Low Price Impact
$(0 < P < 20)$ * Liquid	-2.47**	-1.40	-1.20**
	(0.97)	(0.89)	(0.59)
$(20 \leq P < 40)$ * Liquid	-4.17*	-3.48*	-2.90**
	(2.46)	(2.06)	(1.44)
$(40 \leq P < 60)$ * Liquid	0.48	0.75	0.49
	(0.75)	(0.71)	(0.67)
$(60 \leq P < 80)$ * Liquid	0.03	1.09	0.31
	(1.12)	(1.05)	(1.00)
$(80 \leq P < 100)$ * Liquid	0.99	-0.09	1.77**
	(1.20)	(1.00)	(0.88)
Small Probabilities	-3.32***	-2.44**	-2.05***
	(1.30)	(1.12)	(0.76)
Large Probabilities	0.50	0.59	0.86*
	(0.61)	(0.55)	(0.52)
Large – Small	3.82***	3.03**	2.90***
	(1.45)	(1.27)	(0.90)
P-value for 20 Controls	0.1685	0.1683	0.1554
R-squared	0.0012	0.0012	0.0012
Expiration Days	1101	1101	1101
Observations	159361	159361	159361

Robust standard errors are in parentheses.
* significant at 10%; ** significant at 5%; *** significant at 1%.

Table V: Returns to Expiration for Liquid Securities Sorted by Liquidity Persistence and Trading Volume

This table reports the results from six ordinary least squares (OLS) regressions of securities' returns to expiration on five dummy variables, Price1 through Price5 (see text or Table I for details). All included securities are based on one-day sports and financial events, and have above-median liquidity based on both bid-ask spreads and post-trade price movement. The regressions in Columns One, Two, and Three include securities with sporadically high liquidity, persistently high liquidity and both groups of securities, respectively. I classify securities with high liquidity in three consecutive data recording periods (over one hour) as persistently liquid. The regressions in Columns One and Two estimate mispricing for sporadically and persistently liquid securities in each of the five pricing categories. The regression in Column Three, which includes both groups of securities, uses five interaction terms to estimate the difference in mispricing between persistently and sporadically liquid securities in the five pricing categories. The regressions in Columns Four, Five, and Six perform analogous tests to assess whether mispricing in liquid securities depends on trading volume. I classify all securities with above-median volume as high volume securities.

	Securities with High Liquidity			Securities with High Liquidity		
	Sporadic Liquidity	Persistent Liquidity	Persistent – Sporadic	Low Volume	High Volume	High – Low
$0 < Price < 20$	-1.73**	-7.30***	-5.57***	-4.22***	-2.71***	1.51
	(0.82)	(0.78)	(0.94)	(1.46)	(0.78)	(1.35)
$20 \leq Price < 40$	-1.82	-8.39	-6.57	0.47	-8.89***	-9.36***
	(1.92)	(5.43)	(4.67)	(3.03)	(3.44)	(3.36)
$40 \leq Price < 60$	-0.46	-0.38	0.08	-0.53	-0.31	0.22
	(0.71)	(1.52)	(1.20)	(0.79)	(1.51)	(1.35)
$60 \leq Price < 80$	1.57	1.78	0.22	1.03	2.43	1.40
	(1.08)	(2.17)	(1.67)	(1.34)	(2.21)	(2.10)
$80 \leq Price < 100$	1.32	3.93	2.61	1.02	2.83**	1.81
	(0.92)	(2.60)	(2.32)	(2.20)	(1.38)	(2.13)
Small Probabilities	-1.78*	-7.84***	-6.07***	-1.88	-5.80***	-3.92**
	(1.06)	(2.69)	(2.29)	(1.66)	(1.77)	(1.77)
Large Probabilities	0.81	1.78	0.97	0.51	1.65	1.14
	(0.60)	(1.38)	(1.09)	(0.95)	(1.17)	(1.11)
Large – Small	2.59**	9.62***	7.04***	2.38	7.45***	5.06**
	(1.16)	(3.02)	(2.55)	(1.89)	(2.09)	(2.08)
R-squared	0.0004	0.0020	0.0011	0.0003	0.0025	0.0013
Expiration Days	1059	857	1059	1028	1000	1059
Observations	33684	26344	60028	30518	29510	60028

Robust standard errors are in parentheses.
* significant at 10%; ** significant at 5%; *** significant at 1%.

Table VI: Realizable Returns to Expiration for Liquid Securities

This table reports the results from six ordinary least squares (OLS) regressions of securities' returns to expiration on five dummy variables, Price1 through Price5. The regressions and results are similar to Table I with two exceptions. First, this table includes only the liquid securities—*i.e.*, those with below-median bid-ask spreads and below-median post-trade price movement. Second, the dependent variable is now realizable returns to expiration rather than returns to expiration. This means that there are two separate regressions for the buyer's and the seller's realizable returns to expiration in this table corresponding to each regression in Table I. I assume that the buyer pays the inside ask price, the seller receives the inside bid price in the computation of realizable returns to expiration, and both buyers and sellers must pay 0.8%, the round-trip commission for a market order on the TradeSports exchange. See text for further details.

	Sports		Financial		Both Types	
	Buyer	Seller	Buyer	Seller	Buyer	Seller
0 < Price < 20	-5.80***	3.42***	-2.10	-0.54	-4.29***	1.80**
	(0.82)	(0.84)	(1.36)	(1.37)	(0.79)	(0.81)
20 ≤ Price < 40	-5.49*	2.94	-4.33	1.59	-5.42*	2.86
	(3.06)	(3.06)	(4.06)	(4.05)	(2.88)	(2.89)
40 ≤ Price < 60	-1.75*	-0.76	10.17**	-12.92***	-1.68*	-0.83
	(0.98)	(0.98)	(3.99)	(4.00)	(0.98)	(0.98)
60 ≤ Price < 80	0.30	-2.81*	9.38***	-12.13***	0.42	-2.92**
	(1.48)	(1.48)	(3.31)	(3.31)	(1.46)	(1.46)
80 ≤ Price < 100	1.24	-3.69**	-0.08	-2.54**	1.06	-3.54***
	(1.55)	(1.55)	(1.02)	(1.02)	(1.35)	(1.35)
Small Probabilities	-5.64***	3.18**	-3.22	0.52	-4.85***	2.33
	(1.59)	(1.59)	(2.29)	(2.29)	(1.50)	(1.50)
Large Probabilities	-0.07	-2.42***	6.49***	-9.20***	-0.07	-2.43***
	(0.90)	(0.90)	(1.92)	(1.92)	(0.85)	(0.85)
Large – Small	5.57***	-5.60***	9.71***	-9.72***	4.79***	-4.76***
	(1.79)	(1.79)	(2.90)	(2.90)	(1.70)	(1.70)
R-squared	0.0017	0.0014	0.0279	0.0416	0.0016	0.0015
Expiration Days	1040	1040	524	524	1059	1059
Observations	58425	58425	1603	1603	60028	60028

Robust standard errors are in parentheses.
* significant at 10%; ** significant at 5%; *** significant at 1%.

Table VII: Predicting How Much Information Prices Incorporate Using Liquidity

This table reports the results from two ordinary least squares (OLS) regressions of the reduction in squared percentage returns to expiration on liquidity and several control variables. I compute the reduction in squared returns over the roughly 30-minute data recording period that follows liquidity and control variable measurement. The key independent variables in the two regressions are the two liquidity measures: logarithms of bid-ask spreads (*LnSpread*) and post-trade price movement (*LnImpact*). Each regression also includes control variables for expected information release through expiration (*ExpInfo*), whether the event is in progress (*InEvent*), two measures for the time elapsed since initial security listing (*LnTime* and *LnMoves*), and the trading volume (*LnVolHr*) and volatility (*LnAbsRet*) during the prior period (see text for details). All regressions include time dummy variables for each data recording period. I compute the coefficients and standard errors from the averages and standard deviations of the coefficients in monthly cross-sectional regressions (Fama and MacBeth (1973)). The standard errors are robust to heteroskedasticity and auto-correlation up to three lags in the time series of the coefficients (Newey and West (1987)).

Dependent Variable	Reduction in Squared % Returns to Expiration	
LnSpread	106.3***	
	(11.9)	
LnImpact		55.7***
		(6.9)
ExpInfo	112.6***	110.0***
	(10.8)	(11.2)
InEvent	-10.3	-6.2
	(8.8)	(8.6)
LnTime	-51.8***	-61.7***
	(10.7)	(10.4)
LnMoves	61.7***	64.3***
	(19.7)	(20.3)
LnVolHr	-5.9	-6.9
	(9.7)	(10.2)
LnAbsRet	36.5**	48.9***
	(17.8)	(17.3)
Monthly Regressions	41	41
Total Expiration Days	1109	1109
Total Observations	160855	160855
Average R-squared	0.0443	0.0392

Robust standard errors are in parentheses.
* significant at 10%; ** significant at 5%; *** significant at 1%.

Figure 1: The S-shaped Mispricing Pattern in Sports and Financial Securities

This figure depicts the estimated returns to expiration of securities based on various one-day events for five equally-spaced pricing categories. The three series in the figure depict the returns of securities based on sports events, financial events, and both types of events. Thus, the figure plots the three sets of coefficient estimates on the five pricing category coefficients shown in Columns One, Two, and Three in Table I (see table for construction).
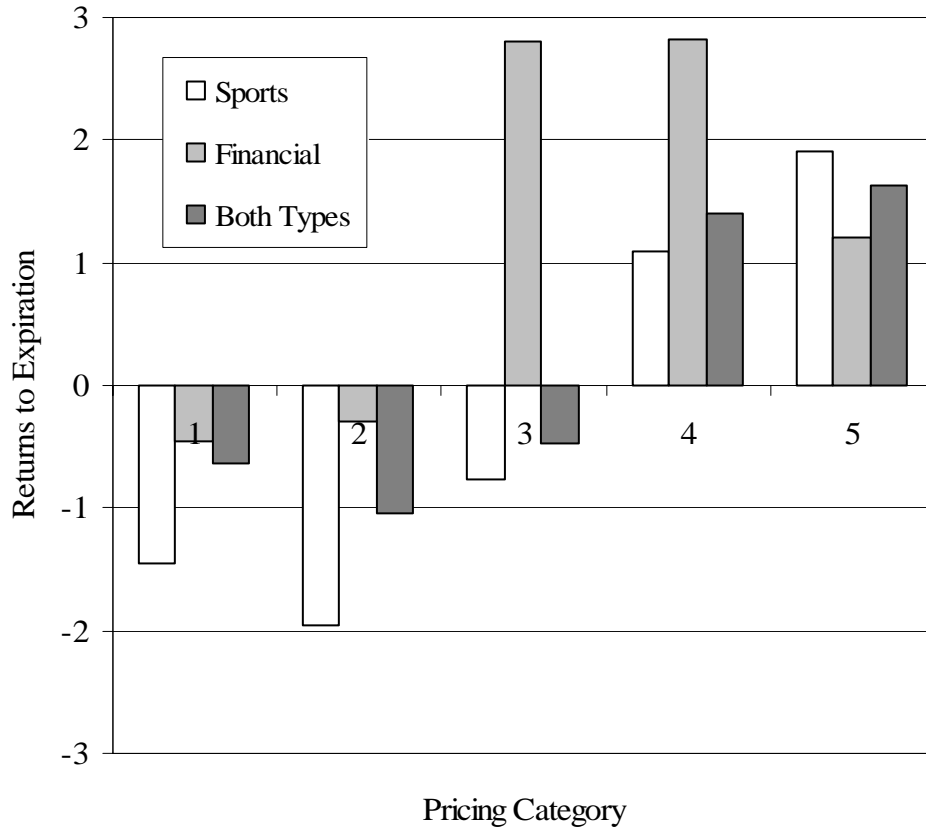
Figure 2: The Effect of Liquidity on Returns to Expiration

This figure depicts the estimated differences between the returns to expiration of securities with differing degrees of liquidity as measured by two proxies—bid-ask spreads and post-trade price movements—for five equally-spaced pricing categories. I plot the four sets of coefficient estimates on the five pricing category interaction terms shown in Columns Three and Six of Table II and Table III (see these tables for construction). These four sets (2x2) of interaction terms measure the effect of spreads and post-trade price movement in sports and financial securities. For spreads, the interaction term is equal to the returns to expiration of low spread minus high spread securities. For post-trade price movement, the interaction term is equal to the returns to expiration of low-impact minus high-impact securities.
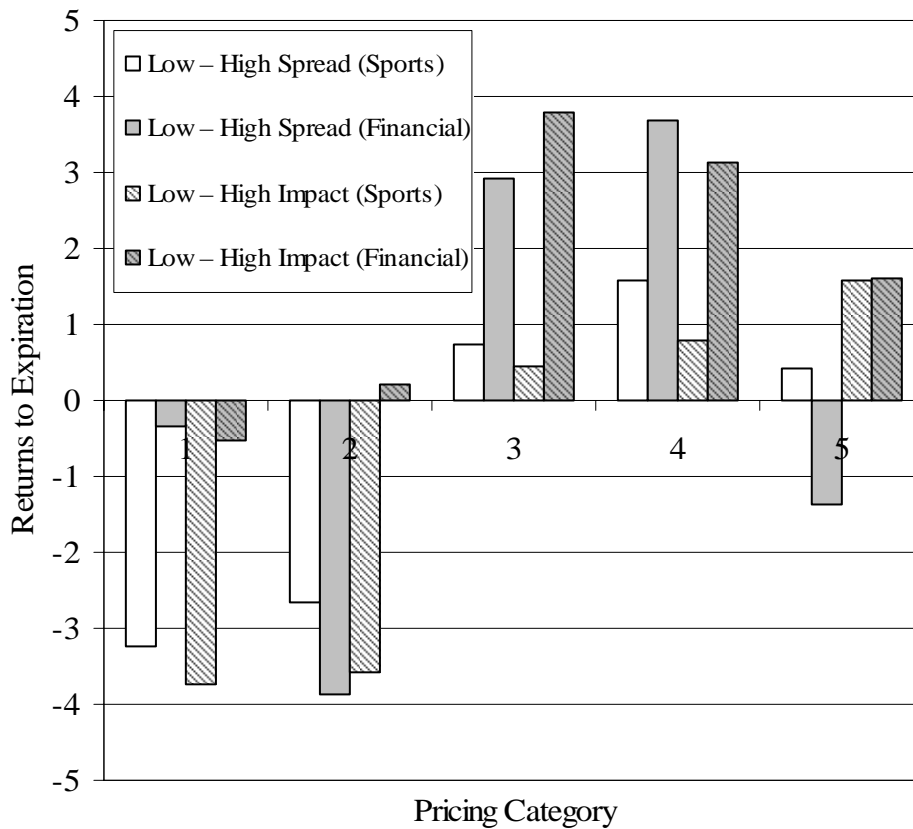
Figure 3: The Effects of Sporadic and Persistent Liquidity on Mispricing

This figure depicts the estimated returns to expiration of securities with low liquidity, sporadic liquidity, and persistent liquidity for five equally-spaced pricing categories. For the three security groupings, I plot the coefficients from three separate regressions of returns to expiration on the five pricing category dummies. Columns One and Two in Table V show the regression estimates for sporadically and persistently liquid securities. The low liquidity estimates come from an analogous regression (see text and tables for further details). All regressions include both one-day sports and financial securities.
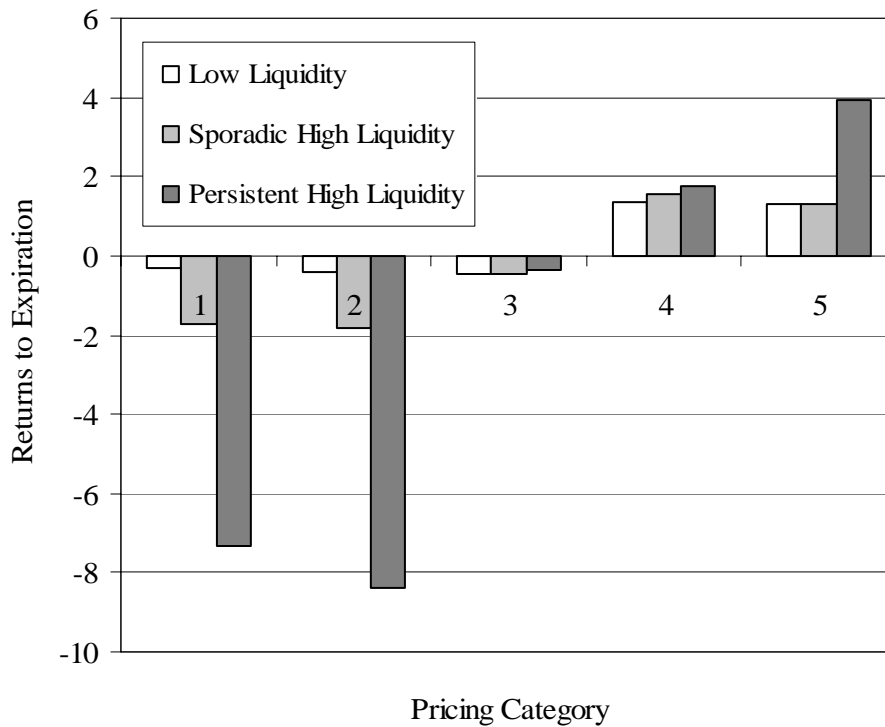
Figure 4: Profitability of Trading in Liquid Sports and Financial Securities

This figure shows the realizable returns to expiration for buyers and sellers of liquid sports and financial securities in each of the five pricing categories. Each line represents one of the four sets of coefficient estimates for the five pricing category dummies. To compute realizable returns, I assume that the buyer pays the inside ask price, the seller receives the inside bid price in the computation of realizable returns to expiration, and both buyers and sellers must pay 0.8%, the round-trip commission for a market order on the TradeSports exchange. The numerical return estimates appear in Columns One through Four in Table VI (see table for further details).
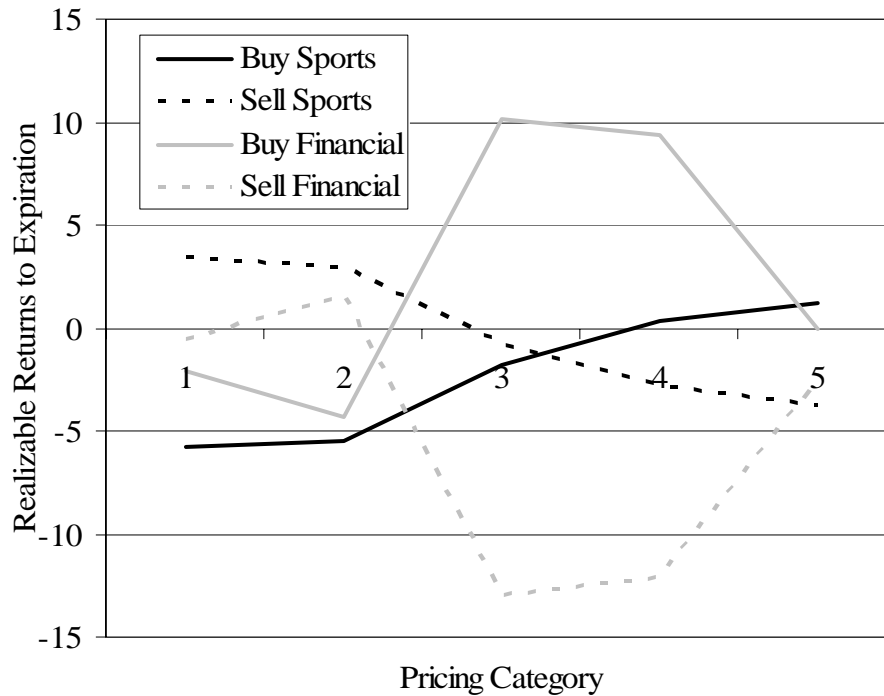
Figure 5: How Prices Incorporate Information around Liquidity Events

The figure visually represents and extends the results in Table VII. I present the cumulative reduction in squared percentage returns to expiration for liquid and illiquid securities in the ten data recording periods surrounding the time of liquidity measurement. The thick lines in the figure depict the difference between the information release during a liquidity event and an illiquidity event. I include only observations that occur after the underlying sports or financial event has begun, as defined by the *InEvent* dummy variable (see text for details).