

Tax Competition for Heterogeneous Firms with Endogenous Entry[†]

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Abstract: This paper models tax competition for mobile firms that are differentiated by the amount of labor needed to cover fixed costs. Because tax competition affects the distribution of firms, it affects both relative equilibrium wages across countries and equilibrium prices. These in turn influence the equilibrium number of firms. From the social planner's perspective, optimal tax rates are harmonized, providing the optimal number of firms, and set such that income is efficiently distributed between private and public consumption. As is common in tax competition models, in the Nash equilibrium tax rates are inefficiently low, yielding underprovision of public goods. Furthermore, there exist a variety of situations in which equilibrium tax rates differ. As a result, too many firms enter the market as governments compete to be the low-tax, high-wage country. This illustrates a new distortion from tax competition and provides an additional benefit from tax harmonization.

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1. Introduction

One of the most important recent innovations to the literature on international trade and foreign direct investment (FDI) has incorporated heterogeneous firms into models of imperfect competition that previously assumed identical firms.¹ We build upon these innovations by incorporating imperfectly competitive heterogeneous firms and endogenous entry into a general equilibrium model of tax competition for mobile investment. Although a handful of tax competition models with imperfect competition exist, firms in these models are identical and their number is exogenous.² In our model, firms differ in their labor productivity. Because of this, low productivity firms will prefer a low cost location even if this has a greater profit tax. Furthermore, the last firm to enter will enter only in the low cost location. Nevertheless, governments still have an incentive to lower taxes as this attracts more profitable firms and raises the real value of local wages. This race to the bottom in taxes leads to the well-known result of public good underprovision. However, in many equilibrium outcomes, it also leads to an overabundance of firms. When too many firms enter, this creates a new inefficiency from tax competition, one that hurts welfare by raising consumer prices such that any gains from additional varieties are wiped out. This provides a new motivation for tax harmonization.

Our use of discrete yet endogenously entering firms is a departure from the standard models of tax competition.³ Typically, one of two approaches is used. The first assumes that investors divide their activities is across locations. Since each location has

¹ Examples in the trade literature include Ghironi and Melitz (2005) and Melitz (2003). Examples in the FDI literature include Nocke and Yeaple (2005), Yeaple (2005), and Helpman, Melitz, and Yeaple (2004).

² Examples include Ferrett and Wooton (2005), Baldwin and Krugman (2004) and Janeba (1998).

³ Wilson (1999), Gresik (2001), and Fuest, Huber, and Mintz (2003) provide overviews of the existing literature.

diminishing marginal returns, even with tax differentials each location receives positive investment levels.⁴ We call this the "continuous investment" approach. The second approach assumes a given number of firms (typically one) for which governments compete. Here, firms choose a single location meaning that countries that do not host have no investment. We call this the "discrete investment" approach.⁵

These modeling assumptions have important implications for strategic behavior and the ability to include public goods in the model. When investment is discrete, competition for the firm amounts to a second price auction in which governments bid their own value for the firm but the winner only pays the second highest value.⁶ This leads to a best response in which governments marginally undercut one another's taxes until all gains from hosting are exhausted. In contrast, when investment is continuous, optimal taxes trade off against the size of the tax base and the share of profits collected by the host government. Thus, there is not the weakly dominant strategy found in the discrete investment case. A key implication of this difference is that it is often impossible to include a necessary public good in the discrete investment case because there typically exist equilibria in which some countries do not host FDI and therefore collect no tax revenue. Thus, while both approaches predict a race to the bottom in taxes, only the continuous investment model can be used to describe the inefficiencies tax competition causes for public goods provision. In our model, however, due to the heterogeneity in their productivities, not all firms flock to the low tax location.⁷ Thus even though a single

⁴ Wilson (1984) and Zodrow and Mieszkowski (1984) are seminal papers in this vein.

⁵ Recent examples include Haufler and Wooton (1999, 2006) and Raff (2004).

⁶ See Black and Hoyt (1989) or Davies and Ellis (forthcoming) for detailed discussions of this second price auction result.

⁷ This is in contrast to Baldwin and Krugman (2004) in which mobile firms all agglomerate in one location or the other depending on relative tax rates.

firm's location is discrete, each government collects positive tax revenues from the firms in equilibrium. Therefore, we are able to discuss public goods provision in a model with discrete investment.

As in the continuous investment and discrete investment models, we also find a race to the bottom when taxes are set non-cooperatively. This occurs because countries find it strictly beneficial to host the high productivity firms because these are the most profitable. To win such firms, a government must set a tax rate lower than the others. Furthermore, once a nation is set to host these productive firms, it has an incentive to set an even lower tax in order to encourage entry in the other country which increases the number of varieties, attracts even more firms to itself, and drives up its own wages. Although this race to the bottom is tempered by the need to raise taxes and provide for a public good, it is nevertheless the case that taxes are inefficiently low and the public good is underprovided. In addition, there are many equilibrium situations in which countries' taxes are unequal. This is because the low tax country, by attracting firms to itself lowers overseas wages and raises prices (part of which must be borne by overseas consumers). This externality leads to an overabundance of varieties and creates a new inefficiency from tax competition, one that has not been explored in the literature. Furthermore, this excessive entry provides a new motivation for tax harmonization.

The paper proceeds as follows. Section 2 lays out the baseline model. Section 3 describes the properties of the Nash equilibria. Section 4 solves the social planner's problem. Section 5 considers the sensitivity of the results to alternative assumptions, including the presence of vertical FDI. Section 6 concludes.

2. The Model

In this section, we present the basic framework of our model. Consider a world with two countries labeled A and B . Each country k has a fixed labor endowment given by \bar{L}_k which is the sole factor of production. Without loss of generality, let $\bar{L}_A \geq \bar{L}_B$. The sequence of moves is the following. First, the two countries simultaneously set their tax rates. Second, firms simultaneously choose which of the countries to locate in and how much to produce. Finally, consumption occurs and payoffs accrue. As is standard, we apply subgame perfection.

2.1 Consumers

Utility from private consumption of the representative consumer in country k is of the Dixit-Stiglitz form:

$$U_k = \left(\int_0^N x_k(i)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \quad (1)$$

where N is the number of varieties and $\sigma > 1$. Denote pre-tax private income in country k I_k . Pre-tax private income is the sum of wage income and pre-tax profits of firms that are located in country k (more on this below).⁸ This is taxed at a rate t_k , thus the same tax rate is applied to wage income and profits.⁹ Consumers then maximize utility subject to their budget constraint:

$$\int_0^N p(i)x_k(i)di \leq (1-t_k)I_k.$$

⁸ Note that this means we are not considering "outside firms", i.e. those coming from a third country. Examples of this literature include Davies (2005) and Bjorvatn and Eckel (2006).

⁹ What if tax rates differ?

As is well-known, the solution to this problem yields a demand function for private consumers for each firm of:

$$x_k(i) = P^{1-\sigma} p(i)^{-\sigma} (1-t_k) I_k$$

where P is a price index of the form:

$$P = \left(\int_0^N p(i)^{1-\sigma} di \right)^{\frac{1}{1-\sigma}}. \quad (2)$$

Tax revenues are used by the government to fund public consumption of these same N goods where the relative valuation of is the same as those given by the consumer's utility function, i.e. the government maximizes a function:

$$G_k = \left(\int_0^N g_k(i)^{\frac{(\sigma-1)}{\sigma}} di \right)^{\frac{\sigma}{(\sigma-1)}}. \quad (3)$$

The government of each country must run a balanced budget, i.e.:

$$\int_0^N p(i) g_k(i) di \leq t_k I_k.$$

One interpretation of government consumption is that the tax revenues are used to support consumption by individuals without income (such as the unemployed or the elderly). Alternatively, this government consumption can represent the consumption of a corrupt government official of the type common in the Leviathan models of taxation.¹⁰ A third interpretation of this is that it represents the government's transformation of the N products into a publicly-provided good. Regardless of the interpretation, government consumption in country k results in a country k public demand for each firm of:

$$g_k(i) = P^{1-\sigma} p(i)^{-\sigma} t_k I_k$$

implying that total demand for each firm from country k is:

¹⁰ Examples of this include Brennan and Buchanan (1980) and Edwards and Keen (1996).

$$X_k(i) = P^{1-\sigma} p(i)^{-\sigma} I_k.$$

We assume that there is no price discrimination between countries (which is guaranteed by free resale and an assumption of zero trade costs).¹¹ Thus, the firm's worldwide demand is:

$$X(i) = P^{1-\sigma} p(i)^{-\sigma} I$$

where $I = I_A + I_B$.

2.2 Firms

A given firm i makes two choices. First it decides in which country to locate (or to not enter at all).¹² Second, if it does enter, it decides how much to produce. Firms make these decisions simultaneously, i.e. taking the location and output of other firms as given. As is also standard, firms take wages as given. Firm i 's after-tax profits when based in country k are given by:

$$\pi_k(i) = (1-t_k)[p(i)q(i) - w_k a(i)q(i) - w_k F] \quad (4)$$

where t_k is again the income tax in country k , w_k is the wage rate in country k , $q(i)$ is firm i 's output, $a(i)$ is firm i 's exogenously endowed productivity parameter and F is the amount of labor firm i requires to cover fixed costs. As in other models of firm heterogeneity, this productivity parameter drives the differences across firms.¹³ We

¹¹ If trade costs are positive, this increases the desirability of locating in the country with the greater income as this is where more of the firm's output will be sold. As shown by Haufler and Wooten (1999) in a model with a single firm, this gives an advantage to this larger economy in the tax competition game. Similarly, this would give an advantage to this country in our model.

¹² A useful conceptualization of the model is to attribute a given firm's productivity to an internationally mobile entrepreneur. This entrepreneur earns rents from this firm-specific asset, i.e. entrepreneurial income amounts to firm's profits. Given a country of residence, an entrepreneur then becomes a part of that country's representative consumer which under this preference structure implies that entrepreneurial income, demand, and welfare then enter the model consistent with the above formulation.

¹³ In an earlier version of the paper (Davies and Eckel, 2007), we consider a variant of the model where productivity is the same across firms but they differ in the labor requirement for the fixed cost. The results are qualitatively identical to those presented here.

assume that this parameter is increasing in the index, i.e. $a(0) < a(i) < a(j)$ for $0 < i < j$

implying that firm 0 is the most productive firm. We assume that

$F < \min\{\overline{L}_A, \overline{L}_B\}$ ensuring that either country is capable of hosting at least one firm with

labor remaining for positive production.¹⁴ Given a location, profit maximization implies

that:

$$q_k(i) = \left(\frac{\sigma}{(\sigma-1)} a(i) w_k \right)^{-\sigma} P^{(\sigma-1)} I \quad (5)$$

yielding a markup over marginal cost:

$$p(i) = \frac{\sigma}{(\sigma-1)} a(i) w_k. \quad (6)$$

This condition allows us to rewrite profits so that at their maximum:

$$\pi_k(i) = (1-t_k) \frac{1}{(\sigma-1)} w_k [a(i)q(i) - (\sigma-1)F] \quad (7)$$

where, for this firm's market to clear, its quantity $q_k(i)$ equals its worldwide demand

$X_k(i)$. For future use, it is important to note that by (5):

$$\frac{\partial a(i)q_k(i)}{\partial i} = (1-\sigma) \left(\frac{\sigma}{(\sigma-1)} a(i) w_k \right)^{-\sigma} P^{(\sigma-1)} I < 0 \quad (8)$$

that is, a firm's labor demand is falling in its index.

In order to more easily describe the distribution of firms and derive best response tax rates, it is useful to make a distinction between the high-tax country and the low-tax country. We will label our countries such that $t_1 \leq t_2$ and refer to country 1 as the low tax

¹⁴ It is important to recognize that our firms are not multinationals according to the standard definition because they have their headquarters (where the fixed cost takes place) and their production in the same country. We consider the case of multinationals in Section 5.

country.¹⁵ If $t_1 < t_2$ and $w_1 \leq w_2$, all firms will locate in country 1 since this location offers both lower taxes and lower costs. As described in detail below, this is incompatible with endogenous wages since it implies an excess labor supply in country 2. Thus $w_1 \geq w_2$ with strict equality only if $t_1 = t_2$. Additional implications are that $p_1 \geq p_2$ and $q_1 \leq q_2$.

For notational convenience define $\theta \equiv \frac{w_2}{w_1} \leq 1$ which holds with equality only when taxes are equal.¹⁶

With free entry, firms enter until the last firm earns zero profits by doing so. This last firm is firm N . Whenever taxes are unequal, we find that:

$$a(N)q_1 - (\sigma - 1)F(N) < a(N)q_2 - (\sigma - 1)F(N)$$

implying that pre-tax profits are greater for this firm in the low-cost country 2. Although country 2 has a higher tax rate, since firm N earns no profits, this is not a deterrent. Note that since this firm earns zero profits, in equilibrium:

$$a(N)q_2(N) = (\sigma - 1)F. \quad (9)$$

Using (5) for a generic firm i producing in country 2 and (9), we see that:

$$q_2(i) = a(i)^{-\sigma} a(N)^{\sigma-1} (\sigma - 1)F. \quad (10)$$

Since all other firms have strictly lower production costs for the same quantity, if they were to enter country 2, they would earn positive profits (and potentially even higher profits by entering country 1). Taking the ratio of the quantity solutions of a given firm in countries 1 and 2 and using (10), allows us to derive that for a given firm i :

¹⁵ Note that whether country A or B corresponds to country 1 depends on their relative taxes. The purpose behind this distinction is to ease the derivation of the best responses for countries A and B .

¹⁶ The potential for different wages does not exist in Baldwin and Krugman (2004) because of an additional, freely traded good produced under constant returns to scale and perfect competition.

$$q_1(i) = \theta^\sigma q_2(i) = a(i)^{-\sigma} a(N)^{\sigma-1} (\sigma-1)F. \quad (11)$$

An important aspect of this solution is that, unlike monopolistic competition with identical technologies across firms, in our setting only the last firm to enter has zero profits. Therefore, because of the heterogeneous productivities, profit taxes have the ability to influence firm location even with free entry. In addition, when tax rates differ, there exists a firm λ that is indifferent between the two countries. Specifically, this firm is given by:

$$(1-t_1) \frac{1}{(\sigma-1)} w_1 [a(\lambda)q_1(\lambda) - (\sigma-1)F] = (1-t_2) \frac{1}{(\sigma-1)} w_2 [a(\lambda)q_2(\lambda) - (\sigma-1)F] \quad (12)$$

or, defining $\tau \equiv \frac{1-t_2}{1-t_1} \leq 1$ and using (11):

$$[\theta^\sigma a(N)^{\sigma-1} - a(\lambda)^{\sigma-1}] = \tau \theta [a(N)^{\sigma-1} - a(\lambda)^{\sigma-1}]. \quad (13)$$

This implies that whenever tax rates differ that there is a distribution of firms such that firms 0 to λ locate in country 1 and λ to N locate in country 2. This is represented graphically in Figure 1. Intuitively, the firms that use the least labor (high productivity firms) seek out the location with the lowest taxes (country 1) whereas those for whom wages costs are relatively more important (low productivity firms) seek out the lowest wage rate (country 2). Firms with index numbers higher than N do not enter. Note that this leads to an agglomeration of relatively productive firms in one country and relatively unproductive firms in the other. This is not, however, due to spillovers across firms but due to the fact that country 1 has a comparative advantage in "tax avoidance" due to its comparatively low tax rate whereas country 2 has a comparative advantage in production by virtue of its relatively low wages.

2.3 Labor Markets and Income

In order to clear the labor market in each country, wages must adjust so that labor supply equals labor demand. As noted above, if country 1 has both lower taxes and (weakly) lower wages, all firms will locate there. This would lead to an excess supply of labor in country 2, pushing w_2 down. As a result, if $t_1 \leq t_2$ labor markets in the two countries will clear only if $w_2 \leq w_1$ with strict equality only when taxes are equal. Adding up across the firms that country 1 hosts, equilibrium in country 1's labor market requires:

$$\bar{L}_1 = \int_0^\lambda a(j)q_1(j)dj + \lambda F,$$

For notational simplicity, define $\mu_1(\lambda) \equiv \lambda^{-1} \int_0^\lambda a(j)^{1-\sigma} dj$. Thus, dropping the argument of

μ_1 we can write:

$$\bar{L}_1 = \lambda \theta^\sigma (\sigma - 1) F a(N)^{\sigma-1} \mu_1 + \lambda F \quad (14)$$

where $\theta^\sigma (\sigma - 1) F a(N)^{\sigma-1} \mu_1$ is the average labor used in production by a firm in country 1.

Similarly, defining $\mu_2(\lambda, N) = (N - \lambda)^{-1} \int_\lambda^N a(j)^{1-\sigma} dj$, in country 2:

$$\bar{L}_2 = (N - \lambda) a(N)^{\sigma-1} (\sigma - 1) F \mu_2 + (N - \lambda) F \quad (15)$$

where $a(N)^{\sigma-1} (\sigma - 1) F \mu_2$ is the average amount of production labor used by its firms.

This then also allows us to write equilibrium values of income and the price index:

$$I_1 = w_1 \lambda F \theta^\sigma a(N)^{\sigma-1} \sigma \mu_1, \quad (16)$$

$$I_2 = w_2 (N - \lambda) a(N)^{\sigma-1} F \sigma \mu_2, \quad (17)$$

and

$$P = \frac{\sigma}{(\sigma-1)} w_1 \left(\lambda \mu_1 + (N - \lambda) \theta^{1-\sigma} \mu_2 \right)^{\frac{1}{1-\sigma}}. \quad (18)$$

2.4 Comparative Statics

We now have a system of three equilibrium equations, one describing the location-indifferent firm λ ((13)) and two describing labor market equilibria ((14) and (15)). We also have three endogenous variables, the index number of the indifferent firm λ , the number of firms N , and relative wages θ . Using this system of equations, we can derive how these variables move with the relative tax variable τ .

Lemma 1: *The index of the indifferent firm and the number of firms are decreasing in τ . Relative wages are increasing in τ .*

Proof: By direct calculation:

$$\frac{d\lambda}{d\tau} = - \frac{\left[a(N)^{\sigma-1} - a(\lambda)^{\sigma-1} \right]}{\Delta a(\lambda)^{\sigma-1}} \sigma(\sigma-1) \lambda \theta^\sigma a(N)^{\sigma-1} \mu_1 \left\{ \sigma + (\sigma-1)^2 (N - \lambda) a(N)^{\sigma-2} a'(N) \mu_2 \right\} < 0 \quad (19)$$

$$\frac{dN}{d\tau} = - \frac{\left[a(N)^{\sigma-1} - a(\lambda)^{\sigma-1} \right]}{\Delta a(\lambda)^{\sigma-1}} \left\{ a(\lambda)^{(\sigma-1)} + a(N)^{\sigma-1} (\sigma-1) \right\} a(\lambda)^{-(\sigma-1)} \sigma(\sigma-1) \lambda \theta^\sigma a(N)^{\sigma-1} \mu_1 < 0 \quad (20)$$

and

$$\begin{aligned} \frac{d\theta}{d\tau} &= \frac{\left[a(N)^{\sigma-1} - a(\lambda)^{\sigma-1} \right]}{\Delta a(\lambda)^{\sigma-1}} \\ &\theta \left\{ \left(a(\lambda)^{(\sigma-1)} + (\sigma-1) \theta^\sigma a(N)^{\sigma-1} \right) a(\lambda)^{-(\sigma-1)} \left(\sigma + (\sigma-1)^2 (N - \lambda) a(N)^{\sigma-2} a'(N) \mu_2 \right) \right. \\ &\left. + \left\{ a(\lambda)^{(\sigma-1)} + a(N)^{\sigma-1} (\sigma-1) \right\} a(\lambda)^{-(\sigma-1)} (\sigma-1)^2 \lambda \theta^\sigma a(N)^{\sigma-2} a'(N) \mu_1 \right\} > 0 \end{aligned} \quad (21)$$

where

$$\begin{aligned} \Delta = & \left\{1 + \Psi^{\sigma-1}(\sigma-1)\right\}(\sigma-1)^2 \theta^{\sigma-1} \lambda a(N)^{\sigma-2} a'(N) \mu_1 \left\{(\sigma-1) \tau \sigma \theta \Psi^{\sigma-1} + \tau \theta\right\} \\ & + \left\{\sigma + (\sigma-1)^2 (N-\lambda) a(N)^{\sigma-2} a'(N) \mu_2\right\} \\ & \left\{(\sigma-1) \Psi^\sigma a(N)^{-1} a'(\lambda) (\theta^\sigma - \tau \theta) \sigma (\sigma-1) \lambda \theta^{\sigma-1} a(N)^{\sigma-1} \mu_1 + ((\sigma-1) \theta^\sigma \Psi^{\sigma-1} + 1)^2 \theta^{-1}\right\} > 0 \end{aligned}$$

Q.E.D.

The intuition behind these comparative statics is straightforward. Suppose that t_1 rises, implying an increase in τ . This rise will lead the initially indifferent firm to strictly prefer country 2. As a result, the indifferent firm's index falls. This shift in firms towards country 2 increases labor demand there, thereby increasing country 2's relative wage θ . This rise in costs in country 2 means that the firm that previously just covered its costs now has negative profits, leading it to exit and N to fall.

2.5 The Distribution of Firms under Equal Taxes

One difficulty with the above analysis is that when taxes are equal, wages must also be equal, otherwise all firms will flock to one location or the other. The difficulty this presents is that at this point, all firms are indifferent between locations. As such, there exist many distributions of the firms across the two locations that are consistent with this equilibrium besides those in which firms agglomerate according to their productivity. Furthermore, even if we use a distribution such that firms zero to λ locate in one country and the remainder locate in the other, there is no obvious way to assign the high productivity firms to one country or the other. Therefore we make the assumption that when tax rates are equal, country A hosts firms zero to λ with probability β and firms λ to N with probability $1-\beta$.

It will be useful to establish certain results regarding the distribution of firms in the equal tax case.

Proposition 1: Assume equal taxes. Then:

- a) Regardless of which country hosts the high productivity firms, N remains the same.
- b) If A has more labor than B , then when A hosts the high (low) productivity firms it hosts more firms in equilibrium than when B hosts the high (low) productivity firms.
- c) A country hosts more firms when hosting the low productivity firms than when it hosts the high productivity firms.
- d) If A hosts the low productivity firms then it hosts more firms in equilibrium than B does.
- e) When a country hosts the high productivity firms, more labor is devoted to production than when it hosts the low productivity firms.
- f) When endowments are equal, the country hosting the high productivity firms hosts fewer firms in equilibrium than the other country does.

Proof: When tax rates are equal (which implies equal wages):

$$\bar{L}_1 = (\sigma - 1)a(N)^{\sigma-1} F \int_0^{\lambda} a(j)^{1-\sigma} dj + \lambda F$$

and

$$\bar{L}_2 = (\sigma - 1)a(N)^{\sigma-1} F \int_{\lambda}^N a(j)^{1-\sigma} dj + (N - \lambda)F .$$

Adding these together,

$$\bar{L}_A + \bar{L}_B = \bar{L}_1 + \bar{L}_2 = (\sigma - 1)a(N)^{\sigma-1} F \int_0^N a(j)^{1-\sigma} dj + NF$$

i.e. the total number of firms is the same regardless of whether the relatively large A hosts the low or the high productivity firms. Denote this number of firms N^* .

Although N^* is independent of who hosts the low fixed costs firms, λ is not.

When country A hosts the high productivity firms the number it hosts is λ_A^* , where:

$$\bar{L}_A = (\sigma - 1)a(N^*)^{\sigma-1} F \int_0^{\lambda_A^*} a(j)^{1-\sigma} dj + \lambda_A^* F .$$

Similarly, when country B hosts the high productivity firms the number it hosts is λ_B^* where:

$$\bar{L}_B = (\sigma - 1)a(N^*)^{\sigma-1} F \int_0^{\lambda_B^*} a(j)^{1-\sigma} dj + \lambda_B^* F .$$

Since $\bar{L}_A \geq \bar{L}_B$ and $a(i)$ is increasing in i , this implies that:

$$\lambda_A^* \geq \lambda_B^* \quad (22)$$

with strict equality only when endowments are equal. This means that in equilibrium A would host at least as many high productivity firms as B would. This in turn implies that:

$$N^* - \lambda_B^* \geq N^* - \lambda_A^* \quad (23)$$

with strict equality only when endowments are equal. This implies that in equilibrium A would host at least as many low productivity firms as B would.

Since by (8) a firm's total labor demand is decreasing in its index, the average firm in country 2 uses less labor than the average firm in country 1 does. As a result, when a given country k hosts the low productivity firms, it must host more firms than when it hosts the high productivity firms in order to exhaust its labor supply, i.e.

$$\lambda_k^* < N^* - \lambda_{-k}^* \quad (24)$$

Combining (22) through (24) implies that because B is no larger than A , when B hosts the high productivity firms, it hosts fewer firms that A does. An additional implication of (24) is that the amount of labor devoted to production is greater when a country hosts the high productivity firms:

$$\lambda_k^* \mu_1(\lambda_k^*) > (N^* - \lambda_{-k}^*) \mu_2(\lambda_k^*, N^*) . \quad (25)$$

Furthermore, when countries have equal endowments, this implies that where

$$\lambda_A^* = \lambda_B^* = \lambda^* :$$

$$\lambda^* - (N^* - \lambda^*) = a(N^*)^{\sigma-1}(\sigma-1) \left[(N^* - \lambda^*)\mu_2 - \lambda^*\mu_1 \right] < 0. \quad (26)$$

Q.E.D.

2.6 Government Objectives

The government of country k maximizes a national welfare function that depends on the utility derived from private consumption and that derived by government consumption. Specifically, national welfare in country k is given by:

$$v_k(t_k, t_{-k}) = U_k^\alpha G_k^{(1-\alpha)} \quad (27)$$

One interpretation of this function is of that of a representative consumer who derives utility from their own private consumption and a publicly-provided good created by a production function given by (3). Alternatively, this can represent a function that weights the utility of income-earners relative to that of those consuming out of tax revenues (be they the unemployed or Leviathan government officials). Defining $T_k = t_k^{(1-\alpha)}(1-t_k)^\alpha$, and using the above results for quantities and prices, when tax rates differ we can write country 1's indirect national welfare as:

$$v_1 = T_1(\sigma-1) \left(\lambda\mu_1 + \theta^{1-\sigma} (N-\lambda)\mu_2 \right)^{\frac{1}{\sigma-1}} \lambda\theta^\sigma a(N)^{\sigma-1} F\mu_1 \quad (28)$$

while that for country 2 is:

$$v_2 = T_2(\sigma-1) \left(\lambda\mu_1 + \theta^{1-\sigma} (N-\lambda)\mu_2 \right)^{\frac{1}{\sigma-1}} (N-\lambda)\theta a(N)^{\sigma-1} F\mu_2. \quad (29)$$

Inspection of these shows that if a country's tax rate equals 1 or 0, regardless of the other country's tax rate, national welfare is zero because all income is allocated to the public or

private sector. For a given pair of tax rates $t_1 = t_2 = t_k$, recall that country k has a probability β of receiving the high productivity firms. Thus, its expected utility is:

$$v_k^*(t_k, t_k) = T_k (\sigma - 1) \left(\int_0^{N^*} a(j)^{1-\sigma} dj \right)^{\frac{1}{\sigma-1}} a(N^*)^{\sigma-1} F \left[\beta \lambda_k^* \mu_1(\lambda_k^*) + (1 - \beta)(N^* - \lambda_{-k}^*) \mu_2(\lambda_{-k}^*, N) \right] \quad (30)$$

where the term in brackets is proportional to the expected amount of labor used in production in country k . Since a country hosts fewer firms when hosting the high productivity firms than when hosting low productivity firms, this implies that labor dedicated to production is greater when hosting the high productivity firms. Thus, for any $t_1 = t_2 = t \in (0, 1)$, $v_2(t, t) < v_k^*(t, t) < v_1(t, t)$ i.e. when taxes are equal national income and national welfare are greater when hosting the high productivity firms.

3. Nash Equilibrium Taxes

With the framework now laid out, we are now ready to derive the best responses for the two countries A and B . To do so, we will begin by examining a country's behavior assuming that it is the low-tax country 1. We will then examine behavior assuming that it is the high-tax country 2. Finally, we will combine the results from each of these to derive the best response for each country A and B .

3.1 Best Response for Country 1

To derive country 1's best response, first suppose that $t_1 < t_2$. Looking at how country 1's welfare (28) moves in τ , using the comparative statics (19), (20), and (21) we see that:

$$\begin{aligned} \frac{dv_1}{d\tau} = & \Gamma^{-1} T_1 \theta^\sigma \left\{ (\theta^{1-\sigma} - 1) \sigma \theta^\sigma a(\lambda)^{1-\sigma} a(N)^{\sigma-1} \lambda \mu_1 \right. \\ & \left. + \left[\sigma - (1 + a(\lambda)^{1-\sigma} a(N)^{\sigma-1} (\sigma - 1)) \right] \theta \lambda \mu_1 + \left[(\sigma - 1) \theta^{1-\sigma} - \theta (\sigma - 1) a(\lambda)^{1-\sigma} a(N)^{\sigma-1} \right] (N - \lambda) \mu_2 \right\} \end{aligned} \quad (31)$$

where

$$\Gamma \equiv \frac{\Delta a(\lambda)^{\sigma-1} (\lambda \mu_1 + \theta^{1-\sigma} (N - \lambda) \mu_2)^{\frac{1}{\sigma-1}}}{\left\{ \sigma + (\sigma - 1)^2 (N - \lambda) a(N)^{\sigma-2} a'(\lambda) \mu_2 \right\} \left[a(N)^{\sigma-1} - a(\lambda)^{\sigma-1} \right] \lambda \mu_1 F(\sigma - 1) a(N)^{\sigma-1}} > 0.$$

Evaluating this at $\tau = 1$ yields:

$$\frac{dv_1}{d\tau} = \Gamma^{-1} T_1 (\sigma - 1) a(\lambda)^{1-\sigma} \left[a(\lambda)^{\sigma-1} - a(N)^{\sigma-1} \right] (\lambda \mu_1 + (N - \lambda) \mu_2) < 0 \quad (32)$$

i.e. country 1 benefits by having $\tau < 1$. Recalling the definition of τ :

$$\frac{d\tau}{dt_1} = \frac{\tau}{1 - t_1} \quad (33)$$

indicating that, ignoring the distribution of income between private and public consumption, country 1 has a dominant strategy of undercutting country 2's tax rate.

The full effect of t_1 on v_1 , however, must also take into account the effect of t_1 on the distribution of income. Therefore the actual first-order condition for country 1 is:

$$\begin{aligned} \frac{dv_1}{dt_1} = & \left[(1 - \alpha) t_1^{-1} - \alpha (1 - t_1)^{-1} \right] T_1 (\sigma - 1) (\lambda \mu_1 + \theta^{1-\sigma} (N - \lambda) \mu_2)^{\frac{1}{\sigma-1}} \lambda F \theta^\sigma a(N)^{\sigma-1} \mu_1 \\ & + \frac{\partial \Gamma^{-1} T_1 \theta^\sigma}{1 - t_1} \left\{ (\theta^{1-\sigma} - 1) \sigma \theta^\sigma a(\lambda)^{1-\sigma} a(N)^{\sigma-1} \lambda \mu_1 + \left[\sigma - (1 + a(\lambda)^{1-\sigma} a(N)^{\sigma-1} (\sigma - 1)) \right] \theta \lambda \mu_1 \right. \\ & \left. + \left[(\sigma - 1) \theta^{1-\sigma} - \theta (\sigma - 1) a(\lambda)^{1-\sigma} a(N)^{\sigma-1} \right] (N - \lambda) \mu_2 \right\}. \end{aligned} \quad (34)$$

In order to understand the best response function $t_1(t_2)$ this implies, it is useful to consider two values of t_2 . First, when $t_2 = 1$, $\tau = 0$, implying that no matter what tax rate country 1 chooses it cannot affect the distribution of firms. In this case, (34) dictates that country 1 will set $t_1(1) = 1 - \alpha$, i.e. it will efficiently allocate income between the public and

private sectors. The intuition behind this is that when $t_2 = 1$, country 1 has no incentive to use its tax to affect firm location. Thus, it uses it purely to achieve the desired income allocation. One item of note is that in this case, since taxes differ, country 1 is indeed the high wage country and, as any positive profit firm will flee country 2 for country 1, country 2 hosts only one firm (firm N).

The second point to consider is the value of t_2 such that if country 1 knew that it would host the low fixed costs firms with certainty that it would be content to match this tax rate. Denote this tax rate \bar{t}_2 which is specifically given for country k by:

$$\bar{t}_{2k} = \frac{(1-\alpha)}{\left(1 + \frac{\Gamma^{-1} a(\lambda_k^*)^{1-\sigma} [a(N^*)^{\sigma-1} - a(\lambda_k^*)^{\sigma-1}]}{(\lambda_k^* \mu_1(\lambda_k^*) + (N^* - \lambda_k^*) \mu_2(\lambda_k^*, N^*))^{\frac{1}{\sigma-1}-1} \lambda_k^* F a(N^*)^{\sigma-1} \mu_1(\lambda_k^*)} \right)} \quad (35)$$

where Γ is a function of λ_k^* and N^* . Note that $\bar{t}_{2k} < 1 - \alpha$. At this \bar{t}_{2k} , although it would be beneficial for 1 to reduce t_1 below \bar{t}_{2k} in terms of affecting τ , this would cause too great a distortion to the distribution of income. However, if this country actually set its tax rate equal to that of the other, it would not know for certain that it would receive the low fixed cost firms. Instead, it would receive an expected income level $v_k^*(\bar{t}_{2k}, \bar{t}_{2k})$ which is strictly less than $v_1(\bar{t}_{2k}, \bar{t}_{2k})$. Thus, actually matching creates a discrete fall in income. An alternative way of recognizing this is to look at the relative income of being country 1 versus that of country 2 when taxes are equal. The ratio of (16) and (17) at this point is:

$$\frac{I_1}{I_2} = \frac{\lambda_k^* \mu_1(\lambda_k^*)}{(N^* - \lambda_k^*) \mu_2(\lambda_k^*, N^*)} > 1 \quad (36)$$

by (25). Therefore, at this point country 1 will prefer to set its tax rate marginally below that for country 2, creating a marginal loss in welfare due to the underprovision of the public good but gaining a discrete benefit to expected welfare by guaranteeing that it hosts high productivity firms. Thus, country 1's best response is such that $t_1(t_2) \leq t_2$ with equality only when $t_2 = 0$. Graphically, this looks as in Figure 2 where for values of $t_2 \leq \bar{t}_2$ the best response lies just to the left of the 45° line. Note that we have not proven the exact shape of the portion above this point however the graph matches results from simulations of specific examples.¹⁷ However, it is straightforward to show that given the strict convexity of the preferences, that for each value of t_2 there is a unique value of $t_1(t_2)$ corresponding to it implying that the best response does not bend backwards.

This desire to undercut the other nation's tax is comparable to that found in models where governments compete over discrete firms. In that class of models, there is a discrete change in welfare generated by undercutting the other country's tax as this guarantees the winning of the firm. A similar motivation is found here. Unlike those models, however, there is also a desire to strictly undercut the other country in order to increase wages and attract more firms. Thus, even though the endogenous variables in our model move continuously, this leads to a dominant strategy not found in models where investment is continuously distributed (i.e. where a country internalizes the tradeoff between tax rates and the tax base).

3.2 Best Response for Country 2

¹⁷ Details on these are available upon request.

As when we derived the best response for country 1, we initially consider the effect of τ on country 2's welfare. Here, we find that:

$$\begin{aligned} \frac{dv_2}{dt_2} = & T_2 \theta a(\lambda)^{1-\sigma} \Gamma^{-1} \left\{ (\theta^{1-\sigma} - 1)(N - \lambda) \mu_2 \sigma \theta^\sigma a(N)^{\sigma-1} \right. \\ & + (\sigma - 1) \theta^\sigma \left(a(N)^{\sigma-1} - a(\lambda)^{(\sigma-1)} \right) \lambda \mu_1 \\ & \left. + \left(\theta^\sigma (\sigma - 1) a(N)^{\sigma-1} - (\theta \sigma - 1) a(\lambda)^{(\sigma-1)} \right) (N - \lambda) \mu_2 \right\} \end{aligned} \quad (37)$$

which, evaluated at $t_1 = t_2$ reduces to:

$$\frac{dv_2}{dt_2} = T_2 a(\lambda)^{1-\sigma} \Gamma^{-1} (\sigma - 1) \left(a(N)^{\sigma-1} - a(\lambda)^{(\sigma-1)} \right) \left[\lambda \mu_1 + (N - \lambda) \mu_2 \right] > 0 \quad (38)$$

i.e. country 2 wants $\tau = 1$ (recall that this is the highest value τ can take). Since

$$\frac{d\tau}{dt_2} = \frac{-1}{1 - t_1} < 0 \quad (39)$$

this implies that country 2 will lower its tax until it matches that of country 1. Looking at the total impact of t_2 on v_2 , we find that:

$$\begin{aligned} \frac{dv_2}{dt_2} = & \left[(1 - \alpha) t_2^{-1} - \alpha (1 - t_2)^{-1} \right] T_2 (\sigma - 1) \left(\lambda \mu_1 + \theta^{1-\sigma} (N - \lambda) \mu_2 \right)^{\frac{1}{\sigma-1}} (N - \lambda) \theta a(N)^{\sigma-1} F \mu_2 \\ & - \frac{T_2 \theta a(\lambda)^{1-\sigma} \Gamma^{-1}}{(1 - t_1)} \left\{ (\theta^{1-\sigma} - 1)(N - \lambda) \mu_2 \sigma \theta^\sigma a(N)^{\sigma-1} \right. \\ & + (\sigma - 1) \theta^\sigma \left(a(N)^{\sigma-1} - a(\lambda)^{(\sigma-1)} \right) \lambda \mu_1 \\ & \left. + \left(\theta^\sigma (\sigma - 1) a(N)^{\sigma-1} - (\theta \sigma - 1) a(\lambda)^{(\sigma-1)} \right) (N - \lambda) \mu_2 \right\}. \end{aligned} \quad (40)$$

As before, it is instructive to consider the best response $t_2(t_1)$ at two key values of t_1 .

First, when $t_1 = 0$, if country 2 matches this tax it devotes no income to the public sector.

The first term then goes to negative infinity implying that this is not a best response.

However, if country 2 allocates income efficiently by setting $t_2 = 1 - \alpha$, the first term goes to zero while the second is negative, imply that this is also not a best response. Thus $0 < t_2(0) < 1 - \alpha$.

Next, similar to country 1's \bar{t}_2 , there exists a value of t_1 denoted by \tilde{t}_1 such that if country 2 knew for sure that it would receive the low productivity firms its first order condition equals zero by setting $t_2(\tilde{t}_1) = \tilde{t}_1$. Specifically, for country k :

$$\tilde{t}_{1k} = \frac{(1 - \alpha)}{1 + \frac{a(\lambda_{-k}^*)^{1-\sigma} \Gamma^{-1} \left(a(N^*)^{\sigma-1} - a(\lambda_{-k}^*)^{(\sigma-1)} \right)}{\left(\lambda_{-k}^* \mu_1(\lambda_{-k}^*) + (N^* - \lambda_{-k}^*) \mu_2(\lambda_{-k}^*, N^*) \right)^{\frac{1}{\sigma-1}} (N^* - \lambda_{-k}^*) a(N^*)^{\sigma-1} F \mu_2(\lambda_{-k}^*, N^*)}} \quad (41)$$

where Γ is a function of N^* and λ_{-k}^* . Note that this is less than $1 - \alpha$.

In practice, however, once country 2 matches its tax rate to that of country 1, it receives a discrete boost in expected income since it now has a positive probability of receiving the high productivity firms. Because of this, there is a strict income advantage to matching tax rates at this point. Therefore there will exist a tax rate \hat{t}_{1k} by country 1 for which country 2 is in fact indifferent between having a higher tax rate with its superior allocation of income and an equal tax rate with a higher expected income level. Given the above discussion, it follows that $0 < \hat{t}_{1k} < \tilde{t}_{1k}$. For values of t_1 beyond this point country 2's best response is to match its tax to that of country 1. Thus, country 2's best response is characterized by $t_2(t_1) \geq t_1$ with strict inequality only when $t_1 < \hat{t}_{1k}$.

Graphically, this is illustrated by Figure 3 where again strict concavity of preferences rules out a backwards-bending best response.

3.3 Equilibrium with Identical Countries

In order to determine the Nash equilibria, it is necessary to derive the best responses for the countries A and B by utilizing the above results for countries 1 and 2. In this section, we consider the case of identical countries where $\bar{L}_A = \bar{L}_B$.

First, consider the case of country A . When $t_B = 0$, country A will find it desirable to set a strictly higher tax rate and allocate some income to the public sector. When $t_B = 1$, however, country A will choose to set a strictly smaller tax rate (in fact one equal to $1 - \alpha$). Thus, for low values of t_B country A will choose to be the high tax country 2 whereas for high values of t_B it will choose to be the low tax country 1. A comparable intuition underlies country B 's best response.

The challenge in describing country A 's best response lies in finding the point at which the switch occurs. When $t_B = \hat{t}_{1A}$ for country A , we know that A is indifferent between maintaining a tax rate higher than \hat{t}_{1A} and matching tax rates *if it is unable to lower its tax further*. However, it can indeed lower its tax further and, given the discussion for country 1, it will find it desirable to do so as this guarantees that it receives the high productivity firms. Thus, country A 's indifference is in fact between having a strictly higher tax rate or a strictly smaller tax rate. Therefore, the jump in country A 's best response will happen when B 's tax rate is $t_B = \bar{t}_{1A} < \hat{t}_{1A}$ implying that there exist two optimal tax rates at this point, i.e. $t_A(\bar{t}_{1A}) = \{\underline{t}_A, \bar{t}_A\}$ where $\underline{t}_A < \bar{t}_{1A} < \bar{t}_A$. The only remaining question is whether \underline{t}_A is marginally less than \bar{t}_{1A} or discretely so, that is whether $\bar{t}_{1A} < \bar{t}_{2A}$ from A 's perspective.

Lemma 2: When countries are identical, $\bar{t}_{1A} = \bar{t}_{1B} < \bar{t}_{2A} = \bar{t}_{2B}$.

Proof: With identical countries, $\lambda_A^* = \lambda_B^*$. Thus, comparing (35) and (41), we see that the difference between \bar{t}_{1k} and \tilde{t}_{1k} is the first has $\lambda_k^* \mu_1(\lambda_k^*)$ in its denominator whereas the second has $(N^* - \lambda_k^*) \mu_2(\lambda_k^*, N^*)$. Since, by (25) $\lambda_k^* \mu_1(\lambda_k^*) > (N^* - \lambda_k^*) \mu_2(\lambda_k^*, N^*)$, this implies that $\bar{t}_{2k} > \tilde{t}_{1k}$. Therefore that point at which the jump in the country 2 best response occurs (i.e. \bar{t}_{1A}) is before the country 1 best response moves discretely away from the 45° line. This implies that country A's best response appears as that in Figure 4. Given the symmetry between countries, the same ranking holds for country B. **Q.E.D.**

This best response combines features of those found in both the continuous and discrete investment models. When B's tax rate is moderate, the dominant factor in A's decision making is the effect of its tax rate on the distribution of firms. As a result, as in the discrete investment models it chooses to undercut B's tax. However, when B's tax rate is very high or very low, A becomes far more cognizant of the tradeoff between the tax base and the allocation of income between sectors. This then leads to behavior comparable to that found in the continuous investment models.

We can now describe the Nash equilibrium for identical countries.

Proposition 2: When countries are identical, there does not exist a Nash equilibrium in pure strategies. Furthermore, no equilibrium outcomes involve efficient public good provision.

Proof: Combining the two best responses together as in Figure 5, by Lemma 2 it is clear that there does not exist a Nash equilibrium in pure strategies. This is because for each country, the highest tax rate at which it is willing to be the discretely high tax country (\bar{t}_1) is less than the lowest tax rate for which the other is willing to be the discretely low tax country (\bar{t}_2). Thus, best responses do not cross and the Nash equilibrium (or equilibria) must be in mixed strategies.

Furthermore, any mixed strategy equilibrium has two properties. First, there are equilibrium outcomes that occur with positive probability under which $t_A \neq t_B$, i.e. for which $\tau < 1$. Second, since neither country sets its tax equal to one with a positive probability, the other country assigns no positive probability to choosing a tax equal to $1 - \alpha$. Thus, regardless of the equilibrium outcome, the public good is underprovided.

Q.E.D.

3.4 Equilibrium with Asymmetric Countries

Now, assume that country *A* has a strictly greater labor endowment than country *B*. Given the above discussion, it is still clear that no pure strategy equilibrium exists for which taxes are equal since both *A* and *B* would have an incentive to lower their taxes (at least marginally) in order to capture the high productivity firms. Also comparable to the symmetric case, any equilibrium outcome will be such that taxes are less than $1 - \alpha$ since neither country assigns a positive probability to choosing a tax rate of 1. This is formalized in the following corollary.

Corollary 1: *When country A is strictly larger than B, there do not exist pure strategy equilibria with equal taxes. Furthermore, there are no equilibrium outcomes for which either country's tax rate is $1 - \alpha$.*

Unlike the symmetric case, there now exists the potential for a pure strategy Nash equilibrium with unequal taxes.

Proposition 3: *Suppose that country A is strictly larger than B. Any pure strategy Nash Equilibrium must be such that $t_A > t_B$. The existence of such a pure strategy equilibrium requires that A's labor endowment be sufficiently large relative to B's.*

Proof: For there to exist a pure strategy Nash equilibrium in which B has the higher tax, then it must be that \bar{t}_{2A} , the lowest tax by B for which country A is willing to be the low tax country is greater than \tilde{t}_{1B} , the highest tax by country A for which country B is willing to be the high tax country, i.e. that:

$$\tilde{t}_{1B} - \bar{t}_{2A} > 0. \quad (42)$$

For this to be true, it must be that:

$$\lambda_A^* \mu_1(\lambda_A^*) < (N^* - \lambda_A^*) \mu_2(\lambda_A^*, N^*). \quad (43)$$

Using (25) for country B, this in turn requires that:

$$\lambda_A^* \mu_1(\lambda_A^*) < (N^* - \lambda_A^*) \mu_2(\lambda_A^*, N^*) < \lambda_B^* \mu_1(\lambda_B^*) \quad (44)$$

which, since $\lambda_A^* > \lambda_B^*$ and $\lambda \mu_1(\lambda)$ is increasing in λ , cannot be true.

Thus, any pure strategy Nash equilibria must be such that the larger country sets the higher tax. For such a thing to occur, a necessary condition is that $\tilde{t}_{1A} - \bar{t}_{2B} > 0$ since only if this is true will there exist tax rates for which A is willing to have a discretely

higher tax and B is willing to have a discretely lower tax (i.e. this is necessary for $\bar{t}_{1A} - \bar{t}_{2B} > 0$). For $\tilde{t}_{1A} - \bar{t}_{2B} > 0$, it must be that:

$$\lambda_B^* \mu_1(\lambda_B^*) < (N^* - \lambda_B^*) \mu_2(\lambda_B^*, N^*). \quad (45)$$

When countries are the same in size, this condition fails. Keeping total labor supply constant but lowering \bar{L}_B also lowers λ_B^* , reducing the left hand side of (45) and increasing the right hand side. Furthermore, as \bar{L}_B approaches zero, λ_B^* does as well. This implies that there is a sufficiently large degree of asymmetry for which $\bar{t}_{1A} - \bar{t}_{2B} > 0$. As a result, only when asymmetries are sufficiently large is there a possibility of a pure strategy Nash equilibrium. ***Q.E.D.***

This then predicts that if there is a pure strategy Nash equilibrium, the larger country will set the higher tax. It is worth noting that this matches the empirical results of Devereux, Lockwood, and Redoano (2005) who find that within the OECD relatively large countries set higher statutory corporate taxes. van der Hoek (2003) finds a similar pattern in European Union taxes. In any case, regardless of whether the Nash equilibria are in pure or mixed strategies since best responses do not cross the 45° line there exist equilibrium outcomes for which taxes are unequal (and that this is the only possibility for pure strategy Nash equilibria). Furthermore, all of these have underprovision of the public good.

4. Social Planner's Problem

Regardless of whether countries are symmetric or not, since all equilibrium outcomes involve tax rates less than $1 - \alpha$, they under provide the public good. This

aspect of tax competition is well known. However, in our model, there is also the potential that additional distortions arise in the number of firms, relative wages, and/or the distribution of firms between countries. To investigate these, we now consider the social planner's problem.

This social planner maximizes a social welfare function that is the sum of the two countries' welfare functions. Specifically, the social planner maximizes:

$$W = v_A(t_A, t_B) + v_B(t_A, t_B). \quad (46)$$

by choosing the two countries' tax rates. Using (28) and (29), this reduces to:

$$W = (\sigma - 1) \theta^\sigma a(N)^{\sigma-1} F(\lambda \mu_1 + \theta^{1-\sigma} (N - \lambda) \mu_2)^{\frac{1}{\sigma-1}} (T_1 \lambda \mu_1 + T_2 (N - \lambda) \mu_2) \quad (47)$$

where λ and N depend on relative taxes or, if taxes are equal, λ depends on the random assignment of the low cost firms. The solution to this is found in our final proposition.

Proposition 4: *The social planner's optimum is to set $t_A = t_B = 1 - \alpha$.*

Proof: First we examine how W behaves in τ , i.e. treating T_1 and T_2 as fixed. Taking the derivative of (47), and evaluating it at $\tau = 1$ (which implies that $T_1 = T_2$):

$$\frac{\partial W}{\partial \tau} = 0. \quad (48)$$

Thus, in order to maximize the real value of worldwide income, the social planner will set taxes equal to one another. Note that this does not specify the level of taxes, merely their relative values.

As in the above analysis, taking τ as given, the impact of country k 's tax rate on its distribution of income is:

$$(1-\alpha)t_k^{-1} - \alpha(1-t_k)^{-1}.$$

Since the total impact of a country's tax rate on worldwide welfare is the combination of its effect on τ and its effect on its distribution of income, this implies that the social planner will set $t_A = t_B = 1 - \alpha$ in order to reach an optimum. Furthermore, note that at this solution, λ falls out of (47), implying that the social planner is indifferent as to which country hosts the low fixed cost firms in equilibrium. ***Q.E.D.***

Thus, the social planner harmonizes taxes and sets them so that the marginal value of income is equalized between the public and private sectors. Since the Nash equilibria involve outcomes for which taxes are unequal and always has taxes set below $1 - \alpha$, this makes it clear that all Nash equilibrium outcomes are inefficient relative to the social planner's problem.

While the underprovision of public goods in the Nash equilibrium is a result found in many models of tax competition, in our model it is perhaps less expected that the world welfare maximum involves tax harmonization. This is because the number of firms in our model is lowest when taxes are equal. With the love for variety Dixit-Stiglitz preferences represent, one might expect that the social planner would implement unequal taxes, thereby creating a low-cost location and encouraging entry. However, creating this low cost country also entails creating a high cost country. This lowers the real income in country 2 sufficiently to destroy any benefits the new varieties create. This is also the root of an externality imposed by the low tax country in the Nash equilibrium since it does not internalize the impact of this on country 2's citizens.

5. Alternative Assumptions

The above results are robust to several alternative assumptions. In this section, we consider several alternative assumptions to the baseline model.

First, we consider alternative firm distributions when taxes are equal. One alternative would be to assume that when taxes are equal that the relatively large country hosts the low productivity firms (thereby guaranteeing that under equal taxes more firms locate in the large country). In this case, country B would be willing to match A 's tax rate for intermediate tax rate levels. Nevertheless, country A will always benefit by setting its tax marginally below B 's instead of matching it. As a result, A 's best response destroys the possibility of pure strategy equilibria with equal taxes. A second alternative firm distribution would be to deviate from the distribution of Figure 1 and instead distribute firms so that the average profits of firms in each country are the same, that average productivities are the same, or that the number of firms are equal. In any case, however, by marginally undercutting the other nation's tax rate, a country again creates a discrete shift in its income by attracting only the most productive (and profitable) firms. Thus again there would not exist pure strategy Nash equilibria with equal taxes. Furthermore it is still a best response to set a tax rate of $1 - \alpha$ only when the other country sets its tax equal to 1. Since the social planner is indifferent to how firms are distributed when $t_A = t_B = 1 - \alpha$, these alternatives do not change the solution to the social planners problem.

Second, we can change the assumption that wage income and profits are taxed equally. If we instead allow for different tax rates, then because labor is exogenously endowed, a wage tax is a non-distortionary lump sum tax. If wage income is sufficiently large so that enough tax revenues can be generated for public use, then it is optimal for

governments to use the wage tax to transfer income between private and public sectors and use its profit tax to attract the desired number of firms. This then separates the need to balance τ against income allocation. Given the above results for a country's preferred τ , it is clear that this leads to a race to the bottom which, unless taxes are bounded from below, implies that profit taxes shoot towards negative infinity. It is worth noting that in this case, if both profit tax rates are bounded at the same point, that this equilibrium is efficient relative to the social planner's problem. This is because the minimum tax rate effectively harmonizes profit taxes and wage taxes distribute income optimally.

Third, we assumed that firm profits accrue entirely to local income. This is akin to not allowing investors in one country to invest in the other. However, this strong of an assumption is not necessary for our results. If we instead replace it with one assuming that the majority of a firm's profits go to local income, all of our results hold. This is because, when taxes are equal, it still strictly benefits a country to undercut the other's tax because of the boost to income this provides. The primary difference is that the discrete gain from doing so is smaller than before because the country only keeps a majority of the profit earned by these high profit firms.

Fourth, we can consider best responses when firms are able to geographically fragment their activities, i.e. become multinationals. In the literature on FDI, there are two broad classes of multinational firms: vertical firms that engage in headquarters activity in one country and production in another (Helpman, 1984) and horizontal firms that have their headquarters in one country but produce in multiple countries (Markusen, 1984). Consistent with this literature, let the location of the fixed cost represents the headquarter activity. In our model, as noted by Markusen (1984), the absence of trade

costs and constant returns to scale in production eliminate the need for multiple production facilities.¹⁸ Thus, if multinationals exist in our model, they are of the vertical type.¹⁹ An important difference between our setting and the standard one is that in the typical model of vertical FDI, multinationals arise due to factor price differences across countries. Typically, headquarter services are skilled-labor intensive relative to production. Therefore if countries differ in their relative endowments and factor prices are not equalized through trade, then the skilled-labor abundant country hosts the headquarter activity and the other country hosts production. In our model, however, there is only one factor of production. Nevertheless, as discussed above, when taxes differ there can still exist wage differences across countries. This provides a motivation for vertical FDI.

In order to explore the implications of vertical FDI, it is necessary to make some assumptions regarding tax jurisdictions. Specifically, we assume that countries only levy taxes on firms headquartered within their borders. We also assume that the parent part of the multinational (i.e. where the fixed cost occurs) pays its subsidiary (where production occurs) w_k per unit of output where k is the country hosting production. This amounts to assuming that there is no ability to transfer price.²⁰ As a result, the only tax base for a given firm is found in the country hosting its headquarters.²¹

¹⁸ In fact, in this setting if there are costs to building each production facility, only single production-location firms will exist.

¹⁹ Evidence of vertical FDI is found by Davies (forthcoming), Braconier, Norback, and Urban (2005), Hanson, Mataloni, and Slaughter (2005), Yeaple (2003), and Feinberg and Keane (2001).

²⁰ We discuss relaxing this momentarily.

²¹ An advantage of this is that it eliminates the need to consider double tax issues, the strategic aspects of which are considered by Bond and Samuelson (1989), Janeba (1995), and Davies (2003). We leave a more realistic treatment of this issue to future work.

When taxes are equal, as before, wages will be equal. Therefore there is no need for firms to fragment their activities and all of the properties above hold. Now suppose that $t_1 < t_2$, which, for labor markets to clear, implies that $w_1 > w_2$. In this case, all firms will seek to locate their production in the low cost country 2 since there are no tax advantages to locating production in country 1. We assume that the labor supply in 2 is large enough to handle this. Unlike production, there are advantages to locating the headquarters in the low-tax country 1. The primary difference this causes is that the indifferent firm λ now pays w_2 on production costs regardless of where it locates its headquarters. Thus, (12) becomes:

$$\left[\theta a(N)^{\sigma-1} - a(\lambda)^{\sigma-1} \right] = \tau \theta \left[a(N)^{\sigma-1} - a(\lambda)^{\sigma-1} \right]. \quad (49)$$

Once again, this implies that low productivity firms will find it advantageous to locate their headquarters in country 2 because the tax savings are small compared to the wage savings on the fixed cost. Furthermore, this yields the same equation determining the last firm to enter ((9)). Now, however, since the cost advantages to locating in country 2 are smaller, the indifferent firm has a higher index than in the baseline model.

Despite this change in the equilibrium λ , there is still a discrete income benefit to undercutting the other nation's tax rate for the same reasons as described above. In fact, since FDI increases the profits of high productivity firms, doing so attracts even more profitable firms than it did before. This then provides a greater income boost than in the baseline model therefore the introduction of FDI only increases the severity of the race to the bottom tax competition. Thus, as in the baseline model, there are no pure strategy Nash equilibria with equal taxes and all equilibrium outcomes will have taxes less than $1 - \alpha$. Since the equilibrium with vertical FDI is the same as the baseline case when taxes

are equal, allowing vertical FDI does not change the solution to the social planner's problem, implying that all Nash equilibrium outcomes are again inefficient.

Finally, it is important to recognize that alternative assumptions on the social welfare function can result in tax harmonization being undesirable from the social planner's perspective. First, if countries have different α 's then harmonization creates distortions for at least one of them vis-à-vis its income distribution. As such, the social planner may choose to set differing tax rates in the two locations. Second, if country's welfares are unequally weighted in the social welfare function, the social planner has two reasons to maintain different tax rates. The first of these is that the social planner is no longer indifferent over which country hosts the high productivity firms. By setting its favored nation's tax just under the other, this ensures that its favored nation will host these firms, providing a boost to the social welfare function. Furthermore, by lowering the favored nation's tax relative to the others, this benefits that country (at the expense of the other) by sending even more firms to it. Thus, using unequal tax rates is a method of shifting income from the high tax to the low tax country. Nevertheless, this is an inefficient way of doing so since lump sum transfers could be used to shift this income without the distortions unequal taxes create.

6. Conclusion

The goal of this paper has been to incorporate recent innovations from the trade literature on mobile firms into a tax competition model. In particular, we have modeled competition between governments for heterogeneous, imperfectly competitive firms with endogenous entry. These new features of the model highlight a heretofore unrecognized

aspect of tax competition – that it can encourage excessive firm entry. This then adds to the typical woe of tax competition, the underprovision of a public good. Furthermore, our framework allows us to study the extent of this problem even in a model where firms choose a single location, something that cannot generally be done in other models with discrete investment.

An implication of our results is that tax coordination, or at least de facto coordination by imposing a minimum tax rate across countries, can improve welfare relative to the Nash equilibrium. This then lends some support to the drive for such coordination by the OECD (1998, 2000) or the European Union (see van der Hoek, 2003, for a discussion). While there are certainly reasons to caution against harmonization (such as varying preferences over public versus private consumption), we hope that our results add further depth to this lively and important debate.

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Figure 1: Distribution of Firms

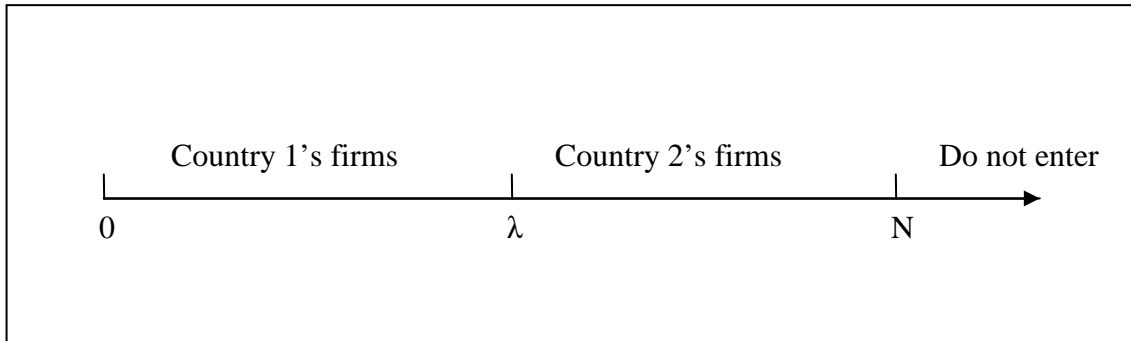


Figure 2: Best Response of Country 1

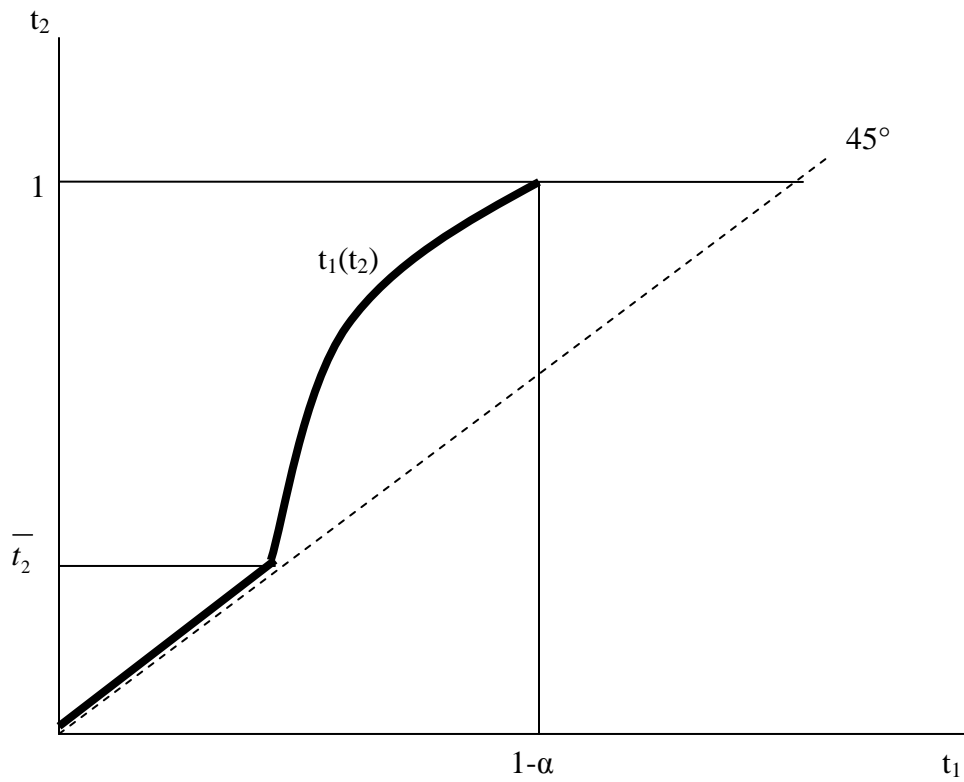


Figure 3: Best Response of Country 2

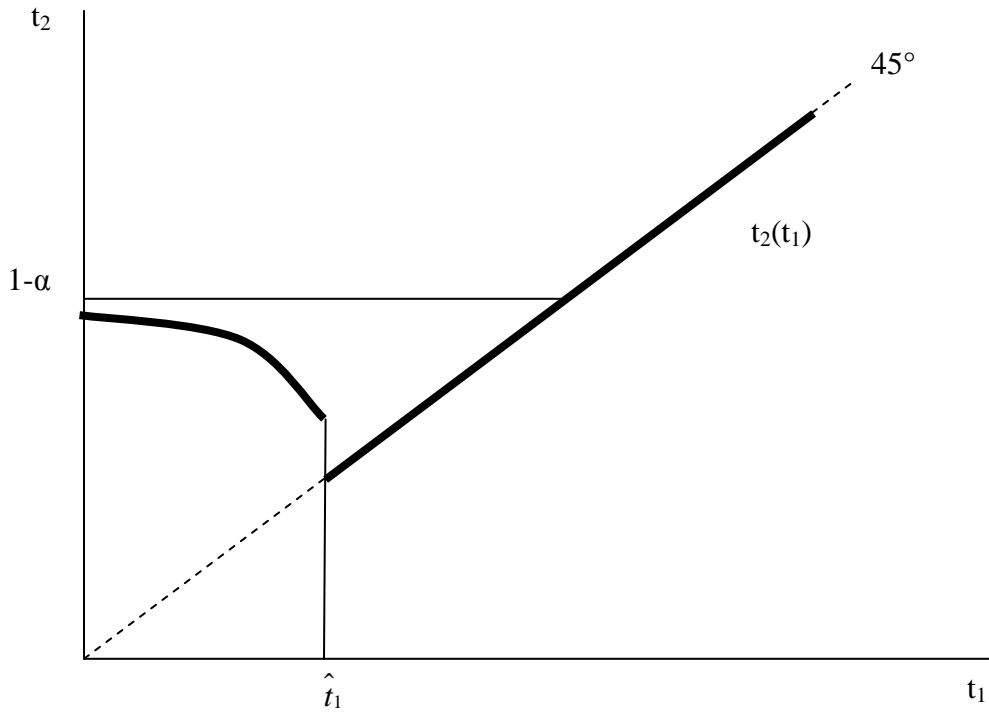


Figure 4: Country A's Best Response when $\bar{L}_A = \bar{L}_B$

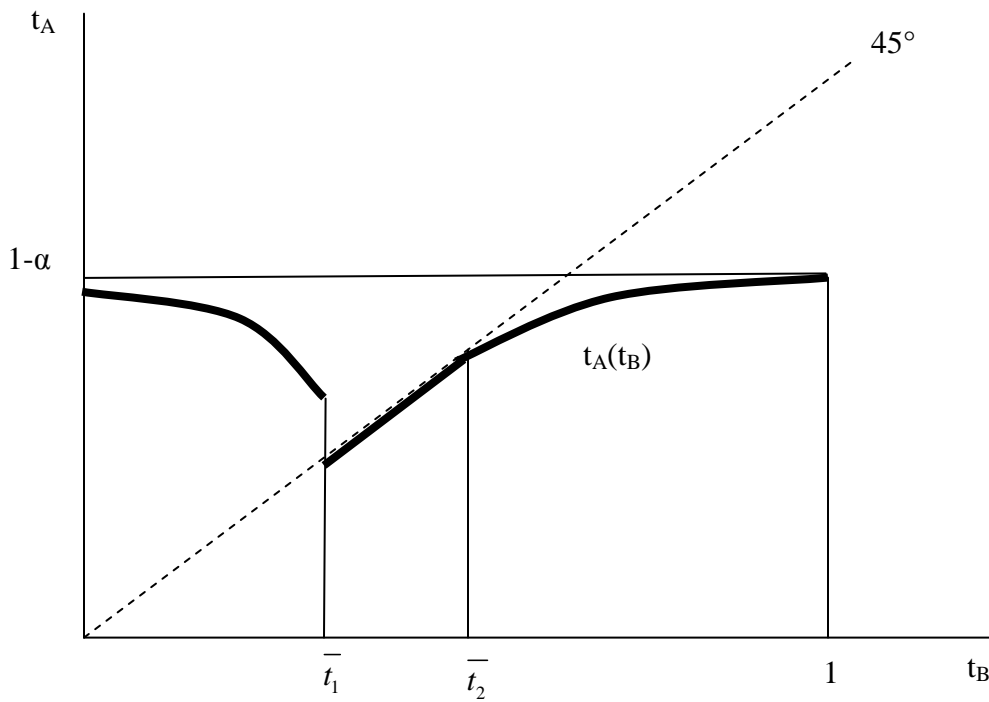


Figure 5: Nash Equilibrium when $\bar{L}_A = \bar{L}_B$

