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Volume Title: The Role Of The Computer In Economic And Social Research

Volume Author/Editor: Nancy D. Ruggles

Volume Publisher:

Volume URL: http://www.nber.org/books/rugg74-1

Publication Date: 1974

Chapter Title: A Simulation Model of the Mexican Educational System

Chapter Author: Ernesto Schiefelbein

Chapter URL: http://www.nber.org/chapters/c6624

Chapter pages in book: (p. 231 - 252)

A SIMULATION MODEL OF THE MEXICAN EDUCATIONAL SYSTEM

ERNESTO SCHIEFELBEIN*
Center for Educational Studies, Mexico

SUMMARY

This paper describes a linear programming model designed to study the Mexican educational system and the effects of alternative educational policies on it. In the course of implementing the model, the principal educational statistical data were found to be inconsistent. In order to overcome these inconsistencies a Markov-type model was constructed to simulate the flow of students at the elementary- and secondary-school levels. By means of this and other information and subjective estimates (Delphi Method) of the remaining parameters, versions of the linear programming model were computed for different periods starting with 1970. The specification of the model has been completed and when estimates of the demand for human resources become available, it will be possible to use the model for analyzing decision-making problems involving uncertainty.

1. INTRODUCTION

There is increasing pressure to make decisions in a rational manner, especially when the decisions affect large groups of the population. With perhaps a certain lag relative to other sectors, this need is now being felt in the educational field. Educational authorities in different countries may differ in the goals which they hope to attain, but all want to reach them with a minimum of resources, that is, in the most efficient way.

In the last decade, there has been great interest in applying the analytical tools of economics to the educational sphere. Specialized literature on educational models is abundant, but the number of works that describe applications to real situations is very limited indeed. This is, perhaps, attributable to the complexities of the interrelationships which must be taken into account in making decisions about educational problems.

As in most social systems, education can only be partially controlled. The educator in an executive position cannot foresee all possible results of each alternative decision, because they depend on events that cannot be predicted ex ante with any degree of accuracy. In other words, the executive who directs the educational system encounters problems of the type known as decision-making under uncertainty.

2. OUTLINE OF WORK

When the Center for Educational Studies undertook the task of examining concrete proposals for action for the Mexican educational system, it had to grapple with these problems of uncertainty. Among the various procedures that could be utilized, the designing of a model capable of systematically exploring the

* The author is currently Director of the Interdisciplinary Research Program in Education [PIIE], Catholic University of Chile.

relevant alternatives was selected. They wanted to devise a tool that would make it possible to show in a simplified form the effects of the various policies which might be implemented in the Mexican educational system. In other words, instead of attempting to calculate *the* optimal solution, the aim was to insure that the executive was consistent in his "guesstimates" and that he at least evaluate the possible effects suggested by the calculations of the model.

Since the global model should be useful to the educational executives (assuming they are educators), requirements such as the following had to be met:

—the model should comprehend the whole system;

—the variables and parameters should have obvious significance for the educators (see Appendixes A, B, C);

—the model should make it possible to obtain results comparable to those habitually obtained;

—the internal effects of the alternatives should be considered, that is, the repercussions within the educational system;

—the variables determined in other activity sectors, outside the educational system should be considered exogenous;

—the results must be obtained the next day.

A fundamental assumption underlying all the work is that the model will be used within a planning context, that is, it will be used by planners who have an extensive knowledge of the situation represented by the model.

The characteristics of the model of the educational system depend on the availability of the data necessary for its implementation. A few preliminary checks of the statistics indicated some inconsistencies; it was necessary to eliminate them with the aid of a Markov-type model of the flow of students. Since the future use which can be made of this model will depend, in part, on additional studies (e.g., of human resources and construction), it can be said that carrying out the objective will involve a system of models that are mutually complementary. Only the global model will be described below in detail.

A special effort was made to determine transition coefficients that would be consistent with the other information used in the model; however, other parameters also need to be estimated with great precision. In any event, the nature of the problem tells us that it will never be possible to reach a complete definition of the situation. That is why the model has been designed without waiting for better sources, and assumptions have been made on qualitative aspects of the model. The Delphi method was used to obtain some of these.

In the first stage, described in this paper, the goal has been to adjust the model so that its functioning adequately represents the reality that will be modeled. In a subsequent stage, it will be possible to include the recently completed studies of human resources, to utilize information from the recent population census, and to make specific studies with the assurance that their results will be incorporated in a model which has been shown to function adequately.

3. Global Model of the Educational System

The educational system is considered in this model as a structure within which the intensity of use of a certain number of educational processes must be determined. Each process, or activity, is defined as the result of rates, or quotients, of the annual inputs of certain factors and the generation of certain results in the corresponding period. Each activity corresponds to a different way of teaching or learning.

The educational system is defined in the model as an aggregate of activities which generate educated persons in each period. To educate by these activities, different resources are used, the supply of which is assumed to be known. The number of people to be educated depends upon economic and social demands which are considered to be exogenously determined.

It is assumed that the variables are continuous, that is, that each process can attain any level within a previously defined range if the corresponding factors (resources) are available.

The optimum of a solution corresponds to the maximum value that a given function can attain while all the restrictions established in the problem are respected. This function, which is utilized as a criterion for determining the optimal solution, is called the objective function. In this problem, both the objective function and the restrictions will be linear expressions.

4. General Characteristics of the Model

In the process of forming persons with different levels of preparation (academic achievement), the educational system utilizes human resources (teachers, students, workers) and other resources (buildings, texts, transportation, and various monetary and nonmonetary expenditures) at relatively constant rates. In order to design the model, it will be necessary to assume that the educational technology implicit in these coefficients will be maintained at least at the level that would be predicted by historic trends.

Another important assumption refers to the influence that education and on-the-job training can have on the productivity of the workers. This will permit relating various types of training to the demand for graduates of the system. The specific form of these relationships will be discussed below.

For each of the periods considered, 65 variables were defined: 20 levels of education; 22 alternatives for increasing the technology of the system; 2 types of teachers; 3 types of school building; 5 types of on-the-job training; 7 levels of skill of labor; and 10 overflow variables defined to facilitate the study of the system's bottlenecks. Notwithstanding this level of aggregation, the model has 646 linear equations, 1,419 variables, and 4,573 elements in the matrix. In order to give a picture of the structure of the model, a tabulation of the equations is presented in Table 1.

Both the objective function and the relationships that define the system are expressed as linear functions, so that the formulation will constitute a special case of the well-known family of linear programming problems (see Appendix D). This will permit us to make use of the well-known properties of this set of problems.

In Table 1 numerous intertemporal relationships are described. They are obtained in part from a model of the flow of students. The students in each year depend upon those who existed the year before, in accord with rates of promotion, repetition, or dropout, and upon the new pupils who are integrated into the system

TABLE 1
SUMMARY OF THE EQUATIONS BY PERIOD

	Classification of the Equations			
Period	Identities (definitions)	Functional Relations	Limits (restrictions)	Initial and Terminal Conditions
Criterion		1		
1970	3	10	10	1
1970–1971	2	24	18	
1971	1	7	10	
1971–1972	5	24	18	
1972	1	7	10	
1972–1973	5	24	18	
1973	1	7	10	
1973–1974	5	24	18	
1974	1	7	10	
1974–1975	5	24	18	
1975	1	7	10	
1975–1976	5	24	18	
1976	1	7	10	
1976–1977	5	24	18	
1977	1	7	10	
1977–1978	5	24	18	
1978	1	7	10	
1978–1979	5	24	18	
1979	1	7	10	
1979–1980		21	18	
Totals	54	311	280	1

during the year. Other intertemporal relations correspond to the equations which determine the best alternative activities for increasing productivity, that is, the efficiency with which the system functions. Equations defining teachers and buildings required in each period constitute the rest of the intertemporal relationships included in this model. Time is considered as being discontinuous and the unit is one year of operation.

In describing the model, the following symbols are used:

—lower case letters: the initial letters of the alphabet are used as indexes or as stock variables; the last letters of the alphabet are used as vectors of activity levels, that is, variables;

—upper case letters: the last letters of the alphabet are used as matrices of variables; the upper case letters with a bar indicate the availability of resources (limits);

—lower case Greek letters: used as parameter vectors;

—upper case Greek letters: used as parameter matrices.

A summary of these equations and a definition of the nomenclature are presented at the end of this paper (see Appendixes A through D).

5. THE OBJECTIVE FUNCTION

It was pointed out that the model furnishes an optimal solution for each trial. The criterion for reaching each solution is the minimization of the operating expenses of the entire educational system in a certain number of periods, while at the same time observing the various conditions imposed by the model.

Differing from the normal usage of linear programming models, no attempt is made to reach a unique optimal solution. Computers make it possible to use the model to obtain families of solutions resulting from systematic changes of the parameters which reflect the assumptions on which each decision is based.[1] These changes make it possible to compute the effects of diverse educational policies on the educational system. In other words, the model, within the restrictions and the optimization criterion that define it, provides a set of solutions for any combination of information, functions, and assumptions which define a given educational system.

Among the solutions computed by the model, it is possible for the executive to select some, on the basis of subjective criteria, which represent a relative optimum, so that he can examine these combinations of assumptions with greater attention. In this manner, the model will help the executive in making decisions in a sphere in which it is very difficult to measure the results.

In the general design of the model in its present form, it is assumed that expenses are discounted at the initial moment. This makes it possible to examine the use of an annual flow of resources. Market prices can be assumed to be constant since marginal variations in the amount of resources used constitute a small fraction of the market value of these goods and there exists a specific restriction that prevents the use of more teachers than those available.[2]

The simpler the projection, the better—especially when it is not possible to establish with clarity the tendency of change. That is why it seems acceptable to use an objective linear function. It is necessary to examine the assumptions each time that a solution which is considered optimal is reached in order to verify whether, in this case, a different function might have been able to change the result substantially.

In accord with what has been stated above, the objective function of the problem would be the following:[3]

$$(1) \qquad C = \sum_{t} \alpha_{1t}p_t + \alpha_{2t}x_t + \sigma_t\bar{y}_t + \beta_t y_t + \gamma_{1t}r_t + \gamma_{2t}q_t + \delta_t v_t + \psi_t u_t + \varepsilon_t w_t + \pi_t z_t$$

If the meaning of each term is examined (see Appendixes A, B, and C), it will be seen that the expression to be minimized is nothing more than the sum of the operating costs (exclusive of teachers) of the kindergarten level (preprimary); plus the operating costs of the rest of the system; plus the expenses of attending to the children with serious problems before their entrance into the primary level; plus the costs of remedial courses and individual attention to those who need it; plus the social costs of children failing to enter the system on time; plus the social costs for premature dropouts; plus the costs of building new capacity for the various levels; plus the remuneration of teachers who work in the entire educational

[1] The selection of values can be made using the Monte Carlo method.

[2] In those solutions in which a rapid change in the inputs per pupil can be inferred, it will be necessary to estimate, subjectively whether the assumption of a linear cost function is met.

[3] Descriptions of the variables and coefficients appear in Appendixes A, B, and C.

system; plus the costs of on-the-job training; and plus the costs for resorting to extraordinary procedures to reach a feasible solution despite the restrictions established by the system's functioning.

The last set of variables enter into the solution solely when it is the only way to reach solutions, since, because of the high cost associated with them, the routine of computation tries all the rest of the variables first before letting these special variables enter; whenever a special variable does appear in the solution, it is a signal that there is some bottleneck in the system which must be uncovered.

The objective function assumes that both existing levels of expenditures for students and their prices will be maintained. Even if the prices are not affected by the quantities demanded by the educational system (with the exception of the demand for educators, which, in turn, depends on the levels of remuneration of the labor market), in a model with a ten-year time horizon it can be assumed that prices may have important variations, but clearly these cannot be foreseen in the base year.

6. The Restrictions

The algebraic expressions and the definitions of the variables and parameters appear in Appendixes A through D. Although the study of equations suffices for understanding the internal logic of the model, the comments below will facilitate identification of the purpose of each equation in the overall functioning of the model.

Equation (2): calculates the total current operating expenses of the regular educational system for each year and avoids letting these total current expenses exceed the current budget of this year. The variables and parameters correspond to those of the objective function. In order to avoid infeasibilities due to the budgetary restriction, a variable is included which, because of its high cost, only enters the solution when it is impossible otherwise to comply simultaneously with the budgetary restriction and the other restrictions of the model. It should be pointed out that equation (2) includes the costs of increasing the efficiency of the system in the future by means of consideration of the $\bar{y}$ variables lagged up to four periods.

Equation (3): calculates total capital expenditures required for the replacement of installed capacity or for the addition of new capacity, in each year, without exceeding the budgeted total for the period. If there is flexibility in the budget, it would be possible to combine this equation with the previous one, or to introduce variables which would permit the transfer of the budget surplus on capital account to the current budget.

Equation (4): calculates enrollment of year t as a function of the enrollment of the previous year and of those who enter the first grade of primary education. The transition coefficients between the two consecutive periods represent the technology of the system. This equation considers the possibility of improving the technology by means of two alternative processes. It is possible to improve the results by making students with problems enter the preprimary level ahead of schedule, or to attain this objective by giving them remedial courses, that is, special treatment, until the problems which impede their normal intellectual development are resolved.

Equation (5): defines enrollment in the preprimary course as the sum of students who require attention in kindergarten, before entering the first year of primary school, in order to avoid failures at the outset of their school life.

Equation (6): limits the annual increases of educational technology to a percentage of total enrollment in each grade in the following year. The resulting figure represents the maximum annual improvement in the efficiency of the system (more promotions or fewer dropouts) which can be attained given the conditions (especially costs) defined in the model for reaching these objectives. The allowed percentage by which the technology of the system can be raised is fixed subjectively by the Delphi Method.

Equation (7): calculates the total supply of personnel that is incorporated in each level of labor and compares this total with the labor demanded. The demand estimate is partially exogenous since, internally, consideration is given to the requirements for the replacement of persons who succeed, by means of accelerated schooling, in passing to a more highly skilled level, as well as to teachers needed in each level. This equation makes it possible to use the surpluses produced in the various qualification levels on the qualification levels immediately lower, where this personnel is included in the corresponding total supply.

Equation (8): calculates the total number of primary and secondary teachers that are needed to take care of students who are enrolled in the regular and remedial courses. It compares this total with the supply of teachers determined in equation (9) and it forces this supply to be equal to, or larger than, the number required. When the minimum level of supply is determined in this equation (supply equals demand), the minimum number of teachers to be trained in the period is also determined in the following equation. In this version it is established that supply and demand are equal in each period, but it would also be possible to define the relationship lagged by one year.

Equation (9): defines the total supply of teachers as the sum of the survivors of the previous period (who have not retired from exercising their profession) plus the teachers trained in that period. However, the teachers trained in the period increase the personnel requirements because they are included in total manpower demand (equation (7)).

Equation (10): calculates the space requirements (buildings) of the students enrolled in the regular educational system. It compares this total buildings requirement with the total capacity of the previous period, increased by the investments of the same period; and it establishes that the capacity must be equal to, or greater than, the requirements. When the minimum requirement is determined in this equation (capacity equal to demand), in the following equation the minimum square meters of building space that must be constructed in the period is determined. The relations between supply and demand could be planned with a lag of one or more periods in order to take into account the time the construction will take. In this version, nonetheless, no lag has been considered.

Equation (11): defines the total capacity available in the period as the sum of the available capacity in the previous period (reduced in accordance with an average rate of depreciation) plus the additions to capacity that are produced. If it is possible to establish how far in advance construction must be initiated, one can consider one or more periods of lag in the pertinent variables.

Equation (12): establishes that a certain proportion of the students of the preprimary level must remain at least two years in that level in order not to have learning problems later on. In the implementation of the model, this equation has only been used to establish that these levels grow by at least a 10 percent annual cumulative rate, given the low proportion initially expected. In utilizing this equation for its original purpose, it is necessary to return to the Delphi Method when appropriate studies are not available.

Equation (13): establishes that the first preparatory grade must grow from period to period. However, in order to prevent this restriction from generating an impossible solution, a variable is included which, because of its high cost, only enters the solution where it is impossible for the model to find any other method of matching the required enrollment in the first grade while satisfying the other restrictions imposed by the model.

Equation (14): establishes certain minimum activity levels for the regular educational system in each period. In implementing the model, this restriction is used only as an initial condition for the first grades in the first period. This equation has great usefulness in establishing initial and terminal conditions of the model which permit a true representation of the reality which is being modeled. These conditions require, usually, flexible functional relationships for their adequate definition.

Equation (15): determines the total number of youths within an age cohort who remain outside the system each year. This number is calculated as the difference between the total number of students enrolled in a certain level of the system and the total number of youths of the age which is normal for that level. This equation facilitates the representation, in the model, of the effects of various policies intended to reduce the number of youths who are marginal to the system in some of the age groups.

Equation (16): determines the dropouts from the first levels of the system. In these levels, it is very difficult to find measuring devices to evaluate the academic achievement of those students. Therefore, the large number of dropouts ought to be attributed to the teachers' expectations. That is why in the model the total number of premature dropouts is designated as one of the possibilities for massive increase in the efficiency with which the regular school system operates.

Equation (17): establishes that the number of pupils in the seven-year-old cohort, which is entering the first year of primary school, must be at least equal to that of the previous period. In this way the social imperative of not reducing the input capacity of the system in any period is represented. If desired, a coefficient could be included by means of which the input capacity would grow at least at the rate of increase of the total population, or of the respective cohort.

Equation (18): establishes a minimum activity level of institutions providing accelerated manpower training. In this way, one can assure the availability of a group of highly qualified instructors and of an operating organization, even though at a minimum level (its cost will correspond, basically, to fixed costs, that is, those that would not vary with changes in volume of activity), which could be easily expanded in the future if circumstances so demand.

7. Estimation of the Parameters

As was pointed out earlier, preliminary examination of the data revealed certain inconsistencies. The tasks of refining the model and of gathering adequate data were carried out simultaneously. A special study was made of those transition rates for which the historic values were seriously questionable.[4] In this way, it was possible to use these rates in the first versions of the model. For the other parameters, the best available data were used, as well as subjective estimates—using simplified versions of the Delphi Method—to complete the information needed in the design of the model.

The information used in the version described in this paper is presented in Tables 2 through 6. It is hoped that, in this way, it will be possible to verify the model's validity by comparing these figures with those available from other sources. Given their provisional nature and limitations of space, the sources for each table are not described.

It can be observed that, for example, the student-teacher relationship used in the model is 37, whereas the historical relationship is approximately 46. There is no intention of absorbing the difference in the ten years which the model considers; rather, it is maintained as a constant throughout the exercise. It is evident that if a better understanding of the educational policy about the student-teacher ratio is achieved, it can be easily represented in the model. A similar treatment was employed for school buildings used in multiple shifts. In order to simplify the problem in this case, the initial (theoretical) capacity was calculated in accord with the numbers per student which will be used in the period. Therefore, the total numbers for capacity which appear in Table 2 exceed reality by a large margin.

Regarding costs per student and teachers' salaries, it was estimated that the annual rate of increase would correspond, approximately, to the discount rate which exists in the Mexican market. Therefore, the initial prices were held constant for the entire period.

8. Testing the Model

Given the nature of the model, no systematic procedure for testing the model could be used. Only the quantitative aspects were tested, comparing actual figures for the 1962–1968 period with the model outcome for the same period. The results are presented in Table 7. The actual figures included in the 1962–1968 period were not used for estimating the transition parameters.

The fit for the primary level is relatively good. All discrepancies between the model's estimates and reality do not exceed 3.2 percent except for three cases in which errors reach −5.7 percent in 1963, +6.3 percent in 1966, and +4.2 percent in 1968. Given that the rates were estimated by adjusting continuous curves so that they would preserve historical tendencies, it can be concluded that these discrepancies are not significant. It might be considered that the differences denote deficiencies in the gathering of statistical data. It is evident, however, that certain tendencies exist among the differences. Initially the estimates tend to be smaller, while during the last years the situation is the reverse. This type of error could

[4] E. Schiefelbein, "Un modelo de simulación del sistema educativo mexicano," *Revista del Centro de Estudios Educativos*, Vol. 1, No. 4, Mexico, 1971.

TABLE 2
VALUE OF THE PARAMETERS IN THE BASE YEAR

Symbol	Description	Value
ϕ_1	Student-teacher ratio at the preprimary level	0.027
ϕ_2	Student-teacher ratio at the primary level	0.03
ϕ_3	Student-teacher ratio at the secondary level	0.071
ϕ_1^1	Student-teacher ratio in primary-level remedial courses	0.05
ϕ_2^1	Student-teacher ratio in secondary-level remedial courses	0.1
θ_1	Square meters per student in the preprimary and primary level	1.85
θ_2	Square meters per student in the secondary level	4.5
θ_3	Square meters per student in the upper level	10.0
η	Maximum annual increment through better technology	0.10
$\bar{v}_1$	Square meters of initial capacity at the primary level	17,370,000
$\bar{v}_2$	Square meters of initial capacity at the middle level	6,770,000
$\bar{v}_3$	Square meters of initial capacity at the upper level	1,950,000
$\hat{w}_2$	Minimum annual activity in accelerated formation at level two	20,000
$\hat{w}_3$	Minimum annual activity in accelerated formation at level three	10,000
$\hat{w}_4$	Minimum annual activity in accelerated formation at level four	5,000
$\hat{w}_5$	Minimum annual activity in accelerated formation at level five	2,000
$\hat{w}_6$	Minimum annual activity in accelerated formation at level six	2,000
ψ_1	Annual cost per full-time teacher at the primary and preprimary level	493
ψ_2	Annual cost per full-time teacher at the middle level	1,420
α_1	Current costs per preprimary student (without teaching salaries)	181
α_{21}	Current costs per primary student (without teaching salaries)	91
α_{22}	Current costs per student of the first middle cycle (without teaching salaries)	760
α_{23}	Current costs per student of the second middle cycle (without teaching salaries)	1,366
α_{24}	Average costs per university student (1st to 4th year); including salaries	8,668
α_{25}	Average cost per 5th year university student; including salaries	9,000
α_{26}	Average cost per 6th year university student; including salaries	10,000
δ_1	Square meter cost of construction in the primary level	580
δ_2	Square meter cost of construction in the middle level	850
δ_3	Square meter cost of construction in the upper level	1,250
σ	Cost of entering students at the primary level	200
β_1	Cost of remedial work, primary	100
β_2	Cost of remedial work, first middle cycle	800
β_3	Cost of remedial work, second middle cycle	1,400
β_4	Cost of remedial work in the 1st to 4th year of the university	9,000
β_5	Cost of remedial work in the 5th year of the university	10,000
β_6	Cost of remedial work in the 6th year of the university	15,000
γ_1	Social cost of failure to enter at the proper time	26
γ_2	Social cost of premature dropout	26
ε_1	Cost of formation in the work of the first level	30,000
ε_2	Cost of formation in the work of the second level	20,000
ε_3	Cost of formation in the work of the third level	60,000
ε_4	Cost of formation in the work of the fourth level	20,000
ε_5	Cost of formation in the work of the fifth level	10,000
π	Cost of special variables	100,000

be corrected by increasing the number of trials in the Markovian model mentioned above.

The fit for the general secondary school is less good. In the lower cycle, differences do not exceed 10 percent, except for five cases concentrated in the years 1963 and 1964. In the higher cycle, in contrast, in several cases differences of 20 percent are reached, including some major ones in the last grade for several years. The great loss of students which occurs along the way until the middle level is reached means that one works with figures which can vary rapidly from year to

TABLE 3
VALUES OF THE LIMITS OF THE RESTRICTIONS IN EACH YEAR

	Budget Available		Manpower Requirements (thousands) by Years						Number of 7-year-old Children (thousands)
Year t	Current Budget C	Capital Budget K	7 to 9 Years W_2	10 to 12 Years W_3	13 and 14 Years W_4	15 and 16 Years W_5	17 Years W_6	18 Years and Over W_7	P
1970	10,700	1,300	200.0	75.0	17.0	4.0	6.0	3.0	1,537
1971	11,450	1,390	210.0	90.0	18.0	4.7	7.1	3.5	1,583
1972	12,250	1,487	248.0	97.0	21.0	12.0	8.4	4.1	1,631
1973	13,108	1,591	293.0	105.0	25.0	16.0	9.9	4.8	1,680
1974	14,025	1,702	346.0	113.0	30.0	20.0	11.7	5.7	1,730
1975	15,006	1,821	408.0	122.0	35.0	24.0	13.8	6.7	1,782
1976	16,056	1,948	481.0	132.0	41.0	28.0	16.3	7.9	1,835
1977	17,180	2,084	568.0	142.0	48.0	32.0	19.2	9.3	1,891
1978	18,382	2,230	670.0	153.0	65.0	36.0	22.7	11.0	1,947
1979	19,669	2,386	790.0	165.0	66.0	44.0	26.8	13.0	2,006

TABLE 4
RATES OF PROMOTION BY LEVEL IN EACH YEAR

	Rates of Promotion by Level																
Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1970	.58	.66	.65	.64	.62	.55	.56	.60	.51	.62	.52	.66	.35	.48	.385	.365	.165
1971	.59	.68	.65	.64	.63	.56	.57	.61	.51	.62	.63	.65	.37	.49	.39	.39	.20
1972	.61	.69	.66	.67	.63	.57	.57	.61	.51	.63	.52	.65	.39	.50	.42	.40	.22
1973	.62	.70	.67	.68	.64	.58	.58	.62	.52	.64	.54	.66	.40	.51	.43	.41	.23
1974	.64	.71	.67	.65	.64	.59	.58	.63	.52	.64	.55	.67	.41	.52	.44	.42	.25
1975	.65	.72	.67	.65	.65	.60	.59	.64	.53	.65	.56	.69	.42	.53	.45	.43	.27
1976	.66	.72	.67	.65	.65	.61	.60	.64	.54	.66	.56	.70	.43	.54	.46	.44	.28
1977	.67	.73	.68	.66	.65	.62	.60	.65	.55	.67	.57	.71	.44	.54	.47	.45	.30
1978	.68	.74	.68	.67	.66	.64	.61	.66	.55	.68	.58	.73	.46	.55	.48	.46	.32
1979	.70	.74	.68	.67	.66	.64	.62	.67	.56	.69	.59	.74	.47	.56	.50	.47	.33

TABLE 5
Rates of Repetition by Level in Each Year

Year	Rates of Repetition by Level																	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1970	.38	.25	.25	.25	.28	.30	.24	.23	.26	.23	.23	.24	.45	.42	.55	.60	.585	.05
1971	.37	.24	.26	.26	.28	.30	.23	.24	.25	.22	.24	.22	.43	.41	.51	.56	.54	.05
1972	.37	.24	.26	.25	.28	.30	.23	.24	.26	.22	.23	.22	.42	.40	.49	.54	.52	.05
1973	.34	.23	.26	.25	.28	.30	.23	.24	.25	.21	.21	.21	.41	.39	.47	.52	.49	.05
1974	.33	.23	.26	.25	.28	.28	.28	.22	.24	.20	.20	.20	.40	.39	.45	.50	.48	.05
1975	.32	.22	.25	.24	.27	.28	.22	.22	.23	.19	.19	.18	.39	.38	.43	.48	.45	.05
1976	.31	.22	.26	.25	.27	.27	.21	.20	.23	.18	.18	.17	.38	.37	.41	.46	.43	.05
1977	.30	.21	.25	.24	.26	.27	.21	.19	.22	.17	.18	.16	.37	.36	.39	.44	.41	.05
1978	.29	.20	.24	.24	.26	.26	.20	.18	.21	.16	.17	.15	.36	.36	.37	.42	.39	.05
1979	.27	.20	.24	.24	.26	.26	.19	.17	.20	.16	.15	.14	.35	.35	.35	.40	.37	.05

TABLE 6
Dropout Rates by Level in Each Year

Year	Dropout Rates by Level																	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1970	.04	.09	.10	.11	.10	.15	.20	.17	.23	.15	.26	.10	.20	.10	.065	.035	.25	.95
1971	.04	.08	.09	.11	.09	.14	.20	.15	.24	.16	.23	.13	.20	.10	.08	.05	.26	.95
1972	.02	.07	.08	.08	.09	.13	.20	.15	.23	.15	.25	.13	.19	.10	.09	.06	.26	.95
1973	.04	.07	.07	.07	.08	.12	.19	.14	.23	.15	.25	.13	.19	.10	.10	.07	.28	.95
1974	.03	.06	.07	.10	.08	.13	.20	.15	.24	.16	.25	.13	.19	.09	.11	.08	.27	.95
1975	.03	.06	.08	.11	.08	.12	.19	.14	.24	.16	.25	.13	.19	.09	.12	.09	.28	.95
1976	.03	.06	.07	.10	.08	.12	.19	.16	.23	.16	.26	.13	.19	.09	.13	.10	.29	.95
1977	.03	.06	.07	.10	.09	.11	.19	.16	.23	.16	.25	.13	.19	.10	.14	.11	.29	.95
1978	.03	.06	.08	.09	.08	.10	.19	.16	.24	.16	.25	.12	.18	.09	.15	.12	.29	.95
1979	.03	.06	.08	.09	.08	.10	.19	.16	.24	.15	.26	.12	.18	.09	.15	.13	.30	.95

TABLE 7
ESTIMATES OF ENROLLMENT BY COURSE FOR THE PERIOD 1962–1968

Grade	1962 Estimated	1962 Actual	1962 % Error	1963 Estimated	1963 Actual	1963 % Error	1964 Estimated	1964 Actual	1964 % Error
Primary:									
First	2,272.5	2,272.5	0.0	2,333.9	2,360.0	1.2	2,377.3	2,416.9	1.6
Second	1,282.3	1,282.5	0.0	1,361.5	1,361.0	0.0	1,458.1	1,453.6	0.4
Third	970.2	868.6	0.1	1,025.5	1,043.5	1.8	1,105.9	1,098.2	0.7
Fourth	678.0	678.4	0.0	746.9	751.4	0.6	803.8	810.4	0.8
Fifth	504.0	508.2	0.8	565.7	576.5	1.9	614.1	627.4	2.1
Sixth	401.0	401.0	0.0	423.0	448.9	5.7	509.0	495.4	2.8
Secondary:									
First	168.7	167.8	0.0	203.4	182.5	11.5	201.6	199.3	1.1
Second	122.0	122.1	0.0	147.5	132.8	11.2	177.1	150.1	18.1
Third	90.7	91.4	0.8	109.8	103.3	6.2	131.9	118.0	11.8
Fourth	49.5	48.8	1.4	59.0	63.0	6.4	72.1	77.1	4.0
Fifth	36.9	37.2	0.8	37.5	50.0	25.0	44.4	49.6	1.1
Sixth	6.5	4.5	44.7	9.6	10.5	8.6	13.6	12.7	7.1
University:									
First	111.8	110.0	1.6	127.8	115.7	10.5	125.8	127.4	12.5
Second	68.1	60.5	12.5	73.3	61.6	18.9	80.7	70.9	13.8
Third	37.9	37.9	0.0	45.1	43.8	2.9	48.2	45.3	6.6
Fourth	20.8	15.6	32.4	22.9	17.9	27.8	25.8	19.5	32.3
Fifth	9.7	8.6	12.8	13.5	11.1	21.6	14.6	11.2	30.2
Sixth	5.3	2.8	88.5	5.8	3.0	93.1	6.4	2.8	–

TABLE 7 (*continued*)

Grade	1965 Estimated	1965 Actual	1965 % Error	1966 Estimated	1966 Actual	1966 % Error	1967 Estimated	1967 Actual	1967 % Error
Primary:									
First	2,393.4	2,399.3	0.3	2,455.2	2,521.9	2.6	2,486.9	2,542.3	1.3
Second	1,558.2	1,536.8	1.4	1,588.0	1,615.1	1.7	1,686.2	1,715.0	1.9
Third	1,191.2	1,166.9	2.1	1,308.2	1,276.4	2.5	1,365.4	1,374.4	0.6
Fourth	891.4	873.5	2.1	961.5	959.8	0.2	1,064.9	1,050.4	1.4
Fifth	683.6	683.2	0.1	794.2	747.8	6.3	833.1	818.6	1.8
Sixth	541.1	551.1	1.8	595.7	602.7	1.2	677.5	658.7	2.8
Secondary:									
First	233.7	231.0	1.2	249.3	253.4	1.6	271.8	269.9	0.7
Second	180.7	171.7	5.2	199.7	195.5	2.2	211.7	208.2	1.7
Third	157.1	138.8	13.2	164.2	154.3	6.4	178.1	170.0	4.8
Fourth	87.9	80.2	9.6	106.4	87.6	21.5	115.9	97.9	18.4
Fifth	54.8	64.1	14.5	67.8	66.2	2.4	83.6	74.3	12.5
Sixth	20.1	22.5	10.6	27.2	31.1	12.5	39.8	30.6	30.0
University:									
First	140.6	152.4	7.7	146.0	151.2	3.4	154.5	49.2	–
Second	78.9	84.7	6.8	83.4	85.1	2.0	84.2	30.0	–
Third	51.4	47.9	7.3	49.2	52.0	5.4	49.5	16.7	–
Fourth	26.5	21.4	23.9	26.1	22.7	14.9	23.8	2.5	–
Fifth	15.6	12.5	24.8	15.0	11.8	27.2	14.0	0.8	–
Sixth	5.4	2.4	–	3.9	2.5	–	3.6	0.1	–

TABLE 7 (*concluded*)

Grade	1968		
	Estimated	Actual	% Error
Primary:			
First	2,464.8	2,530.4	3.0
Second	1,811.0	1,738.4	4.2
Third	1,480.8	1,438.5	2.9
Fourth	1,140.1	1,122.0	1.6
Fifth	914.8	886.8	3.2
Sixth	715.6	713.3	0.3
Secondary:			
First	302.3	278.9	8.7
Second	224.9	225.7	0.3
Third	188.7	185.8	1.6
Fourth	127.6	108.1	14.4
Fifth	94.2	87.7	7.1
Sixth	57.6	39.0	48.8
University:			
First	168.7	158.5	6.4
Second	85.1	91.1	6.8
Third	49.1	79.0	38.1
Fourth	22.1	30.2	26.6
Fifth	12.1	3.0	–
Sixth	3.3	3.0	–

NOTE: The percentage of error is equal to the quotient $\frac{\text{Estimated}}{\text{Actual}}$.

SOURCE: The estimated figures are computed from the model (see text). The actual figures have been estimated by the Center for Educational Studies.

year. This partially explains the deficiencies of the fit. One should remember, at the same time, that the middle-level rates were not fitted through the simulation model, since at this stage all attention had to be given to the primary level. Analysis of the differences leads to the conclusion that it would be possible to improve the fit in the middle level of the educational system substantially by a certain number of additional trials in the model.

The situation for technical education is similar to that of the general secondary system. It can be observed, in this case, that the figures for 1967 reflect a failure in the statistical information, since an abrupt decline in enrollment of all courses is produced. As in the previous case, a part of the remaining discrepancies could be eliminated by means of simulation of new sets of rates.

In all cases, however, it can be asserted that the observed margins of error in the various transition rates are narrow. They would involve an increase or decrease of less than 5 percent in each of these rates. This, in turn, makes it possible to determine that a large underestimation exists in the repetition rates calculated on the basis of the available continuous statistics on repeaters. Although the repetition rate for the first year in 1965 is reduced to 35 percent, it is still considerably greater than the figure of 21.1 percent which appears in the continuous statistics. However, some evidence exists that repetition would be higher than 40 percent.

In any case, the objective of the model was to provide not accurate forecasts, but a tool for thinking of as many secondary effects as possible. The model is not solved in order to get *the* solution to be implemented, but to obtain families of solutions (sensitivity analysis) according to alternative sets of assumed policies.

When the model is used, for example, to study the possible effects of eliminating actual repetition of 40 percent of the first-grade enrollments, all the structure of the system must be changed. It follows that most of the historical relations are distorted. It makes no sense, therefore, to attempt a more detailed validation of the model.

9. Usefulness of the Model

As has been pointed out earlier, only the first stage of work has been completed. The model has been computed for different periods and its satisfactory functioning has been verified. It has been possible to confirm that in using the model for more than six periods, computation time tends to increase excessively, reaching close to one hour for each solution. In the future, it would probably be useful to reduce the size of the model.

For the time being, use of the model to represent the effects of the various alternative policies is not intended, since first the quality of certain estimates will have to be improved, especially those which refer to demand for manpower with different grades of qualification. It is known that work is being done along this line. It is probable that the data will soon be available to permit proper use of the model. The tabulation and analysis of the population census carried out in 1970 could, for example, clarify some important data, such as the values of the transition rates.

In the results of preliminary versions, it can, nonetheless, be pointed out that given the high rates of repetition, the model tends to include in the solution the activities of on-the-job training and of technological improvements. These activities represent alternatives to the regular system that have considerably higher annual costs per student than the regular system. The remedial classes cost almost three times as much as the regular ones, and four times more than the regular classes of preprimary school. Nevertheless, when computing the relative efficiency of the activities in terms of graduates, remedial classes are preferred. These results show the necessity of revising the data used and, should they be adequate, the desirability of modifying actual teaching practices.

Another indirect result of this first stage of the program lies in pointing out the need of concentrating efforts in collecting and analyzing a fairly limited number of statistics that would appear to be key ones in the decision-making process in the Mexican educational system. This concentration of effort should permit us to count very shortly on having the necessary information.

Once the revised version of the parameters is available, it would be desirable to use a model with five annual periods plus a terminal period of five years in order to reduce computer time and to keep a ten-year time horizon. This solution would maintain the ability to study the long-term effects of decisions that affect the system's day-by-day operation.

Using the model to show the repercussions of various policies on the variables included, it will be possible to construct transformation curves among different "products" of education, or to calculate constant cost curves among the various activities which generate these products.

Mention is always made of the desirability of considering secondary effects in evaluating educational projects, but actual computation is not frequently done. The frame of the model, nonetheless, permits computation of the reduction in costs which would be gained in the future because of the improved technology generated by the investment of the plan. The present value of the planned solution can be compared with the actual costs. The difference in the cost of using traditional techniques permits the calculation of an implicit discount rate which can be compared with the corresponding rates of other alternative projects.

Each time that results are found which have positive or negative characteristics of special interest, it will be possible to create work groups commissioned to study the problem in depth. The members of the group would have at their disposal, as a starting point for their work, the set of interrelationships that generated the model's results and that call for special attention. Their critical analysis would permit an in-depth study, at times with a creative approach, of the problems facing the system.

It should be remembered, in concluding this section, that the work already done confirms that "models cannot replace the executive or planner." The model does not constitute a sufficient basis (although it is a necessary one) for making decisions. Its main utility lies in checking intuitions, confirming the consistency of the assumptions, inspiring new solutions by computing unforeseen results and, quite importantly, in forcing explicit definitions of the essential elements that influence the decision. The model cannot be used mechanically. The solutions of the model can only be interpreted in light of a thorough understanding of the total situation into which the particular aspect being examined with the help of the model is being inserted.

It should be pointed out that the proposed model has certain features which differentiate it from previous models. It includes alternative methods of giving education, that is, various educational technologies; and it offers the possibility of assigning social costs for students not covered by the system, as well as for students who drop out prematurely. As in previous models, this one can consider the relations between regular education and on-the-job training, and the minimum levels of education subjectively desirable; and it can provide indications, through shadow prices, of the critical points which reduce the efficiency of the system.

10. Conclusions

The construction of models makes it possible to obtain results both in the design stage and in the utilization of the model as a manipulable representation of reality. Although some partial conclusions have been described in the previous sections, it is worthwhile to present here a general conclusion.

In designing a model of the Mexican educational system, it was possible to discover inconsistencies. The major problem appeared in the magnitude calculated for repetition rates, compared with the continuous statistics. It is mentioned in several available studies that the repetition in the first grade of Mexican primary school would be approximately 30 percent; but when this rate was used in the model, results were obtained which differed appreciably from reality. Later, the conclusion was reached that the repetition rate is greater than 40 percent. This result leads to the suggestion of the advisability of carefully revising educational policy, since it means that the attendance of at least one-million Mexican children is wasted in the first grade of primary school alone, with a loss of more than five hundred million pesos.

Testing the functioning of the model made it possible to concentrate efforts on collecting a relatively reduced set of statistical data. It would be possible to emphasize, on this ground, the statistics which facilitate the rationality of decisions, rather than those giving global orientations of the system.

In order to overcome traditional excuses of lack of statistics on the qualitative aspects of the educational system, recourse was had to the subjective judgement of experts. For this reason very simple versions of the Delphi Method were used. This procedure can also be considerably refined in the future in order to examine the possible effects of policies which would involve fairly large innovations in the functioning of the system.

The computations of the model showed that feasible solutions were obtained under the restrictions specified. No comments have been made about the numerical results, since the results of the studies of manpower now in progress still have to be incorporated into the model. At this time, it may be mentioned only that alternative activities to the system of regular education are included in the solution. As the inclusion of these activities in the solution involves a considerable cost per student, it is inferred that the regular system operates at an extremely low efficiency level (small percentage of graduates).

It is imperative to allocate resources, in the initial periods, to increasing the efficiency with which the system operates. There would not be any educational policy that would have greater importance than reducing the repetition in the first grades of the system. It is evident that if one out of every two children repeats the first grade, these children will be taking up space in the first grade in the following year, instead of having progressed a grade in their studies.

As in other models of equal size and complexity, it is possible to ascertain that the executive cannot foresee all the secondary effects of his decisions. In this sense a model such as the one described in this paper permits the person who sets the policy of the educational system to have systematic assistance in evaluating the possible effects of his decisions. The model cannot replace the executive in charge of the system, but it appears to be a necessary tool to improve the rationality of the system.

The time for running this type of model is fairly large and its initial conclusions must be considered more as suggestions for verification than for their value in themselves. The conference represents an excellent opportunity to confront the present formulation with the ideas of the other participants.

Appendix A: Definitions of the Variables Used in Period t

The following variables are defined in the order in which they appear in the equations.

p Column vector (2×1) of the activity levels p_j. Each activity level represents the number of children attending in the jth preprimary level.

x Column vector (18×1) of activity levels x_j. Each activity level represents the number of students at level j of the regular educational system (primary, secondary, and university).

$\bar{y}$ Column vector (4×1) of activity levels $\bar{y}_j$. Each activity level represents the number of students promoted to the jth educational level (in excess of historal trends) thanks to a preparatory course for entering the first year of primary school. The activity level of these variables shows the increase, with respect to the number of students who are usually promoted to grade j, necessary to match the set of restrictions, that can be attained with minimum cost.

y Column vector (18×1) of activity levels y_j. Each activity level represents the number of students promoted to level j (in excess of the historical tendencies of promotion) thanks to remedial attention to their learning problems.

r Total number of children who cannot enter the first grade of the educational system in the period in which they become seven years old.

q Total number of students who drop out from the first three levels of the system due to deficiencies in the quality of the education that they receive, or because their family forces them to work.

v Column vector (3×1) of activity levels v_j. Each activity level represents the number of m^2 built in period t for the jth educational level.

$\bar{u}$ Column vector (2×1) of activity levels u_j. Each activity level represents the additional number of teachers that have to be trained (in the regular system or on-the-job) to satisfy the requirements of the jth educational level.

w Column vector (5×1) of activity levels w_j. Each activity level represents the number of workers, trained on-the-job (including teachers trained on-the-job), with the jth quality level.

z Column vector (10×1) of activity levels z_j. Each activity level represents an overflow variable that, because of the high cost assigned to it in the objective function, only enters the solution when a feasible solution cannot be obtained in another manner.

s Total number of children who enter the educational system in the period that they have their seventh birthday.

In addition to the variables that are defined in each period t, the following variable is used as a result of the objective function which simultaneously considers the total amount of time included in the model.

c The present value (discounting to the year one) of the operational expenses (of all levels) of the educational system, during t periods, necessary to satisfy all restrictions that are described in the text. The expenses include current expenses as well as those of the investments necessary to enlarge the system's capacity to match the restrictions specified.

Appendix B: Definition of the Coefficients Used

α_1 Row vector (1 × 2) of coefficients α_{1j}. Each coefficient represents the current annual costs per student in the jth preprimary level.

α_2 Row vector (1 × 18) of coefficient α_{2j}. Each coefficient represents the current annual costs per student in the jth level of the regular educational system.

σ Row vector (1 × 4) of coefficients σ_j. Each coefficient represents the annual monetary costs per student (promoted in excess of the historical trend) of the jth educational level. This vector reflects the cost of anticipatory attention to those students who would have problems in entering the first grade of primary schools without previous training.

β Row vector (1 × 10) of coefficients β_j. Each coefficient represents the annual current cost of avoiding the repetition of a pupil in level j by having him attend remedial courses. It reflects the cost of introducing better technologies in the system in order to improve the promotion rates of the corresponding level.

γ_1 Social cost for each student who cannot be accepted by the school system when he reaches the normal age for entry. Initially, it is estimated at a very reduced value. It is included in the model with the aim of being able to represent in the future the effects of assigning various values to this parameter.

γ_2 Social cost per student who drops out unnecessarily during the first years of the system. Initially, a very reduced value is assigned to it. It is included to study, in the future, the effects of changes in the value of this parameter.

δ Row vector (1 × 3) of the coefficients δ_j. Each coefficient represents construction costs per m² in the jth educational level.

ψ Row vector (1 × 2) of the coefficients ψ_j. Each coefficient represents the annual costs for a teacher who works in the jth level.

ε Row vector (1 × 5) of the coefficients ε_j. Each coefficient represents the annual costs per worker who has been trained on-the-job for the jth level.

π Row vector (1 × 10) of the coefficients π_j. Each coefficient corresponds to a fairly high number, in order to avoid, if possible, having the corresponding variable remain in the solution.

Z Diagonal matrix (18 × 18) of the dropout rates ζ_{1j} of the jth level of the educational system.

H Diagonal matrix (18 × 8) of the coefficients η_j. Each coefficient represents the maximum increase in the number of promotions from the jth educational level (educated guess).

Λ Matrix (n × n), with ones in the diagonal above the main one and zeros in the remaining elements. The product of this matrix and a column vector is a vector, element j being the element $j + 1$ of the multiplied vector. The last element of the new vector is zero.

λ_q Column vector (n × 1) each of whose elements is 1.

M Diagonal matrix (18 × 18) of the coefficients μ_j. Each coefficient represents the proportion of the total number of students who drop out from the jth educational level and who join the labor force.

ρ Row vector (1 × 18) of the coefficients ρ_j. Each coefficient represents the minimum percentage of the enrollment of the preprimary school level who have to remain in that level for two years.

Φ Diagonal matrix (18 × 18) of coefficients ϕ_{1j}. Each coefficient represents the number of teachers per student in the jth educational level. Each coefficient is defined as the reciprocal of the student-teacher relation.

θ Diagonal matrix (18 × 18) of coefficients θ_{2j}. Each coefficient represents the number of m² per student in the jth educational level.

Ω Matrix (18 × 18) of transition rates. The elements ω_{ij} of the matrix correspond to the transition rates at the jth level. The elements $\omega_{j+1,j}$ correspond to promotion rates from the jth level to the level $j + 1$. The dropout rates remain included in the matrix M.

I Row vector (1 × 3) of coefficients i_j. Each coefficient represents the annual depreciation rate on buildings.

Appendix C: Definition of the Available Resources and of the Limits

C Maximum budget available for year t.

K Maximum capital budget available for year t.

P Number of children (population) aged seven in the period.

$\bar{U}$ Column vector (2 × 1) of coefficient $\bar{u}_j$. Each coefficient represents the number of teachers who teach in the jth educational level at the beginning of the initial period.

$\bar{V}$ Column vector (3 × 1) of coefficients $\bar{v}_j$. Each coefficient represents the number of m² available at the beginning of the initial period for the jth educational level.

$\bar{W}$ Column vector (7 × 1) of coefficients w_j. Each coefficient represents the number of workers (including teachers) required in the jth educational level in the period.

$\hat{W}$ Column vector (5 × 1) of coefficients $\hat{w}_j$. Each coefficient represents the minimum number of workers that have to be trained on the job in the period in order to maintain an activity level which permits expanding the operations up to the superior (maximum) range of the following period (educated guess).

Appendix D: Definitions of Equations and Inequalities

(1) $$\text{Min}: C = \sum_t \alpha_1 p + \alpha_2 x + \sigma\bar{y} + \beta y + \gamma_1 r + \gamma_2 q + \delta v + \psi u + \varepsilon w + \pi z$$

(2) $$\alpha x_t + \beta y_t + \beta y_{t-1} + \beta y_{t-2} + \psi u_t + \varepsilon w_t - z_t \leq \bar{C}_t$$

(3) $$\delta v_t \leq K_t$$

(4) $$\Omega_{t-1}\chi_{t-1} + s_{t-1} + \bar{y}_{t-1} - \Lambda\bar{y}_{t-1} + y_{t-1} = x_t$$

(5) $$p_t = \bar{y}_t + \bar{y}_{t+1} + \bar{y}_{t+2} + \bar{y}_{t+3}$$

(6) $$\bar{y}_t + y_t \leq H_t x_{t-1}$$

(7) $$M_t Z_t x_t - \bar{y}_t - y_t - w_t - \Lambda w_t - h + \Lambda h + \bar{u}_t - \Lambda\bar{u}_t + z_t \geq \bar{w}$$

(8) $$\Phi_1 x_t + \Phi'_1 y_t = u_t$$

(9) $$\bar{u}_t + \Upsilon u_{t-1} = u_t$$

(10) $$\theta x_t + \theta' y_t - v_t - iV_{t-1} = \bar{V}$$

(11) $$v_t + iV_{t-1} = V_t$$

(12) $$x_{1,t-1} + x_{2,t-1} \leq \rho x_{1,t}$$

(13) $$x_t + z_t \geq x_{t-1}$$

(14) $$x \geq \bar{P}$$

(15) $$s + r \geq P$$

(16) $$Z_t x_t - q_t \leq 0$$

(17) $$s_t \leq s_{t+1}$$

(18) $$w - z \leq \hat{w}$$