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THE ANATOMY OF MARKET POWER IN ELECTRICITY MARKETS WITH HYDROPOWER AS A DOMINATING TECHNOLOGY

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THE ANATOMY OF MARKET POWER IN ELECTRICITY MARKETS WITH HYDROPOWER AS A DOMINATING TECHNOLOGY*

by

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Abstract: The problem of optimal management of a water reservoir by a hydropower producer is necessarily a dynamic one since water can be transferred between periods. A hydropower producer being a monopolist cannot reduce output in the classical way without spilling water. He will follow a strategy of setting marginal revenues equal between time periods and thus shift water from relatively inelastic periods to relatively elastic ones. If the monopolist has thermal capacity the strategy is the same, but the utilization of thermal capacity is reduced. If the monopolist has control over external trade import is reduced and export increased compared with the social solution. Technical constraints of limited reservoir and interconnector capacity and a competitive fringe may reduce markedly the consequences of exercising market power.

Key words: Hydropower, thermal power, market power, flexibility-corrected price, competitive fringe.

JEL-classification: C61, D42, Q41

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1. Introduction

The deregulation of the electricity power production system in many countries since the early 1990ies has stimulated interest in the possibilities of producers behaving strategically. The classical implication of use of market power that production is reduced compared with perfect competition, also holds for electricity markets being supplied by conventional thermal power. Typical base-load plants like nuclear power plants do not have the same physical opportunities due to long and expensive start-up and close-down times. Systems with a significant contribution from hydropower with storage of water have not been studied so much. However, hydropower plays a significant part in many countries. About 20 % of the world's electricity is produced by hydro power, and 1/3 of countries in the world depend on hydropower for more than 50 % of their electricity generation (www.hydropower.org). Hydropower with water storage has features that set it apart from other generating technologies concerning possibilities of exercising market power. The almost costless instantaneous change in hydro generation within the effect capacities makes it perfect for strategic actions in competition with thermal generators with both costs and time lags involved in changing production levels of the latter. In countries with day ahead spot markets hydro producers interact daily and they all know that operating output-depending costs are zero, the opportunity cost is represented by future expected market prices and they may hold quite similar expectations. This may facilitate collusion. In the case of hydropower production can only be reduced by using less water. This may lead to spillage of water when reservoirs are limited and inflows positive. Spilling water has the same logic as burning coffee beans to support the coffee price of a cartel, but it is also easy observable and may be met with regulatory action, since spilling water from reservoirs is obviously not part of a social solution (if technically avoidable). The reason for the concern about potential market power abuse of hydro producers is that it may be used without any spilling of water and not so easy to detect by regulators, because market power is typically exercised by a *reallocation* of release of water on periods compared with what would be the socially desired release pattern. Measuring existence of market power by comparing price and marginal costs does not work for hydropower since variable cost is virtually zero. The relevant variable cost is the opportunity cost of water, but this is an expected variable and not directly observable.

Although there is some recent literature covering market power by hydro producers, the topic deserves a closer scrutiny and systematic review. Use of market power by hydro producers is covered in Ambec and Doucet (2003) and Crampes and Moreaux (2001) using very simplifying assumptions. A two-period model is considered in both models and the standard result of a monopoly following the strategy of equalising marginal revenues of the periods, resulting in a reallocation of water from periods with relative inelastic demand to periods with relatively more elastic demand, is established. A constraint on the transferability of water from one period to the next is not considered. Borenstein et al. (2002) investigated the possible use of market power by hydro producers when thermal capacities are also present at the backdrop of the California crisis. The formal model is the same as the model in Bushnell (2003) dealing with strategic scheduling of the hydro producer with different assumptions about the behaviour of the thermal producers. When a monopolist controls thermal capacities, the equalisation of the marginal revenue rule over the periods is confirmed.

In Section 2 the case of a hydro monopoly is analysed based on a basic water availability constraint establishing the nature of water shifting. Discrete time is used here and in the rest of the paper. Opening up for export- and import is studied in Section 3, both without and with limits on the amount of trade. The monopolist maintains the control of import, but takes the export price as given. The case of reservoir constraints not so readily found in the literature is analysed in Section 4. The case with both trade- and reservoir constraints is addressed in Section 5, and the case of a monopoly having both hydro and conventional thermal capacity is studied in Section 6. A hydropower firm with conventional thermal plants acting as a competitive fringe is analysed in Section 7 and some conclusions are offered in Section 8.

2. Monopoly

In order to expose the strategies of a hydro monopolist we start with the simplest possible case and then increase the complexities later. As a starting point we assume that all hydro producers are part of a monopoly and simplify further by considering the monopolist as a single production unit. We assume that the monopolist knows the period demand functions $p_t(e_t^H)$ on price form with standard properties. The amount of electricity produced by the monopolist in period t is e_t^H. The profit maximisation problem of the monopolist in the basic case of a single water availability constraint is:

$$
\begin{aligned}
& Max \sum_{t=1}^{T} p_t(e_t^H) \cdot e_t^H \\
& s.t. \\
& \sum_{t=1}^{T} e_t^H \leq W \\
& W, T \text{ given}
\end{aligned}
\qquad (1)
$$

For simplicity discounting is not performed. The horizon is T and for operational planning may be one to five years. The periods may be as disaggregated as hours, but are usually weeks. Water W is measured in electricity units. To use a constraint as specified above builds on the reservoir capacity never becoming constrained, and that all available water arrives in the first period. This is not so unrealistic in the case of Norway when over 2/3 of the yearly inflow comes from melting in a few weeks accumulation of snow. It is usual to assume that variable operating costs of hydropower can be neglected, and fixed cost will not play any part in the pure management problem of utilising a given amount of water, *W*.

The Lagrangian for problem (1) is:

$$
\sum_{t=1}^{T} p_t(e_t^H) \cdot e_t^H - l\,(\sum_{t=1}^{T} e_t^H - W) \qquad (2)
$$

The necessary first order conditions are:

$$
\begin{aligned}
& \frac{\partial L}{\partial e_t^H} = p_t'(e_t^H) e_t^H + p_t(e_t^H) - l \leq 0 \ \ (= 0 \text{ for } e_t^H > 0),\ t = 1,..,T \\
& l \geq 0 \ (= 0 \text{ for } \sum_{t=1}^{T} e_t^H < W)
\end{aligned}
\qquad (3)
$$

Assuming that the monopolist will produce electricity in all periods the conditions may be written:

$$
p_t(e_t^H)(1 + \breve{h}_t) = p_{t'}(e_{t'}^H)(1 + \breve{h}_{t'}) = l\,,\ \ t, t' = 1,..,T \qquad (4)
$$

In the expression for the marginal revenue of increasing production we have introduced the *demand flexibility,* $\breve{h}_t = p_t' e_t^H / p_t$ (the inverse of the demand elasticity), which is negative. The condition is that the marginal revenues, expressed as *flexibility-corrected* *prices*, should be equal for all the periods and equal to the shadow price on stored water. As in the textbook monopoly case the absolute value of the demand flexibilities (demand elasticities) must be less (greater) than, or equal to, one for a unique solution to exist. The short-run demand may in general be on the inelastic side, so the condition on the price elasticities is not necessarily so innocent. Prices may become quite high in order for the monopolist to be able to push demand to the elastic part of the demand function, and in the case of inelastic demand with vertical demand curve the monopoly solution characterised by (3) does not exist. Equality of marginal revenues between periods implies that the period with the relatively most elastic demand i.e. the smallest absolute value of the demand flexibility $\breve{\eta}_t$, at the optimal quantities of electricity obtains the smallest market price. From (4) we have:

$$\begin{aligned} & p_t(e_t^H) = p_{t'}(e_{t'}^H)\frac{1+\breve{\eta}_{t'}(e_{t'}^H)}{1+\breve{\eta}_t(e_t^H)} \Rightarrow \\ & p_t(e_t^H) < p_{t'}(e_{t'}^H) \text{ if } \left|\breve{\eta}_t(e_t^H)\right| < \left|\breve{\eta}_{t'}(e_{t'}^H)\right|,\ t,t' = 1,..,T,\ t \neq t' \end{aligned} \tag{5}$$

The benchmark social planning case uses consumer and producer surplus, $\sum_{t=1}^{T}\int_{z=0}^{e_t^H} p_t(z)dz$, as objective function while the monopolist only considers producer surplus, $\sum_{t=1}^{T} p_t(e_t^H)e_t^H$. The difference between the monopoly solution (3) or (4) and the social solution is that the flexibility-corrected price is substituted for the price. Compared with the solution in the social planning case the monopolist can only obtain higher profit than by using the common social if the demand functions differ over periods. If the demand functions are identical for the periods it follows from (3) that the flexibility-corrected prices become equal, and therefore the prices will be equal and equal to the common price in the social solution. However, the shadow value on the water resource becomes less than this price, reflecting that a monopolist considers the marginal revenue as the opportunity cost of using water. This difference may have implications in a dynamic setting of investment in new capacity. A monopoly will tend to expand less facing positive shifts in demand.

If water is left unused we have from (3) that the shadow price of water is zero. Since the shadow price of water is a scalar this implies that the flexibility-corrected prices must be equal to zero for all periods and hence the price flexibilities equal to 1.

An illustration in the case of two periods, where linear demand curves are used, is provided in Figure 1 in the form of a *bathtub diagram* (Førsund, 2005). The broken lines are the marginal revenue curves. The length AD of the floor of the bathtub indicates the available water. We have that in the illustration the marginal revenue curves intersect at a positive value, i.e. it will not be optimal for the monopolist to leave any water unused. This value is the shadow value on water. But this result depends on the form of the demand functions. If we have unused water as an optimal solution, then the shadow water value is zero. Going vertically up to the demand curves from the intersection point of the marginal revenue curves gives us the monopoly prices for the two periods.

In Figure 1 the thin dotted horizontal line $p_1^S p_2^S$ and the corresponding water allocation by the by the point M^S indicates the social solution. The shadow value of water is smaller in the monopoly case than in the social optimal case. If all water is to be used we must have in general that at least one monopoly price is lower than the social price. (Notice that this is not sufficient for all water to be used.) In this case, for the quantity corresponding to the lowest monopoly price the marginal revenue must be lower than the social price for the period in question and consequently the common shadow value

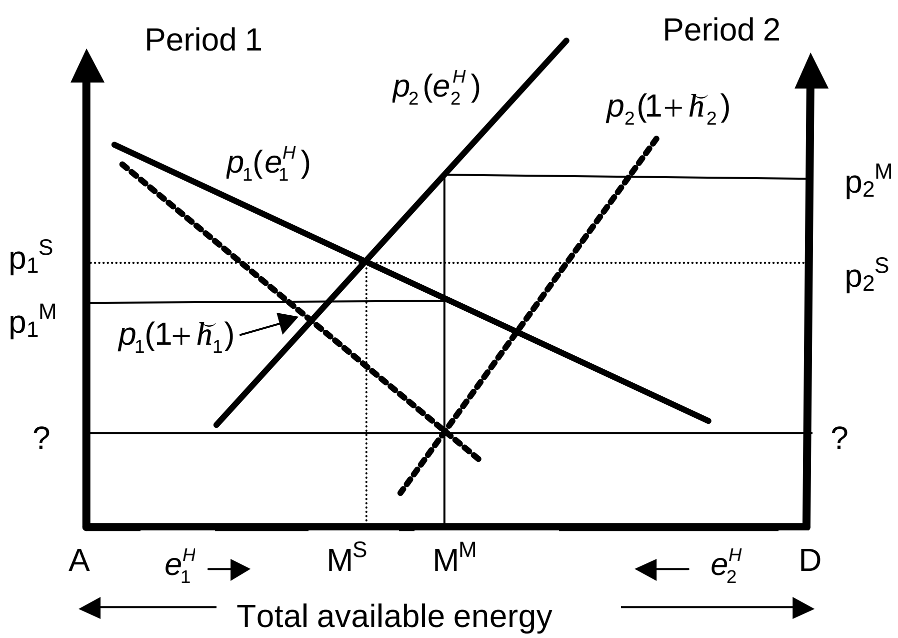

Figure 1. The basic monopoly case

of water in the monopoly case must in general be smaller than the shadow value in the social planning case. If water remains unused we have that the shadow value of water is zero, according to the complementary slackness condition in (3).

An important general result is that in the case of monopoly the market prices become *different* for the periods in contrast to the constant price in the social optimal solution indicated by the dotted horizontal line $p_1^S p_2^S$. For the period with the most inelastic demand, period 2, the price becomes higher than the social optimal price, and for the most elastic period, period 1, the price becomes smaller, in accordance with (5). Thus we have a general *shifting* in the utilisation of water from periods with relative inelastic demand to periods with relative elastic demand. The water allocation in Figure 1 moves from the point M^S in the social case to M^M in the monopoly case. Although the total electricity supply over the two periods is the same as in the social case the monopolist increases his profit by selling more in the most elastic period, and then partially reducing his revenue indicated by the area $(p_1^S - p_1^M)AM^M$ on the sales in period 1, but recouping more than this in increased revenue in period 2, indicated by the area $(p_2^M - p_2^S)M^MD$.

The monopolist will leave water unused if it is optimal to set marginal revenues equal to zero. Note that since we have only one shadow price on the water resource if marginal revenue is to be zero in one period the marginal revenues have to be zero in all the other periods, too, when water is used in all periods. By changing the slope of the demand curves in Figure 1 slightly this case is illustrated in Figure 2. The marginal revenue curves do not intersect within the bathtub, and becomes zero at M_1 and M_2 respectively for the two periods. Period 1 has the relatively most elastic demand and more electricity is sold than in the social solution reducing the monopoly price below the social price, as indicated by the position of the horizontal dotted line for the social case. In period 2 the available water is not fully utilised; the amount M_1M_2 is left unprocessed. The monopoly price is far above the social price.

Since unused water is easy to observe it may be of interest to see what the monopoly solution will be if a condition of full use of the available water is made. Technically this means that the water resource constraint is made into an equality constraint so the sign

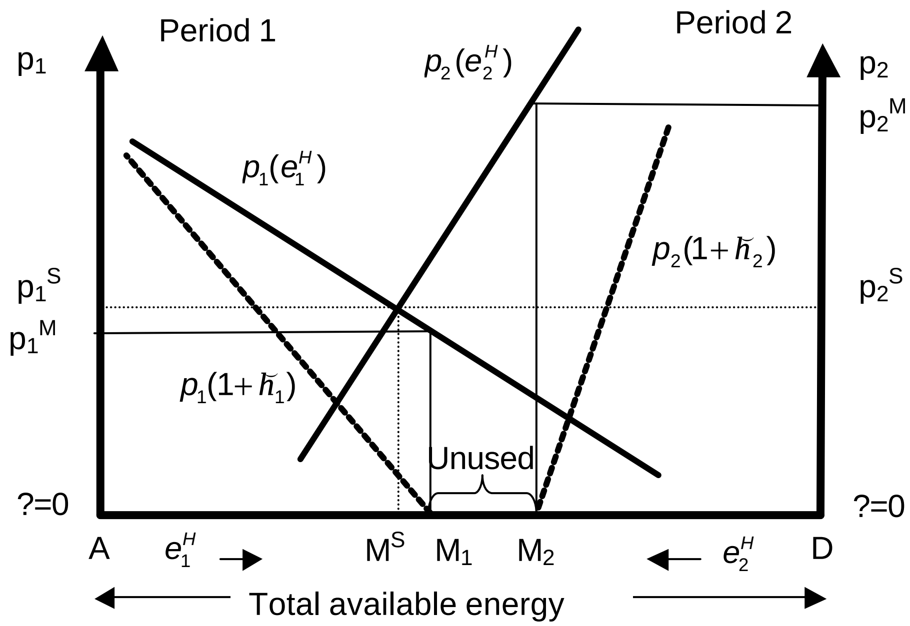

Figure 2. Unused water in the monopoly case

of the shadow price *?* in (2) is not restricted anymore and the last condition in (3) is dropped. Marginal revenues should still be equal and equal to the water shadow price. Using the same demand functions and total water availability as in Figure 2 the solution with the water constraint as an equality constraint means that the marginal revenues become negative, and more water is used in both periods, resulting in lower prices in both periods and still unequal prices, as shown in Figure 3. The monopoly - profit is reduced from the situation with inequality constraint on water use (indicated by the thin dotted lines).

3. Monopoly and trade

A hydro region with a regional monopoly may engage in electricity trade with neighbouring regions. Let us call a region for a country for ease. We will look at a situation where the monopolist controls both import and export, but takes the import/export prices as given. Unlimited trade will be assumed. Although this is unrealistic it will serve as a benchmark for introducing restrictions on the interconnector

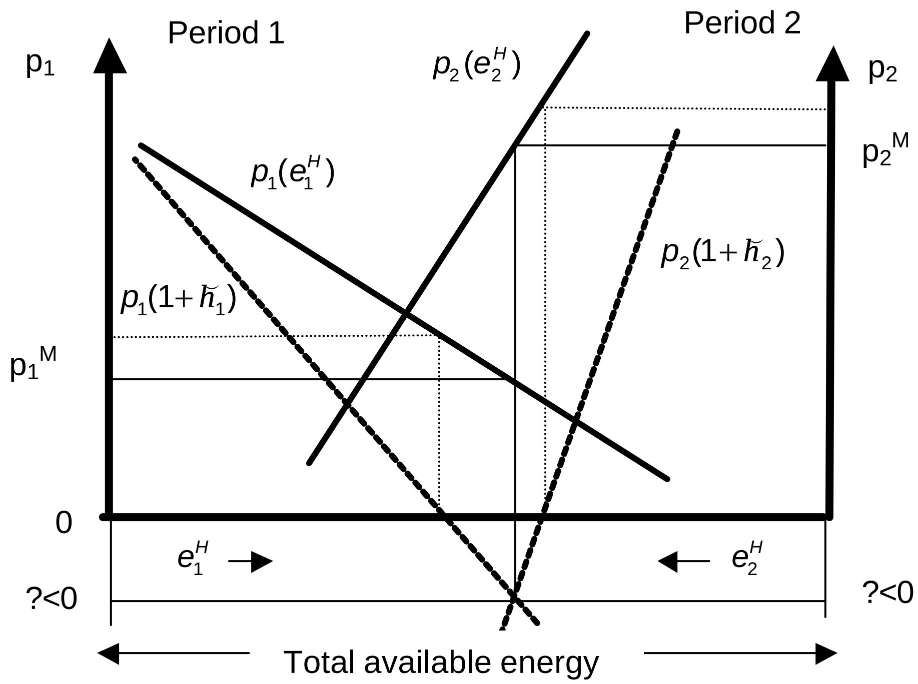

Figure 3. Monopoly with full resource-use constraint

capacity later. Extending model (1) we have the monopoly profit maximisation problem adding export revenues or subtracting import outlays from the home profit function:

$$
\begin{aligned}
&Max \sum_{t=1}^{T} p_t(x_t)x_t + p_t^{XI}e_t^{XI} \\
&s.t. \\
&x_t = e_t^H - e_t^{XI} \qquad\qquad (6)\\
&\sum_{t=1}^{T} e_t^H \leq W \\
&T, W, p_t^{XI} \text{ given, } e_t^{XI} \text{ free, } t = 1,..,T
\end{aligned}
$$

Here p_t^{XI} is the export/import price (prices are equal and transmission cost is disregarded) and e_t^{XI} is export if positive and import if negative. The first restriction in (6) is the energy balance; the consumption x_t at home may be supplied by locally produced hydro or by imports. Inserting the energy balance that holds as an equality constraint yields the Lagrangian:

$$
\begin{aligned}
L = &\sum_{t=1}^{T} p_t(e_t^H - e_t^{XI}) \cdot (e_t^H - e_t^{XI}) + p_t^{XI}e_t^{XI} \\
&- l\,(\sum_{t=1}^{T} e_t^H - W)
\end{aligned} \qquad (7)
$$

The necessary first-order conditions are:

$$\frac{\partial L}{\partial e_t^H} = p_t'(e_t^H - e_t^{XI})\cdot(e_t^H - e_t^{XI}) + p_t(e_t^H - e_t^{XI}) - l \le 0 \ (=0 \ for \ e_t^H > 0)$$

$$\frac{\partial L}{\partial e_t^{XI}} = -p_t'(e_t^H - e_t^{XI})\cdot(e_t^H - e_t^{XI}) - p_t(e_t^H - e_t^{XI}) + p_t^{XI} = 0 \qquad (8)$$

$$l \ge 0 \ (=0 \text{ for} \sum_{t=1}^{T} e_t^H < W)$$

We assume that the amount of electricity consumed locally is positive in all periods (i.e. $x_t > 0$) and that the export/import prices are all different. The second condition in (8) holds with equality since the export/import variable is not constrained in sign. Since the monopolist will not waste an export opportunity to positive a price the shadow price on water will be positive. If hydro is used in an import period then the first condition in (8) holds with equality implying that the flexibility-corrected home market price, $p_t(1+\breve{h}_t)$, is equal to the shadow price on water. The second condition tells us that the flexibility-corrected price is always equal to the import price. But since the export/import prices are different the shadow price on water can only be determined by *one* flexibility-corrected price. We know that in an export period we must also use hydro at home due to the assumption of positive consumption at home of electricity in all periods. Therefore in an export period the flexibility-corrected price is also equal to the shadow price on water. Due to lack of any restriction on trade it is the highest export price period that will become the *only* export period, and in all other periods there will be imports and no use of hydro at home. This means that in import periods the flexibility-corrected price is *less* than the shadow price on water.

An illustration is provided in Figure 4. Since the import price by construction is lowest in period 1 this period will be the import period. The amount of import is determined by the intersection of the marginal revenue curve and the import price line. The home market price will be higher than the import price in the standard way of a monopoly. Import may be regarded as an alternative way to using hydro to "produce" electricity (marginal revenue is set equal to the marginal production cost; the import price). In the export period the use at home of hydro is determined by the intersection of the marginal revenue curve and the export price line. Export is residually determined as the rest of the available water. The shadow price of water is equal to the export price. Comparing

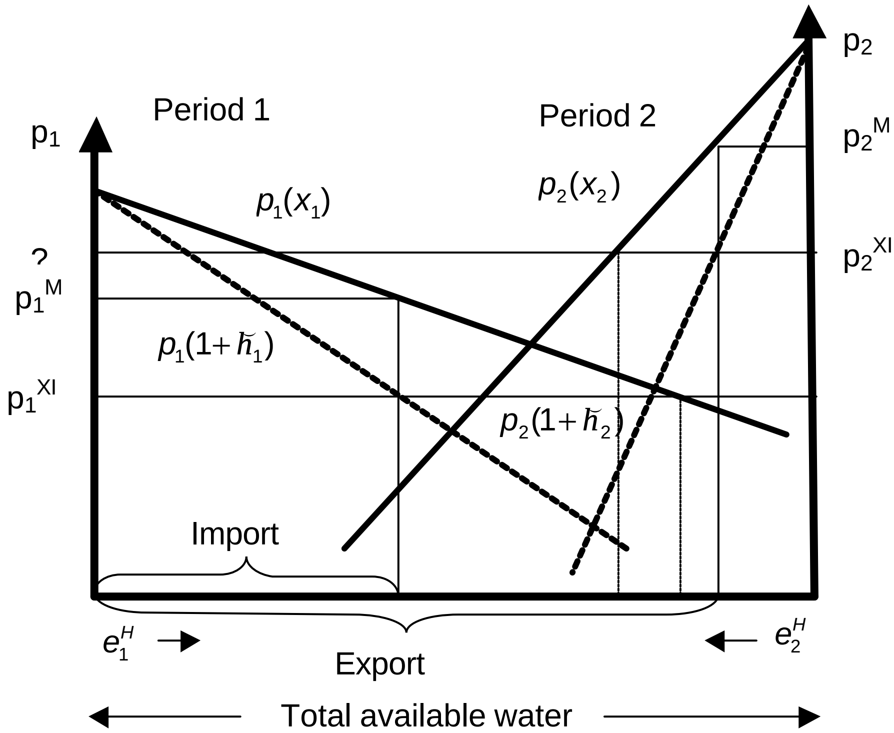

Figure 4. Monopoly and trade without restrictions

the monopoly solution with the socially optimal solution the latter is indicated by the vertical broken lines from the intersection of period 1 demand curve with the import price for this period, and the intersection of period 2 demand curve with the export price for that period. The import and export periods will be the same. The shadow price on water will be the same in the two solutions, but import will be considerably reduced in the monopoly case resulting in a higher home price than the import price. In the export period the monopoly will export more water and restrict correspondingly the use of water for electricity production at home resulting in a home price higher than the export price. The monopolist is playing price discrimination between two markets.

Constraining the trade

Constraining the amount traded due to limited interconnector capacity makes for a more realistic situation. The monopoly optimisation problem in the case of restrictions on trade is:

$$
\begin{aligned}
& Max \sum_{t=1}^{T} p_t(x_t)x_t + p_t^{XI}e_t^{XI} \\
& s.t. \\
& x_t = e_t^H - e_t^{XI} \\
& \sum_{t=1}^{T} e_t^H \leq W \\
& -\overline{e}^{XI} \leq e_t^{XI} \leq \overline{e}^{XI} \\
& T, W, p_t^{XI}, \overline{e}^{XI} \text{given, } t = 1,..,T
\end{aligned}
\tag{9}
$$

The restriction on trade can be expressed by one restriction on export and another on import, remembering that import is negative and export positive. Inserting the energy balance that holds as an equality constraint yields the Lagrangian:

$$
\begin{aligned}
L = & \sum_{t=1}^{T} p_t(e_t^H - e_t^{XI}) \cdot (e_t^H - e_t^{XI}) + p_t^{XI}e_t^{XI} \\
& - l\,(\sum_{t=1}^{T} e_t^H - W) \\
& - \sum_{t=1}^{T} a_t(e_t^{XI} - \overline{e}^{XI}) \\
& - \sum_{t=1}^{T} b_t(-e_t^{XI} - \overline{e}^{XI})
\end{aligned}
\tag{10}
$$

The necessary first-order conditions are:

$$
\begin{aligned}
& \frac{\partial L}{\partial e_t^H} = p_t'(e_t^H - e_t^{XI}) \cdot (e_t^H - e_t^{XI}) + p_t(e_t^H - e_t^{XI}) - l \leq 0 \; (= 0 \text{ for } e_t^H > 0) \\
& \frac{\partial L}{\partial e_t^{XI}} = -p_t'(e_t^H - e_t^{XI}) \cdot (e_t^H - e_t^{XI}) - p_t(e_t^H - e_t^{XI}) - a_t + b_t + p_t^{XI} = 0 \\
& l \geq 0 \; (= 0 \text{ for} \sum_{t=1}^{T} e_t^H < W) \\
& a_t \geq 0 \; (= 0 \text{ for } e_t^{XI} < \overline{e}^{XI}) \\
& b_t \geq 0 \; (= 0 \text{ for } -e_t^{XI} < \overline{e}^{XI})
\end{aligned}
\tag{11}
$$

We maintain the assumptions that the amount of electricity consumed at home, x_t, is positive in all periods and that the export/import prices are all different. Looking at the second condition, since we either have import or export in a period the shadow prices on the upper and lower constraint cannot both be positive at the same time, but they may both be zero if the constraints are not binding.

We have by assumption that in an export period we must also use hydro at home. Therefore in an export period the flexibility-corrected price is also equal to the shadow price on water. The second condition in (11) tells us that the flexibility-corrected price is equal to the export price minus the shadow price on the export constraint. It will be arbitrary if export in each period of export is exactly equal to the constraint. In general there will therefore be a period when the export possibility is not fully utilised. We will call this period the *marginal export period*. But in this period the shadow price on water is equal to the export price. Denoting the period when the marginal export period occurs for $t*$ we have:

$$p_{t*}(1+\hbar_{t*}) = l = p_{t*}^{XI} - a_{t*} = p_{t*}^{XI} \tag{12}$$

But the shadow price on the water resource is a scalar. It is therefore the marginal export period that determines this shadow price. For all the export periods with a binding constraint the shadow price on the upper constraint comes in positive satisfying the second equality in (11) for a general t belonging to the export periods (i.e. the periods when the export price is higher than the price for the marginal export period). The shadow prices are determined such that the difference between export price and the corresponding shadow price is constant and equal to the shadow price on water.

If hydro is used in an import period then the first condition in (11) holds with equality implying that the flexibility-corrected home market price $p_t(1+\hbar_t)$ is equal to the shadow price on water. The second condition tells us that the flexibility-corrected price is always equal to the import price plus the shadow price on the upper constraint on import, yielding:

$$p_t(1+\hbar_t) = l = p_t^{XI} + b_t \tag{13}$$

But by assumption $p_{t*}^{XI} > p_t^{XI}$ for all periods being import periods. This means that hydro cannot be used in the home market in import periods unless the total import capacity is used. If hydro is not used in import periods the flexibility-corrected price is in a regular case lower than the shadow value on water and the import price is lower than the shadow value of water.

An illustration is provided in Figure 5. Since the import price is lowest in period 1 this period will be the import period. The original bathtub walls are drawn with solid lines. Both import- and export- capacities will be fully utilised. Since the import/export price is lowest in period 1 this will be the import period. The import capacity is added to the hydro wall to the left and marked with the broken vertical line. The demand- and

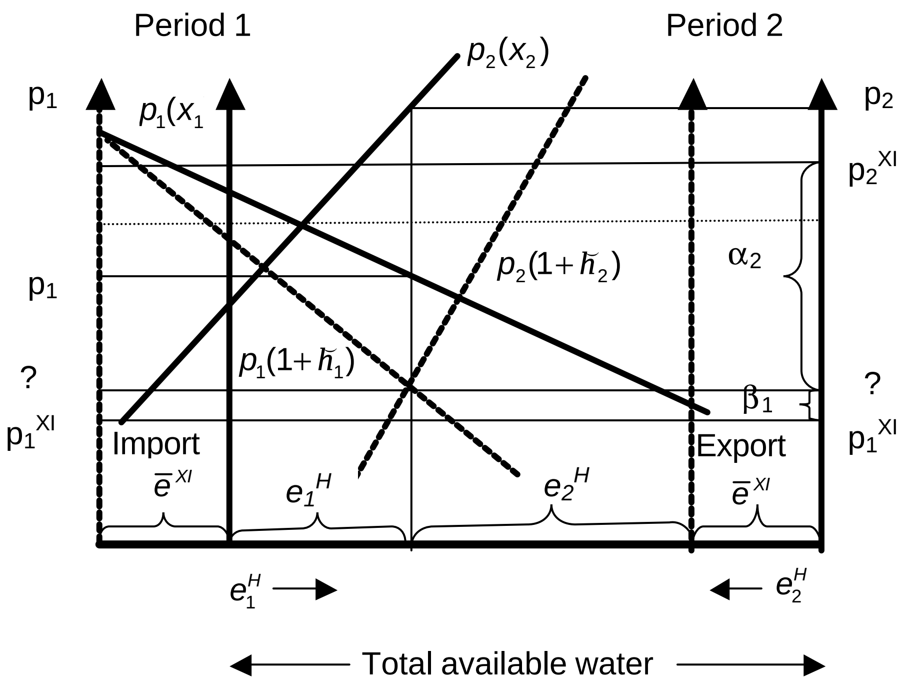

Figure 5. Monopoly and trade with constraints

marginal revenue- curves are shifted to the left and anchored on the "import wall." In the export period 2 the hydro wall is shifted to the left with the length of the export constraint marked with the broken vertical line to the left of the hydro wall. This amount will be exported. The demand- and marginal revenue- curves are shifted to the left with the length of the export constraint and anchored on the broken vertical line. The flexibility-corrected prices are equal and equal to the shadow price on water. The home price becomes higher than the export price in the export period, and the home price becomes higher than the import price in the import period. The connection between the shadow price on water, the import/export prices and the shadow prices on the trade constraints are shown in the figure.

Comparing with the social solution we have in that case that both import- and export trade capacity will be fully utilized, but that the home price will be equal for the two periods indicated by the dotted horizontal line through the point of intersection between the demand curves for the two periods. The monopolist will use more water at home in the relative more price-elastic demand period 1 and accept a lower price than for the social solution (but higher than the import price), but then having less water left for the relatively inelastic period he will realise a higher price than both the social price and the export price.

4. Monopoly with reservoir constraint

Limited transferability of water between periods is the most realistic situation for hydropower. An upper limit, $\bar{R}$, on the reservoir will be introduced together with an accompanying water-accumulation equation. This states that the reservoir at the end of period t, R_t, must be less (if overflow) or equal to the amount R_{t-1} inherited from period t-1, plus the inflow w_t during period t and subtracted the amount used for electricity production corresponding to e_t^H. The water variables are all measured in electricity units. The profit maximisation problem is:

$$
\begin{aligned}
&Max \sum_{t=1}^{T} p_t(e_t^H)\cdot e_t^H \\
&s.t. \\
&R_t \le R_{t-1} + w_t - e_t^H \qquad (14)\\
&R_t \le \bar{R} \\
&R_t, w_t, e_t^H \ge 0\ ,\ \ t = 1,..,T \\
&T,\ R_o,\ \bar{R} \text{ given}, R_T \text{ free}
\end{aligned}
$$

The Lagrangian is:

$$
\begin{aligned}
L = &\sum_{t=1}^{T} p_t(e_t^H)e_t^H \\
&-\sum_{t=1}^{T} l_t(R_t - R_{t-1} - w_t + e_t^H) \qquad (15)\\
&-\sum_{t=1}^{T} g_t(R_t - \bar{R})
\end{aligned}
$$

The necessary first order conditions are:

$$
\begin{aligned}
&\frac{\partial L}{\partial e_t^H} = p_t'(e_t^H)e_t^H + p_t(e_t^H) - l_t \le 0\ (= 0 \text{ for } e_t^H > 0) \\
&\frac{\partial L}{\partial R_t} = -l_t + l_{t+1} - g_t \le 0\ (= 0 \text{ for } R_t > 0) \qquad (16)\\
&l_t \ge 0\,(0 = \text{for } R_t < R_{t-1} + w_t - e_t^H) \\
&g_t \ge 0\,(0 = \text{for } R_t < \bar{R}) \qquad , \qquad t = 1,..,T
\end{aligned}
$$

Assuming electricity is always supplied and introducing the demand flexibility, $\hbar_t = p_t' e_t^H / p_t$, the first-order conditions read:

$$
\begin{aligned}
&p_t'(e_t^H)e_t^H + p_t(e_t^H) - l_t = p_t(e_t^H)(1+\hbar_t) - l_t = 0 \\
&-l_t + l_{t+1} - g_t \leq \ (= 0 \text{ for } R_t > 0), \qquad t = 1,..,T
\end{aligned}
\qquad (17)
$$

Comparing with the solution of the corresponding social planning problem the marginal revenue is again substituted for the marginal willingness to pay (the price). The flexibility-corrected price is set equal to the water value, but the water values are period specific, so marginal revenue may now differ over time. The second condition in (16) or (17), showing the dynamics of the water value, is qualitatively the same as in the social planning case. By *backward induction* we can find the path of development for the water value. A general feature is that if the reservoir neither is threatened with overflow nor runs empty, the water value will remain constant and equal to the value in the terminal period. But in the monopoly case market the prices may fluctuate from period to period depending on changing demand functions.

The terminal water value may become zero. It means that some water may be unused in the terminal period. If the upper reservoir constraint is not binding in the preceding period *T*-1 the water value will also be zero in this period implying that the flexibility-corrected price is zero and water may be added to the reservoir handed to the terminal period. The water value can only become positive if there is a period where it is optimal to use up all available water. If this period is *t*, then we have from (17) that $l_t \geq l_{t+1} = 0.$ The regular case will be that the water value for period *t* becomes positive. In the case of a full reservoir in a period where all the later periods have zero water values the water value cannot become less than zero. The shadow price on the upper constraint is in this case zero. Nothing is gained by expanding the reservoir limit marginally. If it is optimal to empty the reservoir in the terminal period, i.e. the marginal revenue is positive, the terminal shadow price on water becomes positive and the story above going backwards is repeated.

The general strategy of the monopolist of shifting water use from relatively inelastic demand periods to relatively elastic ones will also prevail in the case of a reservoir constraint. Let us first assume that the monopolist will not find it profitable to spill any water, i.e. that the marginal revenues stay positive. The constraint on the reservoir capacity will in general lead to the monopoly prices being closer to the prices in the social solution if the constraint is binding in the latter case. If it is optimal for a monopolist to have the upper constraint on the reservoir binding in a period, then this means that he must charge the market price given by the intersection of the demand curve and the vertical reservoir constraint in order to sell the available water. If the same amount of water is available as in the social case then the monopoly price must be equal to the price in social optimum. The shadow value of water must adjust downwards for this to be possible. The monopolist follows the general strategy of using more water in elastic periods and having less water for the more inelastic periods. How this strategy interacts with storing more or less water than in the social planning case is connected to whether the reservoir build-up periods and the draw-down periods coincide with relatively elastic- or inelastic periods. If build-up periods coincide with relatively elastic demand periods there will be a tendency to reduce the number of periods with binding reservoir constraint. Maximal storing may become more seldom the optimal strategy for a monopolist.

In the two-period illustration in Figure 6 the available water, including inflow and initial filling, in period 1 is AC and the inflow in period 2 is CD. The reservoir capacity is BC.

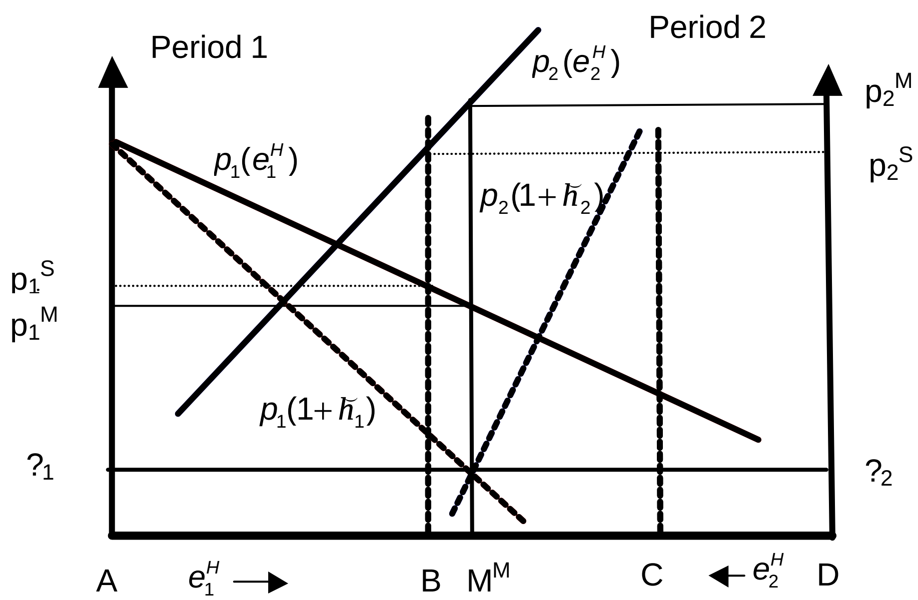

Figure 6. Monopoly with reservoir constraint

The build-up period is period 1 with the most elastic demand. The reservoir constraint is not binding in the monopoly case, but was binding in the social optimal solution, as indicated by the dotted horizontal price lines intersecting the vertical reservoir constraint through B, and we have no spillage. The allocation point for water is moved from B in the social case to M^M in the monopoly case. We note that the monopoly price in period 1 with the relatively most elastic demand becomes lower than the social optimal price with a binding reservoir constraint, and the monopoly price in period 2 with relatively inelastic demand becomes higher than in the social optimal case. This is the general effect of shifting of water from periods with relative inelastic demand to periods with relatively elastic demand in the case of market power. The areas representing reduced income in period 1 and increased income in period 2 can easily be identified in Figure 6. Notice that the price differences are now quite reduced compared with the case of no reservoir constraint.

It is often assumed that high demand period, e.g. a peak period, is the period with relatively most inelastic demand (Borenstein et al., 2002). However, this is an empirical question and should not be assumed without further investigations. Also in peak demand periods there are substitution possibilities for consumers. In a summer period without both heating and cooling the substitution possibilities are much more restricted, so it may as well be such periods that have the most inelastic demand as peak periods. The monopolist is utilising differences in demand elasticities and not differences in absolute demand.

A monopolist will experience a binding reservoir constraint as in the social case illustrated in Figure 6 if the intersection of marginal revenue curves is to the left of the vertical through B representing the reservoir constraint (the demand curves have to be slightly redrawn to obtain this case). In a two-period case with the same availability of water in the first period with the binding reservoir constraint the monopolist cannot do better than adopt the social solution although the demand in period 1 is more elastic.

Spilling of water can only take place in a period when the reservoir is filled up to the limit. The spilling then occurs if marginal revenue becomes zero before all available water in addition to the full reservoir is processed. Figure 7 illustrates such a case for the build-up period 1 having a less elastic demand than the draw-down period 2. The

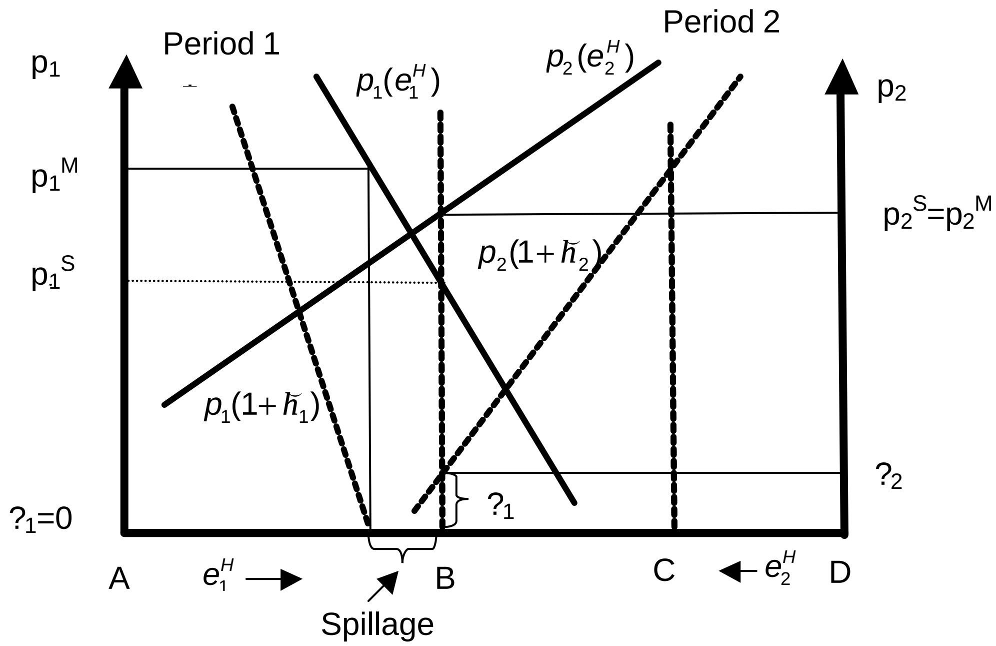

Figure 7. Monopoly with reservoir constraint and spillage

symbols have otherwise identical interpretations with Figure 6. The marginal revenue becomes zero before all available water AB in addition to a full reservoir BC is processed, resulting in a spillage in period 1. The water value becomes zero according to the second condition in (17). The monopoly price is markedly increased compared with the social planning price, indicated by the thin horizontal dotted line from the intersection point between the demand curve for period 1 and the thick vertical broken line from B being the reservoir wall. However, since the marginal revenue curve for period 2 is hitting the reservoir wall at a positive value the monopolist will utilise all available water in period 2, implying he has to charge the same price as in the social planning case. There is a positive value of the shadow price on the reservoir constraint in period 1 equal to the differences between the water values for the two periods. Since the water value for period 1 is zero due to the overflow, the shadow price on the reservoir constraint become equal to the water value in period 2. If the reservoir could be expanded the monopolist will increase his profit with this amount at the margin. If period 2 is a peak period we see that the monopolist is not increasing the price in this period, but in the off-peak period because this period is relatively more inelastic.

5. Monopoly with trade- and reservoir-constraints

We will now combine trade and restriction on the reservoir. The monopoly optimisation problem in the case of restrictions both on trade and reservoir is:

$$
\begin{aligned}
& Max \sum_{t=1}^{T}(p_t(x_t)x_t + p_t^{XI}e_t^{XI}) \\
& s.t. \\
& x_t = e_t^H - e_t^{XI} \\
& -\bar{e}^{XI} \le e_t^{XI} \le \bar{e}^{XI}, \\
& R_t \le R_{t-1} + w_t - e_t^H \\
& R_t \le \bar{R} \\
& x_t, e_t^H, R_t \ge 0 \\
& \bar{R}, \bar{e}^{XI}, p_t^{XI} \text{ given, } e_t^{XI} \text{ free, } t = 1,..T
\end{aligned} \tag{18}
$$

Inserting the energy balance that holds as an equality constraint yields the Lagrangian:

$$
\begin{aligned}
L = & \sum_{t=1}^{T} p_t(e_t^H - e_t^{XI})\cdot(e_t^H - e_t^{XI}) + p_t^{XI}e_t^{XI} \\
& -\sum_{t=1}^{T} l_t(R_t - R_{t-1} - w_t + e_t^H) \\
& -\sum_{t=1}^{T} g_t(R_t - \bar{R}) \\
& -\sum_{t=1}^{T} a_t(e_t^{XI} - \bar{e}^{XI}) \\
& -\sum_{t=1}^{T} b_t(-e_t^{XI} - \bar{e}^{XI})
\end{aligned} \tag{19}
$$

The necessary first-order conditions are:

$$
\begin{aligned}
& \frac{\partial L}{\partial e_t^H} = p_t'(e_t^H - e_t^{XI})\cdot(e_t^H - e_t^{XI}) + p_t(e_t^H - e_t^{XI}) - l_t \le 0 \;(= 0 \text{ for } e_t^H > 0) \\
& \frac{\partial L}{\partial e_t^{XI}} = -p_t'(e_t^H - e_t^{XI})\cdot(e_t^H - e_t^{XI}) - p_t(e_t^H - e_t^{XI}) - a_t + b_t + p_t^{XI} = 0 \\
& \frac{\partial L}{\partial R_t} = -l_t + l_{t+1} - g_t \le 0 \;(= 0 \text{ for } R_t > 0) \\
& l_t \ge 0 \;(= 0 \text{ for } R_t < R_{t-1} + w_t - e_t^H) \\
& g_t \ge 0 \;(= 0 \text{ for } R_t < \bar{R}) \\
& a_t \ge 0 \;(= 0 \text{ for } e_t^{XI} < \bar{e}^{XI}) \\
& b_t \ge 0 \;(= 0 \text{ for } e_t^{XI} < -\bar{e}^{XI})
\end{aligned} \tag{20}
$$

The change from the case of trade without reservoir restriction is that the water values are now period specific. Two consecutive water values are connected through the value of the shadow price on the reservoir constraint, as seen from the third condition in (20). The possibility of overflow may restrict import of electricity since water is used until the marginal revenue becomes zero if that is necessary to avoid overflow. In export periods home price may be driven further up because there is a limit on the transfer from the previous period. If the reservoir constraint does not become binding we are back to the solution without a reservoir constraint.

A bathtub illustration for two periods is provided in Figure 8. The figure is based on Figure 5. Since the import price is lowest in period 1 this period will again be the import period. Available water including inflow to the reservoir in period 1 is AC and inflow in period 2 is CD. The size of the reservoir is BC, indicated by $\bar{R}$, and the thinner solid vertical lines from B and C represents the reservoir. The reservoir is introduced from C to the left to B because our problem for two periods is how much water to leave to

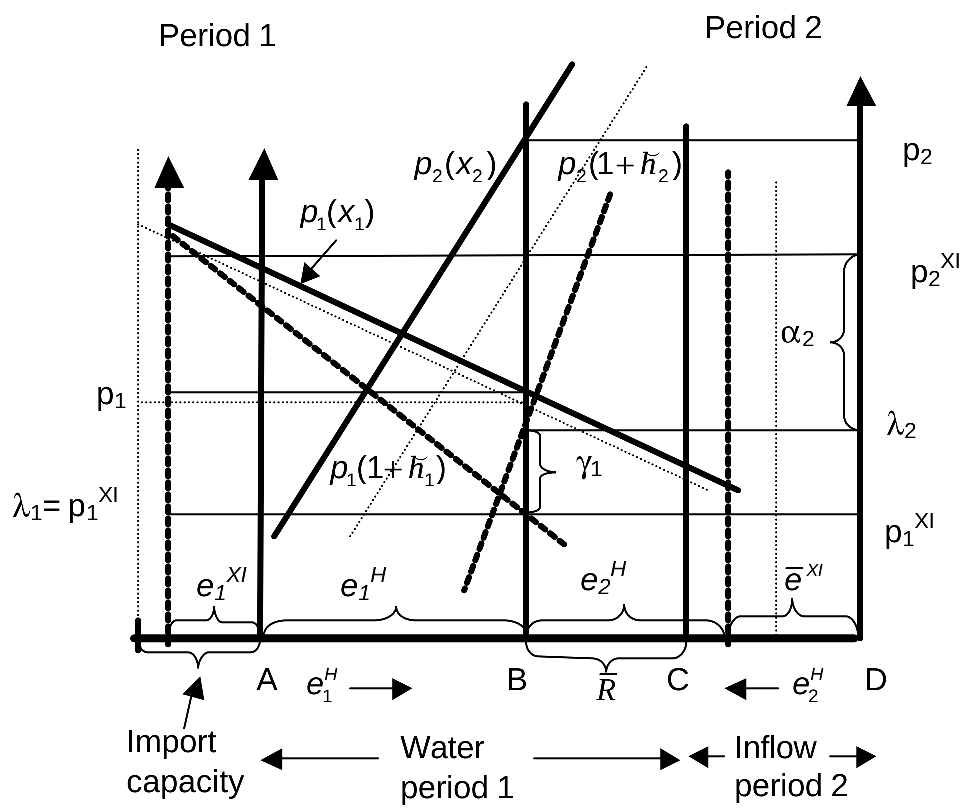

Figure 8. Monopoly and trade-and reservoir constraints

period 2. The import constraint is indicated to the left of the hydropower wall drawn with a solid line. In our case the full import capacity will not be utilised. But the full export capacity will be used, and this capacity is indicated by the first thick broken line to the left of the right-hand hydro wall drawn with a solid line. The final layout of the figure is the result of two stages for the two periods' curves. In the first stage the demand-and marginal revenue- curves are anchored to the hydropower walls. The optimality conditions for the import period tell us that the marginal revenue curve should pass through the intersection between the import price line and the hydro wall through B. The demand- and marginal revenue curves are shifted horizontally to the left to allow this, and the stopping point is where the import wall is erected. If more import is tried the marginal revenue will become smaller than the import price. At least water AB has to be used home in period 1, and the market price matching this amount is higher than the import price. Therefore import is introduced until the marginal revenue is equal to the import price. Remember the analogy between imports and another technology for producing electricity. The final market price is found the usual way of moving vertically up to the demand curve. Since the import capacity is not fully utilised the shadow price b_1 on this capacity is zero. The water value becomes equal to the import price for this period. The maximal amount of water is transferred to period 2. Checking period 2 there is in the first stage enough water to utilise the export capacity fully. The thick vertical broken line to the left of the hydropower wall then indicates the reduced availability for hydropower at home, and the demand- and marginal revenue-curve are shifted horizontally to the left and anchored to the new wall. The intersection of the vertical water storage wall from B and the marginal revenue curve for period 2 then gives the water value for period 2. The home price is found by the intersection of the hydropower storage line and the demand curve. The shadow price on the reservoir capacity is the difference between the two periods' water values and is indicated in the figure. Since the export capacity is fully utilised its shadow price is positive and indicated as the difference between the export price and the water value for period 2.

Entering thin dotted lines for the solution of the social case facilitates a comparison with the monopoly case. The import and export periods remain the same. The import capacity will now be fully utilized, so the demand curve for period 1 will be anchored at this import-extended wall, illustrated by the thin dotted vertical line to the left of the wall in the monopoly case. In addition all water that cannot be transferred to period 2 will be used at home in the import period, resulting in slightly more use of water in the social case in period 1 and a slightly lower price than in the monopoly case. In period 2 the full export capacity will not be used since using it will leave so little water to be consumed at home that the market price will increase above the exogenous export price. Only the amount will be exported that lead to the same price at home as the export price. The demand curve for period 2 must therefore pass through the intersection point of the export price line and the vertical storage wall from B. The demand curve is anchored (not shown in the figure) at the thin vertical dotted line to the right of the monopoly anchoring indicating the optimal export in the social case. In our illustration monopoly leads to a shift away from imports and over to exports. Since import is reduced the monopoly price is (slightly) higher in the import period. Since the same total amount of water is transferred to period 2 in the monopoly case the increased export leads to a (markedly) higher home price and a reduced consumption. The export period has the relatively most inelastic demand.

6. Monopoly with hydro and thermal plants

Hydro is in most countries combined with thermal capacity. Let us assume that a monopolist has full control over both hydro and thermal capacity. The thermal capacity is aggregated into a sector capacity by using an aggregate merit-order cost function with positive and increasing marginal cost (Førsund, 2005). For simplicity the cost function is the same for all periods. We will investigate how the monopolist utilises the two types of electricity technologies compared with the social solution. We assume that the monopolist is free to reduce production, e_t^{Th}, from the thermal units as he sees in his interest. A limited thermal capacity, $\bar{e}^{Th}$, is assumed. The simplest restriction on hydro production of a total available amount of water is used. The demand functions are $p_t(x_t)$, where x_t is the electricity demand supplied both by hydro and thermal capacity. The optimisation problem is:

$$Max \quad \sum_{t=1}^{T}[(p_t(x_t)x_t - c(e_t^{Th})] \ s.t.$$
$$x_t = e_t^H + e_t^{Th}$$
$$\sum_{t=1}^{T} e_t^H \le W \qquad (21)$$
$$e_t^{Th} \le \overline{e}^{Th}$$
$$x_t, e_t^H, e_t^{Th} \ge 0, \quad t = 1,..,T$$
$$T, W, \overline{e}^{Th} \text{ given}$$

Substituting for total energy the Lagrangian is

$$L = \sum_{t=1}^{T}[p_t(e_t^H + e_t^{Th})(e_t^H + e_t^{Th}) - c(e_t^{Th})]$$
$$-\sum_{t=1}^{T} q_t(e_t^{Th} - \overline{e}^{Th}) \qquad (22)$$
$$-l\,(\sum_{t=1}^{T} e_t^H - W)$$

The necessary conditions are:

$$\frac{\partial L}{\partial e_t^H} = p_t'(e_t^H + e_t^{Th})(e_t^H + e_t^{Th}) + p_t(e_t^H + e_t^{Th}) - l \le 0 \ \ (= 0 \text{ for } e_t^H > 0)$$
$$\frac{\partial L}{\partial e_t^{Th}} = p_t'(e_t^H + e_t^{Th})(e_t^H + e_t^{Th}) + p_t(e_t^H + e_t^{Th}) - c'(e_t^{Th}) - q_t \le 0 \ \ (= 0 \text{ for } e_t^{Th} > 0) \qquad (23)$$
$$l \ge 0 \ (= 0 \text{ for } \sum_{t=1}^{T} e_t^H < W)$$
$$q_t \ge 0 \ (= 0 \text{ for } e_t^{Th} < \overline{e}^{Th})$$

Concentrating on periods where both hydro and thermal are used the general result is that marginal revenue substitutes for the marginal willingness to pay in the social optimal solution:

$$p_t(x_t)(1 + \hbar_t) = l = c'(e_t^{Th}) + q_t \qquad (24)$$

The monopoly solution for a period is illustrated in Figure 9. If the monopolist's water value is OB in a period total energy supplied is indicated by the intersection of the horizontal water value line BB' and the marginal revenue curve, yielding quantity Oe^H and monopoly price p^M. Both thermal and hydro capacity will be used according to the marginal revenue condition (24). The thermal capacity will be Oe^{Th}, determined by the intersection between the marginal cost curve and the water value line BB' at b, and the hydro capacity (Oe^H - Oe^{Th}). The thermal capacity is not exhausted, so the shadow price on thermal capacity is zero.

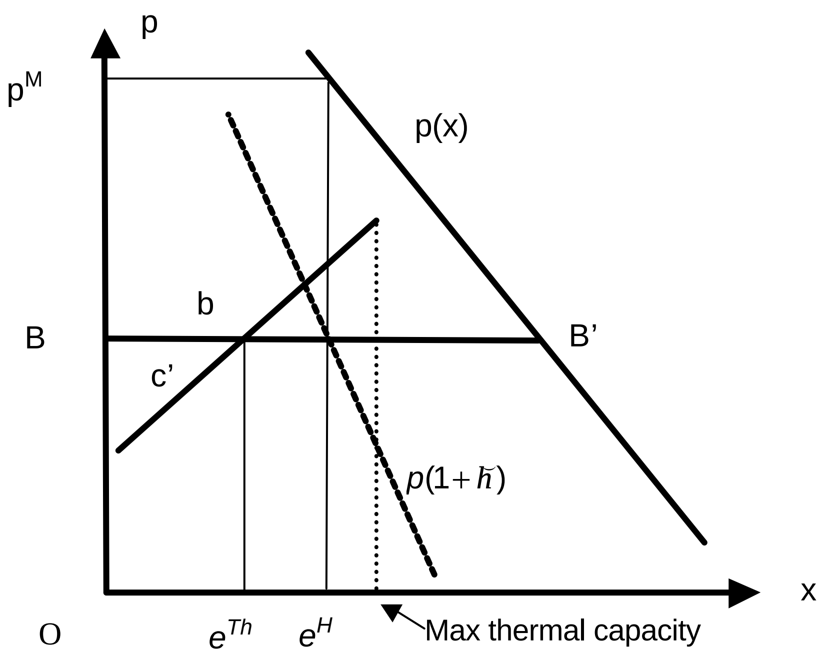

Figure 9. Monopoly. Hydro and thermal capacity

For two periods we may again use the bathtub diagram to illustrate the allocation of the two types of power on the two periods. In Figure 10 the length of the hydro bathtub, BD, is extended at each end with the thermal capacity. The thermal marginal cost

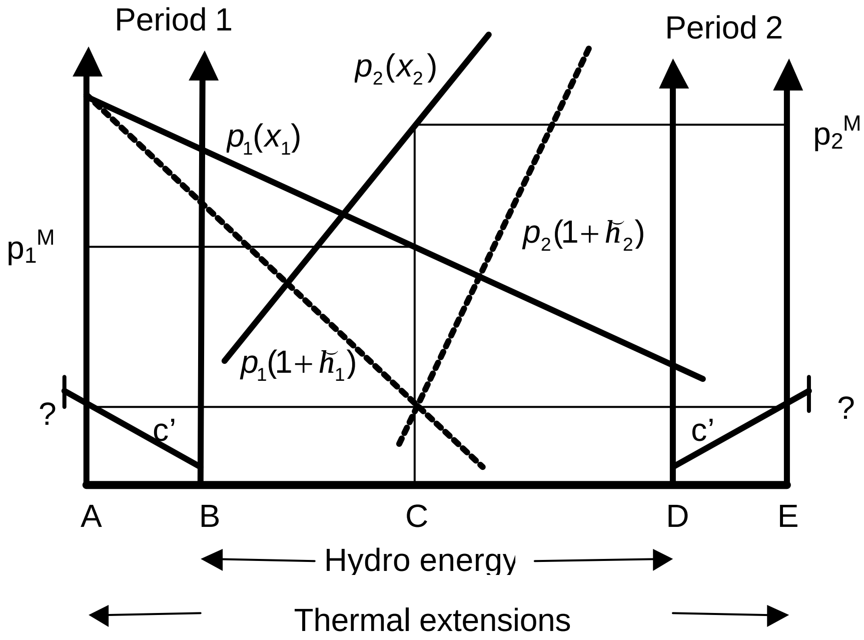

Figure 10. Two periods and monopoly, hydro and thermal

functions are anchored at the hydro walls and extending to the left out to the capacity limit, indicated by a short vertical line, for period 1 and to the right for period 2. Using the result (24), with the shadow price on the thermal capacity constraint being zero, we have that the thermal extension of the bathtub is equal at each end; with AB in period 1 and DE in period 2 and AB = DE. The equilibrium allocation is at point C, resulting in an allocation of AB thermal and BC hydro in period 1, and CD hydro and DE thermal in period 2. Introducing a reservoir constraint as in Figure 6 will not change the solution for the case of an intersection of the marginal revenue curves within the area delimited with the lines from B and C in that figure showing the storage possibilities. A monopolist will equate the water value with the marginal cost of thermal, and not the market price. The use of thermal capacity may be reduced in all periods and will be base load unless a hydro reservoir constraint is binding. For such periods thermal capacity will also be used as peak.

7. Dominant firm with a competitive fringe

A pure monopoly in the electricity market is not so common. There may be a dominating firm in terms of market share, but there will often be many smaller firms acting as price takers in the market. The existence of such a competitive fringe reduces the possibility of using market power because the fringe firms will supply according to the market price. For simplicity we will model the dominant firm by using the hydro model (1) without a reservoir constraint, but with a total water constraint, and model the competitive fringe by introducing a thermal sector represented by a cost function, as in the previous section, but without imposing a capacity constraint for the time being.

The optimisation problem for the dominating hydro producer is:

$$
\begin{aligned}
&Max \quad \sum_{t=1}^{T} p_t(x_t)e_t^H \\
&s.t. \\
&x_t = e_t^H + e_t^{Th} \\
&\sum_{t=1}^{T} e_t^H \le W \\
&p_t(x_t) = c'(e_t^{Th}) \\
&x_t, e_t^H, e_t^{Th} \ge 0, \quad t = 1,..,T \\
&T, W \text{ given}
\end{aligned} \tag{25}
$$

The third constraint in (25) represents the behaviour of the competitive fringe. It supplies according to the price-taking profit maximising condition of equating market price with marginal costs. We can most conveniently proceed in the standard textbook way by using the third condition to derive the relationship between the supply of the fringe and the dominant producer's supply of hydroelectricity. If the hydro producer supplies more the market price *cet. par.* goes down, but then the fringe contracts its output, assuming that the marginal cost is increasing. Differentiating

$$p_t(e_t^H + e_t^{Th}) = c'(e_t^{Th})\,(t = 1,..,T) \tag{26}$$

yields:

$$
\begin{aligned}
&p_t'(e_t^H + e_t^{Th})(de_t^H + de_t^{Th}) = c''(e_t^{Th})de_t^{Th} \Rightarrow \\
&\frac{de_t^{Th}}{de_t^H} = \frac{-p_t'(e_t^H + e_t^{Th})}{p_t'(e_t^H + e_t^{Th}) - c''(e_t^{Th})} < 0 \quad (t = 1,..,T)
\end{aligned} \tag{27}
$$

Equation (26) defines implicitly the fringe output as a function of the output of the dominant firm. The relationship can be expressed by

$$e_t^{Th} = f_t(e_t^H),\ f_t' < 0\ (t = 1,..,T) \tag{28}$$

Using the energy balance and the relationship between fringe output and output of the dominating firm yields a more compact problem than (25) with the Lagrangian as

$$
\begin{aligned}
L = &\sum_{t=1}^{T} p_t(e_t^H + f_t(e_t^H))e_t^H \\
&- l\,(\sum_{t=1}^{T} e_t^H - W)
\end{aligned} \tag{29}
$$

The first-order conditions are:

$$\frac{\partial L}{\partial e_t^H} = p_t(e_t^H + e_t^{Th}) + p_t'(e_t^H + e_t^{Th})e_t^H(1 + \frac{de_t^{Th}}{de_t^H}) - l \le 0 \ (= 0 \text{ for } e_t^H > 0)$$
$$l \ge 0 \ (= 0 \text{ for } \sum_{t=1}^{T} e_t^H < W) \ , \quad t = 1,..,T \tag{30}$$

The last bracketed term, $(1 + de_t^{Th} / de_t^H)$, on the rhs of the first condition in (30) is positive, but less than 1, resulting in the conditional marginal revenue becoming less than the price. Using (27) yields

$$1 + \frac{de_t^{Th}}{de_t^H} = 1 + \frac{-p_t'(e_t^H + e_t^{Th})}{p_t'(e_t^H + e_t^{Th}) - c''(e_t^{Th})} = \frac{-c''(e_t^{Th})}{p_t'(e_t^H + e_t^{Th}) - c''(e_t^{Th})} > 0 \tag{31}$$

The marginal revenue of the dominant firm is now reflecting the behaviour of the fringe. We have that the value of the conditional marginal revenue is closer to the market price, but still below this value compared with the expression for monopoly marginal revenue. Rearranging the first-order condition in (30) yields the following expression for the conditional marginal revenue:

$$MR_{t|p_t'=c'} = p_t(1 + \breve{\eta}_t \frac{e_t^H}{e_t^H + e_t^{Th}}) + p_t' \frac{de_t^{Th}}{de_t^H} e_t^H , \ t = 1,..,T \tag{32}$$

The conditional marginal revenue function is closer to the demand function than the monopolist's marginal revenue function due to two factors: the market share of the dominant firm is less than one in the first expression in (32) reducing the impact of the demand flexibility, and the second expression involving the quantity reaction of the fringe is positive.

When the dominant firm is producing (30) tells us that the marginal revenues conditional upon the behaviour of the fringe shall all be equal and equal to the shadow price on water. It seems reasonable to assume that the dominant firm produces in all periods. Zero production implies that the shadow value of water is greater than the marginal cost of the fringe providing the whole market quantity. We will disregard this possibility.

Figure 11 provides an illustration in the two-period case. The broken lines are the conditional marginal revenue curves. The optimal solution is characterised by these conditional marginal revenues being equal and equal to the shadow price on water. The use of the fringe thermal capacity is governed by the equality of the market price and

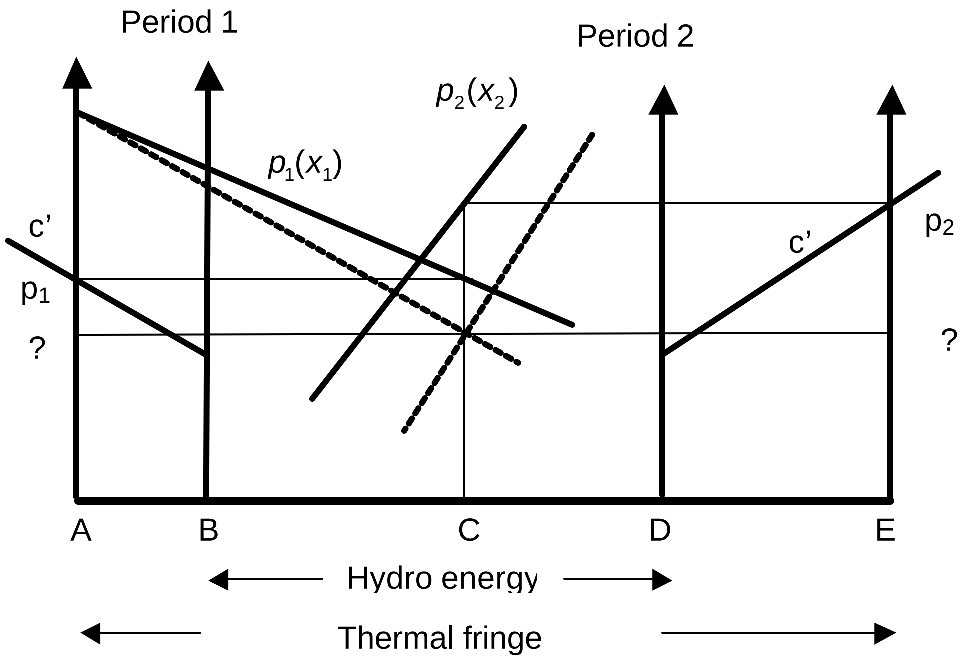

Figure 11. Dominant hydro and a thermal fringe

the marginal cost. The demand- and conditional marginal revenue curves are anchored on the thermal walls, being endogenously determined, extending the energy bathtub. The thermal cost functions are anchored on the hydro bathtub walls. The use of thermal capacity, AB, in the relatively elastic period 1 is smaller than the use DE in the more inelastic period 2. In the illustration more hydro, BC, is used in period 1 than in period 2 using CD. The market prices differ and the price is highest in the more inelastic period. Compared with the monopoly case the impact of the fringe is clearly to make the prices become more equal. A larger fringe capacity will be used in the more inelastic period forcing the market price down. This reduces the effectiveness of shifting water from period 2 to period 1.

A constraint on the thermal capacity of the fringe will be an advantage for the dominant hydro firm if the constraint becomes binding. The first-order profit-maximising conditions for the price-taking fringe in the case of a capacity constraint are:

$$p_t(x_t) = c_t'(e_t^{Th}) + q_t$$
$$q_t \geq 0 \ (= 0 \text{ for } e_t^{Th} < \bar{e}^{Th}) \qquad (33)$$

The capacity constraint is $\bar{e}^{Th}$ and its shadow price $?_t$. The capacity restriction implies the following response of the fringe:

$$e_t^{Th} = \bar{e}^{Th} \text{ for } p_t(x_t) \geq \bar{p} = c'(\bar{e}^{Th}) \tag{34}$$

In the case of $p_t(x_t) \geq \bar{p}$ the first-order condition (30) for the dominant firm becomes

$$\begin{aligned} & p_t(e_t^H + \bar{e}^{Th}) + p_t'(e_t^H + \bar{e}^{Th})e_t^H = \\ & p_t(1 + \bar{h}_t \frac{e_t^H}{e_t^H + \bar{e}^{Th}}) = l \ , \ t = 1,..,T \end{aligned} \tag{35}$$

assuming that the dominating firm is producing. The conditional marginal revenue function shifts further away from the demand function. But since the demand flexibility is multiplied with the market share of the dominating firm this implies that the conditional marginal revenue function does not shift down as far as to the monopoly marginal revenue function.

In the two-period case the situation can be illustrated as in Figure 12, building upon Figure 11. Total hydro resource is BD. The capacity of the thermal fringe is indicated by the small vertical line at the end of the marginal cost curve outside the thermal wall in period 1. The thermal capacity constraint is binding in period 2, but not in period 1. The demand- and marginal revenue curves for period 2 are now anchored on the thermal

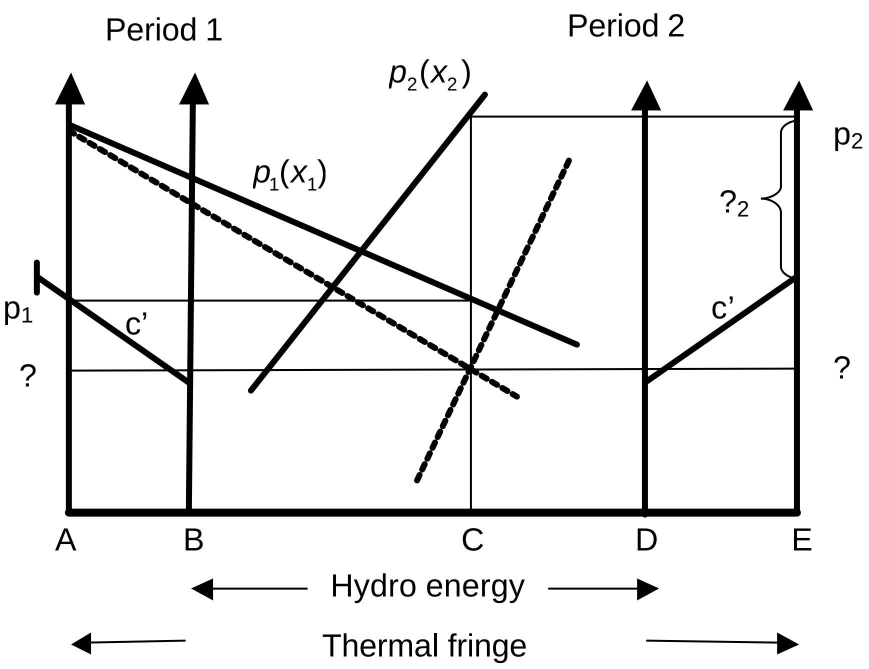

Figure 12. Dominant firm and constraint on the fringe output

wall dictated by the capacity constraint. The shift to the marginal revenue curve defined in (35) valid when the fringe output is constrained, is shown by the greater distance between the demand- and the marginal revenue curve. The opposite direction of shifts for the demand- and revenue curves implies an increase in period 2 price. Total supply is CE, the fringe supplies its maximal capacity DE and the dominant firm supplies CD. In period 1 the thermal capacity is not fully utilised and the conditional marginal revenue curve follows from (30) and lies relative closer to the demand curve as in Figure 11. The fringe supplies AB, less than its capacity, and the dominant firm supplies BC. When allocating water between the two periods the dominant hydro firm strikes a balance between marginal income from the two periods taking into consideration the lack of quantity response from the fringe in period 2 with full capacity utilisation and the contracting response in period 1 if more water is shifted to this period. The shadow price on the thermal capacity constraint in period 2 is shown in the figure and is the difference between the market price and the marginal cost at full capacity. Thus it measures the revenue to the fringe of expanding capacity marginally. The size of the capacity shadow price is also an indication for the dominant firm of the advantage enjoyed due to the fringe being capacity constrained.

Hydro producers as a fringe

Hydro producers can also constitute a fringe. However, the behaviour of the fringe can lead to analytical problems finding an optimal solution to the profit maximising problem of the dominating firm. Assuming that the fringe has at its disposal an amount of water corresponding to W_F and has enough reservoir capacity to be perfectly flexible as to in which period to use the water, the fringe will use all its water in the period with the highest price. Although it is a fringe and therefore W_F may be considerably smaller than W_D, where W_D is the dominant firm's water resource, it can still have a considerable market share if all its water is used just in one period. It may happen that for a relatively high fringe water resource the solution is forced to be the social optimal solution with equal price for all periods.

Introducing reservoir constraints for the fringe may introduce some market power for the dominant firm. But it is then also logical to introduce a reservoir constraint for the dominant firm. We will not develop such an analysis further, but just mention that in

Norway the reservoir capacity is quite concentrated on a small number of firms. Small hydropower firms tend to have relatively less reservoir capacity, thus opening up for the possibility of a group of dominating firms to exercise some market power.

Oligopolistic markets

It may easily become difficult to analyse oligopolistic markets involving hydro producers analytically. The basic problem is that such analyses have to be dynamic due to the basic dynamic nature of optimal adjustments of hydro producers with reservoir capacity. Even a duopoly involving a hydro firm and a thermal firm may become intractable without assuming special functional forms for the demand- and cost functions considering only two periods (Crampes and Moreaux, 2001). Since there is zero variable cost in the hydro case Bertrand competition moving prices is of special interest. A hydro producer can more easily drive down the price in the short run and force thermal capacity out and use water in order to create more scarcity in later periods. We will not attempt to develop such analyses here.

8. Concluding remarks

The basic strategy of a hydro producer exercising market power is to shift water trying to make marginal revenues equal. Water is shifted away from periods with relatively inelastic demand to periods with relative elastic demand. The elasticity of demand may not follow a pattern of high demand peak periods being inelastic and low demand off-peak periods being elastic, as often assumed in the literature. This is an empirical question. A crucial question is also the period length to which a demand function refers. Are we talking about hours or longer time periods? In applied analyses shifts in the load duration curve is often used to aggregate into periods longer than one hour.

Introducing technical constraints on the maneuverability of the hydro producer reduces generally the consequences of exercising market power. This is especially the case if a regulator prohibits spilling of water. But even when a monopolist is lead to use the social prices the shadow price of water is always below the social water value. This may have implication for investments in production capacity.

Opening up for trade is often advocated as a way to increase competition. However, if a monopolist has control over the trade there is a systematic shift away from imports to exports and with the home price being higher than the trade prices. Introducing constraints on the interconnector capacity facilitating trade reduces markedly the consequences of a monopoly maintaining control with trade. This is somewhat paradoxical since increased trading possibilities are regarded positively, but it just underlines the importance of keeping trade away from the control of a monopolist.

If a monopolist also has thermal capacity his water shifting strategy is maintained. However, the classical case of reducing production is now seen by the reduction of use of thermal capacity compared with the social solution. This result is coupled to the lower shadow price of water in the monopoly case, since the use of thermal capacity is governed by the equality of this shadow price and the marginal thermal cost.

The existence of a competitive fringe reduces the possibility of exercising market power by a dominating hydro producer just as in the standard textbook case. If the fringe consist of hydro firms it may even lead to social prices being followed, since a hydro fringe will seek to use its water in the highest price period.

Market interactions between few competitors involving hydro firms will be dynamic in nature due to reservoirs and solving analytically such market games may easily become intractable even making drastic simplifying assumptions.

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