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Christian Keuschnigg,
University of St. Gallen (IFF-HSG), CEPR and CESifo
Soren Bo Nielsen,
Copenhagen Business School, EPRU, CEPR and CESifo

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Department of Economics University of St. Gallen

Editor: Prof. Jörg Baumberger
University of St. Gallen
Department of Economics
Bodanstr. 1
CH-9000 St. Gallen
Phone +41 71 224 22 41
Fax +41 71 224 28 85
Email joerg.baumberger@unisg.ch
Publisher: Department of Economics
University of St. Gallen
Bodanstrasse 8
CH-9000 St. Gallen
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Christian Keuschnigg
University of St. Gallen (IFF-HSG), CEPR and CESifo
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Copenhagen Business School, EPRU, CEPR and CESifo

Main Author's address:	Christian Keuschnigg	
	IFF-HSG	
	Varnbuelstrasse 19	
	9000/ St. Gallen	
	Tel.	+41 71 224 25 20
	Email	christian.keuschnigg@unisg.ch
	Website	www.iff.unisg.ch
Co-Author's address:	Soren Bo Nielsen	
	Copenhagen Business School	
	Solbjerg Plads 3	
	DK-2000 Frederiksberg	
	Tel.	+45 3815 2596
	Email	sbn.eco@cbs.dk
	Website	www.cbs.dk

Abstract

In financing start-up firms, venture capitalists carefully select among alternative projects, design incentive compatible financial contracts and support portfolio companies with value enhancing managerial advice. This paper considers how venture capitalists can induce self-selection among entrepreneurial firms with different qualities and growth potential by designing appropriate contracts and offering managerial support. We study the efficiency of the competitive market equilibrium with respect to the level and quality of entrepreneurship and the level of effort by entrepreneurs and venture capitalists. We also provide comparative static results with respect to basic preference and technology parameters.

Keywords

Venture capital, entrepreneurship, self-selection, moral hazard.

JEL Classification

D82, G24, M13.

1 Introduction

Venture capital backed firms are more innovative, grow larger and create more value than other bank financed firms. Kortum and Lerner (2000) find that a disproportionately large share of industrial innovation originates in firms financed with venture capital (VC).[1] Based on a sample of start-up firms in Silicon valley, Hellmann and Puri (2000, 2002) estimate the value added of venture capitalists (VCs) to company development. The strategic advice and the monitoring activities of VCs promote the commercialization of portfolio companies and helps them to exploit better their growth potential.[2] It has often been argued, however, that much of the superior performance of VC backed compared to bank financed companies might not be due to the value added activities of VCs. It might rather result from the fact that VCs are simply more successful in selecting the more promising firms in the pool of all start-ups. Kaplan and Stromberg (2001, 2004) indeed point to the importance of both the screening and advising activities of VCs. Quite consistent with this, the empirical study of Sorensen (2005) finds that about 50 percent of the extra performance of VC backed firms is due to the advisory support of VC firms, and the rest to their screening activities.

A good descriptive model of the VC industry should therefore pay due attention to the implications of both selection and advice for the quality of VC financing. We build on our own previous research in modeling the productive contribution of VCs to their portfolio companies in terms of advice and managerial support (Kanniainen and Keuschnigg, 2003, and Keuschnigg and Nielsen, 2003, 2004a,b). We now extend this research by allowing for quality differences among projects as is emphasized in the literature of adverse selection.[3] There are three main differences with this literature: (i) we allow for only two qualities instead of a continuum to simplify the model; (ii) we combine this with a

[1] See Da Rin, Nicodano and Sembenelli (2005) on innovation financing with VC and the impact of public policy in Europe.

[2] See Gompers and Lerner (1999) for a standard reference on empirical work on VC financing and Kanniainen and Keuschnigg (2005) for a collection of policy oriented contributions.

[3] See e.g. DeMeza and Webb (1987) and the survey of the subsequent literature by De Meza (2002). Boadway and Keen (2004) synthesize different models which mostly consider pooling equilibria. For an analyis of separating equilibria see Innes (1990) and Fuest and Tillessen (2005), among others.

double moral hazard problem after a contract is signed. The moral hazard relates to the entrepreneur's managerial effort in building the company and the VC's advisory support to magnify company growth. And (iii), we combine this with the self-selection model of Hall (2005) to endogenize entry into entrepreneurship. The result is a quite tractable model that rationalizes the use of convertible debt in VC financing and allows to consider the characteristics and efficiency of market equilibrium.[4] We show how certain structural parameters of the VC industry lead VCs to adjust their convertible debt contracts to attract a better selection of firms and to assure optimal incentives for managerial effort and advisory support. We are thus able to characterize the quantity and quality of VC financed entrepreneurship.

When start-ups invent new and untested products, the technological risk in making the product ready for production and the market potential of the innovation may be larger or smaller. It is thus assumed that ideas of entrepreneurs have either high or low quality, leading to large or small market potential in case of success. Entrepreneurs have not enough own capital and therefore need outside finance to start the firm. Initially, neither the entrepreneur nor the VC know the true quality of the project. Entrepreneurs, however, receive an informative signal on the potential of their project that allows them to revise their prior expectations. Depending on the financing contracts on offer, agents self-select into entrepreneurship if they receive a sufficiently good signal indicating that their project is likely to be a high quality one. Once a firm is started and the collaboration between the VC and entrepreneur begins, the project quality becomes known. The probability of success of either type may still be advanced by managerial effort and VC advice, either to increase expected profits or to cut losses. On average, bad quality firms result in a loss and high quality firms yield profits.

To shed light on the policy implications of the model, we clarify the welfare properties of the market equilibrium with respect to self-selected entry of entrepreneurs and the managerial and advisory effort levels. Entry results in an average quality of start-ups while effort levels determine the success probability of each type of firm. We find that

[4]See Casamatta (2003) and Schmidt (2003) for theoretical analysis on the role of convertible instruments. Kaplan and Stromberg (2003) and Cumming (2005) document the empirical importance of convertible instruments in VC financing.

the double moral hazard between entrepreneurs and VCs leads to an inefficiently low effort level. The reasons for this underinvestment are well known from our own previous research, or by the analysis of Schmidt (2003). Since an entrepreneur has insufficient own resources to start a project, he needs outside finance and must share the returns with the financier. Consequently, during the start-up phase, the entrepreneur and VC must each bear the full cost of their own effort but must share the returns to effort among them.

On the other hand, entrepreneurial entry is excessive. VCs incur a loss on bad projects and need to cross-subsidize them with profits from good projects. Limited liability prevents that entrepreneurs could pay for these losses. As a result of cross-subsidization, entrepreneurs who are endowed with a bad project with relatively high probability get a too favorable deal. They are thus too eager to start firms that should not get started. This result is in line with the adverse selection models of DeMeza and Webb (1987, 2002). The policy implications are immediate. The model calls for higher quality but fewer numbers of start-up firms. To improve the efficiency of the market equilibrium, one needs to look for policies that are able to stimulate effort but at the same time to reduce entry into VC backed entrepreneurship.

The next section sets up the model and analyzes the constrained optimal allocation. Section 3 derives competitive market equilibrium and investigates its efficiency properties by comparing to the optimal allocation. Section 4 discusses the results of comparative static analysis of the industry equilibrium with respect to key taste and technology parameters. Section 5 concludes and points out possible avenues for future research.

2 The Model

Potential entrepreneurs have ideas for new products, good and bad ones, but are endowed with little own wealth k. To start a firm, they need outside finance since the necessary capital investment $I > k$ exceeds own capital. Entrepreneurs not only lack own capital but also managerial experience. They would benefit from professional support of a seasoned VC who has industry knowledge, can give advice and add value by sharing his own managerial know-how. VC financing of new firms involves the following sequence of events: (i) VCs offer outside financing, covering at least $I - k$, and announce the type of contracts that they offer. (ii) Agents have an idea for a project which may be good or bad, $j \in \{G, B\}$. The true share of good projects is ε but agents do not know in advance whether their project is good. They receive a signal y indicating that the project is good with probability q. A higher signal value means a higher probability of the project being good. If y is sufficiently large, agents opt for entrepreneurship and apply for outside finance. If not, they turn to alternative employment in industry, earning a fixed wage w and deriving end of period wealth $w + k$. Interest on assets is normalized to zero. (iii) After the business is launched and investment I is sunk, the true quality of the project is revealed to both VC and entrepreneur as a result of their early collaboration. (iv) Knowing quality, agents spend effort and advice to boost the success probability. Even if the project turned out to have low potential, effort is spent in order to limit expected losses. (v) Good and bad projects yield outcome v_G or v_B and the payments according to the terms of contract are executed. The model is solved backwards.

2.1 Venture Capital Financing

If a project succeeds, it generates a value $v_G > v_B$ on the output market. When it fails, revenue is zero, leaving expected revenue $p_j v_j$. The success probability p_j is specific to the type which is revealed after the project starts but before full effort is expended. The success of the firm depends on the entrepreneur's effort and the degree of VC support. To simplify, we assume that managerial effort is discrete and can be either high or low, $l_j \in \{0, 1\}$. The entrepreneur's effort is critical for the success chances of the firm. The company would always fail if the entrepreneur shirks and puts in low effort. VC advice a_j is a more gradual matter and is treated as a continuous variable. Both efforts determine the success probability

$$p_j = p^j (l_j, a_j) = l_j \cdot (a_j)^{\alpha}, \quad 0 < \alpha < 1. \tag{1}$$

The key condition is that p_j is increasing in each effort level and strictly concave in VC advice. Efforts are complements. The marginal return p_l^j of managerial effort increases if the VC puts in more advice, $p_{la}^j = \alpha a_j^{\alpha-1} > 0$.

When the VC offers a contract, she does not yet know the type of the company but she anticipates that she will learn the project type after starting collaboration. The contract thus includes a convertible option that can be exercised when this information becomes available after the investment is sunk but before the outcome is realized. The contract consists of (i) a credit $I - k$, (ii) a share s_B of profits if the project turns out bad,[5] and (iii) an option to increase the equity stake to $s_G > s_B$ at a conversion price b. The option can be exercised after the project is started and its type is revealed to both parties. It will naturally be profitable to exercise the option when the project turns out good. In all cases, both agents receive a zero repayment if the project fails and no revenue is generated. The VC thus gets paid back only in case of success.

After a contract is signed, the project specific profit shares s_j are fixed. It is assumed that the early cooperation of the team reveals the potential of the project. Knowing the type of the project and the corresponding profit share after the VC has exercised her conversion option, efforts a_j and l_j can be tailored to it, giving a success rate p_j as in (1). During the start-up period, agents choose their own effort, taking the input by the other party as given, and thereby maximize the rent

$$\begin{aligned} R_j^E &= \max\nolimits_{l_j} \; p^j(l_j, a_j) \cdot (1 - s_j) \cdot v_j - \beta l_j, \\ R_j^F &= \max\nolimits_{a_j} \; p^j(l_j, a_j) \cdot s_j \cdot v_j - \gamma a_j, \\ R_j &= R_j^E + R_j^F = p^j(l_j, a_j) \cdot v_j - \beta l_j - \gamma a_j. \end{aligned} \tag{2}$$

The upper indices E and F refer to entrepreneurs and VC financiers. Since the success probability is concave in advice, VCs can expect a rent which compensates them for their effort cost and earlier pecuniary expenses.

After a deal is struck, the firm must make a uniform capital investment I where a part k is financed out of own equity and $I - k$ by the VC. Apart from this credit, the contract specifies profit shares s_j and fixed payments b_j. When exercising the conversion option, the VC thus pays an extra $b_G = b$. Naturally, when the project is revealed as bad, the VC does not convert, yielding $b_B = 0$. Optimality would in fact require a negative price ($b_B < 0$) or a payment from the entrepreneur to the VC which is prevented by limited

[5] The amouont $s_B v_B$ can also be interpreted as a debt repayment, leaving $v_B - s_B v_B$ to E.

liability. Taking account of the optional payment b_j, the entrepreneur expects a total value $R_j^E + b_j$ while the VC claims $R_j^F - b_j - (I - k)$.

Agents are endowed with good and bad ideas. However, nobody knows in advance whether an idea is good or bad, not even the entrepreneur herself. The true proportion of good ideas in the population is ε, a fraction $1 - \varepsilon$ of agents are endowed with a bad idea. The share ε is the prior probability for a high quality project which is the same for all potential entrepreneurs. However, agents receive a signal y that is positively correlated with project quality. If an agent receives a high value of the signal, her idea is very likely to be good. She updates her perceived probability of a good idea to $q(y) > \varepsilon$, giving a probability $1 - q$ of being stuck with a bad idea. If an agent receives a bad signal instead, she expects to have a good project with a low probability $q(y) < \varepsilon$. Even though all are identical in other respects, they continuously differ in the signal received and therefore in the expected project quality q. Agents who received a better signal are also more probable to have a good idea, $q' > q$ for $y' > y$. Given a marginal probability q of an agent with signal y, one obtains an average probability $Q > q$ of having a good project where the average is taken over all agents with signals $y' > y$. Appendix A explains details. From now on, we define the quality of a project as the probability that it is of a good type.

It is important to note that the average quality increases with the marginal quality, $dQ/dq > 0$. In fact, by assuming a specific functional form of the distribution of signals in the population as in (A.8), one can conveniently derive a closed form expression for average quality,

$$Q = \frac{q}{q + (1 - q)\theta} > q, \quad \theta < 1, \tag{3}$$

where θ parameterizes the information content of the signal. If $\theta = 1$, the signal would not be informative and agents could not update their prior probability ε so that all would expect to have a good project with the same probability $q = \varepsilon$, implying $Q = \varepsilon$ as well. If the signal is informative, then the perceived probability of a good project $q(y)$ increases with the signal received, and so does the average probability Q taken over all agents with even better signals.

When an agent considers entrepreneurship, the type of the project is not yet known. She thus reckons with two possible events: (i) With probability q, her project is good and has value $R_G^E + b_G$. (ii) With probability $1-q$, it is bad but is financed nevertheless, yielding $R_B^E + b_B$, where $b_B \geqslant 0$. In both cases, the entrepreneur gives up k to pay part of I. The signal must be sufficiently good to warrant entry, $q\left(R_G^E + b_G\right) + (1-q)\left(R_B^E + b_B\right) \geq w + k$. To simplify notation, we will write q_j meaning $q_G = q$ and $q_B = 1-q$. The entrepreneur's surplus over her outside option is

$$\pi^E = \sum_{j\in\{G,B\}} q_j \left(R_j^E + b_j\right) - k - w \geq 0. \tag{4}$$

The VC's surplus from a project of type q is π^F. The VC finances only the part $I-k$ of what the entrepreneur cannot finance herself. Comparing with (4) yields a joint surplus $\pi = \pi^E + \pi^F$,

$$\begin{aligned}
\pi^E &= \textstyle\sum_j q_j \cdot \left(R_j^E + b_j\right) - k - w, \\
\pi^F &= \textstyle\sum_j q_j \cdot \left(R_j^F - b_j\right) - (I-k), \\
\pi &= \pi^F + \pi^E = \textstyle\sum_j q_j \cdot R_j - w - I.
\end{aligned} \tag{5}$$

To get the average surplus over all projects financed, substitute q and $1-q$ from (A.2) into (5) and multiply the result by the denominator of (A.2). Next, integrate over $y' \geqslant y$ and note $H_j(y)$ as well as $E(y) = \int_y^\infty e(y')\,dy'$ by (A.1-3). Divide by $E(y)$ and note the definition of $Q = Q(y) = Q(\phi(q))$ in (A.3). Define the average surplus *per applicant* by $\Pi^E \equiv \int_y^\infty \pi^E(y')\,e(y')\,dy' \big/ E(y)$. Again we use the notation Q_j with $Q_G = Q$ and $Q_B = 1-Q$. The result is

$$\begin{aligned}
\Pi^E &= \textstyle\sum_j Q_j \cdot \left(R_j^E + b_j\right) - k - w, \\
\Pi^F &= \textstyle\sum_j Q_j \cdot \left(R_j^F - b_j\right) - (I-k), \\
\Pi &= \Pi^E + \Pi^F = \textstyle\sum_j Q_j \cdot R_j - w - I.
\end{aligned} \tag{6}$$

We close the model by deriving GDP and thereby obtain a welfare measure. To this end, substitute (2) into (6) to get

$$\begin{aligned}
\Pi^E &= \textstyle\sum_j Q_j \cdot \left[(1-s_j)\,p_j v_j - \beta l_j + b_j\right] - k - w, \\
\Pi^F &= \textstyle\sum_j Q_j \cdot \left[s_j p_j v_j - \gamma a_j - b_j\right] - (I-k), \\
\Pi &= \textstyle\sum_j Q_j \cdot \left[p_j v_j - \beta l_j - \gamma a_j\right] - w - I.
\end{aligned} \tag{7}$$

With population normalized to unity, the number of entrepreneurs obtaining VC financing is equal to the number of applications, $E < 1$. A share Q of them is endowed with a good project and a share $1 - Q$ with a bad one. GDP, or end of period income, is k plus the output of workers, each producing w, and of start-up firms, after subtracting various costs. The welfare measure must also take account of all non-pecuniary effort costs. Welfare is $Y = w + k + \Pi E$. Upon substitution of ΠE and using the definition of $L + E = 1$,

$$Y = (w + k) L + \left[\sum_j Q_j \cdot (p_j v_j - \beta l_j - \gamma a_j) - (I - k) \right] \cdot E. \tag{8}$$

GDP consists of wealth plus output of workers, $(w + k) L$, plus output of (good and bad) entrepreneurial firms, net of start-up costs $(I - k) E$. Subtracting effort costs yields welfare Y.

2.2 Efficient Allocation

The constrained optimal allocation q^*, l_j^*, a_j^* maximizes welfare $Y = w + k + \Pi E$ by directly allocating resources subject to the restriction that the government does not know more than private parties. Since w and k are exogenous, we need to consider only ΠE. Substituting Π from (6) and noting $Q_j E = \varepsilon_j E_j(y)$ by (A.3) and $E = \sum_j \varepsilon_j E_j(y)$ yields

$$\Pi E = \sum_j \varepsilon_j E_j(y) \cdot [p(l_j, a_j) v_j - \beta l_j - \gamma a_j] - (I + w) E. \tag{9}$$

Maximize Y subject to $y = \phi(q)$ as defined in (A.2-4). Use $dE_j/dy = -e_j$, $\varepsilon_j e_j = q_j e$ and $\varepsilon_j E_j = Q_j E$ when calculating the first derivatives,

$$\begin{aligned} l_j^* &: \frac{dY}{dl_j} = \left[p_l^j \cdot v_j - \beta\right] \cdot Q_j E \geqslant 0 \quad \Rightarrow \quad l_j^* = 1, \\ a_j^* &: \frac{dY}{da_j} = \left[p_a^j \cdot v_j - \gamma\right] \cdot Q_j E = 0, \\ q^* &: \frac{dY}{dq} = -\left[\sum_j q_j \cdot (p_j v_j - \gamma a_j - \beta l_j) - I - w\right] \cdot e\phi'(q) = 0. \end{aligned} \tag{10}$$

The first two conditions determine efficient effort levels. Since $p_{la}^j > 0$, efforts are complements. If the VC advises more intensively, the success probability increases which strengthens entrepreneur's incentives for high effort. Conversely, if the entrepreneur shirks, the project never succeeds and yields no revenue. That cannot be optimal and must be ruled out. Given the functional form in (1), optimal VC advice is calculated by the second condition above. Advice is positive only if entrepreneurial effort is high. Hence, $p_l^j v_j \geqslant \beta$ must hold by the first condition. Substituting optimal advice, this condition gives a restriction on parameters to assure an interior solution,

$$a_j^* = (v_j \cdot \alpha/\gamma)^{1/(1-\alpha)}, \qquad p_l^j \left(a_j^*, 1\right) v_j \geqslant \beta \quad \Leftrightarrow \quad v_j \geqslant (\gamma/\alpha)^{\alpha} \beta^{1-\alpha}. \tag{11}$$

Advice is higher with the good project. Hence, if the condition for high managerial effort is fulfilled for the bad project, it is a fortiori fulfilled for the good project as well.

One should note that the first two conditions result from maximizing the joint rent in (2), yielding $R'(v_j) = p_j > 0$. Rents strictly increase in project value v_j on account of the envelope theorem with respect to advice. We denote by R_j^* the joint rent with the first-best levels of effort. The condition for optimal entry thus emerges as

$$q^* : \quad \frac{dY}{dq} = -\left[\sum_j q_j \cdot R_j^* - I - w\right] \cdot e\phi'(q) = 0. \tag{12}$$

Using $q_G = q$ and $q_B = 1 - q$, socially optimal entry is given by the marginal quality

$$q^* = \frac{I + w - R_B^*}{R_G^* - R_B^*}. \tag{13}$$

This condition reveals that bad projects must make a loss for there to be a well defined interior solution of the constrained optimal allocation. We henceforth assume

$$R_B^* - I - w < 0. \tag{14}$$

3 Competitive Market Equilibrium

The model is solved backwards. (i) We solve for effort and advice and find the resulting rents to effort. (ii) We characterize the overall surplus and find the optimal VC contract given by profit shares s_j and the conversion price b. Shares s_j reflect profit maximization of VCs with respect to each individual project when the implications for effort are correctly anticipated. To highlight the role of the limited liability constraint, we start with unrestricted prices b_j by allowing b_B to be negative. Imposing limited liability $b_B = 0$ then gives the relevant case where $b = b_G$ stands for the conversion price that is due whenever the VC upgrades her stake to $s_G > s_B$. The prices b_j result from competition among VCs for good projects and their desire to deter bad types which result in a loss. On average, competitive VCs must at least break even on their investments. (iii) We finally determine entrepreneurial entry as it results from self-selection based on signals and the offered contracts.

3.1 Effort, Advice and Profit Shares

The solution is by backward induction. Given profit shares, entrepreneurs and VCs strive to maximize the rent from their productive inputs to the company. When maximizing (2), each party takes the action of the other party as given. The incentive compatibility conditions are the first order condition with respect to advice and the inequality resulting from the discrete comparison of rents for low and high managerial effort. Quite obviously, the VC leaves a high enough a profit share $1 - s_j$ to the entrepreneur to assure her critical effort. Otherwise there would be no revenue at all:

$$a_j : p_a^j (l_j, a_j) s_j v_j = \gamma, \quad l_j : p_l^j (l_j, a_j) (1 - s_j) v_j - \beta \geqslant 0. \tag{15}$$

The VC chooses optimal advice, taking the previously announced profit shares s_j and the entrepreneur's effort as given. Advice increases with her own share. The entrepreneur acts the same. Her share must be high enough to induce high effort, $l_j = 1$.

A reduction in s_j would reduce the VC's profit and incentives for advice without any extra gain on the entrepreneur's critical input. As long as managerial effort remains high, the VC can increase her profit by raising her own share, $dR_j^F d/s_j = p_j v_j > 0$ by the envelope theorem. Profit maximization thus leads the VC to raise s_j as much as possible until the entrepreneur's incentive compatibility condition in (15) binds with equality. Raising the share beyond this value would lead to a discontinuous drop in rents to zero since the entrepreneur would not provide her critical effort anymore. The equilibrium profit share and level of VC support are thus given by the two constraints in (15) holding with equality. Using the functional form in (1), we compute

$$a_j = (s_j v_j \alpha/\gamma)^{1/(1-\alpha)} < a^*. \tag{16}$$

The value of the profit share pins down the level of advice which is smaller than the first best level noted in (11). From a social perspective, the VC should be full residual claimant on her input. However, the need to provide incentives to the entrepreneur limits her share in the market equilibrium, leading to underinvestment in advice. Knowing the VC's stake s_j and her level of support a_j, we can infer her rent.

Proposition 1 *(a) Given the form in (1), the VC's profit shares fulfill* $1 > s_G > s_B > \alpha$. *(b) The VC's share* s_j *and the level of advice* a_j *increase in project value* v_j *but decline in marginal effort costs* β *of entrepreneurs and* γ *of VCs. (c) The entrepreneur's rent* R_j^E *is zero, her profit share exactly compensates for managerial effort cost. The VC gets the entire joint rent* R_j *which increases with project value but falls with marginal effort costs.*

Proof. See Appendix B. ■

Noting $a_j \cdot p_a^j = \alpha \cdot p_j$ as implied by (1) and using the optimality condition (15) to substitute for γ yields a convenient closed form for the VC's rent

$$R_j^F = (1-\alpha)\, s_j p_j v_j, \quad R_j^E = (1-s_j)\, p_j v_j - \beta = 0. \tag{17}$$

The entrepreneur cannot earn any rent since the success probability is linear in managerial effort. Her rent thus coincides with the incentive compatibility constraint in (15) which was shown to be binding in equilibrium. The entrepreneur must nevertheless appropriate a surplus to make her willing to forgo other career opportunities. Her expected surplus derives from selling the share s_j to the VC at a price that will exceed the capital costs of the venture by an amount b_j to be determined below. With $R_j^E = 0$, the financier's rent coincides with the joint rent, $R_j^F = R_j$.[6] Log-linearizing (17) and substituting (B.2) and (B.4) for the effort to determine the impact on the success probability yields

$$\hat{R}_j = \hat{R}_j^F = \hat{s}_j + \hat{p}_j + \hat{v}_j = \frac{1}{s_j - \alpha}\left[\hat{v}_j - (1-s_j)\,\hat{\beta} - \alpha\hat{\gamma}\right]. \tag{18}$$

[6]Hence, the solution of the profit share as stated in Proposition 1 reflects the principle of Pareto-optimality as in Inderst and Mueller (2004).

3.2 Competition Among VCs

The profit shares are chosen to maximize the VC's rent which coincides with the joint rent. The entrepreneur keeps a minimum share, but no more than that, to assure her critical contribution to the firm without which no return would be possible at all. The VC should receive the largest possible stake to bring her as close as possible to being the full residual claimant on the returns to advice. This is required to bring the privately determined level of advice as close as possible to the socially optimal level. On the other hand, VCs must compete for financing start-up firms by offering an overall attractive package to entrepreneurs. They can do so by offering a high price $b_j + I - k$ for their share s_j which must cover at least the unfinanced part of start-up cost but can also include a lump-sum, success independent component b_j. In offering a price, VCs aim to attract good projects and to deter bad ones, especially if the bad ones result in a loss. Since VCs can convert their profit shares s_j after the firm is started and quality is revealed, they can also differentiate the prices b_j, where $b_G - b_B$ is the conversion price to be paid if the option to increase the share from s_B to s_G is exercised.

The VC's surplus from a project of type j is $\pi_j^F = R_j^F - b_j - (I - k)$. VCs try to attract entrepreneurs with good signals who are very likely to have a good project, by offering a high conversion price resulting in a high value of b_G for the same share s_G, possibly until $\pi_G^F = 0$. Similarly, VCs will offer a very low price b_B to deter entrepreneurs with low signals who are likely to be endowed with a bad project that might result in a loss. If even the bad project is profitable, the VC competes for this project by offering a low but positive price b_B. If it is unprofitable, she would ask for a negative price, i.e. a payment from entrepreneur to VC, to avoid losses that would have to be covered by cross-subsidization. However, limited liability $b_j \geqslant 0$ prevents this since entrepreneurs have already invested their entire wealth k in the project and simply have no funds left. If $b_B = 0$ still results in a loss, the VC might raise her share s_B to cut losses but this it not possible either because raising it would violate the managerial incentive constraint and provoke a certain failure. So the VC cannot cut her losses on a bad project any further than paying the lowest possible price $b_B = 0$ after $I - k$ is sunk and project quality is revealed. When the project turns out bad, she does not exercise the conversion option and must incur a loss. In this case, the VC will have to make strictly positive profits on good projects to cover her losses by means of cross subsidization. The VC must break even at least on average. To sum up, one must distinguish two cases:

$$\begin{aligned} &(a) \quad b_j > 0; \quad R_j^F - (I-k) = b_j, \\ &(b) \quad b_G > 0; \quad R_G^F - (I-k) > b_G, \\ &\qquad\ b_B = 0; \quad R_B^F - (I-k) < 0. \end{aligned} \tag{19}$$

In case (a), both projects are profitable and the VC competes with prices b_j that allow her to break even on each project separately and, hence, on average as well. In case (b), only the good project is profitable and the bad one makes a loss. She thus sets the price for a bad project to the lowest possible value $b_B = 0$ and offers a conversion price such that she makes a strictly positive profit on good projects and just breaks even on average. In this case, she cannot avoid cross-subsidizing from good to bad projects.

Knowing the expected surplus derived from a given contract, entrepreneurs decide upon entry. The marginal entrant must have received a sufficiently good signal. She expects to be endowed with a good project with a minimum probability q that makes her indifferent between entrepreneurship and an alternative career. Entry of the marginal entrepreneur with quality q determines the average probability $Q(q)$ as explained in Appendix A and in (3). Since the VC might in fact face any entrant, she must consider the average probability. Given Q, competition among VCs forces them to raise prices b_j for high quality ventures and cut them for bad ones until they just break even. The zero profit condition is $\Pi^F = \sum_j Q_j \pi_j^F = 0$, or

$$Q \cdot \left(R_G^F - b_G\right) + (1-Q) \cdot \left(R_B^F - b_B\right) = I - k. \tag{20}$$

This average break even condition must hold in all cases but it can be fulfilled in two distinct ways. First, if both projects are profitable, $b_j > 0$ for all j, the VC competes for each project separately and breaks even on each of them individually. Obviously, she breaks even on average as well. There is no cross-subsidization. Second, if the bad project is unprofitable as in case (b) of (19), then $b_B = 0$ on account of limited liability. Consequently, the VC makes losses on bad projects which must be covered with profits from good ones. The VC can break even only on average where competition for good projects would raise the conversion price $b = b_G$ up from zero until (20) binds.

The type specific profit shares together with prices b_j can be understood as a very simple representation of convertible debt. The interpretation rests on the fact that debt and equity are really the same in our framework if we consider only a single project. The VC provides a total amount of funds equal to $b_j + I - k$ to pay for capital expenses and to compensate for the foregone outside option that is not covered by the entrepreneur sharing in the profit.[7] The VC gets the return either as a profit share or a debt repayment. Define $s_B v_B \equiv D_B > b_B + I - k$ as the debt repayment of a bad project that goes to the VC if the project succeeds. The debt repayment D_B exceeds the initial credit by an implicit interest which must cover the credit losses from failed projects as well as any effort costs by the VC. Repayment of debt leaves all residual profits $(1 - s_B) v_B = v_B - D_B$ to the entrepreneur. This way we can understand the project specific profit share as debt for a bad project, allowing the VC to convert to a higher equity share $s_G > s_B$ if the project turns out good and if she is willing to pay the conversion price $b_G - b_B$. The conversion is done after the project is started and quality is revealed. Converting to a higher share reinforces the incentives to advise more intensively the good project and to add more value as it should be done from the social viewpoint.

Proposition 2 *(a) When bad projects are profitable, competition among VCs raises prices $b_j + I - k$ for the profit shares s_j. The entire surplus goes to entrepreneurs. VCs break even separately on each project type without cross-subsidization. (b) When bad projects are unprofitable and limited liability binds, high quality entrepreneurs obtain less and low quality entrepreneurs more than the joint surplus. VCs make positive profits on high quality ventures that subsidize losses on low quality projects. They break even on average.*

Proof. Discussion of equations (19) and (20). ■

3.3 Self-Selection of Entrepreneurs

Agents who received sufficiently good signals, start a firm and apply for VC financing. Using $q_G = q$ and $q_B = 1 - q$, condition (4) is $q\left(R_G^E + b_G\right) + (1 - q)\left(R_B^E + b_B\right) = w + k$.

[7] In fact, the conversion price must cover the entire outside option $w + k$ since the profit share just suffices to compensate for managerial effort cost and leaves a zero rent to the entrepreneur.

The marginal entrant thus comes with a project that is good with probability q and bad with probability $1-q$. This marginal quality is

$$\pi^E = 0 \quad \Rightarrow \quad q = \frac{w + k - R_B^E - b_B}{R_G^E + b_G - R_B^E - b_B}. \tag{21}$$

Although we know that an entrepreneur obtains no rent beyond the compensation for effort, $R_j^E = 0$, we keep these terms for better interpretation. The more realistic case of loss-making bad projects gives $b_B = 0$ and $b_G = b$. With these simplifications, the critical quality would be $q = (w + k)/b$.

Having chosen rent maximizing equity stakes, and taking average quality as given, VCs compete for projects by setting prices b_j. Offering higher prices b_j encourages entry by lowering the critical quality q. Use $R_j^E = R_j - R_j^F$ and write the denominator in (21) as $R_G^E + b_G - R_B^E - b_B = R_G - R_B - \nabla$ with ∇ defined below. Taking the derivative yields

$$\begin{aligned} \frac{dq}{db_G} &= -\frac{q}{R_G - R_B - \nabla} < 0, \\ \frac{dq}{db_B} &= -\frac{1-q}{R_G - R_B - \nabla} < 0, \\ \nabla &\equiv \left(R_G^F - b_G\right) - \left(R_B^F - b_B\right). \end{aligned} \tag{22}$$

Raising the conversion price b_G lowers the critical probability q and thereby encourages entry. The denominator stands for the income difference that the entrepreneur would realize if she could exchange a bad for a good project, and ∇ gives the corresponding income difference of the VC. Quite intuitively, the entrepreneur's gain $R_G - R_B - \nabla$ is the total gain minus the VC's share in the income gain.

When bad project are unprofitable to the VC, condition (19.b) applies and limited liability binds, $b_B = 0$, which creates a need for cross-subsidization. In this case, the conversion price $b = b_G$ is fixed by the VC's average break even condition in (20),

$$Q \cdot \nabla + R_B^F - b_B - (I - k) = 0. \tag{23}$$

When bad projects are unprofitable, (19.b) implies $\nabla > 0$. If limited liability were not imposed, then the VC would compete with two positive prices b_j until she breaks even on each project separately as in (19.a), implying $\nabla = 0$. By (20), the VC would obviously break even also on average.

3.4 Efficiency

We first turn to the identity of the marginal entrepreneur and the implied efficiency of market entry. One could directly compare the constrained optimal allocation q^* in (13) with the market allocation in (21). Since q^* follows from $Y_q = 0$, we evaluate instead the welfare derivative Y_q at the market allocation. If this derivative is zero, market entry is optimal. Rewrite the break even condition (4) of the marginal entrepreneur, using $R_j^E = R_j - R_j^F$ and the definition of ∇ in (22),

$$\pi^E = q \cdot [R_G - R_B - \nabla] + \left[R_B - \left(R_B^F - b_B\right)\right] - k - w = 0. \tag{24}$$

Substituting this for w in (12), as it is evaluated in private equilibrium, yields

$$Y_q = -\left[q \cdot \nabla + R_B^F - b_B - (I - k)\right] \cdot e\phi'(q). \tag{25}$$

Finally, replacing $I - k$ by the VC's break even condition (23) gives

$$Y_q = (Q - q) \cdot \nabla \cdot e\phi'(q). \tag{26}$$

The positive correlation of the signal with project quality implies $\phi'(q) > 0$, see (A.6), where $y = \phi(q)$ is the inverted relationship of (A.2). We also have $Q > q$. The welfare result with respect to entry thus depends on the sign of ∇ which follows from (19). If, as in case (a), limited liability is not binding, the VC breaks even on each project separately without cross-subsidization which gives $\nabla = 0$. Entry is efficient in this case. If case (b) applies and the limited liability condition is binding, then $\nabla > 0$. Preventing entry of the marginal entrepreneur and thereby raising the value of the marginal probability q would boost welfare, $Y_q > 0$ by (26). With limited liability, the VC must cross-subsidize from good to bad projects. Low quality entrepreneurs thus get a too favorably deal which results in excess entry.

Further intuition about the reasons for excess entry is obtained by writing π^E as in (24) and using (5) to obtain $\pi^E = \pi - q\nabla - \left[R_B^F - b_B - (I - k)\right]$. Replacing the square bracket by the VC's zero profit condition in (23) results in $\pi^E = \pi + (Q - q) \cdot \nabla$. If there is no cross-subsidization, $\nabla = 0$ and the entrepreneur gets the entire social surplus, $\pi^E = \pi$.

She obtains the surplus by obtaining high prices b_j since the rent to effort is set to zero in the interest of maximum revenue per project. If instead the limited liability constraint necessitates cross-subsidization, we have $\nabla > 0$. Consequently, good entrepreneurs with a high probability $q > Q$ get less than the social surplus while low quality entrepreneurs with $q < Q$ get more than the joint surplus which induces excess entry.

The other distortion in this model might be the level of VC advice. Managerial effort cannot be distorted since it was assumed to be discrete. High effort is critical for the survival of the firm. The entrepreneur's shirking would result in business failure with certainty. If she reduced her effort below this critical level, the success probability would drop to zero. Hence, the VC will always assure the socially optimal high effort $l = 1$. Nevertheless, it is instructive to treat managerial effort as continuous for the moment. According to (10), the overall welfare impact is $dY = Y_q \cdot dq + \sum_j \left(Y_{a_j} \cdot da_j + Y_{l_j} \cdot dl_j\right)$. Substituting γ in the square bracket of Y_a in (10) by the private optimality condition in (15), and substituting similarly for β in Y_l, yields

$$dY = (Q - q) \nabla \phi' (q) edq + \sum_j \left[(1 - s_j) p_a^j v_j \cdot da + s_j p_l^j v_j \cdot dl\right] Q_j E. \qquad (27)$$

We have already discussed the distorted excess entry of entrepreneurs. The other two terms indicate inefficiently low effort by the VC and, if managerial effort l_j were continuous, by the entrepreneur. Welfare would increase if VC advice could be stimulated. In putting in more advice, the VC creates extra value $p_a^j v_j$ but can appropriate only a share $s_j p_a^j v_j$ since she must cede a share $1 - s_j$ to the entrepreneur to secure her cooperation. The difference between the social and private returns to VC advice is a spill-over to entrepreneurs that the VC does not take account of when she decides about her own input to the company. Advice is thus inefficiently low. A symmetric argument applies to entrepreneur's managerial effort. The social return to effort is $p_l^j v_j$ but the entrepreneur gets only a share $(1 - s_j) p_l^j v_j$. The rest accrues to the VC to compensate her for advice and the financial funds $I - k$. If her effort were continuous, the entrepreneur would underinvest in effort as well. However, we have much simplified our model by assuming discrete effort so that managerial effort cannot be reduced below the critical, efficient level $l^* = 1$, i.e. $dl = 0$ in the above formula. Otherwise the company would fail for sure. Hence, the entrepreneur's effort is always efficient in this model.

Proposition 3 *(a) VC advice is inefficiently low in equilibrium. (b) When limited liability binds for unprofitable projects, entry is excessive on the low quality margin.*

Proof. See equation (27) and its discussion. ■

It is instructive to see under what conditions the above mentioned distortions in VC financing could be avoided. If we had a budget breaking third party as in Holmstrom (1982) which can also be replicated as a tax transfer mechanism supplied by the government as in Keuschnigg and Nielsen (2003), advice could be made efficient. Basically, the mechanism subsidizes the VC's revenues ex post until she is full residual claimant on the returns to advisory effort, and finances the subsidy by a tax ex ante. If, in addition, the entrepreneur could be made a residual claimant on the project type which is different from being residual claimant on the returns to effort at the moral hazard stage, entry would be efficient as well. The entrepreneur would be residual claimant on the project type if she appropriated the entire income difference between good and bad projects. In competitive market equilibrium, this would be possible only if low quality projects would be profitable as well, and is prevented otherwise by the limited liability constraint.

4 Comparative Statics

Equilibrium in the unrestricted case is fully recursive. The VC breaks even separately on each venture, yielding $b_j = R_j^F - I + k > 0$ even for the bad project. Since the entrepreneur's rent on effort is zero by the incentive compatibility constraint, the VC gets the entire joint rent, $R_j^E = 0$ and $R_j^F = R_j$. The entrepreneur, however, appropriates the total surplus $\pi_j^E = \pi_j$ by asking prices b_j. Competitive VCs bid up prices until their own surplus from financing the project is exhausted, $\pi_j^F = 0$. Substituting these prices into (21) reveals the quality of the marginal entrant, $q = (w + I - R_B) / (R_G - R_B)$. This is the same formula as for socially optimal quality q^* in (13). Marginal quality, however, will not be the same since VC advice is inefficiently low, implying that $R_j < R_j^*$. Nevertheless, according to (27), welfare cannot be improved by encouraging entry since $\nabla = 0$ when limited liability is not binding. The complete optimum is obtained only by encouraging

VC advice until $a = a^*$ which yields $R_j = R_j^*$ and therefore $q = q^*$ as well. This completes the solution of the unconstrained case since all other variables are implied.

More realistic is the case where bad projects result in a loss, implying $b_B = 0$ on account of limited liability and $b_G = b$. One is left with two unknowns, q and b, and two restrictions: free entry of entrepreneurs in (21) and the average break-even condition of VCs in (23). Using $\nabla = R_G - R_B - b > 0$ yields

$$VC^D : q = (w + k) / b, \quad VC^S : Q(q, \theta) \cdot (R_G - R_B - b) = I - k - R_B. \tag{28}$$

Entry of entrepreneurs creates demand for VC finance. The identity of the marginal entrepreneur, as given by marginal quality q, is a downward sloping function of the conversion price b. Figure 1 illustrates. The break-even condition of competitive VCs in (23) stands for the supply of VC. VCs take entry and average quality as given and respond by offering a conversion price. The supply function gives the break even price b for any given quality of the pool of applicants such that $\sum_j Q(q_j) \pi_j^F = 0$. Since average and marginal qualities are positively related, it is an upward sloping function of q. When the quality of projects increases, in the sense that any given venture is more likely to be of the profitable type, then the financier's expected surplus rises. She competes by offering an even higher conversion price until she hits the break even condition. A higher price means that the VC can offer finance only to fewer firms with better quality. The supply of VC declines on account of an increase in the required marginal quality.

The comparative statics can be entirely understood with Figure 1. The signs noted below the exogenous variables in the supply and demand schedules indicate in which direction the curves are shifted when the variable is increased. Consider, for example, an increase in the market values v_j of ventures. By proposition 1, rents R_j from managing and advising a company increase which boosts the surplus π_j^F of the VC. Given an average quality in the pool of applicants, VCs bid up the conversion price to attract more business, until they break even. The supply schedule shifts to the right (not drawn). When the VC is willing to pay a higher price when converting, the deal becomes more attractive to entrepreneurs and attracts additional entry at the lower quality margin. Since the supply schedule in Figure 1 is unchanged, a new equilibrium results with a higher conversion price and more entry at the lower quality margin. Average quality thus declines.

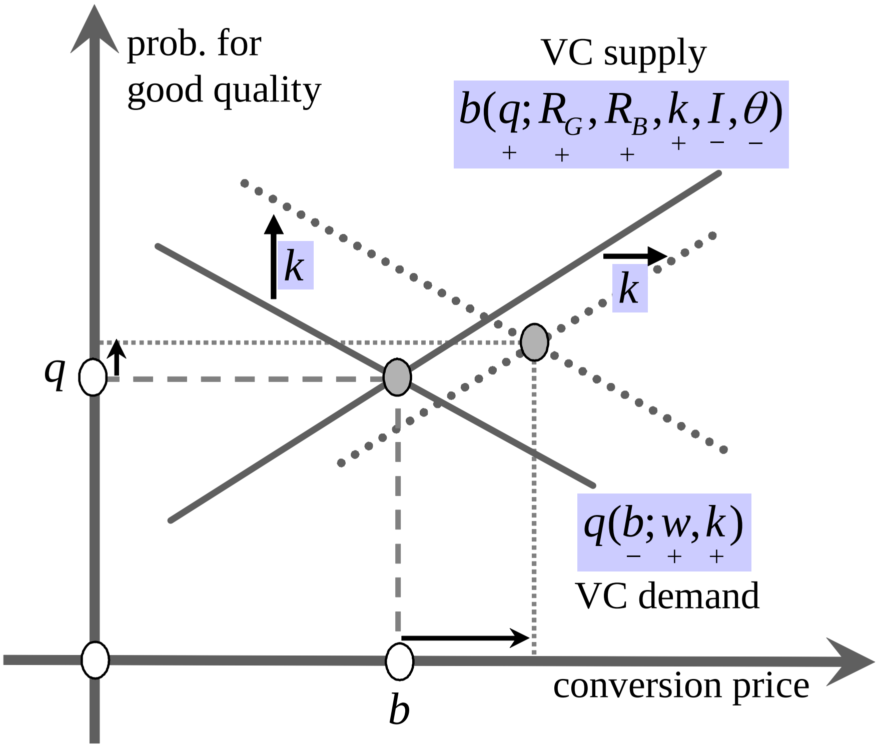

Fig. 1: Demand and Supply of Venture Capital

As another example, consider a change in the informativeness of signals. When market and technological uncertainty increases, the potential of a new idea is more difficult to assess. The signals received by entrepreneurs become less informative which is captured by the parameter $\theta < 1$ increasing towards one, see Appendix A. Consequently, people will not be able to revise very much their expectations of having a good project relative to what they believe in advance. The average and marginal probabilities $Q > q$ of being endowed with a good business idea will move closer to the prior probability ε. In fact, if the signal were completely uninformative ($\theta = 1$), all would believe the same, implying $Q = q = \varepsilon$. A lower information content of the signal reduces the possibility of self-selection into entrepreneurship. In particular, the wedge between average and marginal quality shrinks as (A.11) in the Appendix shows. Hence, for any given q, a lower information content reduces the average quality in the pool of applicants and thereby forces VCs to offer less favorable deals by reducing the conversion price. The supply schedule shifts to the left. Accordingly, entrepreneurs become more hesitant to start their own business. In the new equilibrium, a lower information content of signals which might be the result of increased technological and market uncertainty, reduces the conversion price b and raises the quality of the marginal entrant. Therefore, a less informative signal reduces entrepreneurship. The effect on average quality as stated in (A.11) seems ambiguous at first sight. On the one hand, a lower information content directly reduces average quality in the pool but the fact that some low quality marginal entrepreneurs stay out works in the opposite direction. According to (C.5) in the appendix, the direct effect dominates. The net effect is a reduction in the average quality of VC backed start-up firms.

As a final experiment, we discuss the consequences of agents being endowed with more assets. All other shocks can be derived unambiguously from Figure 1 and are not discussed in more detail. Since an increase in own assets shifts up the demand schedule but also shifts to the right the supply schedule, the effects on entry are ambiguous at first sight. When more own capital is at stake, the opportunity cost of starting a firm rises and thereby shifts up the demand curve for VC. The fear of loosing own capital in case of business failure raises the required marginal quality q and diminishes demand for VC. On the other hand, more own capital reduces the need for external financing. Given that the VC finances a smaller amount $I - k$, she will compete with a higher conversion price b to acquire the same profit shares s_j. This in itself shifts the supply schedule to the right and would attract more rather than fewer entrepreneurs. In the Appendix, we show in (C.3-4) that the net effect is positive. When more own equity is at stake, agents require a more reliable signal for high quality since the VC contract rewards them only if the quality turns out to be good. When the entrepreneur comes with more equity, VCs bear less downside risk. They can reduce their losses from bad projects which would allow for more financing at the lower quality margin. The entrepreneur's income will be low even when the low potential firm successfully reaches maturity. Income will be just sufficient to compensate for managerial effort during the start-up phase but no more than that. Entrepreneurs are exposed more to the downside risk when they are endowed with more equity. The demand effect dominates. More own equity results in entry of fewer firms with higher quality on average. Demand for VC falls.[8]

[8]This result is best appreciated when considering coexistance of VC and bank financing, see for example Ueda (2004). It is an established result that firms with little collateral turn to VC while high

The results are now summarized for the case that limited liability binds:

Proposition 4 *(a) Higher market values v_j or lower marginal effort costs β and γ lead to a higher conversion price, more entry and lower average quality. (b) More own capital k and a larger outside wage w result in a higher price, less entry and lower average quality. (c) A lower information content of signals (θ rises, reflecting increased technological and market uncertainty) reduces the conversion price, entry and average quality. A larger capital investment I has the same effects, except that average quality increases.*

Proof. Proposition 1, Figure 1 and Appendix C. ■

5 Conclusions

Despite of the fact that project selection is presumably as important as the value added role of VCs for the performance of VC backed compared to other firms, the literature has largely focussed on the value added role. This paper proposed a descriptive model of VC financing when entrepreneurs propose projects with high or low market potential. Our model of the VC industry features (i) an advisory role of VCs that is conditional on the quality of the venture which is revealed after the firm is started and close cooperation begins, (ii) a financial contract that is interpreted as a simple form of convertible debt, and (iii) self-selection into entrepreneurship of those persons who are more likely to be endowed with a high potential business idea.

The proposed framework replicates a number of important stylized facts in VC financing. In our model, VCs acquire a larger stake in high quality ventures and advise them more intensively. Since self-selection of entrepreneurs according to quality is imperfect, they are inevitably stuck with low potential firms as well. Should a venture turn out to be of low quality, the firm will receive less intensive but still positive VC support which is mainly motivated to cut losses from that firm. The VC contract to be announced before quality of applicants becomes known must serve different functions. The profit shares

equity firms prefer bank financing.

should be chosen to allocate incentives for managerial and advisory effort in a value maximizing way, implying a high VC share if the project turns out good, and a low share if it is a looser. The prices that are offered for these shares must be high enough to pay for capital expenses. They should also be chosen to induce the right self-selection and to attract those entrepreneurs who are more likely to be endowed with a profitable business idea. Since the contract must be posted before quality is known, it must allow for sufficient flexibility. Our model rationalizes a simple form of convertible debt, one of the most widely used contracts in VC financing. The contract posts a low profit share that is suitable for low quality firms at a price that covers no more than the capital expenses that can not be financed out of the entrepreneur's own pocket. In addition, the contract includes an option to convert, at a prespecified conversion price, to a higher equity stake should the firm turn out to be a profitable type. The conversion is exercised and the conversion price paid after the firm is started and project quality is revealed.

The contract induces the right self-selection because it rewards entrepreneurs much more when the project in fact turns out to be of high potential. In this case, the VC converts to the high profit share and pays a conversion price that additionally rewards the entrepreneur for having delivered a high potential project. In fact, competition among VCs for good projects bids up the conversion price to the largest possible extent until VCs hit their break even condition. Entrepreneurs thereby appropriate most of the surplus from good projects while they obtain no surplus from a bad project when the VC doesn't exercise her option. The contract is therefore particularly attractive for entrepreneurs who have received a good signal and are, thus, very likely to be endowed with a high quality project, compared to others who have received a less inviting signal.

When considering the efficiency of the market equilibrium, we have identified two conditions for an efficient allocation of effort and entry. First, optimal effort requires that an agent is full residual claimant on the returns to her own effort input as the literature on double moral hazard has emphasized. This is not possible since the entrepreneur needs external financing and thus must share profits with the VC. Advice is therefore inefficiently low in our model. Second, efficient entry requires that the entrepreneur is full residual claimant on the project type, meaning that she gets all the extra return when moving from a bad to a good project. Again this is not possible when bad projects are unprofitable to the VC and limited liability binds so that the entrepreneur cannot be asked to pay for the loss. In this case, the VC must cross-subsidize from good to bad projects which implies that low quality entrepreneurs get a too favorable deal. Hence, entry is excessive at the low quality margin.

Given these results on efficiency, the policy implications are in principle clear. Policy should find ways to stimulate VC effort by favorably treating ex post returns on projects. On the other hand, policies should try to restrict entry of marginal entrepreneurs who are rather likely to be endowed with low quality projects. What specific policy instruments could achieve these goals is left for future research. It also seems that the proposed framework is sufficiently tractable. It should thus be explored whether this self-selection framework lends itself to consider the coexistence of bank and VC financed firms. It would be particularly interesting to see whether such an extended framework can explain the differential performance of bank versus VC financed firms, and to what extent it is due to selection effects rather than the value added role of VCs. Recent empirical work by Sorensen (2005) has disentangled the reasons for such differential performance. Applied theoretical work could try to identify those structural parameters that shift the relative importance of selection versus advice in one or the other way.

Appendix

A Signals and Probabilities

Projects are either good and bad, $j \in \{G, B\}$. The true proportion of good ideas is ε. In the beginning, agents do not know the type but receive an informative signal y that is positively correlated with project quality. A good signal is thus received by the good type with higher probability than by the bad type. The distribution of signals in the population is

$$E_j(y) = \int_y^\infty e_j(y')\,dy', \quad E_j'(y) = -e_j(y). \tag{A.1}$$

The *marginal probability* of having a good project is

$$q = \Pr(G|y) = \frac{\varepsilon e_G(y)}{e}, \quad e \equiv \varepsilon e_G(y) + (1-\varepsilon) e_B(y). \tag{A.2}$$

The average probability of observing a specific signal y by any of the two types is e. Good types are much less likely than bad types to receive a low signal, implying $e_G(y) < e_B(y)$ for y small. In contrast, high signal values are more frequently received by good types, implying $e_G(y) > e_B(y)$ for y large. Consequently, the marginal probability increases in the signal value.

The *average probability* for a good idea among all agents with signals $y' > y$ is

$$Q = \Pr(G|y' \geqslant y) = \frac{\varepsilon E_G(y)}{E}, \quad E \equiv \varepsilon E_G(y) + (1-\varepsilon) E_B(y). \tag{A.3}$$

The total number of signals $y' \geqslant y$ received by all agents is E.

Agents pursue entrepreneurship only if they perceive a sufficiently high probability q of having a good project. They will thus need to receive a sufficiently good signal y. Inverting (A.2) establishes a correspondence $y = \phi(q)$. By (A.3), the average probability Q of a good project among all agents with signals better than $y = \phi(q)$ is

$$Q = \Pr(G|y' \geq \phi(q)). \tag{A.4}$$

Entry decision thus establishes a cut-off value or marginal probability q and an average probability $Q > q$ of a good project in the entire pool of applicants.

We now determine the impact of the marginal entrepreneur with probability q on the average probability Q. Using (A.2) and (A.3), we first calculate

$$\frac{dQ}{dy} = (Q - q) \cdot \frac{e}{E} > 0. \tag{A.5}$$

The effect is positive with increasing signals and probabilities, yielding $Q > q$. It will also be useful to get, from equation (A.2),

$$\frac{dq}{dy} = q \cdot (1-q) \cdot \left[\frac{e'_G}{e_G} - \frac{e'_B}{e_B}\right] > 0. \tag{A.6}$$

The sign of the square bracket must be positive for the signal to be informative of the true quality as will be shown below. Divide (A.5) by (A.6) to obtain the desired effect,

$$\frac{dQ}{dq} = \frac{(Q-q) \cdot e/E}{q \cdot (1-q) \cdot [e'_G/e_G - e'_B/e_B]} > 0. \tag{A.7}$$

We now use a special functional form for the density of signals. The parameter θ conveniently measures the informativeness of signals,

$$e_G(y) = \theta \exp(-\theta y) = \theta E_G(y), \quad e_B(y) = \exp(-y) = E_B(y), \quad \theta < 1. \tag{A.8}$$

If $\theta = 1$, then $e_G = e_B$ and $E_G = E_B$. In this case, $q = Q = \varepsilon$ by (A.2-3), and the signal is not informative. If $\theta < 1$, the good type receives a low signal with smaller density than the bad type, $e_G(0) = \theta < e_B(0) = 1$. The density of higher signals falls more rapidly with the bad type so that $e_G(y) > e_B(y)$ for y large enough. The two density functions cross exactly once. The relative slopes are governed by θ, implying

$$e'_G/e_G - e'_B/e_B = 1 - \theta. \tag{A.9}$$

The functional form in (A.8) allows for a convenient closed form solution of the relation between average and marginal quality which also shows how this relation depends on the informativeness parameter. Substitute $e_G = \theta E_G$ and $e_B = E_B$ from (A.8) into (A.2), divide the numerator and denominator by E and use the definition of Q in (A.3),

$$q = \frac{\theta Q}{\theta Q + 1 - Q} \quad \Rightarrow \quad Q = \frac{q}{q + (1-q)\theta} > q. \tag{A.10}$$

The informativeness assumption $\theta < 1$ implies $Q > q$. If the signal were not informative, then average and marginal quality would be equal, $Q = q$, as argued before. The elasticity of Q with respect to q follows from the log-linearization where the hat notation indicates a relative change, $\hat{Q} \equiv d \ln Q = dQ/Q$. Rewrite (A.10) as $(1-Q)q = (1-q)\theta Q$, we obtain the log-linear form

$$\hat{Q} = \mu \cdot \hat{q} - (1-Q) \cdot \hat{\theta}, \quad \mu \equiv (1-Q)/(1-q). \tag{A.11}$$

As a consistency check, we use (A.9) and write the coefficient of (A.7), $\hat{Q} = \mu \hat{q}$, as $\mu = \frac{(Q-q)e/E}{(1-q)(1-\theta)Q}$. Using again (A.8) to rewrite e yields $e/E = \theta Q + (1-Q)$. Rearranging (A.10) as $(1-Q)q = (1-q)\theta Q$, one obtains $\theta Q = (1-Q)q/(1-q)$, and thereby $(1-\theta)Q = (Q-q)/(1-q)$. Using these expressions to replace θQ in the numerator and $(1-\theta)Q$ in the denominator yields $\mu = \frac{(Q-q)[\theta Q + (1-Q)]}{(1-q)(1-\theta)Q} = \frac{1-Q}{1-q}$ as in (A.11).

B Proof of Proposition 1

Part (a): Section 3.1 showed that a_j and s_j satisfy the two conditions in (15) with equality. Using (1) and substituting (16) into the condition on l_j in (15) yields

$$z_j \equiv (1-s_j)^{1-\alpha}(s_j)^{\alpha} = \beta^{1-\alpha}(\gamma/\alpha)^{\alpha}/v_j, \quad \frac{dz_j}{ds_j} = \frac{\alpha - s_j}{(1-s_j)^{\alpha}(s_j)^{1-\alpha}}. \tag{B.1}$$

This equation implicitly determines the profit share. The z_j-function returns a zero for values $s_j = 0$ and $s_j = 1$, and is positive and concave in between. Its slope turns from positive to negative as s_j starts from zero and moves beyond α. With an interior solution, there are two values for s_j of which the larger is the relevant profit maximizing one by the arguments in the paragraph preceding (16). Hence, the slope of the z_j-function must be negative at the optimal value of s_j, implying $\alpha < s_j$. The inequality $s_G > s_B$ follows from the fact that a higher value $v_G > v_B$ reduces the r.h.s. of (B.1) and shifts down the horizontal line which intersects the z-function.

Part (b): We show this by linearizing the system in (15). The notation $\hat{a} \equiv da/a$ indicates a percentage change where da is the absolute deviation from an initial value of a. The functional form (1) yields together with the equilibrium value $l_j = 1$,

$$p_j = l_j \cdot (a_j)^{\alpha} \quad \Rightarrow \quad a_j \cdot p_a^j = \alpha \cdot p_j, \quad \hat{p}_j = \alpha \hat{a}_j, \quad \hat{p}_a^j = -(1-\alpha)\hat{a}_j. \tag{B.2}$$

The comparative static effects of shocks to exogenous parameters can be uncovered by log-linearization of (15). Using (B.1) yields

$$(1-\alpha)\hat{a}_j = \hat{s}_j + \hat{v}_j - \hat{\gamma}, \quad \frac{s_j}{1-s_j}\hat{s}_j = \alpha\hat{a}_j + \hat{v}_j - \hat{\beta}. \tag{B.3}$$

The first equation shows how a VC increases advice upon receiving a larger profit share, the second relates to the entrepreneur's incentives. If she receives more advice, her own incentives for effort can be assured with a lower share $1-s_j$, or a higher share s_j for the VC. Solving (B.3) for the two unknowns and noting $s_j > \alpha$ by part (a) yields

$$\hat{a}_j = \frac{1}{s_j - \alpha}\left[\hat{v}_j - s_j\hat{\gamma} - (1-s_j)\hat{\beta}\right], \quad \hat{s}_j = \frac{1-s_j}{s_j-\alpha}\left[\hat{v}_j - \alpha\hat{\gamma} - (1-\alpha)\hat{\beta}\right]. \tag{B.4}$$

Part (c): This part is shown in equations (18) and (19) of the main text.

C Comparative Statics

Log-linearize (28) to obtain comparative static results. Define the share $\delta \equiv k/(w+k)$ of own capital in the total opportunity cost of entrepreneurs. The share of foregone wages is $1-\delta = w/(w+k)$. Use this to obtain the log-linearized form of the demand schedule. The relative change in average quality was already shown in (A.11). Use this together with $\nabla = R_G - R_B - b > 0$ (or $\nabla\hat{\nabla} = R_G\hat{R}_G - R_B\hat{R}_B - b\hat{b}$) in the log-linearized form of the supply schedule in (28), $Q\nabla\left(\hat{Q}+\hat{\nabla}\right) = I\hat{I} - k\hat{k} - R_B\hat{R}_B$, which gives

$$\begin{aligned} \hat{q} &= -\hat{b} + (1-\delta)\,\hat{w} + \delta\hat{k}, \\ \hat{b} &= \left[Q\nabla\mu\hat{q} - I\hat{I} + k\hat{k} - (1-Q)\,Q\nabla\hat{\theta} + QR_G\hat{R}_G + (1-Q)\,R_B\hat{R}_B\right]/\,(Qb)\,. \end{aligned} \tag{C.1}$$

Substituting the demand function $\hat{q}$ into the supply function yields

$$\hat{b} = \frac{(1-\delta)\,Q\nabla\mu\hat{w} + (\delta Q\nabla\mu + k)\,\hat{k} - (1-Q)\,Q\nabla\hat{\theta} - I\hat{I} + \sum_j Q_j R_j \hat{R}_j}{(b+\nabla\mu)\,Q}. \tag{C.2}$$

Substituting back into the demand schedule $\hat{q}$ yields the marginal entrant,

$$\hat{q} = \frac{(1-\delta)\,Qb\hat{w} + (bQ-w-k)\,\delta\hat{k} + (1-Q)\,Q\nabla\hat{\theta} + I\hat{I} - \sum_j Q_j R_j \hat{R}_j}{(b+\nabla\mu)\,Q}. \tag{C.3}$$

All results are unique except for the impact of k. Substituting the demand schedule in (28) for $w+k$ yields

$$Qb - w - k = (Q-q)\,b > 0. \tag{C.4}$$

The effect of θ on average quality seems ambiguous at first sight. Substituting the effect in (C.3) into (A.11) and finally replacing μ yields, after some manipulations,

$$\hat{q} = \frac{1-Q}{b+\nabla\mu}\nabla\hat{\theta}, \quad \hat{Q} = -\frac{(1-Q)\,b}{b+\nabla\mu}\cdot\hat{\theta}. \tag{C.5}$$

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