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Sufficient Information in Structural VARs

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Sufficient Information in Structural VARs^{*}

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Abstract

We derive necessary and sufficient conditions under which a set of variables is *information-ally sufficient*, i.e. contains enough information to estimate the structural shocks with a VAR model. Based on such conditions, we provide a procedure to test for informational sufficiency. If sufficiency is rejected, we propose a strategy to amend the VAR. Our method can be applied to FAVAR models and can be used to determine how many factors to include in such models. We apply our procedure to a VAR including TFP, unemployment and per-capita hours worked. We find that the three variables are not informationally sufficient. When adding missing information, the effects of technology shocks change dramatically.

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Keywords: Structural VAR, non-fundamentalness, information, FAVAR models, technology shocks.

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1 Introduction

Since Sims (1980)'s seminal paper, Structural Vector Autoregression (SVAR) models have become extremely popular for structural and policy analysis. The idea behind these models is that structural economic shocks can be found as linear combinations of the residuals of the linear projection of a vector of variables onto their past values, i.e. are innovations with respect to the econometrician's information set. Therefore, an obvious requirement for the analysis to be meaningful is that the variables used in the VAR convey all of the relevant information. Such *informational sufficiency* is implicitly assumed in any VAR application.

But is this assumption always sensible? Unfortunately the answer is no. The basic problem is that, while agents typically have access to rich information, VAR techniques allow to handle a limited number of variables. If the econometrician's information set does not span that of the agents the structural shocks are non-fundamental and cannot be obtained from a VAR (Hansen and Sargent, 1991, Lippi and Reichlin, 1993, 1994, Chari, Kehoe and Mcgrattan, 2008). Fernandez-Villaverde, Rubio-Ramirez, Sargent and Watson (2007) shows theoretical cases in which VAR techniques fail. Fiscal foresight and news shocks are two important examples (Leeper, Walker and Yang, 2008, Yang, 2008, Forni and Gambetti, 2010, Forni, Gambetti and Sala, 2010, Gambetti, 2010).

At now there is no generally accepted and systematic way to verify whether a specific VAR suffers from this informational problem. In this paper we provide a testing procedure which is relatively easy to implement and valid under fairly general conditions. Moreover, we also we propose a strategy to amend the VAR when informational sufficiency is rejected.

Our main theoretical result is a necessary and sufficient condition for informational sufficiency, which is derived under the assumption that the economy admits a state space representation. Such condition is that there are no state variables that Granger cause the variables included in the VAR. The intuition is that the state variables convey all of the relevant information; therefore, if they do not help to predict a vector, such vector must contain the same information.

Based on this result, we suggest the following procedure. First, estimate the space spanned by the state variables of the economy by using the principal components of a large dataset, containing all available macroeconomic information. Second, test whether the estimated principal components Granger cause the variables included in the VAR. The variables are informationally sufficient if and only if the null hypothesis of no Granger causality is not rejected.

If a set of variables is not sufficient, we propose to estimate either a factor model, or a Factor Augmented VAR model (FAVAR), where the original set of variables is enlarged with the principal components above. Our test can be applied recursively to the FAVAR in order to determine how many factors to retain. The number of factors is the minimum number such that the extended vector is informationally sufficient. To our knowledge, this is the first method suitable for FAVAR models.

As an additional result, we show that, even if the VAR is not informational sufficient to recover all of the structural shocks, still a single shock of interest can be correctly identified and estimated. In order for this to be the case, a necessary condition is that the shock must be orthogonal to the past of the state variables. This result can be used to test for structuralness of a shock as follows. First, identify and estimate the shock. Then test for orthogonality between this shock and the lags of the principal components. If the null of orthogonality is rejected, then the shock obtained from the VAR cannot be structural.

As an application we study technology shocks in the US. We test whether a smallscale VAR model, such as those typically used to study the effects of technology shocks, is informationally sufficient. Specifically, we use a VAR with total factor productivity, the unemployment rate and per-capita hours worked. We find that these three variables are Granger caused by the first two principal components of a large dataset of US macroeconomic variables. Therefore we add such principal components to the VAR and show that the remaining principal components do not Granger cause the factor augmented VAR, meaning that the information conveyed in the FAVAR is sufficient. Finally, we identify the technology shock as the only one driving total factor productivity in the long run, in both the original and the augmented VAR. Differences in the results in the two models are dramatic. While in the original VAR technology shocks increase hours and reduce unemployment, in the augmented VAR results are reversed: hours reduce and unemployment increases. Consistently with the test outcome, adding further factors does not change results any more. In the augmented model, investment and GDP react very sluggishly to the shock, prices fall and the real wage increases. Overall the result are hard to reconcile with the view that technology shocks are an important source of business cycle fluctuations.

The remainder of the paper is organized as follows. Section 2 presents theoretical results, as well as our proposed testing procedures. Section 3 discusses the application. Section 4 concludes. Appendix A reports the proofs. Appendix B reports information about the data used in the empirical application.

2 Theory

2.1 The macroeconomy

Let us start from the following MA representation of the macroeconomy.

Assumption 1 (MA representation). The n-dimensional vector x_t of stationary macroeconomic time series satisfies

$$x_t = F(L)u_t,\tag{1}$$

where u_t is a q-dimensional, orthonormal white noise vector of structural macroeconomic shocks and F(L) is an $n \times q$ matrix of impulse response functions, i.e. squaresummable linear filters in the non-negative powers of the lag operator L, such that rank (F(z)) = q for some complex number z.

Representation (1) can be thought of as the representation of a macroeconomic equilibrium. Consider for instance the state-space representation studied in Fernandez-Villaverde *et al.* (2007), i.e.

$$s_t = As_{t-1} + Bu_t \tag{2}$$

$$x_t = Cs_{t-1} + Du_t \tag{3}$$

where s_t is an *r*-dimensional vector of stationary "state" variables, $q \leq r \leq n, A, B$, *C* and *D* are conformable matrices of parameters, *B* has a left inverse B^{-1} such that $B^{-1}B = I_q$. Pre-multiplying (2) by B^{-1} we get $u_t = B^{-1}(I - AL)s_t$. Substituting this into (3) and rearranging gives

$$x_t = (DB^{-1} + (C - DB^{-1}A)L)s_t.$$
(4)

Stationarity of s_t ensures invertibility of (2), so that $s_t = (I - AL)^{-1}Bu_t$. Combining this with (4) we get the MA representation

$$x_t = \left(DB^{-1} + (C - DB^{-1}A)L\right)(I - AL)^{-1}Bu_t,$$
(5)

which is a special case of (1).

The assumption on the rank of F(L) ensures that the representation is not redundant in the sense that there are no representations with a smaller number of shocks.

2.2 Sufficient information

Let us now define the information sets of the econometrician and the VAR, and the concept of sufficient information.

To begin, we assume that the SVAR econometrician observes x_t , possibly with error. Allowing for a measurement error (which can be zero), besides being an interesting generalization *per se*, will enable us to establish a link between the VAR model and the factor model introduced below, and extend our results to FAVAR models. Precisely:

Assumption 2. (Econometrician's information set) The econometrician information set \mathcal{X}_t^* is given by the closed linear space spanned by present and past values of the variables in x_t^* (in symbols $\mathcal{X}_t^* = \overline{\operatorname{span}}(x_{1t}^*, \dots, x_{nt}^*))$, where

$$x_t^* = x_t + \xi_t = F(L)u_t + \xi_t, \tag{6}$$

 ξ_t being a (possibly zero) vector of measurement errors, orthogonal to u_{jt-k} , $j = 1, \ldots, q$, any k, and ξ_{t-k} , k > 0.

In practice the number of observable variables n is very large, so that the econometrician needs to reduce it in order to estimate a VAR. The VAR information set is then spanned by an s-dimensional sub-vector of x_t^* , or more, generally, an s-dimensional linear combination of x_t^* , say $z_t^* = Wx_t^*$ (with s not necessarily equal to q). Considering also linear combinations will enable us to apply our results to the principal components of the variables and therefore to the FAVAR model.

Assumption 3 (VAR information set). The information set of the VAR is $\mathcal{Z}_t^* = \overline{\operatorname{span}}(z_{1t-k}^*, \ldots, z_{st-k}^*, k \ge 0)$, where $z_t^* = Wx_t^*$, W being $s \times n$.

Now, consider the theoretical projection equation of z_t^* on its past history, i.e.

$$z_t^* = P(z_t^* | \mathcal{Z}_{t-1}^*) + \epsilon_t.$$

$$\tag{7}$$

The SVAR methodology consists in (a) estimating a VAR to get ϵ_t ; (b) attempting to get the structural shocks as linear combinations of the estimated entries of ϵ_t . Hence a key property of z_t^* and the related information set is that the entries of ϵ_t span the structural shocks, i.e. the information in the history of z_t^* is sufficient to estimate the shocks. We call such property "sufficient information".

Definition 1 (Sufficient information). We say that z_t^* and the related information set \mathcal{Z}_t^* contain "sufficient information" if and only if there exist a matrix M such that $u_t = M \epsilon_t$.

It is important to stress that sufficiency, defined in this way, is related only to the variables in z_t^* and has nothing to do with the choice of a proper identification scheme. The correct identification of M is a further problem, which in general does make sense only if sufficiency holds true.

2.3 The relation with fundamentalness

Informational sufficiency is closely related to "fundamentalness". In this section we clarify the relation between the two concepts.¹

From (6) and the definition of z_t^* we get

$$z_t^* = WF(L)u_t + W\xi_t = z_t + W\xi_t.$$
 (8)

¹Some important references about fundamentalness are Hansen and Sargent (1991), Lippi and Reichlin (1993, 1994), Chari, Kehoe and McGrattan (2008), Fernandez-Villaverde *et al.* (2007).

Definition 2 (Fundamentalness). We say that u_t is fundamental for $w_t = Hx_t$, and the MA representation $w_t = HF(L)u_t$ is fundamental, if and only if $u_t \in W_t = \overline{\text{span}}(w_{1t-k}, \ldots, w_{mt-k}, k \ge 0)$ (i.e. $\mathcal{U}_t = \overline{\text{span}}(u_{1t-k}, \ldots, u_{qt-k}, k \ge 0) = \mathcal{W}_t$).

The following proposition formally establishes the relation between fundamentalnes and sufficiency.

Proposition 1. The information in z_t^* is sufficient if and only if there is a matrix R such that (a) $\tilde{z}_t = Rz_t^* = Rz_t$ and (b) u_t is fundamental for \tilde{z}_t .

For the proof see Appendix A. Proposition 1 says that, for z_t^* being sufficient, there must be a linear transformation of z_t^* which is free of measurement errors and have a fundamental representation in the structural shocks. Therefore, informational sufficiency is almost equivalent to fundamentalness plus absence of errors. If errors are small, informational sufficiency and fundamentalness essentially coincide; if, on the contrary, a VAR includes variables with large errors, information may be insufficient even if fundamentalness of z_t is met.

To conclude this subsection, let us observe that, in the particular case of F(L) being a matrix of rational functions, fundamentalness of u_t for w_t , along with fundamentalness of the associated MA representation $w_t = HF(L)u_t$ is equivalent to the following condition (see e.g. Rozanov, 1967, Ch. 2).

Condition R. The rank of HF(z) is q for all z such that |z| < 1.

Considering equation (5) and the case $w_t = x_t$, condition R is satisfied if and only if D is invertible and the eigenvalues of $A - BD^{-1}C$ are strictly less than one in modulus, which is Condition 1 of Villaverde *et al.* (2007).

2.4 Testable implications

Here we derive testable implications of sufficient information. A first relevant result is the following.

Proposition 2. If x_t^* Granger causes z_t^* , then z_t^* is not informationally sufficient.

For the proof see Appendix A. The intuition is that, if a set of variables is sufficient, than it contains all of the existing information, so that no other variable or set of variables can Granger cause it.

Proposition 2 can be useful in practice. In particular, if the econometrician believes that a given variable in x_t^* , say v_t , conveys relevant information, he can check whether v_t Granger causes z_t^* as a vector. If v_t Granger causes z_t^* , the VAR with z_t^* is misspecified.²

 $^{^{2}}$ Observe that, according to Proposition 2, identification is not required to perform the test, consistently with the fact that sufficient information, as observed above, is independent of the identification scheme.

However, Proposition 2 has an important limitation in that, being only a necessary condition, it can be used to reject sufficiency but not to validate it. Clearly, testing all of the variables in x_t^* would be close to a validation, but unfortunately this is not feasible, since in practice x_t^* is of high dimension. On the one hand, we cannot use all of the variables simultaneously; on the other hand, testing each one of them separately would yield, with very high probability, to reject sufficiency even if z_t^* is informationally sufficient, owing to Type I error.

We can provide a sufficient condition by assuming the state space representation above, i.e. by replacing Assumption 1 with the more restrictive Assumption 1':

Assumption 1' (ABCD representation). The vector x_t of macroeconomic time series satisfies equations (2) and (3).

It is easily seen from equations (6) and (4) that x_t^* follows the static factor model

$$x_t^* = Gf_t + \xi_t,\tag{9}$$

where $G = (DB^{-1} \quad C - DB^{-1}A)$ and $f_t = (s'_t \quad s'_{t-1})'$.

In addition to the above assumption, to derive the main result of the paper we need a condition ensuring that the dynamic rank of z_t^* is no less than q and that z_t^* is predictable to some extent. Precisely,

Assumption 4. There exists a summable sequence $\{c_k\}_{k=1}^{\infty}$ such that $R = W \sum_{k=1}^{\infty} c_k F_k$ has rank q.

The following proposition establishes a necessary and sufficient condition for informational sufficiency.

Proposition 3. Let K be any non-singular $p \times p$ matrix, p being the dimension of f_t . z_t^* is informationally sufficient if and only if $g_t = Kf_t$ does not Granger cause z_t^* .

For the proof see Appendix A. The intuition for sufficiency is that, under Assumption 1', the factors are informationally sufficient; therefore they Granger cause every predictable vector, unless such vector contain the same information.

Notice that assumption 4 rules out two cases. First, that z_t^* has a fundamental representation in a number of shocks less than q, e.g. $z_t^* = a(L)u_{1t}$, q > 1. In this case z_t^* is not Granger caused but it is obviously not informationally sufficient since it does not contain information about all the q shocks. Second, that the entries of z_t^* are contemporaneous linear combinations of the entries of u_t plus a measurement error. In this case z_t^* is not Granger caused since it is unpredictable, but is not informationally sufficient because of the measurement error.

Proposition 3 is useful in that, besides providing a sufficient condition, allows us to summarize the signals in the large dimensional vector x_t into a relatively small number of factors (the entries of g_t). Such factors are unobservable, but, under suitable assumptions, can be consistently estimated by the principal components \hat{g}_t , as both the number of variables and the number of time observations go to infinity (Stock and Watson, 2002b; Forni *et al.* 2009).

2.5 The testing procedure

Proposition 3 provides the theoretical basis for the following testing procedure.

- 1. Take a large data set x_t^* , capturing all of the relevant macroeconomic information.
- 2. Set a maximum number of factors P and compute the first P principal components of x_t^* .
- 3. Perform Granger causation tests to see whether the first h principal components, $h = 1, \ldots, P$, Granger cause z_t^* . If the null of no Granger causality is never rejected, z_t^* is informationally sufficient. Otherwise, sufficiency is rejected.

If informational sufficiency is rejected, we cannot use the VAR to identify all of the structural shocks. However, partial identification could still provide correct results, as shown in the following subsection.

2.6 Structuralness of a single shock

Even if informational sufficiency is rejected, z_t^* could be sufficient to get a single shock of interest, say u_{1t} , or a subset of shocks $u_{1t}, \ldots, u_{jt}, j < q$. This is important in that for many applications the econometrician is interested in identifying just a single shock.

To see this, consider the following example

$$z_{1t}^* = u_{1t} + u_{2t-1}$$
$$z_{2t}^* = u_{1t} - u_{2t-1}$$

In this case z_t^* is not sufficient for u_t by Proposition 1. In fact, since the determinant of the MA filter has a zero in zero, the MA representation is non-fundamental by Condition R. Indeed, it is easily seen that u_{2t} cannot be recovered from the present and the past of z_t^* . Nevertheless, z_t^* is sufficient for u_{1t} , since $z_{1t}^* + z_{2t}^* = 2u_{1t}$.

By Assumption 1 the structural shocks are unpredictable, i.e. u_{jt} , $j = 1, \ldots, q$ is orthogonal to x_{t-k}^* , k > 0, and the lagged factors f_{t-k} , k > 0. Therefore, after having identified the shock of interest, we can verify whether it can be a structural shock by testing for orthogonality with respect to the past of the principal components. If orthogonality is rejected the shock cannot be a structural shock.³

Let us stress however that orthogonality is only a necessary condition for structuralness. Hence even if it is not rejected, it is safer to change the information set as suggested below.

2.7 Amending the VAR information set

What should the econometrician do if sufficient information is rejected? Assumption 1' guarantees that $g_t = K f_t$ is informationally sufficient. Hence a possible solution is to estimate a VAR with the principal components \hat{g}_t and use it to estimate the whole factor model (9) along the lines of Forni *et al.* (2009).

Alternatively, we can extend the vector of variables appearing in the original VAR (or some of them) by adding principal components and estimate a FAVAR model. To this end, a crucial problem is to establish the number of factors to include.

Since by Assumption 3 also linear combinations of the x^* 's can be included in the vector z_t^* , our testing procedure can be applied to the FAVAR model to see whether it is informationally sufficient or not. Moreover, it can be used to determine the number of factors. The idea is to add the principal components one at a time in decreasing order, apply recursively the Granger causation test and stop when informational sufficiency is no longer rejected. Precisely, we propose the following procedure.

- 1. Take $w_t^h = (z_t^{*'} \ \hat{g}_{1t} \ \cdots \ \hat{g}_{ht})'$ and test for sufficiency of w_t^h as explained above, for $h = 1, \ldots, P$.
- 2. Retain p principal components if w_t^p is informationally sufficient whereas w_t^1, \ldots, w_t^{p-1} are not.

Note that existing information criteria, like Bai and Ng (2002) or Onatski (2010), being designed for pure unobservable factor models, are ill-suited for the FAVAR framework. In particular, the number of principal components needed in a FAVAR model may be smaller than the number of principal components needed in a factor model, since valuable information is already provided by the variables in z_t^* . To our knowledge, this is the first method specifically designed for FAVAR models.

The approach also allows the econometrician to consistently estimate the impulse response functions of all the x's. In fact, the x's are linear combinations of the factors (see equation (9)) and therefore, if w_t^p is informationally sufficient, are also linear

³Ramey (2009) applies a version of this test to check whether the fiscal policy shock obtained with a SVAR \acute{a} la Perotti (2007) is structural. She however does not use the principal components, but the forecast of public expenditure from the survey of professional forecasters.

combinations of the entries of w_t^p , say $x_t = Qw_t^p$. Hence the responses of x_t can be estimated as $\hat{Q}\hat{B}(L)$ where the entries of \hat{Q} are the coefficients of the OLS projection of x_t^* on w_t^p and the entries of $\hat{B}(L)$ are the estimated impulse response functions of the enlarged VAR. In addition, a key implication is that the shocks of interest can be identified by imposing restrictions on variables which are not included in the VAR. This is very useful since restrictions on the principal components would be rather difficult to interpret.

2.8 Relations with the literature

Our work is closely related to Forni and Reichlin (1996), Giannone and Reichlin (2006), Forni, Giannone, Lippi and Reichlin (2009), Fernandez-Villaverde *et al.* (2007) and the FAVAR literature originated by Bernanke *et al.* (2005).

Fernandez-Villaverde *et al.* (2007) derive a necessary and sufficient condition for fundamentalness; our condition is different in that it can be tested without resorting to any particular economic model.

Forni and Reichlin (1996) and Giannone and Reichlin (2006) derive a necessary condition essentially equivalent to Proposition 2 above; Giannone and Reichlin (2006) propose a Granger causality test based on it. The problem of this test is that, being based on a necessary condition, it is not conclusive if the null is not rejected. Moreover, its general applicability is limited by the fact that there is no indication about which variables to use. The crucial novelty with respect to the above work is then the sufficiency result in Proposition 3 and the related identification of a set of regressors for the Granger causality test.

Forni *et al.* (2009) propose an informal way to check for fundamentalness by looking at the roots of the determinant of the matrix of impulse-response functions obtained by estimating a factor model. The shortcoming of this method is that it checks for sufficiency of the common components of the variables, rather than the variables themselves; hence results are reliable only if the idiosyncratic component is small.

Finally, our contribution with respect to the FAVAR literature is twofold. On the one hand, we explicitly show a results which was so far only conjectured in literature, namely that the FAVAR model may solve the non-fundamentalness problem. On the other hand, we provide a procedure to check whether a given FAVAR is informationally sufficient and determine the number of factors.

3 An Application to Technology Shocks

3.1 Technology shocks and the business cycle

Do technology shock explain aggregate fluctuations? Despite the huge amount of works that have addressed this question over the last years, no consensus has been reached. The empirical evidence is mixed. In his seminal paper, Gali (1999) finds a very modest role for technology shocks as a source of economic fluctuations. The result echoes the finding in Blanchard and Quah (1989) that aggregate supply shocks are not important for the business cycle. On the contrary other authors, see for instance Christiano, Eichenbaum and Vigfusson (2003) and Beaudry and Portier (2006), provide evidence that technology shocks are capable of generating sizable fluctuations in macroeconomic aggregates.

Most of the existing evidence about the effects of technology shocks is obtained using small-scale VAR models. In many cases only two or three variables are used. Here, as an application of our testing procedure, we investigate whether a small scale model conveys enough information to identify the shocks, in particular the technology shock.

We consider the vector z_t^* including the growth rate of total factor productivity (TFP_t) , the unemployment rate (u_t) and the logs of per capita hours worked (h_t) . The space spanned by the state variables of the economy is estimated by using the principal components of a large dataset of US macroeconomic variables.⁴

3.2 Testing for informational sufficiency

We apply our testing procedure to this VAR. We use the Gelper and Croux (2007) multivariate extension of the out-of-sample Granger causality test proposed by Harvey *et al.*(1998).

Table 1 shows the results. The first column of panel A shows the p-value of the test of the null hypothesis that the first principal component does not Granger cause z_t^* . The hypothesis is strongly rejected suggesting that the three variables do not contain sufficient information to correctly recovering the structural shocks. The second column of A shows the p-values of the test of the null hypothesis that the VAR augmented by the first principal component, i.e. $w_t^1 = (z'_t \ \hat{g}_{1t})'$, is not Granger caused by the remaining principal components from the second to the *j*-th, $j = 2, \ldots, P$. For instance the third element of the column, i.e. 0.405, is the p-value obtained by testing that $(\hat{g}_{2t} \ \hat{g}_{3t})'$ does not Granger cause w_t^1 . We reject that the principal components from the second up to the eleventh do not Granger cause w_t^1 at the 5% level, suggesting

⁴See Appendix B for the precise definition and the treatment of the variables used in the dataset.

that not even w_t^1 is informationally sufficient. However we can not reject that w_t^2 is informationally sufficient since it is never Granger caused by the remaining principal components. Augmenting z_t^* with the first two principal components is sufficient to obtain the structural shocks, including the technology shock.

3.3 Testing for structuralness of the technology shock

As observed in subsection 2.6, even if the VAR is not informationally sufficient, still it could be possible to identify the technology shock. To check whether this is the case, we identify the technology shock, following Beaudry and Portier (2006), as the only one affecting total factor productivity in the long run. Then we test whether the shock is orthogonal to the past of the estimated principal components. Precisely, we run a regression of the estimated shock on the lagged principal components and perform an F-test of the null hypothesis that the coefficients are jointly zero. The first column of B in Table 1 displays the p-value of the test when only the first principal component is included as a regressor. The hypothesis is strongly rejected suggesting that the shock obtained from the original VAR is not structural.

Then we implement the same identification in the VARs for w_t^1 and w_t^2 and run the same orthogonality test. The second column reports the p-values for w_t^1 . The null that the second principal component does not predict the shock is rejected at the 10% but not the 5% level. The hypothesis that the shock is orthogonal to the principal components from the second up to the eighth is strongly rejected. Finally, orthogonality is never rejected for the w_t^2 specification, consistently with the results of panel A.

3.4 Information and impulse response functions

Next we study the consequences of insufficient information in terms of impulse response functions. In particular, we investigate to what extent the effects of technology shocks change by augmenting the original VAR with the principal components. According to the results of the test, impulse response functions are expected to change when adding the first two principal components, but should remain essentially unchanged when adding further components.

Figure 1 shows the impulse response functions. The left column plots the impulse response functions for the three varables, total factor productivity, unemployment and per capita hours, for all the sixteen specifications $z_t^*, w_t^1, \ldots, w_t^{15}$. The solid line with dots represents the impulse response functions estimated with z_t^* . The line with crosses represents the impulse response functions estimated with w_t^2 . The remaining lines are the estimated responses of the other models. The effects are expressed in percentage terms. The right column displays for the three variables the impact effect (dots),

the effect at 1 year (crosses), 2 years (circles) and in the long run (diamonds). The horizontal axis displays the number of principal components included in the VAR.

The VAR without principal components predicts that the technology shock increases per-capita hours worked and reduces unemployment. Such results are in line with the theoretical predictions of standard RBC models and the empirical findings of Christiano, Eichenbaum and Vigfusson (2003) and Beaudry and Portier (2006). Total factor productivity reacts positively on impact and stays roughly constant afterward, with no delay in the diffusion process.

The picture changes dramatically when adding the principal components. The effects on both unemployment and hours change sign. Now, unemployment increases and hours reduce so that technology becomes contractionary. Moreover, the impact effect of productivity reduces substantially while the long run effect is roughly unchanged so that the diffusion process is substantially slower in line with the S-shape view and the recent news shocks literature (Beaudry and Portier, 2006, and Schmitt-Grohe and Uribe, 2008).

Notice that, consistently with the results of the test, models including more than two principal components, all deliver the same impulse response functions. This can also be seen from the right panels of Figure 1. Impulse response functions change radically by adding the first principal component, and to a lesser extent by adding the second one, but are roughly constant from that point onward.

Figure 2 plots the impulse response functions of some variables of interest for the specification w_t^2 . The solid line represents the point estimate while the dotted lines are the 68% confidence bands. Investment and GDP do not react significantly on impact and start to increase significantly only after a few quarters, reaching their maximal level after about two years. The shape of the response of consumption is similar to that of investment and GDP (although the impact effect is slightly negative). The GDP deflator reduces immediately while real wages immediately increase.

Overall the picture that emerges is hard to reconcile with the view that technology shocks are an important source of business cycle fluctuations.

4 Conclusions

This paper derives necessary and sufficient conditions under which a set of variables is *informationally sufficient*, i.e. contains enough information to estimate the structural shocks with a VAR model. Based on such conditions, a procedure to test for informational sufficiency is proposed. Moreover, a test is provided to verify whether a single shock obtained with partial identification is a structural shock. Finally, the paper shows how to amend the model if informational sufficiency is rejected. The idea is to estimate a FAVAR, where the number of factors is determined by applying recursively the sufficient information test.

Our testing procedures are applied to a three-variable VAR including TFP, unemployment and per-capita hours worked. It is found that the VAR is not informationally sufficient, and the technology shock, identified as the only one affecting TFP in the long run, is not a structural shock. When amending the model by adding missing information, informational sufficiency and structuralness cannot be rejected. Results in terms of impulse response functions change dramatically: the reaction of both unemployment and hours worked changes sign, so that a positive shock becomes contractionary, and the response of TFP becomes S-shaped, in accordance with the recent "news" shock literature.

Appendix A: Proofs

Proof of Proposition 1. Sufficiency of (a) and (b) is obvious. As for necessity, let us assume that z_t^* is sufficient. Then from equation (6) and Assumption 3 we have $P(z_t^*|\mathcal{Z}_{t-1}^*) = \sum_{k=1}^{\infty} F_k u_{t-k}$ and $\epsilon_t = WF_0 u_t + W\xi_t$, because $u_{t-k} \in \mathcal{Z}_{t-1}^*$ and both u_t and ξ_t are orthogonal to \mathcal{Z}_{t-1}^* . Since $u_t = M\epsilon_t$, $MW\xi_t = 0$, so that the variancecovariance matrix of $W\xi_t$, say Σ , has rank $l \leq s - q$. Hence $\Sigma = Q'Q$, Q being a $l \times s$ matrix of rank l. Let S be the l-dimensional subspace of \mathbf{R}^s spanned by the rows of Q and R be any $s - l \times s$ matrix whose rows span the orthogonal complement of S. Clearly $RW\xi_t = 0$ and $\tilde{z}_t = Rz_t^* = Rz_t$. Notice also that RWF_0 has a left inverse N, so that $u_t = NR\epsilon_t = M\epsilon_t$, since otherwise u_t cannot be a linear transformation of ϵ_t , contrary to the informational sufficiency assumption.

Coming to (b), we want to show that $R\epsilon_t$ is the residual of the projection of \tilde{z}_t onto its own past, so that $u_t = NR\epsilon_t \in \tilde{Z}_t = \overline{\operatorname{span}}(\tilde{z}_{jt-k}, j = 1, \ldots, s - l, k \geq 0)$. Let $\bar{z}_t = Qz_t^*$ and P_K be the projection of \tilde{z}_t on the lags $1, \ldots, K$ of \tilde{z}_t and \bar{z}_t , i.e. $P_K = P(\tilde{z}_t | \tilde{z}_{jt-k}, \bar{z}_{ht-k}, j = 1, \ldots, s - l, h = 1, \ldots, l, k = 1, \ldots, K)$. First, observe that z_t^* is a linear combination of the entries of \tilde{z}_t and \bar{z}_t , since the matrix (R' Q')' is nonsingular by construction. Hence $P_K \to P(\tilde{z}_t | Z_{t-1}^*) = R \sum_{k=1}^{\infty} F_k u_{t-k}$ in mean square as $K \to \infty$. On the other hand, $P_K = C_K(L)\tilde{z}_{t-1} + D_K(L)Qz_{t-1} + D_K(L)QW\xi_{t-1}$. Therefore the latter term must go to zero in mean square as $K \to \infty$. But $QW\xi_{t-1}$ is white noise by Assumption 2 and its variance-covariance matrix $Q\Sigma Q'$ is non-singular by construction, so that all entries of $D_K(L)$ go to zero in sum of squares as $K \to \infty$. But then the latter projection is equal to $R\sum_{k=1}^{\infty} F_k u_{t-k}$, and the corresponding residual is $R\epsilon_t$.

Proof of Proposition 2. Assume that z_t^* is sufficient, so that $u_t = M\epsilon_t$. Then $u_{jt-k} \in \mathcal{Z}_{t-1}^*$ for k > 0. It follows that $P(z_t^* | \mathcal{Z}_{t-1}^*) = W(F_1 u_{t-1} + F_2 u_{t-2} + ...)$ and $\epsilon_t = WF_0 u_t + W\xi_t$. Hence ϵ_t is orthogonal to both u_{t-k} , k > 0, and, by serial uncorrelation of ξ_t (Assumption 2), ξ_{t-k} , k > 0. Therefore $\epsilon_t \perp x_{t-k}^*$, k > 0 and x_t^* does not Granger cause z_t^* .

Proof of Proposition 3. Let us assume that z_t^* is sufficient, i.e. $u_t = M\epsilon_t$. Then ϵ_t is orthogonal to u_{t-k} , k > 0 and therefore to g_{t-k} , k > 0. Hence $P(z_t^*|\mathcal{Z}_{t-1}^*) = P(z_t^*|z_{jt-k}^*, g_{it-k}, j = 1, \ldots, s, i = 1, \ldots, p, k > 0)$, so that g_t does not Granger cause z_t^* . Regarding the opposite implication, let us assume that g_t does not Granger cause z_t^* . We have $P(z_t^*|\mathcal{Z}_{t-1}^*) = P(z_t^*|z_{jt-k}^*, g_{it-k}, j = 1, \ldots, s, i = 1, \ldots, p, k > 0)$. But the latter projection is equal to $P(z_t^*|u_{jt-k}, j = 1, \ldots, q, k > 0) = W \sum_{k=1}^{\infty} F_k u_{t-k} = \zeta_t$, since ζ_t belongs to $\overline{\text{span}}(z_{jt-k}^*, g_{it-k}, j = 1, \ldots, s, i = 1, \ldots, p, k > 0)$ and $z_t^* - \zeta_t$ is orthogonal to such space because of Assumption 2. On the other hand, $\zeta_t =$

$$\begin{split} P(z_t^* | \mathcal{Z}_{t-1}^*) &= \sum_{k=1}^{\infty} A_k \epsilon_{t-k}. \text{ Projecting both sums on } \overline{\text{span}}(\epsilon_{it-k}, u_{it-k}, i = 1, \dots, s, j = 1, \dots, r) \text{ we get } WF_k u_{t-k} &= A_k \epsilon_{t-k} \text{ for all } k, \text{ so that } WF_k u_t = A_k \epsilon_t \text{ for all } k \text{ and } V u_t = (W\sum_{k=1}^{\infty} c_k F_k) u_t = (\sum_{k=1}^{\infty} c_k A_k) \epsilon_t \text{ for any sequence } c_k, k = 1, \dots, \infty. \text{ Assumption } 4 \text{ ensures that } V \text{ has a left inverse } V^*, \text{ so that } u_t = V^* (\sum_{k=1}^{\infty} c_k A_k) \epsilon_t. \end{split}$$

Appendix B: Data

Transformations: 1=levels, 2= first differences of the original series, $4 = \log s$ of the original series, 5 = first differences of the logs of the original series .

no.series	Transf.	Mnemonic	Long Label			
1	5	GDPC1	Real Gross Domestic Product, 1 Decimal			
2	5	GNPC96	Real Gross National Product			
3	5	NICUR/GDPDEF	National Income/GDPDEF			
4	5	DPIC96	Real Disposable Personal Income			
5	5	OUTNFB	Nonfarm Business Sector: Output			
6	5	FINSLC1	Real Final Sales of Domestic Product, 1 Decimal			
7	5	FPIC1	Real Private Fixed Investment, 1 Decimal			
8	5	PRFIC1	Real Private Residential Fixed Investment, 1 Decimal			
9	5	PNFIC1	Real Private Nonresidential Fixed Investment, 1 Decimal			
10	5	GPDIC1	Real Gross Private Domestic Investment, 1 Decimal			
11	5	PCECC96	Real Personal Consumption Expenditures			
12	5	PCNDGC96	Real Personal Consumption Expenditures: Nondurable Goods			
13	5	PCDGCC96	Real Personal Consumption Expenditures: Durable Goods			
14	5	PCESVC96	Real Personal Consumption Expenditures: Services			
15	5	GPSAVE/GDPDEF	Gross Private Saving/GDP Deflator			
16	5	FGCEC1	Real Federal Consumption Expenditures & Gross Investment, 1 Decimal			
17	5	FGEXPND/GDPDEF	Federal Government: Current Expenditures/ GDP deflator			
18	5	FGRECPT/GDPDEF	Federal Government Current Receipts/ GDP deflator			
19	2	FGDEF	Federal Real Expend-Real Receipts			
20	1	CBIC1	Real Change in Private Inventories, 1 Decimal			
21	5	EXPGSC1	Real Exports of Goods & Services, 1 Decimal			
22	5	IMPGSC1	Real Imports of Goods & Services, 1 Decimal			
23	5	CP/GDPDEF	Corporate Profits After Tax/GDP deflator			
24	5	NFCPATAX/GDPDEF	Nonfinancial Corporate Business: Profits After Tax/GDP deflator			
25	5	CNCF/GDPDEF	Corporate Net Cash Flow/GDP deflator			
26	5	DIVIDEND/GDPDEF	Net Corporate Dividends/GDP deflator			
27	5	HOANBS	Nonfarm Business Sector: Hours of All Persons			
28	5	OPHNFB	Nonfarm Business Sector: Output Per Hour of All Persons			
29	5	UNLPNBS	Nonfarm Business Sector: Unit Nonlabor Payments			
30	5	ULCNFB	Nonfarm Business Sector: Unit Labor Cost			
31	5	WASCUR/CPI	Compensation of Employees: Wages & Salary Accruals/CPI			
32	1	COMPNFB	Nonfarm Business Sector: Compensation Per Hour			
33	5	COMPRNFB	Nonfarm Business Sector: Real Compensation Per Hour			
34	1	GDPCTPI	Gross Domestic Product: Chain-type Price Index			
35	1	GNPCTPI	Gross National Product: Chain-type Price Index			
36	1	GDPDEF	Gross Domestic Product: Implicit Price Deflator			
37	1	GNPDEF	Gross National Product: Implicit Price Deflator			

no.series	Transf.	Mnemonic	Long Label			
38	5	INDPRO	Industrial Production Index			
39	5	IPBUSEQ	Industrial Production: Business Equipment			
40	5	IPCONGD	Industrial Production: Consumer Goods			
41	5	IPDCONGD	Industrial Production: Durable Consumer Goods			
42	5	IPFINAL	Industrial Production: Final Products (Market Group)			
43	5	IPMAT	Industrial Production: Materials			
44	5	IPNCONGD	Industrial Production: Nondurable Consumer Goods			
45	1	AWHMAN	Average Weekly Hours: Manufacturing			
46	1	AWOTMAN	Average Weekly Hours: Overtime: Manufacturing			
47	2	CIVPART	Civilian Participation Rate			
48	5	CLF16OV	Civilian Labor Force			
49	5	CE16OV	Civilian Employment			
50	5	USPRIV	All Employees: Total Private Industries			
51	5	USGOOD	All Employees: Goods-Producing Industries			
52	5	SRVPRD	All Employees: Service-Providing Industries			
53	5	UNEMPLOY	Unemployed			
54	1	UEMPMEAN	Average (Mean) Duration of Unemployment			
55	1	UNRATE	Civilian Unemployment Rate			
56	5	HOUST	Housing Starts: Total: New Privately Owned Housing Units Started			
57	1	FEDFUNDS	Effective Federal Funds Rate			
58	1	TB3MS	3-Month Treasury Bill: Secondary Market Rate			
59	1	GS1	1-Year Treasury Constant Maturity Rate			
60	1	GS10	10-Year Treasury Constant Maturity Rate			
61	1	AAA	Moody's Seasoned Aaa Corporate Bond Yield			
62	1	BAA	Moody's Seasoned Baa Corporate Bond Yield			
63	1	MPRIME	Bank Prime Loan Rate			
64	5	BOGNONBR	Non-Borrowed Reserves of Depository Institutions			
65	5	TRARR	Board of Governors Total Reserves, Adjusted for Changes in Reserve			
66	5	BOGAMBSL	Board of Governors Monetary Base, Adjusted for Changes in Reserve			
67	5	M1SL	M1 Money Stock			
68	5	M2MSL	M2 Minus			
69	5	M2SL	M2 Money Stock			
70	5	BUSLOANS	Commercial and Industrial Loans at All Commercial Banks			
71	5	CONSUMER	Consumer (Individual) Loans at All Commercial Banks			
72	5	LOANINV	Total Loans and Investments at All Commercial Banks			
73	5	REALLN	Real Estate Loans at All Commercial Banks			
74	5	TOTALSL	Total Consumer Credit Outstanding			
75	5	CPIAUCSL	Consumer Price Index For All Urban Consumers: All Items			
76	5	CPIULFSL	Consumer Price Index for All Urban Consumers: All Items Less Food			
77	5	CPILEGSL	Consumer Price Index for All Urban Consumers: All Items Less Energy			

no.series	Transf.	Mnemonic	Long Label			
78	5	CPILFESL	Consumer Price Index for All Urban Consumers: All Items Less Food & Energy			
79	5	CPIENGSL	Consumer Price Index for All Urban Consumers: Energy			
80	5	CPIUFDSL	Consumer Price Index for All Urban Consumers: Food			
81	5	PPICPE	Producer Price Index Finished Goods: Capital Equipment			
82	5	PPICRM	Producer Price Index: Crude Materials for Further Processing			
83	5	PPIFCG	Producer Price Index: Finished Consumer Goods			
84	5	PPIFGS	Producer Price Index: Finished Goods			
85	5	OILPRICE	Spot Oil Price: West Texas Intermediate			
86	5	USSHRPRCF	US Dow Jones Industrials Share Price Index (EP) NADJ			
87	5	US500STK	US Standard & Poor's Index if 500 Common Stocks			
88	5	USI62F	US Share Price Index NADJ			
89	5	USNOIDN.D	US Manufacturers New Orders for Non Defense Capital Goods (BCI 27)			
90	5	USCNORCGD	US New Orders of Consumer Goods & Materials (BCI 8) CONA			
91	1	USNAPMNO	US ISM Manufacturers Survey: New Orders Index SADJ			
92	5	USVACTOTO	US Index of Help Wanted Advertising VOLA			
93	5	USCYLEAD	US The Conference Board Leading Economic Indicators Index SADJ			
94	5	USECRIWLH	US Economic Cycle Research Institute Weekly Leading Index			
95	1	GS10-FEDFUNDS				
96	1	GS1-FEDFUNDS				
97	1	BAA-FEDFUNDS				
98	5	GEXPND/GDPDEF	Government Current Expenditures/ GDP deflator			
99	5	GRECPT/GDPDEF	Government Current Receipts/ GDP deflator			
100	2	GDEF	Government Real Expend-Real Receipts			
101	5	GCEC1	Real Government Consumption Expenditures & Gross Investment, 1 Decimal			
102	1		Fernald's TFP growth CU adjusted			
103	1		Fernald's TFP growth			
104	5		DOW JOONES/GDP DEFL			
105	5		S&P/GDP DEFL			
106	1		Fernald's TFP growth - Investment			
107	1		Fernald's TFP growth - Consumption			
108	1		Fernald's TFP growth CU - Investment			
109	1		Fernald's TFP growth CU - Consumption			
110	1		Personal Finance Current			
111	1		Personal Finance Expected			
112	1		Business Condition 12 Months			
113	1		Business Condition 5 Years			
114	1		Buying Conditions			
115	1		Consumer's sentiment: Current Index			
116	1		Consumer's sentiment: Expected Index			
117	4		Per-capita hours worked (HOANBS/Civilian Polulation 16 and over)			

References

- Bai, J., and S. Ng (2002). Determining the number of factors in approximate factor models, Econometrica 70, 191-221.
- [2] Beaudry, P. and F. Portier (2006). Stock Prices, News, and Economic Fluctuations American Economic Review, 96(4): 1293-1307.
- [3] Bernanke, B. S., J. Boivin and P. Eliasz (2005). Measuring Monetary Policy: A Factor Augmented Autoregressive (FAVAR) Approach, The Quarterly Journal of Economics 120, 387-422.
- [4] Blanchard, O. J., and D. Quah (1989). The Dynamic Effects of Aggregate Demand and Supply Disturbances. American Economic Review, 79(4): 655-73
- [5] Chari, V.V., Kehoe, P.J. and E.R. McGrattan (2008). Are structural VARs with long-run restrictions useful in developing business cycle theory? Journal of Monetary Economics 55: 1337-1352.
- [6] Christiano, L., M. Eichenbaum and R. Vigfusson (2003), "What Happens After a Technology Shock?", NBER Working Papers 9819.
- [7] Fernandez-Villaverde J., J.F. Rubio-Ramirez, T.J. Sargent and M.W. Watson (2007). ABCs (and Ds) of Understanding VARs. American Economic Review, American Economic Association, 97(3):1021-1026.
- [8] Forni, M. and L. Gambetti (2010). Fiscal Foresight and the Effects of Government Spending, CEPR DP series no. 7840.
- [9] Forni, Gambetti, L. and L. Sala (2010). News shocks do not drive the business cycle. Mimeo
- [10] Forni, M., D. Giannone, M. Lippi and L. Reichlin (2009). Opening the Black Box: Structural Factor Models with Large Cross-Sections, Econometric Theory 25, 1319-1347.
- [11] Forni M. and L., Reichlin (1996). Dynamic Common Factors in Large Cross-Sections, Empirical Economics 21, 27-42.
- [12] Galí, J. (1999), "Technology, Employment and the Business Cycle: Do Technology Shocks Explain Aggregate Fluctuations?", American Economic Review, 89(1): 249-271.
- [13] Gambetti, L. (2010). Fiscal Policy, Foresight and the Trade Balance in the U.S. UFAE and IAE Working Papers 852.10.

- [14] Gelper, S. and C. Croux (2007), Multivariate out-of-sample tests for Granger causality, Computational Statistics & Data Analysis 51, 3319-3329.
- [15] Giannone, D., and L. Reichlin (2006). Does Information Help Recovering Structural Shocks from Past Observations? Journal of the European Economic Association, 4(2-3), 455465.
- [16] Hansen, L.P., and T.J. Sargent (1991). Two problems in interpreting vector autoregressions. In Rational Expectations Econometrics, L.P. Hansen and T.J. Sargent, eds. Boulder: Westview, pp.77-119.
- [17] Harvey, D. I., Leybourne, S. J. and P. Newbold, (1998) Tests for Forecast Encompassing, Journal of Business & Economic Statistics 16, 254-259.
- [18] Leeper, E.M., Walker, T.B. and S.S. Yang (2008). Fiscal Foresight: Analytics and Econometrics, NBER Working Paper No. 14028.
- [19] Leeper, E.M., Walker, T.B. (2009). Information Flows and News Driven Business Cycles, mimeo Indiana University
- [20] Lippi, M. and L. Reichlin (1993). The Dynamic Effects of Aggregate Demand and Supply Disturbances: Comment, American Economic Review 83, 644-652.
- [21] Lippi, M. and L. Reichlin (1994). VAR analysis, non fundamental representation, Blaschke matrices, Journal of Econometrics 63, 307-325.
- [22] Onatski, A. (2010). Determining the Number of Factors from Empirical Distribution of Eigenvalues, The Review of Economics and Statistics 92, 1004-1016.
- [23] Rozanov, Yu. (1967). Stationary Random processes. San Francisco: Holden Day
- [24] Schmitt-Grohe, S and M., Uribe (2008). What's News in Business Cycles, NBER WP 14215
- [25] Sims, C. A. (1980). Macroeconomics and Reality. Econometrica, 48(1), 148.
- [26] Stock, J.H. and M.W. Watson (2002a). Macroeconomic Forecasting Using Diffusion Indexes, Journal of Business and Economic Statistics 20, 147-162.
- [27] Stock, J.H. and M.W. Watson (2002b). Forecasting Using Principal Components from a Large Number of Predictors, Journal of the American Statistical Association 97, 1167-1179.
- [28] Yang, S.S. (2008). Quantifying tax effects under policy foresight. Journal of Monetary Economics, 52(8):1557-1568.

Tables

		Δ		B		
i	~*	21 au1	au ²	~*	u^1	²
J	z_t	w_t	<i>w</i> _t	z_t	w_t	w_t
1	0.000	—	-	0.005	_	—
2	-	0.480	-	_	0.055	_
3		0.405	0.475	_	0.113	0.977
4	-	0.620	0.375	—	0.091	0.452
5	—	0.125	0.250	—	0.115	0.581
6	_	0.105	0.500	_	0.142	0.641
7	_	0.125	0.545	_	0.126	0.186
8	—	0.285	0.785	—	0.027	0.197
9	_	0.125	0.705	_	_	0.216
10	_	0.085	0.450	_	_	0.207
11	—	0.050	0.660	—	—	0.148
12	_	—	0.355	_	_	0.186
13	-	_	0.395	-	_	0.239
14	-	_	0.560	-	-	0.279
15	-	_	0.720	-	—	0.337

Table 1: p-values A: Test for informational sufficiency B: Test for structuralness of
the technology shock.

Figures



Figure 1: Impulse response functions



Figure 2: Impulse response functions.

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