SHOULD UI BENEFITS REALLY FALL OVER TIME?

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Abstract

The issue of whether unemployment benefits should increase or decrease over the unemployment spell is analyzed in an analytically tractable model allowing moral hazard, adverse selection and hidden savings. Analytical results show that when the search productivity of unemployed is constant over the unemployment spell, benefits should typically increase or be constant. The only exception is when there is moral hazard and no hidden savings. In general, adverse selection problems calls for increasing benefits, moral hazard problems for constant benefits and decreasing search productivity for decreasing benefits.

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1 Introduction

The seminal paper by Shavell and Weiss (1979) characterized the optimal design of unemployment insurance (UI) when search activity is unobservable. Under such a moral hazard problem, they concluded that unemployment benefits should decline over the period the individual remains unemployed. Intuitively, the fear of lower future consumption increases the incentives for engaging in costly search, since active search reduces the probability of having to endure this future lower consumption.

Much more recently, this analysis was extended by Hopenhayn and Nicolini (1997), allowing the insurer to control the consumption profile also for working individuals, using a history dependent wage tax. They confirm the previous results that optimal unemployment benefits should decline over time. A key assumption in both papers is that the insurer can fully control the individual's consumption – usually interpreted as the individual having no access to markets for saving and borrowing and no alternative sources of income. A large part of the welfare gains from introducing the optimal UI plan is due to the insurer acting as a substitute bank *vis-à-vis* the individual. More importantly, there are *a priori* reasons for believing the assumption that consumption can be fully controlled by the insurer to be important for the result that UI benefits should decrease over time. First, it is well known that precautionary savings are a good substitute for insurance against short spells of unemployment (see, e.g. Hassler and Rodríguez Mora (1999)). Second, when individuals self-insure by building precautionary buffers, consumption follows a profile qualitatively similar to the optimal path in Hopenhayn and Nicolini (1997) – i) falling during unemployment as the buffer is depleted and ii) lower consumption paths for individuals with a history of many and long unemployment spells.

There is empirical evidence indicating that precautionary saving is used in order to self-insure against unemployment risk. Using PSID, Gruber (1997) finds that, in absence of UI, consumption falls by 22% when an individual become unemployed, showing that individuals are able to smooth consumption also when there is no UI. Similarly, Engen and Gruber (2001) shows that UI crowds out financial savings, indicating that households use the financial markets for self-insure against unemployment risk.[1] The assumption that the insurer can perfectly control individual consumption is thus not realistic. Furthermore, we will in this paper argue that neither is it innocuous.

In the literature on UI design, two main approaches have been used. With optimal contract theory, optimal UI design can be analyzed with few restrictions on the form of the insurance scheme. Despite obvious advantages in terms of generality, this framework also has some drawbacks. It has proven difficult to relax the assumption of no hidden savings, only recently progress has been made in this respect (see Pavoni (2001), Arpad and Pavoni (2002) and Werning (2002)).

[1] Also if access to the capital market is limited, alternative means to smooth consumption may exist, see e.g., Cullen and Gruber (2000).

Furthermore, it is inherently very difficult to handle multiple incentive problems and adverse selection using optimal contract theory. Numerical analysis is the second main approach and, e.g., Abdulkadiroglu, Kuruscu and Sahin (2002) show that benefits should not necessarily be decreasing while Heer (2000) reaches the opposite conclusion. Although numerical analysis allows more realistic models and quantitative predictions, it is typically difficult to understand the mechanisms behind the results and to make conjectures about their generality.

In this paper, we will follow a third route. We will restrict individual preferences to be in the class of constant absolute risk aversion and like, e.g., Fredriksson and Holmlund (2001) focus on two-tier benefit systems allowing different benefits for the short-run and long-run unemployed. In particular, while maintaining the assumption that consumption cannot be controlled as in Pavoni (2001) and Werning (2002), we, in contrast, do not allow benefits to be conditioned on the entire individual employment history.[2] While our assumptions come at some cost of reduced generality, they also provide substantial benefits. We can analytically characterize optimal benefits using standard economic tools when different forms of asymmetric information coexist. Our results can be graphically represented and easily interpreted.

As in the previous literature, we will consider the moral hazard problem arising from a costly but unverifiable search activity. Furthermore, we argue that another informational problem, largely neglected in the previous literature on optimal UI design, calls for attention. Specifically, we will consider the case when some, but not all, unemployed can increase the probability of being hired by undertaking a costly investment, e.g., by retraining or moving to a more a location with better employment prospects. Under the assumption that the insurer is unable to observe who has this option a realistic adverse selection problem arises.

There are reasons to believe that the adverse selection problem is of quantitative importance. For instance, Bartel (1979) documents that the proportion of moves in the U.S. caused by the decision to change jobs is one-half of all migration decisions for young workers and one third of all migration decisions for workers above the age of 45. Other empirical documentations of the link between unemployment and geographical mobility are DaVanzo (1978), Pissarides and Wadsworth (1989) and McCormick (1997). Furthermore, geographical mobility is substantially lower in continental Europe, possibly due to higher unemployment insurance – a mechanism explored in Hassler, Rodríguez Mora, Storesletten and Zilibotti (2002). Therefore, we argue that it is important to analyze how the informational problem associated with geographical mobility and similar cases affect how UI should be designed.

In all cases, also when both types of asymmetric information are jointly present, we provide analytical results. The key to analytical tractability under hidden savings is that with constant absolute risk-aversion, search incentives are independent of asset holdings. However, the results

[2] The restriction to two-tier systems can arguably be interesting in itself, since many real world UI systems have this feature, possibly due to political restrictions on system complexity.

will crucially depend on riskaversion and access to markets for saving and borrowing and can provide qualitative insights that may prove to be valuable also for more general preferences. Therefore, we believe that our results can complement both numerical analysis and the optimal contract framework.

The paper is organized as follows; the basic structure of the model is presented in section 2, the cases of moral hazard and adverse selection are analyzed separately in sections 3 and 4 respectively. In section 5, we analyze first the case of moral hazard on the job and finally the case when moral hazard and adverse selection are allowed simultaneously and section 6 concludes. Some proofs are provided in the appendix, while others are available upon request.

2 The model

Consider an economy in continuous time where individuals can be employed or unemployed. They have access to a capital market with an exogenous return r, equal to the subjective discount rate (possibly including a positive probability of dying). An employed individual is said to be in state 1, receiving an exogenous income, w. She loses her job with instantaneous probability q, and enters into state 2, where she receives benefits, denoted b_2. To analyze the issue of whether unemployment benefits should be increasing or decreasing, we allow two benefit levels, b_2 and b_3.[3] If $b_2 > (<)\, b_3$, we say that benefits are decreasing (increasing) over time. The latter benefit level is given to individuals in state 3, who are denoted as long-term unemployed, while those in state 2 are called short-term unemployed. To facilitate a simple presentation of the results, we assume that an individual in state 2 enters state 3 with a constant instantaneous probability f.[4] Our main interpretation is that state 3 is an administrative state associated with long unemployment duration; consequently, as a baseline case, we assume individuals who search to have the same hiring rates, h, in the two unemployment states. However, for completeness, we will also consider the case when hiring rates are different in the two states.

Unemployed individuals can affect their hiring rate by costly and unobservable search activity, creating informational problems making full insurance infeasible. Specifically, we will consider two cases. The first case is that search is costly, and unless individuals search, they will remain unemployed. The second case is that unemployed individuals can make a costly investment increasing their chances of becoming employed. However, this cost is prohibitively high for some

[3]Extending the analysis to any finite number of benefit levels is straightforward.

[4]This assumption implies that seach incentives remain constant as long as the individual remain in state 2. Notice that it is not the random duration that matters, but that incentives remain constant in the short-term unemployment state. An alternative would be to use discrete time and assume that short-term UI benefits are paid for one period only as done by e.g., Cahuc and Lehmann (2000). Assuming that UI benefits change after some fixed period of time would make search incentives depend on the remaining time of current benefits and considerably complicate the analysis with little gain.

individuals. Therefore, there is an adverse selection problem, where individuals with high costs needs insurance while those with low costs should be induced to search.

Individuals maximize their intertemporal utility, given by

$$E \int_0^\infty e^{-rt} U(c_t)\, dt,$$

where c_t is consumption at time t and r the subjective discount rate. In order to facilitate analytical solutions when individuals have access to markets for saving and borrowing, we choose the CARA utility function

$$U(c_t) \equiv -e^{-\gamma c_t},$$

where γ is the coefficient of absolute risk aversion.

All individuals are born (enter the labor market) as employed without assets and are identical at that point. The purpose of this paper is to characterize optimal unemployment insurance under moral hazard. To his end, we want to remove other motives for unemployment benefits, in particular transfer motives and therefore assume that individuals face an actuarially fair insurance. This means that when an individual enters the labor force, the expected present discounted value of the benefits she will receive during her life-time exactly balances the expected present discounted value of her contributions. An alternative interpretation of actuarial fairness is that in a decentralized equilibrium, actuarial fairness is identical to a break-even condition for insurance companies, which would be satisfied under perfect competition.

Without loss of generality, we let individuals pay lump-sum taxes, denoted τ. We denote the average discounted probabilities (ADP's) of being in state 2 and 3, respectively, by

$$\Pi_2 \equiv r \int_0^\infty e^{-rt} \mu_{2,t} dt,$$
$$\Pi_3 \equiv r \int_0^\infty e^{-rt} \mu_{3,t} dt.$$

where $\mu_{2,t}$ and $\mu_{3,t}$ are the probabilities of being short term and long term unemployed at time t, respectively, conditioned on being employed at time zero. Solving for the ADP's in the base line case when hiring rates are the same in both states yields[5]

[5]If, instead, hiring rates are h_2 and h_3, we have

$$\Pi_2 = \frac{(h_3 + r)\, q}{(\rho_2 - r)(\rho_1 - r)},$$
$$\Pi_3 = \frac{fq}{(\rho_2 - r)(\rho_1 - r)},$$

where ρ_1 and ρ_2 are the roots of the system and given by

$$\rho_{1,2} = -\frac{F \pm \sqrt{F^2 - 4\left(qf + h_3\left(f + h_2 + q\right)\right)}}{2} < 0,$$

where $F \equiv f + q + h_3 + h_2$

$$\Pi_2 = q\frac{h+r}{(r+h+q)(r+h+f)},$$
$$\Pi_3 = \Pi_2\frac{f}{h+r}.$$

The actuarial fairness requirement of the UI system can then be written

$$\tau = \Pi_2 b_2 + \Pi_3 b_3. \tag{1}$$

2.1 Search costs

The insurer's ability to provide insurance is hampered by asymmetric information. First, we assume search activity to be costly – a cost of m per unit of time must be paid, otherwise the hiring probability is zero. We may consider this cost as representing the opportunity cost of searching, arising from some alternative economic activity. Whether the agent actually searches or not is assumed to her own private information. Second, we assume that an unemployed individual can undertake a costly investment, (re-training or moving). The cost is either low, $\tilde{m}$ (with probability p) or prohibitively high. For simplicity, we assume that if the unemployed pays the cost, she is immediately rehired. Otherwise, she remains unemployed and decides whether to search for a new job. To make the problem interesting, we assume parameters to be such that it is optimal to induce search for all unemployed and investment for individuals with low costs.

3 Moral hazard

We start the analysis by assuming that individuals cannot save or borrow. The value function of an employed individual, conditional on her searching when unemployed, is then given by

$$V_1 = -e^{\gamma\tau}e^{-\gamma w}\frac{1-\Pi_2-\Pi_3+\Pi_2 e^{\gamma\Delta_2}+\Pi_3 e^{\gamma\Delta_3}}{r} \tag{2}$$

where $\Delta_2 \equiv c_1 - c_2, \Delta_3 \equiv c_1 - c_3$ denotes the reduction in consumption for the short- and long-run unemployed, relative to employed. It is straightforward to verify that that individuals prefer flat benefit schedules under imperfect insurance.[6] To see this, note that the slope of an indifference curve in (Δ_2, Δ_3) space is given by

$$\frac{d\Delta_2}{d\Delta_3}|_{V_1=\bar{V}} = -\frac{\Pi_3(1-\Pi_3)e^{\gamma\Delta_3} - \left(1-\Pi_2-\Pi_3+\Pi_2 e^{\gamma\Delta_2}\right)\Pi_3}{\Pi_2(1-\Pi_2)e^{\gamma\Delta_2} - \left(1-\Pi_2-\Pi_3+\Pi_3 e^{\gamma\Delta_3}\right)\Pi_2}.$$

When $\Delta_3 = \Delta_2$, this simplifies to $-\frac{\Pi_3}{\Pi_2}$.Given a tax rate, the slope of the budget constraint is also $-\frac{\Pi_3}{\Pi_2}$. Thus, $\Delta_2 = \Delta_3$, requiring $b_2 = b_3$ is the optimum. In other words;

[6] Without additional constraints, full insurance is, of course, optimal. We therefore consider the optimal benefit profile given a tax level insufficient to provide full insurance.

Proposition 1 *When no moral hazard problem exists and there is no market for saving and borrowing, UI benefits should be constant over time.*

Intuitively, when $\Delta_2 = \Delta_3$ and thus marginal utility is the same in states 2 and 3, the marginal rate of substitution between Δ_2 and Δ_3, i.e., the slope of the indifference curve, equal minus the ratio of the ADP's of the two state, that is $-\frac{\Pi_3}{\Pi_2}$.Furthermore, $-\frac{\Pi_3}{\Pi_2}$ is also the rate of transformation between benefits in the two states implied by the budget restriction (1).

Let us now introduce moral hazard by allowing individuals to abstain from searching if the search incentive is too weak. In order to derive the incentive compatibility constraints, we first note that the incentive compatible value functions for the two states, V_2 and V_3, are given by

$$V_2 = -e^{\gamma\tau}e^{-\gamma w}\frac{\frac{h}{h+r}\left(1-\Pi_2-\Pi_3\right)+\frac{r+q}{q}\Pi_2 e^{\gamma\Delta_2}+\frac{r+q}{q}\Pi_3 e^{\gamma\Delta_3}}{r} \tag{3}$$

$$V_3 = -e^{\gamma\tau}e^{-\gamma w}\frac{\frac{h}{h+r}\left(1-\Pi_2-\Pi_3\right)+\frac{h}{h+r}\Pi_2 e^{\gamma\Delta_2}+\left(1+\frac{r(r+h+f+q)}{qf}\right)\Pi_3 e^{\gamma\Delta_3}}{r},$$

where $\Delta_2 = w - b_2 + m$ and $\Delta_3 = w - b_3 + m$ since we continue to assume no saving or borrowing.

It the individual does not search in the current state, the corresponding value functions, are

$$V_{3,n} = \frac{-e^{-\gamma(b_3-\tau)}}{r}, V_{2,n} = \frac{-e^{-\gamma(b_2-\tau)}+fV_3}{r+f},$$

where $V_{2,n}$ is conditioned on searching in state 3.

The incentive compatibility constraint for the long-term unemployed (IC3) is then

$$V_3 \geq V_{3,n}.$$

Using the above definitions, this can be rewritten as

$$\left(1-\Pi_2-\Pi_3\right)\left(1-e^{-\gamma\Delta_3}\right)+\Pi_2\left(1-e^{\gamma(\Delta_2-\Delta_3)}\right) \geq \left(1+\frac{r}{h}\right)\left(1-e^{-\gamma m}\right). \tag{4}$$

When the IC3 constraint is satisfied with equality, we have an increasing relationship between Δ_2 and Δ_3;

$$\frac{d\Delta_2}{d\Delta_3}|_{IC3} = \frac{1-\Pi_3}{\Pi_2} - \left(\frac{1-\Pi_2-\Pi_3}{\Pi_2}\right)\left(1-e^{-\gamma\Delta_2}\right) \geq 1,$$

where we should note that $\frac{\Pi_2}{1-\Pi_3}$ is the slope of the IC3 constraint under risk neutrality and/or under perfect insurance. Thus, higher search costs require higher Δ_3 and/or lower Δ_2 to induce search. The positive effect on search incentives of higher consumption in state 3 is the "entitlement" effect (see Mortensen (1977)) – higher benefits for short-term unemployed *increase* the search incentives for the long-term unemployed since the latter first need to become employed to be entitled to these higher benefits.

Now, consider the short-term unemployed. The incentive compatibility constraint for these individuals (IC2) is

$$V_2 \geq V_{2,n}$$

which we can write as

$$(1-\Pi_2-\Pi_3)\left(1-e^{-\gamma\Delta_2}\right)+\frac{f}{q}\Pi_2\left(e^{\gamma(\Delta_3-\Delta_2)}-1\right)\geq\left(1+\frac{r}{h}\right)\left(1-e^{-\gamma m}\right). \tag{5}$$

Along the IC2 constraint, we have

$$\begin{aligned}\frac{d\Delta_2}{d\Delta_3}|_{IC2}&=-\frac{\Pi_3}{\Pi_2}\frac{1}{1-\left(1+\frac{\Pi_3}{\Pi_2}\right)\left(1-e^{-\gamma\Delta_3}\right)}\\&<-\frac{\Pi_3}{\Pi_2}\forall\Delta_3\in\left(0,\frac{1}{\gamma}\ln\left(1+\frac{\Pi_2}{\Pi_3}\right)\right),\end{aligned} \tag{6}$$

where $-\frac{\Pi_3}{\Pi_2}=-\frac{f}{h+r}$ is the slope of the IC2 constraint under risk-neutrality and/or under perfect insurance.[7]

The slope of the IC2 constraint is negative since a decrease in Δ_2 achieved by an increase in b_2 reduces search incentives for the short-run unemployed and therefore needs to be compensated by an increase in Δ_3 achieved by a decrease in b_3.

Note that when $\Delta_3=\Delta_2$, the two constraints coincide at $(1-\Pi_2-\Pi_3)\left(1-e^{-\gamma\Delta_2}\right)\geq\left(1-e^{-\gamma m}\right)\frac{r+h}{h}$, implying that

$$\Delta_2\geq\frac{-\ln\left(1-\left(1-e^{-\gamma m}\right)\left(1+\frac{q+r}{h}\right)\right)}{\gamma}\equiv\bar{\Delta}.$$

The optimal contract maximizes V_1 as given by (2) over Δ_2 and Δ_3, subject to the incentive constraints (4) and (5) and the fairness constraint (1).

As wee see from (6), the slope of IC2 is necessarily steeper than the indifference curve at $\Delta_2=\Delta_3=\bar{\Delta}$, whenever risk-aversion is strictly positive and a strictly positive m prevents full insurance. Therefore, Δ_3 should optimally be larger than Δ_2, requiring a downward-sloping benefit schedule.

We depict our results in Figure 1 and summarize in the following proposition;

Proposition 2 *Under moral hazard and without markets for saving and borrowing, UI benefits should be decreasing over time.*

[7] As Δ_3 approaches $\frac{1}{\gamma}\ln\left(1+\frac{\Pi_2}{\Pi_3}\right)$, the IC2 curve becomes vertical. However, if $m<-\frac{1}{\gamma}\ln\left(1-\frac{h}{q}\Pi_2\right)$, the IC2 curve has a finite slope in the positive quadrant of the space Δ_2,Δ_3. To see this, set $\Delta_2=0$ in the IC2 constraint, solve for Δ_3 as a function of m and set this expression equal to $\frac{1}{\gamma}\ln\left(1+\frac{\Pi_2}{\Pi_3}\right)$.

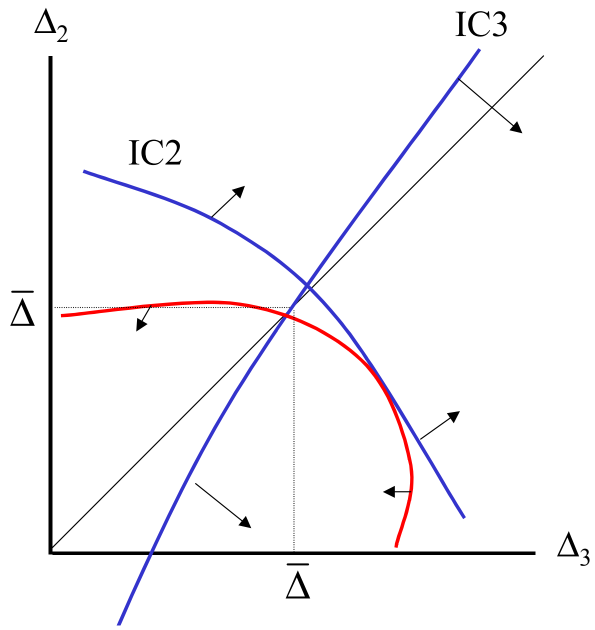

Figure 1. Moral hazard and no savings.

This result is qualitatively similar to those of Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997), but the intuition is somewhat different. At $\Delta_2 = \Delta_3$, or equivalently $b_2 = b_3$, a non-deviator is indifferent to small actuarial changes in b_2 and b_3 since her marginal utility is the same in both states. In other words, the indifference curve has slope $-\Delta_3/\Delta_2$. Consider now an individual who deviates by not searching in state 2 but will search in state 3. Clearly, she will have higher marginal utility in state 3 than in state 2. Therefore, she will be harmed by increases in actuarially fair reductions in b_3. Optimal benefits use this difference in relative preferences for b_2 and b_3 to discourage deviation.

3.1 Saving

Consider now the case when individuals can self-insure via precautionary savings. As above, we assume that there is a cost of searching and that the search only takes place if there are sufficiently strong search incentives. The value function for the three types are,

$$V_j\left(A_t\right) = -\frac{1}{r}e^{-\gamma r A_t}e^{-\gamma c_j}, j \in \{1, 2, 3\},$$

and consumption is

$$c_{t,j} = rA_t + c_j \; j \in \{1, 2, 3\}.$$

Note our abuse of notation; from now on, we let c_j denote consumption net of permanent income from current asset holding. The Bellman equation for the employed is satisfied if the constants c_j, satisfy

$$c_1 = w - \tau - \frac{q\left(e^{\gamma\Delta_2} - 1\right)}{\gamma r}, \tag{7}$$

$$c_2 = b_2 - m - \tau + \frac{h\left(1 - e^{-\gamma\Delta_2}\right)}{\gamma r} - \frac{f\left(e^{\gamma(\Delta_3 - \Delta_2)} - 1\right)}{\gamma r},$$

$$c_3 = b_3 - m - \tau + \frac{h\left(1 - e^{-\gamma\Delta_3}\right)}{\gamma r}.$$

It is convenient to rewrite the second two equations of (7) as

$$b_2 = w - \Delta_2 + m - \frac{q\left(e^{\gamma\Delta_2} - 1\right)}{\gamma r} - \frac{h\left(1 - e^{-\gamma\Delta_2}\right)}{\gamma r} + \frac{f\left(e^{\gamma(\Delta_3 - \Delta_2)} - 1\right)}{\gamma r}, \tag{8}$$

$$b_3 = w - \Delta_3 + m - \frac{q\left(e^{\gamma\Delta_2} - 1\right)}{\gamma r} - \frac{h\left(1 - e^{-\gamma\Delta_3}\right)}{\gamma r}.$$

Notice that as in the no-savings case, there is a one-to-one mapping from (b_2, b_3) to (Δ_2, Δ_3), although in the case of savings, it in no longer by construction but a result of individual optimization. As we will see, it is convenient to write the problem as if the insurer directly chooses Δ_2 and Δ_3, constrained by individual optimal optimization as given by (8).

Consider first the problem of choosing benefits, disregarding the incentive compatibility constraints. The optimal contract maximizes c_1 (and therefore $V_1(A)$ for any A) as given by (7) over Δ_2 and Δ_3 subject to individual consumption choices as given by (8) and the fairness constraint (1).[8] It follows immediately that whenever Δ_2 is positive in optimum (less than full insurance) individuals prefer upward sloping benefits. To see this, note that the first-order condition for Δ_3 simply minimizes taxes over Δ_3, i.e., sets

$$\frac{\partial\tau}{\partial\Delta_3} = \Pi_2\left(\frac{f}{r}e^{\gamma(\Delta_3 - \Delta_2)}\right) - \Pi_3\left(1 + \frac{h}{r}e^{-\gamma\Delta_3}\right) = 0.$$

Clearly, this is not satisfied at $\Delta_3 = \Delta_2$, unless there is full insurance ($\Delta_3 = \Delta_2 = 0$). Instead, b_3 should be increased until

$$\Delta_3 = \Delta_2 + \frac{1}{\gamma}\ln\left(1 - \frac{h}{h+r}\left(1 - e^{-\gamma\Delta_3}\right)\right) < \Delta_2. \tag{9}$$

Now, let us consider the incentive constraints. A long-run unemployed who does not search consumes $b_3 - \tau + rA_t$ for ever, yielding a value of $-\frac{1}{r}e^{-\gamma r A_t}e^{-\gamma(b_3-\tau)}$. The IC3 constraint is therefore

$$-\frac{1}{r}e^{-\gamma r A_t}e^{-\gamma c_3} \geq -\frac{1}{r}e^{-\gamma r A_t}e^{-\gamma(b_3-\tau)},$$

$$c_3 \geq b_3 - \tau.$$

As we see, total consumption ($c_3 + rA_t$), must be at least as large as net income ($b_3 - \tau + rA_t$). This means that incentives have to be at least large enough to make the individual willing to

[8] As in the no-savings case, full insurance is of course optimal without any additional restrictions, so we assume taxes are fixed at a level insufficient for providing full insurance.

borrow to finance the search cost. This, in turn, means that consumption necessarily falls as long as the individual remains unemployed.

Using (7), IC3 can be written as

$$\Delta_3 \geq \frac{\ln\left(\frac{h}{h-\gamma rm}\right)}{\gamma} \equiv \hat{\Delta}(h). \tag{10}$$

Note that while the search incentive in general depends on the extent to which the value function increases when employment is gained, in this case, the incentive constraint can be written as only depending on the extent to which *consumption* increases at re-employment. In particular, consumption in state 2 and b_2 have no separate effects on search behavior in state 3. This does of course not mean that only b_3 matters for search incentives. On the contrary, both b_2 and b_3 affect consumption in all states, as seen in (7). However, individual optimization and access to markets for saving and borrowing imply the value function to be a monotonous transformation of consumption. Thus, the wedge between consumption in the two states is a sufficient statistic to determine if search incentives are sufficiently strong. Furthermore, note also that to induce search we need $\gamma rm < h$.

For the short term unemployed, we compute the value associated with no search in state 2, conditioned on searching in state 3. This is $-\frac{e^{-\gamma rA_t}e^{-\gamma c_{2,n}}}{r}$ where $c_{2,n}$ satisfies

$$c_{2,n} = b_2 - \tau + \frac{f\left(1-e^{-\gamma(c_3-c_{2,n})}\right)}{\gamma r}.$$

The IC2 constraint is $c_2 \geq c_{2,n}$,which can be written as

$$\Delta_2 \geq \hat{\Delta}(h). \tag{11}$$

The IC2 constraint is independent of Δ_3 for the same reasons the IC3 constraint is independent of Δ_2, as discussed above. The optimal insurance contract should then be chosen to maximize c_1 as given by (7) over Δ_2 and Δ_3, subject to the incentive constraints (10) and (11), individual consumption choices (8) and the actuarial fairness constraint (1).

Since the indifference curve in (Δ_2, Δ_3) has a negative slope at $\Delta_2 = \Delta_3$, the optimal contract is the point at which IC3 crosses IC2.[9] There we have $\Delta_2 = \Delta_2 = \hat{\Delta}(h)$ and $b_2 = b_3 = w - \hat{\Delta}(h) - \frac{mq}{h-\gamma rm}$.Substituting this into (7) yields, $c_2 = c_3 = b_2 - \tau$. We depict our results in Figure 2 and and summarize as follows

Proposition 3 *Under moral hazard and with markets for saving and borrowing, UI benefits should be constant over states at* $b_2 = b_3 = w - \hat{\Delta}(h) - \frac{mq}{h-\gamma rm}$. *The consumption of the unemployed is equal to their income. The search cost is financed by borrowing, implying falling consumption over the unemployment spell. Optimal incentive compatible benefits increase in h and decrease and in m.*

[9]The slope of an indiffence curve at $\Delta_2 = \Delta_3$ is given by $-\frac{\Pi_3 h\left(1-e^{-\gamma\Delta_2}\right)}{\Pi_2 f+(1-\Pi_2-\Pi_3)q\left(e^{\gamma\Delta_2}-1\right)+\Pi_2 h\left(1-e^{-\gamma\Delta_2}\right)} \leq 0.$

Proof: Follows from the text, expect for the last result, which is derived from differentiating $w - \hat{\Delta}(h) - \frac{mq}{h-\gamma rm}$ and using $h > \gamma rm$.

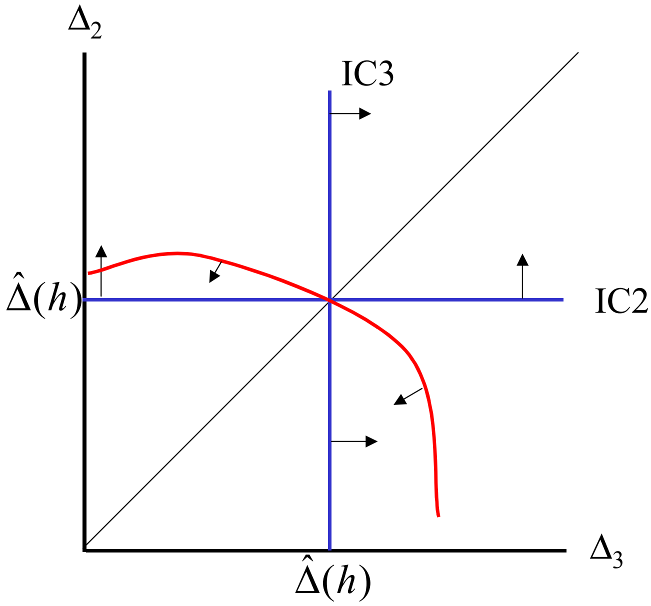

Figure 2. Moral hazard with savings.

To obtain some intuition for the results, we once more note that in general, search incentives arise from a comparison of expected lifetime utility (the value function) under different search strategies. When individuals have access to a capital market for saving and borrowing, however, there is a one-to-one mapping between consumption and the value function. In contrast to the no savings case, we cannot increase the search incentive in state 3 for a given level of Δ_3 by reducing Δ_2. Similarly, in state 2, and given Δ_2 the search incentive cannot be strengthened by increasing Δ_3. In other words, the two constraints are independent and both should be satisfied with equality. As we have assumed hiring rates to be the same in both states, benefits should be constant.[10]

4 Adverse selection

As above, the value functions are of the form $-\frac{1}{r}e^{-\gamma(rA_t+c_j)}$ for the three states indexed by j, and consumption is given by $rA_t + c_j$ unless the individual invests, in which case assets fall discontinuously by $\tilde{m}$. Recalling that individuals loosing their job have the option of investing with probability p, it is straightforward to show that the incentive compatible consumption

[10] In fact, in an unpublished paper, Werning (2002) shows in a similiar setting that constant benefits are optimal under CARA utility in a general class of UI-schemes.

constants must satisfy

$$\begin{aligned} c_1 &= w - \tau - q\frac{pe^{\gamma r\tilde{m}} + (1-p)\, e^{\gamma \Delta_2} - 1}{\gamma r} \\ c_2 &= b_2 - \tau + h\frac{1 - e^{-\gamma\Delta_2}}{\gamma r} - f\frac{e^{\gamma(\Delta_3 - \Delta_2)} - 1}{\gamma r} \\ c_3 &= b_3 - \tau + h\frac{1 - e^{-\gamma\Delta_3}}{\gamma r}. \end{aligned} \tag{12}$$

Noting that when the incentive constraints are satisfied, the flow into unemployment is $(1-p)\,q$, we find that the ADPs of being short-term and long-term unemployed, respectively, are given by

$$\begin{aligned} \bar{\Pi}_2 &\equiv q\,(1-p)\,\frac{h+r}{(r+h+q\,(1-p))\,(r+h+f)}, \\ \bar{\Pi}_3 &\equiv \bar{\Pi}_2\frac{f}{h+r}. \end{aligned} \tag{13}$$

Now, the incentive compatibility constraint under adverse selection (ICA) is that individuals with a low cost should pay the investment cost. This can be written as

$$\begin{gathered} -\frac{1}{r}e^{-\gamma(r(A_t - \tilde{m}) + c_1)} \geq -\frac{1}{r}e^{-\gamma(rA_t + c_2)}, \\ \Delta_2 \geq r\tilde{m}, \end{gathered} \tag{14}$$

which is independent of assets. Now, since the insurance is actuarially fair and individuals are risk averse, the ICA condition will surely bind at the optimal tax rate, in which case $r\tilde{m} = \Delta_2$, giving $c_1 = w - \tau - q\frac{e^{\gamma r\tilde{m}}-1}{\gamma r}$, and

$$\begin{aligned} b_2 &= w - r\tilde{m} - q\frac{e^{\gamma r\tilde{m}} - 1}{\gamma r} - \left(h\frac{1 - e^{-\gamma r\tilde{m}}}{\gamma r} - f\frac{e^{\gamma(\Delta_3 - r\tilde{m})} - 1}{\gamma r}\right), \\ b_3 &= w - \Delta_3 - q\frac{e^{\gamma r\tilde{m}} - 1}{\gamma r} - h\frac{1 - e^{-\gamma\Delta_3}}{\gamma r}. \end{aligned} \tag{15}$$

Optimal benefits then maximize c_1 as given by (12), subject to the incentive constraint (14), the individual consumption choices (15) and the actuarial fairness condition $\tau = \bar{\Pi}_2 b_2 + \bar{\Pi}_3 b_3$. Clearly, ICA will bind, fixing the difference in consumption between states 1 and 2 ($\Delta_2 = r\tilde{m}$), implying that $c_1 = w - \tau - q\frac{e^{\gamma r\tilde{m}}-1}{\gamma r}$. Therefore, the problem reduces to minimize taxes, given the constraints. The first-order condition for this problem can be written

$$e^{\gamma(\Delta_3 - r\tilde{m})} - \left(1 - \frac{h}{h+r}\left(1 - e^{-\gamma\Delta_3}\right)\right) = 0, \tag{16}$$

The LHS of this is increasing in Δ_3, negative at $\Delta_3 = 0$ and positive at $\Delta_3 = r\tilde{m}$. The solution to the first order condition, given by

$$\Delta_3 = -\frac{\ln\left(\sqrt{\left(\frac{r}{2h}\right)^2 + e^{-\gamma rm}\left(1 + \frac{r}{h}\right)} - \frac{r}{2h}\right)}{\gamma}, \tag{17}$$

is therefore such that $0 \leq \Delta_3 \leq \Delta_2 = r\tilde{m}$, with equality only if $\tilde{m} = 0$. Using (15), we find that

$$b_2 - b_3 = (\Delta_3 - r\tilde{m}) + f\frac{e^{\gamma(\Delta_3 - r\tilde{m})} - 1}{\gamma r} + h\left(\frac{e^{-\gamma r\tilde{m}} - e^{-\gamma\Delta_3}}{\gamma r}\right) < 0. \tag{18}$$

Our results are depicted in Figure 3, and summarized in the following proposition;

Proposition 4 *Under adverse selection and access to markets for saving and borrowing, benefits should increase over time, but not to full insurance.*

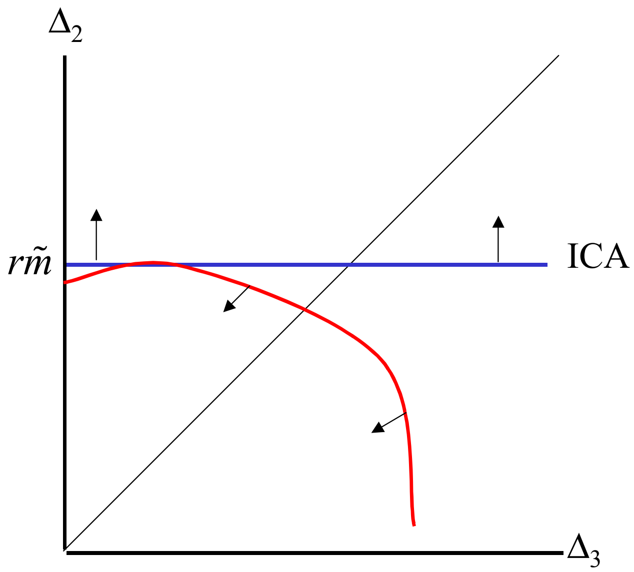

Figure 3. Adverse selection with savings.

The intuition here is that the IC constraint associated with the adverse selection problem puts a wedge between the value of being employed and short-run unemployed, and therefore between consumption in these states. However, the relative preferences for b_2 and b_3 is the same for a deviator and a non-deviator. In contrast to the case of moral hazard and no savings, differences in relative preferences of the two benefit levels cannot be used to strengthen the incentive to invest. Therefore, there is no point in not satisfying the preference for upward sloping benefits in the case when investment costs are high. In a sense, insurance should be (constrained) efficient in the choice of relative insurance for long-term and short-term insurance. However, full insurance for long-term unemployed cannot be optimal since a marginal reallocation from long-term to short-term unemployed, when the former but not the latter have full insurance, must improve the constrained efficiency of the insurance.

4.1 Adverse selection and no saving

To understand the results on adverse selection, we want to analyze the case of no savings in a setting as close to the case of savings as possible. This poses a technical problem, since investments are hard to model when there are no savings. To keep as close to the savings case as possible, in particular that the investment cost is monetary and that there are three employment states only, we make the following assumption; the investment cost is a loss of income $\tilde{m}$ during a short period of time. Formally, we assume the period of lower consumption to be a unitary masspoint of time.[11] This assumption corresponds to a discrete time case when consumption falls by an amount $\tilde{m}$ during one period if the investment is undertaken. This means that the value function falls by an amount $\left(e^{-\gamma(w-\tau-\tilde{m})} - e^{-\gamma(w-\tau)}\right)$ if the individual decides to undertake the investment and the cost is low.

Now, the incentive compatibility constraint $V_1 - \left(e^{-\gamma(w-\tau-\tilde{m})} - e^{-\gamma(w-\tau)}\right) \geq V_2$ can be written as[12]

$$\bar{\Pi}_2 \left(e^{\gamma\Delta_2} - 1\right) + \bar{\Pi}_3 \left(e^{\gamma\Delta_3} - 1\right) \geq \left(e^{\gamma\tilde{m}} - 1\right) q\,(1-p)\,\frac{h+r+q}{h+r+q\,(1-p)}$$

Clearly, the slope of this constraint is the same as the indifference curve if and only if $\Delta_3 = \Delta_2$. Thus, benefits should be flat under adverse selection and no savings.

Proposition 5 *Under adverse selection and no access to markets for saving and borrowing, benefits should be constant over time.*

The intuition for our results is the same as under savings; the IC constraint puts a wedge between the value of the employed and the short-term unemployed. However, this does not call for not satisfying the preferences of the unemployed with high investment costs, which is to have constant benefits in the case of no savings. The insurance should be efficient in the relative insurance of the long- and short-term unemployed.

[11] Alternative assumptions would require an additional state.

[12] The expressions for the value functions are given in the appendix.

5 Some extensions

5.1 Moral hazard on the job

The model is very easy to extend to allow for, e.g., moral hazard on the job. Suppose that there is an effort cost $\hat{m}$ associated with working. If an individual produce effort, she will loose her job with probability q. If shirking, there is a detection probability of δ. Then, the incentive compatible consumption constants are,

$$\begin{aligned} c_1 &= w - \hat{m} - \tau - \frac{q\left(e^{\gamma\Delta_2} - 1\right)}{\gamma r}, \\ c_2 &= b_2 - \tau + \frac{h\left(1 - e^{-\gamma\Delta_2}\right)}{\gamma r} - \frac{f\left(e^{\gamma(\Delta_3 - \Delta_2)} - 1\right)}{\gamma r}, \\ c_3 &= b_3 - \tau + \frac{h\left(1 - e^{-\gamma\Delta_3}\right)}{\gamma r}. \end{aligned} \tag{19}$$

while c_2 and c_3 are given by (12). An individual who considers deviation in the current employment, instead has optimal labor income consumption given by

$$c_{1,n} = w - \tau - \frac{(q+\delta)\left(e^{\gamma(c_{1,n} - c_2)} - 1\right)}{\gamma r}.$$

To induce effort, we require $c_1 \geq c_{1,n}$, which can be written

$$\Delta_2 \geq \frac{1}{\gamma} \ln\left(1 + \frac{\gamma r \hat{m}}{\delta}\right) \equiv \hat{\Delta}.$$

Clearly, this will bind at optimal benefits, implying, $c_1 = w - \tau - \hat{m}\left(1 + \frac{q}{\delta}\right)$. Imperfect observability of effort, i.e., $\delta < \infty$, reduces consumption relative to the first best by an amount $\hat{m}q/\delta$.

As in the case of adverse selection, optimal benefits minimize taxes over Δ_3. The first order constraint for this is as (16) with $r\tilde{m}$ replaced by $\hat{\Delta}$, with a solution

$$\Delta_3 = -\frac{\ln\left(\sqrt{\left(\frac{r}{2h}\right)^2 + e^{-\gamma\hat{\Delta}}\left(1 + \frac{r}{h}\right)} - \frac{r}{2h}\right)}{\gamma}. \tag{20}$$

implying increasing benefits.

5.2 Moral hazard, adverse selection and different hiring rates

Finally, let us analyze the more general case when hiring rates are allowed to be different in the two unemployment states (denoted h_2 and h_3), and when we have both moral hazard and adverse selection.[13] The consumption constants satisfy

$$
\begin{aligned}
c_1 &= w - \tau - q\frac{pe^{\gamma r\tilde{m}} + (1-p)\, e^{\gamma\Delta_2} - 1}{\gamma r}, \\
c_2 &= b_2 - m - \tau + h_2\frac{1 - e^{-\gamma\Delta_2}}{\gamma r} - f\frac{e^{\gamma(\Delta_3 - \Delta_2)} - 1}{\gamma r}, \\
c_3 &= b_3 - m - \tau + h_3\frac{1 - e^{-\gamma\Delta_3}}{\gamma r},
\end{aligned} \tag{21}
$$

the ICA constraint remains $\Delta_2 \geq r\tilde{m}$ and the IC2 and IC3 constraint are $\Delta_2 \geq \hat{\Delta}(h_2)$ and $\Delta_3 \geq \hat{\Delta}(h_3)$, respectively.

We now have two cases. First, when the adverse selection problem is small, specifically, if $\hat{\Delta}(h_2) \geq r\tilde{m}$, the ICA constraint is satisfied whenever IC2 is satisfied. Then, the optimal contract sets $\Delta_2 = \hat{\Delta}(h_2)$ and $\Delta_3 = \hat{\Delta}(h_3)$, implying from (21) that

$$
\begin{aligned}
b_3 &= w - \hat{\Delta}(h_3) - q\frac{pe^{\gamma r\tilde{m}} + (1-p)\left(\frac{h_2}{h_2 - \gamma r m}\right) - 1}{\gamma r} \\
b_2 &= w - \hat{\Delta}(h_2) - q\frac{pe^{\gamma r\tilde{m}} + (1-p)\left(\frac{h_2}{h_2 - \gamma r m}\right) - 1}{\gamma r} + fm\frac{h_2 - h_3}{h_2 h_3 - mr\gamma h_2}.
\end{aligned}
$$

Since $\hat{\Delta}'(h) < 0$, we have that $h_2 > (\leq)\, h_3 \implies b_2 - b_3 = \hat{\Delta}(h_3) - \hat{\Delta}(h_2) + fm\frac{h_2 - h_3}{h_2 h_3 - mr\gamma h_2} > (\leq)\, 0$.

Second, if the adverse selection problem is relatively strong, i.e., $\hat{\Delta}(h_2) < r\tilde{m}$, IC2 is satisfied when ICA is satisfied. Then, the optimal contract is $\Delta_2 = r\tilde{m}$ and $\Delta_3 = \hat{\Delta}(h_3)$. Using this in (21) implies,

$$
\begin{aligned}
b_3 &= w - q\frac{e^{\gamma r\tilde{m}} - 1}{\gamma r} - \hat{\Delta}(h_3) \\
b_2 &= w - r\tilde{m} + m - h_2\frac{1 - e^{-\gamma r\tilde{m}}}{\gamma r} + f\frac{\frac{h_3}{h_3 - \gamma r m} e^{-\gamma r\tilde{m}} - 1}{\gamma r} - q\frac{e^{\gamma r\tilde{m}} - 1}{\gamma r}
\end{aligned}
$$

In this case,

$$
b_2 - b_3 = m - r\tilde{m} + h_2\frac{e^{-\gamma r\tilde{m}} - 1}{\gamma r} + f\frac{\frac{h_3}{h_3 - \gamma r m} e^{-\gamma r\tilde{m}} - 1}{\gamma r} + \hat{\Delta}(h_3).
$$

As we see, the benefit profile is more increasing (more negative $b_2 - b_3$) the smaller are m and h_2 and the larger are $\tilde{m}$ and h_3. In other words, an adverse selection problem that is strong relative to the moral hazard problem and increasing (decreasing) hiring rates, calls for increasing benefits. When $\hat{\Delta}(h_2) = r\tilde{m}$ and $h_2 = h_3$, we already know that optimally, $b_2 = b_3$. Consequently, if $\tilde{m}$ is increased from this point, the benefit profile becomes upward-sloping. Furthermore, for a sufficiently low h_3, benefits should be downward sloping. The results are

[13] If there is both an adverse selection constraint and an incentive compatibility constraint for moral hazard in effort, the former (latter) is the relevant constraint if $\frac{1}{\gamma}\ln\frac{\gamma r\tilde{m}}{\delta} < (>) r\tilde{m}$.

depicted in Figure 4. An increase in $\tilde{m}$ shifts the ICA constraint upwards, similarly, a decrease in h_2 shifts IC2 upwards.

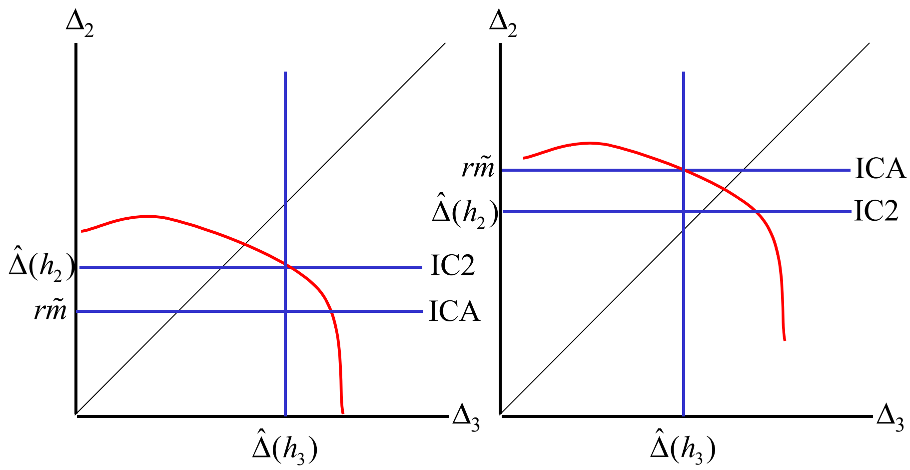

Figure 4. Strong moral hazard and falling hiring rates (left). Strong adverse selection and increasing hiring rates (right).

Summarizing

Proposition 6 *When the adverse selection problem is relatively small ($\hat{\Delta}\left(h_2\right) \geq r\tilde{m}$), benefits should be decreasing iff the hiring rates are decreasing ($h_2 > h_3$).*
When the adverse selection problem is relatively high, $\hat{\Delta}\left(h_2\right) < r\tilde{m}$, benefits should be increasing for non-decreasing hiring rates. For sufficiently decreasing hiring rates, benefits should be decreasing.

6 Conclusion

In this paper, we have provided a tractable model where risk averse individuals face unemployment risk that cannot be completely insured against due to various forms of asymmetric information. It has been shown that access to savings has important qualitative effects on the time profile of optimal unemployment benefits. The model provided a number of analytical results.

First, access to savings imply that individuals tend to prefer increasing benefits. Since individual assets are depleted during the unemployment spell, consumption tends to fall and marginal utility increase. Therefore, it is particularly important to have good insurance against long unemployment spells. In other words, precautionary savings is a good (bad) substitute for short (long) spells of unemployment.

Second, moral hazard problems arising from unobservable search effort may call for decreasing benefits if the insurer can control individual consumption, i.e., when there is no (hidden) savings. However, if, realistically, the insurer cannot control consumption, this is no longer necessarily the case. The reason is that individual consumption choices imply that search incentives have a one-to-one relation to the expected consumption increase associated with finding a job. If search productivity and the search cost are constant over time, the incentive to search should also be constant. This calls for a consumption increase at employment that is independent of the duration of the unemployment period, which is implemented by constant benefits. Specifically, search incentives should be strong enough to induce the individual to borrow (or dissave) to finance the search cost. The reason for this result is that a deviator, who does not search, would consume his current income and the non-deviator must have at least as high consumption for incentive compatibility to be satisfied.

Third, we have analyzed the case when individuals can affect the hiring probability by an up-front investment, e.g., retraining or moving. When the adverse selection problem arising from this is strong, the benefit profile is optimally increasing. The intuition here is straightforward – the adverse selection problem calls for a separation of individuals with low and high investment costs. However, since a deviator with low costs have the same relative preference for long-run and short-run benefits, a benefit profile that provides an inefficient insurance should not be used to provide incentives to invest. Instead the unemployed individuals' preference for increasing benefits should be satisfied.

Finally, the benefit profile is sensitive to how search productivity changes over the unemployment spell. If search productivity tends to fall, benefits should also be falling.

Let us conclude by some speculations on the consequences of allowing constant relative risk-aversion. In such a case, the analysis is greatly complicated by the fact that, in general, search incentives would depend on asset holdings. Therefore, incentive compatibility would not in general be consistent with a finite number of benefits that are independent of individual asset holdings. However, the intuition for the results in this paper appear not to be related to such effects. For example, the preference for increasing benefits depends on the fact that individual assets are depleted during unemployment, which is true for general specifications of utility, in particular for CRRA, as shown in e.g., Hassler and Rodríguez Mora (1999). Similarly, the result that incentives have to be large enough to induce borrowing to finance the search cost relies on the existence of capital markets rather than on the exact specification of preferences. Therefore, the mechanisms analyzed here are likely be present also under more general preference specifications. However, since search incentives in general depend on asset holdings and the duration of unemployment is likely to be correlated with the individual's asset holdings, unobservability of the latter may have consequences for optimal benefit time profiles. For example, if the search incentives are reinforced as wealth decumulates and individuals with long unemployment spells are likely to have less wealth, this might call for increasing benefits.

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7 Appendix

Recursive proof of (2) and (3). The value functions must satisfy

$$V_1 = \frac{-e^{\gamma\tau}e^{-\gamma w} + qV_2}{r+q},$$
$$V_2 = \frac{-e^{\gamma\tau}e^{-\gamma b_2} + hV_1 + fV_3}{r+h+f},$$
$$V_3 = \frac{-e^{\gamma\tau}e^{-\gamma b_3} + hV_1}{r+h}.$$

Solving these equations yields (2) and (3).

7.1 Savings

Guessing that the value function is $-e^{-\gamma(rA_t+c_j)}$ for $j \in \{1,2,3\}$, the Bellman equation for the employed is

$$-\frac{1}{r}e^{-\gamma(rA_t+c_1)} = \max_c -e^{-\gamma(rA_t+c)}dt - (1-rdt)\left[(1-qdt)\frac{1}{r}e^{-\gamma(rA_{t+dt}+c_1)} + qdt\frac{1}{r}e^{-\gamma(rA_{t+dt}+c_2)}\right].$$

Using first-order linear approximations and dividing by $e^{-\gamma r A_t}$, this becomes

$$-\frac{1}{r}e^{-\gamma c_1} = \max_c -e^{-\gamma c}dt$$
$$-(1-rdt)\left[(1-qdt)\frac{1}{r}e^{-\gamma c_1}\left(1-\gamma r\left(w-c-\tau\right)dt\right) + qdt\frac{1}{r}e^{-\gamma c_2}\left(1-\gamma r\left(w-\tau-c\right)dt\right)\right]$$

Adding $\frac{1}{r}e^{-\gamma c_1}$ to both sides, dividing by dt and letting dt approach zero, yields

$$0 = \max_c\left\{-re^{-\gamma(c-c_1)} + r + \gamma r\left(w-c-\tau\right) + q\left(1-e^{-\gamma(c_2-c_1)}\right)\right\}. \tag{22}$$

Similarly, for the short-term and long-run unemployed, we obtain

$$0 = \max_c\left\{-re^{-\gamma(c-c_2)} + r + \gamma r\left(b_2-c-m-\tau\right) + h + f - he^{-\gamma(c_1-c_2)} - fe^{-\gamma(c_3-c_2)}\right\}, \tag{23}$$
$$0 = \max_c\left\{-re^{-\gamma(c-c_3)} + r + \gamma r\left(b_3-c-m-\tau\right) + h + he^{-\gamma(c_1-c_3)}\right\}.$$

Equations (22) and (23) are maximized at $c = c_j$, implying that for the Bellman equation to be satisfied, the constants c_j, must satisfy

$$c_1 = w - \tau - \frac{q\left(e^{\gamma\Delta_2}-1\right)}{\gamma r}$$
$$c_2 = b_2 - m - \tau + \frac{h\left(1-e^{-\gamma\Delta_2}\right)}{\gamma r} - \frac{f\left(e^{\gamma(\Delta_3-\Delta_2)}-1\right)}{\gamma r}$$
$$c_3 = b_3 - m - \tau + \frac{h\left(1-e^{-\gamma\Delta_3}\right)}{\gamma r}.$$

The IC2 constraint is given by

$$c_2 - c_{2,n} \geq 0.$$

Furthermore,

$$\begin{aligned}
c_2 - c_{2,n} &= \left(-m + \frac{h\left(1-e^{-\gamma\Delta_2}\right)}{\gamma r} - \frac{f\left(e^{\gamma(\Delta_3-\Delta_2)} - e^{-\gamma(c_3-c_{2,n})}\right)}{\gamma r}\right) \qquad (24)\\
&= \left(-m + \frac{h}{\gamma r}\left(1-e^{-\gamma\Delta_2}\right) - \frac{f}{\gamma r}e^{\gamma(\Delta_3-\Delta_2)}\left(1-e^{-\gamma(c_2-c_{2,n})}\right)\right)\\
&\equiv R\left(c_2-c_{2,n}\right)
\end{aligned}$$

Clearly, R is a monotonously decreasing function that has an horizontal asymptote at $-m + \frac{h}{\gamma r}\left(1-e^{-\gamma\Delta_2}\right) - \frac{f}{\gamma r}e^{\gamma(\Delta_3-\Delta_2)}$ (achieved as $c_2 - c_{2,n}$ approaches infinity), approaches infinity as $c_2 - c_{2,n}$ approaches minus infinity and $R(0) = -m + \frac{h}{\gamma r}\left(1-e^{-\gamma\Delta_2}\right)$. The solution to (24) is the unique fixed-point of R.This value is non-negative if and only if $-m + \frac{h}{\gamma r}\left(1-e^{-\gamma\Delta_2}\right) \geq 0$. So

$$c_2 \geq c_{2,n} \qquad \Leftrightarrow \qquad \Delta_2 \geq \hat{\Delta}(h) = -\frac{\ln\left(1-\frac{\gamma r m}{h}\right)}{\gamma}$$

To find the indifference curves, we consider the problem

$$\begin{aligned}
&\max_{\Delta_2,\Delta_3} w - \tau + \frac{q\left(1-e^{\gamma\Delta_2}\right)}{\gamma r}\\
&s.t \text{ IC2, IC3}\\
&\tau = \Pi_2 b_2 + \Pi_3 b_3\\
&b_2 = w - \Delta_2 + m - \frac{q\left(e^{\gamma\Delta_2}-1\right)}{\gamma r} - \frac{h\left(1-e^{-\gamma\Delta_2}\right)}{\gamma r} + \frac{f\left(e^{\gamma(\Delta_3-\Delta_2)}-1\right)}{\gamma r}\\
&b_3 = w - \Delta_3 + m - \frac{q\left(e^{\gamma\Delta_2}-1\right)}{\gamma r} - \frac{h\left(1-e^{-\gamma\Delta_3}\right)}{\gamma r}.
\end{aligned}$$

The three last constraints imply that

$$\begin{aligned}
\frac{\partial\tau}{\partial\Delta_2} &= -\Pi_2\left(1 + \frac{q}{r}e^{\gamma\Delta_2} + \frac{h}{r}e^{-\gamma\Delta_2} + \frac{f}{r}e^{\gamma(\Delta_3-\Delta_2)}\right) - \Pi_3\frac{q}{r}e^{\gamma\Delta_2}\\
\frac{\partial\tau}{\partial\Delta_3} &= \Pi_2\left(\frac{f}{r}e^{\gamma(\Delta_3-\Delta_2)}\right) - \Pi_3\left(1 + \frac{h}{r}e^{-\gamma\Delta_3}\right).
\end{aligned}$$

Then,

$$\begin{aligned}
\frac{d\Delta_2}{d\Delta_3}|_{\bar{U}} &= \frac{-\frac{\partial\tau}{\partial\Delta_3}}{\left(\frac{\partial\tau}{\partial\Delta_2} + q\frac{e^{\gamma\Delta_2}}{r}\right)}\\
&= -\frac{\Pi_2\left(\frac{f}{r}e^{\gamma(\Delta_3-\Delta_2)}\right) - \Pi_3\left(1+\frac{h}{r}e^{-\gamma\Delta_3}\right)}{-\Pi_2\left(1+\frac{q}{r}e^{\gamma\Delta_2} + \frac{h}{r}e^{-\gamma\Delta_2} + \frac{f}{r}e^{\gamma(\Delta_3-\Delta_2)}\right) - \Pi_3\frac{q}{r}e^{\gamma\Delta_2} + q\frac{e^{\gamma\Delta_2}}{r}},
\end{aligned}$$

which is strictly positive when $\Delta_3 = 0$, given that $\Delta_2 > 0$.

7.2 Moral hazard on the job

The incentive constraint $c_1 - c_{1,n} \geq 0$, where

$$\begin{aligned} c_1 - c_{1,n} &= -\hat{m} - \frac{q\left(e^{\gamma\Delta_2} - 1\right)}{\gamma r} + \frac{(q+\delta)\left(e^{\gamma(c_{1,n}-c_2)} - 1\right)}{\gamma r} \\ &= -\hat{m} - \frac{q\left(e^{\gamma\Delta_2} - 1\right)}{\gamma r} + \frac{(q+\delta)\left(e^{-\gamma(c_1-c_{1,n})}e^{\gamma\Delta_2} - 1\right)}{\gamma r} \\ &\equiv \hat{R}\left(c_1 - c_{1,n}\right). \end{aligned}$$

Clearly, $\hat{R}$ is a monotonously decreasing function that has an horizontal asymptote at $-\hat{m} - \frac{q\left(e^{\gamma\Delta_2}-1\right)}{\gamma r} - \frac{q+\delta}{\gamma r}$ (achieved as $c_1 - c_{1,n}$ approaches infinity), approaches infinity as $c_1 - c_{1,n}$ approaches minus infinity and $R(0) = -\hat{m} + \frac{\delta\left(e^{\gamma\Delta_2}-1\right)}{\gamma r}$.

The solution to $c_1 - c_{1,n}$ is the unique fixed-point of $\hat{R}$.This value is non-negative if and only if $-\hat{m} + \frac{\delta\left(e^{\gamma\Delta_2}-1\right)}{\gamma r} \geq 0$, or, equivalently, $\Delta_2 \geq \frac{1}{\gamma}\ln\left(1 + \frac{\gamma r\hat{m}}{\delta}\right)$.

7.3 Adverse selection and no saving

We can write the value functions when the individuals are undertaking the investments as

$$\begin{aligned} V_1 &= -e^{\gamma\tau}e^{-\gamma w}\frac{1 - qp\left(1 - e^{\gamma m}\right)}{r+q} + q\frac{(1-p)V_2}{r+q} \\ V_2 &= -\frac{e^{\gamma\tau}e^{-\gamma b_2}}{r+h+f} + \frac{hV_1 + fV_3}{r+h+f} \\ V_3 &= -\frac{e^{\gamma\tau}e^{-\gamma b_2}}{r+h} + \frac{hV_1}{r+h}. \end{aligned}$$

Solving this yields

$$\begin{aligned} V_1 &= -e^{\gamma\tau}e^{-\gamma w}\frac{\left(1 - \bar{\Pi}_2 - \bar{\Pi}_3\right)\left(1 - qp\left(1 - e^{\gamma\tilde{m}}\right)\right) + \bar{\Pi}_2 e^{\gamma\Delta_2} + \bar{\Pi}_3 e^{\gamma\Delta_3}}{r} \\ V_2 &= -e^{\gamma\tau}e^{-\gamma w}\left(\frac{\frac{h}{h+r}\left(1 - qp\left(1 - e^{\gamma\tilde{m}}\right)\right)\left(1 - \bar{\Pi}_2 - \bar{\Pi}_3\right)}{r}\right) \\ &\quad - e^{\gamma\tau}e^{-\gamma w}\left(\frac{\frac{r+q(1-p)}{q(1-p)}\bar{\Pi}_2 e^{\gamma\Delta_2} + \frac{r+q(1-p)}{q(1-p)}\bar{\Pi}_3 e^{\gamma\Delta_3}}{r}\right), \\ V_3 &= -e^{\gamma\tau}e^{-\gamma w}\left(\frac{\frac{h}{h+r}\left(1 - qp\left(1 - e^{\gamma\tilde{m}}\right)\right)\left(1 - \bar{\Pi}_2 - \bar{\Pi}_3\right)}{r}\right) \\ &\quad - e^{\gamma\tau}e^{-\gamma w}\left(\frac{\frac{h}{h+r}\bar{\Pi}_2 e^{\gamma\Delta_2} + \left(1 + \frac{r(r+h+f+q(1-p))}{(1-p)qf}\right)\Pi_3 e^{\gamma\Delta_3}}{r}\right), \end{aligned}$$

where $\bar{\Pi}_2$ and $\bar{\Pi}_3$ are defined in (13).

The positive solution to

$$e^{\gamma(\Delta_3 - r\tilde{m})} - \left(1 - \frac{h}{h+r}\left(1 - e^{-\gamma\Delta_3}\right)\right) = 0$$

is

$$\Delta_2 - \frac{\ln\left(\sqrt{\left(\frac{r}{2h}\right)^2 + e^{-\gamma r\tilde{m}}\left(1+\frac{r}{h}\right)} - \frac{r}{2h}\right)}{\gamma}$$

which is verified by substitution. This is zero at $m = 0$, and increasing in $\tilde{m}$.Furthermore, since $e^{\gamma(\Delta_3 - r\tilde{m})} - \left(1 - \frac{h}{h+r}\left(1 - e^{-\gamma\Delta_3}\right)\right)$ i) increases in Δ_3, ii) is negative at $\Delta_3 = 0$ and iii) is positive at $\Delta_3 = r\tilde{m}$, the solution is larger than zero but smaller than $r\tilde{m}$.

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