Equity Issuance and Dividend Policy under Commitment*
Work in Progress

Alexis Anagnostopoulos, SUNY Stony Brook†

Eva Cárceles-Poveda, SUNY Stony Brook‡

Albert Marcet, London School of Economics and IAE, CSIC§

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Abstract. This paper studies a model of corporate finance in which firms use stock issuance to finance investment. Since the firm recognizes the relationship between future dividends and stock prices, future variables enter in the constraints and optimal policy is in general time inconsistent. We discuss the nature of time inconsistency and show that it arises because managers promise to incorporate value maximization gradually into their objective function. This shows how one could change managers' incentives in order to enforce the optimal contract under full commitment. We then characterize several cases where time consistency arises and we study different examples where policy is time inconsistent. This allows us to address some outstanding issues in the literature about dividend policy and equity issuance. In particular, our results suggest that growing firms that can credibly commit will pay lower dividends at the beginning and promise higher dividends in the future, consistent with empirical evidence. Our results also suggests that compensation that is tied to stock options creates incentives to inflate prices and pay lower dividends. This is consistent with the empirical evidence of increased stock option compensation and payout through repurchases instead to dividends during the last decades.

Keywords: Stock Issuance; time inconsistency; dividend policy

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†*Correspondence:* Department of Economics, Stony Brook University, NY-11794-4384. *Email:* aanagnostopo@notes.cc.sunysb.edu.

‡*Correspondence:* Department of Economics, Stony Brook University, NY-11794-4384. *Email:* ecarcelespov@gmail.com.

§*Correspondence:* Department of Economics, The London School of Economics and Political Science, Houghton Street, London WC2A 2AE, UK; Tel: +44(0)20-7955-7480. E-mail: # a.marcet@lse.ac.uk.

1. Introduction

The study of consumers' savings and portfolio choices in dynamic stochastic models has progressed enormously in the last thirty years. However, modelling firms' savings and portfolio choices in a dynamic framework has received less attention. Issues on firms' financing have been often assumed away, sometimes by the introduction of complete markets, sometimes by simply assuming that consumers take the decision of how much physical capital to accumulate. This situation is unsatisfactory, since the form of financing matters for the equilibrium quantities under incomplete markets, and we know that consumers do not actually decide on the firm's investment. Moreover, issues such as stocks repurchases, dividend payments, and their interaction with investment cannot be analyzed in such a framework. There is a large empirical literature on firm financing, but it often lacks explicit dynamic modelling, making it hard to formulate hypotheses to be tested with time series data.

Some papers have started to close this gap and explicit modelling has been used to address issues such as firm dynamics in an infinite horizon setting or the effects of financing frictions on investment (see e.g. Hopenhayn (1992), Cooley and Quadrini (2001), Covas and Den-Haan (2007), Cooley, Marimon and Quadrini (2004), Quadrini and Jermann (2005), Gomes (2000, 2003)). In this paper, we contribute to this literature by focusing on equity financing and dividend policy in a dynamic setting.

Our work differs from the previous literature in several aspects. First, since the focus is on equity financing, we abstract from the debt/equity choice. The choice of the firm is, then, about how many stocks to issue (or repurchase), how much to pay out as dividends and how much to invest. Second, most dynamic analyses of financial policy share the common features of ruling out repurchases and obtaining the pecking order theory. In this setting, the pecking order theory would imply that firms will use outside equity only if internal funds are not enough to finance the optimal level of investment. As a consequence, firms will not increase dividends while they are issuing equity. Given the empirical implausibility of this result, we focus on cases under which the pecking order theory does not hold. To do this, we introduce financing frictions and consider different firm objectives under which managers are in conflict with the objective of market investors. These objectives can be interpreted as different managerial compensations and one way to theoretically rationalize them is to consider them as a reduced form agency issues, which we do not model explicitly.

We first point out the following. A firm that is deciding how much equity to sell (or repurchase) in a competitive stock market and, simultaneously, how much dividends to pay should recognize that the stock market imposes a mapping from future dividends to current stock prices, namely, that stock prices equal the discounted sum of dividends. We label this relationship the price-dividend mapping (PD mapping henceforth). Put differently, a rational firm understands that future dividends influence the current stock price and thus constrain how much funding can be obtained and therefore how much can be invested by issuing stocks today. The literature has avoided considering this link by assuming value maximization as a firm objective and/or very particular financing frictions. In these cases, the PD mapping does not constraint the firm's investment choice but we show that it does so in general.

As is well known, if future decision variables (in this case future dividends) constrain today's choices, the problem of the firm might not be recursive in the natural state variables, standard dynamic programming does not apply and the Bellman equation does not hold. The problem of the firm is then of a similar nature to a problem of optimal macroeconomic policy and the optimal solution is generally time inconsistent. Usually, the reason for time inconsistency is that a manager who needs equity financing will promise to pay high dividends in the future to buyers of newly issued stock. Later on, however, if the manager could reoptimize, he will have an incentive to lower current dividend and promise high dividends in the future. In this way the manager can fund current investment and the future high dividends drive the stock price up today. This gives rise to a number of issues on how to formulate and solve the model and how to apply it in several setups.

The first issue to address is of a technical nature, since standard dynamic programming is not appropriate. Using the approach recursive lagrangeans in Marcet and Marimon (2009) the model is formulated recursively by adding the sum of past lagrange multipliers as a co-state variable. This recursive formulation facilitates numerical solutions, theoretical analysis and it provides insights as to the nature of time inconsistency. In particular, it shows that the objective that implements the full commitment solution will compensate the manager according to his own objective function but it will also assign a time-varying weight to the (investors') objective of value maximization. On the one hand, if a re-optimization is possible, this formulation illustrates that the manager will have incentives to set the weights on value maximization to zero. On the other hand, it shows that the "optimal compensation" enforcing the full commitment policy would have to incorporate gradually the objective of investors into the managers' objective.[1]

We argue that the lack of recursivity and the time inconsistency problem is a very general feature arising in most setups in which a firm uses stock issuance to finance its investment. Similarly, it would also arise if a firm uses stock repurchases to distribute profits to the stockholders. We then show that time consistency can be recovered in the following cases: *a)* there is agreement between managers and shareholders (value maximization) or *b)* the manager is compensated only with cash flows. These results are useful to a researcher who wishes to ignore issues of time consistency. This researcher can set up his/her model as one of these special cases and, using our results, can use the standard set of state variables that are dictated by the Bellman equation.

The previous findings complement the literature on firm dynamics and financing frictions mentioned earlier, where the PD mapping is not mentioned in spite of the fact that firms use equity issuance. At first sight it might look as though these papers consider firms that ignore the PD mapping. Instead, they assume value maximization and fall therefore into the categories mentioned above where the issue of time inconsistency is absent.[2] While these models derive important implications of financing frictions for investment, analyzing financial policy with value maximizing firms is very limiting. First, from an empirical point of view, the papers imply a pecking order result: firms finance their investment by only issuing equity in the initial periods and only later pay dividends. As mentioned before, this behavior is not validated by observed firm behavior, since many firms actually pay dividends and issue stocks in the same period.[3] Second, from a theoretical point of view, the literature on general equilibrium and production with incomplete markets (see McGill and Quinzii (1996) for a compendium) has pointed out that value maximization might not be agreed upon heterogeneous shareholders and it is therefore not a universally valid principle.[4] In other words, many issues of interest can only be addressed if there is a conflict of interest between managers and investors.

[1] There is a vast literature where the optimal compensation contract is explicitly modelled and a recently growing literature studies this in dynamic contexts (see e.g. Danthine and Donaldson (2008), Atkeson and Cole (2005), DeMarzo et al (2008) or Cooley et al (2009)).

[2] Many macro models have assumed a fixed dividend rule (for example, dividends equal earnings) or they have assumed no equity issuance. Of course, the issues we discuss do not arise in these models, since the PD mapping plays no role under these restrictions.

[3] This poses no direct problem to the results in the papers just mentioned, since they focus on the behavior of firms' investment.

[4] This literature has proposed various alternatives to determine endogenously the objective function of the firm. Some of the first proposed solution concepts were by Drèze (1994) and Grossman and Hart (1979). We take from this literature the observation that value maximization is not to arise as a natural objective of the firm so that, according to our results, time inconsistency is likely to arise in equity financing setups. Unlike this literature, however, we assume a certain objective of the firm, although in the examples we consider we do check that the environment justifies the objective assumed.

Finally, we use our setup to analyze several examples that differ in the manager's compensation and the source of disagreement between managers and shareholders. We first consider a simple case of internal versus external investors. We assume that an internal investor controls the firm by holding a sufficient amount of stocks, acts as manager and finances investment by selling equity to investors. To capture the idea that the manager's interests are closely linked to the firm we assume that his/her only income comes from the dividend yield of his/her shares. To capture the idea that investors draw income from many sources we assume they are risk-neutral. We find that with enough transaction costs, the optimal policy for a growing firm that can credibly commit is to pay low dividends in the initial periods and higher dividends in the future. However, if the firm cannot commit, such a profile would not be credible and dividend payments decrease over time. These results indicate that commitment could be part of the explanation for the observation that growing firms pay low dividends.

The previous example can also be interpreted as one where managers are compensated with stocks. According to the survey by Murphy (1999), however, other important components of US CEO compensation include a fixed salary, a bonus linked to performance and stock options. Given this, we study other examples where the compensation corresponds to different combinations of these. First, we assume that managers are compensated with bonuses linked to cash flows and a fixed salary component. In the absence of disagreement, optimal policy is time consistency and we therefore assume that there is a probability that the manager gets fired every period. This generates a conflict of interest between investors and managers suffering from short termism. In this case, if the manager is allowed to reoptimize, we show that he will try to lower the optimal capital and pay lower dividends. Finally, a last example assumes that managers are compensated with bonuses linked to cash flows as well as one period stock options. In this case, managers have strong incentives to inflate stock prices and reduce the dividend payout, a fact that is consistent with the increase in stock option compensation and the substitution of dividend payouts with repurchases during the last decades.

To summarize. Our theoretical results show that time inconsistency is not a problem when managers agree with the shareholders or when they are compensated only with bonuses linked to cash flows. However, the previous examples illustrate that small deviations from cash flow compensation lead to time inconsistency when firms use equity financing. In the absence of institutions that can enforce full commitment, this could potentially make equity financing very costly, potentially providing an explanation for several empirical regularities. For example, our framework could partly explain why in some countries (like Germany) there are relatively new firms listed in the stock market, while the opposite happens in countries like the US, where the institutional framework makes the problem of time consistency less severe through common leverage buyouts.

Our work is related to several other strands in the literature. The link between future dividends and stock prices is implicitly recognized in the seminal work of Modigliani and Miller (1961). In their work, firms realize how their initial value is determined by future dividend payments and this assumes that firms understand the PD mapping in period zero. The difference is that, in our paper, due to market incompleteness, the PD mapping influences choices every period. As to the issue of time inconsistency, the only explicit mention in the literature of corporate finance we have found is in Miller and Rock (1985), who study a two period model with private information. In contrast, our paper assumes full information but disagreement between stockholders and managers. Given this, our work provides an alternative channel for the presence of time inconsistency.

There are several issues which our paper does not address. While we compare the solution with and without commitment for some of our examples, our focus is on the case where firms have full commitment. This is obviously an extreme assumption. The alternative is to assume that firms have no commitment and they follow time consistent policies, in which case one could use the solution techniques designed by Klein, Krusell and Ríos-Rull (2007) in a number of papers on optimal fiscal policy. This is also an extreme assumption that is not validated by informal observation, since CEO's often justify paying lower dividends in a given period by promising investments and, therefore, future dividends. We believe that the assumption of full commitment is the most reasonable place to start. The issues of commitment and how to support it by reputation, institutional design, or manager compensation can only be even discussed if we have the full commitment solution at hand.

Other important issues we do not address are why dividends are smooth or why dividends are paid at all. There is a very basic reason why firms pay dividends in our setup: under rational expectations and non-bubble prices zero dividends in all periods imply zero stock prices, so that the firm would be unable to finance itself with stock. As to dividend smoothing, some versions of our model simply impose that firm managers dislike dividend variability. This could be thought of as capturing an optimal contract in which the firm's board of directors has solved an agency problem determining that the payout to the manager should be according to dividend payments or the stock price of the firm. This has been justified in various ways by the literature of hidden information but we take the compensation as given and discuss other issues. These are all interesting issues that we leave for future research.

The paper is organized as follows. The model is presented in Section 2. Section 3 presents the main theoretical results. It formulates the problem recursively under full commitment, it discusses the nature of time inconsistency and it presents an example with an analytical solution where a proof of time inconsistency is provided. Section 4 characterizes the cases under which the solution is time consistent. Section 5 present other examples that differ in the compensation of the manager. Finally, Section 5 summarizes and concludes.

2. The Model

We first set up a generic model where firms take into account the PD-mapping. This model can be taken literally as a partial equilibrium model or the firm behavior can be embedded in a general equilibrium setting.

2.1. The Choice Set of the Manager. Time is discrete and indexed by $t = 0, 1, 2...$Firms take as given an exogenous stochastic process $\mathbf{z} = \{z_t\}_{t=0}^{\infty}$. $z_t \in \mathbb{R}_+^2$ includes (θ_t, m_t), where m_t is an exogenous market discount factor to be endogeneized later and θ_t is an exogenous productivity shock that follows a Markov process. As usual, z denotes a possible realization of z_t, while z^t denotes histories of $\mathbf{z}$ up to period t. Moreover, Ω^t denotes the set of possible histories z^t and Ω denotes the state space of $\mathbf{z}$. The firm has full information and observes z^t in period t.

Each period, the firm acquires investment goods and converts them into capital to be used next period. The firm produces a good using capital as the only input with the production function $\theta_t f(k_{t-1})$. The price of investment goods is constant and normalized to one. Capital depreciates at a constant rate η and the cash flow of the firm is therefore given by:

$$n_t = \theta_t f(k_{t-1}) + (1-\eta)k_{t-1} - k_t \tag{1}$$

where $k_t - (1-\eta)k_{t-1}$ is gross investment.

Each period, the firm can obtain external financing by issuing new stocks that are traded at the (per share) stock price p_t. Letting s_t be the amount of stocks in the firm, total equity financing received by the firm at time t is $p_t(s_t - s_{t-1})$. The firm distributes a dividend per share of d_t and it can also repurchase stocks. A constraint relating the dividend and price will be specified later.

The choice variables of the firm at each period are capital, stocks issued, dividend and stock price. Denoting these choice variables by $x_t \equiv (k_t, s_t, d_t, p_t)$, the firm chooses sequences

$\{x_t\}_{t=0}^{\infty}$, where each x_t maps Ω^t into R_+^4. Throughout the text, we denote $\mathbf{x} = \{x_t\}_{t=0}^{\infty}$ and $\mathbf{k} = \{k_t\}_{t=0}^{\infty}$.

We introduce incomplete markets in an extreme way by assuming that the firm can not issue or hold any asset other than its own stock. In addition it has to pay costs $\mathcal{C}_t \equiv \mathcal{C}(x_t, x_{t-1}, z_t)$ for a fixed function $\mathcal{C}$. This general formulation encompasses capital adjustment costs, financial transaction costs of issuing or repurchasing equity, costs of changing dividends and many other frictions that have been considered in the literature. It will also include the compensation of the manager when this is given by a fixed function of the firm's performance.

To simplify notation, we define net cash flows (net of financing, adjustment and managerial costs) as $n_t^c = n_t - \mathcal{C}_t$. The previous elements consolidate in the following budget constraint of the firm:

$$d_t s_{t-1} + k_t - (1-\eta)k_{t-1} \le \theta_t f(k_{t-1}) + p_t\,(s_t - s_{t-1}) - \mathcal{C}(x_t, x_{t-1}, z_t) \tag{2}$$

or, equivalently,

$$d_t s_{t-1} \le n_t^c + p_t\,(s_t - s_{t-1})$$

in addition to the non negativity constraints $k_t \ge 0$ and $d_t \ge 0$.

This formulation implies that a start up firm can increase its capital in two ways. It can retain earnings and pay low dividends (internal investment) or it can use equity financing (setting $s_t > s_{t-1}$). Each of these options may imply different financing costs and manager compensations which are summarized by $\mathcal{C}_t$. Similarly, in the face of a negative (positive) shock, the firm can sustain a level of investment by lowering (increasing) dividends or by issuing new (repurchasing old) equity. In general, however, the firm will not be able to invest optimally, as if it had access to complete markets due to two sources of frictions: *i)* it can only accumulate one asset (namely, the firm's own stock) and *ii)* it faces financial costs reflected by $\mathcal{C}_t$. Most aspects of this formulation are in line with the recent literature on dynamic corporate finance. While many of the papers in the literature do not have both a dividend and equity choice, similar costs are introduced.

2.2. The PD Mapping. One fundamental difference between this paper and the previous literature is that we include the stock price p in the choice set of the firm. However, this choice is not unrestricted. Since the new stocks are purchased by external investors the manager has to choose combinations of prices and dividends such that the investors will indeed purchase the stock. We assume that investors purchase the stock if the following condition holds:

$$p_t = \delta E_t m_{t+1}\,(p_{t+1} + d_{t+1}) \tag{3}$$

Most dynamic stochastic models under rational expectations imply the above relationship, where m_t is the stochastic discount factor for the marginal investor. Throughout the paper, we assume that the firm takes this discount factor as given[5]. Obviously, the standard case of risk neutral investors with a constant discount factor corresponds to $m_t = 1$.

The firm also realizes that the transversality condition of the investors' problem has to be satisfied. Formally, let s_t^I be the stocks owned by investors, where $s_t^I = s_t - s^m$ and s^m are the stocks of the manager, which are assumed to be fixed. The firm understands that the following transversality condition holds a.s.:

$$\lim_{j\to\infty} E_t \delta^j M_t^j p_{t+j} s_{t+j}^I = 0 \tag{4}$$

[5] This is justified, for example, if there is a continuum of identical firms subject to the same shock or, more generally, if each firm has a minuscule impact on the consumption of the market stockholders.

where M_t^j is the compounded stochastic discount factor:

$$M_t^j \equiv \prod_{\tau=1}^{j} m_{t+\tau}$$

As usual, this transversality condition requires that the firm chooses a path for prices and stocks that satisfies one of two conditions. Either the stock price grows at a rate lower than the inverse of the discount factor or the number of stocks in the hands of investors goes to zero. In other words, under the above constraint, the firm understands that it could afford to never pay dividends, but in this case the firm would have to compromise to retire all stock outstanding sufficiently fast by setting $\lim_{j\to\infty} E_t s_{t+j}^I = 0$. This is because with zero dividends (3) would imply that the stock price goes to infinity at the rate δ^{-1} so that in order for (4) to hold the firm has to repurchase eventually all stock.

In the remainder of the paper, we only consider cases where the firm never repurchases *all* of the stock that has been previously issued. We make this assumption so that the stock price is the discounted present value of dividends. This is assumed for simplicity. In principle, we could impose (3) and (4) directly, but this would complicate the analysis. More precisely, we assume the firm acts as though a constraint:

$$s_t^I \geq \overline{s} \tag{5}$$

holds for all periods, for some positive $\overline{s}$. Some firms in the real world do not behave in this way and indeed internal investors sometimes repurchase *all* the firm's stock in order to avoid interferences from outside investors. But this does not happen very often. Once a firm becomes public it often stays public. We conjecture that this constraint would arise endogenously in setups where the costs for a large firm of re-entering the stock market after retiring all the stock would be very large, but we do not pursue this possibility in this paper. Furthermore, there are legal limits and other barriers to repurchases. Imposing (5) is therefore a relevant possibility.

Clearly, this inequality together with (4) implies that $\lim_{j\to\infty} E_t \delta^j M_t^j p_{t+j} = 0$ a.s. for all t. Given this and condition (3), we can write the stock prices as follows:

$$p_t = E_t \sum_{j=1}^{\infty} \delta^j M_t^j d_{t+j} \tag{6}$$

Equation (6) maps future dividends into today's stock price in a standard way. Throughout the paper, we call this relationship the price-dividend mapping or PD mapping.

As mentioned earlier, the fact that the manager recognizes that (6) constrains his choice of investment is what distinguishes this paper from the literature. In previous papers on dynamic corporate finance this relation is not written explicitly. More precisely, the papers in the literature only introduce in the constraints of the firm the value of equity issued $e_t \equiv p_t (s_t - s_{t-1})$ and the value of total dividend payments $D_t = d_t s_{t-1}$ without distinguishing between s and p or s and d. Moreover, the costs and frictions are also functions of e and D. In this case, we have that $x_t = (k_t, e_t, D_t)$ and the budget constraint of the firm becomes:

$$D_t + k_t - (1-\eta)k_{t-1} \leq \theta_t f(k_{t-1}) + e_t - \mathcal{C}(x_t, x_{t-1}, z_t)$$

Later on, we will discuss the conditions under which this is equivalent to our setting.

Finally, we would like to point out that the fact that firms consider the PD mapping as a constraint in their feasible set corresponds to a standard definition of competitive behavior under incomplete markets and rational expectations. This needs some more careful justification because, at first sight, it might seem that there is an element of monopolistic behavior in the problem defined above, since firms choose stock prices. But the fact is that a firm that behaves competitively in the stock market should choose stock prices and dividends subject to the PD mapping.[6]

To explain this point in detail let us build an analogy by considering two types of firms who face slightly different financing environment as the firm considered above. First consider a firm that has to finance investment under incomplete markets but the firm can only issue bonds of two different maturities. Say, the firm can only issue short bonds that mature in one period and long bonds that mature in N periods, for a given $N > 1$. Both are real riskless bonds that pay one unit of consumption at maturity. Assume, for simplicity, that the firm never buys back any of these bonds, so that the budget constraint of the firm is

$$b_{t-1}^1 + b_{t-N}^N \leq n_t^c + p_t^{b,1} b_t^1 + p_t^{b,N} b_t^N$$

where b_t^1, b_t^N are the amount of short and long bonds issued by the firm at time t and $p_t^{b,1}$, $p_t^{b,N}$ are the corresponding bond prices. It should be uncontroversial to claim that a standard definition of competitive equilibrium in this case would entail assuming that the firm chooses $\left\{b_t^1, b_t^N, n_t^c\right\}$ taking as given the price process $\left\{p_t^{b,1}, p_t^{b,N}\right\}$. The firm chooses the total cost of the portfolio of bonds issued $p_t^{b,1} b_t^1 + p_t^{b,N} b_t^N$, but it is obviously behaving competitively since it takes prices as given.

Suppose now that we change this model very slightly. In particular, let's assume that the firm issues BP_t units of a portfolio of bonds. Investors can purchase units of this portfolio from the firm, but the short or long bonds can not be purchased separately. Let us denote the units of the short bond by sh_t so that $(1 - sh_t)$ is the share of the long bond in each portfolio. The firm can choose the share of long and short bonds sh_t, and it can choose the amount of bond portfolios issued BP_t each period. The firm sells each unit of portfolio of bonds for a price PP_t. Let us call this a *bond-portfolio-financing (*BPF) firm and let us assume again there is no buyback of previously issued bonds.

In this setup the firm has to repay $sh_{t-1}BP_{t-1}$ short bonds plus $(1 - sh_{t-N}) BP_{t-N}$ long bonds in period t. Assuming again there is no buyback of previously issued bonds, the budget constraint of a BPF firm is

$$sh_{t-1}BP_{t-1} + (1 - sh_{t-N}) BP_{t-N} \leq n_t^c + BP_t \; PP_t$$

A general equilibrium model will, in general, deliver that the following holds in equilibrium

$$PP_t = sh_t \, p_t^{b,1} + (1 - sh_t) \, p_t^{b,N} \tag{7}$$

A natural definition of competitive behavior for a rational BPF firm would say that the firm takes the process $\left\{p_t^{b,1}, p_t^{b,N}\right\}$ and it takes as given that (7) holds. The firm can choose the share sh_t and change the price of the portfolio PP_t accordingly, but the firm behaves competitively in the bond market because it takes bond prices and the mapping (7) as given. In fact, the BPF firm is not doing anything different from the firm issuing only long and short bonds described above, it is just packaging the bonds differently. Equation (7) should then become a constraint in the BPF firm's problem and PP_t would become a choice variable in the firm's problem. In other words, a rational BPF firm is behaving competitively by choosing PP_t subject to (7).[7]

[6] We have found that economists who are active in the finance or corporate finance literature find the assumption quite natural. On the other hand economists active in the macroeconomics literature sometimes view this assumption as though we include an element of monopolistic behavior on the part of the firm. It is to the later, that the following discussion is mainly addressed.

[7] To make an even more basic analogy: consider a competitive firm that produces two goods jointly. For

The previous example shows that a BPF firm that took PP_t as given, independent of sh_t, would behave rationally. By the same reasoning, a stock financing firm that ignored how the choice of future dividends influences today's stock price would behave irrationally. Consider now our setting. From a competitive equilibrium point of view a stock is a composite asset, equivalent to a portfolio of many contingent claims that deliver a consumption good in each future period and for each possible realization. The dividend $d_t(z^t)$ is the amount of contingent claims that pay one unit of consumption at t if z^t occurs. The discount factor $\delta^j M_t^j$ is the price of a contingent claim that pays one consumption unit in period $t + j$ in terms of period t consumption units. By choosing a history of dividends d, firms in our paper offer a different composite portfolio s, and firms simply understand how the market values the different elements of this asset. This is reflected in the fact that the price-dividend mapping (6) is a constraint in the firms' problem.

A similar reasoning has been used in the corporate finance literature before. Most notably, the Modigliani-Miller theorem only works if firms understand how future dividends map into the initial stock price and, therefore, into the value of the firm. More generally, the definition of firms' objective functions under incomplete markets proposed by Dreze (1974), Grossman and Hart (1979) and recently by Bisin et al (2010) take for granted that a firm understands (or conjecture) how their future dividend choice map into the current initial stock price. As in the present paper, this literature also assumes that firms take the stochastic discount factor as given. We simply extend the same reasoning to all periods and assume that when the stock price appears in the firm's budget constraint in period t (2) the firm understands how this p_t is linked to future dividends.

2.3. Feasible Allocations. We state that a nonnegative sequence $\mathbf{x} = (\mathbf{k}, \mathbf{s}, \mathbf{d}, \mathbf{p})$ is feasible if it satisfies

$$d_t s_{t-1} \leq n_t^c + p_t \left(s_t - s_{t-1}\right) \tag{8}$$

$$n_t^c \equiv \theta_t f(k_{t-1}) - k_t + (1-\eta)k_{t-1} - \mathcal{C}(x_t, x_{t-1}, z_t) \tag{9}$$

$$p_t = \delta E_t M_t^1 \left(p_{t+1} + d_{t+1}\right) = E_t \sum_{j=1}^{\infty} \delta^j M_t^j d_{t+j} \tag{10}$$

for all $t = 0, 1, ...$ a.s. and the transversality condition (4) holds.[8]

One of the key differences between our setting and previous papers in dynamic corporate finance is that due to constraint (10) future choice variables influence today's feasible set. More precisely, future dividends influence today's value of equity and, therefore, today's investment. This means that standard dynamic programming does not apply and we have to

example, consider a winery that produces white and red wine. The winery sells bottles of red and white wine in neatly packaged wooden boxes, 6 bottles in each box. The firm chooses how many bottles of red or white wine go in each box. It would be natural to assume that a competitive firm should recognize that the price of the box depends on how many bottles of each kind are included. The firm can in a way choose the price of the 6-bottle box, by choosing how many bottles of each kind are included, but this is compatible with price-taking behavior since the firm takes as given the way that the market of wine drinkers values boxes with a different number of whites or reds.

[8] Our formulation is consistent with stock splits being irrelevant. A stock split occurs in actual corporations when a firm decides that each stock previously issued is converted into, say, two stocks. In this case each shareholder will now receive half the dividends per share it would have received otherwise but it now has twice as many shares. Stock splits are performed in the real world for accounting reasons and in a framework like ours they should be irrelevant. In fact, we could introduce a stock split by assuming the firm can give ν_t shares for each previously owned share, then s_{t-1} should be replaced by $\nu_t s_{t-1}$ in the above budget constraint, dividends, prices and future stocks would be divided by ν_t and financing costs and objectives are unchanged by such an action. Our formulation of (8), however, does not introduce such a ν. It assumes that each old share is always one old share, while newly issued equity $(s_t - s_{t-1})$ is sold in the market and it is not given out to former shareholders. Therefore we simply rule out stock splits.

resort to other formulations in order to formulate the model recursively. A similar difficulty is commonly found in the macro literature on Ramsey equilibria of optimal fiscal and monetary policy and models of risk sharing with dynamic participation constraints. Throughout the paper we will make analogies to this literature to clarify the nature of the results.

We now discuss some features of the firms' feasible set. The first result rewrites the period-by-period constraint (8) in a present value discounted form.[9]

Lemma 1. *A non negative sequence $x = (k, s, d, p)$ is feasible if and only if*

$$(d_t + p_t)\, s_{t-1} = E_t \sum_{j=0}^{\infty} \delta^j M_t^j n_{t+j}^c \tag{11}$$

for all $t = 0, 1, ...$ a.s.

The proof uses forward substitution in (8) and it is standard, except that one needs to check that equation (10) also holds. This result will be useful in various parts of the paper and it shows how incomplete markets restricts the firms' choice. As is well known, if the firm would have access to complete markets it would be enough to impose equation (11) only in period $t = 0$, namely

$$(d_0 + p_0)\, s_{-1} = E_0 \sum_{j=0}^{\infty} \delta^j \; M_0^j \; n_j^c \tag{12}$$

But in the presence of incomplete markets equation (11) has to hold for all t and all realizations of uncertainty. This means that in addition to (38) the firm faces many more constraints. These additional constraints are called measurability conditions and they are stated precisely in the appendix. We discuss this in what follows with an example.

Consider the case of risk neutral investors $m_t = 1$ and no financing costs $\mathcal{C}_t \equiv 0$. Given any cash flow $\mathbf{n}$, a constant stream of dividends $d_t = d = (1 - \delta)\, E_0 \sum_{j=0}^{\infty} \delta^j n_j$ for all t does satisfy (38) for the stock price $p_t = p = d\frac{\delta}{1-\delta}$ and, therefore, (p, d) would be a feasible choice if the firm would have access to complete markets. Indeed, this would be the optimal choice if the manager would have a concave utility function depending on dividends. The budget constraint of the firm at $t \geq 1$ will only be satisfied if:

$$s_{t-1} = \frac{E_t \sum_{j=0}^{\infty} \delta^j n_{t+j}}{d + p}$$

Clearly, if cash flows are stochastic, the right hand side of this equation depends on information up to t, while the left side can only be chosen contingent on information up to $t - 1$. Alternatively, if the firm can only use stocks to save or disave (8) would imply $s_t = -\frac{n_t^c}{p} + \delta^{-1} s_{t-1}$, which gives an explosive solution for s_t and therefore is not feasible (it violates the transversality condition of the investors) whenever cash flows are stochastic. Therefore, constant dividends is not a feasible policy under the sort of incomplete markets that we consider in this paper, but it would be a feasible policy with complete markets.

Since we seek a theory of equity issuance and dividend policy we have to depart sufficiently from complete markets so as to break the Modigliani Miller result of irrelevance of financial policy. We already mentioned that we depart from complete markets in two ways: *i)* only one asset is available to the firm (the firm's own stock), *ii)* there are financial frictions $\mathcal{C}_t$. The following proposition shows that *i)* alone is not enough to break the Modigliani-Miller result.

[9] The proof of this lemma and of all the other results throughout the paper is provided in Appendix A.

Proposition 1. *Assume $\mathcal{C}_t \equiv 0$.*

1. *Any sequence* $\mathbf{k}$ *with non-negative discounted cash flows, i.e. if* $E_t \sum_{j=0}^{\infty} \delta^j M_t^j n_{t+j}^c \geq 0$ *a.s. for all* t *is feasible.*

2. *For any* $\mathbf{k}$ *as in part 1. there are many feasible choices for the financial variables* $(\mathbf{d}, \mathbf{s}, \mathbf{p})$ *that are compatible with* $\mathbf{k}$.

3. *The* $\mathbf{k}$ *that maximizes firm's value is feasible.*

This proposition establishes that in the absence of frictions the firm can effectively complete the markets by appropriately using financial policy. Furthermore, many financial choices on equity and dividend are feasible for a given capital sequence. In other words, the first best that maximizes value can be achieved as under complete markets. One of the implications of this is that in order to break the Modigliani-Miller irrelevance result, we have to assume that $\mathcal{C}_t \neq 0$ and/or depart from the objective of value maximization.

2.4. Firm Behavior. We assume that the objective of the firm manager is to maximize

$$E_0 \sum_{t=0}^{\infty} \delta^t V(x_t, x_{t-1}, z_t) \tag{13}$$

where V is some given function V. The previous function encompasses many issues that arise in corporate finance and firm financing and it includes different objective functions that we will use in the examples throughout the paper. For example, we will look at a case of internal versus external investors, a case with short termism of the manager and a case where compensation includes stock options.

The firm's problem is to choose $\mathbf{x}$ in order to maximize this function subject to (8)-(10) and the investors' transversality condition, taking s_{-1}, k_{-1} and the processes for z as given. Our solution assumes full commitment on the part of the firm to the preannounced policy, since the firm decides in period zero what the solution will be in all future periods for any possible contingency. Later on, we will show that if the manager could reoptimize in the future, he would change the preannounced plan. The above firm behavior can be embedded in a general equilibrium model in a standard way, namely, by assuming that the stochastic discount factors are determined in equilibrium by the marginal investors. In this case,

$$m_t = \frac{u'(c_t)}{u'(c_{t-1})}$$

Finally, note that we take V as given and thus avoid the issues of endogeneizing corporate control as discussed in the literature on general equilibrium, incomplete markets and endogenous production. This literature points out that the objective of the firm is not well defined in an incomplete markets setting and that heterogeneous stockholders will generally disagree as to what should be the objective of the firm. In short, each shareholder has a different stochastic discount factor and each would like the firm to maximize the value of the firm according to their own stochastic discount factor. Various alternatives have been postulated in this literature to determine the firm's objective and they postulate different equilibrium concepts (see e.g. Drèze (1974), Grossman and Hart (1979) and Bisin et al (2010)). In this paper V is not endogeneized, but our analysis could be used in order to determine V endogenously in a fully dynamic model by implementing the equilibrium concepts in the literature.

3. Recursive Formulation and Time Inconsistency

Due to the presence of constraint (6) in the problem of the firm, the Bellman equation does not hold and, in general, the optimal choice is not a time invariant function of the natural state variables. Formally, denoting the optimal choice x_t^*, there is no reason to expect that $x_t^* = F(x_{t-1}^*, z_t)$ for some time invariant function F[10]. This happens in many other models where future variables appear in the current choice set. Moreover, as pointed out by Kydland and Prescott (1977), the solution is likely to be time inconsistent, in the sense that the solution promised by the firm in period zero is such that, if in a future period τ the firm is allowed to re-optimize, the decision will in general be different from the optimum initially promised.

The assumption of full commitment on the part of a firm manager may be a questionable description of the way managers behave. Arguably, investors may not believe blindly what firm managers state. Our focus on the full commitment solution is justifiable. First of all, this is the first best given the technology and financial frictions, so that this will always be a benchmark for any solution under some sort of partial commitment. Second, only by making commitment an issue can we study how to build commitment in a firm's financial policy. As we will show later, depending on the objective function for the firm and the form of financial frictions, the optimal policy under full commitment happens to be time consistent, and we can study what incentives the manager can receive to seek a time consistent policy. Third, while full commitment may be a questionable assumption, any other assumption on the commitment of the manager is also questionable. Perhaps managers do not fully commit as in the solution studied in this paper, but managers certainly do not default constantly on past promises, since this carries costs of various kinds. Given this, full commitment is as good a place to start. [11]

3.1. Recursive Formulation. We now show how the problem of the rational firm can be written recursively following the approach of Marcet and Marimon (2009). To have a general setting we assume that all the inequalities faced by the firm are summarized by the function $\mathcal{B}_t \equiv \mathcal{B}(x_t, x_{t-1}, z_t)$. The Lagrangian for the firm's problem can then be written as:

$$L = E_0 \sum_{t=0}^{\infty} \delta^t \left[V(x_t, x_{t-1}, z_t) + \lambda_t \left(E_t \sum_{j=1}^{\infty} \delta^j M_t^j d_{t+j} - p_t \right) \right.$$
$$\left. + \gamma_t \left(n_t^c + p_t(s_t - s_{t-1}) - d_t s_{t-1} - \mathcal{C}_t \right) - \xi_t \mathcal{B}_t \right]$$

where γ_t and ξ_t are the multipliers associated with the period t budget constraint and the non negativity constraints and λ_t is the multiplier on the PD mapping (10). To obtain a recursive Lagrangian, one has to rewrite the previous Lagrangian so that future variables do not appear in the current return function. This can be achieved as follows. In the first line of the Lagrangian (the second line can stay as is), apply first the law of iterated expectations so that the term E_t above disappears. Second, group terms depending on d_t to obtain:

$$L = E_0 \sum_{t=0}^{\infty} \delta^t \left[V(x_t, x_{t-1}, z_t) + d_t \left(M_{t-1}^1 \lambda_{t-1} + ... + M_0^t \lambda_0 \right) - \lambda_t p_t \ ... \right.$$

Then, we can introduce a new variable μ with the following law of motion for $t \geq 0$:

$$\mu_t = \mu_{t-1} m_t + \lambda_t \text{ with } \mu_{-1} = 0 \tag{14}$$

[10] Most commonly the state variables would not include p_{t-1} and d_{t-1}. Given the general objective and frictions discussed earlier, it could happen that those appear as natural state variables. See later for examples where this is the case.

[11] SeeKlein, Krusell and Rios-Rull (2007) or Domínguez (2009) for applications to fiscal policy under no commitment and see Debortoli and Nunes (2008) for applications to government debt policy with partial commitment.

while the Lagrangian can be written as:

$$L = E_0 \sum_{t=0}^{\infty} \delta^t \left[V(x_t, x_{t-1}, z_t) + \mu_{t-1} m_t d_t - \lambda_t p_t \right.$$
$$\left. + \gamma_t \left(n_t^c + p_t(s_t - s_{t-1}) - d_t s_{t-1} - \mathcal{C}_t \right) - \xi_t \mathcal{B}_t \right]$$

After rewriting the problem in this way it is clear that future variables do not enter today's objective function and that now only past μ's appear in the objective. This suggests that the optimal choice (with the additional assumption that (θ, m) is Markovian) satisfies:

$$(x_t^*, \lambda_t^*) = F(x_{t-1}^*, z_t, \mu_{t-1}^*)$$

for a time-invatiant policy function F. Marcet and Marimon (2008) provide conditions guaranteeing that this is indeed the case and they show a saddle point functional equation that plays the role of the Bellman equation. This means that solving the model amounts to finding a policy function F that satisfies either the saddle point functional equation or the first order conditions for optimality. There are various methods to find this policy function numerically. The key observation here is that μ now becomes a state variable (sometimes called a co-state variable) and that the multiplier λ needs to be added to the list of decision variables. Given the initial conditions x_{-1}, z_0, and adding the initial conditions $m_0 = 1, \mu_{-1} = 0$, this policy rule, together with (14), determines the whole optimal path.

Note that the multiplier μ_{t-1} captures the promises that have been made in the past about future dividends. Past promises bind the current choice for d_t. Since there are no past promises to be kept at the beginning of time, the optimal choice entails setting $\mu_{-1} = 0$. However, at $t = 1$, there is an inherited promise from period 0, $\mu_0^* = \lambda_0^*$, which summarizes the cost of the promises made about future dividends. Since the firm is fully committed to the optimal plan it will have to remember the promise made in all past periods about today's dividend payments, and this promise is summarized in the value μ_{t-1}. Similarly, as we consider dividends further away in the future (d_2, d_3 etc.), these are linked with promises made in past periods. As reflected by its law of motion, the co-state μ_{t-1} adds up all of these past promises and summarizes them in a single number.

3.2. Time Inconsistency. In what follows, we discuss the issue of time inconsistency that can potentially arise when firms are rational. Intuitively, the reason for time inconsistency is the following. A firm that wishes to issue stock today will in general have incentives to announce that it will postpone dividend payments. By paying a low dividend in the first period and by promising high future dividends today's per-share stock price is higher and more funds can be raised by today's equity issuance. However, in future dates, after some investors have already bought the firm's stock, the manager will have an incentive to deviate from the previously announced policy, namely, the firm will announce that today's dividends will be lower than had been promised, but that they will be higher in the future. This is because adjusting today's dividends will not affect the current stock price, since it depends only on future dividends. Thus, there is a permanent pressure to announce a temporary reduction of dividends, relative to what had been promised.

To show this formally, we first describe a standard definition of time consistency. Denote the full commitment solution by x_t^*. Given a time period $\tau > 0$ and a realization of uncertainty z_τ, define the "time-τ reoptimization problem" as:

$$\max_{\{x_t\}_{t=\tau}^{\infty}} E_\tau \sum_{t=0}^{\infty} \delta^t V(x_{t+\tau}, x_{t+\tau-1}, z_{t+\tau}) \text{ s.t.}$$

$$\text{(8)-(10) for all } t \geq \tau$$
$$\text{given } x_{\tau-1} = x_{\tau-1}^*$$

where x_t is now a function of histories of realizations from τ to t. This is the solution that would arise if, having followed the full commitment solution up to time τ, the manager could re-optimize and choose the best solution from then on, ignoring the plans that were involved in the solution $\mathbf{x}^*$. Denote the solution to this problem by $\mathbf{x}^{\tau *} = \{x_t^{\tau *}\}_{t=\tau}^{\infty}$.

Definition 1. *We say that the problem is time consistent at time τ if $\mathbf{x}^{\tau *} = \{x_t^*\}_{t=\tau}^{\infty}$* a.s.. *We say that the problem is time consistent if it is time consistent for all $\tau > 0$.*

Time inconsistency arises if the problem is not time consistent. The fact that time inconsistency may arise in the present setup is reflected formally in the recursive formulation. If the manager reoptimized in period τ, he would want to follow a policy that implies resetting $\mu_{\tau-1} = 0$ and following the optimal policy F from then on, since this is the solution to the full commitment problem. But if the manager is fully committed to the pre-announced policy, he will plug in $\mu^*_{\tau-1}$ in the policy function F.

Time inconsistency is usually seen as a source of instability. The full commitment solution can only be implemented if the manager can convince investors that he/she will indeed follow the full commitment plan and that it will never reoptimize. In the presence of time inconsistency, the manager will try to establish credibility that he will follow the full commitment solution, but of course the temptation to default on past promises and to reoptimize is very strong and it may undermine how the whole system works.

The method of Marcet and Marimon provides another way to express the problem of time inconsistency and, at the same time, of discussing how the incentives of managers should change in order to restore time consistency. In particular, Marcet and Marimon show that the full commitment solution $\mathbf{x}^*$ would arise as a solution of the reoptimization problem if the objective function is modified appropriately. In our application this translates into the following statement. Consider a "reoptimization problem" with the following objective function:

$$E_\tau\left(\sum_{t=0}^{\infty}\delta^t V(x_{t+\tau}, x_{t+\tau-1}, z_{t+\tau})\right) + \frac{\mu^*_{\tau-1}}{s^*_{\tau-1}}(p_\tau + d_\tau) \tag{15}$$

while all constraints stay as in the previous reoptimization problem. It turns out that the solution to this modified reoptimization problem is the original full commitment solution $\{x_t^*\}_{t=\tau}^{\infty}$. In other words, if the manager was able to reoptimize but his incentives would somehow change so that instead of caring only about V he would now care about a linear combination of his own objective and the value of the firm, with a weight in the value of the firm that is equal to $\mu^*_{\tau-1}/s^*_{\tau-1}$, then the manager would decide to maintain the previously stated promises. This suggests a possible solution of the time inconsistency problem. The objective of market investors should be incorporate gradually to the one of the manager. This amounts to saying that investors' preferences should play a larger role in older firms. Such a compensation can be interpreted as an "optimal contract", in the sense that it would eliminate the problem of time inconsistency.

In the next section, we analyze an example that demonstrates all the issues we have been discussing.

3.3. Example 1: A Dominant Shareholder. In what follows, we consider an example in which the manager owns a fixed number of stocks in the firm and has no other sources of income. Formally, assume that managers hold a (fixed) number of stocks s^m and let the stocks held by investors be given by s_t^I, so that the total number of stocks in the economy is equal to $s_t = s_t^I + s^m$. The problem of the investors implies the PD mapping and their consumption is equal to $c_t^I = n_t^c$. Moreover, the consumption of the manager is equal to

$c_{m,t} = d_t s^m$ and he maximizes the following objective:[12]

$$E_0 \sum_{t=0}^{\infty} \delta^t v(d_t s^m)$$

for some increasing and concave utility function v.

In general, v can be justified as the contract that the manager has been offered to give him or her incentives to manage the firm properly. In a setting in which the optimal payout and investment are not observable, the manager is restricted from overinvesting or diverting funds by linking his compensation to the payout. In other words, there may be a signalling problem or hidden action mechanism in the background, that prompts the firm to offer a reward to the manager that is tied to the dividend. Indeed, many firms offer stocks or options as a form of payment to managers and managers are not allowed to sell these assets for a long time. We concentrate on the optimal stock issuance policy given v, but we can think of v as a reduced form of an incentive problem that we take as exogenous here but that, ideally, would be endogenized.

Another interpretation of this utility function is that there are two types of stockholders: market stockholders and internal stockholders. Market stockholders would correspond to the investors (households) in this setting. Internal stockholders are somehow tied to this firm, either because they founded the firm, or because their human capital is particularly useful in this firm; they run the firm and they decide how much to invest and how many stocks to issue, while the utility $v(d_t)$ represents their direct preferences on the firm's performance.

Consider the simple case where $m_t = 1$ and no financing frictions $\mathcal{C}_t \equiv 0$. The first order conditions for optimality imply:

$$v'(d_t) = \gamma_t s_{t-1} - m_t \mu_{t-1} \tag{16}$$

$$\gamma_t p_t = E_t[\gamma_{t+1}(d_{t+1} + p_{t+1})] \tag{17}$$

$$\gamma_t = \delta E_t \left[\gamma_{t+1}\left(\theta_{t+1} f'(k_t) + 1 - \eta\right)\right] \tag{18}$$

$$\lambda_t = \gamma_t(s_t - s_{t-1}) \tag{19}$$

The second and third equations represent the stock Euler equation (17) and the capital Euler equation (18) respectively, which are fairly standard. The last condition is the first order condition for the stock price and it allows us to write the co-state as:

$$\mu_t = \mu_{t-1} m_t + \gamma_t(s_t - s_{t-1}) \text{ with } \mu_{-1} = 0. \tag{20}$$

We focus on the condition describing the optimal dividend choice (16). As we see, a marginal increase in d_t yields a direct utility benefit of $v'(d_t)$ but it has a cost in terms of lost resources at t that is equal to $\gamma_t s_{t-1}$. A rational firm takes into account the fact that the dividend choice at time t will affect stock prices in all previous periods. In particular, a marginal increase in d_t also implies increases in the stock prices of all previous periods and this in turn affects the resources available in previous periods. If the firm has been issuing stocks we see from (20) that we can expect $\mu_{t-1} > 0$, implying that more funds were raised in the past for the same level of stock issuance given a higher dividend at t. Conversely, if the firm has been repurchasing stocks in the past ($\mu_{t-1} < 0$), a dividend increase today has a negative effect on past resources. A fully rational firm needs to take into account all these effects when deciding the optimal dividend and equity issuance policies.

[12] Whereas this formulation assumes that the stocks of the manager are fixed, he can change his proportion in the firm by modifying the total number of stocks through issues and repurchases. Issues arising from the trade of shares between managers and shareholders are also discussed in Gorton and He (2006). Their focus is more on the interaction of agency issues and asset pricing and less on financial policy and investment.

Consider now a firm that ignores the PD mapping. Throughout the paper, we denote this firm as naive. It can be easily checked that the first order conditions of such a firm are like the ones above but setting $\mu_{t-1} = 0$ in all periods. This shows how the naive firm will ignore the links among the periods and it will, in general, achieve a lower objective value than the fully rational firm.

The previous discussion clarifies again how the multiplier μ_{t-1} summarizes the effect of a marginal change in d_t on all previous periods' resources and it can be positive or negative depending on the history of stock issuance and repurchase. Even though the whole past history is needed to make decisions at any point in time t, the recursive contracts formulation of Marcet and Marimon allows us to summarize all the relevant information in just one variable, μ_{t-1}. The nature of time inconsistency is that the firm will always be tempted to follow a policy where μ is re-set to zero and only the fact that the firm is fully committed will prevent this from happening. Next, we provide an analytical version of the example that compares the full commitment with the no commitment solution.

Full Commitment. We now analyze a version of the example above for which we can obtain an analytical solution. In this example, the friction consists of a maximum amount of stock that can be issued in the first periods. This can be justified by the presence of transaction costs or due to the manager disliking that too many stocks are distributed, since this would cause a loss of his control in the firm. The manager solves:

$$\max_{\{d_t, s_t, k_t\}} \sum_{t=0}^{\infty} \delta^t v(d_t s^m) \text{ s.t.}$$

$$d_t s_{t-1} + k_t - (1-\eta)k_{t-1} = p_t(s_t - s_{t-1}) + f(k_{t-1})$$

$$s_t - s_{t-1} \le \Delta,\ k_{-1}, s_{-1} \text{ given}$$

$$p_t = \sum_{j=1}^{\infty} \delta^j d_{t+j}$$

where $s_t = s_t^I + s^m$ and $\Delta > 0$ is a fixed constant limiting the amount of stocks that can be issued. We assume that initial capital is much lower than the steady state capital. Formally, the steady state capital, which we denote by k^{GR} for 'golden rule', satisfies:

$$1 = \delta \left[f'(k^{GR}) + 1 - \eta \right]$$

and we assume that $k_{-1} < k^{GR}$.

No Bounds on Stock Issuance: $\Delta = \infty$. In the absence of uncertainty, the firm would be able to achieve the complete market solution if the constraint on stock issuance was not present. That is, if $\Delta = \infty$, the manager would be able to issue a sufficiently large amount of stocks in the first period to finance the desired accumulation of capital at $t = 0$, achieving the first best capital in one step. In fact, the manager would be able to complete the markets with stock issuance so that $k_t = k^{GR}$ for all $t \ge 0$ and dividends would be perfectly smoothed. For the case with $v(.) = \log(.)$ the analytical solution is provided below.

Result 1. *When $\Delta = \infty$, $v(.) = \log(.)$ and $k_{-1} < k_s$, the allocations are, for $t \geq 0$*[13]*:*

$$k_t = k^{GR} = \left[\frac{\frac{1}{\delta} - 1 + \eta}{\alpha} \right]^{\frac{1}{\alpha - 1}}$$

$$s_t = \bar{s} = \frac{n^{GR}}{(1-\delta)\left(k_{-1}^{\alpha} + (1-\eta)k_{-1} - k^{GR}\right) + \delta n^{GR}} s_{-1}$$

$$d_t = \bar{d} = \frac{n^{GR}}{\bar{s}} \text{ and } p_t = p = \frac{\delta}{1-\delta}\bar{d}$$

The proof of these results is provided in the appendix. The above allocations imply that firms issue stocks in the first period and invest enough to jump to the optimal level of capital immediately. This implies that there is no time inconsistency. In what follows, we explore the solution in the presence of a relatively tight bound Δ.

Bounds on Stock Issuance: $\Delta < \infty$. Suppose there is a bound on stock issuance $\Delta < \infty$. For any $\Delta > 0$, there is a point in time after which the bound is not binding any longer. That is, capital has grown enough so that it is close to the steady state and one last period of stock issuance (that does not violate the issuance bound) is enough to reach the steady state. Suppose this happens after T periods. Then

$$s_t - s_{t-1} \leq \Delta \text{ for } 0 \leq t \leq T - 1$$

Starting at period $t = T$ and given s_{T-1} and k_{T-1}, the continuation problem is one where bounds on stock issuance are not binding any more and the solution given by the one in the previous section. The optimal policy for this setup turns out to be time inconsistent, as shown by Proposition 2 below.

Proposition 2. *In a production economy with no uncertainty, bounds on stock issuance for the initial T periods and initial capital lower than the steady state, the problem is time inconsistent.*

The previous proposition shows that the example with the issuance bound and risk averse firms exhibits time inconsistency. This illustrates that a crucial factor generating time inconsistency in the model is that there is disagreement between the shareholders and the manager of the firm. It would be interesting to know how the firm would set its policy if could not credibly commit. This is investigated in what follows.

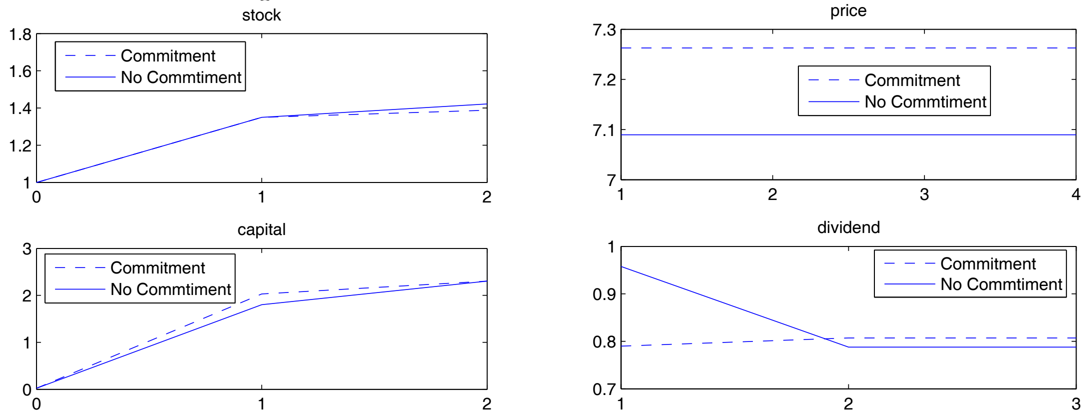
No Commitment. We now compare the full commitment solution with the one that would arise under no commitment. To simplify things, we assume that the bound is only binding for one period $t = 1$. In this case, the solution for $t \geq 2$ is the same as under full commitment and it would have an identical path if the initial conditions were the same. However, the solution for period $t = 1$ will differ if the firm cannot commit. Since we do not have an analytical solution, we have depicted the path for some of the endogenous variables in the graph below. We consider a startup firm, that is, a firm that starts at a very low level of capital, $k_{-1} < k^{GR}$.

Consider first the solution under commitment. The figure reflects that the firm can promise higher dividends in the future and lower dividends in the first period, which allows for higher stock prices and a higher growth. Note that the bounds on stock issuance are binding in the first period both under commitment and no commitment. However, if the firm can credibly commit, it can obtain higher levels of external finance. In particular, by promising lower dividends in the first period and higher dividends in the future, it ensures

[13] See the appendix for details and for a comparison with the naive solution.

that the competitive price for its stock is higher and thus its external finance is higher for the same level of stock issuance.

Figure 1: Commitment versus no Commitment



The figure also reflects that the dividend policy looks very different if the firm cannot credibly commit to the pre-announced policy. In this case, an increasing profile of dividends would not be credible and we see that dividends decrease over time. Intuitively, the firm will have incentives to lower dividends in the future and promising an increasing sequence would not be sustainable. Given this, the shareholders are compensated in the first period with a higher dividend per share.

Two important implications arise from this comparison. First, the commitment solution seems to be more relevant empirically, growing firms typically pay lower dividends when young and more when mature. Second, our framework suggests that commitment is an additional motive explaining this empirical observation.

Although time-inconsistency will arise in general in the presence of a constraint such as the PD mapping, we can characterize some cases under which the solution is time consistent. These cases are discussed in the following section.

4. Time Consistency

It should be clear that, in general, the solution is time inconsistent in the presence of a constraint such as the PD mapping. However, time consistency can be recovered sometimes. It turns out that this happens when the objective function of the manager depends only on cash flows net of financing frictions, $n_t^c = d_t s_{t-1} - p_t (s_t - s_{t-1})$, namely,

$$V(x_t, x_{t-1}, z_t) = V(d_t s_{t-1} - p_t (s_t - s_{t-1}), z_t) \tag{21}$$

Propositions 3 and 4 characterize these settings.

Proposition 3: *Let* $V(x_t, x_{t-1}, z_t) = V(d_t s_{t-1} - p_t (s_t - s_{t-1}), z_t)$ *and define* $V_t' = V_1(d_t s_{t-1} - p_t (s_t - s_{t-1}), z_t)$. *The full commitment solution is time consistent in period* τ *if either*:

a) There is agreement between managers and shareholders, namely, $V_t' = M_0^t$.

b) The PD mapping is not a binding constraint until τ. *Formally,* $\mu_{\tau-1} = 0$.

The proof of proposition 3 relies on showing that, when reoptimizing at period $t = \tau$, the same prices and allocations can be supported with a suitable renormalization of multipliers if conditions a) or b) are satisfied. Several remarks are worth noting. First, part a) of the proposition holds if there is agreement between the managers and the shareholders. This will arise under value maximization, which is a particular case of (21) that satisfies:

$$V(d_t s_{t-1} - p_t (s_t - s_{t-1}), z_t) = \prod_{i=1}^{t} m_i (d_t s_{t-1} - p_t (s_t - s_{t-1})) = M_0^t (d_t s_{t-1} - p_t (s_t - s_{t-1}))$$

Given this, the objective of the firm becomes:

$$E_0 \sum_{t=0}^{\infty} \delta^t M_0^t \left[d_t s_{t-1} - p_t \left(s_t - s_{t-1}\right)\right] \tag{22}$$

First, an important feature of the formulation in (**??**) is that it uses the PD mapping. To see this note that the (cum-dividend) value of the firm at time $t = 0$ is given by $(p_0 + d_0)s_{-1}$. Using the PD mapping and the period by period budget constraint of the firm, we can then re-write the value of the firm as follows:

$$\begin{aligned}(p_0 + d_0)s_{-1} &= n_0^c + p_0 s_0 = n_0^c + \delta E_0 M_0^1 \left(p_1 + d_1\right) s_0 \\ &= E_0 \sum_{t=0}^{\infty} \delta^t M_0^t n_t^c = E_0 \sum_{t=0}^{\infty} \delta^t M_0^t \left(d_t s_{t-1} - p_t \left(s_t - s_{t-1}\right)\right) \end{aligned} \tag{23}$$

To make the point clearer, notice that a manager who ignored the price dividend mapping and who literally maximized the value of the firm taking stock prices as given would treat p_0 as outside of his control and decide that the optimum is to pay everything out as dividends today and close down the firm in one period. A manager could only go from $(p_0 + d_0)s_{-1}$ to (**??**) or (23) if he understood the link between future dividends and p_0 every period.

Second, under this objective, it becomes clear that $V_t' = M_0^t$ and therefore

$$\frac{V_{t+1}'}{V_t'} = \frac{M_0^{t+1}}{M_0^t} = m_{t+1} = \frac{u'\left(c_{t+1}\right)}{u'\left(c_t\right)} = \frac{u'\left(n_{t+1}^c\right)}{u'\left(n_t^c\right)} \tag{24}$$

Thus, the marginal rate of substitution of the manager is the same as the one of market investors at the optimum, who consume the net cash flows net of financing costs, $c_t = n_t^c$. Conversely, if $V_t' = M_0^t$, implying (24), then it must be the case that $V\left(\left(d_t s_{t-1} - p_t\left(s_t - s_{t-1}\right)\right), z_t\right) = M_0^t\left(d_t s_{t-1} - p_t\left(s_t - s_{t-1}\right)\right)$. It is in that sense that we identify value maximization with agreement between managers and shareholders.

As for part b) of Proposition 3, also shows that agreement between the manager and the shareholders is sufficient but not necessary for the solution to be time consistent. In other words, time consistency can also arise under disagreement. An example of this case is discussed in Proposition 4 below.

Proposition 4: *If $V\left(x_t, x_{t-1}, z_t\right) = V\left(d_t s_{t-1}, p_t\left(s_t - s_{t-1}\right), z_t\right)$ and frictions are symmetric, in the sense that they are of the form $\mathcal{C}_t = C\left(k_t, k_{t-1}, p_t\left(s_t - s_{t-1}\right), d_t s_{t-1}, , z_t\right)$* and *$\mathcal{B}_t = \mathcal{B}\left(k_t, k_{t-1}, p_t\left(s_t - s_{t-1}\right), d_t s_{t-1}, z_t\right)$, then $\mu_t = 0$ for all t.*

The previous proposition states that time consistency will arise if both the compensation of the manager and the financing frictions are symmetric. We label a compensation or a friction symmetric if it affects stocks and per share dividends equally or if they depend on the total value of issuance $p_t\left(s_t - s_{t-1}\right)$ and/or the total value of dividends $d_t s_{t-1}$. The most common example of a symmetric compensation is cash flow compensation, since $n_t^c = d_t s_{t-1} - p_t\left(s_t - s_{t-1}\right)$. Examples of symmetric frictions are restrictions on repurchases, $p_t\left(s_t - s_{t-1}\right) \geq 0$, issuance costs $C\left(p_t\left(s_t - s_{t-1}\right)\right) = \left[p_t\left(s_t - s_{t-1}\right)\right]^n$ for $n \geq 1$ or minimum dividend payments, $d_t s_{t-1} \geq 0$. In contrast, examples of asymmetric frictions would be a limit on the number of stocks issued, $s_t - s_{t-1} \leq \Delta$ per share dividend targets $\tau_d\left(d_t - d\right)^n$ for $n \geq 1$ or costs in changing per share dividends, $\tau_d\left(d_t - d_{t-1}\right)^n$ for $n \geq 1$.

This result is particularly important, since the literature typically assumes symmetric frictions and compensation linked to cash flows. In fact, the literature has considered what we denote as DE-problem, given by

$$\max_{\{k_t,e_t,D_t\}} E_0 \sum_{t=0}^{\infty} \beta^t V(k_t, k_{t-1}, D_t - e_t, z_t) \text{ s.t.}$$

$$\begin{aligned} D_t + k_t &= e_t + \theta_t f(k_{t-1}) + (1-\eta)\, k_{t-1} - \mathcal{C}(k_t, k_{t-1}, D_t, e_t, z_t) \\ 0 &\geq \mathcal{B}\,(k_t, k_{t-1}, D_t, e_t, z_t) \end{aligned} \tag{DE}$$

where $D_t = d_t s_{t-1}$ and $e_t = p_t\,(s_t - s_{t-1})$. The proposition then shows that this problem is equivalent to what we call the OP-problem:

$$\max_{\{d_t,s_t,k_t,p_t\}} E_0 \sum_{t=0}^{\infty} \beta^t V(k_t, k_{t-1} D_t - e_t, z_t) \text{ s.t.}$$

$$\begin{aligned} d_t s_{t-1} + k_t &= p_t\,(s_t - s_{t-1}) + \theta_t f(k_{t-1}) + (1-\eta)\, k_{t-1} - \mathcal{C}(k_t, k_{t-1}, D_t, e_t, z_t) \\ p_t &= E_t \sum_{j=1}^{\infty} \beta^j M_t^j d_{t+j} \\ 0 &\geq \mathcal{B}\,(k_t, k_{t-1}, D_t, e_t, z_t) \end{aligned} \tag{OP}$$

An important implication of this equivalence is that the issues we discuss do not arise, since the price dividend mapping is redundant. Note also that the DE problem is actually the model considered by Gomes (2000) and Gomes at all (2003), who assume that $V(k_t, k_{t-1} D_t - e_t, z_t) = M_0^t\,(D_t - e_t)$, a cost of issuing equity, a no repurchase constraint and a lower bound on total dividends[14]. Proposition 4 shows then that these authors are justified in focusing on the naive case. On the other hand, the solution displays the well known pecking order result, implying that firms do not increase their dividend payments while they are issuing equity. This, and the fact that value maximization is not validated by the data leads us to study other firm objectives that introduce a conflict of interest between managers and shareholders and are more in line with the empirical observations on manager compensation. We do this in the next section.

Finally, Proposition 5 below shows that one cannot ignore the PD mapping if frictions are asymmetric, even under value maximization. In this case, it turns out that $\mu_t \neq 0$. Our second example illustrates this.

Proposition 5. *If $\mu_t \neq 0$ for some t, then policies are different if the PD mapping is ignored.*

4.1. Example 2: Value Maximization with Asymmetric Frictions. In this example, we assume value maximization as in Gomes(2000) and Gomes et all (2003) but introduce costs of changing per share dividends. This is intended to capture (in an admittedly crude fashion) the observation that per share dividends are very persistent, in the sense that they are very infrequently changed. This is an example in which the naive and rational solutions do not coincide in spite of the fact that the rational solution is time consistent. Formally, the firm solves:

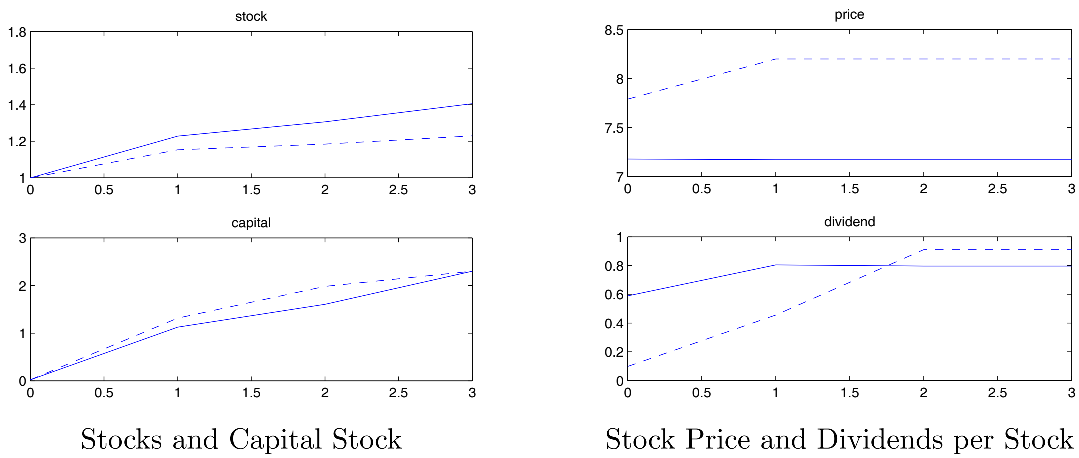
$$\max_{\{d_t,k_t,s_t\}} \sum_{t=0}^{\infty} \delta^t \left[d_t s_{t-1} - p_t\,(s_t - s_{t-1})\right] \text{ s.t.}$$

[14] Variations of this are analyzed in Cooley and Quadrini (2001), Covas and Den-Haan (2007), Quadrini and Jermann (2005) and Gomes et al (2003) amongst others.

$$
\begin{aligned}
d_t s_{t-1} &= F(k_t) - k_t + (1-\delta)k_{t-1} + p_t\,(s_t - s_{t-1}) \\
&\quad -\tau p_t^2\,(s_t - s_{t-1})^2 - \tau_d\,(d_t - d_{t-1})^2 \\
d_t s_{t-1} &\geq 0,\ p_t\,(s_t - s_{t-1}) \geq 0 \\
p_t &= \sum_{j=1}^{\infty} \delta^j d_{t+j}
\end{aligned}
$$

The time path for some of the endogenous variables in the model is displayed in the figure below. For comparison, we also depict the solution for a naive firm that ignores the PD mapping. As reflected by the figure, the two solution can differ substantially. When the PD mapping is taken into account, dividends are lower at first and higher in the future, leading to higher stock prices every period.

Figure 2: Value maximization with costs in changing dividends



The previous results illustrate that time consistency arises in very specific cases and in particular when shareholders are compensated according to symmetric payments, like cash flows, or when shareholders and managers agree, for example when the objective of managers is value maximization. These are the standard assumptions in the literature. In the next section, we show that small departures from these assumptions lead to time inconsistency. We model these departures so that they reflect empirically observed CEO compensation schemes. The first one assumes that managers are compensated through bonus payments linked to cash flows and a fixed salary, while the second assumes that compensation is based on bonuses linked to cash flows and stock options. In addition, we also study a case with asymmetric frictions to illustrate the importance of Proposition 5.

5. Examples of CEO Compensation

This section studies specific examples that reflect empirically observed compensation schemes. For simplicity, all the examples throughout this section assume risk neutral households and no uncertainty. The initial capital stock of the firm is lower than the steady state value, which is reached after a finite number of periods T. This implies that the firm is growing over time. Given this, the role of stock issuance is, precisely, to provide funding to invest in capital so that the firm can operate at the optimal level given by the golden rule. Since firms will achieve the optimal capital immediately (after one period) in the absence of frictions, we introduce such frictions.

According to the survey by Murphy (1999), the main components of CEO compensation in US are (i) a fixed part or base salary, (ii) a bonus mostly based on yearly performance, with the most common measure being accounting profits, (iii) stock options, which are typically non tradable and now constitute the largest component of compensation in US and (iv) other forms of compensation, including restricted stock. Murphy also notes that stock options have typically a strike price equal to the market value on date of grant and they reward only price appreciation (no dividends). Following the evidence, the objectives we consider are combinations of the different components in (i)-(iv). We have already analyzed objectives linked to cash flows only (component (ii)), a special case of which is value maximization. Moreover, we have studied a firm objective that links compensation to per share dividends, corresponding to component (iv). Next, we study a firm objective that links compensation to cash flows and stock options, corresponding to a combination of components (ii) and (iii) and to cash flows and a base salary, corresponding to combinations of components (i) and (ii).

5.1. Example 3: Bonus Compensation with a Base Salary. We consider an example with infinitely many periods that can be solved analytically. We motivate the main analysis from the point of view of a manager that has a shorter-term view than the investors. But the analysis is obviously extendable to a case of two types of agents, the investors with a higher discount factor than the manager. In essence this is a case where managers discount more heavily the future, but it maintains all the standard assumptions of corporate finance models, namely, the manager is rewarded according to cash flow.[15]

The technology of the firm is as in the main text. There is no uncertainty, and cash flows are:

$$n_t = f(k_{t-1}) - k_t + (1-\eta)k_{t-1} \tag{25}$$

In the absence of frictions, financial policy is indeterminate. Therefore, we assume a bound on issuance that determines the amount of stocks issued in the first period s_0, where the initial stocks $s_{-1} = 0$ are normalized to zero for simplicity. As we will show later, the firm will have no incentives to issue equity in subsequent periods.

Investors are risk-neutral, that is $m_t = 1$. The manager has the same utility function as investors, that is, the manager is risk neutral and has discount factor δ. The manager is rewarded according to a fixed wage and a bonus linked to earnings. So the payment for the manager in period t is:

$$\chi n_t + a$$

for fixed constants χ and a. That the manager is rewarded in this way can be justified in various ways. First, empirically, as manager's compensation is often a combination of a fixed pay and a bonus linked to performance. Second, some papers on the optimal contract literature find that optimal incentives imply this kind of reward.

Each period, there is a probability $1-\rho \leq 1$ that he will be fired, in which case we assume that it has a fixed outside option U^m from then on. This setup would arise endogenously in various ways, for example, i) the founding manager has invented the technology used in the production of the firm, he is the only one who can run this technology initially and *ii)* with probability $1-\rho$, the stockholders find how to manage the technology by themselves and they can coordinate to fire the manager. It is implicit that we only consider problems where the value of the objective function for the manager is higher than the manager's utility from engaging in some other activity. The idea is the manager's human capital is tightly linked to this firm and he is much less productive working in some other activity. This guarantees that the manager will stay as manager unless he is fired.

The only uncertainty in the model is the firing decision. As is well established, in this case we can view the manager as choosing a deterministic sequence that maximizes

[15] With this compensation, the solution will be time consistent if we do not introduce any source of disagreement.

$\sum_{j=0}^{\infty} (\delta\rho)^j \ \left[\chi n_j + a + (1-\rho)U^M\right]$ or, equivalently, that the manager maximizes:

$$\text{Max}_{\{s_t,p_t,k_t,d_t\}} \sum_{j=0}^{\infty} (\delta\rho)^j \, n_j \text{ s.t.}$$

$$p_t = \sum_{j=1}^{\infty} \delta^j d_{t+j} \tag{26}$$

$$d_t s_{t-1} = (1-\chi)n_t - a + p_t\,(s_t - s_{t-1}) \tag{27}$$

$$d_t \geq 0,\; k_t \geq 0 \tag{28}$$

For simplicity we assume that the firm itself fully commits to the preannounced dividend policy, so that the dividend paid by the firm is the one chosen by the manager, regardless of whether or not he has been fired. Therefore the price satisfies the usual PD mapping for the dividends chosen by the manager.

Denote by $\overline{k}^I$ the optimal capital from the point of view of the investor, that is, for $1 = \delta(f'(\overline{k}^I) + 1 - \eta)$. Similarly let $\overline{k}^M$ be the optimal capital from the point of view of the manager, that is $1 = \delta\rho(f'(\overline{k}^M) + 1 - \eta)$. If the investors were to run the firm, capital would be equal to $\overline{k}^I$ in all periods, and if managers can invest free of any financial constraint they will choose capital $\overline{k}^M$ in all periods. We make the usual assumption that:

$$(1-\chi)\left(f(k_{-1}) + (1-\eta)k_{-1} - \overline{k}^I + \delta\frac{f(\overline{k}^I) - \eta\overline{k}^I}{1-\delta}\right) - \frac{a}{1-\delta} > 0 \tag{29}$$

that is, the value of the firm is be positive if capital would be accumulated according to investor's preferences. This assumption insures that the problem of the manager is well defined, since there is a positive stock issuance that is feasible if capital is accumulated according to the investor's preferences, so that the feasible set is non-empty.

As mentioned earlier, we assume that the initial capital low enough that the initial cash flow is negative, so that the firm indeed needs some financing to start growing. Clearly, there is no reason to pay any dividends in the initial period, since this dividend does not yield any return to the manager, so that the optimal choice is $d_0 = 0$. In the following periods, however, dividends will have to be positive in order to sustain a positive stock price.

Full Commitment. The solution to the problem under full commitment differs in two cases.

Case 1: $(1-\chi)\left(f(\overline{k}^M) - \eta\overline{k}^M\right) - a \geq 0$. This case insures that the value of the firm from period 1 onwards is positive if the manager invests according to the investor's preferences. In this case the manager will implement his own first best. The manager has to invest a large amount in the first period, for this he needs to issue some equity. The excess production in periods $t \geq 1$ is enough to guarantee future dividend payments insuring that investors will pay a positive price for these stocks initially issued. In this case the fact that equity has to be issued does not impose a cost on the manager.

This case arises for several parameter values: *i)* the technology of the firm is very productive or initial capital is quite high, so that it is worthwhile for investors to put their money in this firm even if the capital is not accumulated according to their first best, or *ii)* the discount factor of managers and stockholders is in fact very similar (that is, ρ close to 1) and in that case the manager chooses something close enough to the investors so that the investors also have a positive value of the firm, or *iii)* manager compensation is low enough. Formally, this case occurs if $f(\overline{k}^M) - \eta\overline{k}^M \to \infty$, or if $\rho \to 1$ or if $a, \chi \to 0$.

It turns out that in this case the manager implements the first best from the point of view of his own discount factor and never invests according to stockholders' preferences. To see this, all we need to check is that implementing the manager's first best $k_t = \overline{k}^M$ for all t is feasible. To prove this we just need to find sequences for stocks, dividend and prices that satisfy budget constraints and PD mapping. Notice that this is the case if we choose stocks $s_t = 1$ and prices

$$p_t = \sum_{i=1}^{\infty} \delta^i n_{t+i} = \delta \frac{\left(f(\overline{k}^M) - \eta \overline{k}^M\right)(1-\chi) - a}{1-\delta} \tag{30}$$

for all $t \geq 0$, and dividends $d_t = \left(f(\overline{k}^M) - \eta \overline{k}^M\right)(1-\chi) - a$ for all $t \geq 1$.

Case 2: $(1-\chi)\left(f(\overline{k}^M) - \eta \overline{k}^M\right) - a < 0$**.** In this second case, the manager does face some constraints because he has to sell the new equity in the stock market. If the manager tried to implement his first choice for capital as he does in case 1, the budget constraint would mean that p_0 would be negative and, obviously, it would be impossible to raise any funds from equity issuance. This case occurs for the contrary the parameter values *i),ii),iii),* mentioned above, that is, when the technology is not highly productive, or when the manager and investor are very different or when the manager compensation is very high.

In case 2 the manager has a non-trivial choice, the investors' preferences will now constrain his choice. Therefore, the manager will have to find a compromise between his interests and those of the investors.[16] A summary of the properties of the solution is as follows. Capital is always between the investors' and the manager's first best:

$$\overline{k}^I > k_t^* > \overline{k}^M \tag{31}$$

showing that the manager has to find a compromise between his and investor's preferences. That is, even if the investors do not run the firm the fact that they have to purchase the newly issued equity forces the manager to invest somewhere in between his and the investors' preferences. Furthermore, we can show that:

$$k_t^* < k_{t+1}^* \tag{32}$$

That is, capital now is increasing and it converges to the investors' first best. Capital increases because this allows the manager to pay ever higher dividends that allows them support a return of δ^{-1} for the stocks. Moreover, the long run capital converges to the investors' first best:

$$k_t^* \to \overline{k}^I \tag{33}$$

In this way the manager can guarantee that when capital stops growing he can pay dividends that satisfy the investors, since in the long run capital is accumulated optimally from the investors' point of view. Hence, the manager can exploit the investment financed by the equity issuance by promising to find a compromise, he will invest more than he would like to and, in the long run, invest completely according to investor's preferences.

Let us now show these properties formally. The Lagrangian is

$$L = \sum_{t=0}^{\infty} (\delta\rho)^t \left[n_t + \gamma_t \left[d_t s_{t-1} - n_t - p_t \left(s_t - s_{t-1} \right) \right] + \lambda_t \left(p_t - \sum_{j=1}^{\infty} \delta^j d_{t+j} \right) \right]$$

[16] There are many parameter configurations for which this can occur, so the case is not vacuous. To see this, notice that $\rho < 1$ implies $\overline{k}^M < \overline{k}^I$, therefore $f(\overline{k}^M) - \eta \overline{k}^M < f(\overline{k}^I) - \eta \overline{k}^I$, so for any technology and discount factors there are many values f a,χ for which Case 2 arises.

where γ is the lagrange multiplier of (**??**) and λ the multiplier of (**??**). FOC with respect to capital, dividends, and price give:

$$\begin{aligned} 1+\gamma_t &= \delta\rho(1+\gamma_{t+1})\left[f'(k_t)+1-\eta\right] \\ \gamma_{t+1}s_t\rho &= \mu_t \end{aligned}$$

for μ with a law of motion

$$\mu_t = \rho^{-1}\mu_{t-1} + \gamma_t(s_t - s_{t-1}) \text{ for all } t \geq 0 \text{ and } \mu_{-1} = 0$$

It is clear that the following solution satisfies the first order condition. We set s_0 equal to the bound on issuance, which is chosen to be equal to 1 for simplicity. Thus, $s_0 = 1$. Set $s_t = s_{t-1}$ for all $t > 0$, hence $\mu_t = \rho^{-t}\gamma_0$ for all $t \geq 0$. Hence first order condition for capital becomes:

$$1 = \delta\rho\left(\frac{1+\rho^{-t-1}\gamma_0}{1+\rho^{-t}\gamma_0}\right)\left(f'(k_t)+1-\eta\right) \tag{34}$$

Notice that given a value for $\gamma_0 > 0$ (34) gives a solution for the whole capital series $\mathbf{k}^*$. Clearly the large bracket is always larger than one, it increases with t, and it converges to ρ^{-1} as $t \to \infty$. This implies, respectively, (31), (32), (33).[17]

Therefore, if the manager can not implement his first best he will initially accumulate less capital than $\overline{k}^I$ but in the long run capital will be set according to investors' preferences. It is easy to see that initial capital is higher than the managers' first best: $k_0^* > \overline{k}^M$ and that k_0^* is closest to $\overline{k}^M$ when the budget constraint is very tight so that the manager really is in need of external financing (i.e., when the lagrange multiplier γ_0 is very large) or when the manger has very different preferences from the investors' (ρ is very low). To solve the model one has to find γ_0 such that the capital series implied by (34) implies an initial value of the firm equal to zero:

$$\sum_{i=0}^{\infty}\delta^i\,\left[f(k_{i-1}^*)+(1-\eta)\,k_{i-1}^*-k_i^*\right](1-\chi)-\frac{a}{1-\delta}=0 \tag{35}$$

The solution for stocks, prices and dividends that satisfies all budget constraints is as follows: $s_t = 1$ for all t, stock prices satisfy

$$p_t = \sum_{t=1}^{\infty}\delta^i\,\left[f(k_{t+i-1}^*)+(1-\eta)\,k_{t+i-1}^*-k_{t+i}^*\right](1-\chi)-\frac{\delta a}{1-\delta} \tag{36}$$

and it is easy to see that setting dividends $d_t = \left[f(k_{t-1}^*)+(1-\eta)\,k_{t-1}^*-k_t^*\right](1-\chi)-a$ the budget constraints and PD mapping hold. In principle, we would need to check that the non-negativity of dividends is satisfied. But we consider a case that k_0^* is sufficiently high so as to guarantee $[f(k_0^*)+(1-\eta)\,k_0^*-k_1^*](1-\chi)-a>0$ and, therefore, that all later dividends are positive as well. This will occur whenever γ_0 is sufficiently high so as to guarantee a quick jump to the investors' first best. In this case, it is also clear that there is no need for equity issuance beyond the first period.

[17] Obviously, we could have analyzed Case 1 from this analysis. In that case the budget constraint is not binding so that $\gamma_0 = 0$ implying $\mu_t = 0$ for all t. Then the large parenthesis in the FOC for capital disappears so that investment is done according to the manager discount factor. But in the current case the budget constraint is binding therefore $\gamma_0 > 0$.

Time Inconsistency. Now we have to consider the choice that the manager would make if he could reoptimize in period τ. If this choice is different from the one planned in period zero under full commitment we say there is time inconsistency. Using the results in Marcet and Marimon (2009), we see that the full commitment solution amounts to reoptimizing at time τ the problem:

$\mu_{t-1}^* = \gamma_0^* \rho^{-t-1}$ we see that the full commitment solution amounts to reoptimizing at time τ the objective:

$$\max_{\{k_t, s_t, p_t, d_t\}_{t=\tau}^{\infty}} \sum_{t=0}^{\infty} (\delta\rho)^t (1-\chi)\ n_{t+\tau} + \gamma_0^* \rho^{-t-1} \sum_{t=0}^{\infty} \delta^t\ n_{t+\tau} \tag{37}$$

where we have substituted for $\mu_{t-1}^* = \gamma_0^* \rho^{-t-1}$, subject to all the constraints. In other words, the problem can always be seen as one of meeting the interests of two agents who have a discounted utility, they both care about a common good and have the same instantaneous utility (namely n_t) but they have different discount factors. These agents receive different weights, initially the weight $(1-\chi)$ on the manager and $\gamma_0^* \rho^{-1}$ is the weight on the investor. As time goes by, the full commitment solution amounts to increasing the weight given to the investor at an exponential rate equal to ρ^{-1}.

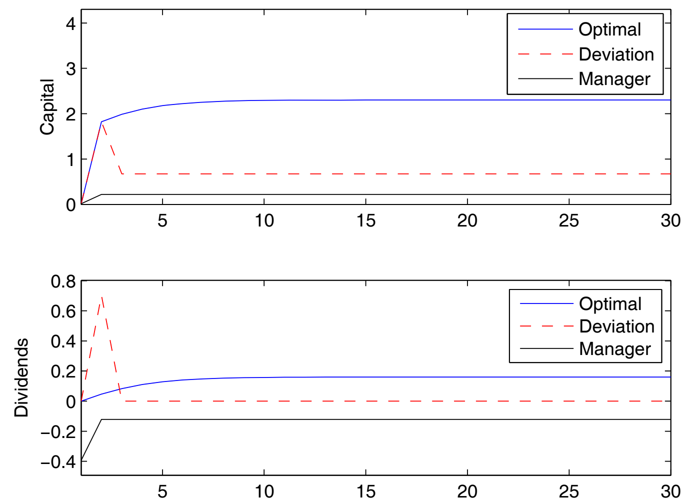
In should be intuitive that in Case 1 the full commitment solution is time consistent. There is nothing to gain from defaulting on past dividend promises if the manager can always implement his own first best and he will pay the dividends promised at time τ because he does not loose anything from paying them. Interpreting this in terms of the problem (37) the objective function that would deliver the full commitment solution is just the objective function of the manager that reoptimizes his own utility, because in case 1 $\gamma_0^* = 0$, reflecting the fact that in this case there is time consistency.

On the other hand, in case 2 $\gamma_0^* \neq 0$ so the objective function that should be reoptimized in order to obtain the full commitment solution gives some weight to the investors and it is different from the one that the manager would maximize if he chose his own first best at τ. Intuitively, what will happen is the following. Recall that the manager had promised to increase capital to make it closer and closer to the investors' first best and, therefore, further and further away from his own first best. Therefore, if the manager can reoptimize, his interest is to "rearrange things" so as to lower capital. The exact level of capital will depend on the value of the capital at reoptimization time $k_{\tau-1}^*$ but in general reoptimization means that the manager will default on the promised prices and dividends and reset capital to a lower value than he had promised. The following figure displays the evolution of the aggregate capital and per share dividend payments under full commitment and in the event of a reoptimization.

The upper panel of the figure displays the aggregate capital stock and the lower panel displays the evolution of the dividend payments. The two solid lines represent the optimum for the manager (black line) and the shareholders (blue line). As already explained earlier, the manager suffers from short termism and wants to invest less than the market investors. If he implemented his optimal investment, however, he would have to pay negative dividends and no one would provide financing to the firm. Given this, he has to compromise and choose a path of capital that converges to the optimal capital of the shareholders in the long run. If the manager is allowed to reoptimize, however, it will choose the lowest possible capital stock that allows him to obtain external finance. This will imply paying positive dividends in the period in which he reoptimizes and zero dividends from then onwards.

The nature of time inconsistency is clear. In period 1, the founding manager will be tempted to renege on promises to stock holders. He will be tempted to pay lower dividends and lower the value of the firm in the future. These lower dividends allow him to invest even less than had been promised. Even though the full commitment solution had lower than optimal investment, the investment under reoptimization is even lower. This allows the manager to have a higher payoff at the time of reoptimization.

Figure 3: Evolution of Capital and Dividends



5.2. Example 4: Bonus Compensation and Stock Options. We now consider a third alternative objective representing the case where managers are compensated through stock options and cash flows[18]. In particular, we assume that managers receive one period options every period at the fixed strike price p^s, which is chosen so that the options are exercised every period. We introduce costly equity issuance and a target for total dividend payout. Both frictions fall under the category of symmetric frictions. The manager solves[19]:

$$\max_{\{d_t,k_t,s_t\}} \sum_{t=0}^{\infty} \delta^t \left[d_t s_{t-1} - p_t \left(s_t - s_{t-1}\right) + \max\left(0, p_t - p^s\right)\right] \text{ s.t.}$$

$$\begin{aligned} d_t s_{t-1} &= F(k_t) - k_t + (1-\delta)k_{t-1} + p_t \left(s_t - s_{t-1}\right) \\ &\quad -\tau p_t^2 \left(s_t - s_{t-1}\right)^2 - \tau_d \left(d_t s_{t-1} - d^{ss} s^{ss}\right)^2 \\ d_t s_{t-1} &\geq 0,\ p_t \left(s_t - s_{t-1}\right) \geq 0,\ p_t = \sum_{j=1}^{\infty} \delta^j d_{t+j} \end{aligned}$$

The following figure displays the evolution of some of the key endogenous variables. Qualitatively, the story here is not any different than in the previous examples. The rational manager pays lower dividends at the beginning and higher dividends in the future, a strategy which allows him to obtain a higher external finance and grow faster. In fact, the naive

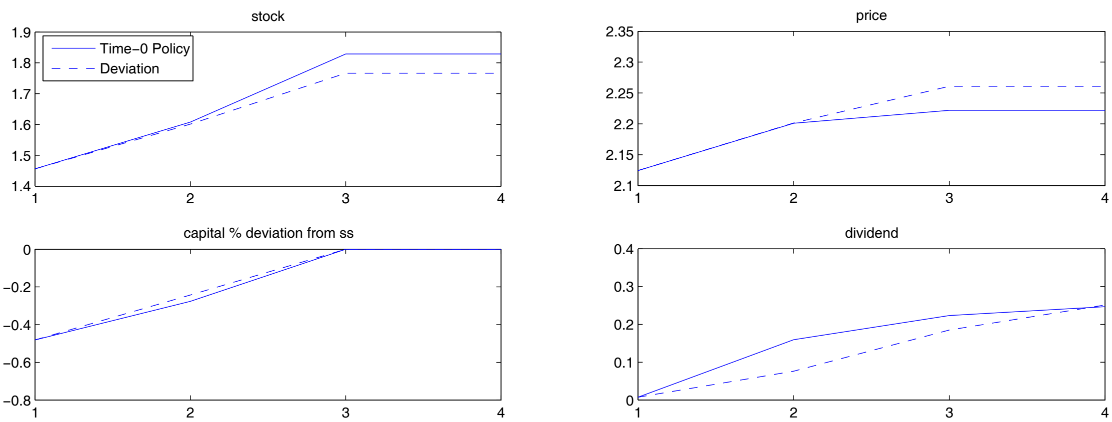
[18] This is the case that is most closely related to the empirical observation on CEO compensation. Recall that the three main components of CEO compensation are bonuses (linked to cash flows), stock opitons and base salary. We omit the base salary component since it makes no qualitative difference.

[19] The presence of the max operator in the objective would in general complicate the maximization problem considerably. We sidestep this issue by ensuring that the option is optimally exercised every period so that the max operator can be ignored.

manager here acts as if there was no stock option component in their compensation since they take prices as given. Naive managers would follow the value maximizing policy while rational manager exploit their decision making power in order to inflate the stock price and, hence, their own compensation.

Using this example, we also attempt to illustrate the nature of time inconsistency. We do this by considering the possibility of deviation in period $t = 2$: After the manager has chosen investment and financial policy for all the future under the assumption of full commitment, we consider what he would change if he were given the opportunity to re-optimize at $t = 2$. At that stage, past choices have already been realized and the manager inherits some levels of capital stock $k_1$and number of outstanding stocks s_1. He also inherits promises made about financial policy in the past (in the form of a positive μ_1), but is allowed to renege on those and set $\mu_1 = 0$.

Figure 5: Compensation linked to cash flows and stock options



As we see. The deviation in dividend policy is clearly aimed at raising stock prices p_2 and p_3. The way this is achieved is by lowering current dividends d_2 which do not affect these prices and promising higher dividends in the future (d^{ss}). By raising stock prices, the manager can raise external finance without too much dilution (s_3 is less under the deviating policy) and also grow faster (k_2 is higher under the deviating policy). As a result, he can deliver the promised higher dividends per share. Clearly the manager is better off by deviating which means the commitment policy is time inconsistent.

6. Conclusions

We have provided a way to formulate and solve a stochastic general equilibrium dynamic model of dividend and stock policy. The aim was to provide a framework within which a number of important issues can be addressed. The model proposed makes explicit the distinction between dividends and stock issuance or repurchases. It is thus well suited to analyze payout policy. In addition, the framework is also available for the analysis of questions regrading the interplay between payout policy and investment.

As a first implication of the theoretical analysis presented in the main section of this paper, we highlight the behavior of growing firms with regard to dividend payments. Typically, startup firms pay little or no dividends, while they funnel resources towards the available productive projects that lead to firm growth. One obvious theoretical explanation of this observation points at financial frictions that do not allow for unlimited funds being raised from external sources. Our framework provides another, complementary mechanism that can explain this observation. The idea is that young firms lack the burden of past promises about dividends and can therefore pay little now, while promising a lot of dividends for the future. This strategy allows them to raise external funds at more favorable prices by inflating the price of their stock. Using the cheaper external funds, they can also grow faster.

Our framework also provides a rationale for why a firm would prefer to use dividends as opposed to repurchases if the full commitment solution is taken as the benchmark case. As mentioned above, the reason is that dividend promises can be used to influence prices towards achieving cheaper external finance, while the same objective cannot be achieved through announcements in stock repurchases.

Finally, our work identifies a potential for time inconsistency in financial policy even in the absence of asymmetric information of the type considered by Miller and Rock (1985). We point out the complications arising from the need for commitment and we provide examples where the full commitment policy is time consistent and others where it is not.

The examples under which optimal policy is time consistent assume that there is agreement between managers and shareholders or that the compensation of managers only consists of bonuses linked to cash flows. In general, when these assumptions are broken, optimal policy is time inconsistent.

One of the examples where policy is time inconsistent assumes that shareholders and managers have different discount factors. In this case we assume that the firm acts to maximize the value according to the discount factor of an agent different from the market investors that purchase the stock each period. Since the firm insures that the PD mapping is satisfied, investors are still willing to hold the stock even though the firm pursues a different objective from the one of the investors. Such setup is quite natural for firms with a group of core shareholders who influence closely the decisions of manager but funds are collected from external shareholders. Another scenario assumes that internal shareholders' income is directly related to the profits distributed by the firm. We assume an extreme case in that internal shareholders are bound to not sell or buy part of the stock (perhaps to retain control of the firm and to insure that he stays on as a manager) and they can not save. In this case the consumption of internal shareholders is proportional to the cum-share dividends. In a different example, we consider an objective function that represents the most common empirically observed CEO compensation by assuming that managers are compensated with bonuses linked to cash flows and one period stock options.

The examples illustrate that time inconsistency would arise in general and that it could be an important aspect in determining the availability of equity financing for firms.

Proof of Lemma 1

To prove Lemma 1, consider a non negative sequence $\mathbf{x}$ that satisfies (8)-(10), together with the transversality condition of investors. Clearly, the period by period budget constraint in (8) together with the PD mapping imply:

$$(d_t + p_t)\, s_{t-1} = E_t \sum_{j=0}^{\infty} \delta^j M_t^j n_{t+j}^c$$

To see this, we only need to use the PD mapping and substitute forward $(p_{t+j}+d_{t+j})s_{t+j-1}$ for $j \geq 1$. To prove the converse, we show that given (11), the budget constraint of the firm in (8) and the PD mapping are satisfied. To see this, note that:

$$\begin{aligned}
(d_t + p_t)\, s_{t-1} &= n_t^c + E_t \sum_{j=1}^{\infty} \delta^j M_t^j n_{t+j}^c \\
&= n_t^c + \delta E_t \left[E_{t+1} \sum_{j=0}^{\infty} \delta^j M_t^{j+1} n_{t+j+1}^c \right] \\
&= n_t^c + \delta E_t M_t^1 \left[n_{t+1}^c + E_{t+1} \sum_{j=1}^{\infty} \delta^j M_{t+1}^{j+1} n_{t+j+1}^c \right] \\
&= n_t^c + \delta E_t M_t^1 \left[(d_{t+1} + p_{t+1})\, s_t \right] \\
&= n_t^c + s_t \delta E_t M_t^1 \left[(d_{t+1} + p_{t+1}) \right]
\end{aligned}$$

Now define $p_t = \delta E_t M_t^1 \left[(d_{t+1} + p_{t+1}) \right]$. This satisfies the PD mapping in (**??**) and it implies that the period by period budget constraint is satisfied, since $(d_t + p_t)\, s_{t-1} = n_t^c + s_t p_t$.

Note that Lemma 1 can also be stated as follows: *A nonnegative sequence $x = (k, s, d, p)$ is feasible if and only if*

$$s_{-1} E_0 \sum_{j=0}^{\infty} \delta^j M_0^j d_j = E_0 \sum_{j=0}^{\infty} \delta^j M_0^j n_j^c \tag{38}$$

$$\frac{N_t}{D_t} \text{ is measurable with respect to information up to } t-1 \text{ for all } t > 0 \tag{39}$$

where D_t and N_t represent the present value of dividends and cash flows net of financing costs respectively:

$$D_t \equiv E_t \sum_{j=0}^{\infty} \delta^j M_t^j d_{t+j} \text{ and } N_t \equiv E_t \sum_{j=0}^{\infty} \delta^j M_t^j n_{t+j}^c$$

In contrast to a framework in which markets are complete, Lemma 1 implies that the period by period budget constraint of the firm is not equivalent to the period zero consolidated budget constraint in (38). Under incomplete markets, the measurability conditions (39) also need to be satisfied.[20] In other words, while many dividend sequences satisfy (38), not all of them are feasible. To prove this version of the Lemma, first it is easy to see that the period-by period constraint in (8) and the price Euler equation from the consumers' problem

[20] The proof of this part follows closely the reasoning of Proposition 1 in Aiyagari, Marcet, Sargent and Seppala (2002).

in (10), together with the no Ponzi scheme assumption, imply (38), (39). It is easy to see that the latter imply:

$$(d_t + p_t)\, s_{t-1} = E_t \sum_{j=0}^{\infty} \delta^j M_t^j n_{t+j}^c \tag{40}$$

where we have used the price dividend mapping to substitute forward $(p_{t+j} + d_{t+j})s_{t+j-1}$ for $j \geq 1$. Since this holds for all $t \geq 0$, the equation evaluated at $t = 0$ implies (38). In addition, using the definitions of N_t and D_t, equation (**??**) implies $\frac{N_t}{D_t} = s_{t-1}$ so that (39) is satisfied.

To prove the converse, we show that given (**??**), (39) and (6), we can construct a sequence of stock holdings such that (8) is satisfied. First, define $S_t \equiv \frac{N_t}{D_t}$ so that S_t is measurable with respect to to information up to $t-1$. Then

$$D_t S_t = n_t^c + E_t \sum_{j=1}^{\infty} \delta^j M_t^j n_{t+j}^c = n_t^c + \delta E_t \left[D_{t+1} S_{t+1}\right]$$

But S_{t+1} is measurable with respect to information up to t, so that $D_t S_t = n_t^c + \delta S_{t+1} E_t \left[D_{t+1}\right]$. Finally, noticing that $D_t = p_t + d_t$, we see that period-by-period budget constraint in (8) is satisfied for $s_{t-1} = S_t = \frac{N_t}{D_t}$.■

Proof of Proposition 1

Lemma 2 directly follows from Lemma 1. Part 1 is straightforward. By Lemma 1, a sequence $\mathbf{k}$ that implies non negative discounted cash flows implies a positive value for the firm and it is therefore feasible.

To prove part 2, consider a sequence $\mathbf{k}$ that implies non negative cash flows and let the cash flow sequence be given by $\{n_t\}_{t=0}^{\infty}$. Consider any choice of stocks $\{\widetilde{s}_t\}_{t=0}^{\infty}$ such that $\widetilde{s}_t \neq 0$ a.s.. Consistent with this choice of $\widetilde{s}$ we can find the associated price to satisfy the following equation:

$$\widetilde{p}_t \widetilde{s}_t = E_t \left(\sum_{j=1}^{\infty} \delta^j M_t^j n_{t+j} \right)$$

and the divided process $\left\{\widetilde{d}\right\}$ to satisfy:

$$\widetilde{d}_t \widetilde{s}_{t-1} + \widetilde{p}_t \widetilde{s}_{t-1} = E_t \sum_{j=0}^{\infty} \delta^j M_t^j n_{t+j}$$

Now we have to show that such a stock, price and dividend processes satisfy budget constraints and pricing equations. First, notice that:

$$\widetilde{p}_t \widetilde{s}_t = E_t \left(E_{t+1} \left(\sum_{j=1}^{\infty} \delta^j M_t^j n_{t+j} \right) \right) = E_t \left(\delta M_t^1 E_{t+1} \sum_{j=0}^{\infty} \delta^j M_{t+1}^j n_{t+j+1} \right)$$

Using the definition of $\widetilde{d}_t \widetilde{s}_{t-1} + \widetilde{p}_t \widetilde{s}_{t-1}$ we also have that

$$\widetilde{p}_t \widetilde{s}_t = E_t \left(\delta M_t^1 \left(\widetilde{d}_{t+1} \widetilde{s}_t + \widetilde{p}_{t+1} \widetilde{s}_t \right) \right)$$

so $\widetilde{s}_t$ cancels out and the PD mapping holds. It is easy to see also that the above choices satisfy the period by period budget constraint of the firm. We can find many other equilibria by changing $\{\widetilde{s}_t\}_{t=0}^{\infty}$.■

Proof of Result 1.

The problem of the firm is:

$$\max \sum_{t=0}^{\infty} \delta^t v\left(d_t s^m\right)$$
$$\text{s.t. } d_t s_{t-1} = p_t\left(s_t - s_{t-1}\right) + k_{t-1}^{\alpha} + (1-\eta)k_{t-1} - k_t$$
$$p_t = \delta\left(d_{t+1} + p_{t+1}\right),\ s_{-1}, k_{-1} \text{ given}$$

The recursive Lagrangian is

$$L = \sum_{t=0}^{\infty} \delta^t \left[v\left(d_t s^m\right) + \mu_{t-1} d_t + \gamma_t\left(k_{t-1}^{\alpha} + (1-\eta)k_{t-1} - k_t - d_t s_{t-1}\right)\right]$$

and the equilibrium conditions are now

$$\begin{aligned}
s^m v'(d_t s^m) &= \gamma_t s_{t-1} - \mu_{t-1} \\
\gamma_t p_t &= \delta\left[\gamma_{t+1}\left(d_{t+1} + p_{t+1}\right)\right] \\
\gamma_t &= \delta \gamma_{t+1}\left(1 - \eta + \alpha k_t^{\alpha-1}\right) \\
p_t &= \delta\left(d_{t+1} + p_{t+1}\right) \\
d_t s_{t-1} &= p_t\left(s_t - s_{t-1}\right) + k_{t-1}^{\alpha} + (1-\eta)k_{t-1} - k_t \\
\mu_t &= \mu_{t-1} + \gamma_t(s_t - s_{t-1})
\end{aligned}$$

We provide an analytical solution to these conditions. The stock Euler together with the price equation imply $\gamma_t = \gamma_{t+1}$ so the stock Euler implies

$$k_t = k^{GR} = \left[\frac{\frac{1}{\delta} - 1 + \eta}{\alpha}\right]^{\frac{1}{\alpha-1}}$$

just like under naive firms. Using the fact that $\gamma_t = \gamma_{t-1}$ for all $t \geq 1$ and the dividend first order conditions we have

$$\begin{aligned}
s^m v'(d_t s^m) - s^m v'(d_{t-1} s^m) &= \gamma_t s_{t-1} - \mu_{t-1} - \gamma_{t-1} s_{t-2} + \mu_{t-2} \\
&= (\gamma_t - \gamma_{t-1}) s_{t-1} = 0
\end{aligned}$$

so $d_t = d_{t-1}$ for all $t \geq 1$. The constant dividend level is found from the time 0 budget constraint

$$d_t = \bar{d} = \frac{(1-\delta)\left(k_{-1}^{\alpha} + (1-\eta)k_{-1} - k^{GR}\right) + \delta n^{GR}}{s_{-1}} \text{ for } t \geq 0$$

Given that, we can use the period 0 dividend first order condition to find γ_t :

$$\gamma_t = \gamma_0 = \frac{s^m v'(\bar{d} s^m)}{s_{-1}}$$

and the price is also constant and equal to $p_t = p = \frac{\delta}{1-\delta}\bar{d}$. We can now compute the stocks from the intertemporal budget constraints for $t \geq 1$

$$(\bar{d} + p)s_{t-1} = \sum_{j=t}^{\infty} \delta^{j-t} n^{GR} = \frac{n^{GR}}{1-\delta} \Rightarrow$$
$$s_{t-1} = \bar{s} \ = \frac{n^{GR}}{\bar{d}} \text{for } t \geq 1$$

It is straightforward to see that

$$\bar{s} = \frac{n^{GR}}{(1-\delta)\left(k_{-1}^{\alpha} + (1-\eta)k_{-1} - k^{GR}\right) + \delta n^{GR}} s_{-1} > s_{-1}$$

as long as $k_{-1} < k^{GR}$. Finally, the multipliers μ_t are constant after period 0 and equal to μ_0

$$\mu_t = \gamma_0(\bar{s} - s_{-1}) > 0 \text{ for } t \geq 0. \blacksquare$$

Proof of Proposition 2.

The first order conditions for the time 0 problem are given by:

$$\mu_t = \mu_{t-1} + \gamma_t(s_t - s_{t-1}) \text{ with } \mu_{-1} = 0$$

$$s^m v'(s^m d_t) = \gamma_t s_{t-1} - \mu_{t-1}$$

along with

$$\begin{aligned}
&\gamma_t = \gamma_{t+1}\delta(f'(k_t) + 1 - \eta) \\
&s_t = s_{-1} + (t+1)\Delta, \quad \text{for } 0 \leq t \leq T-1
\end{aligned}$$

We now consider whether a re-optimization in future periods would lead the firm to deviate from the dividend plans announced in period zero. We use the superscript R to denote the solution if the firm re-optimizes in period $t = 1$. The conditions for capital and the stock are the same as before. On the other hand, we have

$$\begin{aligned}
&\mu_t^R = \mu_{t-1}^R + \gamma_t^R(s_t^R - s_{t-1}^R) \text{ for } t \geq 1 \\
&\mu_0^R = 0
\end{aligned}$$

$$s^m v'(s^m d_t^R) = \gamma_t^R s_{t-1}^R - \mu_{t-1}^R$$

This implies that the following equation holds for $t > 1$:

$$v'(d_t^R) = v'(d_{t-1}^R) + (\gamma_t^R - \gamma_{t-1}^R)s_{t-1}^R$$

In addition, since the firm re-optimizes at $t = 1$, we have

$$v'(d_1^R) = \gamma_1^R s_0$$

Suppose that the re-optimization choices are the same as the original ones, i.e. $d_t^R = d_t$, $s_t^R = s_t$ and $k_t^R = k_t$ for $t \geq 1$. We now show that this leads to a contradiction. If the re-optimized choices are the same as originally, the following must hold

$$v'(d_1) = \gamma_1^R s_0 \tag{41}$$

$$\gamma_2 - \gamma_1 = \frac{v'(d_2) - v'(d_1)}{s_1} = \gamma_2^R - \gamma_1^R \tag{42}$$

In addition, for these choices of γ to be compatible with the same choice for capital in period 1, the following equation must also be satisfied:

$$\gamma_2^R \delta(f'(k_1) + 1 - \eta) = \gamma_1^R$$

but this cannot happen. In fact, if (42) holds, we have $\gamma_2^R = \gamma_1^R - \gamma_1 + \gamma_2$ so that we need the following to be true

$$\begin{aligned}
\gamma_1^R &= \gamma_2^R \delta(f'(k_1) + 1 - \eta) = (\gamma_1^R - \gamma_1 + \gamma_2)\delta(f'(k_1) + 1 - \eta) \\
&= (\gamma_1^R - \gamma_1)\delta(f'(k_1) + 1 - \eta) + \gamma_1
\end{aligned}$$

The last expression can only be equal to γ_1^R if either $\delta(f'(k_1)+1-\eta)=1$ or $\gamma_1^R=\gamma_1$. The first condition arises when capital is optimal, a case which gives rise to time consistency as shown in Proposition 3 below, but which we have excluded above by the choice of a low initial capital and an upper bound on issuance Δ that is binding for at least two periods (period 0 and 1). The second case can be excluded by the formulae for γ_1^R in (41) and for γ_1 in the original problem, since $\mu_0 \neq 0$. Therefore the re-optimized solution cannot be the same as the original one and the time zero policy is time inconsistent in this example.■

Proof of Proposition 3.

We consider the following problem:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t V\left[d_t s_{t-1} - p_t\left(s_t - s_{t-1}\right), z_t\right]$$

s.t.

$$d_t s_{t-1} = F(k_{t-1}) + (1-\delta)k_{t-1} - k_t + p_t\left(s_t - s_{t-1}\right) - \mathcal{C}(k_t, k_{t-1}, s_t, s_{t-1}, d_t, d_{t-1}, p_t, p_{t-1})$$

$$\begin{aligned} p_t &= E_t \sum_{j=1}^{\infty} \beta^j M_t^j d_{t+j} \\ \mathcal{B}(k_t, k_{t-1}, s_t, s_{t-1}, d_t, d_{t-1}, p_t, p_{t-1}) &\leq 0 \end{aligned}$$

The Lagrangian is:

$$\begin{aligned} L &= \max E_0 \sum_{t=0}^{\infty} \beta^t \left[V\left[d_t s_{t-1} - p_t\left(s_t - s_{t-1}\right), z_t\right] - \gamma_{2t} p_t + \mu_{1,t-1} m_t d_t\right] + \\ &\quad + \sum_{t=0}^{\infty} \beta^t \gamma_t \left[F(k_{t-1}) + (1-\delta)k_{t-1} - k_t + p_t\left(s_t - s_{t-1}\right)\right] \\ &\quad + \sum_{t=0}^{\infty} \beta^t \gamma_t \left[-\mathcal{C}(k_t, k_{t-1}, s_t, s_{t-1}, d_t, d_{t-1}, p_t, p_{t-1}) - d_t s_{t-1}\right] + \\ &\quad - \sum_{t=0}^{\infty} \beta^t \xi_t \mathcal{B}(k_t, k_{t-1}, s_t, s_{t-1}, d_t, d_{t-1}, p_t, p_{t-1}) \\ \mu_{1,t} &= m_t \mu_{1,t-1} + \gamma_{2t} \end{aligned}$$

Let $V_t' = V_1\left[d_t s_{t-1} - p_t\left(s_t - s_{t-1}\right), z_t\right]$, $\mathcal{C}_{x,t} = \frac{\partial \mathcal{C}(k_t, k_{t-1}, s_t, s_{t-1}, d_t, d_{t-1}, p_t, p_{t-1})}{\partial x}$ and $\mathcal{B}_{x,t} = \frac{\partial \mathcal{B}(k_t, k_{t-1}, s_t, s_{t-1}, d_t, d_{t-1}, p_t, p_{t-1})}{\partial x}$ where x stands in for any of the arguments of $\mathcal{C}$ and $\mathcal{B}$. The FOC are:

$$\gamma_t\left(1 + \mathcal{C}_{k_t,t}\right) + \xi_t \mathcal{B}_{k_t,t} = \beta E_t\left[\gamma_{t+1}\left(1 - \delta + F'(k_t) - \mathcal{C}_{k_t,t+1}\right) - \xi_{t+1}\mathcal{B}_{k_t,t+1}\right]$$

$$\begin{aligned} 0 &= V_t' s_{t-1} - \left[\gamma_t s_{t-1} + \gamma_t \mathcal{C}_{d_t,t} + \xi_t \mathcal{B}_{d_t,t}\right] - \beta E_t\left[\gamma_{t+1}\mathcal{C}_{d_t,t+1} + \xi_{t+1}\mathcal{B}_{d_t,t+1}\right] + \mu_{1,t-1} m_t \\ &\quad \left[p_t\left(\gamma_t - V_t'\right) - \gamma_t \mathcal{C}_{s_t,t} - \xi_t \mathcal{B}_{s_t,t}\right] \\ &= \beta E_t\left[p_{t+1}\left(\gamma_{t+1} - V_{t+1}'\right) + \gamma_{t+1}\mathcal{C}_{s_t,t+1} + \xi_{t+1}\mathcal{B}_{s_t,t+1} + d_{t+1}\left(\gamma_{t+1} - V_{t+1}'\right)\right] \\ \gamma_{2t} &= \left(s_t - s_{t-1}\right)\left(\gamma_t - V_t'\right) - \gamma_t \mathcal{C}_{p_t,t} - \xi_t \mathcal{B}_{s_{pt},t} - \beta E_t\left[\gamma_{t+1}\mathcal{C}_{p_t,t+1} + \xi_{t+1}\mathcal{B}_{s_{pt},t+1}\right] \\ \mu_{1,t} &= m_t \mu_{1,t-1} + \gamma_{2t} \end{aligned}$$

the last two can be used to get rid of γ_{2t}

$$\mu_{1,t} = m_t \mu_{1,t-1} + \left(s_t - s_{t-1}\right)\left(\gamma_t - V_t'\right) - \gamma_t \mathcal{C}_{p_t,t} - \xi_t \mathcal{B}_{s_{pt},t} - \beta E_t\left[\gamma_{t+1}\mathcal{C}_{p_t,t+1} + \xi_{t+1}\mathcal{B}_{s_{pt},t+1}\right]$$

We now show that the problem is time consistent in period s if either i) $\mu_{1,s-1}=0$ or if ii) $V(n_t^c,z_t)=M_0^t n_t^c$ or $V_t'=M_0^t$. To do this, suppose $\left\{m_t^*,k_t^*,s_t^*,p_t^*,d_t^*,\gamma_t^*,\xi_t^*,\mu_{1,t}^*\right\}_{t=0}^{\infty}$ solve the above problem given $k_{-1},s_{-1},d_{-1},p_{-1},\mu_{1,-1}=0$ and consider reoptimization at $t=s$ given k_{s-1}^*, $s_{s-1}^*,d_{s-1}^*,p_{s-1}^*$ and $\mu_{1,s-1}^{**}=0$ $\left(\text{possibly different from } \mu_{1,s-1}^*\right)$. We will show that the new choices $\left\{m_t^{**},k_t^{**},s_t^{**},p_t^{**},d_t^{**},\gamma_t^{**},\xi_t^{**},\mu_{1,t}^{**}\right\}_{t=s}^{\infty}$ given by

$$\begin{aligned}
m_t^{**} &= m_t^*,k_t^{**}=k_t^*,s_t^{**}=s_t^*,p_t^{**}=p_t^*,d_t^{**}=d_t^* \\
\gamma_t^{**} &= \frac{\gamma_t^*}{1+\frac{\mu_{1,s-1}^*}{s_{s-1}^*}},\xi_t^{**}=\frac{\xi_t^*}{1+\frac{\mu_{1,s-1}^*}{s_{s-1}^*}}, \\
\mu_{1,t}^{**} &= \frac{\mu_{1,t}^*-\frac{\mu_{1,s-1}^*}{s_{s-1}^*}M_0^{*t}s_t^*}{1+\frac{\mu_{1,s-1}^*}{s_{s-1}^*}}
\end{aligned}$$

for all $t\geq s$, satisfy all the first order conditions of the reoptimization problem as long as $V_t'=M_0^t$. Clearly, the capital FOC is satisfied for the same allocations since all multipliers are simply divided by a constant which can be cancelled out. Plugging the above relationships in the stock Euler of the re-optimization problem gives:

$$\begin{aligned}
& p_t^*\left(\gamma_t^*-\left(1+\frac{\mu_{1,s-1}^*}{s_{s-1}^*}\right)V_t^{*\prime}\right)-\left[\gamma_t^*\mathcal{C}_{s_t,t}^*+\xi_t^*\mathcal{B}_{s_t,t}^*\right] \\
=\; & \beta E_t p_{t+1}^*\left(\gamma_{t+1}^*-\left(1+\frac{\mu_{1,s-1}^*}{s_{s-1}^*}\right)V_{t+1}^{*\prime}\right)+\beta E_t\left[\gamma_{t+1}^*\mathcal{C}_{s_t,t+1}^*+\xi_{t+1}^*\mathcal{B}_{s_t,t+1}^*\right] \\
& +\beta E_t d_{t+1}^*\left(\gamma_{t+1}^*-\left(1+\frac{\mu_{1,s-1}^*}{s_{s-1}^*}\right)V_{t+1}^{*\prime}\right)
\end{aligned}$$

and using the original stock Euler

$$\begin{aligned}
p_t^*\frac{\mu_{1,s-1}^*}{s_{s-1}^*}V_t^{*\prime} &= \beta E_t p_{t+1}^*\frac{\mu_{1,s-1}^*}{s_{s-1}^*}V_{t+1}^{*\prime}+\beta E_t d_{t+1}^*\frac{\mu_{1,s-1}^*}{s_{s-1}^*}V_{t+1}^{*\prime}\Leftrightarrow \\
p_t^*V_t^{*\prime} &= \beta E_t\left(p_{t+1}^*+d_{t+1}^*\right)V_{t+1}^{*\prime} \text{ or } \mu_{1,s-1}^*=0
\end{aligned}$$

Using the PD mapping, this is clearly true if $V_t'=M_0^t$ so the stock euler is also satisfied.

We need to show also that the μ_{1t} law of motion and the dividend FOC are satisfied in the re-optimization problem. The dividend FOC is satisfied by choosing μ_{1t}^{**} appropriately. Here is how this is done:

$$\begin{aligned}
0 &= V_t^{**\prime}s_{t-1}^{**}-\left[\gamma_t^{**}s_{t-1}^{**}+\gamma_t^{**}\mathcal{C}_{d_t,t}^{**}+\xi_t^{**}\mathcal{B}_{d_t,t}^{**}\right]-\beta E_t\left[\gamma_{t+1}^{**}\mathcal{C}_{d_t,t+1}^{**}+\xi_{t+1}^{**}\mathcal{B}_{d_t,t+1}^{**}\right]+\mu_{1,t-1}^{**}m_t^{**}\Leftrightarrow \\
0 &= V_t^{**\prime}s_{t-1}^{**}+\frac{-\left[\gamma_t^*s_{t-1}^{**}+\gamma_t^*\mathcal{C}_{d_t,t}^{**}+\xi_t^*\mathcal{B}_{d_t,t}^{**}\right]-\beta E_t\left[\gamma_{t+1}^*\mathcal{C}_{d_t,t+1}^{**}+\xi_{t+1}^*\mathcal{B}_{d_t,t+1}^{**}\right]}{1+\frac{\mu_{1,s-1}^*}{s_{s-1}^*}}+\mu_{1,t-1}^{**}m_t^{**}\Leftrightarrow \\
0 &= \left(1+\frac{\mu_{1,s-1}^*}{s_{s-1}^*}\right)V_t^{*\prime}s_{t-1}^*-\left[\gamma_t^*s_{t-1}^*+\gamma_t^*\mathcal{C}_{d_t,t}^*+\xi_t^*\mathcal{B}_{d_t,t}^*\right]-\beta E_t\left[\gamma_{t+1}^*\mathcal{C}_{d_t,t+1}^*+\xi_{t+1}^*\mathcal{B}_{d_t,t+1}^*\right] \\
&\quad +\left(1+\frac{\mu_{1,s-1}^*}{s_{s-1}^*}\right)\mu_{1,t-1}^{**}m_t^*
\end{aligned}$$

using the $*$ dividend fOC we can derive the relationship between $\mu_{1,t-1}^{**}$ and $\mu_{1,t-1}^*$ as

$$\begin{aligned}
0 &= \frac{\mu_{1,s-1}^*}{s_{s-1}^*}V_t^{*\prime}s_{t-1}^*-\mu_{1,t-1}^*m_t^*+\left(1+\frac{\mu_{1,s-1}^*}{s_{s-1}^*}\right)\mu_{1,t-1}^{**}m_t^*\Rightarrow \\
\mu_{1,t-1}^{**} &= \frac{\mu_{1,t-1}^*-\frac{\mu_{1,s-1}^*}{s_{s-1}^*}\frac{V_t^{*\prime}}{m_t^*}s_{t-1}^*}{1+\frac{\mu_{1,s-1}^*}{s_{s-1}^*}}
\end{aligned}$$

This renormalization of multipliers, although it delivers the right result, is not really feasible because it requires future variables. But under value maximization, $\frac{V_t^{*\prime}}{m_t^*} = \frac{M_0^{t*}}{m_t^*} = M_0^{*t-1}$ is known at period $t-1$. The renormalization required then is

$$\mu_{1,t}^{**} = \frac{\mu_{1,t}^* - \frac{\mu_{1,s-1}^*}{s_{s-1}^*} M_0^{*t} s_t^*}{1 + \frac{\mu_{1,s-1}^*}{s_{s-1}^*}}$$

Finally, we need to check the law of motion for $\mu_{1,t}$ holds:

$$\begin{aligned}
\mu_{1,t}^{**} &= m_t^{**}\mu_{1,t-1}^{**} + \left(s_t^{**} - s_{t-1}^{**}\right)\left(\gamma_t^{**} - V_t^{**\prime}\right) - \gamma_t^{**}\mathcal{C}_{pt,t}^{**} - \xi_t^{**}\mathcal{B}_{s_{pt},t}^{**} \\
&\quad -\beta E_t\left[\gamma_{t+1}^{**}\mathcal{C}_{pt,t+1}^{**} + \xi_{t+1}^{**}\mathcal{B}_{s_{pt},t+1}^{**}\right] \\
&\Leftrightarrow \; \mu_{1,t}^* - \frac{\mu_{1,s-1}^*}{s_{s-1}^*} M_0^{*t} s_t^* = m_t^*\left(\mu_{1,t-1}^* - \frac{\mu_{1,s-1}^*}{s_{s-1}^*} M_0^{*t-1} s_{t-1}^*\right) \\
&\quad + \left(s_t^* - s_{t-1}^*\right)\left(\gamma_t^* - \left(1 + \frac{\mu_{1,s-1}^*}{s_{s-1}^*}\right) V_t^{*\prime}\right) - \gamma_t^*\mathcal{C}_{pt,t}^* + \xi_t^*\mathcal{B}_{s_{pt},t}^* - \beta E_t\left[\gamma_{t+1}^*\mathcal{C}_{pt,t+1}^* + \xi_{t+1}^*\mathcal{B}_{s_{pt},t+1}^*\right] \\
&\Leftrightarrow \; -\frac{\mu_{1,s-1}^*}{s_{s-1}^*} M_0^{*t} s_t^* = m_t^*\left(-\frac{\mu_{1,s-1}^*}{s_{s-1}^*} M_0^{*t-1} s_{t-1}^*\right) + \left(s_t^* - s_{t-1}^*\right)\left(-\frac{\mu_{1,s-1}^*}{s_{s-1}^*} V_t^{*\prime}\right) \\
&\Leftrightarrow \; \mu_{1,s-1}^* M_0^{*t}\left(s_t^* - s_{t-1}^*\right) = \left(s_t^* - s_{t-1}^*\right)\mu_{1,s-1}^* V_t^{*\prime}
\end{aligned}$$

This last condition is true if either $\mu_{1,s-1}^* = 0$ or $V_t' = M_0^t$.■

Proof of Proposition 4.

To prove proposition 2, we first establish the equivalence between the DE and the OP problems. Recall that the OP-problem is given by:

$$\max_{\{k_t, s_t, d_t, p_t\}} E_0 \sum_{t=0}^{\infty} \beta^t V(k_t, k_{t-1} D_t, e_t, z_t) \text{ s.t.} \tag{43}$$

$$d_t s_{t-1} + k_t - (1-\eta)k_{t-1} = p_t\left(s_t - s_{t-1}\right) + \theta_t f(k_{t-1}) - \mathcal{C}\left(k_t, k_{t-1}, D_t, e_t\right) \tag{44}$$

$$p_t = E_t \sum_{j=1}^{\infty} M_t^j \delta^j d_{t+j} \tag{45}$$

where total dividends D_t and new equity e_t are defined as

$$\begin{aligned} D_t &\equiv d_t s_{t-1} \\ e_t &\equiv p_t(s_t - s_{t-1}) \end{aligned} \tag{46}$$

Notice that given constraint (44) $D_t - e_t = n_t - \mathcal{C}_t$ are cash flows. We now prove the following properties of the solution of the OP-problem $\{k_t^*, s_t^*, d_t^*\}$.

1. $\{k_t^*, s_t^*, d_t^*\}$ is recursive in the natural state variables $(\theta_t, k_{t-1}, s_{t-1})$. In particular, it has the following recursive structure:

$$\begin{aligned} k_t^* &= F^k(k_{t-1}^*, \theta_t) \\ \begin{bmatrix} s_t^* \\ d_t^* \end{bmatrix} &= F^{sd}(k_{t-1}^*, \theta_t, s_{t-1}^*) \end{aligned}$$

   for time-invariant functions $F^k : R^2 \to R$ and $F^{sd} : R^3 \to R^2$

2. $\{k_t^*, s_t^*, d_t^*\}$ is time consistent

3. $\{k_t^*, s_t^*, d_t^*\}$ coincides with the solution to the problem of the naive manager

The proof of this result is based on the fact that the OP-problem is equivalent to the following DE-problem:

$$\max_{\{k_t, e_t, D_t\}} E_0 \sum_{t=0}^{\infty} \delta^t V(k_t, k_{t-1}, D_t, e_t, z_t) \text{ s.t.} \tag{47}$$

$$D_t + k_t - (1-\eta)k_{t-1} = e_t + \theta_t f(k_{t-1}) - \mathcal{C}(k_t, k_{t-1}, D_t, e_t) \tag{48}$$

We first prove the following.

a Given a sequence $\{k_t, s_t, d_t\}$ that is feasible in the original problem we can find $\{e_t, D_t\}$ that satisfies (46) and that is feasible in the DE problem for the same k series.

b Conversely, given $\{k_t, e_t, D_t\}$ that is feasible in the DE problem we can find a $\{s_t, d_t\}$ that satisfies (46), and that is feasible in the original problem for the same k series.

Part a) follows immediately from choosing $\{e_t, D_t\}$ that satisfies (46), plugging the results in (44) and observing it satisfies the only constraint in DE problem. For part b), given $\{k_t, e_t, D_t\}$ we build a process $\{s_t, d_t\}$ in the following way: first build the series of cash flows:

$$n_t^c \equiv \theta_t f(k_{t-1}) + (1-\eta)k_{t-1} - \mathcal{C}(k_t, k_{t-1}, D_t, e_t) - k_t$$

Then build $\{s_t, d_t\}$ recursively as follows. At any period $t \geq 0$, given s_{t-1} and the process $\{k_t, e_t, D_t\}$ find (s_t, d_t, p_t) for a given realization as follows:

$$d_t = D_t / s_{t-1} \tag{49}$$

$$p_t = \left(E_t \sum_{j=0}^{\infty} \delta^j M_t^j n_{t+j} - D_t \right) \frac{1}{s_{t-1}} \tag{50}$$

$$s_t = \frac{e_t}{p_t} + s_{t-1} \tag{51}$$

With this solution we get s_t and can construct $(s_{t+1}, d_{t+1}, p_{t+1})$ and so on. It is clear that in this manner one can build a whole process $\{s_t, d_t, p_t\}$. Now we have

$$\begin{aligned}(p_t + d_t)s_{t-1} &= n_t^c + E_t \sum_{j=1}^{\infty} \delta^j M_t^j n_{t+j}^c & (52)\\ &= n_t^c + \delta E_t M_t^1 \left(n_{t+1}^c + E_{t+1} \sum_{j=1}^{\infty} \delta^j M_{t+1}^{j+1} n_{t+1+j}^c \right) \\ &= n_t^c + \delta E_t M_t^1 \left(p_{t+1} + d_{t+1} \right) s_t & (53)\end{aligned}$$

where the first equality follows from the fact that the process so constructed satisfies (49) and (50), the second equality uses the law of iterated expectations and simple algebra and the third equality uses (52) for period $t+1$ inside the expectation. On the other hand we have

$$(p_t + d_t)s_{t-1} = D_t - e_t + p_t s_t = n_t^c + p_t s_t$$

where the first equality follows from (46) and the second from the definition of cash flows. This together with (53) implies that:

$$p_t = \delta E_t M_t^1 \left(p_{t+1} + d_{t+1} \right) = E_t \left(\sum_{j=1}^{\infty} \delta^j M_t^j d_{t+j} \right)$$

so that (45) is also satisfied. In sum, all the constraints of the OP-problem are satisfied for the series that satisfies (49) to (51) and this proves part b).

Now note that given a sequence $\{k_t, s_t, d_t\}$, for the feasible sequence $\{e_t, D_t\}$ that is alluded to in part a), we have

$$E_0 \sum_{t=0}^{\infty} \delta^t V(k_t, k_{t-1}, D_t, e_t, z_t) = E_0 \sum_{t=0}^{\infty} \delta^t V(k_t, k_{t-1}, d_t s_{t-1}, p_t(s_t - s_{t-1}), z_t)$$

The same holds for any feasible sequence $\{k_t, e_t, D_t\}$ and the corresponding sequence $\{s_t, d_t\}$ that is mentioned in part b). This is because, in both cases, $D_t = d_t s_{t-1}$ and $e_t = p_t(s_t - s_{t-1})$. Therefore the maximum value of the original problem coincides with the maximum value of the DE problem. Formally, letting $\{k_t^{**}, e_t^{**}, D_t^{**}\}$ denote the solution of the original problem and let $\{k_t^*, s_t^*, d_t^*\}$ denote the solution to the original problem. Then,

$$E_0 \sum_{t=0}^{\infty} \delta^t V(k_t^{**}, k_{t-1}^{**}, D_t^{**}, e_t^{**}) = E_0 \sum_{t=0}^{\infty} \delta^t V(k_t^*, k_{t-1}^*, d_t^* s_{t-1}^*, p_t^*(s_t^* - s_{t-1}^*))$$

Now it is clear that the solution to the DE problem is recursive in the standard dynamic programming sense, since only past values of k (in addition the shock θ) constrain the feasible set for current e, D, k, therefore the optimal solution for the DE problem has the form

$$(k_t^{**}, e_t^{**}, D_t^{**}) = F^{DE}(k_{t-1}^{**}, \theta_t) \text{ for all } t, \text{ a.s.} \tag{54}$$

for some time-invariant policy function $F^{DE} : R^2 \to R^3$.

This means that the sequence $\{k_t, s_t, d_t\}$ corresponding to $\{e_t^{**}, D_t^{**}, k_t^{**}\}$ according to part b) of the results mentioned above achieves the maximum in the original problem. Since this corresponding $\{s_t, d_t\}$ sequence satisfies (49) to (51) then it is clear that for any t the variables (s_t, d_t) are a function of $(e_t^{**}, D_t^{**}, k_t^{**})$ and also of s_{t-1}^*, therefore, combining (49) to (51) with (54) we have

$$(s_t^*, d_t^*) = F(k_{t-1}^{**}, \theta_t, s_{t-1}^*) \tag{55}$$

for a time invariant function F. This proves part 1 of the proposition.

For part 2, consider the case where the manager reoptimizes at time $\bar{t}$ taking as given the "initial" state variables $(k_{\bar{t}-1}^{**}, \theta_{\bar{t}}, s_{\bar{t}-1}^*)$. We simply state that by a similar argument as above the reoptimized *original* problem is equivalent with the reoptimized *DE* problem. Since the DE problem satisfies a standard Bellman equation this problem is time consistent and the reoptimized series for k, e, D coincides with the original optimum announced at time zero for the DE problem $\{k_t^{**}, e_t^{**}, D_t^{**}\}_{t=\bar{t}}^{\infty}$. It is clear that the corresponding d, s series would also coincide with the preannounced one, so there is time consistency.

For part 3 of the proposition, note that the naive problem simply does not take into account (45) as a constraint. This means that the optimum of the DE problem is consistent with a series k, p, d, s that satisfies all constraints in the naive problem, since this problem simply has one fewer constraint than the original problem, namely (45). Therefore the maximum of the OP-problem is also the maximum of the naive problem.

Proof of Proposition 5.

We now show that the naive solution (N) is equal to the rational solution (R) iff $\mu_t = 0$ for all t. First, if $\mu_t^* = 0$ for all t, then the N and R fOC are the same. Conversely, suppose that $\{k_t, s_t, p_t, d_t, \gamma_t, \mu_t\}_{t=0}^{\infty}$ solve the N problem so that

$$\begin{aligned}
\gamma_t \left(1 + \mathcal{C}_{k_t,t}\right) &= \beta M_t^1 \left[\gamma_{t+1} \left(\theta_t f'(k_t) + 1 - \eta - \mathcal{C}_{k_t,t+1}\right)\right] \\
0 &= V_t' s_{t-1} - \gamma_t s_{t-1} - \gamma_t \mathcal{C}_{d_t,t} - \beta \gamma_{t+1} M_t^1 \mathcal{C}_{d_t,t+1} \\
p_t \left(\gamma_t - V_t'\right) - \gamma_t \mathcal{C}_{s_t,t} &= \beta M_t^1 p_{t+1} \left(\gamma_{t+1} - V_{t+1}'\right) + \beta M_t^1 \gamma_{t+1} \mathcal{C}_{s_t,t+1} \\
&\quad + \beta M_t^1 d_{t+1} \left(\gamma_{t+1} - V_{t+1}'\right)
\end{aligned}$$

Suppose that the same $\{k_t, s_t, p_t, d_t\}_{t=0}^{\infty}$ solve the rational problem, that is

$$\gamma_t^R \left(1 + \mathcal{C}_{k_t,t}\right) = \beta M_t^1 \left[\gamma_{t+1}^R \left(\theta_t f(k_t) + 1 - \eta - \mathcal{C}_{k_t,t+1}\right)\right]$$

$$\begin{aligned} p_t \left(\gamma_t^R - V_t'\right) - \gamma_t^R \mathcal{C}_{s_t,t} &= \beta M_t^1 p_{t+1} \left(\gamma_{t+1}^R - V_{t+1}'\right) \\ &\quad + \beta M_t^1 \gamma_{t+1}^R \mathcal{C}_{s_t,t+1} + \beta M_t^1 d_{t+1} \left(\gamma_{t+1}^R - V_{t+1}'\right) \end{aligned}$$

$$0 = V_t' s_{t-1} - \gamma_t^R s_{t-1} - \gamma_t^R \mathcal{C}_{d_t,t} - \beta M_t^1 \gamma_{t+1}^R \mathcal{C}_{d_t,t+1} + \mu_{1,t-1}$$

$$\mu_{1,t} = \mu_{1,t-1} + (s_t - s_{t-1}) \left(\gamma_t^R - V_t'\right) - \gamma_t^R \mathcal{C}_{p_t,t} - \beta \gamma_{t+1}^R M_t^1 \mathcal{C}_{p_t,t+1}$$

Clearly it has to be the case that $\frac{\gamma_{t+1}^R}{\gamma_t^R} = \frac{\gamma_{t+1}}{\gamma_t}$ for all t. Using the two dividend fOC and subtracting one from the other

$$\left(1 - \frac{\gamma_t^R}{\gamma_t}\right) \left[V_t' s_{t-1} + \mu_{1,t-1}\right] + \mu_{1,t-1} = 0$$

At $t = 0$, this implies $\left(1 - \frac{\gamma_0^R}{\gamma_0}\right) V_0' s_{-1} = 0$. Assuming $V_0' \neq 0$, this implies that $\gamma_0^{FR} = \gamma_0$. By the previous condition, this also implies $\gamma_t^R = \gamma_t$ for all t. As a result, from the R dividend first order condition for any t, it must be that $\mu_t = 0$.■

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