# Industrial Structure, Trade and Regional Economics : Market Segmentation

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January, 2010

*Abstract*

We consider a general equilibrium model *a la* Bhaskar (*Review of Economic Studies* 2002): there are complementarities across sectors, each of which comprise (many) heterogenous monopolistically competitive firms. Bhaskar's model is extended in two directions: production requires capital, and labour markets are segmented. Labour market segmentation models the difficulties of labour migrating across international barriers (in a trade context) or from a poor region to a richer one (in a regional context), whilst the assumption of a single capital market means that capital flows freely between countries or regions. The model is solved analytically and a closed form solution is provided. Adding labour market segmentation to Bhaskar's two-tier industrial structure allows us to study, *inter alia*, the impact of competition regulations on wages and financial flows both in the regional and international context, and the output, welfare and financial implications of relaxing immigration laws. The analytical approach adopted allows us, not only to sign the effect of policies, but also to quantify their effects. Introducing capital as a factor of production improves the realism of the model and refines its empirically testable implications.

## 1. INTRODUCTION

Our work mainly focus on two points, both relevant in current economic studies: demand elasticity heterogeneity across sectors and labour market segmentation.

In the market, we generally find goods that are subject to different degrees of substitutability. This may be due to several reasons: the number of firms operating in an industry, the elasticity of substitution in consumption between products, the information level on the goods, and market regulations in each sector. Demand elasticity and then markup heterogeneity has been studied by F.Bilbiie,F. Ghironi and M. Melitz (2006), that prove that efficiency requires markups to be synchronized across goods, and resources are inefficiently

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underallocated on the production of the good with higher mark-ups (that is consumption) respect to leisure, for which mark-up is absent. [2]

Furthermore, our model includes labour market segmentation at the industry/region level. This is to study the difficulties of labour migrating across international barriers (in a trade context) or from a poor region to a richer one (in a regional context). Focusing the attention on the mobility of labour gives us the possibility to address relevant issues about migration policies and to understand who gains and who looses by allowing labour to freely move across boarders and, so doing, leading to homogenization of wages. As an example, we might think of the enlargement of the European Union to Turkey and/or to other Eastern Europe countries, that is currently challenging economists and politicians about opportunities and costs. At a national level ,instead, we might think of policies able to lead to an efficient distribution of labour across either regions or industrial sectors.

Our set up is an extension of the general equilibrium framework by Bhaskar (Review of Economic Studies, 2002), that introduces the distinction between producer and industry. That structure has been chosen as industrial sectors and regional areas are thought of as being complementary to each other through specialization[3].There are two input factors : capital and labour. We start assuming capital as free mobile, whilst labour is segmented at the industry level and not allowed to move across industries. Then, we relax limitations for labour to move and assume labour to be free mobile, such as capital. Comparing the results from the two settings allows us to calculate whether and to which extent labour integration leads to welfare gains or losses, both at the global economy level and at the industry/region level. Problems related to economic integration have already been addressed, by focusing on distributive implications (see H.Cremer, P. Pestiau, 1996; A.S. Kessler, C. Lulfesmann, G.M. Myers, 2002). Instead, we mostly study integration by looking at the results in terms of efficiency.

By setting a multi-industry (multi-regional) production economy, we prove that if within industry demand elasticities are equal, then full integrated model's input factors distribution exactly matches the social planner solution; otherwise, particular exogenous distributions of that input resource can correct the distortion due to mark-up heterogeneity across industries and give a larger global production. Moreover, our analysis is made analytically more realistic by taking into account firm level productivity heterogeneity. Thus, allocation of input factors is led by both relative productivities of firms/industries and the industry level mark-up degrees.

Finally, our setting is able to contribute to some of the major trade literature over the last years (see M. Melitz, 2003; M.Melitz, G.Ottaviano, 2008, J. Eaton,

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[2] P. Epifani and G. Gancia (2008) have studied this problem in a multi-sector economy: they first provide empirical evidence of the intersectoral mark-up asymmetry in the U.S. and then prove that the latter, due to trade or other distortions, leads to intersectoral misallocations.Their starting point is that "if the degree of monopoly is the same for all final products, there is no monopolistic competition alteration from the optimum at all" (Lerner, 1934).

[3] In a trade context, complementarity of sectors follows the Armington assumption : the more substitutable goods from different sectors, the more differences between them can affect trade.

S. Kortum, 2002), by taking into account three types of asymmetries altogether: productivity (at the firm level), labour endowement (that is the labour amount available at the industry/region level in a segmented economy) and demand elasticity (at the industry/region level). Each one of those heterogeneities can crucially affect the competitiveness in a trade context: industries/regions with a larger demand elasticity, a larger aggregate technical efficiency and a larger labour endowment are likely to be more competitive through lower production costs and, then, prices.

The structure of the paper follows : section 2 outlines the preferences and the technology assumed in the model; section 3 provides the equilibrium in an economy in which capital is allowed to be free mobile across industries, whilst labour is segmented at the industry level; section 4 shows another version of the model, in which both capital and labour are assumed to be free mobile; section 5 shows the social planner solution; section 6 illustrates the effects of labour market liberalization in a two-country economy; section 7 concludes the paper by summarizing the main results.

## 2. OUTLINE OF THE MODEL

The model extends the framework implemented by Bhaskar (2002, RES), by adding capital as a production factor and segmenting the labour market at the industry level. Furthermore, we allow for demand elasticity heterogeneity across industries (regions).

We present a pure production economy. There is one good produced at the global level by a standard CES production function. The latter has as arguments industry/region level products coming from standard CES production functions as well. Intermediate inputs are produced by heterogenous monopolistically competitive firms having to hire both the input factors, that means labour and capital. A large number of firms operate in each region/industry and the number of varieties within each sector is allowed to differ.

We set two different economies[4]: in the first one, labour is segmented at the industry/region level; in the second one, labour is allowed to be free mobile such as capital is in both the settings. The two models' equilibria turn out to be crucially different in terms of production, intermediate inputs costs,and factors allocation. In particular, in the first framework wages differ across industries, whilst in the fully integrated economy equilibrium wage is unique such as the rental in both the settings.

There is one representative agent, which inelastically supplies an exogenously fixed amount of capital $\overline{K}$ through all the economy and a fixed amount of labour, that is $\overline{L}_s$ to the single industries in the first setting, and $\overline{L}$ through all the economy in the second one.

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[4] We have also derived one more setting in which both capital and labour are segmented at the sector level. We can provide that on request.

*Preferences and technology*

2.1 *Global and industry sector*

The global sector, operating under perfect competition, uses the goods produced at the industry/region level to produce the final good according to the standard CES aggregation function,

$$Y = \left[ \frac{1}{S^{1-\delta}} \sum_{s=1}^{S} Y_s^{\delta} \right]^{\frac{1}{\delta}}, \qquad \delta \in (0,1), \tag{1}$$

where Y is the aggregate output of the global sector. The latter will make zero profit, as well as the industry/region sector, since the production function shows constant returns to scale in the inputs $Y_s$ , and since this sector is perfectly competitive, . The elasticity of substitution between any two products is $1/(\delta - 1)$. The global sector generates a derived demand for the differentiated products produced by industries, which will be shown next.

The industry/region sector also produces according a standard CES aggregation function using the specialized inputs:

$$Y_s = \left[ \frac{1}{N_s^{1-\rho_s}} \sum_{n=1}^{N_s} q_{n,s}^{\rho_s} \right]^{\frac{1}{\rho_s}}, \qquad \rho_s \in (0,1), \tag{2}$$

where $Y_s$ is the aggregate output of the industry. Even in this case, since the production function shows constant returns and this sector is perfectly competitive, the profit made will be zero. The elasticity of substitution between any two products is $1/(\rho_s - 1)$, and it is allowed to vary. The industry/region sector creates a derived demand for the differentiated products, which will be shown next.

2.2 *Derived demand*

Cost minimization of (1) generates the derived demand for the industry/region level good $Y_s$ as a function of its own price $\widetilde{p}_s$, of the price of the final good p and of the total output of the final good Y

$$Y_s \equiv \left[ \frac{p}{\widetilde{p}_s} \right]^{\frac{1}{1-\delta}} \frac{Y}{S}, \qquad \text{with} \quad \frac{1}{p^{\nu}} \equiv \frac{1}{S} \sum_{s=1}^{S} \frac{1}{\widetilde{p}_s^{\nu}}, \tag{3}$$

with $\nu \equiv \delta/(1-\delta) \in \mathbf{R}_+$.

Cost minimization of (2) generates the derived demand for the specialized input $q_{n,s}$ as a function of its own price $\widetilde{p}_{n,s}$, of the price of the region level good $\widetilde{p}_s$ and of the total output of the regional good $Y_s$,

$$q_{n,s} \equiv \left[\frac{\widetilde{p}_s}{\widetilde{p}_{n,s}}\right]^{\frac{1}{1-\rho_s}} \frac{Y_s}{N_s}, \qquad \text{with} \quad \frac{1}{\widetilde{p}_s^{\mu_s}} \equiv \frac{1}{N_s} \sum_{n=1}^{N_s} \frac{1}{\widetilde{p}_{n,s}^{\mu_s}}, \qquad (4)$$

with $\mu_s \equiv \rho_s / (1 - \rho_s) \in \mathbf{R}_+$.

Putting the derived demand for the industry good (3) into the latter yields

$$q_{n,s} \equiv \left[\frac{\widetilde{p}_s}{\widetilde{p}_{n,s}}\right]^{\frac{1}{1-\rho_s}} \left[\frac{p}{\widetilde{p}_s}\right]^{\frac{1}{1-\delta}} \frac{Y}{SN_s}.$$

### 2.3 *Specialized firms*

In each industry/region, there is a fixed large number $N^5{}_s$ of monopolistically competitive firms, each one producing a differentiate product. Firm $n_s$ produces according to a standard Cobb-Douglas production function which takes capital and labour as inputs

$$q_{n,s} = \theta_{n,s} k_{n,s}^{\gamma} l_{n,s}^{1-\gamma}, \qquad\qquad \gamma \in (0,1), \qquad (5)$$

where $q_{n,s}$ is its output, $\theta_{n,s}$ is its technical efficiency and $k_{n,s}$, $l_{n,s}$ are the inputs of, respectively, capital and
labour used up by the firm.

## 3. LABOUR SEGMENTED ECONOMY

### 3.1 *Welfare*

In each industry/region, the welfare function is given by summing up firms'profits, labour and capital incomes:

$$I_s = \sum_{n=1}^{N_s} \widetilde{\pi}_{n,s} + \widetilde{i}\widehat{K}_s + \widetilde{w}_s \overline{L}_s, \qquad (6)$$

where $\widehat{K}_s$ is the amount of capital assumed to be owned by sector s. At the global level, the welfare is given by adding up the industry/region level welfares.

### 3.2 *Producer optimization*

The monopoly firms have to hire physical capital and labour on competitive labour markets, because they do not own any input factor. The nominal profit of firm $n_s$ is

$$\widetilde{\pi}_{n,s} \equiv \widetilde{p}_{n,s} q_{n,s} - \left(\widetilde{i} k_{n,s} + \widetilde{w}_s l_{n,s}\right),$$

where $\widetilde{p}_{n,s}$ is the nominal price of the product, $\widetilde{w}_s$ is the nominal wage rate in sector s, and $\widetilde{i}$ is the nominal rental. Notice that, as the labour market is

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[5] The number of firms is allowed to vary across sectors.

segmented, nominal wages turn out to be different across industries/regions. The interest rate, instead, is the same over all sectors, as capital is assumed to be free mobile.

The firm n,s faces the demand curve shown before (**??**) and seeks to maximize its profit

$$\max_{k_{n,s}, l_{n,s}} \widetilde{p}_{n,s} q_{n,s} - \left( \widetilde{i} k_{n,s} + \widetilde{w}_s l_{n,s} \right), \quad \text{subject to} \quad \theta_{n,s} k_{n,s}^{\gamma} l_{n,s}^{1-\gamma} = q_{n,s}. \tag{7}$$

Each monopolistically competitive firm faces the minimization problem in (7), in order to derive the minimal unit cost. The result is that each agent within the same industry/region will choose to operate with the same capital/labour ratio $\kappa_s$

$$\frac{k_{n,s}}{l_{n,s}} = \frac{\gamma}{1-\gamma} \frac{w_s}{i} = \kappa_s, \tag{8}$$

The derived demand for labour and capital by the firm n,s producing $q_{n,s}$ is

$$k_{n,s} = \frac{\kappa_s^{1-\gamma} q_{n,s}}{\theta_{n,s}} \quad \text{and} \quad l_{n,s} = \frac{q_{n,s}}{\theta_{n,s} \kappa_s^{\gamma}}, \tag{9}$$

Capital $\overline{K}$ is exogenously given at the global level. In order to make both the labour and the capital markets clear, the following must hold

$$\overline{K} = \sum_{s=1}^{S} \kappa_s \overline{L}_s. \tag{10}$$

### 3.3 Optimal prices

As a monopolist, the intermediate input firm seeks to maximize its profit by charging a price $\widetilde{p}_{n,s}$ that is equal to a fixed mark-up $(1/\rho_s)$ over cost. Using the optimal capital/labour ratio (8) and the derived demand for capital and labour (9) yields the price charged by firm n,s

$$\widetilde{p}_{n,s} = \frac{w_s}{(1-\gamma)\rho_s \kappa_s^{\gamma} \theta_{n,s}} p. \tag{11}$$

The industry/region level aggregate productivity is given by

$$\theta_s^{\mu_s} = \frac{1}{N_s} \sum_{n=1}^{N_s} \theta_{n,s}^{\mu_s}. \tag{12}$$

Using the latter and the sector level price aggregation (4) yields that within a sector s, the firm relative price is inversely proportional to the firm relative productivity.

$$\frac{\widetilde{p}_{n,s}}{\widetilde{p}_s} = \frac{\theta_s}{\theta_{n,s}}. \tag{13}$$

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Using the latter, the global sector production function (1), the derived demand for specialized inputs (4), the optimal capital-labour ratio (8), the input market clearing condition (10),the optimal price charged by firm n,s (11), and the assumption that labour is inelastically supplied at the region level, we find that

$$\frac{\widetilde{p}_s}{p} = \frac{\left[\frac{1}{S}\sum_{s=1}^{S}\bar{L}_s^{\frac{\delta(1-\gamma)}{1-\gamma\delta}}\theta_s^{\frac{\delta}{1-\gamma\delta}}\rho_s^{\frac{\gamma\delta}{1-\gamma\delta}}\right]^{\frac{1}{\nu}}}{\bar{L}_s^{\frac{(1-\delta)(1-\gamma)}{1-\gamma\delta}}\theta_s^{\frac{(1-\delta)}{1-\gamma\delta}}\rho_s^{\frac{\gamma(1-\delta)}{1-\gamma\delta}}}. \tag{14}$$

Thus, the industry level relative price turns out to be an decreasing function of the labour inelastically supplied in that sector, the aggregate productivity and the within industry demand elasticity.

Rearranging the optimal price ratio at the sector level (11) for the real wage and substituting the result into the optimal capital-labour ratio (8), and then what we find into the input market clearing equation (10), and recalling (14), we can derive the real rental rate, that turns out to be a decreasing function of the capital stock :

$$i = \frac{\gamma\left[\frac{1}{S}\sum_{s=1}^{S}\bar{L}_s^{\frac{\delta(1-\gamma)}{1-\gamma\delta}}\theta_s^{\frac{\delta}{1-\gamma\delta}}\rho_s^{\frac{\gamma\delta}{1-\gamma\delta}}\right]^{\frac{1}{\nu}}\left[\sum_{s=1}^{S}\rho_s^{\frac{1}{1-\delta\gamma}}\theta_s^{\frac{\delta}{1-\delta\gamma}}\bar{L}_s^{\frac{\delta(1-\gamma)}{1-\gamma\delta}}\right]^{1-\gamma}}{\bar{K}^{1-\gamma}}. \tag{15}$$

Using the latter, the optimal capital/labour ratio (8) and the sector relative price (14) gives the real wage rate

$$w_s = \frac{(1-\gamma)\,\bar{K}^{\gamma}\left[\frac{1}{S}\sum_{s=1}^{S}\bar{L}_s^{\frac{\delta(1-\gamma)}{1-\gamma\delta}}\theta_s^{\frac{\delta}{1-\gamma\delta}}\rho_s^{\frac{\gamma\delta}{1-\gamma\delta}}\right]^{\frac{1}{\nu}}\rho_s^{\frac{1}{1-\delta\gamma}}\theta_s^{\frac{\delta}{1-\delta\gamma}}}{\bar{L}_s^{\frac{(1-\delta)}{1-\gamma\delta}}\left[\sum_{s=1}^{S}\rho_s^{\frac{1}{1-\delta\gamma}}\theta_s^{\frac{\delta}{1-\delta\gamma}}\bar{L}_s^{\frac{\delta(1-\gamma)}{1-\gamma\delta}}\right]^{\gamma}}. \tag{16}$$

Industry/region level wage results to be an increasing function of the within industry demand elasticity and aggregate productivities[6]. Thus, relatively more competitive industries are expected to show higher real wages.

### 3.4 *Factor allocation and final output*

Substituting the price ratios (13) and (14) into the derived demand for specialized input (4), and using the latter to replace $q_{n,s}$ into the firms' demand for labour (9),and aggregating labour demand over all monopoly firm by recalling that labour is inelastically supplied at the sector level,$\bar{L}_s$, we can derive the global production

$$\bar{L}_s = \sum_{n=1}^{N_s} l_{n,s} \Longleftrightarrow Y = S\bar{L}_s\kappa_s^{\gamma}\theta_s\left[\frac{\widetilde{p}_s}{p}\right]^{\frac{1}{1-\delta}}. \tag{17}$$

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[6] We have shown that by numerical exemples, that are available upon request.

Considering the optimal capital/labour ratio (8),the equations for real rental rate and the real wage rate (15 and 16),the relative price at the global level (14), the final good production is

$$Y = \frac{\bar{K}^{\gamma}\left[\sum_{s=1}^{S} \bar{L}_s^{\frac{\delta(1-\gamma)}{1-\gamma\delta}} \rho_s^{\frac{\gamma\delta}{1-\gamma\delta}} \theta_s^{\frac{\delta}{1-\gamma\delta}}\right]^{\frac{1}{\delta}}}{S^{\frac{1-\delta}{\delta}}\left[\sum_{s=1}^{S} \bar{L}_s^{\frac{\delta(1-\gamma)}{1-\gamma\delta}} \rho_s^{\frac{1}{1-\gamma\delta}} \theta_s^{\frac{\delta}{1-\delta\gamma}}\right]^{\gamma}}. \tag{18}$$

From the latter and the relative prices (13 and 14), we can derive the production at the firm level

$$q_{n,s} = \frac{\theta_{n,s}^{\frac{1}{1-\rho_s}} \bar{L}_s^{\frac{1-\gamma}{1-\gamma\delta}} \theta_s^{\frac{\gamma\delta-\rho_s}{(1-\gamma\delta)(1-\rho_s)}} \bar{K}^{\gamma} \rho_s^{\frac{\gamma}{1-\gamma\delta}}}{\left[\sum_{s=1}^{S} \rho_s^{\frac{1}{1-\gamma\delta}} \theta_s^{\frac{\delta}{1-\delta\gamma}} \bar{L}_s^{\frac{\delta(1-\gamma)}{1-\gamma\delta}}\right]^{\gamma} N_s}. \tag{19}$$

Substituting the firm level production (19) ,the optimal capital-labour ratio (8), along with the real rental rate and the real wage rate (15 and 16), into the derived demand for labour and capital (9), we find the optimal factor allocation

$$k_{n,s} = \frac{\theta_{n,s}^{\frac{\rho_s}{1-\rho_s}} \bar{L}_s^{\frac{(1-\gamma)\delta}{1-\gamma\delta}} \theta_s^{\frac{\rho_s\delta(\gamma-1)+\delta-\rho_s}{(1-\gamma\delta)(1-\rho_s)}} \bar{K} \rho_s^{\frac{1}{1-\gamma\delta}}}{\left(\sum_{s=1}^{S} \rho_s^{\frac{1}{1-\gamma\delta}} \theta_s^{\frac{\delta}{1-\delta\gamma}} \bar{L}_s^{\frac{\delta(1-\gamma)}{1-\gamma\delta}}\right) N_s}, \tag{20}$$

$$l_{n,s} = \frac{\bar{L}_s \theta_{n,s}^{\frac{\rho_s}{1-\rho_s}}}{N_s \theta_s^{\frac{\rho_s}{1-\rho_s}}}. \tag{21}$$

At the firm level, capital allocation turns out to be a function of all the parameters ruling this economy. Instead, labour is allocated according the average size and relative productivity of the firm within the sector[7].

Particularly, labour distribution is affected only by the parameters at the sector level: this result exactly matches what we would find in a one sector industry[8].

## 4. INTEGRATED ECONOMY
### 4.1 *Welfare*
At the industry/region level, the welfare function is given by summing up firms' profit, labour and capital income

$$I_s^{*9} = \sum_{n=1}^{N_s} \widetilde{\pi}_{n,s}^* + \widetilde{i}^* \widehat{K}_s + \widetilde{w}^* \overline{L}_s, \tag{22}$$

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[7]We can show that in an economy in which both capital and labour are segmented at the sector level, the two input factors are distributed as labour in (21), that means they both turn out to be functions of the firm's relative productivity and its average size.

[8]See Abadir and Talmain (RES, 2002).

[9]We use asterisk * to indicate the variables from the labour integrated model, and differentiate them from the ones coming from the first setting.

where $\widehat{K}_s$ is the capital owned by sector s. At the global level, the welfare is given by adding up the region level welfares.

### 4.2 *Producer optimization*

The monopoly firms have to hire physical and labour on competitive labour markets, because they do not own any capital. The nominal profit of firm n,s is

$$\widetilde{\pi}^*_{n,s} \equiv \widetilde{p}^*_{n,s} q^*_{n,s} - \left( \widetilde{i}^* k^*_{n,s} + \widetilde{w}^* l^*_{n,s} \right),$$

where $\widetilde{p}^*_{n,s}$ is the nominal price of the product, $\widetilde{w}^*$ is the nominal wage rate, and $\widetilde{i}^*$ is the nominal rental. As both the labour and the capital market are fully integrated, the wage rate and the rental rate are unique over all the economy.

The firm faces a demand curve that we will derive next and seek to maximize its profit

$$\max_{k_{n,s},l_{n,s}} \widetilde{p}^*_{n,s} q^*_{n,s} - \left( \widetilde{i}^* k^*_{n,s} + \widetilde{w}^* l^*_{n,s} \right), \quad \text{subject to} \ \ \theta_{n,s} k^{*\gamma}_{n,s} l^{*1-\gamma}_{n,s} = q^*_{n,s}. \quad (23)$$

Each monopolistically competitive firm faces the minimization problem in (23), in order to derive the minimal unit cost. The result is that each agent over all the economy will choose to operate with the same capital/labour ratio $\kappa$ :

$$\frac{k^*_{n,s}}{l^*_{n,s}} = \frac{\gamma}{1-\gamma} \frac{w^*}{i^*} = \kappa. \quad (24)$$

The derived demand for labour and capital by the firm n,s producing $q^*_{n,s}$ is

$$k^*_{n,s} = \frac{\kappa^{1-\gamma} q^*_{n,s}}{\theta_{n,s}} \quad , \quad l^*_{n,s} = \frac{q^*_{n,s}}{\theta_{n,s} \kappa^\gamma}. \quad (25)$$

### 4.3 *Optimal prices*

As a monopolist, the intermediate input firm seeks to maximize its profit by charging a price $\widetilde{p}^*_{n,s}$ that is equal to a fixed mark-up $(1/\rho_s)$ over cost. Using the optimal capital/labour ratio (24) and the derived demand for capital and labour (25) yields the price charged by firm n,s

$$\widetilde{p}^*_{n,s} = \frac{w^*}{(1-\gamma) \rho_s \kappa^\gamma \theta_{n,s}} p^*. \quad (26)$$

Using the sector level price aggregation (4) and the sector level productivity aggregation (12) yields that within a sector the relative price inversely proportional to the relative productivity:

$$\frac{\widetilde{p}^*_{n,s}}{\widetilde{p}^*_s} = \frac{\theta_s}{\theta_{n,s}}. \quad (27)$$

Aggregating (26) at the sector level and using (12) yields:

$$\widetilde{p}^*_s = \frac{w^*}{(1-\gamma)\,\rho_s\kappa^\gamma\theta_s}p^*.$$

First aggregating the latter at the sector level, and then dividing that by the result gives

$$\frac{p^*_s}{p^*} = \frac{\left(\frac{1}{S}\right)^{\frac{1}{\nu}}\left[\sum_{s=1}^{S}(\theta_s\rho_s)^\nu\right]^{\frac{1}{\nu}}}{\theta_s\rho_s}. \tag{28}$$

In this case, the sector relative price turns out to be an increasing function of industry level aggregate productivity and demand elasticity[10].

Using the labour and capital demand (25), the derived demand for specialized good (4), the sector level productivity aggregation (12) and the assumption that both capital and labour are inelastically supplied at the global level, we derive that the optimal capital/labour ratio is simply equal to the ratio between the exogenous global capital stock ($\overline{K}$) and the exogenous global labour stock ($\overline{L}$). From the latter and the equation of the price charged by firms (26), it turns out that the real rental rate and the real wage rate are respectively

$$i^* = \gamma\left(\frac{\overline{L}}{\overline{K}}\right)^{1-\gamma}\left[\frac{1}{S}\sum_{s=1}^{S}(\theta_s\rho_s)^\nu\right]^{\frac{1}{\nu}}, \tag{29}$$

$$w^* = (1-\gamma)\left(\frac{\overline{K}}{\overline{L}}\right)^{\gamma}\left[\frac{1}{S}\sum_{s=1}^{S}(\theta_s\rho_s)^\nu\right]^{\frac{1}{\nu}}. \tag{30}$$

Both the real wage and the real interest rate are functions of the aggregate capital and the aggregate labour; moreover, they result to be increasing functions of an aggregation of industry level aggregate productivities and demand elasticities.

### 4.4 *Factor allocation and final output*

Substituting the price ratios (27 and 28) into the derived demand for specialized input (4), and putting the result along with the optimal capital-labour ratio ($\overline{K}/\overline{L}$) into one of the input aggregations yields the final good production

$$Y^* = \frac{S^{\frac{\delta-1}{\delta}}\overline{K}^\gamma\overline{L}^{1-\gamma}\left[\sum_{s=1}^{S}\theta_s^{\frac{\delta}{(1-\delta)}}\rho_s^{\frac{\delta}{1-\delta}}\right]^{\frac{1}{\delta}}}{\sum_{s=1}^{S}\theta_s^{\frac{\delta}{1-\delta}}\rho_s^{\frac{1}{1-\delta}}}. \tag{31}$$

Using the latter and the relative prices (27 and 28), we can derive the firm n,s' production

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[10]See note 6.

$$q_{n,s}^* = \frac{\theta_s^{\frac{\delta-\rho_s}{(1-\delta)(1-\rho_s)}} \rho_s^{\frac{1}{1-\delta}} \theta_{n,s}^{\frac{1}{1-\rho_s}} \overline{K}^\gamma \overline{L}^{1-\gamma}}{N_s \left[ \sum_{s=1}^{S} \theta_s^{\frac{\delta}{1-\delta}} \rho_s^{\frac{1}{1-\delta}} \right]}. \tag{32}$$

The distribution of production at the firm level is led by the industry/region level aggregate productivity and demand elasticity and, moreover, by firm's productivity.

Finally, substituting the firm level production and the optimal capital/labour ratio into the derived demand for labour and capital (25) yields the factor allocation in the integrated economy

$$k_{n,s}^* = \frac{\theta_{n,s}^{\frac{\rho_s}{1-\rho_s}} \rho_s^{\frac{1}{1-\delta}} \theta_s^{\frac{\delta-\rho_s}{(1-\delta)(1-\rho_s)}} \overline{K}}{N_s \sum_{s=1}^{S} \theta_s^{\frac{\delta}{1-\delta}} \rho_s^{\frac{1}{1-\delta}}}, \tag{33}$$

$$l_{n,s}^* = \frac{\theta_{n,s}^{\frac{\rho_s}{1-\rho_s}} \rho_s^{\frac{1}{1-\delta}} \theta_s^{\frac{\delta-\rho_s}{(1-\delta)(1-\rho_s)}} \overline{L}}{N_s \sum_{s=1}^{S} \theta_s^{\frac{\delta}{1-\delta}} \rho_s^{\frac{1}{1-\delta}}}. \tag{34}$$

Both capital and labour allocations turn out to be a function of all the parameters of this economy. Therefore, labour is allocated in a crucially different way respect to the segmented economy; particularly, it is also an increasing function of the within sector demand elasticity: as long as firms operating in different sectors have different mark-ups, they distort labour allocation asymmetrically.

5. SOCIAL PLANNER SOLUTION

The social planner optimizes the distribution of input factors over the economy. Assuming both capital and labour to be inelastically supplied at the global level, she faces the following optimization problem :

$$\max_{\left\{ \{k_{n,s}, l_{n,s}\}_{n=1}^{N_s} \right\}_{s=1}^{S}} Y = \left[ \frac{1}{S^{1-\delta}} \sum_{s=1}^{S} \left( \frac{1}{N_s^{1-\rho_s}} \sum_{n}^{N_s} \left( \theta_{n,s} k_{n,s}^\gamma l_{n,s}^{1-\gamma} \right)^{\rho_s} \right)^{\frac{\delta}{\rho_s}} \right]^{\frac{1}{\delta}}.$$

Capital and labour turn out to be distributed at the firm level as follows:

$$k_{n,s} = \frac{\overline{K} \theta_s^{\frac{\delta-\rho_s}{(1-\rho_s)(1-\delta)}} \theta_{n,s}^{\frac{\rho_s}{1-\rho_s}}}{S N_s \theta^{\frac{\delta}{1-\delta}}}, \tag{35}$$

$$l_{n,s} = \frac{\overline{L} \theta_s^{\frac{\delta-\rho_s}{(1-\rho_s)(1-\delta)}} \theta_{n,s}^{\frac{\rho_s}{1-\rho_s}}}{S N_s \theta^{\frac{\delta}{1-\delta}}}.$$

The model with full integration of input factors matches what found by the social planner if within sector demand elasticities are equal. If intrasector

mark-ups differ, there is going to be distortion in the allocation of inputs, that is overallocation in industries/regions with higher demand elasticity and, on the contrary, underallocation where mark-ups are higher.

Furthermore, distribution of production factors according to the model in which labour is assumed to be segmented at the sector level also matches (35), if within sector demand elasticities are the same and, moreover, labour supplied at the sector level is equal to

$$\overline{L}_s = \frac{\overline{L}}{S} \left(\frac{\theta_s}{\theta}\right)^{\frac{\delta}{1-\delta}} {}^{11}.$$

## 6. A TWO-COUNTRY ECONOMY

Thinking of what we have called so far industries or regions as countries gives us the opportunity to study what would happen in a two-country model in case of labour market integration. In this economy, capital is assumed to be free mobile, while labour force is initially assumed to be contrained at original population level ($N_1$ and $N_2$), and then , in a second stage, it is allowed to freely move across boarders by migration.

**LABOUR FORCE DISTRIBUTION BETWEEN TWO COUNTRIES**



**1st setting:labour supply constrained at the original population level**

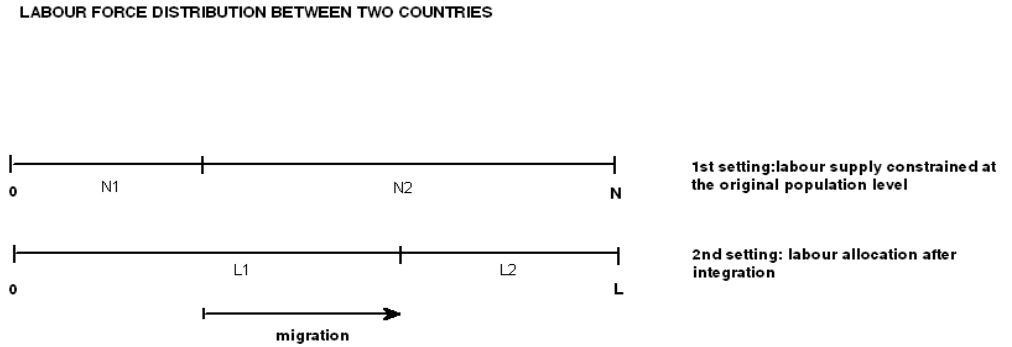**2nd setting: labour allocation after integration**

FIGURE 1

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[11] The right-hand side of that equation results by aggregating the labour demand given by (35) at the sector level.

We distinguish between two different scenarios: in the first one, countries face the same within-boarder demand elasticity and differ only by size (that is population); in the second case, instead, one country has a larger demand elasticity than the other one.

We graphically show the welfare level at both the global and the country level for different distribution of the global population across the two countries. In each country, population can be lower, equal or larger than the allocation of labour according to the integrated labour setting. The number of producers is assumed to be the same, such as firm level productivities. Depending on the starting population "status", countries are going to face immigration (if the population is lower the labour allocation dictated by the integrated setting) or emigration (if the population is larger than the amount of labour expected after liberalization of the labour market).

The distribution of the workforce across the two countries is going to affect the three components of the welfare, that means labour income, capital income and profits. Capital ownership is assumed to be equally shared by the two countries[12]; however, capital income changes according to the workforce distribution that affects the interest rate equilibrium. In the segmented economy wages differ across countries: the larger the population, the lower the wage in the respective country is. Thus, labour income for the two countries is going to be affected by labour market liberalization through both labour moves and changes in wages (that becomes higher in the country facing emigration and lower in that facing immigration).[13] Production distribution across firms operating in the two countries results to be affected by changes in the labour allocation ; changes in the labour and capital distribution and then, input factor costs, are going to affect firms' profits.

### 6.1 *First scenario: mark-up homogeneity*

As shown in Figure 2[14], a labour segmented economy as a whole can never perform better than a fully integrated one, whatever the distribution of population across the two countries is. The two economies reach the same outcome as the population distribution is equal to the labour force distribution dictated by the labour integrated economy.

Figure 3 shows that the less populated country can be either better or worse off in an integrated economy, depending on the distribution of population. Particularly, that country has larger profits and capital income in an integrated economy, but lower labour income, that is due to a lower number of workers. As the population becomes more and more similar to the one of the other country, the gains in terms of profits and capital income are not able to offset the

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[12] We can also show what happens in case of different distribution of the capital ownership across the two countries. However, at the moment we prefer focusing on labour market asymmetries.

[13] Notice that if people cross the boarder, they are still sending what thet earn abroad to their original country by remittences.

[14] Figures are provided in the appendix at the end of the paper.

labour income loss in an integrated context anymore ,as the wage becomes lower and lower, so that the overall economy results to be worse off.

On the other hand,as shown in Figure 4, the more populated country turns out to be always better off as it moves out of the segmented economy. Nevertheless, as the population gets closer and closer to the one of the other country, the positive gap between the integrated economy income and the segmented economy one becomes lower and lower; the latter is because the gain in terms of labour income becomes thinner and thinner as the population decreases.

### 6.2 *Second scenario:mark-up heterogeneity*

Figure 5 shows that for some distributions of population across the two countries, the segmented economy can perform better than the integrated one. Of course, the incomes of the two economies are equal at the point in which the population distribution exactly matches the distribution of the labour force according to free mobile labour setting. The inefficiency implied by integration for some distributions of the populations is due to the cross country mark-up heterogeneity, that leads to asymmetric distortion of factor allocation and, particularly, to overallocation of labour in the more competitive economy. This inefficiency seems to be corrected as labour is segmented at the country level.

Figure 6 shows that in an integrated economy the higher demand elasticity country is worse off as long as the population is lower than the labour force dictated by the free mobile labour context. As the population becomes larger and larger, profits, capital income and labour income turn out to be larger in the integrated economy, leading to an overall gain. Finally, looking at Figure 7, the lower demand elasticity country seems to never be better off in a segmented economy.

### 7. CONCLUSIONS

We find that as long as industry sectors or regions differs only by labour supply, a labour segmented economy cannot perform better than a fully integrated one, in which labour,as well as capital,is allowed to freely move sectors/regions. However, if within sector mark-ups also differ, labour allocation is distorted asymmetrically across sectors possibly leading to inefficiency at the global level. The latter  is due to an overallocation of factors into the relatively more competitive sectors/ regions with lower mark-ups.

In a trade context, as long as countries have the same degree of internal competitiveness, labour market integration can lead to an efficiency improvement through a better allocation of labour according to relative productivities of firms. Nevertheless, if countries, such as regions or industrial sectors, differ by demand elasticity, then integration is likely to lead to an excess of labour into the more competitive market.

This result can be relevant in order to understand difficulties that are currently faced by countries to get into the European Union, such as Turkey or some Eastern Europe countries.

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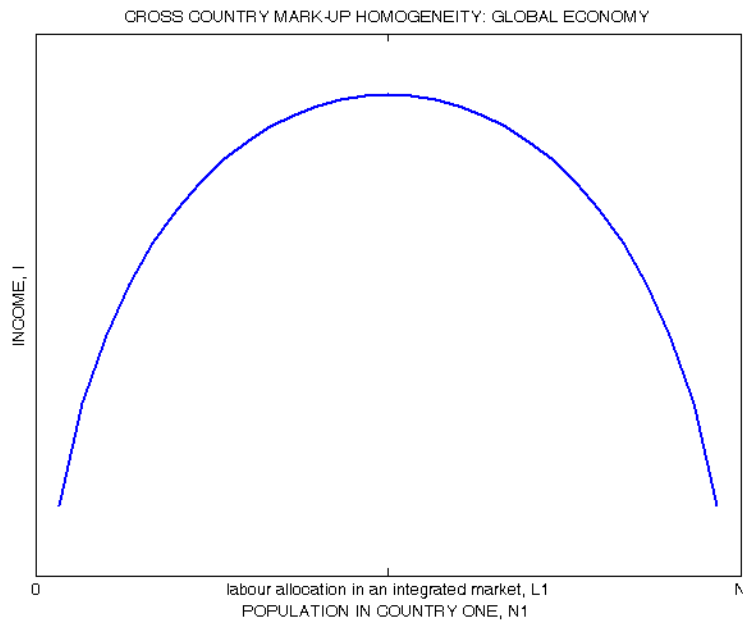Melitz M., G. Ottaviano (2008): "Market Size, Trade, Productivity", *Review of Economic Studies*
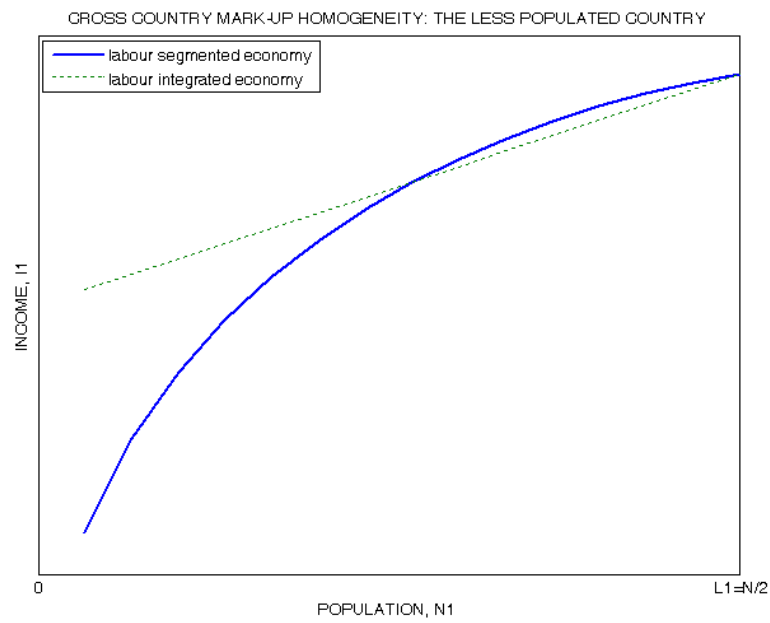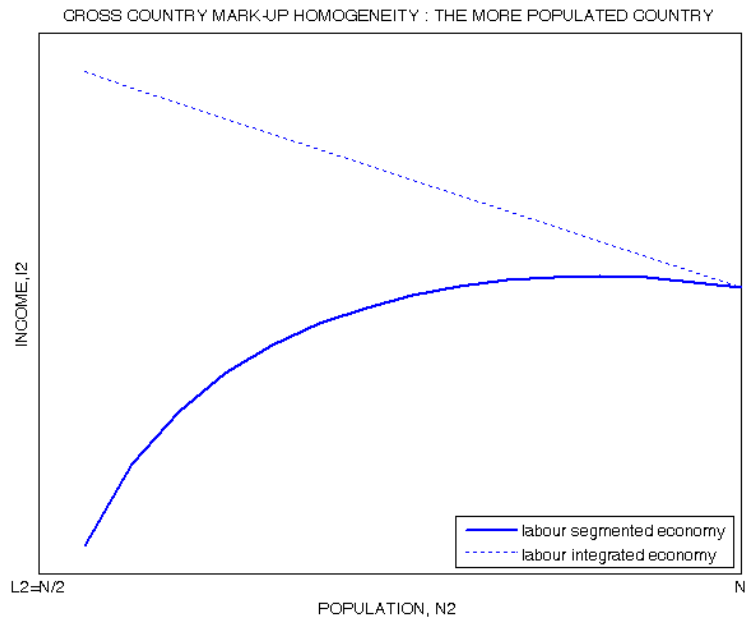
*Appendix*

CROSS COUNTRY MARK-UP HOMOGENEITY: GLOBAL ECONOMY



INCOME, I

labour allocation in an integrated market, L1
POPULATION IN COUNTRY ONE, N1

FIGURE 2

CROSS COUNTRY MARK-UP HOMOGENEITY: THE LESS POPULATED COUNTRY



labour segmented economy
labour integrated economy

INCOME, I1

POPULATION, N1

FIGURE 3

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CROSS COUNTRY MARK-UP HOMOGENEITY : THE MORE POPULATED COUNTRY

INCOME, I2

L2=N/2

N

POPULATION, N2

- labour segmented economy
- - - labour integrated economy

FIGURE 4

CROSS COUNTRY MARK-UP HETEROGENEITY : GLOBAL ECONOMY

INCOME, I

0

labour allocation in an integrated market, L1

N

POPULATION IN THE COUNTRY WITH LOWER MARK-UP, N1

FIGURE 5

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CROSS COUNTRY MARK-UP HETEROGENEITY: COUNTRY WITH LOWER MARK-UP



FIGURE 6

CROSS COUNTRY MARK- UP HETEROGENEITY : COUNTRY WITH HIGHER MARK-UP



FIGURE 7

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