Patrick M Crowley

How do you make a time series sing like a choir?

Using the Hilbert-Huang transform to extract embedded frequencies from economic of financial time series

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This research was mostly completed while the author was visiting the Bank of Finland during the summer of 2008. I would like to thank those who attended the Bank of Finland Macro Workshops and Jouko Vilmunen, Juha Kilponen and Zhaohua Wu for their comments. I am also grateful to Karin Blank of the US National Aeronautical and Space Administration (NASA) for help with using the HHT. Data provision was by Oskari Vähämaa. Finally, I wish to acknowledge that the title of this paper is a direct 'riff' on Lau and Weng (1995).

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ISBN 978-952-462-552-4
ISSN 0785-3572
(print)

ISBN 978-952-462-553-1
ISSN 1456-6184
(online)

Helsinki 2009

How do you make a time series sing like a choir?

Using the Hilbert-Huang transform to extract embedded frequencies from economic or financial time series

Bank of Finland Research
Discussion Papers 32/2009

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Abstract

The Hilbert-Huang transform (HHT) was developed late last century but has still to be introduced to the vast majority of economists. The HHT transform is a way of extracting the frequency mode features of cycles embedded in any time series using an adaptive data method that can be applied without making any assumptions about stationarity or linear data-generating properties. This paper introduces economists to the two constituent parts of the HHT transform, namely empirical mode decomposition (EMD) and Hilbert spectral analysis. Illustrative applications using HHT are also made to two financial and three economic time series.

Keywords: business cycles, growth cycles, Hilbert-Huang transform (HHT), empirical mode decomposition (EMD), economic time series, non-stationarity, spectral analysis

JEL classification numbers: C49, E0

Mikä selittää aikasarjan havaitun vaihtelun?

Hilbertin-Huangin muunnoksen käyttäminen talous- ja rahoitusaikasarjojen vaihtelun dekomponoinnissa taajuusalueella

Suomen Pankin keskustelualoitteita 32/2009

Patrick M. Crowley
Rahapolitiikka- ja tutkimusosasto

Tiivistelmä

Hilbertin-Huangin muunnos kehitettiin 1900-luvun lopulla, mutta menetelmä on vielä melkoisen tuntematon valtaosalle ekonomisteja. Muunnoksen avulla aikasarjan vaihtelu voidaan tilastollisesti hajottaa sarjaan sisältyviin taajuuskomponentteihin. Hilbertin-Huangin muunnos on oikeastaan adaptiiviseen tilastomenetelmään perustuva algoritmi, jota voidaan käyttää ilman rajoittavia oletuksia – kuten stationaarisuus tai lineaarisuus – aineiston tuottavasta mekanismista. Tässä työssä esitellään Hilbertin-Huangin muunnoksen kahta keskeistä elementtiä, tyyppiarvon (moodin) empiiristä dekomponointia ja Hilbertin spektraalianalyysia. Muunnoksen käyttöä havainnollistetaan lisäksi soveltamalla sitä kahden taloudellisen ja kolmen rahoitusaikasarjan analysointiin.

Avainsanat: suhdannevaihtelut, kasvusyklit, Hilbertin-Huangin muunnos, tyyppiarvon empiirinen hajotelma, taloudellinen aikasarja, epästationaarisuus, spektraalianalyysi

JEL-luokittelu: C49, E0

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1 Introduction

Empirical analysis using tools from the frequency domain is a woefully under-researched area in economics. Economics tends to focus on the time dimension of empirical macroeconomics, but developments in frequency domain techniques have extended far beyond the simple spectral analysis originally introduced into the mainstream by Granger through his Granger and Hatanka (1964) and subsequent Granger (1966) contribution. Developments in signal processing and other disciplines have taken place which now give the researcher much more advanced techniques than the very basic spectral analysis that is usually glossed over in a typical graduate econometrics course. Time-frequency analysis has been the area where most value-added is likely to be found for economists, notably in wavelet analysis (see Crowley, 2007), and more recently empirical mode decomposition, the subject matter of this article. The latter, although already over 10 years old, has been, to my knowledge, completely ignored by economists. This article seeks to rectify this deleterious situation, by introducing economists to the Hilbert-Huang transform (HHT) which includes empirical mode decomposition (EMD), a relatively new and innovative technique together with a novel new variant that was introduced last year.

What makes frequency domain analysis important in empirical macroeconomic and financial analysis? Simply put, the time horizon and the interrelationships between macroeconomic and financial variables at different time horizons. In time-series analysis we often search (by using different econometric specifications) for the most appropriate 'fit' for the time-series data at hand, and thus to better understand the evolution of the series over time and the drivers behind the series. In time-frequency domain we can take this one step further – we can attempt to understand the evolution of the series over different time horizons and the drivers behind the series at different time horizons. Given the ongoing developments in time-frequency analysis there is a possibility that we might also be able to uncover meaningful sub-series in the data operating at different frequencies.

The only applications of HHT and EMD to economic and financial time series to date can be found in Huang and Shen (2005), Zhang, Lai, and Wang (2007) and Crowley and Mayes (2008), and of these only one so far appears in journal format. Section 2 explains the technique of empirical mode decomposition, section 3 applies the technique to economic and financial time series, while section 4 concludes.

2 The Hilbert-Huang transform and empirical mode decomposition

2.1 Background

Identifying the different frequencies at work in economic variables was pioneered by Granger (1966) and has recently been updated by Levy and Dezhbakhsh (2003b) and Levy and Dezhbakhsh (2003a). Both of these articles use traditional spectral analysis techniques, but this approach assumes that time series are stationary and linearly generated, so although the findings are consistent with Granger's this is hardly surprising, given that spectral analysis should not be applied to non-stationary time series. Both wavelet analysis and HHT allow the use of non-stationary data, and although wavelet analysis assumes that variables are linearly generated, HHT does not. Table 1 gives a summary of the different frequency domain methods available to researchers and the implicit assumptions used by each method.

One of the main problems with using traditional spectral analysis is that economic and financial variables and rarely both globally and locally stationary, and although using time-varying spectral analysis is clearly superior to using traditional spectral analysis spurious results may still result from local non-stationarities and from asymmetries in cycles. Wavelet analysis (see Crowley, 2007) is clearly superior to spectral analysis for dealing with most economic and financial variables, but problems still exist, as with discrete wavelet analysis cycles might not always lie within the dyadic frequency ranges imposed by scale separation, and so might lie on the border between these ranges hence with some 'bleeding' between the scales might appear in more than one crystal. Also with continuous wavelet analysis there might be problems of frequency resolution and there are also usually only symmetric wavelet functions available 'off the shelf', limiting the usefulness for economic and financial series.

The EMD method introduced by Huang, Shen, Long, Wu, Shih, Zheng, Yen, Tung and Liu (1998), resolves many of these problems by allowing non-stationary series (both globally and locally), non-linearly generated processes and also asymmetric cycles. The method has subsequently been applied to many areas in physics, mechanics, engineering, astronomy and the environmental sciences, and the US National Aeronautical and Space Administration (NASA) has taken great interest in the new technology, patenting a special application of the method. Unlike both spectral methods and wavelets, the EMD method is entirely empirically based – it has no formal mathematical basis, but rather attempts to break down the series according to how many frequencies are apparent in the data – in other words it allows the data speak for itself rather than imposing certain *a priori* beliefs about which frequencies are present at any time within a series.

	Time series	**Spectral**	**Time-varying spectral**	**DWT**	**CWT**	**HHT/EMD**
Basis?	A priori	A priori	A priori	A priori	A priori	A posteriori adaptive
Domain?	Time	Frequency	Frequency through time	Time-frequency	Time-frequency	Time
Stationary?	Yes	Yes	Yes within each window	No	No	No
Linearly generated?	Yes	Yes	Yes	Yes	Yes	No
Mathematical underpinning?	Yes	Yes	Yes	Yes	Yes	No, empirical
Asymmetric cycles?	Yes	Yes	No	Yes	Yes	Yes

Table 1: Summary of frequency domain methods

Since being introduced a decade ago, a small group of researchers have extended and modified EMD, in a series of publications, notably Huang and Shen (2005), Wu and Huang (2008) and Huang and Wu (2008), and have recently launched a journal[1] to provide a publication outlet for applications using EMD and to further advance the EMD methodology.

2.2 Methodology

EMD is actually part of a two-step procedure referred to as the Hilbert-Huang transform (HHT[2]):

1. Do EMD to obtain intrinsic mode functions (IMFs); and
2. use the Hilbert spectrum to assess instantaneous frequency for each IMF.

The Hilbert transform is not new, but EMD is. The main advantage of using EMD over other frequency domain techniques is that it not only identifies separate processes at work in a series, but it also separates each of these out and resolves them in time-frequency space. The IMFs should satisfy the following properties: (1) in the whole data set, the number of extrema and the number of zero crossings must either equal or differ at most by one; and (2) at any point, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero. An exhaustive introduction and comparison with continuous wavelet analysis is provided in Huang, Shen, Long, Wu, Shih, Zheng, Yen, Tung, and Liu (1998).

These processes or IMFs could be meaningful in that they represent the separate processes operating at different frequencies embedded within the data. To quote Huang and Wu (2008), (p. 19):

> HHT offers a potentially viable method for nonlinear and nonstationary data analysis, especially for time-frequency-energy representations. It has been tested widely in various applications other than geophysical research but only empirically. In all the cases studied, HHT gives results much sharper than most of the traditional analysis methods. And in most cases, it reveals true physical meanings.

The essence of the method is to identify the intrinsic oscillatory modes by their characteristic time scales in the data empirically, and then decompose the data accordingly. EMD does not impose any *a priori* conditions on the data (such as stationarity or linearity), but rather allows the data to speak for itself. EMD sifts the data by identifying maxima and minima in the data so as to identify cycles within the data at different frequencies, using a spline algorithm as follows:

[1] Journal of Adaptive Data Analysis.

[2] Named after Norden Huang who invented the EMD part of the process and David Hilbert who is the mathematician who originated the notion of a Hilbert spectrum. The methodology was designated HHT by NASA.

i) identify maxima and minima of $x(t)$

ii) generate upper and lower envelopes with cubic spline interpolation $e_{\min}(t)$ and $e_{\max}(t)$.

iii) calculate mean of upper and lower envelopes

$$m(t) = (e_{\max}(t) + e_{\min}(t))/2 \tag{2.1}$$

– this process is shown in figure 1.

iv) the mean is then subtracted from the series to yield a difference variable, $d(t)$

$$d(t) = x(t) - m(t) \tag{2.2}$$

v) if the stopping criterion (SC)

$$\sum_{t=1}^{T} \frac{[d_j(t) - d_{j+1}(t)]^2}{d_j^2(t)} < SC \tag{2.3}$$

is met, where $d_j(t)$ is the result from the jth iteration, then denote $d(t)$ as the ith IMF and replace $x(t)$ with the residual

$$r(t) = x(t) - d(t) \tag{2.4}$$

vi) if the stopping criterion it is not an IMF, replace $x(t)$ with $d(t)$.

vii) repeat steps i) to v) until residual $r_n(t)$ has at most only one local extremum or becomes a monotonic function from which no more IMFs can be extracted.

The EMD process can also be illustrated by a diagrammatic flow chart, as in figure 2. The resultant decomposition of the series can be written as

$$x(t) = \sum_{j=1}^{n} c_j(t) + r_n(t) \tag{2.5}$$

where $c_j(t)$ represents the jth IMF.

Once the IMFs have been obtained, given the fact that (unlike traditional spectral analysis which typically uses Fourier analysis with constant frequency) variable frequency cycles can occur, it is more appropriate to use measures of instantaneous frequency and amplitude. This follows on from the observation that cycles can either change within a single period (known as ‘intrawave’

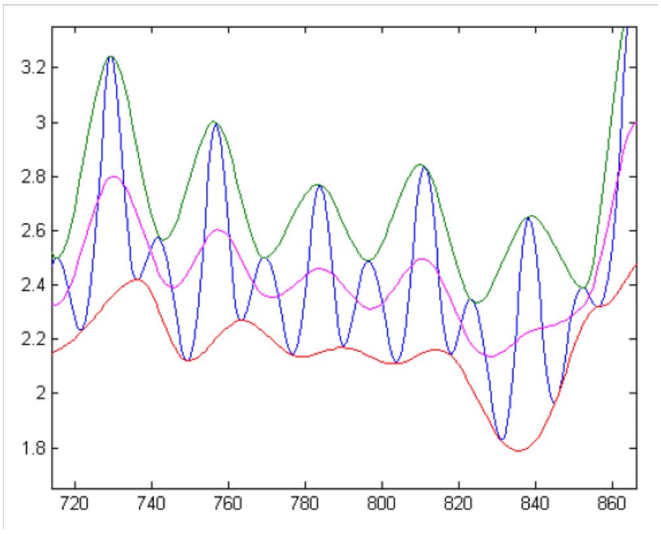

Figure 1: The spline-envelope process under EMD for a hypothetical series

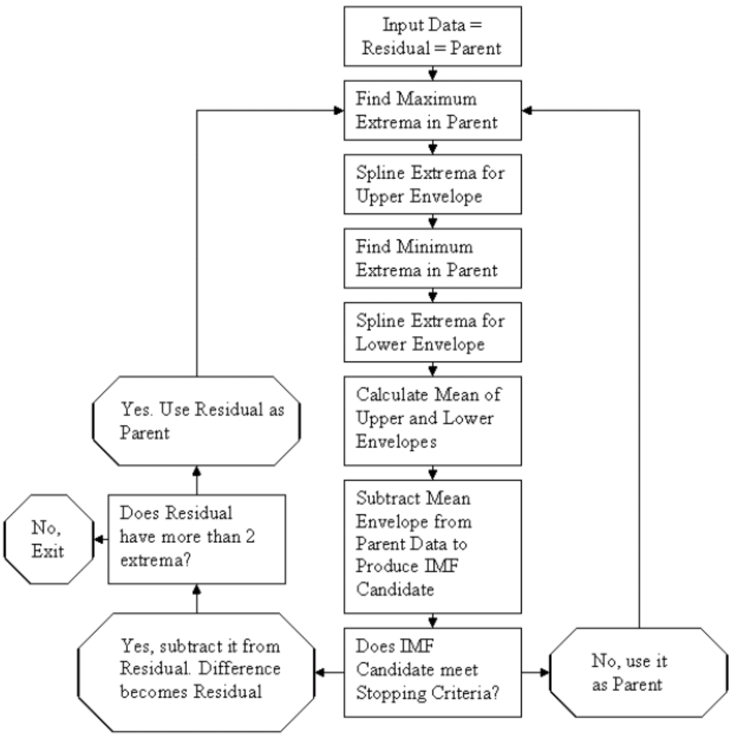

Figure 2: Flow chart of EMD sifting process

frequency modulation) or between cycles (known as 'interwave' frequency modulation), or with a combination of both types of modulation. Spectral analysis can detect the latter, particularly when using time-varying spectral analysis, but it cannot detect the former, and yet the former is likely, particularly with the non-linear types of processes that characterize economics.

The Hilbert spectrum lends itself directly to the task of estimating instantaneous frequency, thus allowing the researcher to account for all types of frequency modulation. In mathematical terms, for any function $x(t)$ of L^p class, its Hilbert transform $y(t)$ is

$$y(t) = \frac{1}{\pi} P \int_{-\infty}^{+\infty} \frac{x(\tau)}{t-\tau} d\tau \tag{2.6}$$

where P is the Cauchy principal value of the singular integral. The Hilbert transform $y(t)$ of any real-valued function $x(t)$ will yield the analytic function

$$z(t) = x(t) + iy(t) = a(t) \exp\left[i\phi(t)\right] \tag{2.7}$$

where $i = \sqrt{-1}$, $a(t)$ represents the amplitude and $\phi(t)$ the phase ($\phi(t) = \arg(x(t))$). $a(t)$ is then given by

$$a(t) = \left(x^2 + y^2\right)^{1/2} \tag{2.8}$$

and

$$\phi(t) = \tan^{-1}\left[\frac{y}{x}\right] \tag{2.9}$$

Instantaneous frequency, ω, then is given by

$$\omega = \frac{d\phi}{dt} \tag{2.10}$$

The instantaneous frequency introduced here is physical and depends on the differentiation of the phase function, which is fully capable of describing not only interwave frequency changes due to nonstationarity but also the intrawave frequency modulation due to nonlinearity. The Hilbert transform as applied to each IMF can now be expressed as

$$z(t) = \sum_{j=1}^{t} a_j(t) \exp\left[i \int \omega_j(t) dt\right] \tag{2.11}$$

so that the instantaneous amplitude $a_j(t)$ can be separately extracted from the instantaneous phase $\omega_j(t)$ for each IMF and combined into a Hilbert (amplitude) spectrum, $H(\omega, t)$ (see Huang and Shen, 2005). The power (or energy) spectrum is given by $[H(\omega, t)]^2$ so accordingly the marginal average power spectrum is

$$h(\omega) = \frac{1}{T} \int_0^T H^2(\omega, t) dt \tag{2.12}$$

2.3 Other issues

The first concern, as with spectral analysis and wavelet analysis, relates to end effects. At the beginning and at the end of the time series the cubic spline is not defined, so where the cubic spline fitting can have large swings. Left by themselves, the end swings can eventually propagate inward and corrupt the whole data span especially in the low-frequency components. A numerical method of adding extra waves to eliminate the end effects has been implemented.

The second concern is with 'mode mixing', which is defined as a single IMF either consisting of signals of widely disparate scales or a signal of a similar scale residing in different IMF components. Usually mode mixing is identified by the frequencies of different (usually adjacent) IMFs intersecting each other. To overcome this problem a new noise-assisted data analysis method was proposed, the ensemble EMD (EEMD), which defines the true IMF components as the mean of an ensemble of trials, each consisting of the signal plus a white noise of finite amplitude. More details can be obtained from Wu and Huang (2008). In this study, as little mode mixing is observed, EEMD is not utilized.

3 Illustrative applications

In this section HTT and EMD are applied to a selection of financial and economic time series. Data sources are listed in an appendix.

3.1 Financial time series

3.1.1 The Dow Jones industrial average

Monthly data for the Dow Jones Industrial Average: 1896–2008 is first used. The data is transformed by taking natural logs, and is displayed in figure 3. The recent fall in the index is clearly significant, but nevertheless the stockmarket crash of 1929 clearly dominates the series. No further transformation of the series is done, and the IMFs obtained as well as the residual are shown in figure 4. Six IMFs are obtained, with the residual shown at the bottom of the panel in red. The residual obviously indicates the trend of the series, with more marked increases in the series shown by two accelerating waves which begin in the 1950s and repeat in the 1990s. Other non-regular cycles are shown in IMFs 3, 4 and 5, while IMFs 1 and 2 appear to contain mostly noise. The quality of the decomposition is dependent on the frequency resolution of the IMFs as noted above, and this should also be apparent from the frequency resolution which is plotted in figure 5. Figure 6 shows the Hilbert spectrum (– with the colour bar on the right hand side of the figure showing that low energy levels are in blue, middling energy levels in red and high energy frequencies in yellow) and it clearly shows that the lower frequency

Figure 3: The Dow-Jones Industrial Average (DJIA) from 1896 to 2008

IMFs tend to generally contain more energy than the higher frequency IMFs, although there are circumstances (for example around 1915 and between 1920 and 1930) when shorter cycles appeared to have more energy. IMF5 appears to correspond most closely to the business cycle, particularly as it has troughs in around 1993 and 2001. Lastly figure 7 shows the marginal Hilbert power spectrum which indicates that the IMF with most consistent energy lies at roughly a 40 year cycle. Beyond this, there is nothing evident in terms of the IMFs from the original data.

3.1.2 The US dollar – British pound exchange rate

Figure 8 gives the US dollar-British pound exchange rate from March 1973 to May 2009 using monthly data, and including the precipitous fall in the pound during the first half of 2009. The data obviously displays some irregular wave-like features, with both irregular amplitude and frequency. Figure 9 shows that the EMD method extracts five IMFs with another IMF likely in the residual – note that unless a full cycle is observed, any incomplete cycles remain in the residual. IMF5 clearly extracts the general cycle directly observed in the data, particularly from 1973 to 1988. Interestingly though four other IMFs are apparent in the data, of various frequency and amplitude. What is also noticeable is that during short-lived period when the pound entered the ERM the volatility observed in IMFs 1 to 3 clearly increased in a synchronized manner, but the lower frequency IMFs were not affected. Also it is interesting to note that the current fall in the value of the pound has been contained solely in IMF3, and does not appear in any other IMF. Figure 10 shows the instantaneous frequency for each IMF and it appears as though the Hilbert transform has achieved a good separation in terms of frequency, leading to

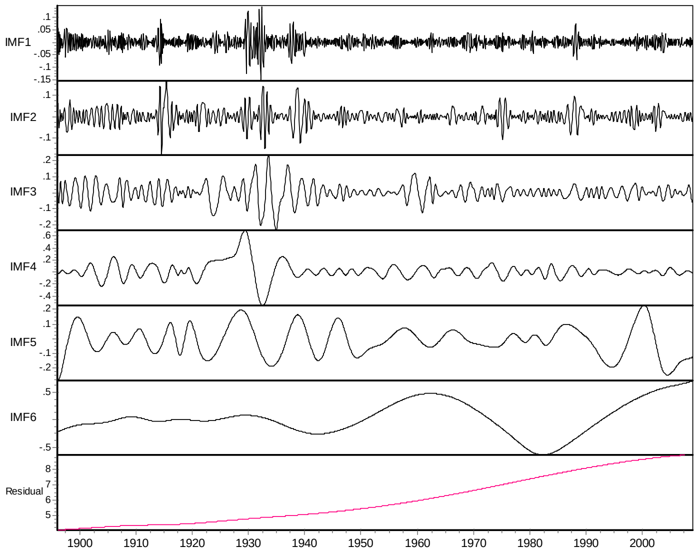

Figure 4: IMFs for DJIA

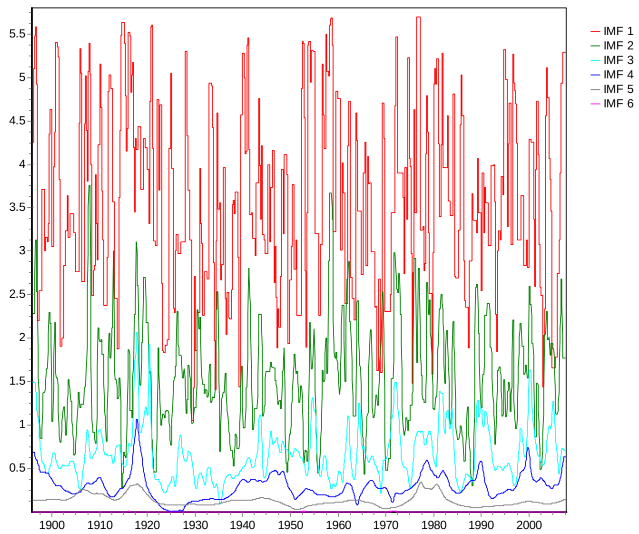

Figure 5: IMF Frequencies for DJIA

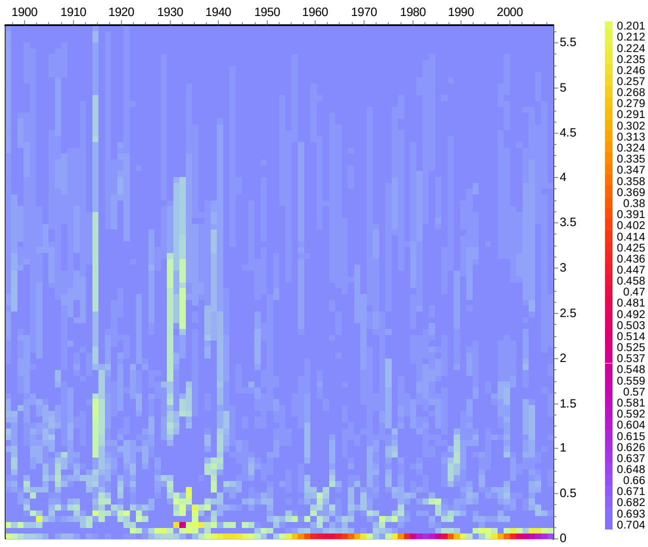

Figure 6: Hilbert spectrum for DJIA IMFs

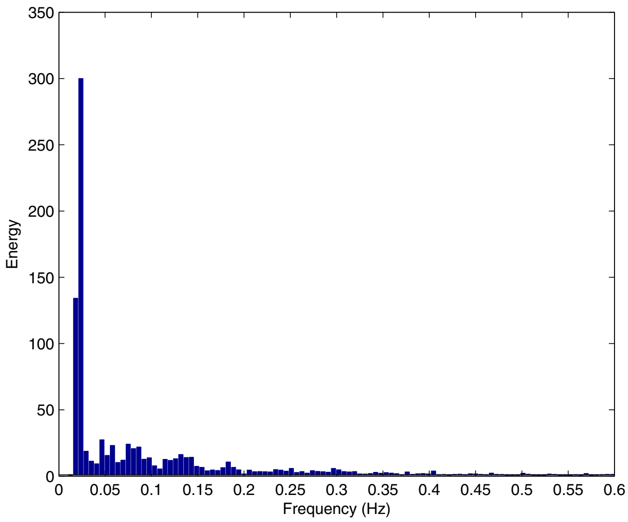

Figure 7: Marginal Hilbert power spectrum for DJIA IMFs

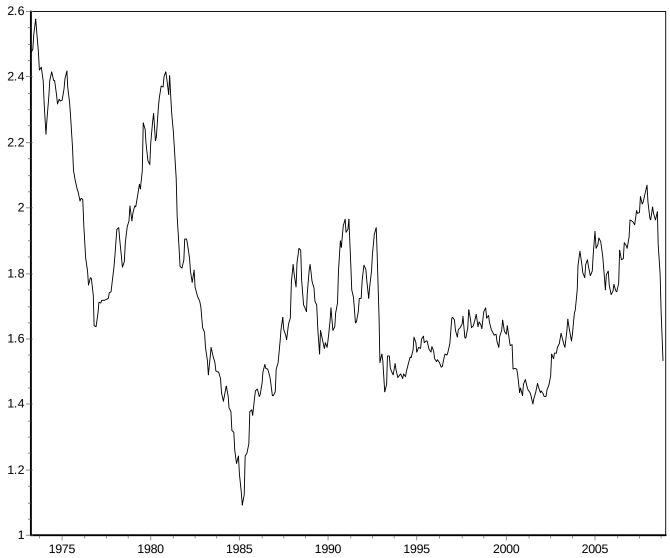

Figure 8: US dollar-British pound exchange rate

well-defined IMFs. There appears to be a very long cycle of around a 20 year duration, and a shorter cycle at about 10 years, and then two variable cycles that range in frequency from between around 6 months to 2 years in length, and lastly a very short cycle with a frequency usually above 6 months. The Hilbert spectrum for the IMFs is shown in figure 11 and as might be expected, shows that the large amplitudes of the longer cycles dominate movements in the series while higher frequency IMFs only appear to gain power over relatively short periods of time (such as in the early 1990s). Lastly figure 12 shows the marginal power spectrum and shows that the two low frequency IMFs hold most of the energy in the series, and dominate the shorter frequencies over the life of the series.

3.2 Economic time series

3.2.1 US industrial production

In figure 13 (natural) log of monthly US industrial production is shown from 1919 to the end of 2008. The series is clearly highly volatile before around 1947 but then appears to exhibit much less volatility in the post-war era. It is well known that recessions in the US usually tend to adversely affect the manufacturing sector much more than other sectors, so the post-war recessions can be very clearly seen in the data. Figure 14 shows the extracted IMFs for the series, five in all, with the business cycle clearly apparent in IMF5, but no cycles detected with lower frequencies than that in the data. This is an important result, as it implies that higher frequency cycles, along with the business cycle account for all fluctuations in industrial production data for the US. With the

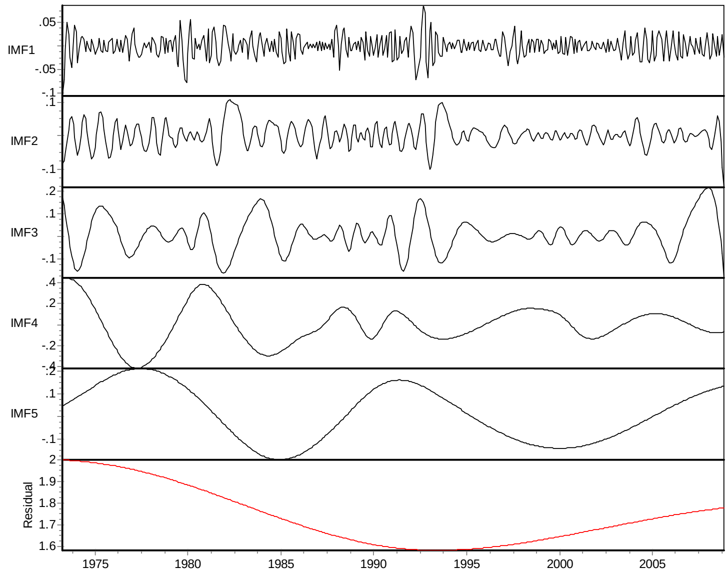

Figure 9: IMFs for the US dollar-British pound exchange rate

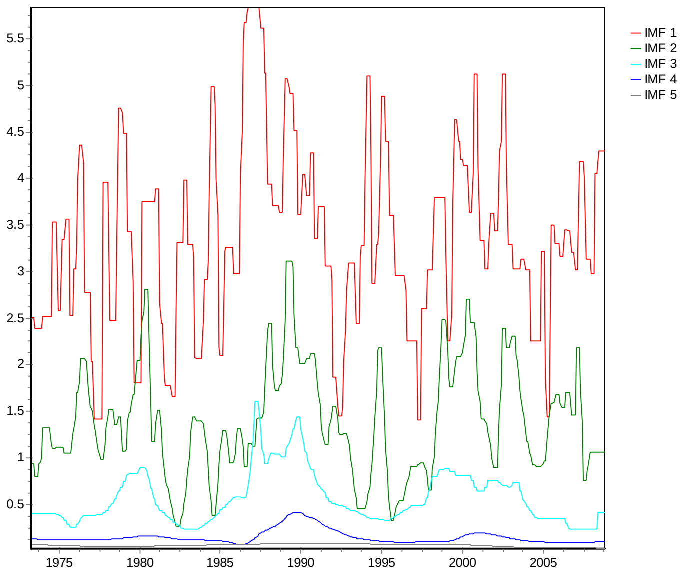

Figure 10: Frequency of IMFs for US dollar-British pound exchange rate

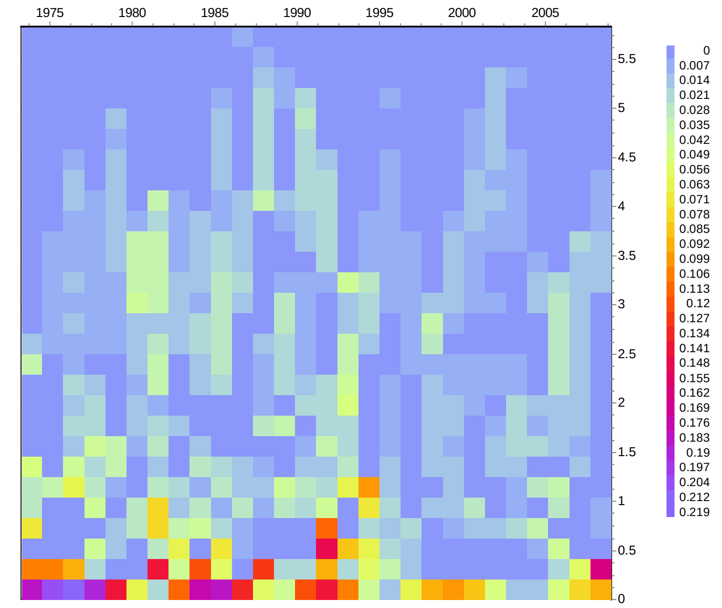

Figure 11: Hilbert spectrum for IMFs of US dollar-British pound exchange rate

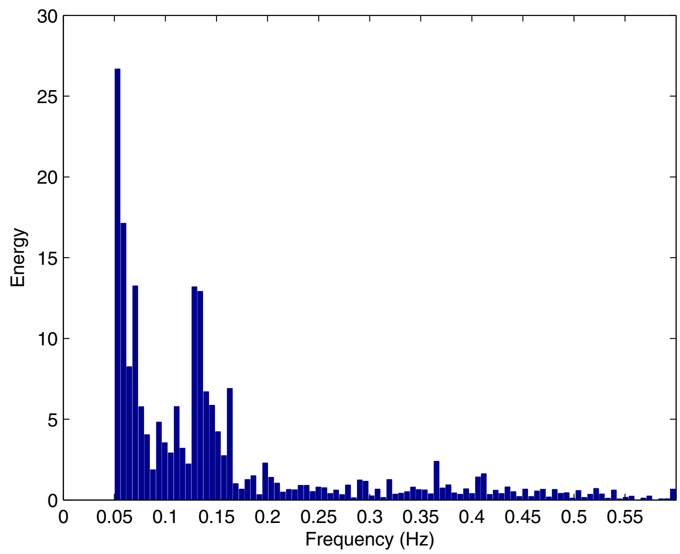

Figure 12: Marginal Hilbert power spectrum for IMFs of the US dollar-British pound exchange rate

higher frequency IMFs, IMF1 shows no significant post-war change, and has very little energy, but IMFs 2 and 3 exhibit reduced volatility from the early 1980s with IMF4 not altering in the 1980s but virtually disappearing from around 1995 onwards. This tends to suggest that the 'great moderation' only affects two cycles in growth in industrial production, but longer term business cycle fluctuations appear not to have been affected. Interestingly the residual which represents the trend in the data is clearly not smooth and is therefore non-linear, with rapid increases in the 1960s, but moderating growth since then.

As figure 15 illustrates, the frequencies of the IMFs are fairly well separated, but there are some instances when 'mode mixing' occurs. What is of interest here is the change in cycle frequencies that clearly happens before and after WWII – this is perhaps best shown by separating out the frequencies for each individual IMF and this is done in figure 16. Before WWII there was a 4–10 year cycle (IMF4) and a stable 20 year cycle (IMF5), but after WWII the cycles change with now roughly a 10 year cycle (IMF5) and a 2.5–5 year cycle (IMF4). Given that this can be treated as a 'stylized' fact, the obvious question to ask is whether this event was brought about by an elimination of the 20 year cycle or whether the cycle (as suggested by the technique here) modulated to a higher frequency after the war.

Lastly, figures 17 and 18 show the Hilbert spectrum for the IMFs and the Marginal Hilbert power spectrum for the IMFs. As expected, lower frequencies contain most energy in the series, and the most noticeable aspect of the Hilbert spectrum is the waning of power in higher frequencies in 1960 and then in the early 1980s. What is also very noticeable is that the energy in the lower frequency cycles has not changed much over time. This has possibly important implications for economic policymakers.- it implies that better economic policymaking can dissipate the energy that resides in high frequency cycles, but to date, it doesn't seem to have affected the energy contained in low frequency cycles, and particularly the IMFs where the business cycle resides.

3.2.2 UK retail price index

The log change in the UK retail price index (RPI) appears to exhibit considerable cyclical fluctuations, as shown in figure 19, but these have tended be less volatile since the early 1990s. In terms of stationarity, inflation is usually considered a non-stationary variable because of inflation persistence, and this can clearly be seen in the clear upward move in the inflation rate during the 1970s.

Figure 20 shows that EMD extracts six IMFs and a inverted U-shaped residual, showing that inflation has dropped to consistently low long-term levels (– despite the fact that in the short term inflation appears to have increased in the last 2 years). IMF1 shows just high frequency noise, but IMF2 appears to include bursts of cyclical volatility,[3] some with large amplitude.

[3]In the signal processing literature these short bursts or packets of volatile movements

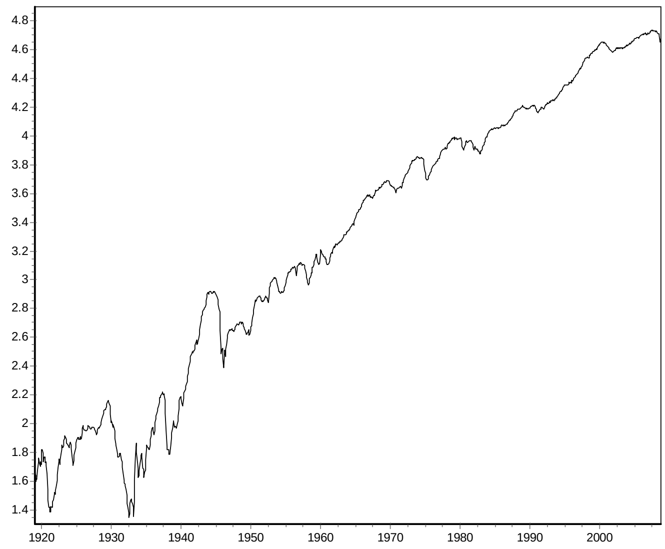

Figure 13: US industrial production (natural log)

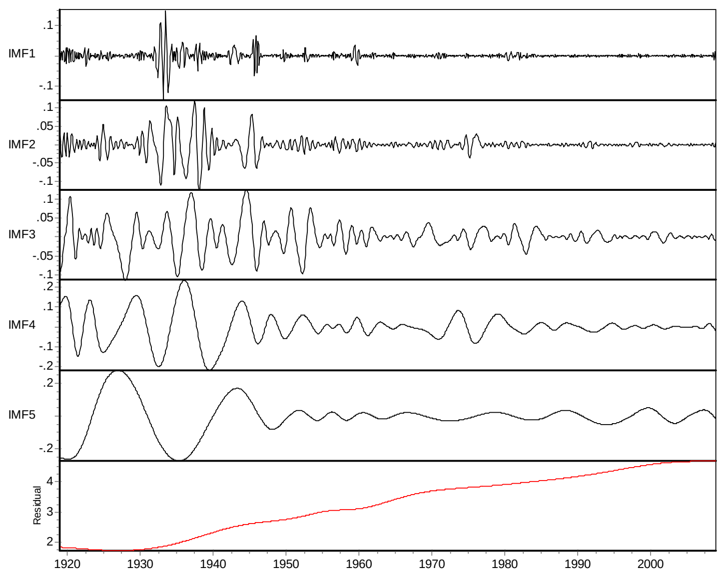

Figure 14: IMFs for US industrial production

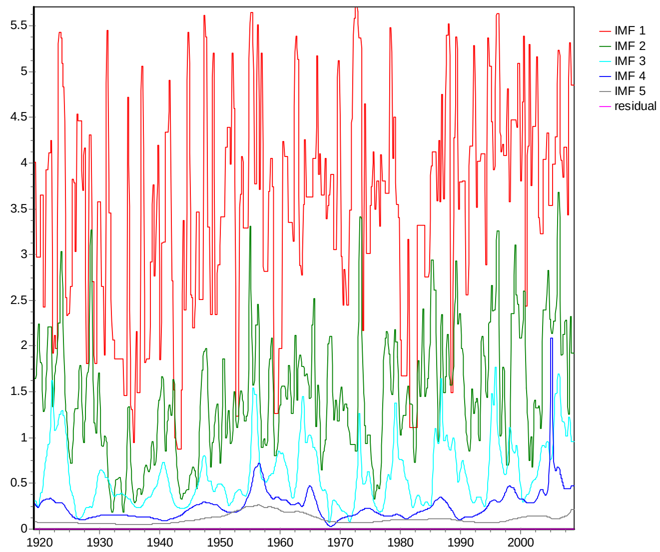

Figure 15: Frequency of IMFs for US industrial production

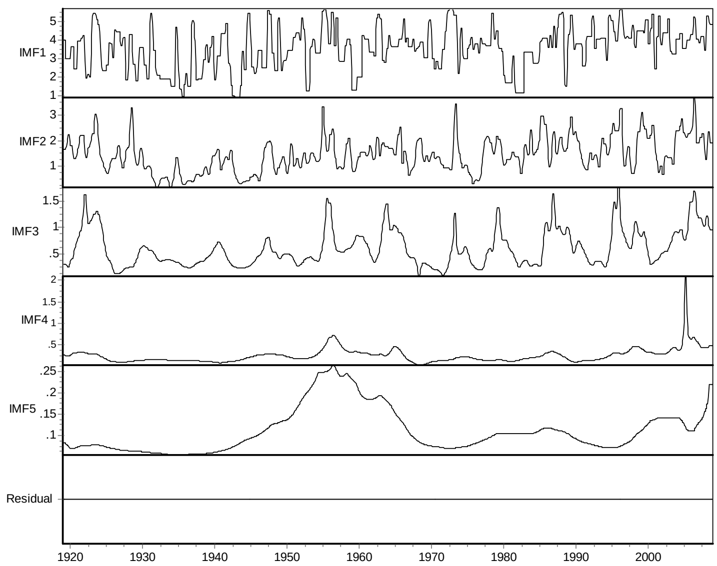

Figure 16: Frequency of individual IMFs for US industrial production

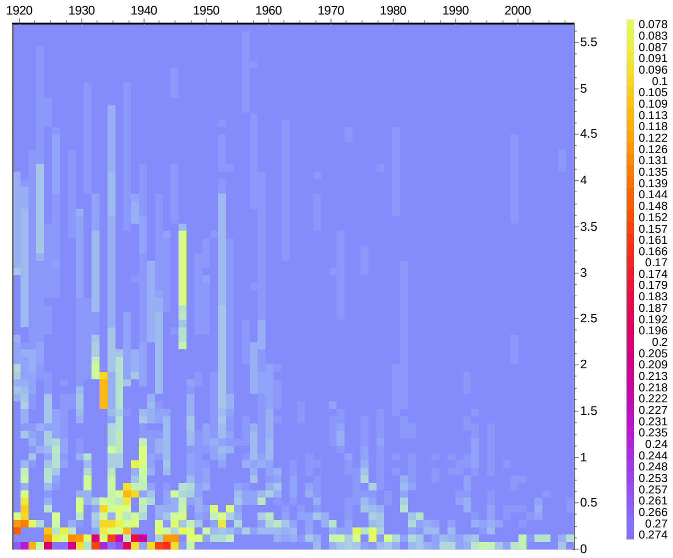

Figure 17: Hilbert spectrum for IMFs of US industrial production

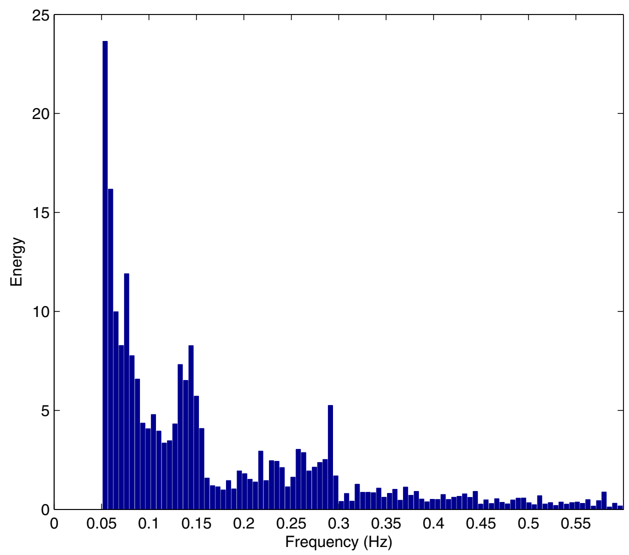

Figure 18: Marginal Hilbert power spectrum for IMFs of US industrial production

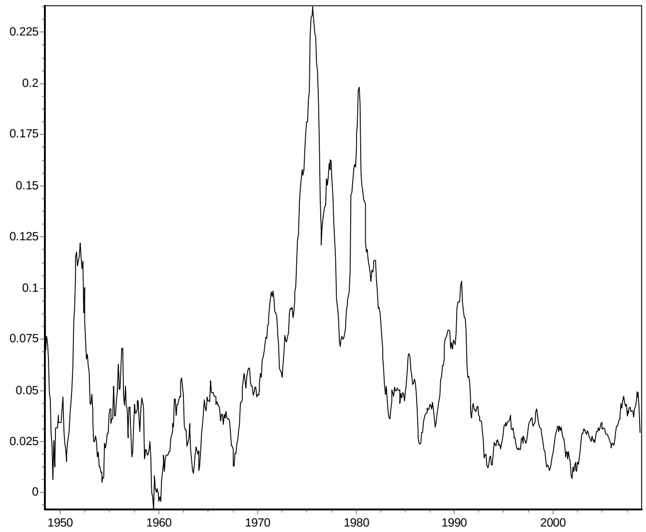

Figure 19: UK log RPI

Also interestingly in IMF2, from 1985 to 1990 there was the appearance of roughly four cycles of similar amplitude but lower frequency than the rest of IMF2 – in fact the frequency of the cycles appears to be more in keeping with the types of cycles observed in IMF3, which are very irregular and of extremely variable frequency. IMFs 4 to 6 contain most of the momentum in the series, with IMF6 containing a 30–35 year cycle, but IMF5 exhibits cycles that are between roughly 10 and 15 years in length, and IMF4 appears to correspond to roughly a business cycle frequency, with inflation being procyclical as expected. What is curious here though is the way that IMFs 4 and 5 appear to disappear from about 1985 until roughly 2005, leaving only IMFs 1 to 3 and 6 in operation during this time.

In terms of frequency, figure 21 suggests that IMFs 1 and 2 are well separated for the most part, but that IMF3 mode mixes with lower frequencies in the 1960s and 1990s and with higher frequencies in the 1970s and 1980s. IMF4 operates at a 4 year cycle while IMF5 coincides with business cycle frequencies of 6 to 10 years. IMF6 operates at very long cycles. As might be expected, IMFs 4 and 5 drop out by 2001, leaving just a long cycle. Figure 22 offers a breakdown by IMF, and here it is apparent that only IMF4 appears to drop out in 2001, but it is now also obvious that IMF6's 30–35 year cycle has been shortening since 1996.

Lastly, the spectra for UK retail price inflation are shown in figures 23 and 24. Clearly the two lower frequency cycles contain most energy but then there is also a two year cycle that is still evident in the data, but the Hilbert spectrum clearly indicates that the cyclical properties of the data are weaker now than they were in the past.

in the series are often called 'chirps'.

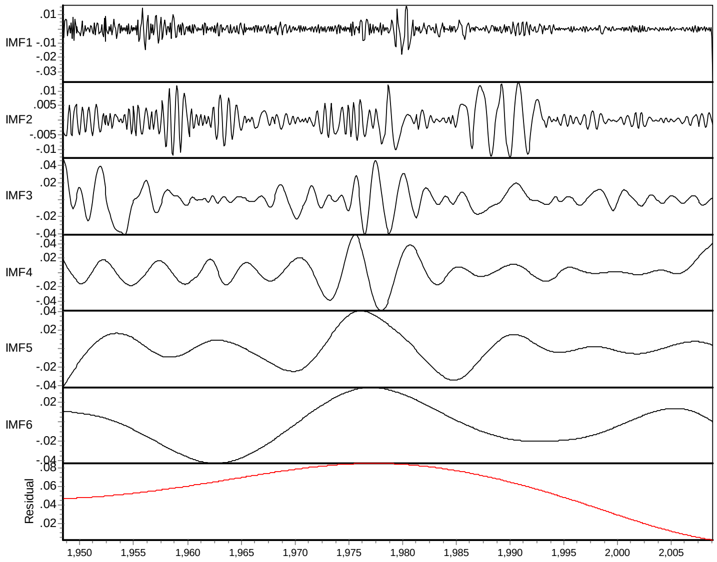

Figure 20: IMFs for UK retail price index

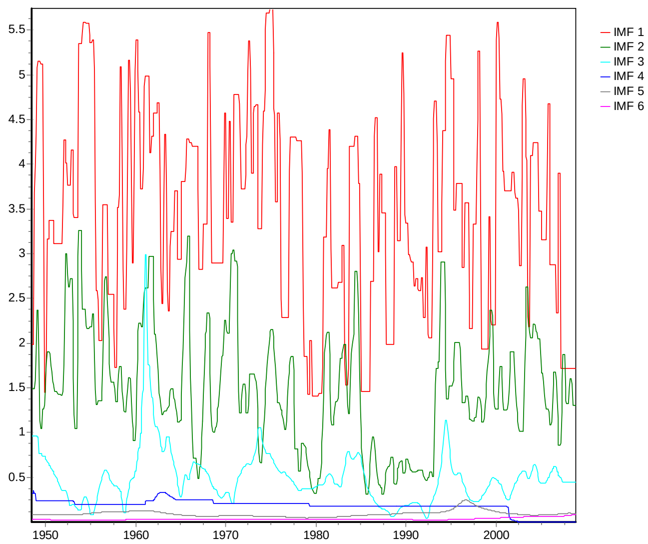

Figure 21: Frequency of IMFs for UK RPI

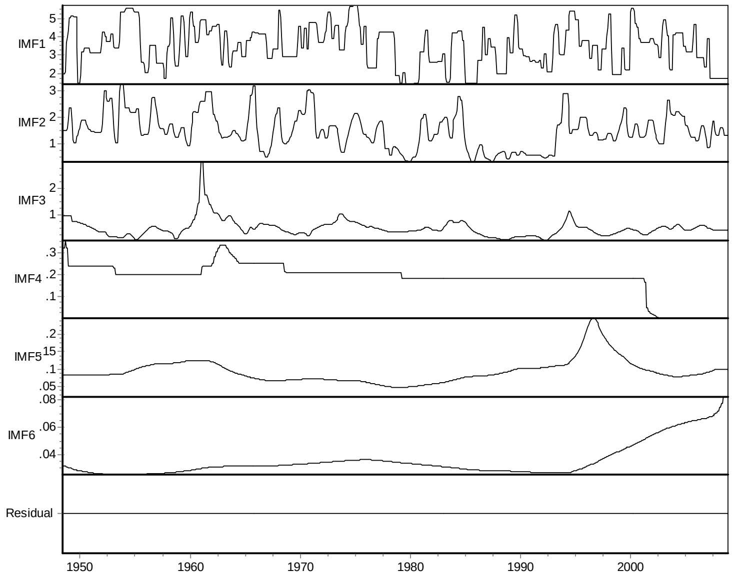

Figure 22: Frequency of individual IMFs for UK RPI

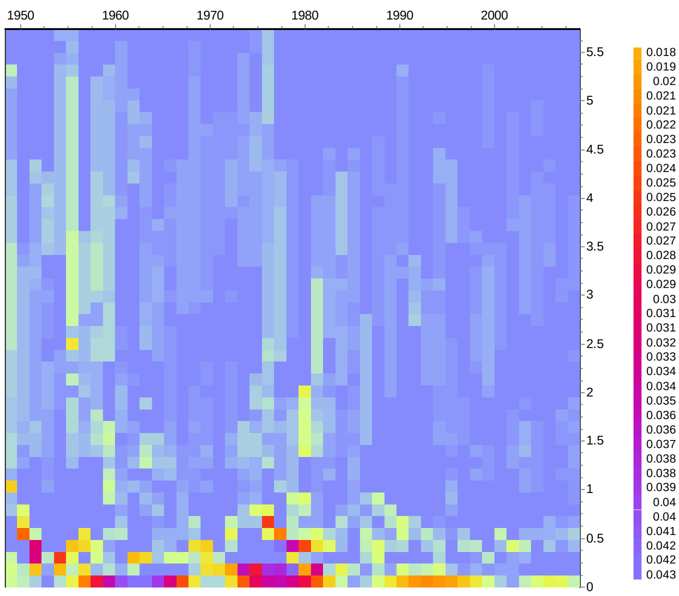

Figure 23: Hilbert spectrum for IMFs of UK RPI

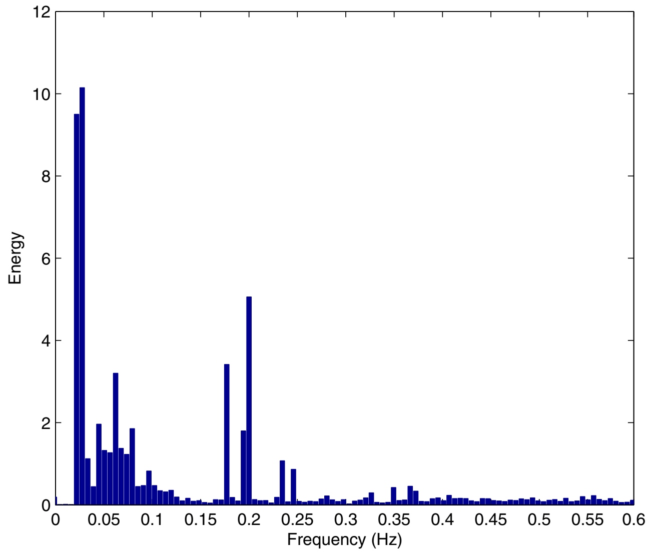

Figure 24: Marginal Hilbert power spectrum for IMFs of UK RPI

3.2.3 UK M1 monetary aggregate

Given that the previous section looked at inflation, it is informative to look at a UK monetary aggregate to see if we get any similar frequency fluctuations evident in the data. Figure 25 shows the series from 1970 quarter 2 through to the end of 2008. The series is clearly volatile, and once again there appears to be local non-stationarity, particularly going from the 1970s to the 1980s. This is not surprising since the 'monetarist' period in UK policymaking began in 1979 and ended in 1982, with a large contraction in the monetary base and high interest rates. There is also another clear 'spike' in the monetary aggregate in 2000, which presumably relates to the downturn in the economy at that time, with a downward 'spike' in the aggregate about a year later. After using EMD, the series decomposes into 6 IMFs, similarly to UK inflation, as shown in figure 26 with a downward trend in the overall data (largely because the large downward 'spike' in the series in the early 1970s is captured in an IMF operating at a higher frequency). IMF1 largely consists of noise, but IMF2 has quite regular cycles for the most part, as does IMF3. Both IMF4 (which has approximately a 4 year cycle) and IMF5 (with around a 7 year cycle) have experienced a reduction in volatility from 1980 onwards, and there is a long cycle (IMF6) operating at roughly a 25 year frequency. Figure 27 shows some mode mixing although for the most part the frequencies of each IMF are resolved separately. There is clearly a two year cycle (IMF3), a three to five year cycle (IMF4), a roughly 10 year cycle that got shorter during the period and then longer (IMF5) and an extremely long cycle (IMF6).

In terms of spectra, figures 28 and 29 show the Hilbert and marginal power spectra for M1. Clearly the largest amount of energy lies in the lower frequencies, but at certain times (for example in the early 1970s, early 1980s and early 2000s), shorter cycles appear to have significant energy, albeit for

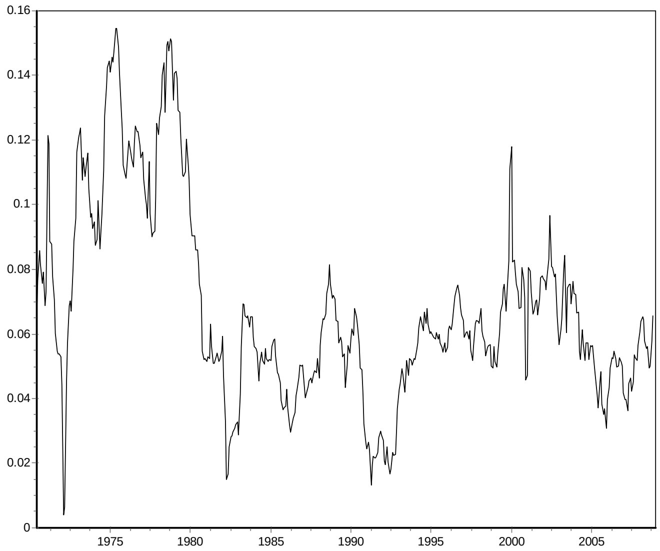

Figure 25: Log change in UK M1 monetary aggregate

relatively short periods of time (– the only exception here being the early 2000s). The marginal spectrum also reflects this, with strong long cycles and less strength in the shorter term cycles.

One interesting corollary of this analysis relates to the long run relationship between money and prices in the UK. There does appear to be long cycles in both inflation and the monetary aggregate for the UK, and with the very long cycle (IMF6) in both cases, inflation appears to lag movements in the monetary aggregate by approximately 2 to 3 years. This mirrors results recently found by Benati (2009). The relationship between other IMFs in the two variables is much less clear though, and merits further investigation.

4 Conclusions

The HHT is a relatively new technique which was introduced just over a decade ago. It offers a new approach in frequency domain analysis by using an adaptive data algorithm which accurately extracts embedded cycles in data. The HHT consists of two stages - first sifting the data using empirical mode decomposition, which extracts the different embedded frequency series (known as IMFs) – and then transforming the data using the Hilbert transform so as to analyse the data in terms of frequency domain measures. The main advantages of the method are that it does not assume stationarity (either globally or locally), and does not assume any data generating process, so can cope with non-linear data. Spectral analysis, the traditional workhorse of frequency domain analysis assumes both stationarity and a linear data generating process so is not suitable for analysis of many economic variables. The method is still in development, with a new variant introduced in 2008, but nevertheless is

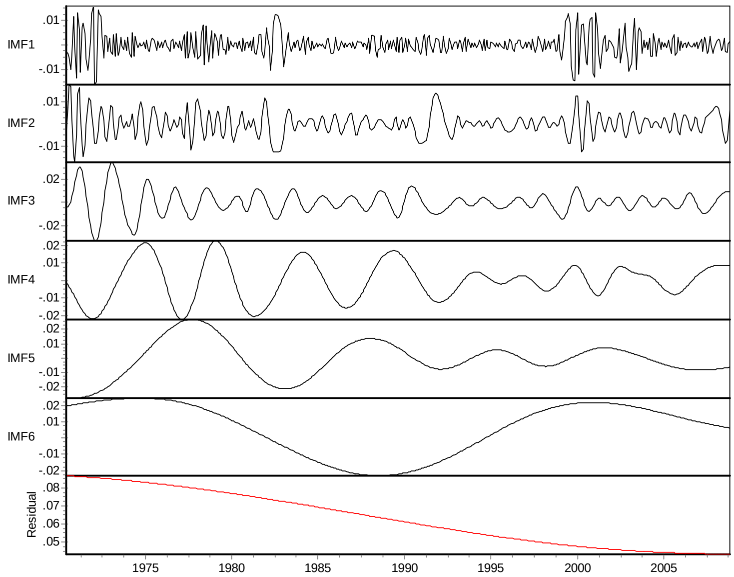

Figure 26: IMFs for UK M1

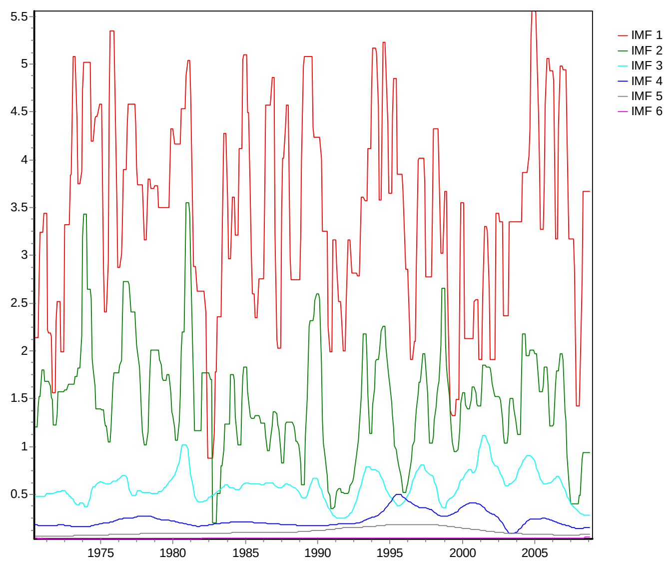

Figure 27: Frequency of IMFs for UK M1

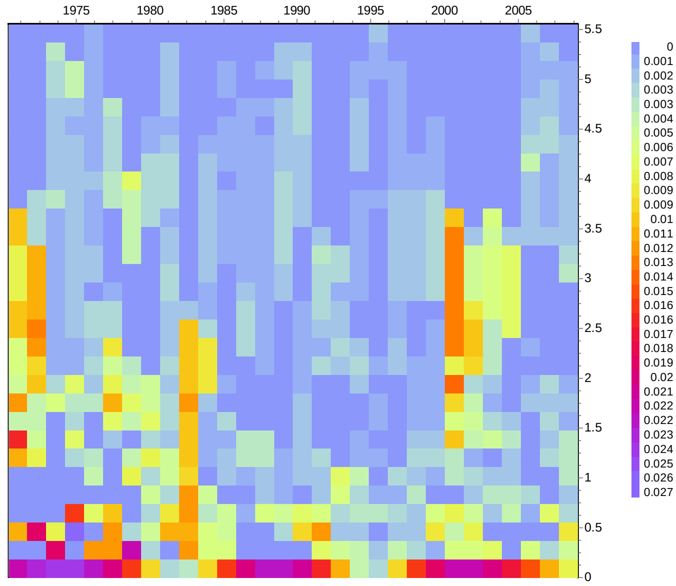

Figure 28: Hilbert spectrum for IMFs of UK M1

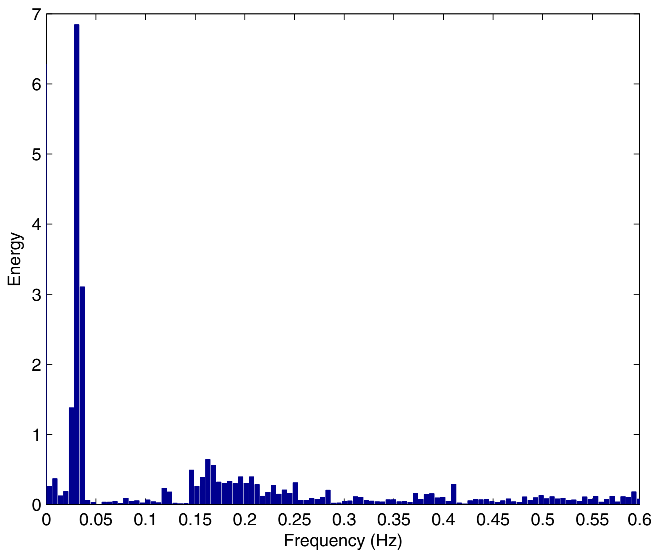

Figure 29: Marginal Hilbert power spectrum for IMFs of UK M1

available for use by economic and financial researchers using widely available software.

Several examples using both economic and financial variables were presented using the software currently available to researchers. The technique revealed several interesting results:

i) with Dow-Jones industrial average stockmarket data, there appears to be a 30–40 year cycle, as well as a cycle operating at or near the business cycle;

ii) with the US dollar-British pound exchange rate, there appears to be a 15–20 year cycle operating;

iii) with US industrial production data, there appears to have been a waning of a 20 year cycle, but a roughly 10 year cycle persists, even though higher frequency cycles have become much less volatile since the 1980s. No longer cycles than 10 year cycles are apparent in the data, which suggests that there is no very long cycle in growth;

iv) with UK inflation and M1 data, there appears to be a long cycle of around 25 years operating in the data – with the monetary aggregate leading the inflation rate by a period of roughly 2 to 3 years.

In terms of future research, there is clearly much that can be done. One forthcoming paper by this author concerns application of the method to US growth data spanning more than a hundred years in order to confirm the lack of a long cycle in growth. Other possibilities are clearly evident – the relationship of prices and money as well as the relationship between consumption and investment, for example.

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Appendices

A. Data sources

The Dow Jones Industrial Average was sourced from the Bank of Finland stockmarket database.

The US dollar – British pound exchange rate was sourced from the Bank of Finland exchange rate database.

US industrial production was sourced from the Bureau of Economic Analysis, Dept of Commerce.

UK RPI was sourced from the National Statistics Office, UK.

UK M1 was sourced from the Bank of England monetary database.

B. Software resources

Software for HHT exists from a variety of different sources:

i) The US National Aviation and Space Agency (NASA) Goddard Space Flight Center has three issued patents, one published patent application, and one copyright on this method. A MATLAB based package (HHT-DPS software) is available for use by licensed researchers directly from NASA and can be obtained through NASA's Technology Transfer Program at http://tco.gsfc.nasa.gov/HHT/index.html or through a subcontracter at http://www.fuentek.com/technologies/hht.htm

ii) Alan Tan has contributed code to MATLAB central which requires both the MATLAB signal processing and spline toolboxes, and this is located at

http://www.mathworks.nl/matlabcentral/fileexchange/19681

iii) Patrick Flandrin has MATLAB/C code on his website at

http://perso.ens-lyon.fr/patrick.flandrin/emd.html

iv) EEMD code is available from the Research Center for Adaptive Data Analysis which is fronted by Dr. Norden Huang in Taiwan. The website is at

http://rcada.ncu.edu.tw/research1.htm

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