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Michael Funke and Michael Paetz

What can an open-economy DSGE model tell us about Hong Kong's housing market?

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Tiivistelmä

Tässä tutkimuksessa kehitetään avotaloutta koskeva yleisen tasapainon (DSGE) malli, jossa ovat mukana asuntomarkkinasektori ja lainausrajoite. Tavanomaisista mallinnustavoista poiketen kotimaiset kotitaloudet voivat sijoittaa ulkomaisiin asuntoihin ja päinvastoin. Malli on estimoitu bayesilaisin metodein käyttäen Hongkongin taloutta koskevia tietoja. Tutkimustulosten mukaan on selvää, että Hongkongin asuntomarkkinat ovat hyvin avoimet ulkomaisille sijoituksille. Lisäksi muutokset luototusasteessa (loan-to-value ratio) ja asuntoihin liittyvissä preferensseissä selittävät merkittävän osan talouden suhdannevaihteluiden volatiliteetista.

Avainsanat: DSGE-mallit, asuntomarkkinat, avotalous, Hongkong

What Can an Open-Economy DSGE Model Tell Us about Hong Kong's Housing Market?

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Abstract

This paper develops an open-economy DSGE model with a housing-market sector and a borrowing constraint. Contrary to standard conventions, domestic households are allowed to invest in foreign housing and vice versa. Using Bayesian methods, the model is applied to data for Hong Kong. The results show that Hong Kong's housing market is quite open to foreign investment, and perhaps more significantly, that variations in the loan-to-value ratio and housing preference shocks largely explain business cycle volatility.

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Keywords: DSGE models, housing, open economy, Hong Kong.

JEL classification: D91, E21, E44, F41.

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1 Introduction

The global financial crisis that began in the US in December 2007 has attracted considerable attention in recent literature. It is now common knowledge that overborrowing of US households, especially to finance housing, had serious consequences for the financial sector and the macroeconomy generally. Indeed, Shiller (2007) sees the housing bubble as the major, if not sole, cause of the sub-prime mortgage crisis and the worldwide economic and financial crisis of 2007-2009.[1] Just as the preceding bubble created dynamics that tended to be self-perpetuating, the dynamics of the crisis were also self-perpetuating, albeit in the opposite direction. Yet, despite the relative importance of the housing in the economy, mainstream macroeconomics treats it either simply as one of many consumption goods or ignores it altogether. Similarly, conventional housing economics research virtually ignores interactions with the macroeconomy. At best, some theoretical and empirical analyses for urban and housing economics include macroeconomic variables as exogenous control variables.

The recent crisis obviously warrants assessment of the housing/business cycle nexus,[2] but the impacts of housing prices on business cycles has not been well understood. For this reason, it is usually omitted from conventional macroeconomic models. With the recent strong growth of housing prices in many countries and the ongoing turbulence in US mortgage markets, there finally appears to be a groundswell of motivation for empirical and normative work to establish and explicate the subtle links between financial markets and the real economy. In this spirit, this paper attempts to assess the impact of housing cycles and financial shocks on business cycles for Hong Kong within a richly specified DSGE model. We have selected a DSGE framework as a shock-accounting device for four reasons: (i) DSGE models provide a flexible framework that can incorporate many economic mechanisms of interest; (ii) unlike conventional ad hoc macro models, DSGE models do not suffer from a lack of detailed microfoundations; (iii) DSGE models allow examination of multiple shocks; and (iv) DSGE models have a well-specified theory for adjustment dynamics that allow for distinct predictions about the dynamic impacts of specific shocks. We thus analyze the importance of the housing market and household credit frictions in richly specified open-economy DSGE models that include housing market features for Hong Kong [Pariés and Notarpietro (2008), Iacoviello (2004), Iacoviello (2005), Iacoviello and Minetti (2008), Calza et al. (2009), and Iacoviello and Neri (2010)].

Hong Kong is modeled here as a small, open economy with a currency board

[1]In fairness, papers disputing the existence of a housing bubble were still being published in late 2006 [see Smith and Smith (2006)].

[2]An important lesson from the recent financial crisis is that credit bubbles like the recent house price bubble can be much more detrimental than bubbles which haven't been financed by debt, such as the dotcom bubble. The reason is that during the bursting of a credit-driven bubble amplification effects magnify the scale of the crisis.

system.[3] The model contains nominal rigidities and collateral constraints. On the empirical front, estimation takes place using Bayesian methods. The addition of the housing sector helps in developing a story of how shifts in housing prices impact GDP, i.e. we quantify the contribution of financial frictions and the housing market to business fluctuations. We devote special attention to the importance of the collateral channel and wealth effects.

The remainder of the paper is organized as follows. Section 2 briefly presents stylized facts on housing prices in Hong Kong. Section 3 describes our open-economy DSGE model, giving proper consideration to Hong Kong's characteristics with emphasis on descriptions of the housing sector. Estimation and inference of the model are laid out in section 4. Section 5 provides concluding remarks.

2 Hong Kong's Property Price History

Hong Kong makes an ideal case study as it has experienced large swings in housing prices in recent decades. Moreover, homeownership is widespread and the bulk of household wealth is tied up in housing.[4] Figure 1 displays the development of Hong Kong's nominal residential property price index over the period 1980Q1–2010Q2.

The Hong Kong housing market experienced two major boom-bust cycles over our sample period. The property bubble that emerged ahead of the Asian financial crisis saw residential property prices climb 65 percent between 1995Q4 and 1997Q3. After the crisis struck, housing prices plummeted 36 percent in just twelve months (October 1997–October 1998) before settling into a more leisurely rate of decline. Residential property prices overall fell 65 percent from their 1997 peak to their low point in 2003Q3. From that time to 2006Q2 residential property prices recovered 57 percent. Housing prices hit a plateau in the first half of 2006, then increased again until 2008Q2. With the recent global financial crisis, housing prices slid 14 percent. The housing market revived again in 2008. The territory-wide housing price index nearly octuples between 1980Q1 (beginning of the sample period) and 2010Q2 (the end of our sample period).[5]

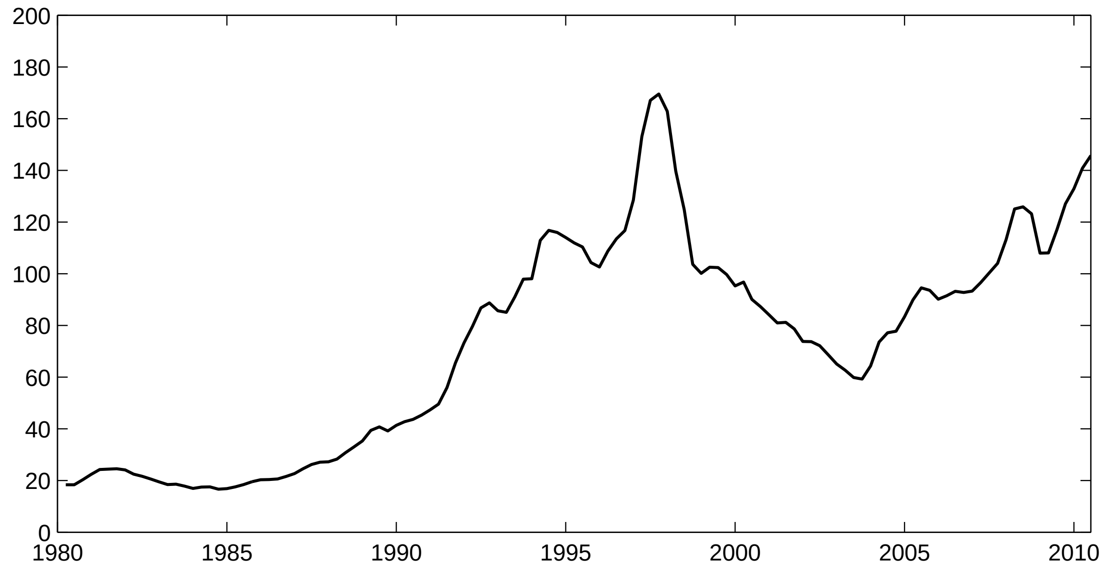
By all accounts, housing prices are much more volatile than GDP, although both variables are correlated.

[3]Hong Kong has one of the world's longest-standing currency board arrangements. Initially, the currency board was adopted as an emergency measure to prevent the HKD from collapsing during a political row between China and the United Kingdom in 1983 over the future of the colony. Locally, Hong Kong's currency board system is known as a "linked exchange rate system."

[4]For a thorough analysis of Hong Kong's housing market, see Ho and Wong (2009) and the literature cited therein.

[5]Hong Kong's housing market has seen a rising influx of mainland Chinese buyers since 2008. Some 30-40 percent of new home sales currently involve buyers from mainland China.

Figure 1: Hong Kong's Residential Property Price Index



Notes: Territory-Wide Residential Property Prices (quarterly data, 1999=100)
Source: Rating and Valuation Department, `http://www.rvd.gov.hk/en/doc/statistics/rvd1_2.pdf`

3 The Model

We begin by describing the relationships we wish to model. As our model is built on a symmetric two-agent, two-sector, open-economy framework, we find ourselves entering some new and uncharted territory. Here, firms produce residential and non-residential goods, while households freely choose how many hours they wish to work in each sector. We assume a two-stage production process. The output of a continuum of intermediate goods producers, acting as monopolistic competitors, is used as input by final goods producers, whose output is traded internationally.

Following the seminal work of Kiyotaki and Moore (1997), households are divided into two groups based on their subjective discount rate. "Impatient" households borrow from "patient" ones. Furthermore, the households derive utility from the consumption of non-residential and residential goods, where the latter are simply called durables and can be either consumed directly or used as a collateral in the mortgage market. This results in an equilibrium with positive private debt and intertemporal trade among households. Credit market frictions are introduced by a binding collateral constraint on borrowers. To capture the exchange-rate peg in Hong Kong, monetary policy is described using a fixed exchange-rate regime.

We initially borrow key ingredients from Iacoviello (2005) and Monacelli (2009).[6] Next, we merge this promising strand of research about housing cycles in DSGE models with the small open-economy framework of Galí and Monacelli (2005). When

[6] As commonly done in the literature, we abstract from modeling capital accumulation [see e.g. Monacelli (2009)].

a variable refers to a single foreign country i, it is denoted by the superscript i. "Rest-of-the-world" variables are denoted by an asterisk. We focus on the domestic economy and only state foreign country relationships if we believe them to be necessary for didactic reasons. For convenience, variables and parameters of the model are summarized in Table 1.

Table 1: Variables and Parameters

X_t	Index of consumption services
C_t	Composite consumption index
D_t	Composite durable consumption index (housing)
$N_{j,t}$	Hours worked in sector j
$N^s_{j,t}$ $(N^b_{j,t})$	Savers' (borrowers') labor supply in sector j $(j = C, D)$
$W_{j,t}$	Wage rate in sector j
$B^b_{H,t}$	Borrowers holdings of domestic real riskless bonds
$B^s_{H,t}$ $(B^s_{F,t})$	Savers holdings of domestic (foreign) real riskless bonds
R_t	Domestic nominal interest rate
$R_{i,t}$	Country i's nominal interest rate
R^*_t	"Rest-of-the-world" nominal interest rate
ψ_t	Marginal value of borrowing
$P_{j,t}$	Price level of sector j
$P_{D/C,t}$	Relative price level of sector D to sector C
$P_{j,i,t}(k)$	Price of sector $j's$ final good k from country i
$\Pi_{j,t}$	Sector specific CPI-inflation rate
$\Pi_{j,H,t}$ $(\Pi_{j,F,t})$	Sector specific domestic and foreign producer price inflation
$Y_{j,t}$	Production of final goods in sector j
Y_t	Aggregate output
NX_t	Net exports
$MC_{j,t}$	Real marginal cost in sector j
$A_{j,t}$	Productivity in sector j
$S_{j,t}$	Sector specific terms of trade $(\frac{P_{j,F,t}}{P_{j,H,t}})$
$\mathfrak{E}_{i,t}$	Nominal exchange rate between home and country i
$\mathfrak{E}_t$	Effective nominal exchange rate
$\mathcal{R}^i_{j,t}$	Sector specific real exchange rate between the home and country i
$\mathcal{R}_{j,t}$	Sector specific effective real exchange rate
ω	Share of impatient households
χ	Fraction of residential goods, which can not be used as collateral
β_l	Discount factor of household type l
σ	Intertemporal elasticity of substitution with respect to consumption
φ_{N_j}	Intertemporal elasticity of substitution with respect to labor in sector j
η_j	Elasticity of substitution between sector j's domestic and foreign goods
α_j	Relative share of sector j's foreign goods in consumption
γ	Relative share of residential goods in consumption
h_C	Habit formation in consumption
δ	Depreciation rate of residential stock
θ_j	Sector specific degree of price rigidity
τ_j	Sector specific degree of backward-looking price setters
ζ_j	Substitution elasticity between sector j's goods produced in foreign countries
ϵ_j	Substitution elasticity between differentiated goods within one country
μ^j_t	time-varying, sector-specific mark-up

In modeling households, we follow the recent strand of literature introduced by Kiyotaki and Moore (1997) and consider agents belonging to our two groups according to their intertemporal discount factor. Households are divided into ω borrowers and $(1-\omega)$ savers, denoted as b and s, respectively. Except for the discount factors, households are assumed to be completely symmetric. The two sectors of the economy, namely residential and non-residential goods, are denoted by the subscripts C and D, respectively.

Borrowers The representative impatient borrower is infinitely-lived and seeks to maximize

$$\max E_0 \sum_{t=0}^{\infty} \beta_b^t \left[\frac{1}{1-\sigma} \left(X_t^b \right)^{1-\sigma} - \frac{1}{1+\varphi} (N_{C,t}^b)^{1+\varphi} - \frac{1}{1+\varphi} (N_{D,t}^b)^{1+\varphi} \right], \tag{1}$$

where E_0 is the conditional expectation operator evaluated at time 0, X_t^b represents a consumption index, $N_{j,t}^b$ represents the labor supply in sector j with φ and σ being the corresponding intertemporal elasticities of substitution $(j = C, H)$ with respect to labor and consumption, respectively, and β_b represents the borrowers discount factor. The composite consumption index is expressed as

$$X_t^b \equiv \left(\tilde{C}_t^b \right)^{\left(1-\gamma \mathcal{E}_t^D\right)} \left(D_t^b \right)^{\gamma \mathcal{E}_t^D}, \tag{2}$$

where $\tilde{C}_t^b \equiv C_t^b - h_c C_{t-1}^b$, C_t^b and D_t^b represent composite indices of non-durable and durable consumption services, respectively, h_C represents habit formation in consumption, γ is the share of housing in consumption, and $\mathcal{E}_t^D \equiv \exp\left(\epsilon_t^D\right)$ is a housing preference shock that affects the marginal rate of substitution between non-residential and residential goods.[7]

Following Pariés and Notarpietro (2008), borrowers can trade nominal riskless bonds, but are unable to tap the international markets to finance their expenditures.[8] Consequently, they face a sequence of budget constraints, given by

$$C_t^b + P_{D/C,t} I_{D,t}^b - B_{H,t}^b = -R_{t-1} \frac{B_{H,t-1}^b}{\Pi_{C,t}} + \sum_{j=C,D} \frac{W_{j,t}^b N_{j,t}^b}{P_{C,t}}, \tag{3}$$

where $\Pi_{C,t+1} \equiv \frac{P_{C,t+1}}{P_{C,t}}$ is the CPI based inflation rate, $B_{H,t}^b$ represents the stock of real domestic debt (denominated with the domestic non-residential price index), R_{t-1} the nominal interest rate (the lending rate on a loan contract issued in $t-1$), $W_{j,t}^b$ the sector-specific wage rate, $I_{D,t}^b \equiv D_t^b - (1-\delta) D_{t-1}^b$ defines housing investments, and δ represents the depreciation rate of the residential stock.[9]

[7] In using a Cobb-Douglas composite consumption index, we implicitly assume a unitary intratemporal elasticity of substitution between housing and non-durable consumption as in e.g. Monacelli (2009) or Pariés and Notarpietro (2008).

[8] This assumption is purely for convenience and does not imply that domestic borrowers do not hold foreign debt as they trade with domestic savers free.

[9] The budget constraint follows from the conventional intratemporal optimization results,

Borrowers do not save and are restricted by the following borrowing constraint

$$R_t B^b_{H,t} \leq (1-\chi)(1-\delta) E_t \left[P_{D/C,t+1} D^b_t \Pi_{C,t+1} \right] \epsilon^{LTV}_t , \tag{4}$$

where χ represents the fraction of residential goods, which can not be used as collateral. Thus, $(1-\chi)$ is a proxy for the loan-to-value ratio (LTV), and ϵ^{LTV}_t reflects variations in the LTV.[10] Equation (4) relates the amount that will be repaid by a borrower in the following period to the expected future value of durable stocks (adjusted for depreciation and the loan-to-value ratio).

The borrowing household maximizes (1) subject to (3) and (4). The FOCs for this optimization problem can be expressed as

$$\begin{aligned}
\frac{W^b_{j,t}}{P_{C,t}} &= \frac{\left(X^b_t\right)^{\sigma} \left(N^b_{j,t}\right)^{\varphi} \left(\tilde{C}^b_t\right)^{\gamma \mathcal{E}^D_t}}{\left(1-\gamma \mathcal{E}^D_t\right) \left(D^b_t\right)^{\gamma \mathcal{E}^D_t}}, j = C, D, && (5)\\
P_{D/C,t} &= \beta_b (1-\delta) E_t \left[\left(\frac{1-\gamma \mathcal{E}^D_{t+1}}{1-\gamma \mathcal{E}^D_t} \right) \left(\frac{X^b_{t+1}}{X^b_t} \right)^{-\sigma} \left(\frac{D^b_{t+1}}{\tilde{C}^b_{t+1}} \right)^{\gamma \mathcal{E}^D_{t+1}} \left(\frac{\tilde{C}^b_t}{D^b_t} \right)^{\gamma \mathcal{E}^D_t} P_{D/C,t+1} \right] \\
&\quad + \left(\frac{\gamma \mathcal{E}^D_t}{1-\gamma \mathcal{E}^D_t} \right) \frac{\tilde{C}^b_t}{D^b_t} + (1-\chi)(1-\delta) \psi_t P_{D/C,t} E_t \left[\Pi_{D,t+1} \right] \varepsilon^{LTV}_t && (6)\\
R_t \psi_t &= 1 - \beta_b E_t \left[\left(\frac{1-\gamma \mathcal{E}^D_{t+1}}{1-\gamma \mathcal{E}^D_t} \right) \left(\frac{X^b_{t+1}}{X^b_t} \right)^{-\sigma} \left(\frac{D^b_{t+1}}{\tilde{C}^b_{t+1}} \right)^{\gamma \mathcal{E}^D_{t+1}} \left(\frac{\tilde{C}^b_t}{D^b_t} \right)^{\gamma \mathcal{E}^D_t} \frac{R_t}{\Pi_{C,t+1}} \right], && (7)
\end{aligned}$$

where $\lambda_t \psi_t$ represent the Lagrangian multiplier on the borrowing constraint, and ψ_t can be interpreted as the marginal value of borrowing.[11] For $\psi_t = 0$, (7) reduces to the standard New Keynesian Euler equation. Thus, a rise in ψ_t represents a tightening of the collateral constraint.[12] The first condition represents the standard labor-leisure trade-off, equating the marginal disutility of an additional unit of labor to the marginal utility received from additional consumption, equation (6) equates the marginal utility of non-durable consumption to the shadow value of durable services, and the last equation is a consumption Euler equation adjusted to capture the borrowing constraint.[13]

$P_{C,H,t} C^b_{H,t} + P_{C,F,t} C^b_{F,t} = P_{C,t} C^b_t$ and $P_{D,H,t} D^b_{H,t} + P_{D,F,t} D^b_{F,t} = P_{D,t} D^b_t$, where $P_{C,H,t}$ and $P_{C,F,t}$ are the aggregate price indices for domestic and foreign goods, and $C^b_{H,t}$, $C^b_{F,t}$, $D^b_{H,t}$ and $D^b_{F,t}$ represents borrower consumption of domestic and foreign non-residential and residential goods, respectively. See Galí and Monacelli (2005) for details.

[10] It can easily be shown that (4) will always be binding in the steady state. See Notarpietro (2007) or Iacoviello (2005) for details.

[11] Note that for a unitary substitution elasticity with respect to consumption ($\sigma = 1$), the FOCs coincide with the equilibrium conditions derived in Monacelli (2009).

[12] Things are not so simple in the real world, of course. Our modeling framework doesn't consider risk of default. Leaving out default risk from the model means we don't have to assume that creditors lend only to housing buyers that can make a substantial downpayment or meet their loan payments.

[13] As pointed out by Monacelli (2009), the shadow value of durables depends on (i) the direct

Savers Patient savers are able to make intertemporal decisions in the standard way. The representative household is infinitely-lived and seeks to maximize

$$\max E_0 \sum_{t=0}^{\infty} \beta_s^t \left[\frac{1}{1-\sigma} \left(X_t^s\right)^{1-\sigma} - \frac{1}{1+\varphi_N} \left(N_{C,t}^s\right)^{1+\varphi} - \frac{1}{1+\varphi} \left(N_{D,t}^s\right)^{1+\varphi} \right],$$

subject to

$$C_t^s + P_{D/C,t} I_{D,t}^s - B_{H,t}^s - \mathfrak{E}_t B_{F,t}^s = -R_{t-1} \frac{B_{H,t-1}^s}{\Pi_{C,t}} - \frac{R_{t-1}^* \mathfrak{E}_t B_{F,t-1}^s}{\Pi_{C,t}} + \sum_{j=C,D} \frac{W_{j,t}^s N_{j,t}^s}{P_{C,t}},$$

where $\mathfrak{E}_t$ represents the nominal exchange rate, $B_{F,t}^s$ foreign bond holdings, R_t^* the foreign interest rate, and all other variables are defined in the same way as for the borrowers. Optimization yields

$$\frac{W_{j,t}^s}{P_{C,t}} = \frac{\left(X_t^s\right)^{\sigma} \left(N_{j,t}^s\right)^{\varphi} \left(\tilde{C}_t^s\right)^{\gamma \mathcal{E}_t^D}}{\left(1-\gamma \mathcal{E}_t^D\right) \left(D_t^s\right)^{\gamma \mathcal{E}_t^D}}, j = C, D, \tag{8}$$

$$P_{D/C,t} = \beta_s (1-\delta) E_t \left[\left(\frac{X_{t+1}^s}{X_t^s}\right)^{-\sigma} \left(\frac{1-\gamma \mathcal{E}_{t+1}^D}{1-\gamma \mathcal{E}_t^D}\right) \left(\frac{D_{t+1}^s}{\tilde{C}_{t+1}^s}\right)^{\gamma \mathcal{E}_{t+1}^D} \left(\frac{\tilde{C}_t^s}{D_t^s}\right)^{\gamma \mathcal{E}_t^D} P_{D/C,t+1} \right] + \left(\frac{\gamma \mathcal{E}_t^D}{1-\gamma \mathcal{E}_t^D}\right) \frac{\tilde{C}_t^s}{D_t^s}, \tag{9}$$

$$1 = \beta_s E_t \left[\left(\frac{1-\gamma \mathcal{E}_{t+1}^D}{1-\gamma \mathcal{E}_t^D}\right) \left(\frac{X_{t+1}^s}{X_t^s}\right)^{-\sigma} \left(\frac{D_{t+1}^s}{\tilde{C}_{t+1}^s}\right)^{\gamma \mathcal{E}_{t+1}^D} \left(\frac{\tilde{C}_t^s}{D_t^s}\right)^{\gamma \mathcal{E}_t^D} \frac{R_t}{\Pi_{C,t+1}} \right], \tag{10}$$

$$1 = \beta_s E_t \left[\left(\frac{1-\gamma \mathcal{E}_{t+1}^D}{1-\gamma \mathcal{E}_t^D}\right) \left(\frac{X_{t+1}^s}{X_t^s}\right)^{-\sigma} \left(\frac{D_{t+1}^s}{\tilde{C}_{t+1}^s}\right)^{\gamma \mathcal{E}_{t+1}^D} \left(\frac{\tilde{C}_t^s}{D_t^s}\right)^{\gamma \mathcal{E}_t^D} \frac{\mathfrak{E}_{t+1}}{\mathfrak{E}_t} \frac{R_t^*}{\Pi_{C,t+1}} \right]. \tag{11}$$

Since patient households do not face a borrowing constraint, the first three equations mirror exactly those of the impatient households for $\psi_t = 0$. Equation (9) equates the purchase price of a durable good to the payoff (the marginal rate of substitution between durable and nondurable consumption), plus the expected resale value, while (10) is now a conventional Euler equation, adjusted for housing in the consumption index.[14] Moreover, the first-order conditions for internationally traded bonds imply the uncovered interest parity

$$\frac{R_t}{R_t^*} = E_t \left[\frac{\mathfrak{E}_{t+1}}{\mathfrak{E}_t} \right]. \tag{12}$$

utility gain of an additional durable unit, (ii) the expected utility from the possibility of expanding future consumption, and (iii) the marginal utility of relaxing the collateral constraint.

[14] Gan (2010) has used a large panel dataset that tracks the housing wealth and credit-card spending of 12,793 individuals in Hong Kong to study the relationship between housing wealth and household consumption. He identified a significant effect of housing wealth on consumption.

User cost interpretation The optimality condition (6) is widely interpreted as equating the marginal rate of substitution between durable and non-durable consumption $\frac{U_{D,t}}{U_{C,t}}$ to the "user cost" of durables z_t:[15]

$$\widehat{z}_t = \Phi^{-1}(1-\delta)\left[\begin{array}{l}\tilde{\Phi}\widehat{p}_{D/C,t} - \beta E_t \widehat{p}_{D/C,t+1} + \beta_s\left(\widehat{r}_t - E_t \pi_{C,t+1}\right) \\ +\left(\beta_s - \beta_b\right)\left(\chi\widehat{\psi}_t - E_t\left[\left(1 - \chi\widehat{\pi}_{D,t+1} - \widehat{\pi}_{C,t+1}\right)\right]\right)\end{array}\right] \tag{13}$$

where $\Phi \equiv 1-(1-\delta)\left[\beta_b + (1-\chi)\left(\beta_s - \beta_b\right)\right]$, $\tilde{\Phi} \equiv \frac{1-(1-\chi)(1-\delta)(\beta_s-\beta_b)}{1-\delta}$, $U\left(X_t^b, N_{C,t}^b, N_{D,t}^b\right) \equiv \frac{\mathcal{E}_t^X}{1-\sigma}\left(X_t^s\right)^{1-\sigma} - \frac{1}{1+\varphi_N}\left(N_{C,t}^s\right)^{1+\varphi} - \frac{1}{1+\varphi}\left(N_{D,t}^s\right)^{1+\varphi}$, and where we used that equilibrium values of the shadow value of capital and the interest rate are pinned down by (7) and (10): $\psi = \beta_s - \beta_b$ and $R = \beta_s^{-1}$. In a standard New Keynesian economy, $\beta_s = \beta_b = \beta$, and the user costs would be reduced to

$$\widehat{z}_t = \frac{1}{(1-\beta(1-\delta))}\left[\widehat{p}_{D/C,t} + \beta(1-\delta)(\widehat{r}_t - E_t\widehat{\pi}_{C_{t+1}} - E_t\widehat{p}_{D/C,t+1})\right]. \tag{14}$$

User cost depends positively on current prices and negatively on expected future prices. Thus, the demand for durables increases when asset appreciation is expected. The latter effect vanishes for $\delta \to 1$, since durability disappears. For $\beta_s > \beta_b$ user cost is also affected by the shadow value of borrowing ψ_t. A tightening of the collateral constraint (an increase in ψ_t) is accompanied, in turn, by an increase in user cost for $\Phi^{-1}(1-\delta)\left(\beta_s - \beta_b\right) > 0$. Hence, a tightening of the collateral constraint implies an increase in user cost when the saver's discount rate (adjusted for the depreciation of durables) is greater than the share of durables that can be used as collateral. This is easier to satisfy for a lower δ (due to higher durability), a smaller loan-to-value ratio $(1-\chi)$ (due to the lower ability to borrow new debt), and a higher saver's discount rate (due to higher willingness of savers to lend).

Some helpful definitions and identities Before proceeding, we offer some helpful definitions and identities used extensively in the following sections. Consumption indices are given by

$$C_t^l \equiv \left[(1-\alpha_C)^{\frac{1}{\eta_C}} C_{H,t}^l(j)^{\frac{\eta_C-1}{\eta_C}} + \alpha_C^{\frac{1}{\eta_C}} C_{F,t}^l(j)^{\frac{\eta_C-1}{\eta_C}}\right]^{\frac{\eta_C}{\eta_C-1}}, \tag{15}$$

$$D_t^l \equiv \left[(1-\alpha_D)^{\frac{1}{\eta_D}} D_{H,t}^l(j)^{\frac{\eta_D-1}{\eta_D}} + \alpha_D^{\frac{1}{\eta_D}} D_{F,t}^l(j)^{\frac{\eta_D-1}{\eta_D}}\right]^{\frac{\eta_D}{\eta_D-1}}, \tag{16}$$

where

$$C_{H,t}^l \equiv \left[\int_0^1 C_{H,t}^l(k)^{\frac{\epsilon_C-1}{\epsilon_C}} dk\right]^{\frac{\epsilon_C}{\epsilon_C-1}}, C_{F,t}^l \equiv \left[\int_0^1 (C_{i,t}^l)^{\frac{\zeta-1}{\zeta}} di\right]^{\frac{\zeta_C}{\zeta_C-1}}, C_{i,t}^l \equiv \left[\int_0^1 C_{i,t}^l(k)^{\frac{\epsilon_C-1}{\epsilon_C}} dk\right]^{\frac{\epsilon_C}{\epsilon_C-1}},$$

$$D_{H,t}^l \equiv \left[\int_0^1 D_{H,t}^l(k)^{\frac{\epsilon_D-1}{\epsilon_D}} dk\right]^{\frac{\epsilon_D}{\epsilon_D-1}}, D_{F,t}^l \equiv \left[\int_0^1 (D_{i,t}^l)^{\frac{\zeta_D-1}{\zeta}} di\right]^{\frac{\zeta_D}{\zeta_D-1}}, D_{i,t}^l \equiv \left[\int_0^1 D_{i,t}^l(k)^{\frac{\epsilon_C-1}{\epsilon_D}} dk\right]^{\frac{\epsilon_D}{\epsilon_D-1}},$$

[15] Following standard notation, lower-case letters denote logs and hats denote percentage deviations from equilibrium ($\widehat{x}_t = \log\left(\frac{X_t}{X}\right)$) unless we explicitly mention a different convention.

and η_j represents the sector-specific intratemporal substitution elasticity between domestic and foreign goods $(j = C, D, l = b, s)$, ζ_j the sector-specific intratemporal substitution elasticity between non-residential goods produced in the "rest-of-the-world," ϵ_j the intratemporal substitution elasticity between differentiated residential goods within one country, and α_j the sector-specific degree of openness.[16] According to (16) we allow domestic borrowers not only to purchase non-residential consumption goods internationally, but also housing. In this respect, we differ from the two-country framework of Pariés and Notarpietro (2008), since we believe spillover effects from foreign countries are important to explain the high volatility of housing prices in Hong Kong.

Consequently, price indices are given by

$$P_{C,t} \equiv \left[(1-\alpha_C)\, P_{C,H,t}^{(1-\eta_C)} + \alpha_C P_{C,F,t}^{(1-\eta_C)}\right]^{\frac{1}{1-\eta_C}}, \tag{17}$$

$$P_{D,t} \equiv \left[(1-\alpha_D)\, P_{D,H,t}^{(1-\eta_C)} + \alpha_D P_{D,F,t}^{(1-\eta_D)}\right]^{\frac{1}{1-\eta_D}}, \tag{18}$$

implying the following demand equations:

$$\begin{aligned} C_{H,t} &= (1-\alpha_C)\left(\frac{P_{C,H,t}}{P_{C,t}}\right)^{-\eta_C} C_t, C_{F,t} = \alpha_C \left(\frac{P_{C,F,t}}{P_{C,t}}\right)^{-\eta_C} C_t, C_{i,t} = \left(\frac{P_{C,i,t}}{P_{C,F,t}}\right)^{-\zeta_C} C_{F,t}, \\ & D_{H,t} - (1-\delta)\, D_{H,t-1} = (1-\alpha_D)\left(\frac{P_{D,H,t}}{P_{D,t}}\right)^{-\eta_D} (D_t - (1-\delta)\, D_{t-1}), \\ & D_{F,t} - (1-\delta)\, D_{F,t-1} = \alpha_D \left(\frac{P_{D,F,t}}{P_{D,t}}\right)^{-\eta_D} (D_t - (1-\delta)\, D_{t-1}), \\ & D_{i,t} - (1-\delta)\, D_{i,t-1} = \left(\frac{P_{D,i,t}}{P_{D,F,t}}\right)^{-\zeta_D} (D_{F,t} - (1-\delta)\, D_{F,t-1}). \end{aligned}$$

The sector-specific bilateral terms of trade between the domestic country and country i represent the price of country i's goods in terms of domestic goods and is given by $S_{j,i,t} = P_{j,i,t}/P_{j,H,t}$. Thus the effective terms of trade is given by

$$S_{C,t} = \frac{P_{C,F,t}}{P_{C,H,t}} = \left(\int_0^1 S_{C,i,t}^{1-\zeta_C} di\right)^{\frac{1}{1-\zeta_C}}, S_{D,t} = \frac{P_{D,F,t}}{P_{D,H,t}} = \left(\int_0^1 S_{D,i,t}^{1-\zeta_D} di\right)^{\frac{1}{1-\zeta_D}}, \tag{19}$$

which can be approximated by $s_{j,t} \equiv \log(S_{j,t}) \approx \int_0^1 s_{j,i,t} di$. Log-linearizing the domestic price indices under the assumption of a symmetric steady state satisfying the PPP implies

$$\widehat{\pi}_{C,t} = \widehat{\pi}_{C,H,t} + \alpha_C \triangle \widehat{s}_{C,t}, \widehat{\pi}_{D,t} = \widehat{\pi}_{D,H,t} + \alpha_D \triangle \widehat{s}_{D,t}. \tag{20}$$

The gap between producer and consumer price inflation in both sectors is propor-

[16]Since $C_t \equiv \omega C_t^b + (1-\omega) C_t^s$ and $D_t \equiv \omega D_t^b + (1-\omega) D_t^s$, we drop the superscripts b and s as all arguments hold for borrowers, savers, and aggregates.

tional to the change in the terms of trade, depending on the openness of the country in both sectors.[17]

Assuming that the LOOP holds on a brand level, we obtain $P_{j,i,t}(k) = \mathfrak{E}_{i,t} P^i_{j,i,t}(k)$ $(\forall i, k \in [0,1])$ $(j = C, D)$, where $P^i_{j,i,t}(k)$ represents the price of non residential good k from country i measured in terms of country i's currency, and $\mathfrak{E}_{i,t}$. Integration over all products k yields $P_{j,i,t} = \mathfrak{E}_{i,t} P^i_{j,i,t}$. Since the foreign sector-specific PPI measured in foreign currency units is given by $P^*_{j,F,t} = \left[\int_0^1 \left(P^i_{j,i,t}\right)^{1-\zeta_j} di\right]^{\frac{1}{1-\zeta_j}}$, and we assume identical preferences without a home bias, we derive

$$P_{j,F,t} = \mathfrak{E}_t P^*_{j,F,t}, P_{j,H,t} = \mathfrak{E}_t P^*_{j,H,t}, P_{j,t} = \mathfrak{E}_t P^*_{j,t}, j = C, D, \tag{21}$$

where $P^*_{j,H,t}$ is defined in the same way as $P^*_{j,F,t}$ and represents the domestic PPI of residential and non-residential goods measured in foreign currency units.

A log-linearization of $P_{j,F,t}$ around a symmetric steady state gives

$$\widehat{p}_{j,F,t} = \int_0^1 \left(\widehat{e}_{i,t} + \widehat{p}^i_{j,i,t}\right) di = \widehat{e}_t + \widehat{p}^*_{j,t}, \tag{22}$$

where $\widehat{p}^*_{j,t}$ represents the sector-specific log world price index.[18] Using this with the definition of the terms of trade gives $\widehat{s}_{j,t} = \widehat{e}_t + \widehat{p}^*_{j,t} - \widehat{p}_{j,H,t}$, $(j = C, D)$, and defines a relationship between the terms of trade in both sectors through the exchange-rate channel, that is

$$\widehat{s}_{C,t} - \widehat{p}^*_{C,t} + \widehat{p}_{C,H,t} = \widehat{s}_{D,t} - \widehat{p}^*_{D,t} + \widehat{p}_{D,H,t}. \tag{23}$$

To derive an relationship between the terms of trade and measures of the real exchange rate, we first define the sector-specific bilateral real exchange rate as $\mathcal{R}^i_{j,t} \equiv \frac{\mathfrak{E}_{i,t} P^i_{j,t}}{P_{j,t}}$ for all $i \in [0,1]$. Defining $\widehat{rer}^i_{j,t} \equiv \log \mathcal{R}^i_{j,t}$ and $\widehat{rer}_{j,t} \equiv \int_0^1 rer^i_{j,t} di$ as the log effective real exchange rate, it follows that

$$\widehat{rer}_{j,t} = \widehat{e}_t + \widehat{p}^*_{j,t} - \widehat{p}_{j,t} = \widehat{s}_{j,t} + \widehat{p}_{j,H,t} - \widehat{p}_{j,t} = (1 - \alpha_j)\, \widehat{s}_{j,t}. \tag{24}$$

International risk-sharing Although borrowers are constrained, we assume savers are able share country-specific risks internationally via the trading of bonds on complete security markets. This implies a proportionate relationship between savers consumption relative to the rest of the world and the real exchange rate. As bonds are internationally tradable, a condition similar to (10) holds for any representative saver in each country i

$$\mathcal{Q}_{t,t+1} = \beta_s \left(\frac{X^{s,i}_{t+1}}{X^{s,i}_t}\right)^{-\sigma} \left(\frac{D^{s,i}_{t+1}}{\tilde{C}^{s,i}_{t+1}}\right)^{\gamma} \left(\frac{\tilde{C}^{s,i}_t}{D^{s,i}_t}\right)^{\gamma} \frac{P^i_{C,t}}{P^i_{C,t+1}} \frac{\mathfrak{E}^i_t}{\mathfrak{E}^i_{t+1}}, \tag{25}$$

[17]For $\alpha_C = \alpha_D = 0$, we derive the closed economy version and consumer and producer prices coincide.

[18]Note that world CPI and PPI are the same as we assume that each country is of measure zero.

where $\mathcal{Q}_{t,t+1}$ is defined by $R_t \equiv \frac{1}{E_t[\mathcal{Q}_{t,t+1}]}$. Equating (25) with (10) yields the following relationship:

$$\left(\frac{X_{t+1}^s}{X_t^s}\right)^{-\sigma}\left(\frac{D_{t+1}^s}{\tilde{C}_{t+1}^s}\right)^{\gamma}\left(\frac{\tilde{C}_t^s}{D_t^s}\right)^{\gamma} = \left(\frac{X_{t+1}^{s,i}}{X_t^{s,i}}\right)^{-\sigma}\left(\frac{D_{t+1}^{s,i}}{\tilde{C}_{t+1}^{s,i}}\right)^{\gamma}\left(\frac{\tilde{C}_t^{s,i}}{D_t^{s,i}}\right)^{\gamma}\frac{\mathcal{R}_t^i}{\mathcal{R}_{t+1}^i},$$

Defining $z_t \equiv \left(\frac{X_t^{s,i}}{X_t^s}\right)^{-\sigma}\left(\frac{\tilde{C}_t^{s,i}}{\tilde{C}_t^s}\right)^{\gamma}\left(\frac{D_t^{s,i}}{D_t^s}\right)^{-\gamma}\mathcal{R}_t^i$, it follows that

$$z_{t+1} = z_t = z_0 = \left(\frac{X_0^{s,i}}{X_0^s}\right)^{-\sigma}\left(\frac{\tilde{C}_0^{s,i}}{\tilde{C}_0^s}\right)^{\gamma}\left(\frac{D_0^{s,i}}{D_0^s}\right)^{\gamma}\mathcal{R}_0^i. \tag{26}$$

Henceforth, and without loss of generality, we assume symmetric initial conditions, implying $z_0 = 1$. Integrating over all countries and using the assumption of symmetry, we obtain

$$\left(X_t^s\right)^{-\sigma}\left(\tilde{C}_t^s\right)^{\gamma}\left(D_t^s\right)^{\gamma} = \left(X_t^{s,*}\right)^{-\sigma}\left(\tilde{C}^{s,*}\right)^{\gamma}\left(D_t^{s,*}\right)^{\gamma}\mathcal{R}_t. \tag{27}$$

Log-linearization using (24) yields a simple relationship that links domestic savers' consumption of durables and non-residential goods to world savers' consumption and the terms of trade. As is familiar from many International Real Business Cycle (IRBC) models, this gives

$$\widehat{\tilde{c}}_t^s - (1-\sigma)\,\gamma\Gamma^{-1}\widehat{d}_t^s = \widehat{\tilde{c}}_t^{s,*} - (1-\sigma)\,\gamma\Gamma^{-1}\widehat{d}_t^{s,*} + (1-\alpha_C)\,\Gamma^{-1}\widehat{s}_{C,t}, \tag{28}$$

with $\Gamma \equiv [\sigma + (1-\sigma)\,\gamma]$, and $\widehat{\tilde{c}}_t^s \equiv \frac{1}{1-h_c}\left(\widehat{c}_t^s - h_c\widehat{c}_{t-1}^s\right)$, $\widehat{\tilde{c}}_t^{s,*} \equiv \frac{1}{1-h_c}\left(\widehat{c}_t^{s,*} - h_c\widehat{c}_{t-1}^{s,*}\right)$.

3.1 Firms

The focal point of this subsection will be the micro-structure of firms. We assume a two-stage production process in each sector, where intermediate goods (wholesale sector) are used to produce final goods (retailers) according to a CES technology.[19]

Retailers Final goods in sector j are produced by aggregating intermediate goods with the following production function:

$$Y_{j,t} = \left(\int_0^1 Y_{j,t}^{\frac{1}{1+\mu_t^j}}(k)\,dk\right)^{1+\mu_t^j}, \tag{29}$$

where $Y_{j,t}$ denotes aggregate output, $Y_{j,t}(k)$ is the input produced by intermediate goods firm k (both expressed in per capita terms) and μ_t^j captures the time-varying, sector-specific mark-up of prices over marginal cost in the wholesale sector. Profit

[19]To retain analytical tractability of the model and retain focus of the discussion, we assume intermediates are non-tradable.

maximizing behavior implies the following demand equations

$$Y_{j,t}(k) = \left(\frac{P_{H,j,t}(k)}{P_{H,j,t}}\right)^{-\frac{1+\mu_t^j}{\mu_t^j}} Y_{j,t}, \tag{30}$$

where $P_{H,j,t}(k)$ is the price of a domestic individual intermediate good k, and the aggregate domestic producer price level in each sector is given by $P_{H,j,t} \equiv \left(\int_0^1 P_{H,j,t}(k)^{\frac{1}{\mu_t^j}} dk\right)^{-\mu_t^j}$.

The wholesale sector At the bottom of the production process, there is a continuum of intermediate goods producers. Production of each intermediate good producer j is assumed to follow a stochastic constant returns to scale production function

$$Y_{j,t}(k) = A_{j,t} N_{j,t}(k), \tag{31}$$

where $A_{j,t}$ denotes sector-specific labour productivity, and $N_{j,t}$ is the labor input.[20]

Real marginal cost in each sector are given by $(W_{j,t}/P_{H,j,t})/MPN_{j,t}$, where $MPN_{j,t}$ represents the marginal product of labor in each sector. By aggregating the optimal labor-leisure decision of borrowers and savers, we derive

$$\frac{W_{j,t}}{P_{C,t}} = \frac{(X_t)^{\sigma}(N_{j,t})^{\varphi}\left(\tilde{C}_t\right)^{\gamma\mathcal{E}_t^D}}{(1-\gamma\mathcal{E}_t^D)(D_t)^{\gamma\mathcal{E}_t^D}\mathcal{E}_t^X}, j = C, D, \tag{32}$$

where $W_{j,t} = \omega W_{j,t}^b + (1-\omega) W_{j,t}^s$ and $N_{j,t} = \omega N_{j,t}^b + (1-\omega) N_{j,t}^s$. Using this, real marginal cost in each sector are represented by

$$MC_{C,t} = \frac{(X_t)^{\sigma}(N_{C,t})^{\varphi}\left(\tilde{C}_t\right)^{\gamma\mathcal{E}_t^D} S_{C,t}^{\alpha_C}}{(1-\gamma\mathcal{E}_t^D)(D_t)^{\gamma\mathcal{E}_t^D} A_{C,t}\mathcal{E}_t^X}, \tag{33}$$

$$MC_{D,t} = \frac{(X_t)^{\sigma}(N_{D,t})^{\varphi}\left(\tilde{C}_t\right)^{\gamma\mathcal{E}_t^D} S_{D,t}^{\alpha_D}}{(1-\gamma\mathcal{E}_t^D)(D_t)^{\gamma\mathcal{E}_t^D} A_{D,t}P_{D/C,t}\mathcal{E}_t^X}. \tag{34}$$

Price-setting Price adjustment of the monopolistically competitive firms is assumed to follow a variant of the memoryless characteristic of Calvo pricing in accordance with Galí and Gertler (1999). A randomly selected fraction of firms in each sector $(1-\theta_j)$ $(j = C, D)$ adjusts prices, while the remaining fraction of firms θ_j does not adjust. In addition, a fraction of $(1-\tau_j)$ firms behaves in a forward-looking way, while the remaining fraction τ_j uses the recent history of the aggregate

[20] Jones (2005) shows that the Cobb-Douglas production function forms a valid approximation in the aggregate for a variety of underlying micro firm production functions.

price index when they set prices. Thus, τ_j is a measure of the degree of backward-looking price-setting.

Defining the sector-specific domestic index for the prices newly set in period t ($\overline{P}^n_{j,H,t}$) as a weighted average of the forward- and backward-looking prices ($P^{bl}_{j,H,t}$) yields

$$P^{1-\epsilon_j}_{j,H,t} = \theta_j P^{1-\epsilon_j}_{j,H,t-1} + (1-\theta_j)\left(\overline{P}^n_{j,H,t}\right)^{1-\epsilon_j}, j = C, D \tag{35}$$

$$\overline{P}^n_{j,H,t} = (1-\tau_j)\, P^{fl}_{j,H,t} + \tau_j P^{bl}_{j,H,t}, j = C, D. \tag{36}$$

We assume that the $P^{bl}_{j,t}$ evolves according to the following equation:

$$P^{bl}_{j,t} = \bar{P}^n_{j,t-1} + \pi_{j,H,t-1}, j = C, D. \tag{37}$$

A backward-looking firm that adjusts at time t simply corrects the average price of last period's price adjustment for inflation. For this correction, it uses the last period's inflation to forecast future inflation. These assumptions ensure that the evolution of prices (i) converges to optimal behavior (as long as inflation is stationary), and (ii) implicitly incorporates future information since $\overline{P}^n_{j,t-1}$ is partly determined by forward-looking price-setters.

As is customary, the above assumptions yield the conventional mark-up rule, whereby firms set the price as a mark-up over current and future real marginal costs and deviations of the time-varying mark-up from its steady state $\left(\widehat{\mu}^j_t\right)$ such that

$$\bar{p}^n_{j,H,t} = \widehat{\mu}^j_t + (1-\beta\theta) \sum_{k=0}^{\infty} (\beta_s \theta_j)^k E_t \left(mc_{t+k} + p_{j,H,t}\right). \tag{38}$$

3.2 Equilibrium

Market clearing for each good k in each sector j of the domestic economy is given by

$$\begin{aligned} & Y_{C,t}(k) = C_{H,t}(k) + \left[\int_0^1 C^i_{H,t}(k)\, di\right] \mathcal{E}^{C,*}_t & (39) \\ = & \left(\frac{P_{C,H,t}}{P_{C,t}}\right)^{-\eta_C} C_t \left[(1-\alpha_C) + \alpha_C \left[\int_0^1 \left(S^i_{C,t} S_{C,i,t}\right)^{\zeta_C - \eta_C} \mathcal{R}^{\eta_C - 1}_{C,i,t}\, di\right] \mathcal{E}^{C,*}_t\right], \\ & Y_{D,t}(k) = D_{H,t}(k) - (1-\delta)\, D_{H,t-1}(k) & (40) \\ & + \left[\int_0^1 D^i_{H,t}(k)\, di - (1-\delta) \int_0^1 D^i_{H,t-1}(k)\, di\right] \mathcal{E}^{D,*}_t \\ = & \left(\frac{P_{D,H,t}}{P_{D,t}}\right)^{-\eta_D} (D_t - (1-\delta)\, D_{t-1}) \left[(1-\alpha_D) + \alpha_D \left[\int_0^1 \left(S^i_{D,t} S_{D,i,t}\right)^{\zeta_D - \eta_D} \mathcal{R}^{\eta_D - 1}_{D,i,t}\, di\right] \mathcal{E}^{D,*}_t\right] \end{aligned}$$

for all $k \in [0,1]$ and all t, where $C^i_{D,t}(k)$ represents country i's demand for the domestic non-residential good k, $S^i_{j,t}$ represents the effective terms of trade of country i in sector j, and $\mathcal{E}^{C,*}_t \equiv \exp(\epsilon^{C,*}_t)$ and $\mathcal{E}^{D,*}_t \equiv \exp(\epsilon^{D,*}_t)$ represent sector-specific foreign demand shocks.[21] Using $\int_0^1 \widehat{s}_{j,t}^i di = 0$ and (24), equations (39) and (41) can be simplified to yield

$$\begin{aligned} \widehat{y}_{C,t} &= \widehat{c}_t + \alpha_C \vartheta_C \widehat{s}_{C,t} + \alpha_C \epsilon_t^{C,*} \\ \widehat{y}_{D,t} &= \frac{1}{\delta}\widehat{d}_t - \frac{(1-\delta)}{\delta}\widehat{d}_{t-1} + \alpha_D \vartheta_D \widehat{s}_{D,t} + \alpha_D \epsilon_t^{D,*}, \end{aligned}$$

where $\vartheta_j \equiv [\zeta_j + (1-\alpha_j)(\eta_j - 1)], j = C, D$.

3.3 Monetary policy

Finally, we adopt a standard formulation for the structure of monetary policy-making under a currency board system.[22] To be specific, we assume a credible exchange rate peg, implying $\widehat{e}_t = 0$. Consequently, monetary policy is conducted to ensure $\Delta\widehat{s}_{C,t} = -\widehat{\pi}_{C,H,t}$, which, in combination with $\Delta\widehat{s}_{D,t} = \Delta\widehat{s}_{C,t} - \widehat{\pi}_{D,H,t} + \widehat{\pi}_{C,H,t}$, eliminates pressure on the exchange rate.

3.4 The log-linearized model

We first transform the model to reach a stationary representation where a steady state exists. After some tedious algebra, the complete log-linearized model is given by the following equations:

$$\begin{aligned} \widehat{p}_{D/C,t} &= \frac{\gamma}{1-\gamma}\frac{\tilde{C}^b}{D^b P_{D/C}}\left(\widehat{\tilde{c}}_t^b - \widehat{d}_t^b\right) + \beta_b(1-\delta)\left[(1-\sigma)\gamma E_t\left(\Delta\widehat{d}_{t+1}^b\right) - \Gamma E_t\left(\Delta\widehat{\tilde{c}}_{t+1}^b\right)\right] \qquad (41) \\ &\quad + \beta_b(1-\delta)\left[E_t\left(\widehat{p}_{D/C,t+1}\right)\right] + \psi(1-\chi)\left(\widehat{\psi}_t + \widehat{p}_{D/C,t} + \widehat{\pi}_{D,t+1} + \epsilon_t^{LTV}\right) \\ &\quad + \left[\frac{\gamma}{(1-\gamma)^2}\frac{\tilde{C}^b}{D^b}P_{D/C}^{-1} - (1-\rho_D)\beta_b(1-\delta)\left((1-\sigma)\gamma\left(\ln D^b - \ln\tilde{C}^b\right) - \frac{\gamma}{1-\gamma}\right)\right]\epsilon_t^D, \\ \widehat{p}_{D/C,t} &= \frac{\gamma}{1-\gamma}\frac{\tilde{C}^s}{D^s P_{D/C}}\left(\widehat{\tilde{c}}_t^s - \widehat{d}_t^s\right) + (1-\delta)\beta_s\left[(1-\sigma)\gamma E_t\left(\Delta\widehat{d}_{t+1}^s\right) - \Gamma E_t\left(\Delta\widehat{\tilde{c}}_{t+1}^s\right)\right] \qquad (42) \end{aligned}$$

[21] We introduce these shocks as we believe foreign demand, especially demand from mainland China, in the housing sector is an important determinant for the output dynamics of Hong Kong.

[22] The stabilizing effect of a currency board arrangement is entirely different from a target zone system. Any holder of paper money can exchange notes for foreign currency at a fixed rate. Since the exchange rate of paper money is fixed, so, too, must be the exchange rate for bank deposits. Any rate differential leads to profitable cash arbitrage that closes the gap. If the prices of the same product in two sub-markets differ, one can buy the product for less in the cheaper sub-market and sell it at a higher price in the other, gaining profit at zero risk. As many market participants would engage in similar arbitrage, the two prices should equalize provided transaction costs are negligible. The second market arbitrage mechanism is interest arbitrage.

If, for example, there is speculation against the currency, funds will flow out of the economy and domestic interest rates will rise. This should reverse the outflow and stabilize the exchange rate. Both market arbitrage mechanisms can be classified as self-reversing market movements and represent the self-adjusting "autopilot" of a currency board arrangement. An in-depth discussion of Hong Kong's currency board, including documentations on the technical details is available at `http://www.info.gov.hk/hkma/eng/currency/link_ex/index.htm`.

$$
\begin{aligned}
& +(1-\delta)\,\beta_s E_t \widehat{p}_{D/C,t+1} \\
& +\left[\gamma(1-\gamma)^2 \frac{\tilde{C}^s}{D^s} P_{D/C}^{-1} - (1-\rho_D)\,\beta_b\,(1-\delta)\left((1-\sigma)\,\gamma\left(\ln D^s - \ln \tilde{C}^s\right) - \frac{\gamma}{1-\gamma}\right)\right]\epsilon_t^D, \\
\widehat{\psi}_t \;=\; & \frac{\beta_b}{\beta_s - \beta_b}\left[\Gamma E_t\left(\Delta \widehat{\tilde{c}}_{t+1}^b\right) - (1-\sigma)\,\gamma E_t\left(\Delta \widehat{d}_{t+1}^b\right) - \left(\widehat{r}_t - \widehat{\pi}_{C,H,t+1} - \alpha_C \Delta \widehat{s}_{C,t+1}\right)\right] - \widehat{r}_t, \\
& +(1-\sigma)\gamma\left[\ln D^b - \ln \tilde{C}^b\right](1-\rho_D)\,\epsilon_t^D \qquad (43)\\
\Gamma \widehat{\tilde{c}}_t^s \;=\; & \Gamma E_t \widehat{\tilde{c}}_{t+1}^s - (1-\sigma)\,\gamma E_t\left(\Delta \widehat{d}_{t+1}^s\right) - \left(\widehat{r}_t - \widehat{\pi}_{C,H,t+1}\right) + \alpha_C \Delta \widehat{s}_{C,t+1}, \qquad (44)\\
\widehat{\tilde{c}}_t^b \;=\; & \frac{B_H^b}{\tilde{C}^b}\left[\widehat{b}_{H,t}^b - \beta_s^{-1}\left(\widehat{r}_{t-1} + \widehat{b}_{H,t-1}^b - \widehat{\pi}_{C,H,t} - \alpha_c \Delta \widehat{s}_{C,t}\right)\right] \\
& -P_{D/C}\frac{D^b}{\tilde{C}^b}\left(\delta \widehat{p}_{D/C,t} + \widehat{d}_t^b - (1-\delta)\,\widehat{d}_{t-1}^b\right) \\
& +\frac{1}{1+\mu_C}\frac{N_C}{\tilde{C}^b}\left(\widehat{\tilde{w}}_{C,t} + \widehat{n}_{C,t}^b\right) + \frac{P_{D/C}}{1+\mu_D}\frac{N_D}{\tilde{C}^b}\left(\widehat{\tilde{w}}_{D,t} + \widehat{n}_{D,t}^b\right), \qquad (45)\\
\widehat{b}_{H,t}^b \;=\; & \widehat{p}_{D/C,t+1} + \widehat{d}_t^b - \left(\widehat{r}_t - E_t\left(\widehat{\pi}_{C,H,t+1} + \alpha_c \Delta s_{C,t+1}\right)\right) + \epsilon_t^{LTV}, \qquad (46)\\
\widehat{y}_{C,t} \;=\; & \widehat{c}_t + \alpha_C \vartheta_C \widehat{s}_{C,t} + \alpha_C \epsilon_t^{C,*} \qquad (47)\\
\widehat{y}_{D,t} \;=\; & \frac{1}{\delta}\left(\widehat{d}_t - (1-\delta)\,\widehat{d}_{t-1}\right) + \alpha_D \vartheta_D \widehat{s}_{D,t} + \alpha_D \epsilon_t^{D,*}, \qquad (48)\\
\widehat{c}_t \;=\; & \omega \frac{C^b}{C}\widehat{c}_t^b + (1-\omega)\frac{C^s}{C}\widehat{c}_t^s, \widehat{d}_t = \omega \frac{D^b}{D}\widehat{d}_t^b + (1-\omega)\frac{D^s}{D}\widehat{d}_t^s, \qquad (49)\\
\widehat{\pi}_{j,H,t} \;=\; & \beta_s \theta_j \phi_j E_t \widehat{\pi}_{j,H,t+1} + \tau_j \phi_j \widehat{\pi}_{j,H,t-1} + \kappa_j \widehat{mc}_{j,t} + \epsilon_t^{\mu_j}, j = C, D, \qquad (50)\\
\widehat{mc}_{C,t} \;=\; & \Gamma \widehat{\tilde{c}}_t - (1-\sigma)\,\gamma \widehat{d}_t + \varphi_N \widehat{n}_{C,t} + \alpha_C \widehat{s}_{C,t} - a_{C,t} \\
& -\left[(1-\sigma)\,\gamma\left(\ln D - \ln \tilde{C}\right) - \frac{\gamma}{1-\gamma}\right]\epsilon_t^D \qquad (51)\\
\widehat{mc}_{D,t} \;=\; & \Gamma \widehat{\tilde{c}}_t - (1-\sigma)\,\gamma \widehat{d}_t + \varphi_N \widehat{n}_{D,t} + \alpha_D \widehat{s}_{D,t} - \widehat{p}_{D/C,t} - a_{D,t} \\
& -\left[(1-\sigma)\,\gamma\left(\ln D - \ln \tilde{C}\right) - \frac{\gamma}{1-\gamma}\right]\epsilon_t^D \qquad (52)\\
\widehat{s}_{C,t} \;=\; & \frac{1}{1-\alpha_C}\left[\Gamma \widehat{\tilde{c}}_t^s - (1-\sigma)\,\gamma \widehat{d}_t^s - \epsilon_t^{RSC}\right], \qquad (53)\\
\widehat{y}_{j,t} \;=\; & \widehat{a}_{j,t} + \widehat{n}_{j,t}, j = C, D, \qquad (54)\\
\widehat{\tilde{w}}_{j,t} \;=\; & \Gamma \widehat{\tilde{c}}_t - (1-\sigma)\,\gamma \widehat{d}_t + \varphi \widehat{n}_{j,t} - \left[(1-\sigma)\,\gamma\left(\ln D - \ln \tilde{C}\right) - \frac{\gamma}{1-\gamma}\right]\epsilon_t^D, \qquad (55)\\
\widehat{\tilde{w}}_{j,t} \;=\; & \Gamma \widehat{\tilde{c}}_t^i - (1-\sigma)\,\gamma \widehat{d}_t^i + \varphi \widehat{n}_{j,t}^i \\
& -\left[(1-\sigma)\,\gamma\left(\ln D^i - \ln \tilde{C}^i\right) - \frac{\gamma}{1-\gamma}\right]\epsilon_t^D, j = C, D, i = b, s, \qquad (56)\\
\widehat{p}_{D/C,t} \;=\; & \widehat{p}_{D/C,t-1} + \widehat{\pi}_{D,H,t} - \widehat{\pi}_{C,H,t} + \alpha_D \Delta \widehat{s}_{D,t} - \alpha_C \Delta \widehat{s}_{C,t}, \qquad (57)\\
\Delta \widehat{s}_{D,t} \;=\; & \Delta \widehat{s}_{C,t} - \widehat{\pi}_{D,H,t} + \widehat{\pi}_{C,H,t}, \Delta \widehat{s}_{C,t} = -\widehat{\pi}_{C,H,t}, \qquad (58)
\end{aligned}
$$

where $\epsilon_t^{RSC} \equiv \Gamma \widehat{\tilde{c}}_t^{s,*} - (1-\sigma)\,\gamma \widehat{\tilde{d}}_t^{s,*}$, $u_t^j \equiv \kappa_j \widehat{\mu}_t^j$, $\tilde{w}_{j,t} \equiv w_{j,t} - p_{C,t}$, and the dynamics of the model are driven by ten orthogonal structural shocks:

$$\epsilon_t^{LTV} = \rho_{LTV}\epsilon_{t-1}^{LTV} + \varepsilon_t^{LTV}, \quad (59)$$
$$\epsilon_t^{\mu_j} = \rho_{\mu_j}^{+}\epsilon_{t-1}^{\mu_j} + \varepsilon_t^{\mu_j} - \rho_{\mu^j}^{-}\varepsilon_{t-1}^{\mu_j}, \quad (60)$$
$$a_{j,t} = \rho_{a_j}a_{j,t-1} + \varepsilon_t^{a_j}, \quad (61)$$
$$\epsilon_t^{RSC} = \rho_{RSC}\epsilon_{t-1}^{RSC} + \varepsilon_t^{RSC}, \quad (62)$$
$$\epsilon_t^{j,*} = \rho_{j,*}\epsilon_{t-1}^{j,*} + \varepsilon_t^{j,*}, \quad (63)$$
$$\epsilon_t^{D} = \rho_{D}\epsilon_{t-1}^{D} + \varepsilon_t^{D}, \quad (64)$$

where $\phi_j \equiv \frac{1}{\theta_j+\tau_j 1-\theta_j(1-\beta_s)}$, $\kappa_j \equiv \frac{(1-\tau_j)(1-\theta_j)(1-\beta_s\theta_j)}{\theta_j+\tau_j 1-\theta_j(1-\beta_s)}$, and all $\varepsilon_t^i \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma_i^2)$.

A positive loan-to-value shock, as defined in (59) leads to a loosening of the borrowing constraint, (60) are sector-specific cost-push shocks that can be justified by exogenous variations of price mark-ups or fluctuations in labor tax income, (61) defines sector-specific technology shocks, and (63) are sector-specific foreign demand shocks. The domestic housing preference shock (64) is defined as an exogenous perturbation to the marginal rate of substitution between residential and non-residential consumption in the utility function. Equation (62) is an aggregated foreign consumption shock on the risk-sharing condition. The last shock influences the real exchange rate (and hence the terms of trade) through the risk-sharing channel, which equates the marginal rates of substitution to the relative prices across countries. The specification of the cost-push shocks follows the philosophy of Smets and Wouters (2007), who argue that ARMA(1,1) processes are useful in capturing high-frequency fluctuations in price mark-ups. Hence, we richly specify the shock structure to allow the DSGE model to explain all possible patterns in the data.

Aggregated price indices can be derived by defining $P_t \equiv P_{C,t}^{1-\gamma}P_{D,t}^{\gamma}$, $P_{H,t} \equiv P_{C,H,t}^{1-\gamma}P_{D,H,t}^{\gamma}$, and aggregated real output (denominated with the aggregated producer price index) is given by $P_{H,t}Y_t = P_{C,H,t}Y_{C,t} + P_{D,H,t}Y_{D,t}$. Log-linearization yields

$$\widehat{\pi}_t = (1-\gamma)\,\widehat{\pi}_{C,t} + \gamma\widehat{\pi}_{D,t}, \widehat{\pi}_{H,t} = (1-\gamma)\,\widehat{\pi}_{H,C,t} + \gamma\widehat{\pi}_{H,D,t}, \quad (65)$$
$$\widehat{y}_t = \frac{P_{D/C}^{-\gamma}C}{Y}\left(\widehat{y}_{C,t} - \gamma\widehat{p}_{D/C,H,t}\right) + \frac{\delta P_{D/C}^{1-\gamma}D}{Y}\left(\widehat{y}_{D,t} + (1-\gamma)\,\widehat{p}_{D/C,H,t}\right), \quad (66)$$

where $Y = P_{D/C}^{-\gamma}C + \delta P_{D/C}^{1-\gamma}D$ and $\widehat{p}_{D/C,H,t} = \widehat{p}_{D/C,t} - \alpha_D\widehat{s}_{D,t} + \alpha_C\widehat{s}_{C,t}$. Moreover, market clearing in the bonds market requires $\omega B_{H,t}^b + (1-\omega)\,B_{H,t}^s = (1-\omega)\,B_{F,t}^s$ and determines the bond holdings of domestic savers.[23]

Next, we test the model's properties on Hong Kong data. For the estimation of the model, we use equations (41)–(66). Our choice of parameter values used in the calibration and estimation stage is explained in the next section.

[23] As we do not explicitly model the world economy, the bond holdings of savers are given by $(1-\omega)\,B_{H,t}^s = \omega B_{H,t}^b$, where $B_{H,t}^b$ is determined by the binding collateral constraint.

4 Estimation and model fit

4.1 Calibration and data

The model is estimated by Bayesian methods. First, fixing parameters and setting priors allows us to introduce pre-sample information and reduce the dimensionality problem associated with the number of parameters. Second, Bayesian methods have important computational advantages over maximum likelihood methods in larger models; simulating the posterior is much easier than maximizing a highly dimensional likelihood.

We start by fixing those parameters that are either notoriously difficult to estimate or are better identified using other information. To get a good fit of the model, we keep the set of such fixed parameters as small as possible. Consequently, we fix the depreciation rate of the residential stock at $\delta = 0.01$, corresponding to an annual rate of 4 percent. The discount factors are fixed at $\beta_s = 0.99$ and $\beta_b = 0.96$, which have become standard values in the literature about borrowing constraints.[24] The implied interest rate, which is pinned down by the savers' intertemporal discount factor, is 4 percent in annual terms. Moreover, the historically established loan-to-value ratio for Hong Kong is about 70 percent.

Concerning the data, we employ quarterly data for eight macroeconomic variables for the sample period 1985Q1−2010Q2: real GDP per capita (Y_t), producer price inflation ($\Pi_{H,t}$), consumer price inflation ($\Pi_{C,t}$), domestic property price inflation ($\Pi_{D,H,t}$), real consumption per capita (C_t), employment (N_t), the 3-month savings deposit rate (R_t), and US output (as a proxy for foreign output Y_t^*).[25] All real variables are seasonally adjusted, using the WIN-X12 interface of the US Census Bureau (`http://www.census.gov/srd/www/winx12/index.html`), and detrended, using an HP filter with smoothing parameter 1600. In accordance with the model, the interest rate is measured in absolute deviations from trend, while all other series are measured in percentage deviations.[26] To account for influences on the actual interest rate that lie beyond the scope of the model, we add a time-varying risk premium disturbance $\epsilon_t^{rp} = \rho_{rp}\epsilon_{t-1}^{rp} + \varepsilon_t^{rp}, \varepsilon_t^{rp} \overset{i.i.d.}{\sim} \mathcal{N}\left(0, \sigma_{rp}^2\right)$ to the deposit rate before estimation: $r_t^{obs} = r_t + \epsilon_t^{rp}$.

[24] See e.g. Pariés and Notarpietro (2008).

[25] US output is used as a proxy for foreign output since the HKD is pegged to the USD. US output is also highly correlated with the output of the major industrialized economies. We believe this data gives a quite good approximation for the rest-of-the-world production of the model.

[26] All series except interest rate are freely available and posted on the website of the Census and Statistics Department of Hong Kong (`http://www.censtatd.gov.hk/hong_kong_statistics/statistical_tables/index.jsp`). The interest rate data are freely available and posted on the HKMA website (`http://www.info.gov.hk/hkma/eng/statistics/index_efdhk.htm`).

4.2 Prior and posterior distributions

As explained in the introduction, we estimate our model using Bayesian methods. Formally, we stack all the model parameters in the vector $\Psi \in \Omega$ and elicit a prior, $p(\Psi)$. The model implies a likelihood $p\left(Y^T|\Psi\right)$ given some observed data, $Y^T = \left\{y^1, \ldots, y^T\right\}$. This yields the posterior distribution $p\left(\Psi|Y^T\right) \propto p\left(Y^T|\Psi\right) p(\Psi)$, where "$\propto$" indicates proportionality. The posterior summarizes uncertainty regarding the parameter values. Under a quadratic loss function, our point estimates are thus the mean of the posterior. Since the posterior is difficult to characterize, we generate draws from it using the Metropolis-Hastings algorithm. We finish by using the resulting empirical distribution to obtain point estimates and standard deviations.

Choosing adequate prior distributions is crucial for the estimation procedure. To let the data decide the parameters, we use loose prior distributions and assume equal prior means across sectors. A detailed description of all prior distributions is given in Table 2. For most priors, we rely on Funke et al. (2011), which includes stock market wealth effects in estimating a DSGE model for Hong Kong. The mark-ups are set to $\mu_j = 0.1$, which is consistent with a substitution elasticity of $\epsilon_j = 11$, as set in Devereux et al. (2006). The degree of openness is set to 0.5. The probabilities of the Calvo lotteries are Beta distributions to keep them bounded between zero and one and are set to 0.67. In other words, we do not force prices in the durable sector to be more flexible than those in the nondurable sector. The persistence priors are set to 0.3. Regarding the consumption habits, we use a prior of $h_C = 0.2$. Contrary to Funke et al. (2011), we do not assume a unitary intertemporal substitution elasticity; we do not fix the inverse of the Frisch elasticity or the elasticities of substitution between domestic goods, and between goods produced in different foreign countries. Instead, we use prior means of $\sigma = \varphi = 1$ and $\zeta_j = \eta_j = 2$ and some fairly high standard deviations. Concerning the share of borrowers ω, we use a prior of 35 percent, which is consistent with estimates of Pariés and Notarpietro (2008) for the US. For the share of durables in aggregate consumption γ, we rely on two household surveys of the Census and Statistics Department of Hong Kong. In 2000, "housing expenditures" accounted for 22.1 percent of total household expenditures; in 2005, the share was about 30.6 percent. Since we estimate the model over a longer time horizon, we choose a value of 25 percent. Finally, prior means of standard deviations and AR(1)-parameters of all shocks are set to 0.1 and 0.7, respectively. Again, our intention is to use a loose prior and let the data speak.

Draws from the unknown distribution of parameters are obtained using the random walk version of the Metropolis-Hastings algorithm in DYNARE and based on two blocks of $250,000$ draws, neglecting the first $10,000$. The results of the estimation procedure are given in Table 2. The high degree of openness in the residential goods sector suggests that Hong Kong's housing market strongly depends on demand from abroad. Foreign demand shocks seem to have a little influence, while the bulk of macroeconomic variability is explained by variations in the loan-to-value ratio

Table 2: Prior and Posterior Distributions

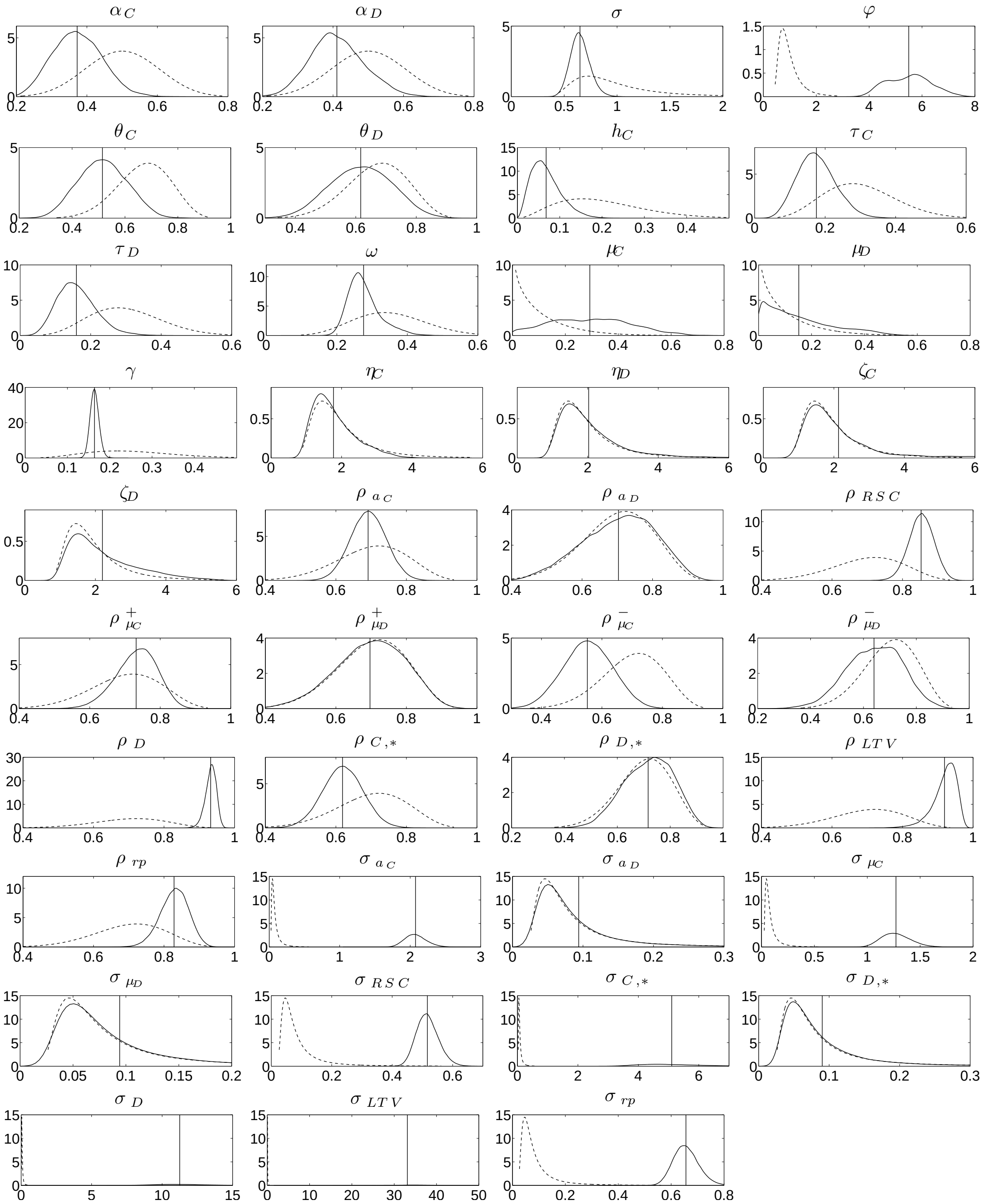
	Prior Distribution			Posterior Distribution		
	Type	Mean	St. Dev.	Mean	Std. Dev.	Conf. Int.
α_C	*beta*	0.5	0.1	0.3724	0.0693	[0.2572,0.4834]
α_D	*beta*	0.5	0.1	0.411	0.0762	[0.2850,0.5374]
σ	*gamma*	1	0.5	0.6495	0.0911	[0.5016,0.7994]
φ	*gamma*	1	0.5	5.4969	0.8209	[4.1254,6.7805]
θ_C	*beta*	0.67	0.1	0.5146	0.0922	[0.3634,0.6666]
θ_D	*beta*	0.67	0.1	0.6162	0.1030	[0.4469,0.7831]
τ_C	*beta*	0.3	0.1	0.1752	0.0539	[0.0867,0.2631]
τ_D	*beta*	0.3	0.1	0.1596	0.0548	[0.0709,0.2455]
h_C	*beta*	0.2	0.1	0.0678	0.0346	[0.0142,0.1217]
μ_C	*beta*	0.1	0.1	0.2927	0.1497	[0.0388,0.5263]
μ_D	*beta*	0.1	0.1	0.1519	0.1245	[0.0000,0.3474]
ω	*beta*	0.35	0.1	0.2760	0.0447	[0.2054,0.3491]
γ	*beta*	0.25	0.1	0.1639	0.0103	[0.1469,0.1806]
η_C	*beta*	2	1	1.7708	0.5903	[0.9011,2.7069]
η_D	*beta*	2	1	2.0268	0.8903	[0.8748,3.1931]
ζ_C	*beta*	2	1	2.1315	1.1810	[0.8269,3.4176]
ζ_D	*beta*	2	1	2.1978	0.9454	[0.9159,3.6823]
ρ_{a_C}	*beta*	0.7	0.1	0.6914	0.0500	[0.6085,0.7746]
ρ_{a_D}	*beta*	0.7	0.1	0.7032	0.130	[0.5326,0.8695]
$\rho^{+}_{\mu_C}$	*beta*	0.7	0.1	0.7316	0.0583	[0.6383,0.8294]
$\rho^{-}_{\mu_C}$	*beta*	0.7	0.1	0.5517	0.0819	[0.4189,0.6882]
$\rho^{+}_{\mu_D}$	*beta*	0.7	0.1	0.6967	0.0995	[0.5407,0.8647]
$\rho^{-}_{\mu_D}$	*beta*	0.7	0.1	0.6400	0.1049	[0.4677,0.8078]
ρ_{RSC}	*beta*	0.7	0.1	0.8527	0.0346	[0.7950,0.9097]
ρ_{LTV}	*beta*	0.7	0.1	0.9191	0.0346	[0.8688,0.9697]
$\rho_{C,*}$	*beta*	0.7	0.1	0.6188	0.0566	[0.5234,0.7108]
$\rho_{D,*}$	*beta*	0.7	0.1	0.7166	0.0933	[0.5673,0.8699]
ρ_D	*beta*	0.7	0.1	0.9323	0.0155	[0.9072,0.957]
ρ_{rp}	*beta*	0.7	0.1	0.8285	0.0400	[0.7639,0.8941]
σ_{a_C}	*gamma*	0.1	2.0	2.0757	0.1510	[1.8273,2.3207]
σ_{a_D}	*gamma*	0.1	2.0	0.0940	0.0943	[0.0226,0.1691]
σ_{RSC}	*gamma*	0.1	2.0	0.5165	0.0361	[0.4563,0.5754]
σ_{μ_C}	*gamma*	0.1	2.0	1.2717	0.1349	[1.0553,1.4923]
σ_{μ_D}	*gamma*	0.1	2.0	0.0941	0.0883	[0.0226,0.1714]
σ_{LTV}	*gamma*	0.1	2.0	33.1204	6.6287	[21.7808,44.3256]
$\sigma_{C,*}$	*gamma*	0.1	2.0	5.1056	1.0766	[3.4536,6.7421]
$\sigma_{D,*}$	*gamma*	0.1	2.0	0.0902	0.0748	[0.0231,0.1656]
σ_D	*gamma*	0.1	2.0	11.2463	1.8397	[8.2790,14.3263]
σ_{rp}	*gamma*	0.1	2.0	0.6559	0.0480	[0.5781,0.7327]

and housing preference shocks. In addition, Figure 2 illustrates the distributions of all estimated parameters and shows that in general they are very informative. We now turn to describing how the model works.

4.3 Properties of the estimated model

In this section, we consider properties and applications of our model to illustrate the contributions that such a model might make to policy analysis. First, we show the

Figure 2: Prior vs. Posterior Distributions, Structural Parameters

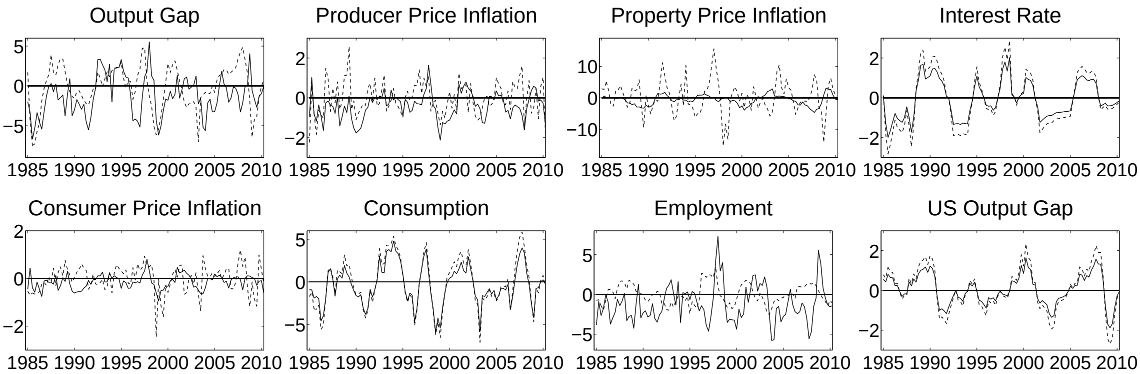


solid line: posterior; **dashed line:** prior; **vertical line:** posterior mean

one-step-ahead predictions of all series used in the estimation in Figure 3. At cyclical frequencies, the model mirrors the main cyclical properties of all series reasonably well.

Another way of looking at what shocks are behind the fluctuations in the data is to simulate the DSGE model with the shocks obtained using the Kalman smoother. Given a set of parameter estimates and the law of motion of the model, we use the

Figure 3: In-Sample One Step Ahead Predictions of the Estimated Model



solid line: forecast; **dashed line:** actual series

observed data to obtain a series of shocks that, given the DSGE model, explain the data.

Figures 4 and 5 show historical shock decompositions that permit analysis of the nature of shocks hitting the key variables of the model over the sample period. The estimates of the shocks are smoothed, i.e. they rely on information contained in the full sample of data. The figures confirm the spillovers from the housing market to the wider economy. As can be seen in the output gap panels, light blue, purple, and orange dominate, i.e. the loan-to-value, housing preference, and foreign output shocks explain the bulk of the variation of the output gap, consumption and employment during the Asian financial crisis of 1997-1998, as well as the global financial crisis of 2007-2009.[27] Notably, technology shocks fail to contribute noticeably to downturns. Thus, in terms of business cycles, was the recession of 2007-2009 all that different from what came before? The results derived here from estimating an open-economy DSGE model suggest an ambitious answer: partly yes and partly no. Compared to previous business cycles, the string of adverse foreign output shocks continued through 2008 into 2009, adding substantially to both the length and severity of the recession. On the other hand, the pattern of disturbances was comparable to the patterns generating previous downturns. The bulk of the marginal value of borrowing movements is also driven by loan-to-value shocks and property prices and housing investment are associated with housing preference shocks. By contrast, the remaining shocks explain little over the sample period. A slightly different picture emerges for consumer and producer price inflation. For those variables, the shock decompositions show that technology shocks had stronger impacts on inflation.

[27]This is consistent with Leamer (2007), who puts the housing sector center stage in most US recessions.

Figure 4: Historical Shock Decomposition I

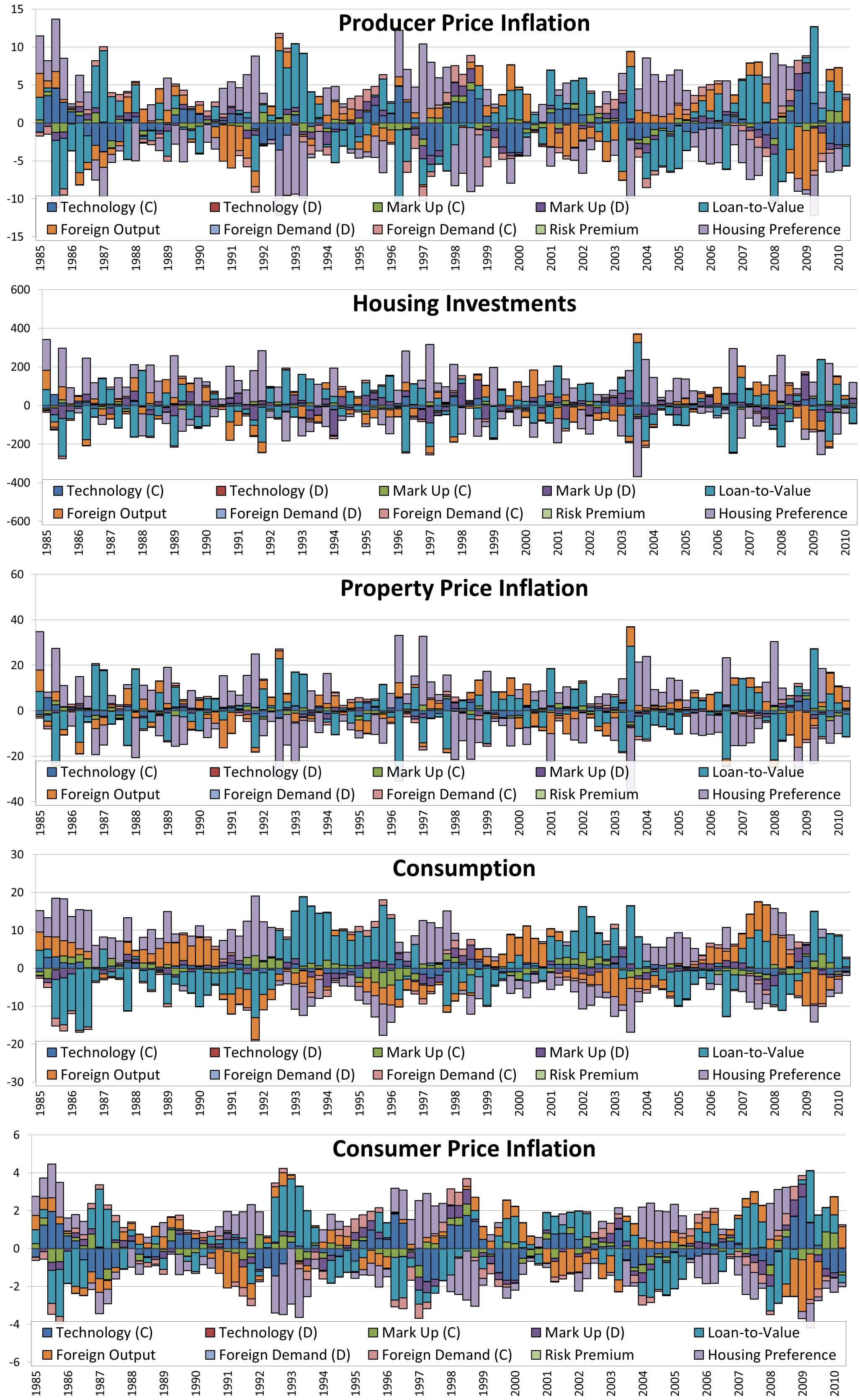
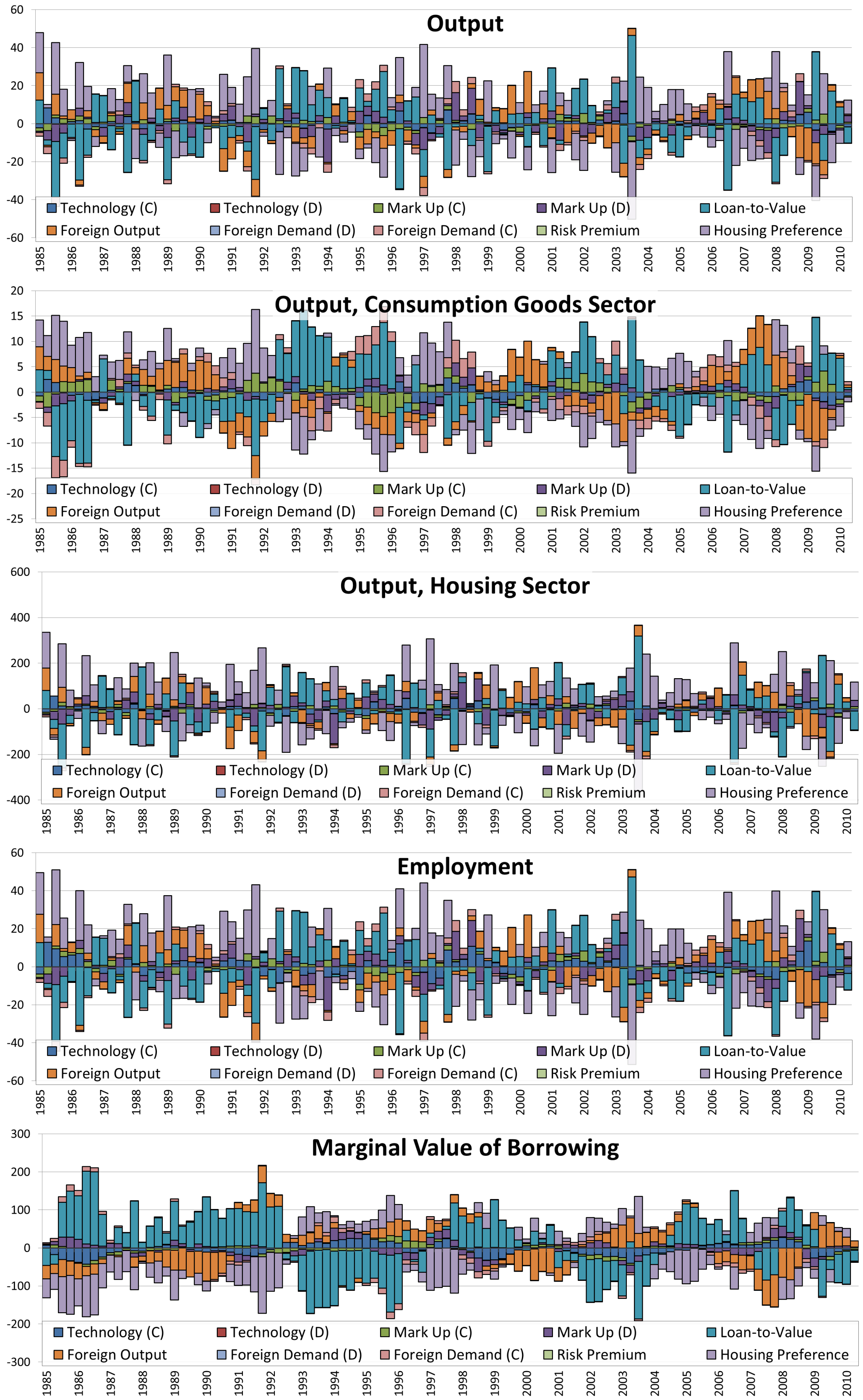


Figure 5: Historical Shock Decomposition II

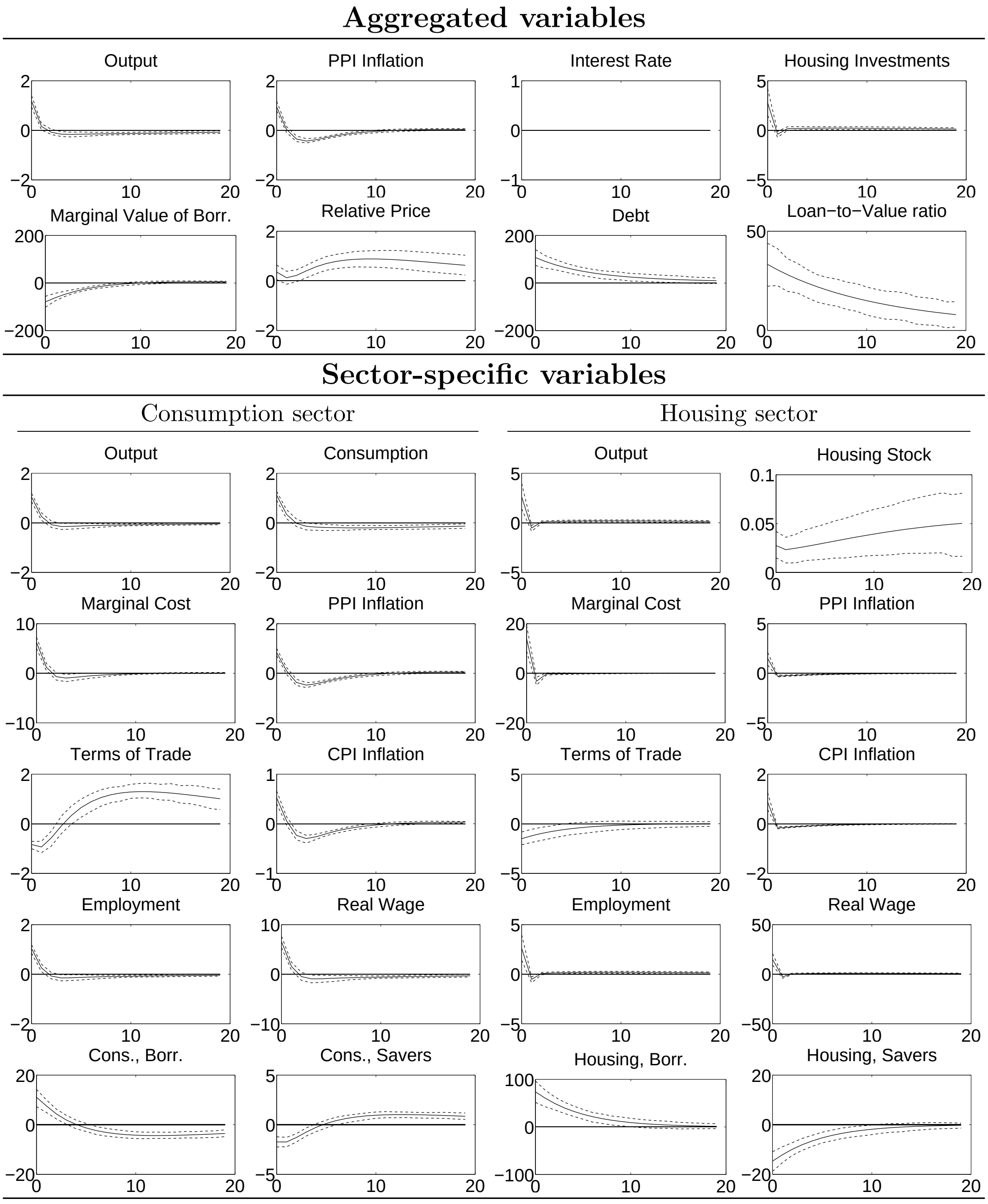


4.4 Impulse response functions

To better understand the propagation mechanisms implied by the model and prove the model implies plausible dynamics, we show impulse responses (mean ±95% confidence interval) for our multi-sector structure with non-housing and housing goods in Figures 6 to 14.

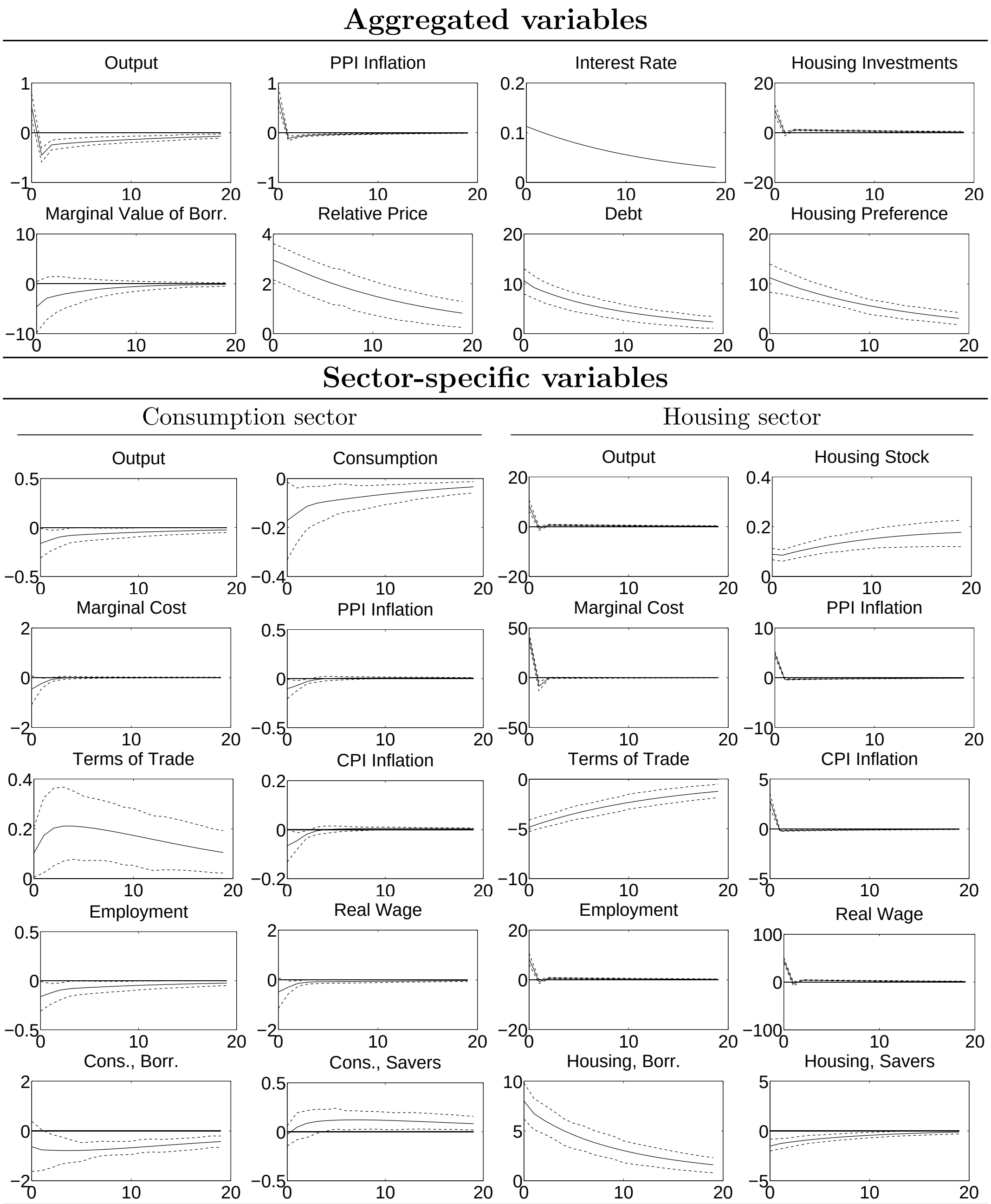
All graphs display intuitive reactions and attest the model to be an appropriate toolbox for analyzing the housing/business cycle nexus. The graphs show that the preference shock acts like a demand-side disturbance, moving output and inflation in the same direction. The cost-push and technology shocks, in turn, act as supply-side disturbances. A productivity shock reduces real marginal cost of firms, enabling them to lower prices of goods. Worthy of emphasis is furthermore that productivity shocks act more strongly on output than inflation. How big are the spillovers from the housing sector to the wider economy? In Figure 6, we present the effects of a loan-to-value shock. Here, both housing investment and GDP increase (as do inflation rates) due to a positive loan-to-value shock that corresponds to an exogenous increase in the availability of funds to borrowers in the economy. Borrowers demand more of both goods, driving up housing prices. In Figure 7, we present the responses to a preference shock in the housing sector. A positive preference shock genrates a surge in housing demand and housing prices with significant spillovers to the rest of the economy. In Figure 11, we present the response to a housing/technology shock. As expected, the shock increases housing investment and decreases housing prices by reducing marginal costs.

Figure 6: Posterior IRFs to a Loan-to-Value Shock $\left(\varepsilon_t^{LTV}\right)$



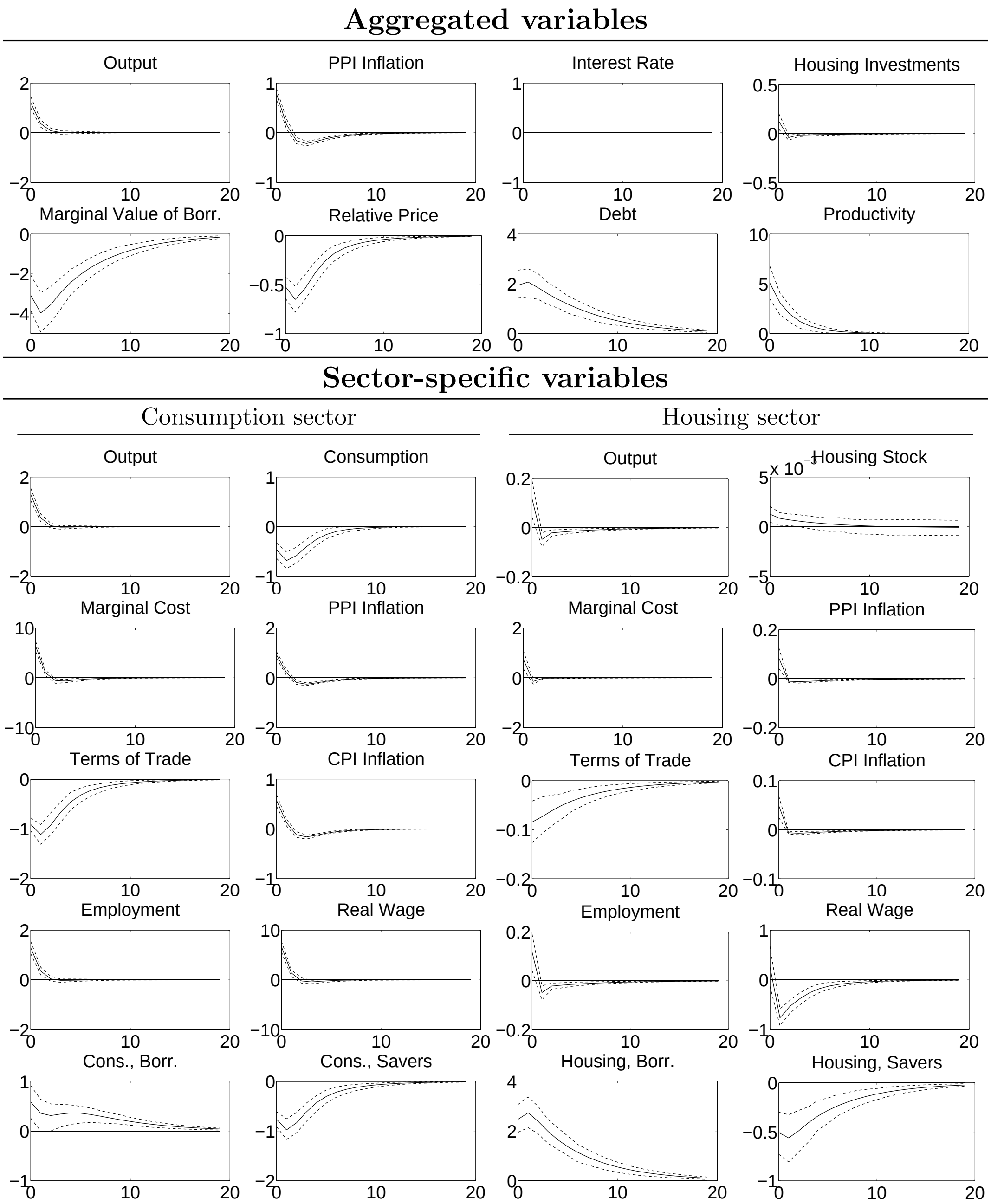
solid line: posterior mean; **dashed lines:** posterior bands

Figure 7: Posterior IRFs to a Housing Preference Shock $\left(\varepsilon_t^D\right)$



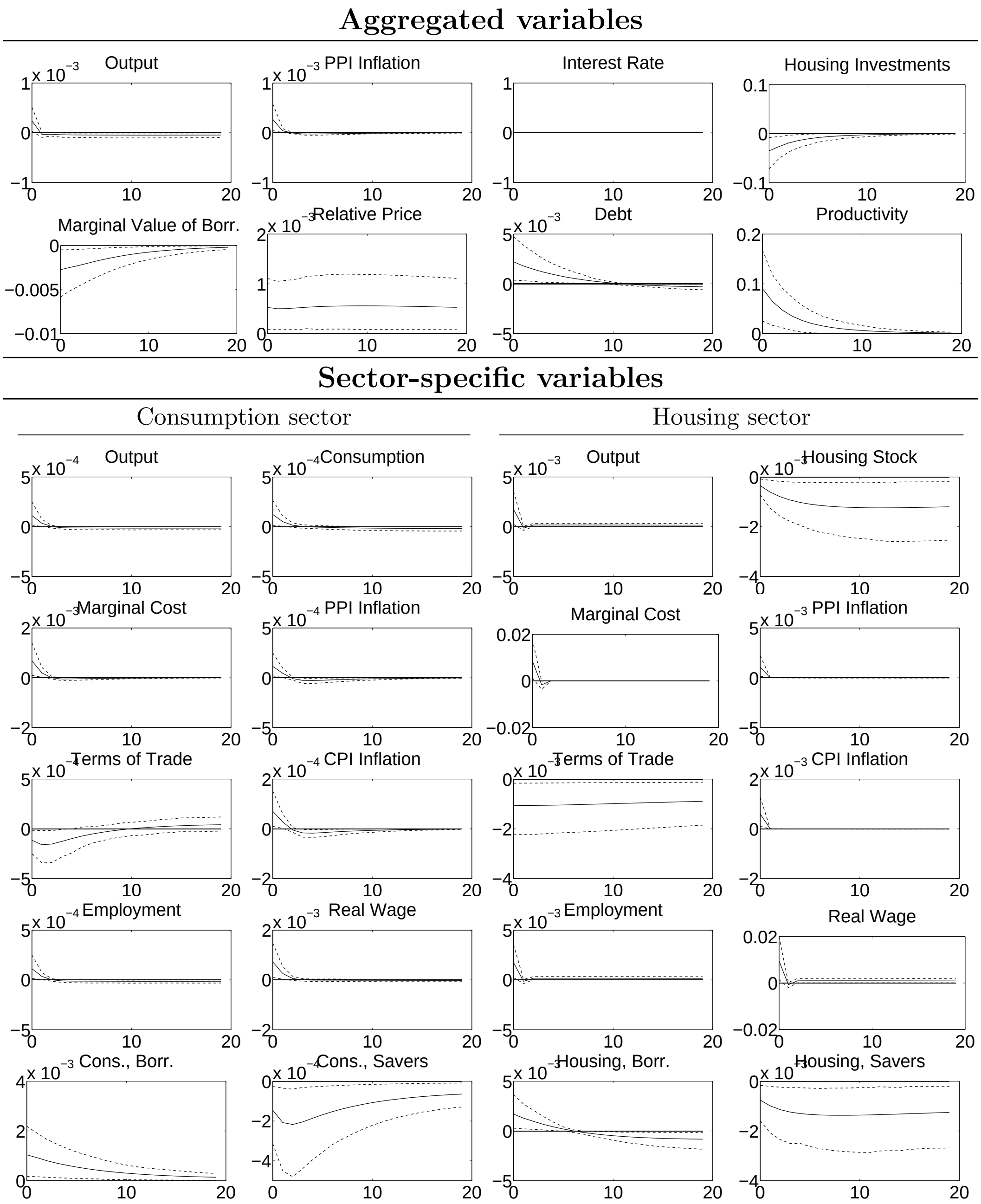
solid line: posterior mean; **dashed lines:** posterior bands

Figure 8: Posterior IRFs to a Foreign Consumption Demand Shock $\left(\varepsilon_t^{C,*}\right)$



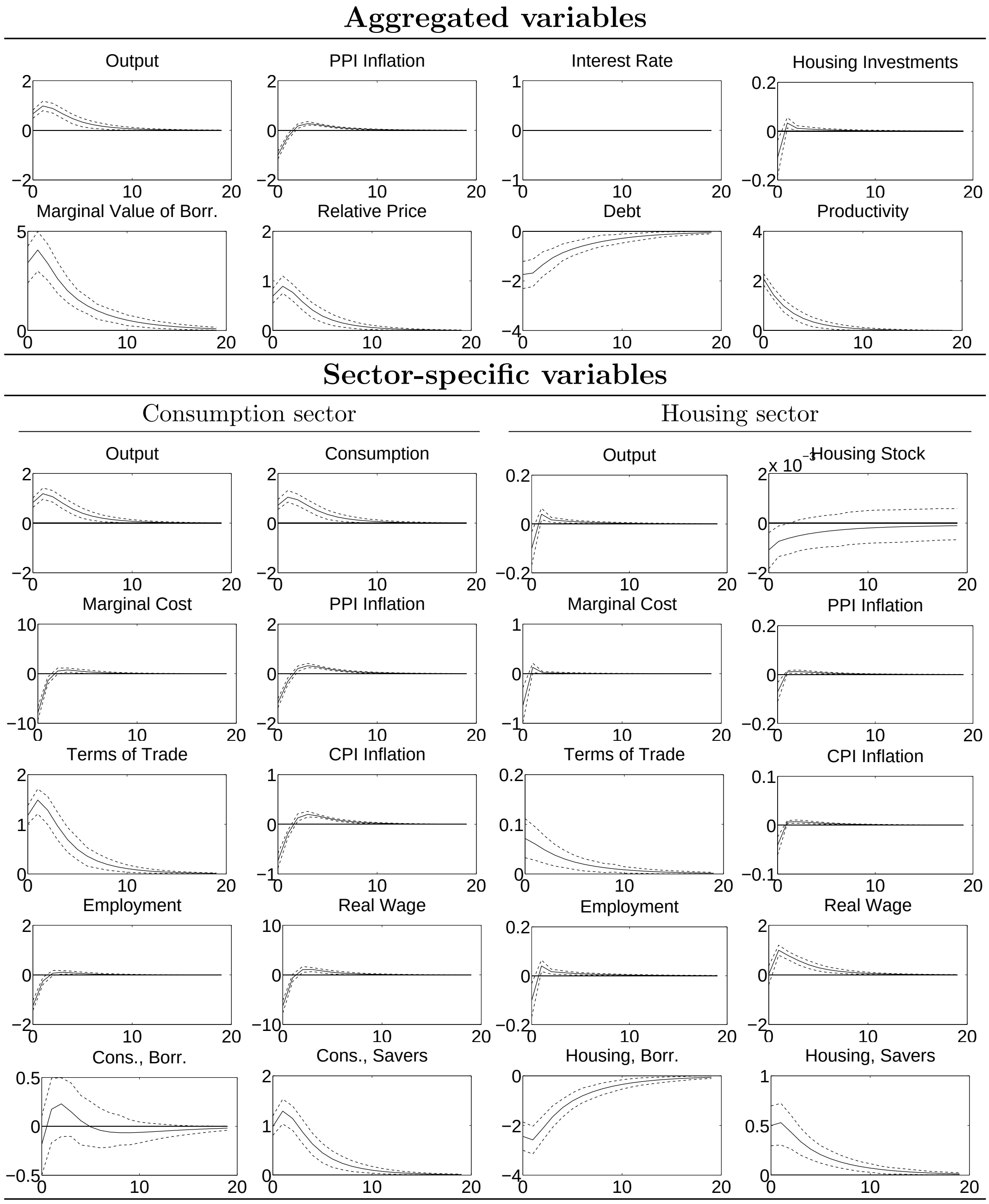
solid line: posterior mean; **dashed lines:** posterior bands

Figure 9: Posterior IRFs to a Foreign Housing Demand Shock $\left(\varepsilon_t^{D,*}\right)$



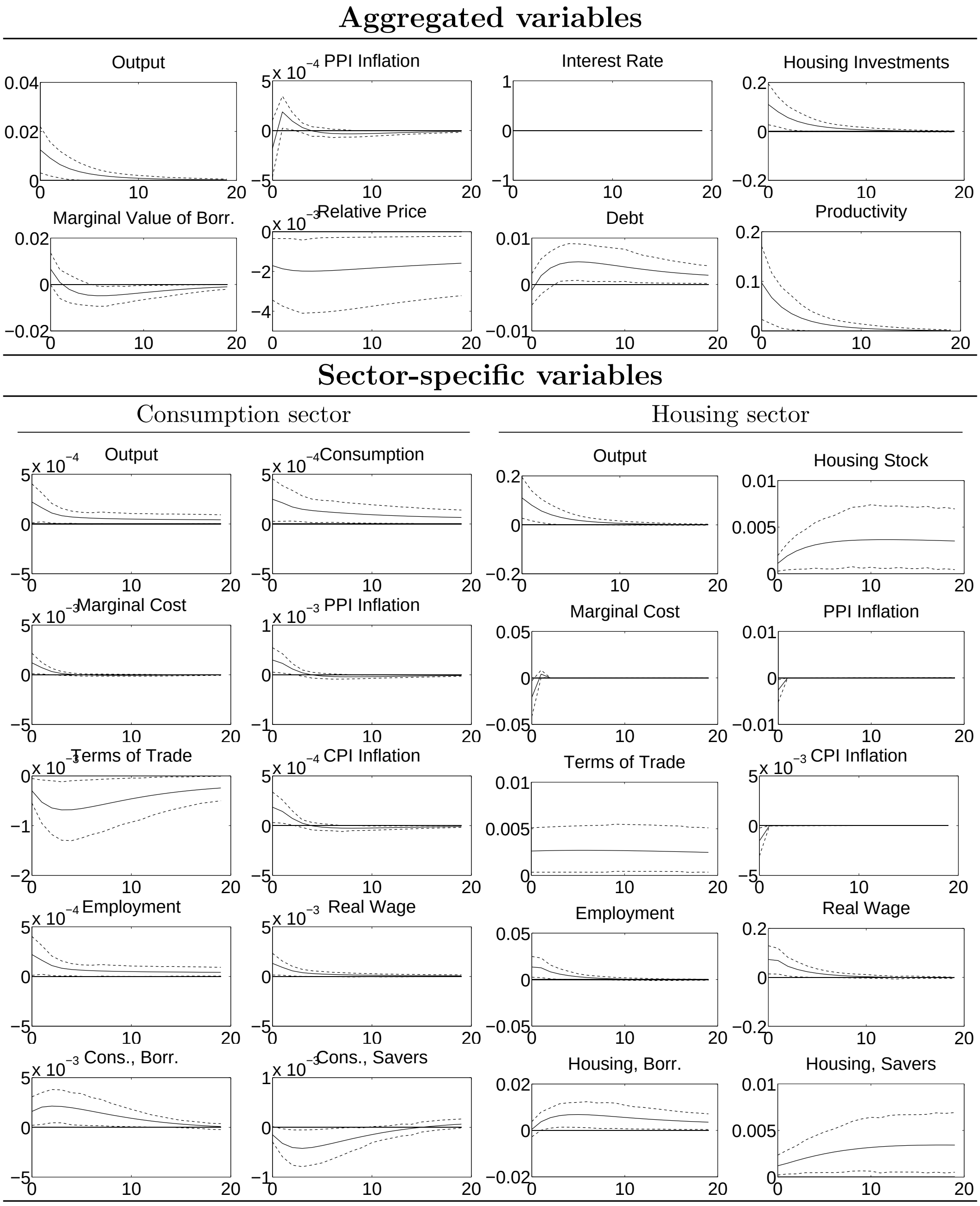
solid line: posterior mean; **dashed lines:** posterior bands

Figure 10: Posterior IRFs to a Productivity Shock, Consumption Sector ($a_{C,t}$)



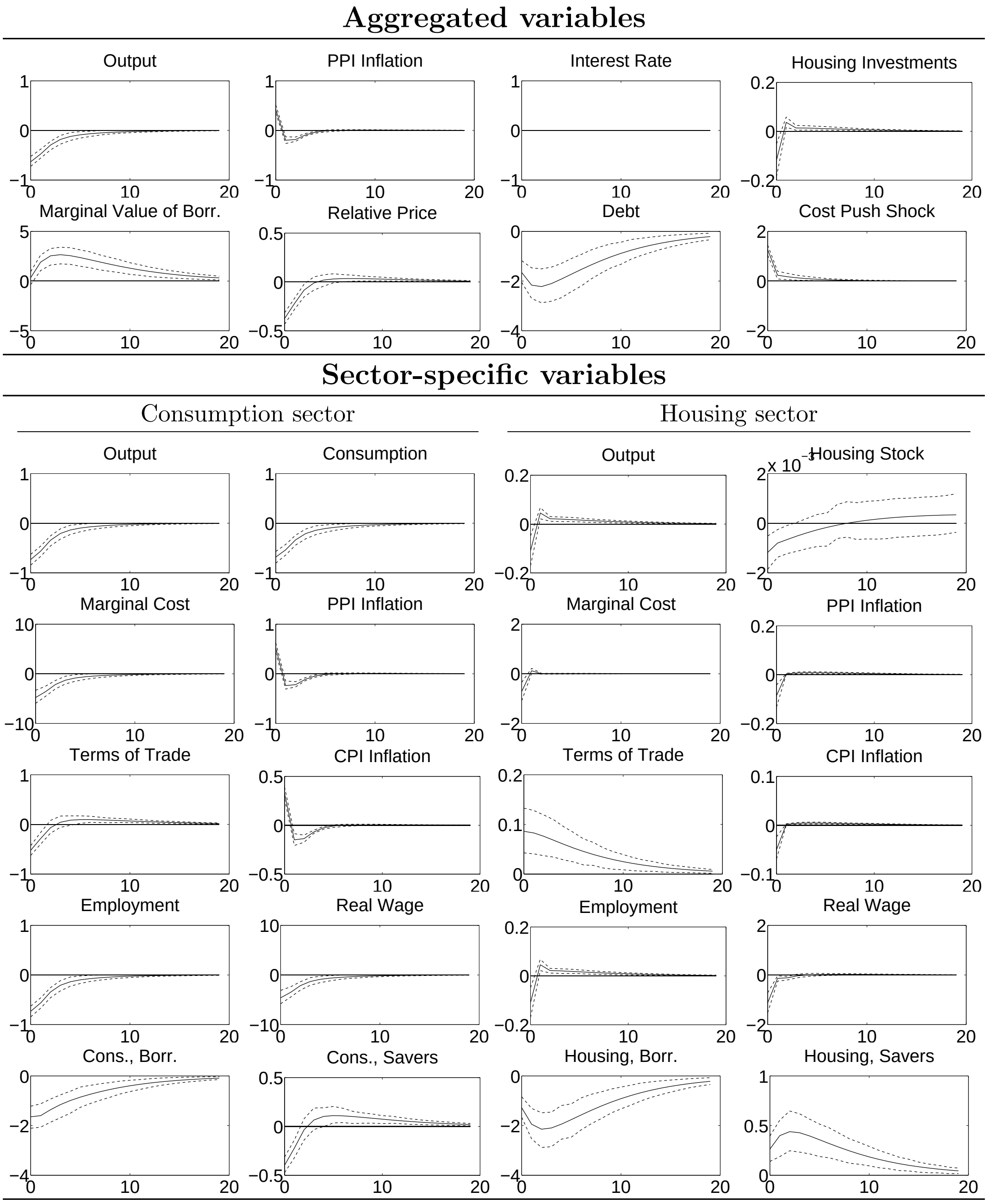
solid line: posterior mean; **dashed lines:** posterior bands

Figure 11: Posterior IRFs to a Productivity Shock, Housing Sector ($a_{D,t}$)



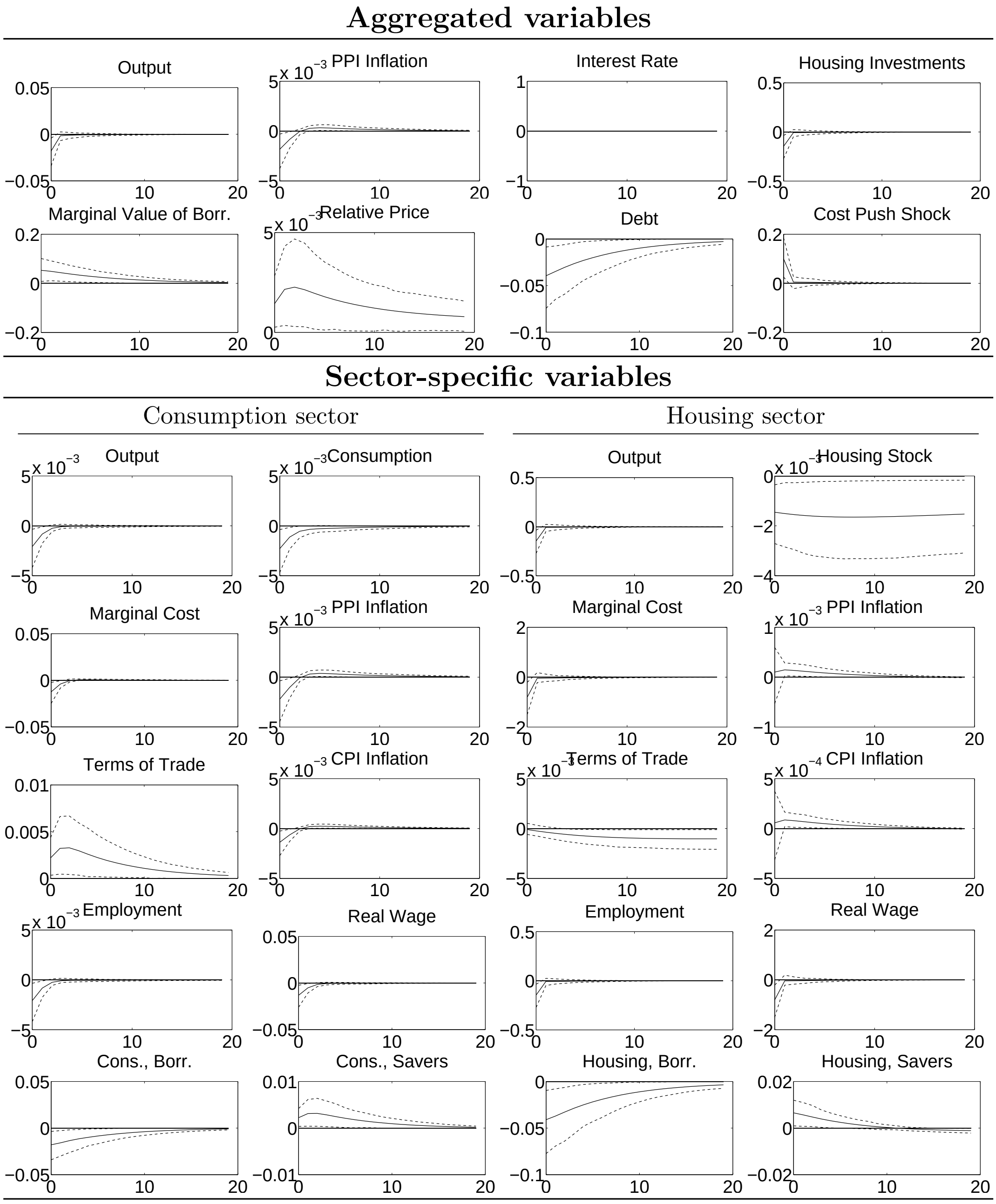
solid line: posterior mean; **dashed lines:** posterior bands

Figure 12: Posterior IRFs to a Cost Push Shock, Consumption Sector $\left(\mu_t^C\right)$



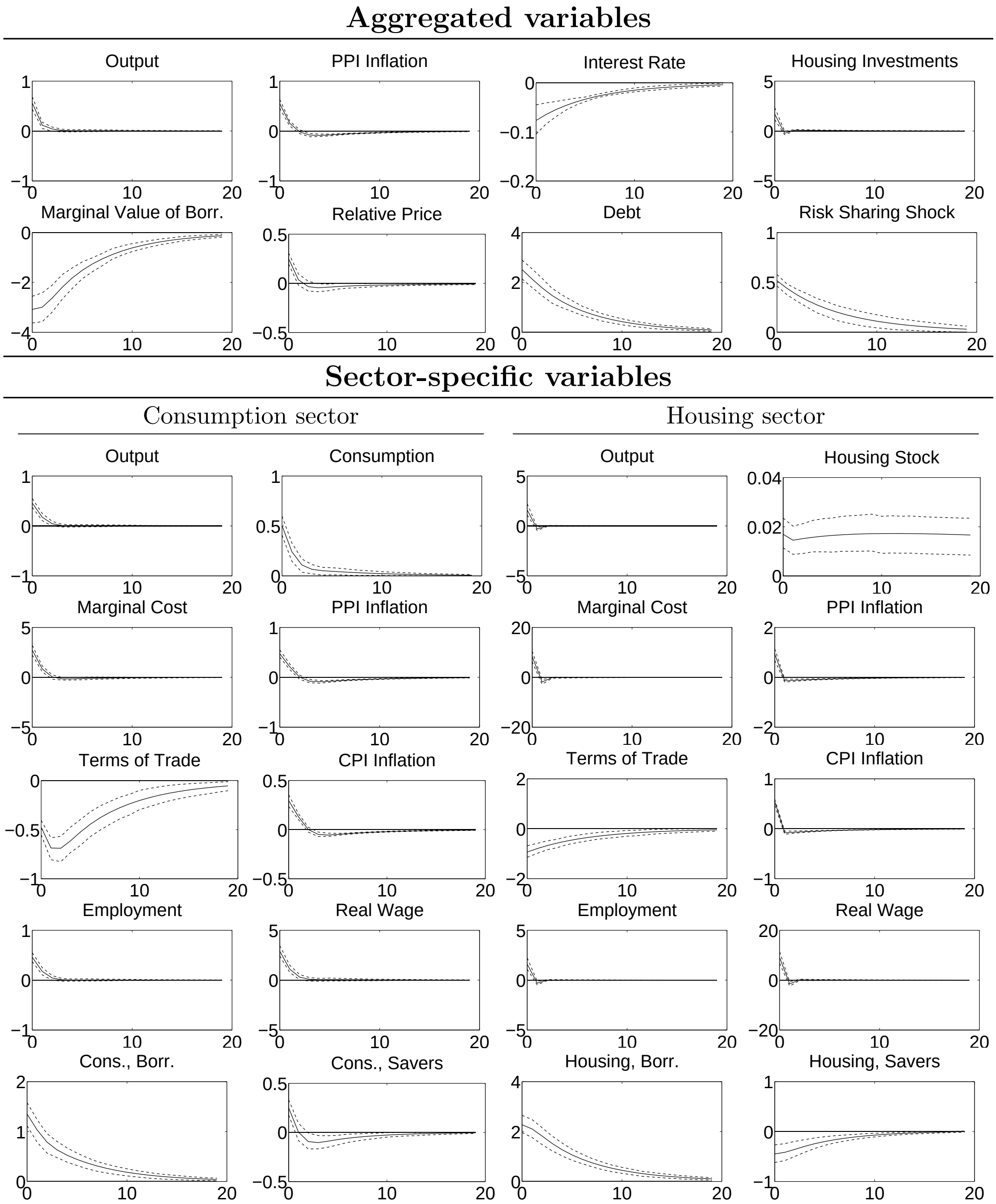
solid line: posterior mean; **dashed lines:** posterior bands

Figure 13: Posterior IRFs to a Cost Push Shock, Housing Sector $\left(\mu_t^D\right)$



solid line: posterior mean; **dashed lines:** posterior bands

Figure 14: Posterior IRFs to a Risk Sharing Shock (y_t^*)



solid line: posterior mean; **dashed lines:** posterior bands

5 Conclusions

The structure of our DSGE model presented above was largely motivated by recent developments in DSGE modeling. It includes extensions that incorporate specific structural characteristics of the Hong Kong economy. Our framework strived to bridge the gap between the business cycle and the housing literature and shed light on issues important for both macro and housing economists.

Generally, our findings show it is possible to extend a benchmark closed-economy model to a two-sector open-economy setup and obtain an empirically plausible model for analysis of the housing/business cycle nexus. The open-economy part of our model reflects the currency board exchange rate regime. Our focus in particular was assessing the relative contribution of preference and cost-push shocks, productivity, domestic and foreign demand shocks, and borrowing constraints (loan-to-value shocks) to explain Hong Kong's business cycle over the period 1985Q1−2010Q2.[28]

We offer two tentative conclusions. First, our results suggest unsurprisingly that Hong Kong's housing market is quite open to foreign investment. Second, and more interestingly, variations in the loan-to-value ratio and housing preference shocks are the most important determinants of domestic property prices, and largely explain business cycle volatility.

The modeling exercise raises the question of whether a DSGE approach is sufficient for modeling the recent financial crisis. At the descriptive level, the model may lack the necessary detail to explain the massive disruption caused by the liquidity spirals and amplification mechanisms, as well as the potential existence of tipping points. Furthermore, although a DSGE model can represent arbitrarily large house price volatility, it is firmly embedded with macroeconomic tradition of designing models to satisfy (local) stability conditions. The key question, however, is not whether we can directly replicate all domino effects and the collapse of the financial system, but whether (i) the cause-and-effect dynamics of the theoretical and empirical framework shed new light on the housing price/macroeconomy nexus, and (ii) the open-economy DSGE model is useful in understanding how exchange rate systems and corresponding monetary policy approaches ameliorate the outcome. We believe this straightforward approach meets these requirements and contributes to our understand of how the housing market influences and is affected by business cycles, monetary policy, and international developments.

The unusual nature of the recent financial crisis and the coincident timing in changes in financial stress and economic activity motivates the use of nonlinear Markov switching models. Such a nonlinear modeling framework makes it possible to categorize financial crisis episodes as a separate regime. Davig and Hakkio (2010) have recently merged the financial accelerator model with a two-regime Markov switching model. The log-linearized financial accelerator model switches between two distinct states. In the first, economic activity is high and financial stress low. In the second distressed state, economic activity is low and financial stress high. In contrast, Chen and Funke (2010) have identified the recent turmoil period as a separate regime in a three-regime Markov switching framework.

Summing up this work in the light of the 2007-2009 global financial crisis, it is clear that the analysis of the consumption/business cycle nexus hold promise for

[28] Since the paper provides interesting illustrative material for the housing/business cycle nexus, we should mention how such findings may apply to other advanced economies. On one hand, Hong Kong's mortgage markets are well-developed; more than 50 percent of all households live in owner-occupied accommodations. On the other hand, households in East Asia typically save more than households in the West and may exhibit different consumption sensitivity.

additional empirical and theoretical work. We hope to explore this in our future modeling work.

Appendix

Steady state To derive the steady state, we postulate a zero-inflation steady state and sector-specific terms-of-trade of value one and assume that the optimal employment subsidy of Galí (2003) is implemented. Since both household types share same preferences and skills, the equilibrium hours worked in each sector are the same for savers and borrowers: $N_C^s = N_C^b = N_C$ and $N_D^s = N_D^b = N_D$. The steady state of the model is represented by the following equations:

$$
\begin{aligned}
R &= \beta_s^{-1}, \\
\psi &= \beta_s - \beta_b, \\
MC_j &= \frac{1}{1+\mu_j}, \\
P_{D/C} &= \frac{1+\mu_D}{1+\mu_C}, \\
\frac{C^b}{D^b} &= \frac{1-\gamma}{\gamma} \frac{1-\beta_b(1-\delta)-(1-\chi)(1-\delta)(\beta_s-\beta_b)}{(1-h_c)\beta_s} P_{D/C}, \\
\frac{C^s}{D^s} &= \frac{1-\gamma}{\gamma} \frac{1-\beta_s(1-\delta)}{1-h_c} P_{D/C}, \\
\frac{B_H^b}{D^b} &= \beta_s(1-\chi)(1-\delta) P_{D/C}, \\
D^b &= \frac{\left(\frac{1}{1+\mu_C}\right) N}{\frac{C^b}{D^b}} + (\delta - (1-\chi)(1-\delta)(\beta_s - 1)) P_{D/C}, \\
C^b &= \frac{C^b}{D^b} D^b, \\
N &= N_C + N_D, \\
N_D &= \frac{\omega\delta \frac{C^s}{D^s}}{\frac{C^s}{D^s}+\delta} D^b + \frac{\delta}{\frac{C^s}{D^s}+\delta} \left(N - \omega C^b\right), \\
D^s &= \frac{1}{(1-\omega)\delta} \left(N_D - \omega\delta D^b\right), \\
C^s &= \frac{1}{1-\omega} \left(N_C - \omega C^b\right), \\
C &= N_C = Y_C, \\
\delta D &= N_D = Y_D.
\end{aligned}
$$

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