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Tiivistelmä

Tässä tutkimuksessa kehitetään ja estimoidaan Hongkongia koskeva dynaaminen stokastinen yleisen tasapainon avotalouden malli. Lyhyellä aikavälillä mallin hinnat ovat jäykkiä, mikä johtuu monopolistisesta kilpailusta sekä porrastetusta uudelleenoptimoinnista. Malliin on lisätty myös varallisuusvaikutus osakemarkkinoiden avulla, minkä uskomme olevan tärkeää. Tämän takia mallissa käytetään jatkuvan nuoruuden oletusta. Mallin parametrit ja havaitsemattomat komponentit estimoidaan käyttäen bayesilaistä suurimman uskottavuuden menetelmää ehdollistettuna aiemmalle tiedolle parametrien arvoista.

Asiasanat: dynaamisen stokastiset mallit, varallisuusvaikutukset, avotalous, Hongkong

Stock Market Wealth Effects in an Estimated DSGE Model for Hong Kong

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Abstract

This paper develops and estimates an open economy dynamic stochastic general equilibrium (DSGE) model of the Hong Kong economy. The model features short-run price rigidities generated by monopolistic competition and staggered reoptimisation. The model is enhanced with wealth effects due to stock price dynamics, which we believe to be important. For this reason we adopt a perpetual youth approach. Model parameters and unobserved components are estimated with a Bayesian maximum likelihood procedure, conditional on prior information concerning the values of parameters.

Keywords: DSGE models, wealth effects, open economy, Hong Kong.

JEL classification: D91, E21, E44, F41.

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1 Introduction

The combination of dynamic stochastic general equilibrium (DSGE) techniques used in the real business cycle literature with the Keynesian rigid prices assumption lies at the heart of the workhorse New Keynesian models of macroeconomic analysis. DSGE models address flaws in the standard Keynesian model and, as equilibrium business cycle theorists like to point out, the accompanying worldview.[1] Moreover, they allow for easy-to-handle small- and mid-scale models with excellent predictive properties by log-linearizing the first-order conditions of optimizing households and firms. For an underlying sticky price framework, these models reveal monetary policy to have real effect over the short-run. Finally, they cannot be criticized like conventional aggregate demand-aggregate supply models for lacking detailed microfoundations. The parameters of the Phillips and IS curves are dependent on the structural preferences of the private sector. The assumption of a fraction of backward-looking price-setters yields to a hybrid model that reflects the persistence of inflation as found for e.g. the US by Sbordone (2002).

DSGE models are also useful in macroeconomic forecasting. For example, Del Negro et al. (2007) apply DSGE models to constrain VARs in a Bayesian setting. Theoretical restrictions from firms and households optimizing behavior subject to their intertemporal budget constraints imply a set of cross-equation restrictions on the VAR parameters. Del Negro et al. (2007) relate these constraints to priors on the model parameters (informative priors pull the parameters toward the theoretical constraints, while ignoring theory tends to diffuse priors). In their simulation study, they find evidence that a DSGE-based VAR yields better out-of-sample forecasting performance for a range of macroeconomic variables than the usual unconstrained VAR.

A vast body of studies seeking to capture Hong Kongs macroeconomic relationships through various ad hoc and partial equilibrium models has been generated in recent years.[2] All these studies, however, omit explicit microfoundations, including the now-standard assessment of optimizing-behavior of rational agents. Our contribution here is an attempt to apply a New Keynesian approach with adjustments to capture Hong Kongs unique macroeconomic features.

Economic transformation has turned Hong Kong into a global financial center with a massive concentration of financial and business services. Much like a city-

[1]The growing body of literature indicates many researchers have found equilibrium business cycle theory worthy of formal analysis. See Clarida et al. (1999) or Goodfriend and King (1997) for surveys. Paul Krugman dismisses the value of this work, noting recently that economists who have spent their entire careers on equilibrium business cycle theory are now discovering, in effect, that they invested their savings with Bernie Madoff. http://krugman.blogs.nytimes.com/2009/01/27/madoff-economics/.

[2]A number of empirical analyses have established a statistically significant relationship between consumption and wealth in Hong Kong. See, for example, Jin (2002), Ha and Leung (2002), Ha et al. (2002) and He et al. (2007).

state, Hong Kongs economic structure resembles a high-income metropolis rather than a typical advanced economy. Its growing integration with Mainland China tracks developments in the adjacent manufacturing-centered Pearl River Delta with which it shares strong trade and service linkages.[3]

Given the relatively stable inflation rates of the last two decades and the rise of financial markets, the latest developments in New Keynesian modeling focus on the interaction of monetary policy and financial stability, and specifically, the question of whether central bankers should respond to stock market dynamics.[4] By enhancing the ideas of Yaari (1965) and Blanchard (1985), we develop a new strand of investigation that gives the possibility to include short-run wealth effects that consider stock-price dynamics in the optimal intertemporal consumption decision (see e.g. Nisticó (2005) and Di Giorgio and Nisticó (2007)). Firms issue equity shares to the public (based on the tree model of Lucas (1978)) and households face a constant probability of economic death.[5] The expected lifetime of a household (creation to dissolution) is equal to the time horizon relevant for decision-making. As long as this time horizon is finite, deviations of stock prices from their long-term equilibrium can impact real variables over the short-run; i.e. households can exit the market during a temporary deviation. As numerous recent empirical studies have shown such effects to be important for the Hong Kong economy, we allow for them using a perpetual youth approach.[6]

The remainder of this paper is organized as follows. Section two briefly presents stylized facts on the Hong Kong economy. Section three describes the model, reflecting these specific characteristics. Estimation and inference of the DSGE model are laid out in section four. We conclude with a summary and evaluation of our findings.

[3]For a discussion of the role of global cities in the globalized production process, see Glaser et al. (2001).

[4]Studies providing a rationale for a reaction to stock-price misalignments include Cecchetti et al. (2000), Cecchetti et al. (2002) and Cecchetti (2003), whereas Bernanke and Gertler (1999) and Bernanke and Gertler (2001) argue that the only desirable reaction to stock prices is the one implicit in the response to expected inflation and output.

[5]Limited planning horizons (or impatience) in economic decision-making can be due to disconnectedness of current households from future generations or the lack of an altruistic bequest motive. They can also occur through imperfect access to financial markets. Recent work has relaxed the assumption of unrestricted financial-market access. In Levchenko (2005), for example, access to international financial markets is restricted to certain households as the paper is focused on explaining the relatively high volatility of consumption in emerging markets.

[6]Liu et al. (2007) find evidence for both anticipated and unanticipated income and wealth effects to influence Hong Kong's consumption, especially after the 1997 financial crisis.

2 Structural characteristics of Hong Kong

As a small, open economy with a US-dollar-based currency board, Hong Kong's domestic interest rates are largely determined exogenously by their US-dollar counterpart adjusted with a risk premium.[7] The nominal effective exchange rate of the Hong Kong dollar follows the movements of the US dollar against currencies of Hong Kong's trading partners.[8] Moreover, the dollar peg and increased integration with Mainland China have increasingly tied the long-run inflation trend to the US and the Mainland.

Wealth effects play an important role in Hong Kong dynamics. Consumption and savings in the special administrative region reflect precautionary incentives to deal with potentially volatile income and capital flows, as well as traditional life-cycle motives. Thus, interest rates impact consumption significantly, while intertemporal substitution is a minor influence due to the relative importance of services with respect to production.

Property and equity prices are important factors in determining household wealth, and thus, domestic demand. It is our expectation here that stock market wealth effects are significant in the case of Hong Kong, so we concentrate on understanding the transmission of stock price dynamics to consumption. This study distinguishes between effects from asset holdings and those from real estate, and focuses solely on the former. Analysis on the influence of housing prices is left to future research.[9]

Neoclassical and New Keynesian economists disagree as to how quickly wages and prices adjust. Neoclassical economists build their macroeconomic theories on the assumption that wages and prices are flexible. Prices clear markets, i.e. balance supply and demand, by adjusting quickly. New Keynesian economists, in contrast, believe that market-clearing models cannot explain short-run economic fluctuations. They advocate models with sticky wages and prices. A reason prices do not adjust immediately to clear markets is that adjusting prices is costly. In the presence of

[7] Under a pure currency board system, the Hong Kong dollar and the US dollar interest rates should move in tandem. Several factors, however, including the expectation that the Hong Kong dollar should also be allowed to reflect renminbi appreciation, has caused the two rates to deviate slightly.

[8] Clearly, the level of American interest rates is not always appropriate for Hong Kong. External shocks can be readily transmitted to the domestic economy through a currency board (i.e. exchange rate policy on auto-pilot) which lacks the cushioning of exchange rate adjustments. Indeed, the shock impacts are often magnified as a result of the Hong Kong economy's openness, so economic costs can be severe. As there is no effective monetary tool for preempting or dealing with the threat of run-away economic overheating, there is even the possibility of exacerbating the amplitude of economic cycles.

[9] An estimation of a DSGE model that includes the housing market (using US data and Bayesian methods) can be found in Iacoviello and Neri (2008). While this is certainly an issue worth pursuing eventually, dealing with the housing market here would require modeling of an explicit housing sector in addition to the conventional consumption sector and an assumption that each household supplies labor to both sectors. Thus, we focus on stock prices and do not investigate the effects of Hong Kong's real estate prices.

this aggregate-demand externality, even small menu costs can make prices sticky, and this stickiness can have large costs for society. Furthermore, the adjustment of prices throughout the economy is staggered. Yetman (2009) provides a recent analysis of the flexibility of prices in Hong Kong. In broad terms, the empirical results indicate that prices of consumer goods and services in the Hong Kong CPI are comparatively flexible. Since Hong Kongs currency board system precludes a systematic policy response to domestic shocks, adjustment has to occur largely through price flexibility. In the estimates below, we treat the degree of persistence as an unknown parameter to be estimated.

3 The Model

The foregoing analysis suggests use of a modeling setup that allows for wealth effects upon consumption. Our modeling framework is mainly based on the closed economy framework with wealth effects of Nisticó (2005) and the seminal work of Galí and Monacelli (2005), who provide a small open economy DSGE model. The study of Lubik and Schorfheide (2007) also offers a useful empirical reference.[10] In order to capture the specific characteristics of the Hong Kong economy, and specifically, the influence of stock prices on consumption, the conventional open economy DSGE framework is reformulated so that the Euler equation for consumption accounts for stock price dynamics. As we define the domestic economy as one among a continuum of infinitesimally small economies, it is of negligible size relative to the rest of the world.[11] The resulting model depends on specific structural parameters such as the openness of the domestic economy, the substitutability of goods of different economies, and the probability that a cohort member will die during a certain period.

Since we focus on the interaction of one economy with the rest of the world, we call this the domestic country and do not use an i-index for variables from this country. Instead, variables with an index $i \in [0, 1]$ refer to one economy i among the continuum making up the rest of the world. The superscript $*$ here refers to all foreign countries (the rest of the world as a whole). In what follows, we describe the microfoundations of foreign countries only where we see necessary. Otherwise, the foreign country counterparts of domestic equations can be derived in a straightforward manner. If not explicitly mentioned, price indices are measured in domestic currency units. To help the reader follow the analytical derivations, Tables 1 - 3 define all variables and parameters used in this section.

Households and firms face the conventional New Keynesian optimization problems based on a competitive goods market (drawn from Dixit and Stiglitz (1977)) and

[10]Although Lubik and Schorfheide (2007) estimate small open economy DSGE models for four countries, their focus is on whether central banks in the studied economies respond to exchange-rate movements.

[11]Since the measure of each economy is zero, its policy decisions have no impact on the rest of the world. For convenience, we assume identical preferences, technology, and market structure.

Table 1: Financial market variables and prices

Variable	Meaning
$Q_t(k)$	equity price of a domestic firm k
$Q_t(Q_t^n)$	domestic (natural) stock price index
D_t	index of domestic real dividend payments
$B_t(B_t^i)$	domestic holdings of domestic (country i's) bonds
$B_t^{(i,H)}\left(B_t^{(i,l)}\right)$	country i's holdings of domestic (country l's) bonds
$D_t(k)$	dividend yields of domestic firm k
$Z_t(k)(Z_t(k,j))$	equity shares of a domestic firm k (held by cohort j)
Ω_t	domestic nominal financial wealth
$F_{t,t+1}$	discount factor on state-contingent claims
$r_t(r_t^*)$	domestic (foreign) interest rate
rr_t^n	real natural rate
$P_t(P_t^i)$	domestic (country i's) CPI
$P_{H,t}(P_{F,t})$	domestic PPI of domestic (imported) goods
$P_{H,t}^i\left(P_{F,t}^i\right)$	country i's PPI of country i's (imported) goods
$P_{H,t}^*\left(P_{F,t}^*\right)$	foreign (domestic) PPI measured in foreign currency
$P_{H,t}(k)\left(P_{i,t}^i(k)\right)$	producer price of domestic (country i's) good k
$P_{i,t}\left(P_{i,t}^i\right)$	country i's PPI in domestic (foreign) currency
$\overline{p}_{H,t}^n$	newly set prices
$p_{H,t}^{fl}\left(p_{H,t}^{bl}\right)$	prices of forward (backward) looking setters
P_t^*	world CPI/PPI
Π_t	gross inflation $(1+\pi_t)$
$\pi_{H,t}$	PPI inflation
$\pi_t(\pi_t^*)$	domestic (foreign) CPI inflation
$S_t(S_t^i)$	domestic (country i's) effective terms of trade $\frac{P_{F,t}}{P_{H,t}}\left(\frac{P_{F,t}^i}{P_{H,t}^i}\right)$
$S_{i,t}$	bilateral terms of trade

sticky price set-up (drawn from Calvo (1983)). In two-stage production, final goods producers (retail sector) use the output of intermediate goods producers (wholesale sector) as input. Final goods producers are assumed to produce competitively using a CES technology consisting of a continuum of nontraded intermediate goods. We further assume that intermediate firms issue equity shares. Thus, households can hold two types of financial assets: state-contingent bonds and equity shares of intermediate firms. If nominal gross returns on internationally tradable state-contingent bonds are equalized across countries (international risk-sharing), we can imply that the Uncovered Interest Parity (UIP) assumption holds. Furthermore, the Law of One Price (LOOP) holds at the brand level, implying Purchasing Power

Table 2: Household variables and additional indexes

Variable	Meaning
Ξ_t^i	nominal effective bilateral exchange rate
$\Xi_t^{(i,l)}$	nominal effective bilateral exchange rate between countries i and l
Ξ_t	nominal effective exchange rate
$\Upsilon_{i,t}$	effective bilateral real exchange rate $\left(\frac{\Xi_t^i P_t^i}{P_t}\right)$
$C_t\,(C_t^*)$	domestic (foreign) consumption index
$C_{H,t}\,(C_{F,t})$	composite domestic demand for domestic (foreign) goods
$C_{i,t}\left(C_{H,t}^i\right)$	composite domestic (country i's) demand for country i's goods
$C_{H,t}(k)\,(C_{F,t}(k))$	domestic (foreign) demand for a domestic good k
$C_{i,t}(k)\left(C_{H,t}^i(k)\right)$	domestic (country i's) demand for country i's good k
N_t	domestic labor
$N_t(k)$	domestic labor for good k
$Y_t\,(Y_t^n)$	domestic (natural) output
Y_t^*	foreign output
$Y_t(k)$	production function of good k
X_t	domestic output gap $\left(\frac{Y_t}{Y_t^n}\right)$
T_t	taxes
$\wp_t$	relative price dispersion among domestic firms
MC_t	domestic marginal costs
W_t	domestic wage rate
H_t	domestic human wealth
A_t	domestic productivity shock
$\triangle_{i,t+i}$	real discount factor

Parity (PPP) at any point in time.[12] To allow for persistence in inflation, we assume a fraction of backward-looking price-setters.

3.1 Households

3.1.1 Individual Optimization

Intertemporal Allocation Following Blanchard (1985), each economy consists of an indefinite number of cohorts facing a constant probability γ of dying each period. The implied expected lifetime of $\frac{1}{\gamma}$ can be interpreted as the effective decision horizon of consumers. As in standard DSGE models, households demand consump-

[12]For simplicity, we assume that LOOP, and therefore PPP, holds. As shown in Lubik and Schorfheide (2005), deviations from PPP result in a Phillips curve relationship between import price inflation and the LOOP price gap.

Table 3: Structural parameters

Parameter	Meaning
ϖ	elasticity of substitution between domestic and foreign goods
ζ	elasticity of substitutibility between goods produced in foreign countries
ε	elasticity of substitution between differentiated goods within one country
γ	probability of dying
α	degree of openness
θ	price rigidity
τ	fraction of backward-looking price-setters
μ	steady state mark-up over a weighted average of expected marginal costs
φ	inverse steady state Frisch elasticity of labor supply
ϵ	conditional covariance between $F_{t,t+1}$ and D_t.
β	discount factor
$\widetilde{\beta}$	steady state discount factor
r	steady state interest rate

tion goods and supply labor.[13] Each cohort j is allowed to buy two types of financial assets: domestic and foreign state-contingent bonds (which holdings of a cohort j are denoted by $B_t(j)$ for domestic bonds and $B_t^i(j)$ for bonds from country i) and equity shares of domestic intermediate firms (denoted as subscript k), for which nominal prices are given by $Q_t(k)$.[14]

Before describing the decision problems of households, we briefly clarify our notation. $Z_t(k,j)$ is defined as the equity shares of a firm k held by the cohort j. The corresponding nominal composite dividend yield is given by $D_t(k)$.[15] Following this notation, the nominal financial wealth of a domestic individual of cohort j is given by

$$\Omega_t(j) \equiv \frac{1}{1-\gamma}\left[\begin{array}{c} B_t(j) + \int_0^1 \Xi_t^i B_t^i(j)\,di \\ + \int_0^1 \left(Q_t(k) + D_t(k)\right) Z_t(k,j)\,dk \end{array}\right], \qquad (1)$$

where Ξ_t^i is the nominal bilateral exchange rate (defined as the domestic price of country i's currency) and firms are normed to one. Following (1), financial wealth

[13]We abstract from money and model a cashless economy. This implicitly assumes the money supply adjusts to clear the market. Thus, we do not need to model money explicitly as here it simply functions as a unit of account.

[14]The simplification that intermediate firms issue shares is purely a convenience. We also exclude the possibility of cohort members buying foreign shares as our focus is wealth effects from the Hong Kong stock market. As government bonds are perfect substitutes, one could summarize all bonds as a single state-contingent bond.

[15]For convenience, we only allow domestic households to buy shares of domestic firms. This is quite appropriate to our modeling strategy as the Hong Kong and the US stock markets are highly correlated. Thus, introducing a foreign stock market into this study would likely add little insight.

includes bond holdings of the previous period and the pay-off on the portfolio of equity shares (dividends plus market value). Since financial wealth also pays off the gross return on the insurance contract, redistributing among surviving consumers, total personal financial wealth is accrued by $\frac{1}{1-\gamma}$.[16]

A domestic representative household of cohort j chooses its optimal amount of consumption, labor supply, and holdings of financial assets to maximize his additively separable utility conditional on survival,

$$E_0\left[\sum_{t=0}^{\infty}\beta^t\left(1-\gamma\right)^t\left[\log C_t\left(j\right)+\log\left(1-N_t\left(j\right)\right)\right]\right], \tag{2}$$

subject to a sequence of dynamic budget constraints (in real terms)

$$\begin{aligned} C_t\left(j\right)+\frac{1}{P_t}E_t\left\{F_{t,t+1}B_{t+1}\left(j\right)\right\}+\int_0^1\frac{\Xi_t^i}{P_t}E_t\left\{F_{t,t+1}B_{t+1}^i\left(j\right)\right\}di \\ +\frac{1}{P_t}\int_0^1 Q_t\left(k\right)Z_{t+1}\left(k,j\right)dk=\frac{W_t}{P_t}N_t\left(j\right)-T_t\left(j\right)+\frac{\Omega_t\left(j\right)}{P_t}, \end{aligned} \tag{3}$$

where W_t and P_t represent the wage rate and the consumption price index (CPI), respectively. $F_{t,t+1}$ is the relevant discount factor for state-contingent claims,[17] T_t are real government taxes, and β is the discount factor of the representative agent.

Domestic human wealth is defined as discounted stream of expected non-tradable income conditional on survival:

$$H_t\left(j\right)\equiv E_t\left\{\sum\nolimits_{k=0}^{\infty}F_{t,t+k}\left(1-\gamma\right)^k\left(\frac{W_t}{P_t}N_t\left(j\right)-T_t\left(j\right)\right)\right\} \tag{4}$$

In period t, the representative household chooses state-contingent sequences for consumption, leisure and share holdings to maximize intertemporal utility subject to budget constraints. In equilibrium, the necessary first-order conditions associated with this utility maximization problem may be stated as (3) and

$$\frac{C_t\left(j\right)}{\left(1-N_t\left(j\right)\right)} = \frac{W_t}{P_t}, \tag{5}$$

$$F_{t,t+1} = \beta E_t\left[\left(\frac{P_t}{P_{t+1}}\right)\left(\frac{C_t\left(j\right)}{C_{t+1}\left(j\right)}\right)\right], \tag{6}$$

$$Q_t\left(k\right) = E_t\left\{F_{t,t+1}\left[Q_{t+1}\left(k\right)+D_{t+1}\left(k\right)\right]\right\}, \tag{7}$$

where (5) represents the optimal intratemporal consumption-leisure decision and (6) is the Euler equation, describing the optimal intertemporal consumption pattern (the equilibrium stochastic discount factor for one-period ahead nominal payoffs equals the time-discounted stochastic growth in the marginal utility of consumption). Furthermore, (7) is the optimal inter-temporal decision concerning the holdings of equity

[16] See Blanchard (1985) for details.

[17] Excluding the possibility of arbitrage, domestic and foreign discount factors on state-contingent claims must be the same.

shares and equates the nominal price of an portfolio to its nominal expected payoff one period ahead, discounted by $F_{t,t+1}$. This defines the stock-price dynamics. The first-order conditions ensure that at utility maximum the representative household cannot benefit from feasible intertemporal consumption reallocations.

Intratemporal Allocation We now proceed to illustrate intratemporal allocation. Consumers consume a composite that is a Dixit-Stiglitz aggregator over differentiated goods. Households may purchase domestic and foreign consumption goods. The composite consumption index of a domestic cohort j is defined as

$$C_t(j) \equiv \left[(1-\alpha)^{\frac{1}{\varpi}} C_{H,t}(j)^{\frac{\varpi-1}{\varpi}} + \alpha^{\frac{1}{\varpi}} C_{F,t}(j)^{\frac{\varpi-1}{\varpi}}\right]^{\frac{\varpi}{\varpi-1}}, \tag{8}$$

where $C_{H,t}(j)$ and $C_{F,t}(j)$ represent the domestic demand for domestic and foreign final goods, respectively, $\varpi > 0$ represents the elasticity of substitution between domestic and foreign goods, and α refers to the share of domestic consumption allocated to imported goods and is thus a natural index of openness. Furthermore, the demand indices are given by the following CES functions

$$C_{H,t}(j) \equiv \left[\int_0^1 C_{H,t}(k,j)^{\frac{\varepsilon-1}{\varepsilon}} dk\right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{9}$$

$$C_{F,t}(j) \equiv \left[\int_0^1 C_{i,t}(j)^{\frac{\zeta-1}{\zeta}} di\right]^{\frac{\zeta}{\zeta-1}}, \tag{10}$$

$$C_{i,t}(j) \equiv \left[\int_0^1 C_{i,t}(k,j)^{\frac{\varepsilon-1}{\varepsilon}} dk\right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{11}$$

where ε denotes the elasticity of substitution between the differentiated goods within one single country and ζ measures the substitutability between goods produced in different foreign countries. $C_{H,t}(k,j)$ and $C_{i,t}(k,j)$ represent the domestic demand of cohort j for a specific domestic product or a specific product from country i, respectively. $C_{i,t}(j)$ is a composite index, summarizing the demand for products from country i, and $C_{F,t}(j)$ is a composite index, summarizing the demand for products from all foreign countries.

Similar to the consumption indices, household's minimization of total expenditures leads to the following producer price indices (PPIs) for domestic and foreign goods,[18]

$$P_{H,t} = \left[\int_0^1 P_{H,t}(k)^{1-\varepsilon} dk\right]^{\frac{1}{1-\varepsilon}}, \tag{12}$$

$$P_{i,t} = \left[\int_0^1 P_{i,t}(k)^{1-\varepsilon} dk\right]^{\frac{1}{1-\varepsilon}}, P_{F,t} = \left[\int_0^1 P_{i,t}^{1-\zeta} di\right]^{\frac{1}{1-\zeta}} \tag{13}$$

[18]To clarify the notation: $P_{H,t}$ is the price index for domestic products, $P_{F,t}$ is the price index for *imported* goods, and $P_{i,t}$ is the price index for imported goods from country i. When we use an additional superscript i (e.g. $P^i_{H,t}$) we are talking about the same indices from country i's perspective. See Galí and Monacelli (2005) for details.

leading to the domestic CPI:

$$P_t \equiv \left[(1-\alpha) P_{H,t}^{(1-\varpi)} + \alpha P_{F,t}^{(1-\varpi)}\right]^{\frac{1}{1-\varpi}}. \tag{14}$$

Following the preceding equations optimal allocation of any given expenditure yields the following demand equations

$$\begin{aligned} C_{H,t}(j) &= (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\varpi} C_t(j), \\ C_{F,t}(j) &= \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\varpi} C_t(j), C_{i,t}(j) = \left(\frac{P_{i,t}}{P_{F,t}}\right)^{-\zeta} C_{F,t}(j). \end{aligned} \tag{15}$$

3.1.2 Some helpful Definitions and Identities

Before proceeding, we offer some helpful definitions and identities used extensively in the following sections.

We start by defining different types of terms of trade relationships. The bilateral terms of trade between the domestic country and country i represents the price of country i's goods in terms of domestic goods and is given by $S_{i,t} = P_{i,t}/P_{H,t}$. Thus the effective terms of trade is given by

$$S_t = \frac{P_{F,t}}{P_{H,t}} = \left(\int_0^1 S_{i,t}^{1-\zeta} di\right)^{\frac{1}{1-\zeta}}, \tag{16}$$

which can be approximated by $s_t \equiv \log(S_t) \approx \int_0^1 s_{i,t} di$. Log-linearizing the domestic CPI under the assumption of a symmetric steady state satisfying the PPP yields

$$p_t = p_{H,t} + \alpha s_t. \tag{17}$$

Using the definition of the domestic PPI, there is a link to the domestic CPI:

$$\pi_t = \pi_{H,t} + \alpha \triangle s_t. \tag{18}$$

We see that the gap between producer and consumer price inflation is proportional to the change in the terms of trade, depending on the openness of the country.[19]

Assuming that the LOOP holds on a brand level, we obtain $P_{i,t}(k) = \Xi_t^i P_{i,t}^i(k)$ $(\forall i, k \in [0,1])$, where $P_{i,t}^i(k)$ represents the price of good k from country i measured in terms of country i's currency. Integration over all products k yields $P_{i,t} = \Xi_t^i P_{i,t}^i$ $(\forall i, k \in [0,1])$, where $P_{i,t}^i \equiv \left[\int_0^1 P_{i,t}^i(k)^{1-\varepsilon} dk\right]^{\frac{1}{1-\varepsilon}}$ represents the composite price index of goods from country i measured in country i's currency. A log-linearization of $P_{F,t}$ around a symmetric steady state gives

$$p_{F,t} = \int_0^1 \left(e_t^i + p_{i,t}^i\right) di = e_t + p_t^*, \tag{19}$$

[19] For $\alpha = 0$ we derive the closed economy version and consumer and producer prices coincide.

where p_t^* represents the log world price index and $e \equiv \log(\Xi)$.[20] Using this with the definition of the terms of trade gives $s_t = e_t + p_t^* - p_{H,t}$. Since the foreign PPI measured in foreign currency units is given by $P_{F,t}^* = \left[\int_0^1 \left(P_{i,t}^i\right)^{1-\zeta} di\right]^{\frac{1}{1-\zeta}}$, the assumption of the LOOP on a brand level, combined with identical preferences and the assumption of no home bias yields the PPP:

$$P_{F,t} = \Xi_t P_{F,t}^*, P_{H,t} = \Xi_t P_{H,t}^*, P_t = \Xi_t P_t^*, \tag{20}$$

where $P_{H,t}^*$ is defined in the same way as $P_{F,t}^*$ and represents the domestic PPI measured in foreign currency units.

3.1.3 International Risk Sharing and the UIP

Assuming complete securities markets gross returns across countries should be equal and first order conditions similar to those of the domestic country should hold in any country, leading to

$$C_t = \varkappa_i C_t^i \Upsilon_{i,t}, \forall t \tag{21}$$

where $\Upsilon_{i,t} = \frac{\Xi_t^i P_t^i}{P_t}$ represents effective bilateral real exchange rate and $\varkappa_i$ is a constant depending on initial conditions. Assuming (without loss of generality) symmetric initial conditions (implying zero net foreign assets) $\varkappa_i = \varkappa = 1$. Taking logs and using the relationship between the terms of trade and the real exchange rate, we obtain

$$c_t = c_t^* + (1 - \alpha) s_t. \tag{22}$$

To exclude arbitrage the nominal gross-return $(1 + r_t)$ on a safe one-period bond paying off one unit of currency in period $t + 1$ must be defined as

$$(1 + r_t) E_t \{F_{t,t+1}\} = 1. \tag{23}$$

Assuming complete financial markets an analogous condition must be fulfilled in any foreign country:

$$\left(1 + r_t^i\right) E_t \left\{\frac{F_{t,t+1}\Xi_t^i}{\Xi_{t+1}^i}\right\} = 1. \tag{24}$$

Defining the nominal effective exchange rate as $\Xi_t = \int_0^1 \Xi_t^i di$, and combining (23) and (24), we get the UIP

$$E_t \left\{F_{t,t+1} \left[(1 + r_t) - (1 + r_t^*) \frac{\Xi_t}{\Xi_{t+1}}\right]\right\} = 0, \tag{25}$$

where r_t^* represents the interest rate for the rest of the world, i.e. $(1 + r_t^*) \equiv \int_0^1 (1 + r_t^i) di$. Log-linearizing (25) yields

$$r_t - r_t^* = E_t (\triangle e_{t+1}), \tag{26}$$

[20] Note that world CPI and PPI are the same as we assume that each country is of measure zero.

which combined with the definition of the terms of trade directly leads to

$$s_t = E_t s_{t+1} + \left(r_t^* - E_t \pi_{t+1}^*\right) - \left(r_t - E_t \pi_{H,t+1}\right). \tag{27}$$

Galí and Monacelli (2005) point out that the UIP follows from international risk sharing and can thus also be derived by combining the risk sharing condition with domestic and world Euler equations.[21]

Furthermore, (23) and (6) lead to the conventional relationship between consumption growth and the interest rate

$$\widetilde{\beta} \equiv \frac{1}{1+r_t} = \beta E_t \left[\left(\frac{P_t}{P_{t+1}} \right) \left(\frac{C_t(j)}{C_{t+1}(j)} \right) \right]. \tag{28}$$

To rule out the possibility of Ponzi schemes we assume that the present value of financial wealth conditional on survival converges to zero:

$$\lim_{k\to\infty} E_t \left\{ F_{t,t+k} (1-\gamma)^k \Omega_{t+k}(j) \right\} = 0. \tag{29}$$

Using (1) and (7) the budget constraint can be modified to a stochastic difference equation in financial wealth

$$P_t C_t(j) + E_t \left\{ F_{t,t+1} (1-\gamma) \Omega_{t+1}(j) \right\} = W_t N_t(j) - P_t T_t + \Omega_t(j). \tag{30}$$

Note that household optimization also implies that the equilibrium discount factor is given by

$$F_{t,t+k} = \beta^k E_t \left[\frac{P_t C_t(j)}{P_{t+k} C_{t+k}(j)} \right] = \prod_{i=0}^{k-1} F_{t+i,t+i+1}. \tag{31}$$

Using this with the no Ponzi condition, the definition of human wealth and the difference equation in financial wealth leads to

$$P_t C_t(j) = \left[1 - \beta(1-\gamma)\right] \left(\Omega_t(j) + H_t(j)\right), \tag{32}$$

where $[1-\beta(1-\gamma)]$ represents the reciprocal of the propensity to consume out of financial and human wealth.[22]

3.1.4 Aggregation Across Cohorts

The aggregate per-capita level of a specific variable G_t is given by the average of all generations, weighted with the cohort sizes: $G_t = \sum_{j=-\infty}^{t} \gamma (1-\gamma)^{t-j} G_t(j)$. We first determine aggregate consumption and production.

[21] The appendix of Galí and Monacelli (2005) demonstrates that (27) can be solved forward, and the terms of trade are a function of current and anticipated real interest rate differentials: $s_t = E_t \sum_{k=0}^{\infty} \left[\left(r_{t+k}^* - E_t \pi_{t+k+1}^*\right) - \left(r_{t+k} - E_t \pi_{H,t+k+1}\right) \right]$.

[22] Solving for consumption follows exactly Nisticó (2005) and Piergallini (2006).

Aggregate Consumption and Production The intratemporal conditions (15) yield

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\varpi} C_t, C_{F,t} = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\varpi} C_t, C_{i,t} = \left(\frac{P_{i,t}}{P_{F,t}}\right)^{-\zeta} C_{F,t}. \quad (33)$$

where C_t can be derived from (8):

$$C_t \equiv \left[(1-\alpha)^{\frac{1}{\varpi}} (C_{H,t})^{\frac{\varpi-1}{\varpi}} + \alpha^{\frac{1}{\varpi}} (C_{F,t})^{\frac{\varpi-1}{\varpi}}\right]^{\frac{\varpi}{\varpi-1}} \quad (34)$$

Note here that the assumption of international risk-sharing implies that $C_t = C_t^i$ $(\forall i)$ equals world per-capita consumption.[23]

Aggregate FOCs Since all equilibrium conditions are linear in cohort-specific variables, the aggregate conditions are simply given by

$$\begin{aligned} \frac{C_t}{(1-N_t)} &= \frac{W_t}{P_t}, & (35)\\ P_t C_t + E_t\{F_{t,t+1}\Omega_{t+1}\} &= W_t N_t - P_t T_t + \Omega_t, & (36)\\ Q_t(k) &= E_t\{F_{t,t+1}[Q_{t+1}(k) + D_{t+1}(k)]\}, & (37)\\ P_t C_t &= [1-\beta(1-\gamma)](\Omega_t + H_t), & (38)\end{aligned}$$

with aggregate nominal financial wealth given by

$$\Omega_t \equiv B_t + \int_0^1 \Xi_t^i B_t^i di + \int_0^1 (Q_t(k) + D_t(k)) Z_t(k) dk. \quad (39)$$

We cancel out the probability of dying with the logical argument that the gross return on the insurance contract must equal one, since it has only redistributive effects.

Combining (36) and (37) describes the dynamic patterns of aggregate consumption, where the effect of financial wealth fades out as the probability of exiting the market goes to zero:

$$\beta P_t C_t = \frac{\gamma}{(1-\gamma)} E_t\{F_{t,t+1}\Omega_{t+1}\} + [1-\beta(1-\gamma)] E_t\{F_{t,t+1}P_{t+1}C_{t+1}\}. \quad (40)$$

3.2 Firms

3.2.1 Technology

For analytical tractability, we assume that a typical domestic firm produces a differentiated good k with a linear production function

$$Y_t(k) = A_t N_t(k), \quad (41)$$

[23]See the appendix of Di Giorgio and Nisticó (2007) for details.

where $A_t \equiv \exp(a_t)$ represents a labor-augmenting productivity shock.[24] Thus, cost minimizing leads to the following expression for firm's real marginal costs:

$$MC_t = (1-\vartheta)\frac{W_t}{P_{H,t}A_t}\exp(\mu_t^p), \tag{42}$$

where ϑ represents a subsidy optimally chosen by the government to correct the monopolistic distortion and μ_t^p is a cost-push shock on marginal costs.[25]

Using brand-specific demand functions and aggregating across domestic brands leads to

$$Y_t \wp_t = A_t N_t, \tag{43}$$

where $\wp_t \equiv \int_0^1 \left(\frac{P_{H,t}(k)}{P_{H,t}}\right)^{-\varepsilon} dk$ measures the relative price dispersion among domestic firms and $N_t = \int_0^1 N_t(k)\,dk$ represents the aggregate per-capita amount of hours worked, which equals labor supply.

3.2.2 Price Setting

Monopolistic firms are assumed to set prices in a Calvo-staggered manner. Calvo-pricing is a parable to account for persistence in inflation. A randomly selected fraction of firms $(1-\theta)$ adjusts prices while the remaining fraction of firms θ does not adjust. In addition, a fraction of $(1-\tau)$ firms behaves in a forward-looking way and the remaining fraction τ uses the recent history of the aggregate price index when they set prices. Thus τ is a measure of the degree of backward-looking price-setting.[26]

For the forward-looking fraction the optimization problem of a firm k has the same form as in Calvo (1983),

$$\max_{\{P_{H,t}(k)\}} E_t \sum_{i=0}^{\infty} \theta^i \triangle_{i,t+i}\left[\frac{P_{H,t}(k)}{P_{H,t+i}}Y_{t+i}(k) - MC_{t+i}Y_{t+i}(k)\right], \tag{44}$$

and yields

$$\left(\frac{p_{H,t}^{fl}}{P_{H,t}}\right) = (1+\mu)\frac{E_t\sum_{i=0}^{\infty}\theta^i\beta^i C_{t+i}Y_{t+i}MC_{t+i}\frac{P_{t+i}}{P_{H,t}}}{E_t\sum_{i=0}^{\infty}\omega^i\beta^i C_{t+i}Y_{t+i}\frac{P_{t+i}}{P_{H,t+i}}}, \tag{45}$$

where $(1+\mu) = \left(\frac{\varepsilon}{\varepsilon-1}\right)$ represents the steady-state markup over a weighted average of expected marginal costs and $\triangle_{i,t+i}$ represents the real discount factor.

Defining the domestic index for the prices newly set in period t $(\overline{p}_{H,t}^n)$ as a weighted average of the forward- and backward-looking prices, and assuming a rule of thumb for the backward-looking price-setters,

$$p_{H,t}^{bl} = \overline{p}_{H,t-1}^n + \pi_{H,t-1}, \tag{46}$$

[24] Following standard practice in New Keynesian models, we do not consider capital (and therefore demand for investment goods). See, for example Clarida et al. (1999).

[25] See Galí (2003) for details.

[26] See Galí and Gertler (1999) for details.

the following equations can be derived using some straightforward algebra:[27]

$$P_{H,t} = \left[\omega P_{H,t-1}^{1-\theta} + (1-\omega)\left(\overline{p}_{H,t}^{n}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}. \tag{47}$$

$$\overline{p}_{H,t}^{n} = (1-\tau)\, p_{H,t}^{fl} + \tau p_{H,t}^{bl}, \tag{48}$$

where $\pi_{H,t} = \ln\left(\frac{P_{H,t}}{P_{H,t-1}}\right)$ represents the domestic producer price inflation.

3.3 Equilibrium

A virtue of the model presented here is its tractability, which benefits both analysis and clarity of intuition. As the model can be solved for an infinite number of steady states, we restrict the initial equilibrium. Without loss of generality, we assume a baseline symmetric steady state with equal and constant consumption and a real exchange rate of value one. Together with the PPP, this also implies a value of one for terms of trade. In what follows, lower-case letters are used for describing percentage deviations from equilibrium values of their upper-case counterparts.

3.3.1 The Demand-Side

Goods market clearing for each good k of the domestic small open economy requires

$$\begin{aligned} Y_t(k) &= C_{H,t}(k) + \int_0^1 C_{H,t}^{i}(k)\, di \\ &= \left(\frac{P_{H,t}(k)}{P_{H,t}}\right)^{-\varepsilon} \left[\begin{array}{c} (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\varpi} C_t \\ +\alpha \int_0^1 \left(\frac{P_{H,t}}{\Xi_t^i P_{F,t}^i}\right)^{-1} \left(\frac{P_{F,t}^i}{P_t^i}\right)^{-\varpi} C_t^i di \end{array}\right]. \end{aligned} \tag{49}$$

for all $k \in [0,1]$ and all t, where $C_{H,t}^{i}(k)$ represents country i's demand for the domestic good k.[28] Using this with the definition of aggregate output gives[29]

$$Y_t = C_t S_t^{\alpha}\left[(1-\alpha) + \alpha \int_0^1 \left(S_t^i S_{i,t}\right)^{\zeta-\varpi} di\right], \tag{50}$$

where S_t^i represents the effective terms of trade of country i ($P_{F,t}^i / P_{H,t}^i$). Following $\int_0^1 s_t^i di = 0$, we get an approximation around a symmetric steady state

$$y_t = c_t + \alpha\omega s_t, \tag{51}$$

where $\omega = \zeta + (1-\alpha)(\varpi - 1)$. Since an analogous condition holds for each country in the world, it follows that $y_t^* = c_t^*$. Furthermore, international risk sharing implies

$$y_t = y_t^* + \frac{1}{\sigma_\alpha} s_t, \tag{52}$$

[27] See Galí and Gertler (1999) for details.

[28] For the derivation of (49), we used (33) and the assumption of symmetric preferences across countries, leading to $C_{H,t}^{i}(k) = \alpha\left(\frac{P_{H,t}(k)}{P_{H,t}^{i}}\right)^{-\varepsilon}\left(\frac{P_{H,t}}{\Xi_t^i P_{F,t}^i}\right)^{-1}\left(\frac{P_{F,t}^i}{P_t^i}\right)^{-\varpi} C_t^i$.

[29] See Galí and Monacelli (2005) for a detailed derivation.

where $\sigma_\alpha = \frac{1}{(1-\alpha)+\alpha\omega}$.

The indices of average real dividend payments and the average stock market capitalization are

$$D_t \equiv \frac{1}{P_t}\int_0^1 D_t(k)\,dk, Q_t \equiv \frac{1}{P_t}\int_0^1 Q_t(k)\,dk. \tag{53}$$

Under the assumption of a balanced world government budget, the equilibrium aggregate world net supply of state-contingent bonds must equal zero. However, in a open economy model, we can approximate around any given distribution of bonds across countries, and thus assume without loss of generality that bond holdings of domestic and foreign households are zero in the initial equilibrium: $B_t + \int_0^1 \Xi_t^i B_t^i di = \frac{1}{\Xi_{t+1}^i} B_{t+1}^{(i,H)} + \int_0^1 \Xi_{t+1}^{(i,l)} B_{t+1}^{(i,l)} dl = 0$. Using this condition with (38), the equilibrium present discounted nominal value of future aggregate nominal financial wealth is equal to the current level of domestic nominal stock-prices:

$$E_t\left\{F_{t,t+1}\Omega_{t+1}\right\} = \int_0^1 Q_t(k)\,dk = Q_t, \tag{54}$$

where we used the intertemporal condition (7) and the logical argument, that $\int_0^1 \Xi_{t+1}^{(i,l)} dl = 1$.[30]

Moreover, the demand side is constrained by

$$D_t = \frac{P_{H,t}}{P_t} Y_t - (1-\vartheta)\frac{W_t}{P_t} N_t. \tag{55}$$

Let Π_t denote the domestic gross inflation rate. Integrating over all firms, we thus get the demand side of the domestic economy, characterized by the following Euler equation:

$$\beta C_t = \frac{\gamma}{(1-\gamma)} Q_t + [1-\beta(1-\gamma)]\, E_t\left\{F_{t,t+1}\Pi_{t+1}C_{t+1}\right\}, \tag{56}$$

and

$$Q_t = E_t\left\{F_{t,t+1}\Pi_{t+1}\left[Q_{t+1}+D_{t+1}\right]\right\}. \tag{57}$$

Furthermore, using $E(xy) = E(x)E(y) + Cov(x,y)$ we derive

$$Q_t = E_t\left\{F_{t,t+1}\right\} E_t\left\{\Pi_{t+1}\left[Q_{t+1}+D_{t+1}\right]\right\} - Q_t\epsilon, \tag{58}$$

where ϵ represents the conditional covariance between the stochastic discount factor and the nominal gross return rate on stocks. This defines the following risk premium:[31]

$$EP_t \equiv E_t\left\{\Pi_{t+1}\left[\frac{Q_{t+1}+D_{t+1}}{Q_t}\right]\right\} - (1+r_t) = (1+r_t)\epsilon. \tag{59}$$

[30] The domestic country is a small country of measure zero, so the bilateral exchange rates in the rest of the world necessarily sum up to one.

[31] In contrast to Nisticó (2005), we do not assume a time varying covariance. Thus, we add a shock representing all non-fundamental factors when estimating the model to improve the estimation of the stock price gap equation. We return to this issue in section 4.

Log-linearizing the demand side of the model gives

$$\begin{aligned} y_t &= \frac{1}{1+\Psi}E_t y_{t+1} + \frac{\Psi}{1+\Psi}q_t - \frac{1}{1+\Psi}\left(r_t - E_t\pi_{t+1} - \rho\right) \\ &\quad -\frac{\alpha\omega}{1+\Psi}E_t\left(\triangle s_{t+1}\right), \qquad (60) \\ w_t - p_{H,t} &= c_t + \varphi n_t, \qquad (61) \\ q_t &= \frac{\widetilde{\beta}}{1+\epsilon}E_t q_{t+1} + \frac{1+\epsilon-\widetilde{\beta}}{1+\epsilon}E_t d_{t+1} - \left(r_t - E_t\pi_{t+1} - \rho\right), \qquad (62) \\ d_t &= \frac{Y}{D}\left(p_{H,t} - p_t + y_t\right) - \frac{WN}{PD}\left(n_t + w_t - p_t\right), \qquad (63) \end{aligned}$$

where $\varphi \equiv \frac{N}{1-N}$ is the inverse of the steady state Frisch elasticity of labor supply, $\Psi \equiv \gamma\frac{1-\beta(1-\gamma)}{1-\gamma}\frac{\Omega}{C}$, and $\rho \equiv \log(\beta)$.

3.3.2 The Supply-Side

Equilibrium in the labor market and the definition of the terms of trade implies

$$MC_t = \frac{C_t P_t \exp(\mu_t^p)}{\left(A_t - Y_t\wp_t\right)P_{H,t}}. \qquad (64)$$

Log-linearizing this by using the assumption of a linear production technology, the optimal labor-leisure decision of the households and (42), equilibrium real marginal costs are given by

$$\begin{aligned} \log\left(MC_t\right) &= y_t^* + \varphi y_t - \left(1+\varphi\right)a_t + s_t + \mu_t^p \qquad (65) \\ &= \left(\sigma_a + \varphi\right)y_t + \left(1-\sigma_a\right)y_t^* - \left(1+\varphi\right)a_t + \mu_t^p. \qquad (66) \end{aligned}$$

In a flexible-price equilibrium all firms set their prices equal to a constant mark-up over marginal costs μ. Thus marginal costs reach their long-run level at each point in time ($mc_t^n = 0$) and the flexible-price equilibrium level of output follows directly from the preceding equation:

$$y_t^n = \Gamma_a a_t - \alpha\Gamma_{y^*}y_t^*, \qquad (67)$$

where $\Gamma_a \equiv \frac{(1+\varphi)}{(\sigma_\alpha+\varphi)}$, $\Gamma_{y^*} = \frac{\Theta\sigma_\alpha}{(\sigma_\alpha+\varphi)}$ and $\Theta \equiv (\zeta-1) + (1-\alpha)(\varpi-1)$. This links real marginal costs with the output gap $x_t \equiv (y_t - y_t^n)$:

$$mc_t = \left(\sigma_\alpha + \varphi\right)x_t + \mu_t^p. \qquad (68)$$

Log-linearizing the price-setting equations (46), (47) and (48) gives the conventional hybrid New Keynesian Phillips curve:

$$\pi_{H,t} = \phi\left(\theta\widetilde{\beta}E_t\pi_{H,t+1} + \tau\pi_{H,t-1}\right) + \lambda mc_t + \varepsilon_t^\mu, \qquad (69)$$

where $\lambda = (1-\tau)(1-\theta)\left(1-\widetilde{\beta}\theta\right)\phi$, $\phi = \left(\theta + \tau\left[1-\theta\left(1-\widetilde{\beta}\right)\right]\right)^{-1}$ and $\varepsilon_t^\mu = \lambda\mu_t^p$.

3.4 The Natural Rate and the Canonical Representation

Exchange rate stability is enshrined as an important objective of monetary policy in Hong Kong. Assuming an exchange rate peg, monetary policy is implicitly given by (the interest rate adjusts endogenously to fulfill the UIP)

$$e_t = \rho_e e_{t-1} + \varepsilon_t^e, \varepsilon_t^e \sim N\left(0, \sigma_e^2\right), \tag{70}$$

where ε_t^e represents an i.i.d. shock on the exchange rate, reflecting variations that are exogenously with respect to our model. As usual in New Keynesian models, we can summarize the model by a few equations, representing percentage deviations of the state variables from their steady state values. The reduced system is given by[32]

$$x_t = \frac{\sigma_\alpha}{\Gamma_0} E_t x_{t+1} + \frac{\Psi}{\Gamma_0}\widehat{q}_t - \frac{1}{\Gamma_0}\left(r_t - E_t \pi_{H,t+1} - rr_t^n\right), \tag{71}$$

$$\widehat{q}_t = \frac{\widetilde{\beta}}{1+\epsilon} E_t \widehat{q}_{t+1} - \frac{\lambda_q}{1+\epsilon} E_t x_{t+1} - \left(r_t - E_t \pi_{H,t+1} - rr_t^n\right) + \eta_t, \tag{72}$$

$$\pi_{H,t} = \phi\left(\theta\widetilde{\beta} E_t \pi_{H,t+1} + \tau \pi_{H,t-1}\right) + \kappa_\alpha x_t + \varepsilon_t^\mu, \tag{73}$$

where $\widehat{q}_t \equiv q_t - q_t^n$, $\Psi \equiv \gamma \frac{1-\beta(1-\gamma)}{1-\gamma}\frac{\Omega}{C}$, $\Gamma_0 \equiv 1 + \Psi - \alpha\Theta\sigma_\alpha$, $\kappa_\alpha \equiv \lambda\left(\sigma_\alpha + \varphi\right)$, $\lambda_q \equiv \left(1+\epsilon-\widetilde{\beta}\right)\left(\frac{1+\varphi-\mu}{\mu} + \alpha\sigma_\alpha\right)$, and η_t is a shock accounting for all non-fundamental movements in the stock price gap. Furthermore, the natural rate is given by[33]

$$\begin{aligned} rr_t^n \equiv\ & \rho + \left[\sigma_\alpha \rho_a + \Psi - \Gamma_0\right]\Gamma_a a_t \\ & + \left[\left(\sigma_\alpha \rho_{y^*} + \Psi - \Gamma_0\right)\Gamma_{y^*} + \Theta\sigma_\alpha\left(\rho_{y^*} - 1\right)\right]\alpha y_t^*, \end{aligned} \tag{74}$$

where we assumed AR(1)-processes for a_t, y_t^* and η_t,

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a, \tag{75}$$

$$y_t^* = \rho_a y_{t-1}^* + \varepsilon_t^{y^*} \tag{76}$$

$$\eta_t = \rho_\eta \eta_{t-1} + \varepsilon_t^\eta, \tag{77}$$

and the ε^is are normally distributed with constant mean and variance σ_i. Finally, following Nisticó (2005), the natural stock price index is given by $q_t^n = y_t^n$.

The reduced-form of the DSGE model is founded on structural parameters that describe the optimal behavior of firms and households. These parameters can be considered independent of the policy regime, and thereby not subject to the Lucas critique.

[32]See appendix for details.

[33]Note that the natural rate rr_t^n is the interest rate compatible with both inflation and output stabilization at their efficient level (flexible-price allocation). For any interest rate rule of the form $r_t = rr_t^n$ the model is undetermined, and for any rule of the form $r_t = rr_t^n + \phi_\pi \pi_t + \phi_x x_t$ the mere threat of reacting to deviations from equilibrium would suffices to keep output and inflation at their efficient level. As we focus on an exchange rate peg, we do not discuss this issue further. Details can be found in e.g. Galí (2008).

4 Estimation and Model Fit

Having a new general equilibrium model for Hong Kong is important in its own right, but it is also important to investigate the mapping of the model into reality using all available data. We use a combination of less formal calibration and estimation in parameterizing the model.

The parameters of the DSGE model are estimated by Bayesian methods, conditional on prior information concerning the values of parameters. Following standard practice, we have removed trend components from the observed macroeconomic variables using the HP-filter.[34] Striking advances have been made in recent years in estimating DSGE models, shifting emphasis in quantitative macroeconomics from calibration exercises to directly estimating the parameters of a structural model and letting the data speak. The Bayesian technique – as forcefully claimed by An and Schorfheide (2007) and Fernández-Villaverde (2009) – is now the standard tool for estimation of DSGE models. Linear approximation methods lead to a state-space representation of the DSGE model that can be analyzed with a Kalman filter. Together with the specification of prior distributions for the parameters, the state-space representation can be translated to form the posterior distribution. Bayesian estimation looks for the parameter that maximizes the posterior, given the prior and the likelihood based on the data. In order to obtain numerically a sequence from the unknown posterior distribution, we employ the Metropolis-Hastings Markov-Chain Monte Carlo algorithm with 50.000 draws. We use the Dynare preprocessor for Matlab in computation.

For the estimation of the model (18), (52), (67), (71) - (77), $x_t = y_t - y_t^n$ and $\triangle(e) = \triangle(s) + \pi_H$ are used, where the last equation follows from the assumption of a small economy. We employ quarterly data for 5 observables for the sample 1981Q1-2007Q3: the real GDP of Hong Kong, the Hang Seng index, the nominal effective exchange rate, the consumer price index of Hong Kong and US GDP.[35] The last series is used as a proxy for foreign demand (y_t^*).

4.1 Calibration

A number of parameters remain fixed throughout the estimation procedure. This is because they are either notoriously difficult to estimate (in the case of substitution elasticities between home and foreign goods and the Frisch wage elasticity of labour supply) or they are better identified using other information. The discount factor β is set to 0.995, implying a nominal interest rate of 2 percent annually. In a

[34]As an alternative, a DSGE model incorporating deterministic or stochastic trends can be estimated. However, as discussed in Clements and Hendry (1999), intermittent structural breaks render such trends empirically inadequate representations of low-frequency variation in observed macroeconomic variables.

[35]It is worth noting that Genberg (2005) has found in a VAR framework that Hong Kong's output growth depends to a large extent on developments in the US.

similar vein, we assume a value of 0.01 for the steady state interest rate r. For the elasticity of substitution between domestic and foreign goods and the substitution elasticity between goods produced in different foreign countries, we choose $\varpi = \zeta = 1$. This is consistent with macro-estimates in the range of 1 to 2 in Collard and Dellas (2002) and the parameterization in Backus et al. (1992).[36] Following the RBC literature, the inverse of the (Frisch) wage elasticity of labour supply ϕ is set to 0.33.[37] The mark-up over expected marginal costs μ is hard to pin down. Below we assume $\mu = 0.1$. Finally, following Nisticó (2005) the steady-state consumption is set at 80 percent of total output and the conditional covariance between the stochastic discount factor and the nominal gross return rate on stocks is assumed to be 0.015.[38] The calibrated parameters are fed into the DSGE model, which is estimated subsequently.

4.2 DSGE Priors and Posterior Distributions

Table 4: Prior and Posterior Distributions of the Structural Parameters

	Prior Distribution			Posterior Distribution			
	Type	Mean	St. Dev.	Mode	St. Dev.	Conf. Int.	Stand. Coeff.
τ	*beta*	0.2	0.1	0.141	0.056	[0.054,0.225]	2.520
θ	*beta*	0.7	0.1	0.684	0.077	[0.560,0.807]	9.064
γ	*beta*	0.03	0.01	0.027	0.010	[0.014,0.046]	3.000
α	*beta*	0.4	0.05	0.141	0.019	[0.110,0.171]	7.420
ρ_a	*beta*	0.7	0.1	0.795	0.037	[0.734,0.855]	21.487
ρ_{y^*}	*beta*	0.7	0.1	0.810	0.045	[0.737,0.884]	18.000
ρ_e	*beta*	0.7	0.1	0.707	0.058	[0.612,0.803]	12.190
ρ_η	*beta*	0.7	0.1	0.775	0.071	[0.637,0.911]	10.920

Detailed descriptions of the prior distributions for the structural DSGE parameters and the shock parameters are given in Table 4 and 5. In selecting the prior distributions for the parameters to be estimated, we are guided by conventions in the literature. For the key parameter γ, we take the value 0.03 for the mean as used by Nisticó (2005) and which corresponds to a financial planning horizon of slightly more than 8 years in a quarterly model. Moreover, this prior is in line with the estimation of Milani (2008), who estimates a version of Nisticó (2005) and derives a planning horizon of 10 years. Given the volatility of inflation in Hong Kong, the

[36] In contrast, higher elasticities of substitution between domestic and foreign goods in the range of 5 to 10 are typically obtained using microdata. Obstfeld and Rogoff (2000) have shown that such high elasticities can explain an observed large home bias in trade.

[37] The value of ϕ has a limited effect on the dynamics of the DSGE model.

[38] We need a value for the steady-state consumption to pin down $\frac{\Omega}{C}$, which determines Ψ. It is easily shown that $\frac{\Omega}{C} = \frac{(1+r)(1+\epsilon)\mu}{(r+(1+r)\epsilon)\frac{C}{Y}}$. See Nisticó (2005) for details.

Table 5: Prior and Posterior Distributions of the Standard Deviations of the Shocks

	Prior Distribution			Posterior Distribution		
	Type	Mean	St. Dev.	Mode	St. Dev.	Conf. Int.
ε^a	*gamma*	2.000	2.000	2.749	0.203	[2.456,3.149]
ε^{y^*}	*gamma*	2.000	2.000	0.485	0.035	[0.432,0.555]
ε^e	*gamma*	2.000	2.000	1.875	0.135	[1.675,2.131]
ε^μ	*gamma*	2.000	2.000	0.610	0.060	[0.524,0.726]
ε^η	*gamma*	2.000	2.000	2.325	1.110	[1.332,5.696]

inflation persistence parameter τ is set at a rather low value of 0.2. Price rigidity is set to the standard value of 0.70, a value that is close to the estimates of Christiano et al. (2005). The degree of openness is set to 0.4, which seems at least plausible for a small open economy like Hong Kong. The persistence parameters for the AR(1) processes of a, y^*, e and η are all set to the standard value of 0.7 and the standard deviation of all shocks is set to 2. These assumptions ensure that the model is roughly consistent with the autocovariance patterns observed in the data. Since all parameters besides the standard deviation of shocks are bounded by the unit interval $[0, 1)$, we use Beta distributions. For the standard deviations we use inverse Gamma distributions as is the standard convention today.[39] We have also examined the sensitivity of results by considering the impact of different prior sizes and prior distributions.

The posterior distributions for the structural parameters are given in Table 4. To facilitate comparisons across variables, we have also divided the posterior mean coefficient by its posterior standard deviation. The posterior standardized coefficient associated with variable x_i allows to assess the relative size and significance of the effect of variable x_i in explaining the dependent variable.[40] In Figure 1 prior and posterior distributions for the parameters and shocks are given. Overall, all parameters seem to be well identified, as shown by the fact that the posterior distribution is not centered on the prior or it is centered but with a smaller dispersion indicating high significance of the estimates. On the price stickiness side, 68 percent of firms don't adjust prices within one quarter. This implies that prices are reoptimized once every three quarters. This is in line with the previous evidence of comparatively flexible prices in Hong Kong (see Yetman (2009)). The structural parameter τ representing the degree of backward-looking price setting is estimated to be 0.141, implying a small share of backward-looking firms. The autoregressive parameters are estimated in the range 0.707 - 0.810, pointing to a high persistence.[41]

[39]See, for example, Castelnuovo and Nisticó (2008). A thorough elicitation of priors is available in Del Negro and Schorfheide (2008).

[40]Brock and Durlauf (2001) have provided a decision-theoretic foundation for using such standardized coefficients in policy analysis.

[41]A widely applied method to assess the validity of estimated DSGE models is to compare them

Figure 1: Prior vs. Posterior Distributions in the Metropolis-Hastings Procedure

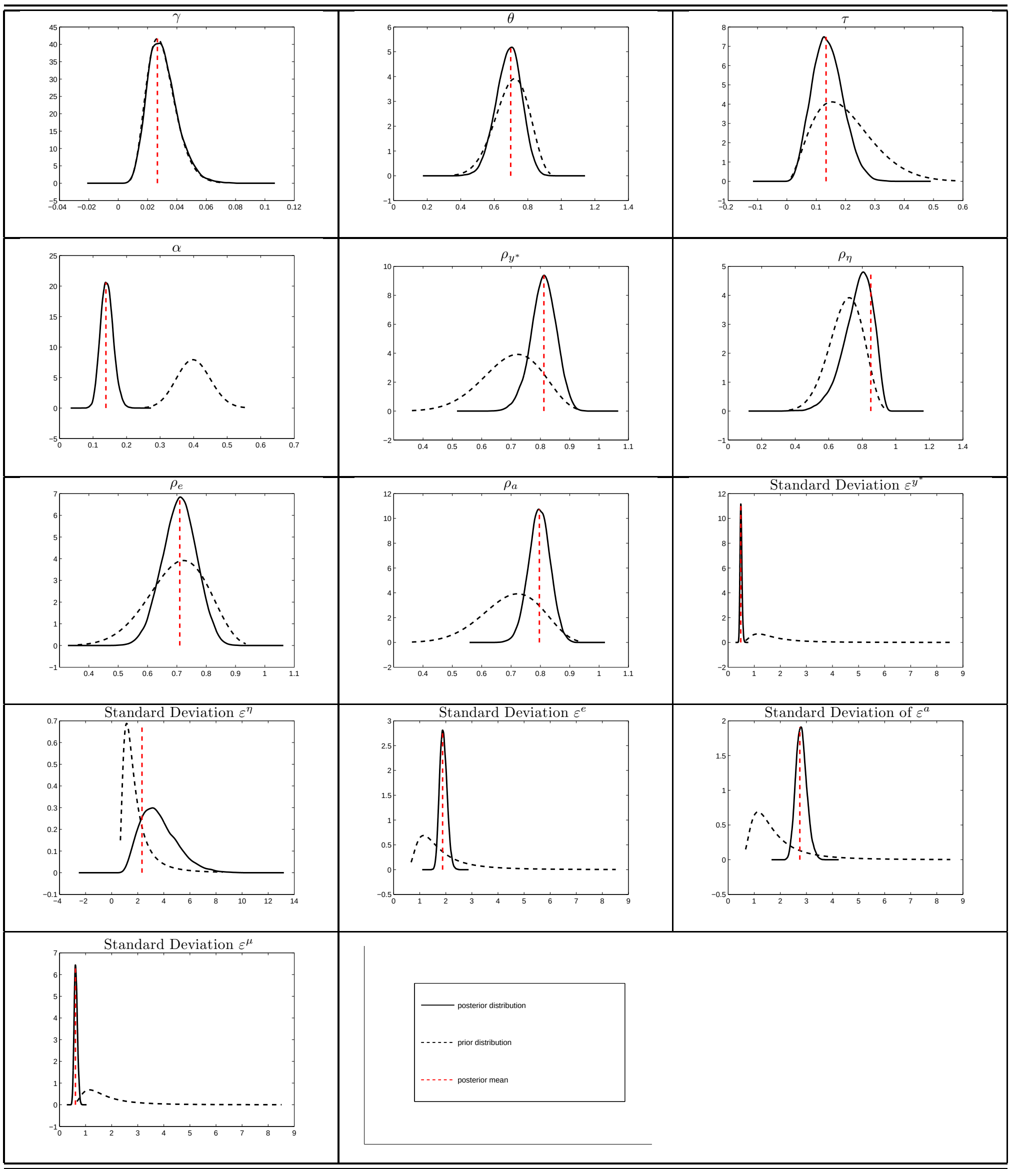
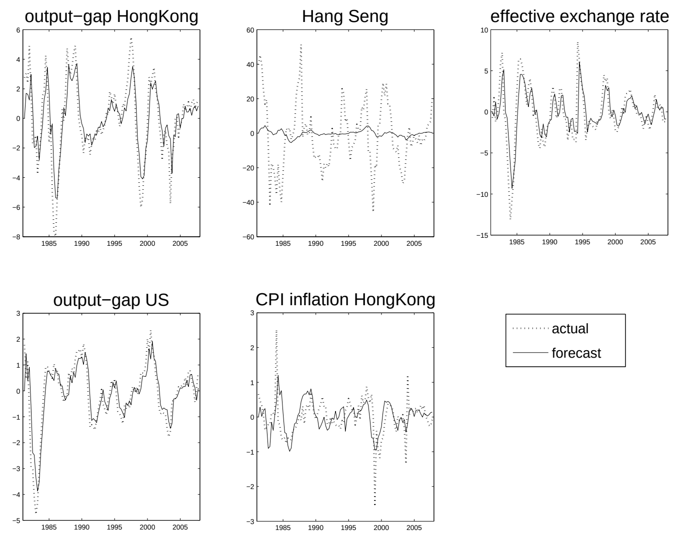


Table 5 summarizes the results on the shock parameters. This gives a first indication on what shocks seem to drive the cyclical variations in our macroeconomic time series. The shocks estimated to have the highest standard deviations are productivity shocks.

with non-structural VARs. Interestingly, Gerlach and Gerlach-Kristen (2006) have found in a semi-structural VAR an AR(1) coefficient for Hong Kong's output gap of 0.68. The estimates are thus quite similar.

4.3 Properties of the Estimated Model

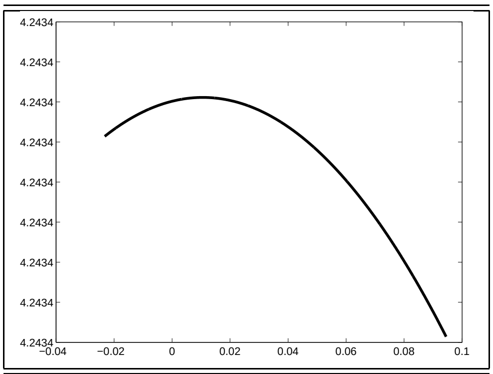
Figure 2: In-Sample One Step Ahead Predictions of the Estimated Model



In this section, we turn to the results from the model, once parameters are obtained from maximizing the posterior over the entire sample period. We do so in three steps. First, in Figure 2 we show the one-step ahead predictions of the model. Obviously, a prerequisite for an estimated DSGE model is a proper empirical representation of the data generating process. The overall impression is that the model is able to mimic the time series, i.e. the results indicate that the model can match the cyclical properties of the data very well. On a cautionary note, however, the model has difficulties accounting for the stock price dynamics and underpredicts the volatility of the Hang Seng index. Besides being difficult to forecast, share prices are far more volatile than the economic fundamentals that supposedly determine them. The high volatility of share prices relative to economic fundamentals is very difficult to replicate in a DSGE model without introducing arbitrary disturbances. The high value of the standard deviation of ε^{η} indicates that the Hang Seng index is also strongly driven by non-fundamental movements.

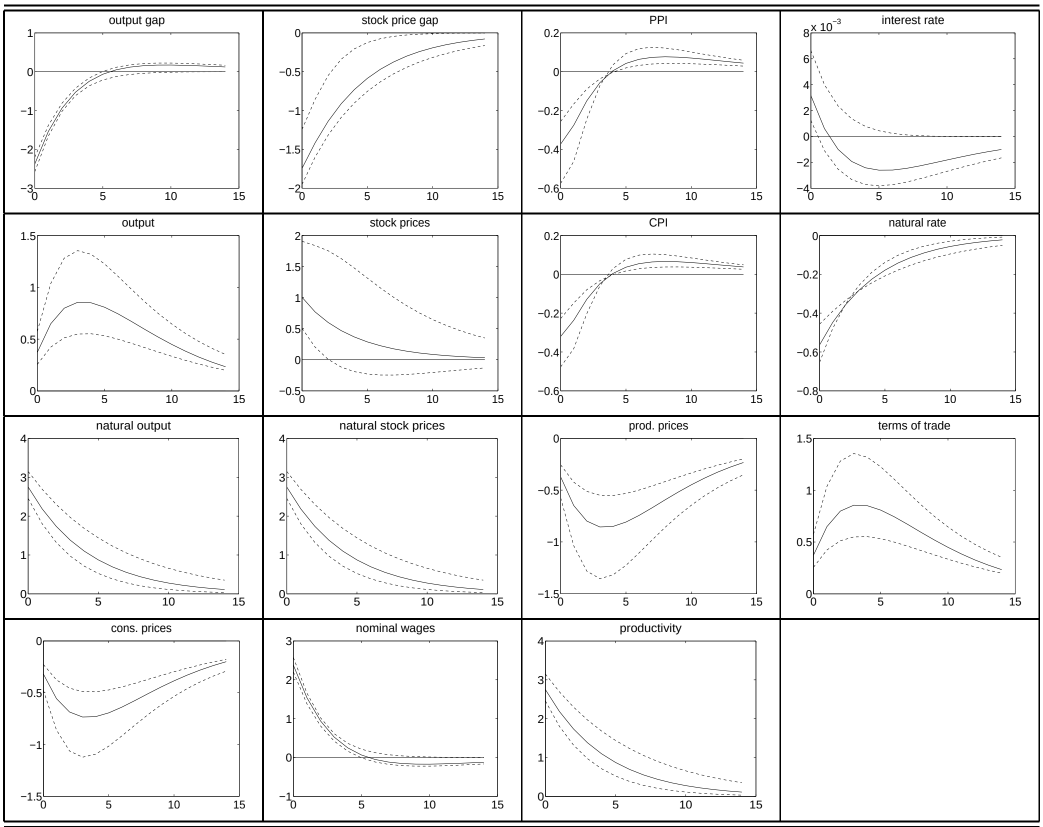
The Bayesian estimation results indicate that agents exhibit a positive elasticity of consumption to share prices ($\frac{\Psi}{\Gamma_0}$). How big are these spillovers from the stock market? Bayesian inference relies upon accurate characterization of the posterior. We have therefore drawn 400.000 draws from the posterior, and tested whether convergence occurred. The results based on the simulated posterior are presented in Figure 3. Overall, we think the test indicates chain convergence such that the numerically simulated posterior is a good approximation. From an economic point of view, the results indicate a small, but positive, wealth effect.

Figure 3: Posterior Density of the Wealth Effect ($\frac{\Psi}{\Gamma_0}$)



Having shown that the model fits the data reasonably well, we next elucidate how the variables react in response to the structural shocks. Figures 4 - 8 below single out the impulse response functions. We present these graphs as they nicely illustrate the functioning of our modeling toolkit.

Figure 4: Productivity Shock



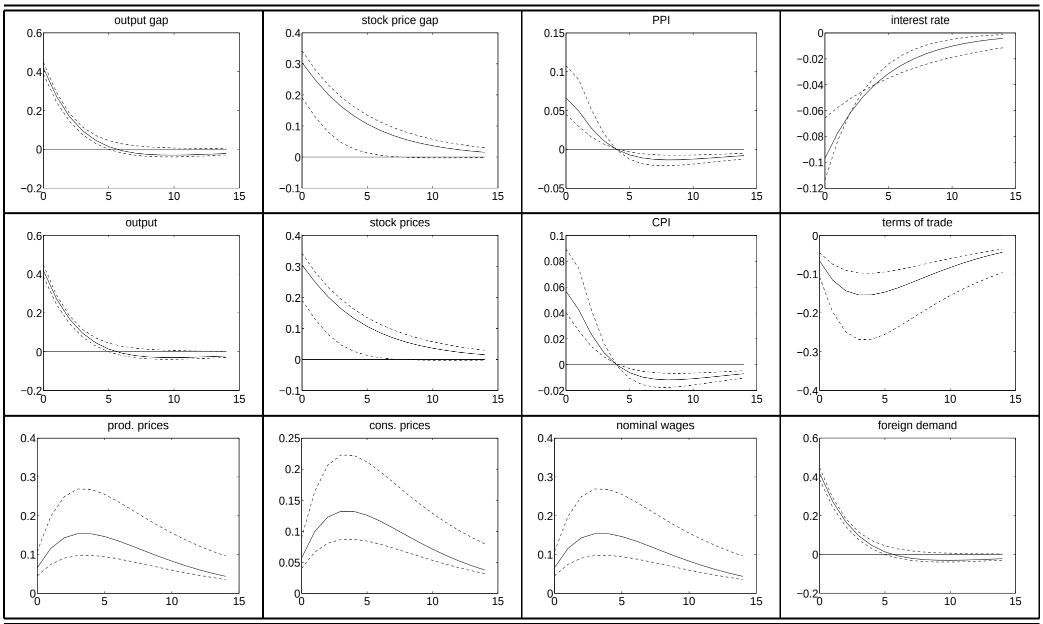
4.4 Impulse Response Functions

Productivity Shock In Figure 4 the impulse responses after a productivity shock are shown (the dashed lines represent the 95% interval), the shock of most interest in a DSGE framework. Apart from the effect of stock price dynamics on consumption and the restrictions on monetary policy due to the exchange rate peg, the responses show the typical characteristics (see e.g. Galí (2008)). An increase in the productivity index leads to an increase in natural output and, as a result, a negative output gap. The productivity shock reduces real marginal costs, enabling firms to lower producer prices. In addition, higher productivity allows firms to pay higher nominal wages.[42] Under a standard New Keynesian model, the central bank would move to accommodate for the improvement of technology by decreasing the interest rate. In the case of an exchange rate peg, however, the central bank's hands are tied. Initially, the interest rate reaction is tiny (in the range of 10^{-3}) and positive as it copes with the upward pressure on the exchange rate from the positive effect on the terms of trade.[43] In the medium term, actual output catches up and the output gap closes.

[42] Nominal wages are simulated using (35) with (41) and (51): $w = p + a - \alpha s$.

[43] This effect on the terms of trade stems from the diminishing producer prices in response to the technology improvement. This is shown in (22) and reflects an assumption of complete security markets.

Figure 5: Foreign Demand Shock

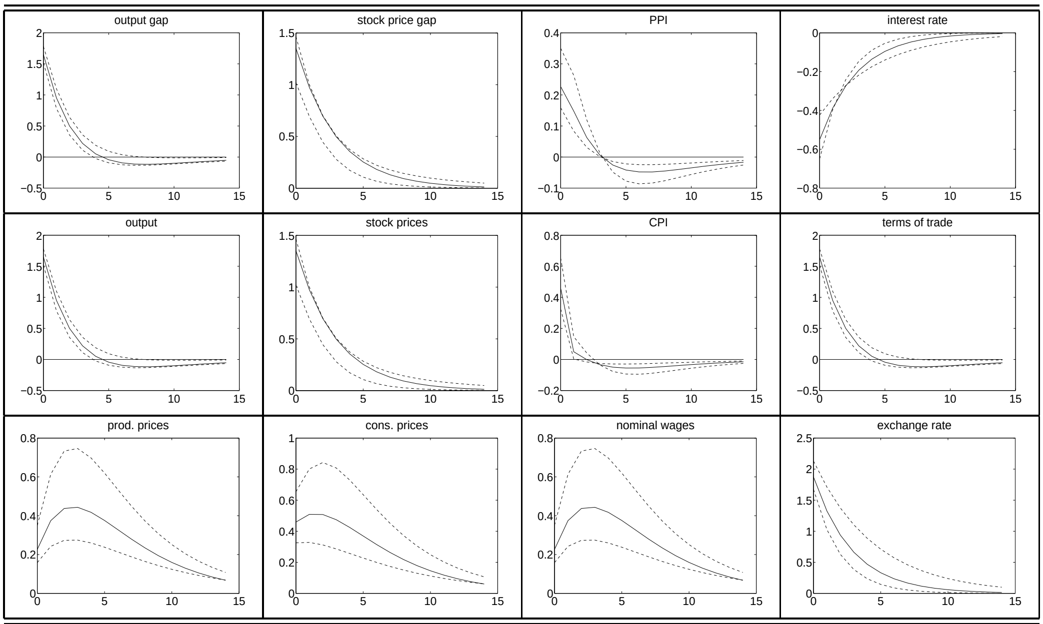


As the natural interest rate (the rate compatible with both output and inflation in equilibrium) declines, the positive "real interest rate gap" $(r_t - E_t\pi_{H,t+1} - rr_t^n)$ leads to a fall in the stock price gap. Comparable to the reaction of the output gap, the increase in productivity leads to an increase in the natural stock prices first, while the actual stock prices lag behind. As the shock diminishes, the interest rate (and all other variables) returns to its steady state.

Foreign Demand Shock The reactions after a foreign demand shock are given in Figure 5.[44] A shock to foreign output increases domestic output due to the boost in foreign demand. Since this is accompanied by a fall in marginal utility, firms seeking to boost production have to increase wages to achieve an increase in hours worked. As a result, real marginal costs and prices increase. The implied fall in the terms of trade (the increase in foreign demand must be due to an improvement in the terms of trade from the foreign perspective) puts downward pressure on the exchange rate, leading the central bank to cut the interest rate. This lower interest rate, in turn, leads to an increase in the output gap, the stock price gap, and the price indexes as shown in the figure.

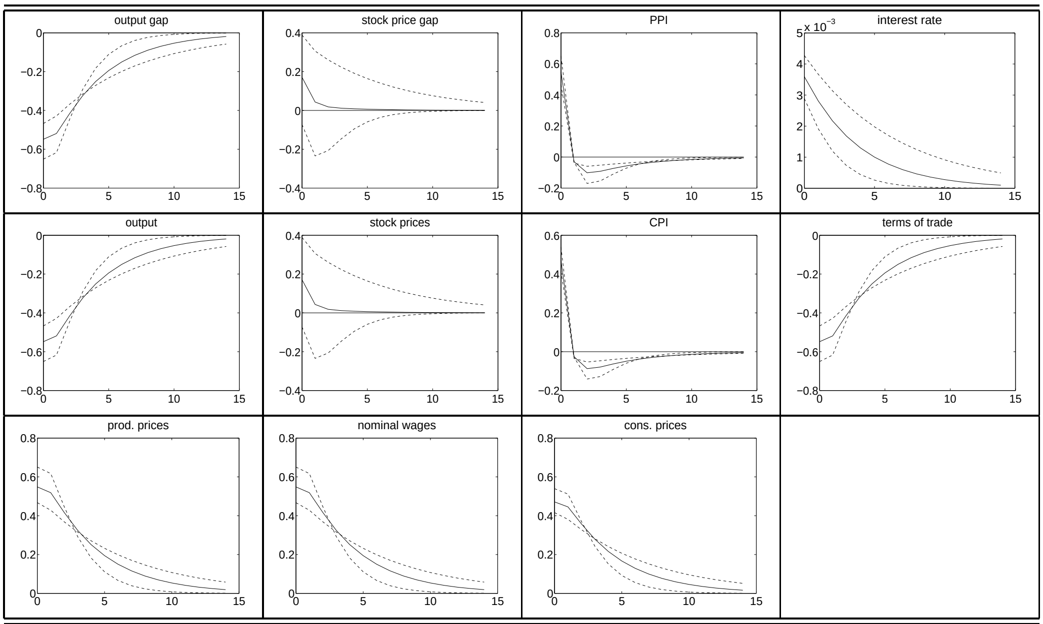
[44]Under the assumption $\varpi = \zeta = 1$, the foreign demand shock has no influence on natural output since all goods are assumed to be perfect substitutes for each other and $\Gamma_{y^*} = 0$. Thus, the effect of an increase in foreign demand has only an effect via the negative influence on the terms of trade.

Figure 6: Exchange Rate Shock



Exchange Rate Shock As the Hong Kong monetary authority follows an exchange rate peg, investigating the dynamics of the economy in response to an exchange rate shock is straightforward. In the underlying model an increase in the exchange rate must be accompanied by a fall in the interest rate, since a depreciation of the exchange rate can only be achieved by a positive interest rate differential and the foreign interest rate is exogenously given. From this perspective, the positive exchange rate shock implies the same responses as a negative monetary policy shock. As the natural values of output, stock prices and the interest rate are unaffected, the output gap equals actual output and stock price dynamics equal the dynamics of the stock price gap. The fall in interest rate generates an economic boom. To recruit more workers, firms must pay higher nominal wages and, as a consequence, prices go up . The terms of trade follow the exchange rate dynamics and reflect the lower import prices for foreign consumers. However, this effect is dampened by the increase in domestic prices. As the interest rate reaches its equilibrium value again, all variables return to their steady states.

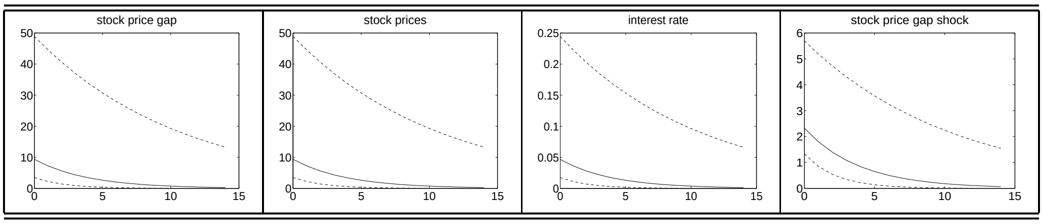
Figure 7: Cost Push Shock



Cost-push Shock The responses after a cost push shock are given in Figure 7. Again, the natural values of output, stock prices and the interest rate are unaffected. The increase in producer prices also affects consumer prices, leading to higher claims from workers and consequently higher nominal wages. In addition, the increase in prices is accompanied by a fall in the terms of trade. Since $\triangle e = \triangle s + \pi_H$, prices and terms of trade dynamics have opposing effects on the exchange rate. However, since the source of disturbance is the increase in the PPI, the cost-push shock generates an upward pressure on the exchange rate. Thus, the central bank needs to increase the interest rate to keep the exchange rate fixed. While the increase in the interest rate brings prices back into equilibrium, it is accompanied by a fall in output. The fall in the output gap has a positive effect on expected real dividends via the marginal cost channel, leading to an increase in stock prices.[45] This represents a stabilizing wealth effect with respect to the output gap.

[45] See (89) in the Appendix for details.

Figure 8: Stock Price Gap Shock



Stock Price Gap Shock Remember we added the stock price gap shock to deal with the difficulty of replicating high volatility in stock price dynamics driven largely by non-fundamental movements or factors that we do not model explicitly. As Table 5 suggests, the responses in Figure 8 show a highly volatile reaction (for the upper bound of the 95% interval stock prices deviate more than 60% from the steady state). As the shock has no influence on natural values, stock prices and the stock price gap show the same dynamics. To eliminate the effect on output, i.e. upward pressure on the terms of trade and the exchange rate, the interest rate must be raised by an amount necessary to keep output (and inflation) in equilibrium.

5 Conclusions

This paper examined the business cycle fluctuations in Hong Kong through a DSGE prism and identified the macroeconomic implications of various structural shocks. Using a small open economy DSGE model based on contemporary economic theory, our Bayesian estimation results pointed to positive and significant wealth effects from stock markets on consumption and business cycles. Although our model is rather general, we restricted its focus to a setup with stock markets and purposely disregarded the role of shifting property prices. This omission was deliberately in an effort to limit the models complexity. Thus, the model treats property prices as being determined by the same factors as stock prices and as having similar impacts. While we believe this to be a reasonable first approximation, this modeling may well miss effects coming through the housing price channel, offering a potentially valuable subject for future research.

A Appendix

A.1 Steady State and Linear Equilibria

A.1.1 Steady State

Following (40) a zero-inflation steady state yields

$$1 = \frac{\gamma \frac{\Omega}{PC}}{(\Sigma - 1)(1 + r)} + \frac{(1-\gamma)\Sigma}{(\Sigma - 1)(1 + r)} = \frac{\gamma \frac{\Omega}{PC}}{(\Sigma - 1)(1 + r)} + \frac{1}{\beta (1 + r)} \tag{78}$$

Furthermore, (40), (38), (59), the production function, equilibrium on the labor market and equation (55) define the following steady state:

$$\Omega = Q + D + \int_0^1 XR^i \left(Q^i + D^i\right) di, \tag{79}$$

$$Q + D = (1+r)\,Q\,(1+\epsilon), \tag{80}$$

$$Y = AN, \tag{81}$$

$$MC = (1-\tau)\frac{W}{AP_H} = \frac{1}{1+\mu}, \tag{82}$$

$$D = \frac{\mu}{1+\mu}\frac{P_H}{P}Y, \tag{83}$$

where we used the optimal subsidy $\vartheta = \frac{\mu}{1+\mu}$ as in Galí (2003).

A.1.2 Approximation

Next, we strip the model of its nonlinearities by linearizing the model at the steady state. Log-linearizing (40) using (23) follows exactly the steps in Nisticó (2005) and yields

$$c_t = \frac{1}{1+\Psi}E_t c_{t+1} + \frac{\Psi}{1+\Psi}q_t - \frac{1}{1+\Psi}\left(r_t - E_t\pi_{t+1} - \rho\right). \tag{84}$$

Substituting consumption with output, using the relationship between the terms of trade and the output differentials and between the CPI and the terms of trade, gives

$$x_t = \frac{\sigma_\alpha}{\Gamma_0}E_t x_{t+1} + \frac{\Psi}{\Gamma_0}\widehat{q}_t - \frac{1}{\Gamma_0}\left(r_t - E_t\pi_{H,t+1} - rr_t^n\right), \tag{85}$$

where

$$\begin{aligned} rr_t^n \equiv\ & \rho + \left[\sigma_\alpha\rho_a + \Psi - \Gamma_0\right]\Gamma_a a_t \\ & + \left[\left(\sigma_\alpha\rho_{y^*} + \Psi - \Gamma_0\right)\Gamma_{y^*} + \Theta\sigma_\alpha\left(\rho_{y^*} - 1\right)\right]\alpha y_t^*, \end{aligned} \tag{86}$$

and $\Gamma_0 \equiv 1 + \Psi - \alpha\Theta\sigma_\alpha$, $\kappa_\alpha \equiv \lambda\left(\sigma_\alpha + \varphi\right)$, $\lambda_q \equiv \left(1 + \epsilon - \widetilde{\beta}\right)\left(\frac{1+\varphi-\mu}{\mu} + \alpha\sigma_\alpha\right)$, $\widehat{q}_t \equiv q_t - q_t^n$.

Rearranging (80) gives

$$\frac{D}{(1+r)\,Q} = 1 + \epsilon - \frac{1}{1+r}, \tag{87}$$

and as a consequence the pricing equation is given by

$$\begin{aligned} (1+\epsilon)\,q_t =\ & \frac{1}{(1+r)}E_t q_{t+1} + \left(1 + \epsilon - \frac{1}{1+r}\right)E_t d_{t+1} \\ & - (1+\epsilon)\left(r_t - E_t\pi_{t+1}\right) - \epsilon\eta_t. \end{aligned} \tag{88}$$

Loglinearizing (55) leads directly to

$$
\begin{aligned}
d_t &= \frac{P_H Y}{PD}\left(p_{H,t} - p_t + y_t\right) - \frac{WN}{PD}\left(w_t + n_t - p_t\right) \\
&= \frac{P_H Y}{PD}\left(p_{H,t} - p_t + y_t\right) - \frac{WN}{PD}\left(y_t + mc_t + p_{H,t} - p_t\right) \\
&= \frac{P_H Y}{PD}\left(y_t - \alpha s_t\right) - \frac{WN}{PD_H}\left(y_t + mc_t - \alpha s_t\right) \\
&= y_t - \alpha s_t - \frac{WN}{PD} mc_t, \qquad (89)
\end{aligned}
$$

where we used the assumption of a linear production technology. Using (83) with the production function and marginal costs yields

$$\frac{WN}{PD} = \frac{WY}{APD} = \frac{1}{\mu}, \qquad (90)$$

implying

$$d_t = y_t - \alpha s_t - \frac{1}{\mu} mc_t. \qquad (91)$$

Using the definition of the output gap and equation (68), we derive a relationship between real dividends, the terms of trade, the output gap, and potential output:

$$d_t = y_t^n - \alpha s_t - \frac{\varphi + \mu}{\mu} x_t. \qquad (92)$$

Plugging this into (88) gives

$$
\begin{aligned}
q_t &= \frac{\widetilde{\beta}}{1+\epsilon} E_t q_{t+1} + \frac{1+\epsilon-\widetilde{\beta}}{1+\epsilon} E_t \left[y_{t+1}^n - \alpha s_{t+1} - \frac{\varphi+\mu}{\mu} x_{t+1} \right] \\
&\quad - \left(r_t - E_t \pi_{t+1}\right). \qquad (93)
\end{aligned}
$$

By following the same steps as described in the appendix of Nisticó (2005), adjusted for the terms of trade, we derive

$$\widehat{q}_t = \frac{\widetilde{\beta}}{1+\epsilon} E_t \widehat{q}_{t+1} - \frac{\lambda_q}{1+\epsilon} E_t x_{t+1} - \left(r_t - E_t \pi_{H,t+1} - rr_t^n\right), \qquad (94)$$

where $\widehat{q}_t \equiv q_t - q_t^n$ and the natural stock price evolves according to $q_t^n = y_t^n$. Adding a shock to account for non-fundamental disturbances (η_t) gives (73).

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