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Tariff Wars and Trade Deals with Costly Government

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Abstract. We study a simple model of tariff wars and trade deals in which government revenue collection and disbursement uses resources. The introduction of costly governments leads to lower non–cooperative tariffs, the possibility that a less costly government may win a tariff war, and fully cooperative trade deals where countries lower tariffs but do not eliminate them, even with lump–sum taxes and transfers.

1. Introduction

The standard theoretical model of non–cooperative tariff determination leads to the familiar tariff–war results (Johnson (1953)). Some authors, including Krugman (1993 pages 61 and 65 and his references), have argued that this standard theoretical result may be inconsistent with observation — non–cooperative tariffs appear to be set more "cooperatively", that is, lower. On the other hand, the traditional customs–union literature presumes free trade within each customs union. Here the reality is that tariffs appear to be set less cooperatively, that is, higher. Even for members of trade blocs "cooperation" appears to be limited in the sense that trade deals while characterized by reciprocal reductions in tariffs are not characterized by the elimination of tariffs on all goods and services. The GATT under the auspices of the WTO furnishes many examples of such trade arrangements. This short paper provides a simple explanation for lower non–cooperative tariffs and higher cooperative tariffs.[1]

Clearly the operation of a government requires resources. So we begin with a standard trade model and assume, for example, that hiring a customs officer costs resources. We show that this simple extension yields two results in the tariff-war

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[1]The literature contains at least two classes of extensions to the traditional models that permit positive tariffs in cooperative trade deals. One is obtained by altering the traditional assumption that benevolent governments act in the *nation's* self–interest. Examples of papers that would fall into this category are Grossman and Helpman (1994), Krugman (1993), and Ethier (1998). A second explanation assumes that trade deals are not fully cooperative in the sense that it is assumed countries cannot make binding international commitments, for example, Bagwell and Staiger (1997).

model.[2] 1) The introduction of a costly government may lower that country's non–cooperative tariff. The logic is that one of the benefits of a tariff is the revenue raised and since an increase in the cost of government reduces the net revenue for a given tariff it also reduces the optimal tariff. Now consider two countries that differ in their costs of government. 2) The country with the less costly government can win a tariff war in the sense that it is better off at the non–cooperative equilibrium than at the laissez–faire free trade allocation. Here the idea is that the country with lower costs will face a lower non–cooperative tariff the more costly is the opposing country's government (result 1); consequently the less costly country can win the tariff war.

We go on to explore an environment where fully cooperative trade deals are possible. We assume that the trade deal must be individually rational. Introducing the possibility of cooperative trade deals into our model of costly governments leads to two further results. Assume that governments are costly, but one government is more costly than the other. Then we show 3) Tariffs will not be zero in a fully cooperative trade deal even with lump–sum taxes and transfers.[3] In the case where the less costly government wins the tariff war (result 2) the free trade allocation is not a potential equilibrium trade deal because it is not individually rational for the less costly government. In such a case, absent revenue collection costs, trade deals would involve free trade and a lump–sum transfer of resources between countries (a compensation mechanism) to make it individually rational and Pareto efficient. But with costly governments the use of a tariff in the less costly country becomes an efficient instrument for transferring resources to that country. The idea is that raising resources in the more costly country to compensate the less costly country may be less efficient than permitting a positive distortionary tariff in the country with the lower revenue collection costs. At free trade, for example, the marginal damage of using the tariff is zero. 4) Fully cooperative trade deals may require each country lowering its tariffs from the non–cooperative level without going to all the way to free trade. The two strands to the logic are that reducing tariffs from their non–cooperative levels is exploiting gains from policy coordination, and not going all the way to zero tariffs follows from result 3.

To summarize, we show adding the assumption that the operation of governments cost resources to a standard trade model may explain lower non–cooperative tariffs and higher cooperative tariffs.

2. Framework and Results

Assume a two-country, two–good, general-equilibrium trade model, in which each country is populated by a large number of identical price–taking individuals. We normalize the population size in each country at unity. Let t_i denote a tariff set by country i and let τ_i be a lump–sum tax (or subsidy) used to balance each country's

[2] To be precise, what we mean by tariff war is the standard one shot non–cooperative game of tariff setting with complete information.

[3] A lump–sum tax is tax that can not be avoided to any extent through any action of an agent. This is why it is not distortionary.

government budget constraint. Denote the utility of each person in country i by $U_i(t_i, t_j, \tau_i)$. Write the government's budget constraint in country i as $R_i(t_i, \tau_i, c_i) = 0$ where c_i is a parameter measuring the costs of collecting and disbursing government revenue. For example, $R_i(t_i, \tau_i, c_i)$ might equal $f_i(t_i) + \tau_i - c_i(|f_i(t_i)| + |\tau_i|)$, $0 < c_i < 1$, where f_i is increasing and concave, or $f_i(t_i) + \tau_i - c_i(f_i(t_i)^2 + \tau_i^2)$.

If each country picks its tariff/tax instruments to maximize U_i subject to R_i, taking as given the tax instruments of the other country, we obtain the standard non–cooperative Nash equilibrium in which typically $t_i > 0$ and $\tau_i < 0$. Each country tries to turn world prices in its favour and to collect sufficient revenue from its tariff to more than compensate for the damage the tariff does to its domestic economy. How is this equilibrium affected by the costs of collecting revenue? To fix ideas suppose $c_1 = c > 0$ and $c_2 = 0$. Have individuals optimize and use the budget constraints in each country to eliminate the taxes and thereby to write utility as $V_1(t_1, t_2, c)$ and $V_2(t_2, t_1)$. Denote first and second partial derivatives by V_i^k and V_i^{kl}. The reaction functions are defined implicitly by $V_1^1(t_1, t_2, c) = 0$ and $V_2^1(t_2, t_1) = 0$. The second-order conditions for each country's optimal tariff problem imply $V_i^{11} < 0$, $i = 1, 2$. Taking the total differential of each reaction function separately and using $V_i^{11} < 0$ we find that the tariffs are strategic complements (substitutes) if $V_i^{12} > 0$ (< 0).

From a total differential of the system of equations we deduce (1) and (2).

$$\frac{dt_1}{dc} = \frac{-V_2^{11}V_1^{13}}{V_1^{11}V_2^{11} - V_1^{12}V_2^{12}} \equiv \frac{-V_2^{11}V_1^{13}}{D} \tag{1}$$

$$\frac{dt_2}{dc} = \frac{V_2^{12}V_1^{13}}{D} \tag{2}$$

Stability of the equilibrium implies $D > 0$. Thus dt_1/dc has the sign of V_1^{13}, where V_1^{13} is the marginal effect of c on the net marginal benefit to country 1 of raising t_1. One of the benefits of employing a tariff is the revenue collected. Since higher c reduces the amount of revenue that a given tariff raises we assume that $V_1^{13} < 0$.

Result 1. With $V_1^{13} < 0$, country 1's optimal tariff is decreasing in c.

The sign of dt_2/dc depends not only on V_1^{13} but also on V_2^{12}. If $V_1^{13} < 0$ and the tariffs are strategic complements (which is true in some trade models) then $dt_2/dc < 0$. Higher costs of revenue collection in country 1 weaken its ability to conduct a tariff war and this could induce country 2 to levy a lower tariff than it would otherwise. In other words both non–cooperative tariffs can be reduced by costly government.[4]

With c set equal to zero and the two countries roughly symmetric the tariff war typically leaves them both worse off than they would be with zero tariffs — both lose the tariff war.[5] As we have observed, with $c > 0$, country 1's incentive to choose

[4]This, of course, is not the only possibility. Below, we present an example with $R_1(t_1, \tau_1, c) = f_1(t_1) + \tau_1 - c(|f_1(t_i)| + |\tau_1|)$ where $dt_1/dc < 0$ and $dt_2/dc = 0$, that is, $V_1^{13} < 0$ and $V_2^{12} = 0$.

[5]Kennan and Riezman (1988) show that if one country is much larger than the other, this can lead to the large country winning the trade war.

a large tariff may be diminished and it may even be optimal for country 1 to set t_1 to zero, for a high enough value of c. If c is so high that the optimal level of $t_1 = 0$ and if, for example, $V_i^{12} = 0$ then if country 2 chooses to have a positive tariff $(V_2(t_2 > 0, 0) > V_2(0,0))$ it will win the tariff war. Here is the reasoning. First, country 2 may choose to have a positive tariff even if c is large because with $V_2^{12} = 0$ country 2's optimal tariff is independent of c. Second, with $V_1^{12} = 0$, country 1's optimal $t_1 = 0$ is independent of t_2. So the choice of $t_2 > 0$ is enough to know country 2 will be better off with the tariff war.

Result 2. The less costly government can win a tariff war.

What are the characteristics of possible cooperative trade agreements satisfying individual rationality, that is, when country 2's utility in the tariff war is higher than it would be with free trade?[6] Efficient trade agreements are solutions to the following problem.

$$\max_{t_1, \tau_1, t_2, \tau_2, S} \quad U_1(t_1, t_2, \tau_1) + \lambda_1 \left(U_2(t_2, t_1, \tau_2) - \overline{U_2}\right) + \lambda_2 \left(R_1(t_1, \tau_1, c) - S\right) + \lambda_3 \left(R_2(t_2, \tau_2, 0) + S\right),$$

where S is a transfer from country 1 to country 2.

It is impossible to provide a complete characterization of the conditions for Pareto efficiency with costly government without a complete model. We provide such a model in the following section, but here we show the possibilities by using the standard logic of optimal taxation.

With no revenue collection costs ($c = 0$) any point on the utility possibility frontier is attainable by setting tariffs to zero and then using a head tax in one country and a head subsidy in the other, together with a transfer from the country with the tax to the country with the subsidy. If country 2 has "won" the tariff war (not because $c > 0$ but say because it is larger than country 1) then in any Pareto-improving and efficient agreement country 1 would levy a head tax and use the revenue so obtained to make a transfer to country 2, and country 2 would distribute the transfer to its citizens through a head subsidy. To repeat, there would be free trade in any Pareto-efficient agreement with $c = 0$.

When c is positive, however, efficient trade agreements lie along a utility possibility frontier, inside the costless government frontier except for one point. The point the two utility possibility frontiers have in common is the free trade point with no transfer between the counties (laissez-faire), for here all taxes/tariffs and hence costs of revenue collection are zero for any c. Figure 1 illustrates utility possibility frontiers with $c = 0$ and with $c > 0$.

[6] The cooperative trade deal here is also the equilibrium outcome of a non–cooperative game of coalition formation (see Burbidge et al. (1997, proposition 1)).

What constitutes optimal tax/tariff combinations along the frontier with $c > 0$? Again, if country 2 has won the tariff war (because say c is sufficiently large), the relevant segment is that section of the frontier where country 2's utility is higher than it is at free trade. So long as costs of revenue collection are symmetric between tariff and head tax revenue (as in the example revenue functions above) it can never be efficient to have a nonzero tariff in country 1. The reason is that country 1's tariff distorts resource allocation whereas its head tax does not, so setting $t_1 = 0$ is optimal. Next, starting at the free trade point, note that there are two ways to raise the welfare of country 2: (i) raise τ_1 above zero and use it to finance a transfer S to country 2; or (ii) raise the tariff rate in country 2, t_2, above zero. In either case the revenue accruing to country 2 would be distributed through the costless head subsidy ($\tau_2 < 0$).[7] Option (i) is somewhat costly because of collection costs; option (ii) is somewhat inefficient because country 2's tariff distorts prices. The choice between (i) and (ii) is a standard optimal tax problem. If it is efficient to use both routes simultaneously it must be that the nature of collection costs is such that the marginal damage to utility per dollar of tax revenue raised is the same for both routes.

Assuming that the marginal collection cost of using τ_1 is positive and since the marginal damage of increasing t_2 at free trade is zero it must be that $t_2 > 0$ in any trade agreement with costly government. This is our third result.

Result 3. With governments that are costly but to differing extents, tariffs are not zero in a fully cooperative trade deal, even with lump-sum taxes and transfers.

Our last result is that one can find examples of initial tariff-war Nash equilibria and associated utility possibility frontiers, with costly government, such that any point along the frontier that is a Pareto improvement (and therefore individually rational) over the tariff war equilibrium has lower tariffs for both countries than tariffs prevailing in the tariff war. We provide an example below.

Result 4. Fully cooperative trade deals may require each country lowering its tariffs from the non–cooperative level without going to all the way to free trade.

3. An Example

Our example is a two-country, three-commodity simplification of Grossman and Helpman (1994), extended to allow for costly governments. The citizens of country $i = 1, 2$ consume X_j^i of good j and have an aggregate endowment A_i of labour and a single specific factor T_i which is essential in the production of good X_i. Utility is quasilinear and has the following form.

$$u(X_0^i, X_1^i, X_2^i) = X_0^i + \ln X_1^i + \ln X_2^i, \text{ for } i = 1, 2 \tag{3}$$

[7] Remember that we have set $c_2 = 0$.

Good 0 is produced in each country from labour alone with this constant returns to scale technology.

$$Q_o^i = L_o^i$$

We assume the technology for the only other good produced by country $i = 1, 2$ has the following particular Cobb–Douglas form.

$$Q_i = (L_i)^{.5}(T_i)^{.5} \text{ for } i = 1, 2 \tag{4}$$

All three goods are traded and we follow the literature in assuming that the trade in good 0 is free. Then by assumption, the imports into country i of good j will equal X_j^i for $i \neq j$.[8] Consumers in country i maximize utility subject to the budget constraint

$$r_i T_i + w_i A_i - \tau_i = P_0 X_0^i + P_i X_i^i + (1 + t_i) P_j X_j^i \text{ for } i = 1, 2 \text{ and } i \neq j, \tag{5}$$

where r_i and w_i are the returns paid by competitive firms to the fixed factor and to labour. Trade takes place and prices are determined by supply and demand; each factor is paid the value of its marginal product. For good 0 in each country we derive $P_o = w_i$. Choosing good 0 to be the numeraire we have $P_0 = w_i = 1$. Assuming the parameters are such that $X_0^i \geq 0$, for $i = 1, 2$, we obtain these demand functions.[9]

$$\begin{aligned} X_i^i &= \frac{1}{P_i} \text{ for } i = 1, 2 \\ X_j^i &= \frac{1}{(1 + t_i) P_j} \text{ for } i = 1, 2 \text{ and } i \neq j \\ X_0^i &= r_i T_i + A_i - 2 - \tau_i \text{ for } i = 1, 2 \end{aligned} \tag{6}$$

The factor-pricing equations are

$$\left. \begin{aligned} \tfrac{1}{2} P_i \left(\tfrac{T_i}{L_i}\right)^{1/2} &= 1 \\ \tfrac{1}{2} P_i \left(\tfrac{L_i}{T_i}\right)^{1/2} &= r_i \end{aligned} \right\} \text{ for } i = 1, 2$$

and equilibrium in the output markets yields

$$\left(\frac{T_i}{L_i}\right)^{1/2} L_i = \frac{1}{P_i} + \frac{1}{(1 + t_j) P_i}, \text{ for } i = 1, 2 \text{ and } i \neq j.$$

[8] This is a further simplification of Grossman and Helpman (1994).

[9] Countries are always denoted by the index i and goods by the index j. But when $j = i$ in an equation we will economize by replacing j by i from the outset.

The first and third equations can be solved in closed form for P_i and L_i and the solutions can then be used in the second equation to solve for r_i, all of which yields:

$$
\begin{aligned}
L_i &= \tfrac{2+t_j}{2(1+t_j)} \text{ for } i \neq j \\
P_i &= \left(\tfrac{2(2+t_j)}{T_i(1+t_j)}\right)^{1/2} \text{ for } i \neq j \\
r_i &= \tfrac{2+t_j}{2T_i(1+t_j)} \text{ for } i \neq j
\end{aligned} \tag{7}
$$

Note from (7) we must assume that ad valorem tariff rates are not so negative that $(1+t_j)$ is negative. Equations (6) and (7) can then be used in (3) to write utility as a function of parameters, head taxes and tariffs.

$$U_i(t_i, t_j, \tau_i) = \frac{2+t_j}{2(1+t_j)} + A_i - \tau_i - 2 - \ln\left[\frac{2[2+t_j]}{T_i(1+t_j)}\right]^{1/2} - \ln\left((1+t_i)\left[\frac{2(2+t_i)}{T_j(1+t_i)}\right]^{1/2}\right)$$

Using (6), the tariff revenue in country i is $t_i P_j X_j^i = \frac{t_i}{(1+t_i)}$, and thus the budget constraints for the governments are:[10]

$$
\begin{aligned}
&\tfrac{t_2}{1+t_2} + \tau_2 = 0 \text{ for country 2} \\
&\tfrac{t_1}{1+t_1} + \tau_1 - w(t_1, \tau_1) = 0, \\
&\text{where } w(t_1, \tau_1) = c\left[\left|\tfrac{t_1}{1+t_1}\right| + |\tau_1|\right], \ c < 1, \text{ for country 1.}
\end{aligned} \tag{8}
$$

At the free trade (laissez–faire) allocation $t_i = 0$, $i = 1, 2$, and therefore each country's utility is $U_i(0,0,0) \equiv u_i^f$, for $i = 1, 2$. In a tariff war country i maximizes $U_i(t_i, t_j, \tau_i)$ subject to (8) by choosing (t_i, τ_i) taking (t_j, τ_j) as given. For country 2 the first–order condition with respect τ_2 implies that the Lagrange multiplier is unity and then using this and the fact that t_2 must be positive for positive consumptions (see (6)) yields a dominant-strategy tariff of[11]

$$t_2^e = \frac{\sqrt{17} - 3}{4} > 0.$$

In appendix A we show that the dominant strategy tariff for country 1 is

$$
\begin{aligned}
t_1^e &= \frac{-3 - 7c + \sqrt{17 - 6c - 7c^2}}{4(1+c)} > 0 \text{ for } 0 \leq c < 1/7 \\
t_1^e &= 0 \text{ for } c \geq 1/7.
\end{aligned}
$$

[10] Note that we can now give the cost of government a formal interpretation as an amount of good 0 or labour (numeraire) consumed by government operations.

[11] This is a dominant strategy because the utility maximizing choices of country i do not depend on the choices of country j.

For $c = 0$, $t_1^e = t_2^e = (\sqrt{17}-3)/4 > 0$ and we derive result 1 from the previous section; t_1^e is a decreasing function of c. The logic of the result is simply that the use of tariff involves costs and benefits where one of the benefits the tariff revenue raised. An increase in c reduces the revenue raised with a given tariff and thus it reduces the equilibrium tariff. At $c = 1/7$ the costs dominate the benefits and thus $t_1^e = 0$.

With the closed forms for t_i, the government budget constraint (8) yields $\tau_1^e = \frac{(1-c)\left(7c+3-\sqrt{(17-6c-7c^2)}\right)}{(1+c)\left(1-3c+\sqrt{(17-6c-7c^2)}\right)}$ and $\tau_2^e = \frac{3-\sqrt{17}}{1+\sqrt{17}}$ (see Appendix A). These then can be used in (3) to determine the dominant strategy equilibrium utilities of $U_i^e(t_i, t_j, \tau_i^e)$. As above, we define "winning a tariff war" as $U_i^e > U_i^f$. Note that the country with a less costly government can win a tariff war. If $c = 1/7$ then $t_1^e = 0$ — the free trade level — irrespective of t_2. Then we derive result 2 from the previous section that $U_2^e > U_2^f$ by the individual rationality of the choice $t_2^e > 0$.

Since t_1^e is a decreasing function of c and U_2^e is a decreasing function of t_1^e, u_2^e increases in c, for $c \in [0, 1/7)$. In fact solving for the c such that

$$u_2^e\left(\frac{\sqrt{17}-3}{4}, \frac{\sqrt{17-6c-7c^2}-3-7c}{4(1+c)}, \frac{3-\sqrt{17}}{1+\sqrt{17}}\right) = U_2^f,$$

one finds that country 2 wins the tariff war for $c > 1/15$.[12]

In Appendix B we characterize world Pareto efficiency for this model, that is, the set of allocations from which it is not possible to make Pareto improvements.

A special case of our work in the appendix is efficiency when $c = 0$. It is characterized by two exchange efficiency conditions—one for each pair of goods and two overall efficiency conditions—one for each pair of goods. In terms of instruments and denoting the costless-government choices by a superscript $*$ the necessary conditions are $t_i = 0$ for $i = 1, 2$. The free-trade (laissez–faire) allocation above is, of course, a Pareto efficient allocation with $c = 0$, but it is just one of a continuum. We show that with $c = 0$, the utility possibility frontier passes through (U_1^f, U_2^f) and has a constant slope of negative one. To achieve allocations where $u_i^* > u_i^f : t_1^* = t_2^* = 0$, $\tau_i^* < 0$, $\tau_j^* > 0$, $i \neq j$, and there is an international transfer of resources from j to i.

In Appendix B we also show that when $c > 0$, Pareto efficiency involves free trade $t_i = 0$ for $i = 1, 2$ only at the free trade allocation. Denoting choices with $c > 0$ by a $**$ superscript, we show that for $U_2^{**} \geq U_2^f : t_1^{**} = 0$, $0 < t_2^{**} \leq \frac{\sqrt{9+8c}-3}{4}$, $\tau_1^{**} \geq 0$, $\tau_2^{**} < 0$ and that there is a transfer of resources from 1 to 2 if $\tau_1^{**} > 0$.

[Place Figure 1 here.]

Figure 1 illustrates the tariff-war equilibria for $0 \leq c \leq 1/7$, the utility possibility frontier with $c = 0$ (which is a straight line), and the utility possibility frontier (hereafter *the* UPF — AC) for $c = 1/7$. All lines are based on $A_i = 3$ and $T_i = 1$, for $i = 1, 2$. The right panel is a blow-up of the marked section in the left panel —

[12] To 10 decimal places it is 0.0671916407 which is slightly greater than 1/15.

$AB'G'$. Along DC', $0 \leq t_2^{**} < \frac{\sqrt{9+8c}-3}{4}$, $\tau_1^{**} = 0$, and an international transfer is not used. Along AF, $t_2^{**} = \frac{\sqrt{9+8c}-3}{4}$, $\tau_1^{**} > 0$ and a transfer is made from country 1 to 2. AXC' is the outer envelope of DC' and AF. The absolute value of the slope of AXC' falls from a value of unity at the free-trade point, A, to $1-c$ at the crossover point, X, and XC' is a straight line. We now provide an explanation of these results.

To raise U_2^{**} from the free-trade allocation the distortion associated with increasing t_2^{**} from 0 is zero when evaluated at the free-trade allocation while the (marginal) cost associated with increasing τ_1^{**} from 0 is $c > 0$. Thus initially the efficient solution is to allow country 2 to use its tariff to increase its well–being. As we require larger U_2^{**}, t_2^{**} is increased with the distortion increasing until $t_2^{**} = \frac{\sqrt{9+8c}-3}{4}$ at which point the distortion associated with increasing t_2^{**} further would be larger than the cost associated with using τ_1^{**}, that is, c. From point X onwards to C' and C, τ_1^{**} is increased from zero and $t_2^{**} = \frac{\sqrt{9+8c}-3}{4}$. This continues to be true for all utility profiles with larger U_2^{**} because cost is linear in τ_1^{**}. Note that increasing τ_1^{**} from 0 always dominates increasing t_1^{**} from 0 as both involve cost but the latter also distorts the prices of goods.

We are now in a position to provide proofs of results 3 and 4 from the previous section for this example.

Result 3. Tariffs are not zero in a fully cooperative trade deal, even with lump-sum taxes and transfers.

Proof: If governments are costless or $c = 0$ then any cooperative trade deal has $t_2^{*} = t_2^{*} = 0$. But with $1/15 < c \leq 1/7$ any cooperative trade deal has $t_2^{**} > 0$.

Result 4. Fully cooperative trade deals may require each country lowering its tariffs from the non–cooperative level without going to all the way to free trade.

Proof: The maximal $t_2^{**} = \frac{\sqrt{9+8c}-3}{4} < \frac{\sqrt{17}-3}{4} = t_2^e$ and $t_1^{**} = 0 < t_1^e = \frac{\sqrt{17-6c-7c^2}-3-7c}{4(1+c)}$ for $1/15 < c \leq 1/7$.

The intuition for these results is contained, of course, in the previous section.

4. CONCLUSIONS

The introduction of governments that use resources leads to lower non–cooperative tariffs, the possibility that a less costly government may win a tariff war, and fully cooperative trade deals where countries lower tariffs from their non-cooperative levels but do not eliminate them, even with the lump–sum taxes and transfers.

Appendix A

The problem of country 1 is

$$\underset{t_1, \tau_1, \lambda_1}{Max} \ \mathcal{L} = u_1(t_1, t_2, \tau_1) + \lambda(t_1/(1+t_1) + \tau_1 - w(t_1, \tau_1)).$$

Observe that the right-hand and left-hand derivatives of $w(t_1, \tau_1)$ differ at $t_1 = 0$ and $\tau_1 = 0$.

$$\frac{\partial w(t_1,\tau_1)}{\partial t_1} = \begin{cases} c/(1+t_1)^2, \ t_1 > 0 \\ -c/(1+t_1)^2, -1 < \ t_1 < 0 \\ undefined, \ t_1 = 0 \end{cases} \text{ and } \frac{\partial w(t_1,\tau_1)}{\partial \tau_1} = \begin{cases} c, \ \tau_1 > 0 \\ -c, \ \tau_1 < 0 \\ undefined, \ \tau_1 = 0 \end{cases}$$

Define

$$\frac{\partial \mathcal{L}}{\partial t_1} \equiv \begin{cases} \left(\frac{\partial \mathcal{L}}{\partial t_1}\right)^R = -\frac{1}{2}\frac{1+2(1+t_1)}{(1+t_1)(2+t_1)} + \frac{\lambda(1-c)}{(1+t_1)^2}, \ t_1 > 0 \\ \left(\frac{\partial \mathcal{L}}{\partial t_1}\right)^L = -\frac{1}{2}\frac{1+2(1+t_1)}{(1+t_1)(2+t_1)} + \frac{\lambda(1+c)}{(1+t_1)^2}, \ t_1 < 0 \end{cases}$$

and

$$\frac{\partial \mathcal{L}}{\partial \tau_1} \equiv \begin{cases} \left(\frac{\partial \mathcal{L}}{\partial \tau_1}\right)^R = -1 + \lambda(1-c), \ \tau_1 > 0 \\ \left(\frac{\partial \mathcal{L}}{\partial \tau_1}\right)^L = -1 + \lambda(1+c), \ \tau_1 < 0. \end{cases}$$

Since $c < 1$ budget balance requires $t_1 \gtreqless 0$ if and only if $\tau_1 \lesseqgtr 0$.

For $t_1 = 0$ and $\tau_1 = 0$ to be a solution then $\left(\frac{\partial \mathcal{L}}{\partial t_1}\right)^R \leq 0$ evaluated at $t_1 = 0$ and $\lambda = 1/(1+c)$ so that it does not pay to increase t_1 from 0 and $\left(\frac{\partial \mathcal{L}}{\partial t_1}\right)^L \geq 0$ evaluated at $t_1 = 0$, and $\lambda = 1/(1-c)$ so that it does not pay to decrease t_1 from 0. Using the derivatives the latter holds for $c \in [0,1)$ and the former holds for $c \geq 1/7$. Thus $t_1^e = 0$ and $\tau_1^e = 0$ for $c \geq 1/7$.

For $t_1 < 0$ and $\tau_1 > 0$ to be solution then $\left(\frac{\partial \mathcal{L}}{\partial \tau_1}\right)^R = 0$ or $\lambda = 1/(1-c)$. Using this in $\left(\frac{\partial \mathcal{L}}{\partial t_1}\right)^L = 0$ then $-\frac{1}{2}\frac{1+2(1+t_1)}{(1+t_1)(2+t_1)} + \frac{\lambda(1+c)}{(1+t_1)^2(1-c)} = 0$. This leads to two roots: $t_1 = \frac{1}{2(2-2c)}\left(-3+7c+\sqrt{(17+6c-7c^2)}\right)$; $t_1 = \frac{1}{2(2-2c)}\left(-3+7c-\sqrt{(17+6c-7c^2)}\right)$. The first is not a solution as it implies $t_1 > 0$ for $c \in (0,1)$. The second is not because it implies $t_1 < -1$ for $c \in (0,1)$ which implies an imaginary number for a price (see (7)).

For $t_1 > 0$ and $\tau_1 < 0$ to be solution then $\left(\frac{\partial \mathcal{L}}{\partial \tau_1}\right)^L = 0$ or $\lambda = 1/(1+c)$. Using this in $\left(\frac{\partial \mathcal{L}}{\partial t_1}\right)^R = 0$ then $-\frac{1}{2}\frac{1+2(1+t_1)}{(1+t_1)(2+t_1)} + \frac{(1-c)}{(1+t_1)^2(1+c)} = 0$. This leads to two roots :

$t_1 = \frac{1}{4(1+c)}\left(\sqrt{(17-6c-7c^2)}-3-7c\right)$, $t_1 = \frac{1}{4(1+c)}\left(-3-7c-\sqrt{(17-6c-7c^2)}\right)$. The second is not a solution as it implies $t_1 < 0$ for $c \in [0,1)$. The first is a solution for $0 \leq c < 1/7$. For $c \geq 1/7$ it is not a solution to this case because it implies $t_1 \leq 0$ which contradicts our assumption here that $t_1 > 0$.

Thus the dominant strategy tariff for country 1 is $t_1^e = \frac{1}{4(1+c)}\left(\sqrt{(17-6c-7c^2)}-3-7c\right)$ for $c < 1/7$ and $t_1^e = 1$ for $c \geq 1/7$. The dominant strategy τ_1^e can then be derived from t_1^e and the budget constraint. We obtain

t_1^e	τ_1^e	Range
$\frac{1}{4(1+c)}\left(\sqrt{(17-6c-7c^2)}-3-7c\right)$	$\frac{(1-c)\left(7c+3-\sqrt{(17-6c-7c^2)}\right)}{(1+c)\left(1-3c+\sqrt{(17-6c-7c^2)}\right)}$	$0 \leq c < 1/7$
0	0	$1/7 \leq c < 1$

Appendix B

We solve the following problem to find Pareto efficient allocations with $c > 0$.

$$\max_{\substack{t_1, t_2, \tau_1, \tau_2,\\ S, \lambda_1, \lambda_2, \lambda_3}} \quad \begin{array}{c} \mathcal{F} = U_1(t_1,t_2,\tau_1) + \lambda_1(U_2(t_2,t_1,\tau_2) - \overline{U_2}) + \\ \lambda_2(t_1/(1+t_1) + \tau_1 - w(t_1,\tau_1) - S) + \lambda_3(R_2(t_2,\tau_2) + S) \end{array},$$

where S is a transfer from country 1 to 2 and $R_2(t_2,\tau_2) = t_2/(1+t_2) + \tau_2$. We are interested in characterizing efficient allocations with $U_2(t_2,t_1,\tau_2) \geq U_2^f$. We assume that the three constraints bind. The first–order conditions for τ_2 and S yield

$$\lambda_1 \frac{\partial U_2}{\partial \tau_2} + \lambda_3 \frac{\partial R_2}{\partial \tau_2} = 0 \tag{9}$$

$$-\lambda_2 + \lambda_3 = 0 \tag{10}$$

or $\lambda_1 = \lambda_2 = \lambda_3 \equiv \lambda$. Using this, the first–order condition for t_2 is

$$-\frac{1}{2\,(2+t_2)\,(1+t_2)^2} + \lambda\left(-\frac{1}{2}\frac{1+2(1+t_2)}{(1+t_2)\,(2+t_2)} + \frac{1}{(1+t_2)^2}\right) = 0. \tag{11}$$

Solving and simplifying yields $\lambda = \frac{1}{3+t_2-2(1+t_2)^2}$.

As in Appendix A define

$$\frac{\partial \mathcal{F}}{\partial t_1} \equiv \begin{cases} \left(\frac{\partial \mathcal{F}}{\partial t_1}\right)^R = -\frac{3+2t_1}{2(1+t_1)(2+t_1)} + \lambda\left(-\frac{1}{2(2+t_1)(1+t_1)^2} + \frac{1-c}{(1+t_1)^2}\right), & t_1 > 0 \\ \left(\frac{\partial \mathcal{F}}{\partial t_1}\right)^L = -\frac{3+2t_1}{2(1+t_1)(2+t_1)} + \lambda\left(-\frac{1}{2(2+t_1)(1+t_1)^2} + \frac{1+c}{(1+t_1)^2}\right), & t_1 < 0 \end{cases}$$

and

$$\frac{\partial \mathcal{F}}{\partial \tau_1} \equiv \begin{cases} \left(\frac{\partial \mathcal{F}}{\partial \tau_1}\right)^R = -1 + \lambda(1-c), \ \tau_1 > 0 \\ \left(\frac{\partial \mathcal{F}}{\partial \tau_1}\right)^L = -1 + \lambda(1+c), \ \tau_1 < 0. \end{cases}$$

For $t_1 = 0$ and $\tau_1 = 0$ to be a solution to the problem $\left(\frac{\partial \mathcal{F}}{\partial t_1}\right)^R \leq 0$, $\left(\frac{\partial \mathcal{F}}{\partial t_1}\right)^L \geq 0$, $\left(\frac{\partial \mathcal{F}}{\partial \tau_1}\right)^R \leq 0$, and $\left(\frac{\partial \mathcal{F}}{\partial \tau_1}\right)^L \geq 0$ evaluated at $t_1 = 0$ and $\lambda = \frac{1}{3+t_2-2(1+t_2)^2}$ so that it does not pay to increase or decrease t_1 from 0 and τ_1 from 0. It is immediate that the free trade point $t_i = 0$ and $\tau_i = 0$ for all i and thus $\lambda = 1$ is a particular efficient allocation, the one for $\overline{U_2} = U_2(0,0,0)$. Further when $c = 0$ the $\left(\frac{\partial \mathcal{F}}{\partial t_1}\right)^R = \left(\frac{\partial \mathcal{F}}{\partial t_1}\right)^L$ and the tariffs are $t_2 = 0$ so that $\lambda = \frac{1}{3+t_2-2(1+t_2)^2} = 1$ and $t_1 = 0$ so that $\left(\frac{\partial \mathcal{F}}{\partial t_1}\right)^R = \left(\frac{\partial \mathcal{F}}{\partial t_1}\right)^L = 0$. Non–zero τ_i and S are used to achieve utility profiles other than that at the free trade allocation.

The tariff war results imply that starting at the free-trade equilibrium $U_2(t_2, t_1, \tau_2)$ is increasing in t_2 for $0 \leq t_2 < \frac{\sqrt{17}-3}{4}$. Since $\lambda = \frac{1}{3+t_2-2(1+t_2)^2}$, λ is also increasing in t_2. With $c > 0$ we have $\left(\frac{\partial \mathcal{F}}{\partial t_1}\right)^R < 0$, $\left(\frac{\partial \mathcal{F}}{\partial t_1}\right)^L > 0$, $\left(\frac{\partial \mathcal{F}}{\partial \tau_1}\right)^R \leq 0$, and $\left(\frac{\partial \mathcal{F}}{\partial \tau_1}\right)^L \geq 0$ at $t_1 = 0$ and for t_2 such that $1 \leq \lambda = \frac{1}{3+t_2-2(1+t_2)^2} \leq \frac{1}{1-c}$ or $0 \leq t_2 \leq \frac{\sqrt{(9+8c)}-3}{4}$. So a solution for utility profiles with $U_2^f \leq U_2(t_2, t_1, \tau_2) \leq U_2\left(\frac{\sqrt{(9+8c)}-3}{4}, 0, \frac{3-\sqrt{(9+8c)}}{1+\sqrt{(9+8c)}}\right)$ has $t_1 = 0$ and $S = \tau_1 = 0$, $\tau_2 = \frac{3-\sqrt{(9+8c)}}{1+\sqrt{(9+8c)}}$, the latter from $R_2(\frac{\sqrt{(9+8c)}-3}{4}, \tau_2) = 0$. If $\frac{1}{3+t_2-2(1+t_2)^2} > 1 - c$ then $\left(\frac{\partial \mathcal{F}}{\partial \tau_1}\right)^R > 0$ so this solution applies only to allocations where $U_2(t_2, t_1, \tau_2) \leq U_2\left(\frac{\sqrt{(9+8c)}-3}{4}, 0, \frac{3-\sqrt{(9+8c)}}{1+\sqrt{(9+8c)}}\right)$.

For $\tau_1 < 0$ to be a solution, $\left(\frac{\partial \mathcal{F}}{\partial \tau_1}\right)^L = 0$ or $\lambda = \frac{1}{1+c} < 1$ which yields $t_2 < 0$ by $\lambda = \frac{1}{3+t_2-2(1+t_2)^2}$. In addition, $t_1 = 0$ as $\left(\frac{\partial \mathcal{F}}{\partial t_1}\right)^R < 0$, $\left(\frac{\partial \mathcal{F}}{\partial t_1}\right)^L > 0$ evaluated at $t_1 = 0$, $\lambda = \frac{1}{1+c}$ and $c > 0$. These imply that $S < 0$ and thus $\tau_2 > 0$ by budget balance. Therefore this allocation has $U_2(t_2, t_1, \tau_2) < U_2^f = U_2(0, 0, 0)$. Since we are interested in the region where $U_2(t_2, t_1, \tau_2) \geq U_2^f$, $\tau_1 < 0$ does not apply.

For $\tau_1 > 0$ to be a solution $\left(\frac{\partial \mathcal{F}}{\partial \tau_1}\right)^R = 0$ or $\lambda = \frac{1}{1-c}$. Given $\lambda = \frac{1}{1-c}$, t_2 is determined by $\lambda = \frac{1}{1-c} = \frac{1}{3+t_2-2(1+t_2)^2}$ or $t_2 = \frac{\sqrt{(9+8c)}-3}{4}$. And with $\lambda = \frac{1}{1-c}$, $\left(\frac{\partial \mathcal{F}}{\partial t_1}\right)^R < 0$, $\left(\frac{\partial \mathcal{F}}{\partial t_1}\right)^L > 0$ evaluated at $t_1 = 0$. Since $\tau_1 > 0$ and $t_1 = 0$ then $S > 0$. Since $S > 0$ and $t_2 = \frac{\sqrt{(9+8c)}-3}{4}$ then $\tau_2 < \frac{3-\sqrt{(9+8c)}}{1+\sqrt{(9+8c)}}$). The budget constraint for country 2 then implies

$U_2(t_2,t_1,\tau_2) > U_2\left(\frac{\sqrt{(9+8c)}-3}{4}, 0, \frac{3-\sqrt{(9+8c)}}{1+\sqrt{(9+8c)}}\right)$ and thus the $\tau_1 > 0$ solution applies only to allocations where $U_2(t_2,t_1,\tau_2) > U_2\left(\frac{\sqrt{(9+8c)}-3}{4}, 0, \frac{3-\sqrt{(9+8c)}}{1+\sqrt{(9+8c)}}\right)$.

Note that $t_1 \neq 0$ leads to a contradiction. For example, $t_1 > 0$ implies $\left(\frac{\partial \mathcal{F}}{\partial t_1}\right)^R = 0$ or $-\frac{3+2t_1}{2(1+t_1)(2+t_1)} + \lambda\left(-\frac{1}{2(2+t_1)(1+t_1)^2} + \frac{1-c}{(1+t_1)^2}\right) = 0$. This in turn implies $\lambda = (1+t_1)\frac{3+2t_1}{(3+2t_1)(1-c)-c} > 1/(1-c)$. But $\lambda > 1/(1-c)$ implies $\left(\frac{\partial \mathcal{F}}{\partial \tau_1}\right)^R > 0$ which implies $\tau_1 > 0$ and then $\lambda = 1/(1-c)$, which contradicts $\lambda > 1/(1-c)$. Likewise, $t_1 < 0$ implies $\left(\frac{\partial \mathcal{F}}{\partial t_1}\right)^L = 0$ or $-\frac{3+2t_1}{2(1+t_1)(2+t_1)} + \lambda\left(-\frac{1}{2(2+t_1)(1+t_1)^2} + \frac{1+c}{(1+t_1)^2}\right) = 0$, which implies $\lambda = (1+t_1)\frac{3+2t_1}{(3+2t_1)(1+c)+c} < 1/(1+c)$, but then $\left(\frac{\partial \mathcal{F}}{\partial \tau_1}\right)^L < 0$ which implies $\tau_1 < 0$ and then $\lambda = 1/(1-c)$, which contradicts $\lambda < 1/(1+c)$. Thus in any efficient solution $t_1 = 1$.

To summarize, for allocations where

$$U_2(0,0,0) < U_2(t_2,t_1,\tau_2) \text{ or } \overline{U_2} \leq U_2\left(\frac{\sqrt{(9+8c)}-3}{4}, 0, \frac{3-\sqrt{(9+8c)}}{1+\sqrt{(9+8c)}}\right)$$

efficiency requires $t_1^{**} = 0$, $S^{**} = \tau_1^{**} = 0$, t_2^{**} given by $U_2\left(t_2^{**}, 0, -t_2^{**}/(1+t_2^{**})\right) = \overline{U_2}$, $\tau_2^{**} = -t_2^{**}/(1+t_2^{**})$ and $\lambda = \frac{1}{3+t_2^{**}-2(1+t_2^{**})^2}$. This corresponds to the curved line segment in Figure 1. For allocations where

$$U_2(t_2,t_1,\tau_2) \text{ or } \overline{U_2} > U_2\left(\frac{\sqrt{(9+8c)}-3}{4}, 0, \frac{3-\sqrt{(9+8c)}}{1+\sqrt{(9+8c)}}\right)$$

it is efficient to have $t_1^{**} = 0$, $t_2^{**} = \frac{\sqrt{(9+8c)}-3}{4}$, $\lambda = \frac{1}{1-c}$, τ_2^{**} given by $U_2\left(\frac{\sqrt{(9+8c)}-3}{4}, 0, \tau_2^{**}\right) = \overline{U_2}$, $S^{**} = -R_2\left(\frac{\sqrt{(9+8c)}-3}{4}, \tau_2^{**}\right)$, and τ_1^{**} given by $\tau_1^{**} - w(0,\tau_1^{**}) - S^{**} = 0$. This corresponds to the straight line segment in Figure 1.

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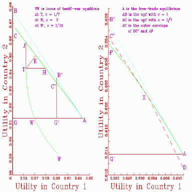

TW is locus of tariff-war equilibria
At T, c = 1/7
At W, c = 0
At W′, c = 1/15
A is the free-trade equilibrium
AB is the upf with c = 0
AC is the upf with c = 1/7
AC is the outer envelope of DC′ and AF
Utility in Country 2
Utility in Country 1

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