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Some Aspects of Incentive-Based Optimal Pricing and Environmental Regulation with Asymmetric Information

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Abstract

The paper aims to analyze the problem of regulating a pollution-generating single product monopolistic firm in the presence of information asymmetry about the firm's cost performance. Following Boyer and Laffont (1999), incentive-based optimal regulation of the firm's price/ output and the environmental performance is characterized when costs are increasing in output and declining in pollution generated during production. Further, the regulatory agency/ legislator may or may not be politically motivated. When he/ she is politically inclined, the process of lobbying assumes that interest groups offer monetary contributions to the regulatory agency or the legislator. These contributions from the lobby help fund election campaigns. Thus, he/ she no longer behaves as a benevolent maximizer of social welfare, but instead maximizes a weighted average of social welfare and welfare of the lobby. Two alternative cases are considered: one, where the lobby represents environmental interests alone, and another, where the lobby stands solely for firm's/ industry's interests. The analysis derives interesting implications for incentive-based regulation of the firm. In general, pricing and environmental performance are distorted for the inefficient firm type under asymmetric information to restrict rents accruing to the efficient firm type. In the presence of the environmental lobby, the politically inclined regulator imposes more stringent environmental regulation under both full information and incomplete information as compared to the no-lobbying case. Interestingly, lobbying by the firm/ industry group also induces the politically motivated regulator to have more restrictive environmental regulation, albeit it now combines it with a higher regulated output for the inefficient firm type under incomplete information vis-à-vis the case of no-lobbying activity.

Keywords: Asymmetric information, incentive regulation, environmental pollution, lobbying.

JEL Classification: D82, D86, L51, Q58.

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1. Introduction

This paper focuses on the problem of incentive-based optimal regulation of pricing/ output and environmental pollution of a polluting single-product monopolistic firm whose cost parameters are not known to the regulator/ legislator. In a regulatory setting, it is plausible to assume that the firm has more information about its cost function, production capabilities and pollution abatement opportunities (or effort toward abatement) than does the regulator. In this paper, we consider the case where firm's costs could be low either due to more efficient technology (firm-type) or due to higher emissions (lower abatement effort). While the regulator can observe costs, he cannot ascribe cost performance to firm's technology type or the level of abatement activity. The optimal regulatory contract specifies output (or price) as well as the allowable pollution level of the monopolistic firm. Further, the analysis is extended to examine whether and how the optimal regulation gets altered in the presence of lobbying by interest groups when the regulator/ legislator is no longer a pure social welfare maximizer. In this case, the regulator/ legislator is influenced by lobbies that operate political contributions to it in return for more favorable regulatory outcomes. Two alternative cases of lobbying are considered: one, where the interest group reflects environmental interest alone and, another, where the firm/ industry lobby is influential for the regulatory process. The environmentalists lobby may prefer a lower level of regulated output and more stringent environmental regulation, while the firm lobby would tend to benefit from a lax environmental regulation and lower output/ higher price. Since the politically inclined regulator would strive to trade-off gains in social welfare (due to a more stringent pollution policy) against loss in rent to the efficient firm as well as loss in rent of the lobby (that would lower his/ her political support) it is not clear *a priori* as to the how lobbying activity would distort the optimal regulatory contract as compared to the case when the regulator is a pure social welfare maximizer.

The emerging regulatory literature has assigned considerable importance to the concerns of asymmetric information, and has utilized a principal-agent framework for the analytical treatment of this issue. For example, Loeb and Magat (1979), Baron and Myerson (1982), Sappington (1983), Guesnerie and Laffont (1984), Maskin and Riley (1984), Riordan (1984), Lewis and Sappington (1988a, 1988b), Laffont and Tirole (1986, 1993), Maggi and Rodriguez-Clare (1995), Laffont and Martimort (2002), Bolton and Dewatripont (2005) all model the principal-agent relationships in the context of economic regulation where the principal is constrained by some private information possessed by the agent, such as that pertaining to the latter's cost, demand, productivity parameter or effort toward cost reduction. Most of these papers derive that due to information asymmetry, the optimal pricing rule and/ or incentive schemes in these second-best situations are distorted as compared to the first-best situation of complete information.

Further, principal-agent frameworks in regulatory literature have been extended to incorporate the concerns of environmental pollution. The application of incentive regulation to environment is appropriate as the regulator does not have complete information about the private benefits that the citizens enjoy if environmental quality is improved and/ or the costs that the firm bears for pollution abatement (see Lewis 1996). Spulber (1988) assumes that the costs of production and pollution abatement are interdependent and firms (operating in a competitive environment) have private information on their costs. The regulatory game involves firms that pursue Bayesian-Nash strategies in communicating with the regulator. The regulatory mechanism specifies both effluent charges and effluent levels for each firm as a function of the cost parameters of all other firms. The trade-off between the benefits of pollution abatement and the benefits from the production of increased market output (and hence higher consumer welfare) is characterized under alternative situations of the budget constraint of the regulator to be binding or not binding. In particular, it is derived that in case of the former, the regulatory mechanism is not individually rational, entailing lower aggregate output and effluent levels under asymmetric information as compared to the full information optimum. Assuming absence of a budget constraint, Lewis (1996) makes important contribution by utilizing incentive-based regulation to evaluate different policy instruments to deal with environmental problems. She shows that optimal mechanism to reduce the rents to the firm results in the loss of efficiency of pollution control. The degree of the regulatory intervention is lowered to limit the rents of the privately informed polluting firms. She considers the optimal regulation of an electric utility's price and pollution. The cost of this firm is unknown to the regulator, although the latter knows the distribution of the unknown technological parameter. Two separate cases are considered – one, where the regulator can observe the level of emissions, and the other where he cannot, since it is costly to do so. It is shown that the amount of output and pollution abatement may be distorted below the efficient level to reduce the rents accruing to the more efficient utility. Boyer and Laffont (1999) also model the regulation of a polluting monopolist firm, which has private knowledge about the cost of realising a public project, and the budget constraint is non-binding. Their results are along the lines of Lewis (1996), where they show that the firm enjoys a rent as it possesses superior information, and the optimal choice of the environmental policy affects this rent. They derive the first- and second-best environmental policy that maximizes social welfare. In the latter case, the regulatory mechanism distorts the level of environmental pollution of the inefficient firm type upward to lower the rent accrual to the more efficient firm type. Lewis and Sappington (1995) follow another line of enquiry. They consider the case where a firm can reduce pollution by restricting its output, however, the capacity to do so differs across firms. They find that it is preferable to regulate output rather than regulating emissions, where the marginal loss in output due to lowering emissions decreases in the level of production. By doing so, the regulator can limit the firm's ability to overstate its cost of foregoing the level of production in terms of lower emissions more effectively than the direct regulation of emissions, as explained by Lewis (1996).

In our paper, we extend the analysis of Spulber (1988) and Boyer and Laffont (1999) to consider regulation of production of a private and differentiated good by a single-product polluting monopolist firm. Unlike Spulber (1988), where each firm takes the market price as given (as the product market is assumed to be competitive), the regulator in our model attempts to control both, output (or price) and the effluent level, of the firm. Moreover, the regulatory mechanism considered here entails no budget balance and the transfers to the firm are funded by distortionary taxes on consumers. Furthermore, our paper differs from Boyer and Laffont (1999) in that instead of a fixed-size public project considered by them, we assume the firm's output to be variable in size. This allows us to characterize optimal regulation of both the price/ output and the environmental performance of the firm.

In addition, to a large extent the above strand of regulatory literature derives optimal regulation in the context of the regulator being benevolent. Boyer and Laffont (1999) is an exception, as their analysis incorporates the role of regulatory capture by the interest groups. If the regulator is assumed to be influenced by special-interest politics, the effect of lobbying on the objective function of the regulator, and hence the implications for the equilibrium regulatory outcomes, needs to be explicitly analyzed. Specifically, Boyer and Laffont (1999) characterize politics by extending their basic model to incorporate the role of regulatory capture by two types of interest groups, represented by stakeholders in the firm and environmentalists respectively. They model the election mechanism explicitly and assume that the environmental policy is delegated to the political majorities pursuing their private agendas. These majorities have different stakes in the information rent of the interest groups and thus favors the groups with which their political interests are aligned. They show that competition between interest groups raises the stakes of political conflict, generating additional distortions and transforms desirable reforms toward delegated incentive mechanism into undesirable reforms. It is shown that different lobby groups may benefit from the capture of the regulator/ government through the size of the information rents that the regulatory mechanism provides due to information asymmetry. For our analysis, we adopt another modeling approach for incorporating lobby behavior as pioneered by Grossman and Helpman (1994). Grossman and Helpman (1994) analyze the behavior of an incumbent government, which is not a benevolent maximizer of social welfare. It is influenced by multiple lobby groups and maximizes a weighted average of pure social welfare and welfare of the lobbies. The lobbies can potentially influence the policy stand of the government by operating campaign contributions that help it win the elections. These contributions induce the government regulator to distort policies in favor of the lobbies. Fredriksson (1997, 1998), Aidt (1998), Maggi and Rodriguez-Clare (1998), Schleich (1999), Damania (2001) and Fredriksson *et al.* (2005) model similar lobbying behavior (in the context of a single lobby or multiple lobbies) to identify optimal pricing and environmental policy. While these papers utilize full information models, we attempt to characterize optimal regulation under asymmetry of information between the regulator and the firm.

Our paper departs from the line of enquiry of Boyer and Laffont (1999) by focusing explicitly on the implication of this lobbying behavior on the optimal regulatory contract emerging in the equilibrium vis-à-vis the benchmark case of no lobbying, and sidesteps the modeling of the election mechanism itself. Further, we consider the effect of one lobby at a time (first, by characterizing the influence of a group that has no stake in the production process but is concerned only about the pollution from production and second, the monopoly firm itself) unlike Boyer and Laffont (1999), who study the effect of competition between these two lobbies in the face of an election process. The effect of politically influential sectors (viz. agriculture) on socially optimal environmental regulation under asymmetric information is also examined by Sheriff (2008). His analysis, however, focuses on the choice of optimal regulatory instruments given both -- informational and political -- constraints.

Given the policy tool-kit comprising regulation of firm's price (or output) and allowable emissions, and provision for government transfers/ subsidies, the key purpose of our paper is to examine how the regulator, maximizing his political support rather than pure social welfare, distorts the optimal regulatory contract. The major findings of the paper are as follows:

1. In the basic model (with no-lobbying activity), both the pricing rule and environmental pollution at the equilibrium are affected by information asymmetry. Further, given the cost-structure, pricing-incentive dichotomy does not hold. That is, incentive concerns are taken care of by using both – the pricing and cost reimbursement rules, induced by incomplete information between the regulator and the firm. Specifically, while the efficient firm's optimal pollution and output is unaffected by information asymmetry, inefficient firm's pollution is distorted upward and its output is altered downward for rent extraction purposes.

2. With lobbying by the environmental group, all the results as in (1) hold. Additionally, a relatively greater weight on the contributions made by the lobby to the regulator, in comparison with pure social welfare, induces the regulator to lower the optimal level of pollution (or adopt more stringent environmental regulation) vis-à-vis the case of no-lobbying. This result holds under both full information and asymmetric information. However, lobbying activity (alone) here has no bearing on the pricing rules or output, with or without full information.
3. With lobbying by the firm/ industry group, the incentive concerns are again taken care of by adjusting both the regulated price and the cost-reimbursement rule i.e. this part of the result in (1) holds. Under asymmetric information, the lobbying activity does not affect the optimal behavior of the efficient-firm type compared to the no-lobby case. But it affects the inefficient type's behavior where, interestingly, the politically inclined regulator restricts the optimal level of pollution and allows for a lower price-cost margin (implying higher regulated output) vis-à-vis the basic case involving no lobbying activity. The rent-seeking regulator may also leave a positive rent to both the firm-types.

The remaining paper is organized as follows. In section 2, the baseline model of incentive-based pricing and environmental regulation is postulated and examined. Here, the regulator acts as a benevolent social welfare maximizer. Next, the analysis is extended to the case where the regulator is politically motivated, and maximizes a weighted sum of private gains and societal welfare. The influence of lobbying by an environmental group is characterized in section 3 and that of the firm/ industry lobby in section 4. In both the cases, we identify the optimal regulatory response of the corruptible regulator. Section 5 concludes.

2. The Basic Model of Environmental and Pricing Regulation of a Single Product Polluting Firm

We consider a monopolistic firm, which produces a differentiated private good with cost function, $C(\beta, d, q)$, where β is the cost characteristic or the efficiency parameter, d is the level of pollution exerted by the firm's production activity and q denotes the level of output. For a given pollution level and output, β measures the efficiency of the firm: a higher β implies higher cost or inefficient-firm type. It is assumed that β is privately known to the firm.

The cost function takes the form, $C = \beta(K - d)q$, which is similar to Boyer and Laffont (1999). C is the total cost and, for a given β and q, we assume that allowing more pollution reduces the cost of the firm. As already stated, for a given d and q, C is increasing in β ($C_\beta > 0$), and $C_{\beta d} < 0$, implying a positive correlation between the ability to produce and ability to reduce pollution. β can take two discrete values $\{\underline{\beta}, \overline{\beta}\}$ with $\overline{\beta} > \underline{\beta}$. K is a constant. Let $\Delta\beta = \overline{\beta} - \underline{\beta}$. $c \equiv \beta(K-d)$ is the marginal cost or the average cost. For a given β, c is decreasing in d. Moreover, $c_\beta > 0$ and $c_{\beta d} < 0$.

The regulator compensates the firm through a transfer. Let t denote the net monetary transfer from the regulator to the firm. Accordingly, the firm's rent is equal to

$$U = t - \beta(K-d)q.$$

We normalize the firm's outside opportunity level of utility or "reservation utility" to zero. Accordingly, the individual rationality or the participation constraint of the firm will be:

$$U \geq 0 \quad \Rightarrow t - \beta(K-d)q \geq 0.$$

Let the good be sold at the (linear) price p. The gross consumer surplus is $S(q)$. We take a general downward sloping demand function $q = Q(p)$ which is invertible, with the inverse demand function $p = P(q) = S'(q)$, as $S(q) = \int_0^q P(\tilde{q})\, d\tilde{q}$. The firm's revenue is $R(q) = P(q)\cdot q$. The aggregate social surplus $V(q)$, is the sum of the net consumers' surplus plus the revenue for the regulator generated by output q of the monopoly firm. The latter is evaluated at the shadow price of public funds denoted by λ, because revenues help the regulator cover the firm's cost and reduce the need for distortionary taxation to operate transfers to the firm. Thus,

$$V(q) = \left[S(q) - R(q)\right] + (1+\lambda)R(q) = S(q) + \lambda R(q) = S(q) + \lambda P(q)\cdot q,$$

where $V(0) = 0$, $V'(q) > 0$ and $V''(q) < 0$. That is, $V(\cdot)$ is concave. Given a downward sloping demand curve, a sufficient condition for $V(\cdot)$ to be concave is that either $P(\cdot)$ be concave or that λ be small enough.[3] The social disutility or damage from pollution is given by the damage function $D(d)$, with $D'(d) > 0$ and $D''(d) > 0$.

The accounting convention is that the regulator collects the revenue from the sales of output and makes the lump sum transfer t to the firm, a part of which are raised through taxation assumed to be

[3] This formulation follows the one in Laffont and Tirole (1993).

distortionary. This inflicts a disutility of $(1+\lambda)t$ on the consumers/taxpayers. The social damage from pollution affects all the consumers equally. That is, the environmental damage is perfectly mixed. The net welfare for the consumers is given by

$$CS = V(q) - D(d) - (1+\lambda)t .$$

We assume that $V(q)$ is large enough so that the production of the good is always desirable from welfare perspective.

In addition, the following implicit assumptions underlie the analysis in this section. First, both the regulator and the firm adopt an optimizing behavior and maximize their individual welfare/ utilities: the regulator maximizes pure social welfare and the firm maximises its private rent. Thus, all the agents are rational. Second, while the regulator does not know the firm's type, it has a prior on or knowledge of the probability distribution of this information, which is also common knowledge. Third, under full information, the regulator maximizes the sum of the net consumers' welfare and the rent of the firm, and under incomplete information, it maximizes the expected value of the same. That is, the regulator follows Bayesian-Nash regulatory strategies. Accordingly, in case of asymmetric information, the regulator moves first (as a Stackelberg leader) anticipating the firm's behavior and specifies a menu of the regulatory contracts. The firm then makes the announcement about its type, the contract is awarded and the firm undertakes production and receives transfers from the regulator. This completes the description of the model. In what follows we characterize the regulatory outcome under full information for the baseline model.

2.1 Full Information

Under full information, the regulator knows the type of the firm i.e. he knows the value of β, and hence, all the components of the cost function. He maximizes social welfare, given by

$$\begin{aligned} W &= CS + U \\ &= V(q) - D(d) - (1+\lambda)t + U . \end{aligned}$$

Substituting for t, this could be expressed as

$$W = V(q) - D(d) - (1+\lambda)\{\beta(K-d)q\} - \lambda U . \qquad (1)$$

Note that in equation (1), the cost of production is evaluated at the shadow price of public funds as by assumption, non-distortionary taxes are absent. The regulator's optimization problem can be written as

$$\underset{d,q,U}{\text{Max}}\ W = V(q) - D(d) - (1+\lambda)\{\beta(K-d)q\} - \lambda U.$$

The first-order conditions are:

$$D'(d) = (1+\lambda)\beta q, \tag{2}$$

$$V'(q) = (1+\lambda)\beta(K-d), \tag{3}$$

$$U = 0. \tag{4}$$

We assume that the second-order conditions, which are: $-D''(d) < 0$, $V''(q) < 0$ and $D''(d)V''(q) + (1+\lambda)^2\beta^2 < 0$, hold.[4]

Equation (2) implies that the regulator sets the optimum pollution at a level where marginal disutility or damage from pollution is equal to its marginal benefit in terms of marginal production cost savings, evaluated at the shadow price of public funds. Equation (3) characterizes the optimum regulated price. According to this equation, the marginal social surplus is equated to the marginal cost of production, again evaluated at the shadow price of public funds.

Differentiating $V(q)$ yields the expression for marginal social surplus as

$$V'(q) = S'(q) + \lambda P'(q)\cdot q + \lambda P(q). \tag{5}$$

Substituting (5) into (3) implies $S'(q) + \lambda P'(q)\cdot q + \lambda P(q) = (1+\lambda)\beta(K-d)$, as $p = P(q) = S'(q)$ and $c = \beta(K-d)$. Rearranging terms yields the optimal pricing rule as

$$\frac{p-c}{p} = \frac{\lambda}{1+\lambda}\cdot\frac{1}{\eta_d}, \tag{6}$$

where $\eta_d = -\dfrac{dq/dp}{q/p}$, is the elasticity of demand. Equation (6) is called the Lerner index, which in this case will be a number between 0 and 1 times the inverse of the elasticity of demand. It implies that the regulated price is set between the competitive price ($p = c$, $\lambda = 0$) and the monopoly price ($L \equiv \dfrac{1}{\eta_d}$, for a large value of λ) as λ is positive and small. It also yields a Ramsey pricing rule where the regulator leaves a positive price-cost margin that depends on the shadow price of public funds. Although the firm does not balance its budget, the positive price-cost margin implies that it is met implicitly. If we denote the equilibrium solution yielded by equations (2) and (3) by $q^*(c)$ and

[4] These three conditions follow from the Hessian being negative semi-definite, implying V"<0, D">0 and both sufficiently large.

the corresponding price by $p^*(c) = P(q^*(c))$, these will be called the Ramsey output and price, respectively. Lastly, equation (4) implies that the regulator leaves no rent to the firm. The intuition being, with positive shadow price of public funds, rent is socially costly. Since the transfer to the firm is financed out of distortionary taxes, the regulator pushes it down to zero. These results conform to the standard results in the regulation of a monopoly firm (see Laffont and Tirole, 1993, pp 133-4).

2.2 Asymmetric Information

Under asymmetry of information, while the regulator does not know the specific type of the firm, he is assumed to have knowledge of the probability distribution of the parameter β. He has prior knowledge of the value of this efficiency parameter characterized by $\nu = \Pr(\beta = \underline{\beta})$. Given our assumption on $V(q)$ being large, the regulator will want both the type of firms to participate. He/ she observes the realised cost C and makes a transfer to the firm. The regulator wants the firm to reveal its true type. Hence the incentive compatibility (IC) condition is imposed, which is tantamount to truth-telling by the firm. It implies that the firm has no incentive to misreport its true type as it enjoys a rent which is at least as high as it would have enjoyed by pretending to be the other type. Thus, the contract preferred by the efficient type $\underline{\beta}$ (respectively type $\overline{\beta}$) is the one designed for the efficient type $\underline{\beta}$ (respectively type $\overline{\beta}$). Let $U(\beta) = t(\beta) - \beta(K - d(\beta))q(\beta)$ denote the rent for the type β when it selects the contract designed for it. For notational simplicity, we use $\underline{t} = t(\underline{\beta})$, $\underline{d} = d(\underline{\beta})$, $\underline{q} = q(\underline{\beta})$ for the efficient type and similar notations for the inefficient type. Then, the IC constraints for the two types are:

$$\underline{t} - \underline{\beta}(K - \underline{d})\underline{q} \geq \bar{t} - \underline{\beta}(K - \overline{d})\overline{q}, \text{ for the } \underline{\beta}\text{-type, and} \tag{7}$$

$$\bar{t} - \overline{\beta}(K - \overline{d})\overline{q} \geq \underline{t} - \overline{\beta}(K - \underline{d})\underline{q} \text{ for the } \overline{\beta}\text{-type.} \tag{8}$$

Next, the individual rationality (IR) constraints imply that the firm will participate irrespective of its type. We write the two IR constraints as

$$\underline{U} = \underline{t} - \underline{\beta}(K - \underline{d})\underline{q} \geq 0, \tag{9}$$

$$\overline{U} = \bar{t} - \overline{\beta}(K - \overline{d})\overline{q} \geq 0. \tag{10}$$

In view of $\underline{t} = \underline{U} + \underline{C}$, and $\bar{t} = \overline{U} + \overline{C}$, the regulator, in this case, maximizes the expected social welfare

$$EW = \nu\left[V\left(\underline{q}\right) - D\left(\underline{d}\right) - \left(1+\lambda\right)\left\{\underline{\beta}\left(K - \underline{d}\right)\underline{q}\right\} - \lambda \underline{U}\right]$$
$$\underline{d}, \overline{d}, \underline{q}, \overline{q}, \underline{U}, \overline{U} \qquad + \left(1-\nu\right)\left[V\left(\overline{q}\right) - D\left(\overline{d}\right) - \left(1+\lambda\right)\left\{\overline{\beta}\left(K - \overline{d}\right)\overline{q}\right\} - \lambda \overline{U}\right], \qquad (11)$$

subject to (7), (8), (9) and (10).

Satisfying the IC for the efficient type and IR for the inefficient type will imply that IR for the efficient type is met. Since $\Delta\beta\left(\equiv \overline{\beta} - \underline{\beta}\right)$ is positive, we use (7) and (10) successively into (9) to show that

$$\underline{U} \geq \bar{t} - \underline{\beta}\left(K - \overline{d}\right)\overline{q} \geq \overline{\beta}\left(K - \overline{d}\right)\overline{q} - \underline{\beta}\left(K - \overline{d}\right)\overline{q} = \Delta\beta\left(K - \overline{d}\right)\overline{q} \geq 0 .$$

Thus (9) is automatically satisfied and we can ignore this constraint. This is because the efficient type can always pretend to be the inefficient type at a lower cost (due to its better technology type). We can write the IC constraint for the efficient type as

$$\underline{U} \geq \bar{t} - \underline{\beta}\left(K - \overline{d}\right)\overline{q} = \overline{U} + \Delta\beta\left(K - \overline{d}\right)\overline{q}, \qquad (12)$$

which implies that the rent of the $\underline{\beta}-$type firm is increasing in the output and declining in the level of pollution of the $\overline{\beta}-$type firm.

The efficient type can always mimic to be the inefficient type by choosing the pollution level, transfer and output of the latter and produce at a cost $\underline{\beta}\left(K - \overline{d}\right)\overline{q}$, which will enable it to enjoy a profit or rent of at least $\Delta\beta\left(K - \overline{d}\right)\overline{q}$. As is evident from the inequality in (12), when the efficient type acts according to its true type, it will receive at least $\Delta\beta\left(K - \overline{d}\right)\overline{q}$ of positive rent even when the inefficient type's rent is fully extracted. Since transfers are distortionary, the rents that accrue to the firm are socially costly. The IC constraint for the efficient type and the IR constraint for the inefficient type would thus bind at the optimum. This leaves us with the reduced form of maximization of the social welfare subject to the constraints (10) and (12),[5] both of which will hold with strict equality at the optimum. Therefore, we substitute $\underline{U} = \Delta\beta\left(K - \overline{d}\right)\overline{q}$ and $\overline{U} = 0$ into (11) and rewrite the regulator's objective function as

[5] We can ignore the IC constraint for the inefficient type (inequality in equation (8)) for the moment and show that it will be satisfied with the solution of the maximization with the binding IC for the efficient type.

$$\underset{\{\underline{d},\overline{d},\underline{q},\overline{q},\}}{\text{Max}}\ EW = \nu\left[V\left(\underline{q}\right) - D\left(\underline{d}\right) - (1+\lambda)\left\{\underline{\beta}\left(K-\underline{d}\right)\underline{q}\right\} - \lambda\Delta\beta\left(K-\overline{d}\right)\overline{q}\right] + (1-\nu)\left[V\left(\overline{q}\right) - D\left(\overline{d}\right) - (1+\lambda)\left\{\overline{\beta}\left(K-\overline{d}\right)\overline{q}\right\}\right].$$

The solution is characterized by the following four first-order conditions

$$D'\left(\underline{d}\right) = (1+\lambda)\underline{\beta}\cdot\underline{q}, \tag{13}$$

$$D'\left(\overline{d}\right) = (1+\lambda)\overline{\beta}\cdot\overline{q} + \frac{\nu}{1-\nu}\lambda\Delta\beta\overline{q}, \tag{14}$$

$$V'\left(\underline{q}\right) = (1+\lambda)\underline{\beta}\left(K-\underline{d}\right), \tag{15}$$

$$V'\left(\overline{q}\right) = (1+\lambda)\overline{\beta}\left(K-\overline{d}\right) + \frac{\nu}{1-\nu}\lambda\Delta\beta\left(K-\overline{d}\right). \tag{16}$$

We provide our observations in terms of the following propositions:

Proposition 1. At the asymmetric information equilibrium, the optimal level of pollution for the inefficient firm type is higher than that of the efficient type. The inefficient firm also sells a smaller output than the efficient firm. That is, $\underline{d} < \overline{d},$ and $\underline{q} > \overline{q},$ or $\dot{d}(\beta) > 0$ and $\dot{q}(\beta) < 0.$

Proof. The above equilibrium conditions in (13)-(16) can be expressed in terms of the parameter β as follows:

$$D'(d) = (1+\lambda)\beta q + \frac{\nu}{1-\nu}\lambda\left(\beta - \underline{\beta}\right)q,$$

$$V'(q) = (1+\lambda)\beta(K-d) + \frac{\nu}{1-\nu}\lambda\left(\beta-\underline{\beta}\right)(K-d),$$

such that as $\beta = \underline{\beta},$ the above two equations collapse to equations (13) and (15) respectively, and at $\beta = \overline{\beta},$ these are as in (14) and (16). The rankings of optimal pollution, output and costs for the two types are derived by doing comparative statics with respect to this parameter.[6] It is found that with the social value function $V(q)$ sufficiently concave and the pollution damage function $D(d)$ sufficiently convex, optimal pollution is increasing in β and optimal output is decreasing in β.[7] However, costs are not monotonic in β. The cost for the inefficient type may be higher or lower than for the efficient type depending on the parameteric configuration of the cost function. Since the positive rent for the efficient type decreases in the pollution level of the inefficient type,

[6] See appendix A for the detailed mathematical proof.
[7] We require $V'' < 0$ and $D'' > 0$ and their absolute magnitude to be sufficiently large for the second-order conditions to hold.

optimal regulation suggests a higher pollution level for the latter so that the rent to the former can be lowered. On the other hand, the rent for the efficient type is an increasing function of the output of the inefficient type. So, the regulator induces a lower equilibrium output for the latter by allowing a higher price-cost margin for it.[8]

Further, we find that

Proposition 2. For the inefficient firm, the optimal pricing rule (as characterized by (16)) and the level of environmental pollution (determined by (14)) are both affected by information asymmetry. For the $\overline{\beta}-$type firm, the choice of the optimal pollution and output are distorted upward and downward respectively under asymmetric information. The behavior of the efficient type, however, is not affected by information asymmetry.

Proof. Similar to the line of the proof of Proposition 1, we can now characterize the move from full information to asymmetric information equilibrium for the $\overline{\beta}-$type firm by expressing the first-order conditions as follows:

$$D'(\overline{d}) = (1+\lambda)\overline{\beta}.\overline{q} + \frac{\nu}{1-\nu}\lambda\xi(\overline{\beta}-\underline{\beta})\overline{q},$$

$$V'(\overline{q}) = (1+\lambda)\overline{\beta}(K-\overline{d}) + \frac{\nu}{1-\nu}\lambda\xi(\overline{\beta}-\underline{\beta})(K-\overline{d}).$$

Note that for $\beta = \overline{\beta}$ and $\xi = 0,$ the above two represent the full information equilibrium conditions (2) and (3) respectively, and as $\xi = 1,$ these collapse into (14) and (16), which are the first-order conditions at the asymmetric information equilibrium. Thus, carrying out the comparative statics with respect to ξ provides an indication of the change in the allowable pollution and output due to a discrete move from full information to asymmetric information equilibrium. It is derived that for the inefficient firm-type, the regulated pollution is raised and output is lowered at the asymmetric information equilibrium as compared to that at the full information equilibrium. This holds since $V'' < 0$ and $D'' > 0$ and their absolute magnitudes are assumed to be large enough.[9] Intuitively, as the rent accruing to the efficient type is increasing in the level of output and decreasing in the level of pollution of the inefficient type, the regulator responds by distorting the latter's pollution level upward and regulated output downward. Thus, the inefficient type's optimal pollution and sale of

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[8] A higher price-cost margin implying a lower level of sales of output directly follows from the inverse demand function.
[9] See Appendix B for a detailed proof.

output are distorted upward and downward respectively to reduce the rent accruing to the efficient type under asymmetric information. The contractual rules governing the optimal price and environmental pollution for the efficient firm-type remain the same as under full information (see the qualitative similarity between equations (13) and (2) and between equations (15) and (3)).

With the optimal solutions as in equations (13)-(16), it is easy to check that the regulatory contract is incentive compatible for the inefficient type, as the IC for the efficient type binds. Expressing (8) as $\overline{U} \geq \underline{t} - \overline{\beta}\left(K - \underline{d}\right)\underline{q} = \underline{U} - \Delta\beta\left(K - \underline{d}\right)\underline{q}$ and substituting (12) with equality on the right hand side, we get the condition

$$0 \geq \Delta\beta\left[\left(K - \overline{d}\right)\overline{q} - \left(K - \underline{d}\right)\underline{q}\right]. \tag{17}$$

This is satisfied at the optimum as $\underline{d} < \overline{d}$, $\underline{q} > \overline{q}$ (from Proposition 1) and $\Delta\beta$ is positive.

The baseline model is now extended to include lobbying activity by a group of people in society who care for the environmental externality arising due to the production activity of the monopoly firm. The lobby has no stake either in the consumption of the polluting commodity or in the production activity of the firm. This helps us abstract away from the effects of lobbying on the consumer surplus and producer surplus, and derives more stark conclusions.[10] The regulator/ legislator receives campaign contributions from this environmental lobby and contributions are used to sway the voters in his favor in an unmodeled election process.

3. Regulation in the Presence of Political Pressure by an Environmental Lobby Group

It is now assumed that from the existing population, one group overcomes the free rider problem of collective action and coordinates its activities to form the environmental lobby. As Aidt (1998) argues in his paper, the implicit assumption is that these people might face lower organizational cost than others to justify how they are able to coordinate perfectly while others do not. Following Aidt (1998) and Boyer and Laffont (1999), we model this as a functionally specialized lobby, driven only by their environmental interest and having no stake in consumption or in the polluting firm. It is the consumers who pay the tax to enable transfers by the regulator. We call them consumer-taxpayers (as in Boyer and Laffont (1999)) to distinguish them from those members of the population who

[10] Other papers such as Aidt (1998) and Boyer and Laffont (1999) also assume this. They, however, model the competition amongst the lobby groups in a regulatory setting, which is beyond the scope of this paper.

constitute the environmental lobby. The consumers are the free riders, as concerns about environmental pollution are addressed entirely by the activity of the self-interested environmental lobby. The net welfare for the taxpayers is the social surplus generated by the production process minus the tax that the regulator collects from them. Since taxation is distortionary, we write the net welfare of taxpayers as:

$$U_1 = V(q) - (1+\lambda)t .$$

The environmental lobby suffers disutility from pollution. As earlier, we take the environmental damage function to be $D = D(d)$, with $D' > 0$, $D'' > 0$. The lobby operates monetary contributions to the regulator to influence him to protect its interest. As in Boyer and Laffont (1999), we assume that the contributions offered by the lobby are a fixed proportion of the rent accruing to them and, for tractability; it is assumed that the contributions are non-distortionary. Denoting the rent for the lobby as U_2, their contribution or offer is given by $o = \mu U_2$, where μ is exogenous. Let the lobby have a fixed endowment E (or income from an exogenous source). The rent for it will then be equal to

$$U_2 = E - D(d) - o .$$

We assume that this fixed endowment or exogenous income of the lobby is large enough for its members to operate the contributions/ bribe to the regulator. Substituting for $o(\cdot)$, we can express the lobby rent as

$$U_2 = \frac{1}{1+\mu}\left[E - D(d)\right].$$

The above equation implies that the contribution made by the lobby, $o = \frac{\mu}{1+\mu}\left[E - D(d)\right]$, will be decreasing in pollution. This implication is compatible with our assumption that the lobby offers a fixed proportion of their rent, which declines as the pollution increases.

3.1 Full Information

As in Maggi and Rodriguez-Clare (1998) we assume that the regulator’s/ legislator’s objective is to maximize a weighted sum of social welfare (sum of the net welfare of the consumers/ taxpayers, the lobby and the net rent for the firm) and contributions from the lobby. Thus, he/ she maximizes

$$\Omega = \alpha W + (1-\alpha)o , \tag{18}$$

where $W = U_1 + U_2 + U$

$= V(q) + \frac{1}{1+\mu}\left[E - D(d)\right] - (1+\lambda)t + U$, where U is the firm's rent as defined in the basic model and $o(.)$ are political contributions as defined earlier in this section. The parameters α and $(1-\alpha)$ in (18) denote the weights that the regulator assigns to social welfare and contributions received by him, respectively. We normalize the objective function and express it as $\Omega' = \frac{\Omega}{\alpha} = W + ao$, where $a = \frac{1-\alpha}{\alpha}$ is the relative weight assigned to the contributions. A higher level of a implies that the regulator is more politically inclined. Substituting for W and $o = \mu U_2$, we can write

$$\Omega' = V(q) + \frac{1+a\mu}{1+\mu}\left[E - D(d)\right] - (1+\lambda)t + U.$$

From our basic model, substitution for t gives the final expression for the objective function as

$$\Omega' = V(q) + \frac{1+a\mu}{1+\mu}\left[E - D(d)\right] - (1+\lambda)\left\{\beta(K-d)q\right\} - \lambda U. \qquad (19)$$

The regulator maximizes the above objective function subject to the constraint that the firm participates ($U \geq 0$). Thus, his/ her problem becomes

$$\underset{d,q,U}{\text{Max}}\ \Omega' = V(q) + \frac{1+a\mu}{1+\mu}\left[E - D(d)\right] - (1+\lambda)\left\{\beta(K-d)q\right\} - \lambda U.$$

The first-order conditions are given by the following three equations:

$$D'(d) = \frac{(1+\lambda)(1+\mu)}{1+a\mu}\beta q, \qquad (20)$$

$$V'(q) = (1+\lambda)\beta(K-d), \qquad (21)$$

$$U = 0. \qquad (22)$$

The second-order condition requires that the Hessian $\begin{bmatrix} -\frac{1+a\mu}{1+\mu}D''(d) & (1+\lambda)\beta \\ (1+\lambda)\beta & V''(q) \end{bmatrix}$ be negative semidefinite.

It is straightforward to see that equations (21) and (22) are analogs of equations (3) and (4) respectively in the basic model with no lobbying. Thus, the rule governing optimum pricing regulation is not affected due to politics. The only difference is that now the behavior of the regulator incorporates his valuation of the environmental lobby's contribution to him/ her and this is captured in equation (20), signifying the extent of adjustment in environmental regulation. This is stated in terms of the next proposition.

Proposition 3. If the regulator assigns a relatively greater weight to contributions from the lobby compared to the weight on pure social welfare, that is, $a > 1$ (which is the most plausible assumption for an electorally motivated regulator) then, under full information, both the pollution and output specified by the regulation mechanism are smaller in the presence of the environmental lobby vis-à-vis the situation of no-lobbying.

Proof. Notably, an equal weight on both – pure social welfare and lobby contributions -- would yield the optimal level of pollution and output exactly equal to that in no-lobby case. That is, for $a = 1$, the right hand side of (20) is equal to that of (2). However, for our case, we have $1-\alpha > \alpha$, implying $a > 1$. Thus, equilibrium pollution and output (with lobbying) yielded by simultaneously solving equations (20) and (21) can be compared with the corresponding solutions with no-lobbying (as derived by solving equations (2) and (3)) by doing comparative statics with respect to the parameter a. It is derived that equilibrium pollution is declining in a, that is, $dd/da < 0$, while, from (21) it is easy to check that, given $V'' < 0$, the equilibrium output is increasing in pollution, or $dq/dd > 0$.[11] Thus, lobbying lowers the equilibrium pollution (as also the regulated output) as compared to no-lobbying. The intuition is that the regulator, driven by the motivation to maximize a weighted sum of social welfare and the welfare of the environmental interest group, will be induced to adopt more stringent environmental regulation when the relative weight assigned to the latter is higher. This is because he/ she can then pocket a larger amount of contributions from the lobby. Since contributions are decreasing in pollution, the regulator lowers the optimal pollution to induce higher contributions. The output reduces as marginal costs increase due to lower pollution, and the regulator now trades-off higher costs with benefits at the margin.

3.2 Asymmetric Information

In this case, the regulator does not know the firm type, but as before, he/ she is assumed to have the knowledge about the distribution of the unknown cost parameter of the firm, β. He/ she maximizes an expected sum of pure social welfare and political contributions offered by the environmental lobby. Since there is no change in the behavior of the firm, the IC and IR constraints will remain the same as in the baseline model. Thus, the regulator solves the following optimization problem:

[11] Appendix C contains the mathematical proof.

$$\begin{aligned}&\text{Max } E\Omega' = \nu\left[V\left(\underline{q}\right)+\frac{1+a\mu}{1+\mu}\left\{E-D\left(\underline{d}\right)\right\}-\left(1+\lambda\right)\left\{\underline{\beta}\left(K-\underline{d}\right)\underline{q}\right\}-\lambda\underline{U}\right]\\ &\underline{d},\overline{d},\underline{q},\overline{q},\underline{U},\overline{U} \qquad +\left(1-\nu\right)\left[V\left(\overline{q}\right)+\frac{1+a\mu}{1+\mu}\left\{E-D\left(\overline{d}\right)\right\}-\left(1+\lambda\right)\left\{\overline{\beta}\left(K-\overline{d}\right)\overline{q}\right\}-\lambda\overline{U}\right],\end{aligned}$$

subject to (7), (8), (9) and (10).

Going by the same argument as provided before, the relevant constraints for the maximization problem will be the IC for the efficient type and the IR for the inefficient type, both of which will bind at the optimum. Substituting these two constraints with strict equality (as the regulatory agency dislikes leaving rent to each type) yields the regulator's problem as

$$\begin{aligned}&\text{Max } E\Omega' = \nu\left[V\left(\underline{q}\right)+\frac{1+a\mu}{1+\mu}\left\{E-D\left(\underline{d}\right)\right\}-\left(1+\lambda\right)\left\{\underline{\beta}\left(K-\underline{d}\right)\underline{q}\right\}-\lambda\Delta\beta\left(K-\overline{d}\right)\overline{q}\right]\\ &\underline{d},\overline{d},\underline{q},\overline{q} \qquad +\left(1-\nu\right)\left[V\left(\overline{q}\right)+\frac{1+a\mu}{1+\mu}\left\{E-D\left(\overline{d}\right)\right\}-\left(1+\lambda\right)\left\{\overline{\beta}\left(K-\overline{d}\right)\overline{q}\right\}\right].\end{aligned}$$

The first order conditions are:

$$D'\left(\underline{d}\right)=\frac{\left(1+\lambda\right)\left(1+\mu\right)}{1+a\mu}\underline{\beta}\cdot\underline{q}\,, \tag{23}$$

$$D'\left(\overline{d}\right)=\frac{\left(1+\lambda\right)\left(1+\mu\right)}{1+a\mu}\overline{\beta}\cdot\overline{q}+\frac{\nu}{1-\nu}\cdot\frac{\lambda\left(1+\mu\right)}{1+a\mu}\Delta\beta\overline{q}\,, \tag{24}$$

$$V'\left(\underline{q}\right)=\left(1+\lambda\right)\underline{\beta}\left(K-\underline{d}\right), \tag{25}$$

$$V'\left(\overline{q}\right)=\left(1+\lambda\right)\overline{\beta}\left(K-\overline{d}\right)+\frac{\nu}{1-\nu}\lambda\Delta\beta\left(K-\overline{d}\right). \tag{26}$$

It is easy to show that even at this political equilibrium (with asymmetric information) results similar to those in propositions 1 and 2 hold. This can be demonstrated by comparing the right hand sides in equations (23) and (24) with that in equation (20) and comparing the same in equations (25) and (26) with that in equation (21). Since, from the IC for the efficient type we have the rent of this type being negatively related to the level of allowable (regulated) pollution of the inefficient type, and rents are socially costly, the optimal response of the regulator is to distort the inefficient type's pollution upward. On the other hand, the level of output of the inefficient type positively affects the rent of the efficient firm-type, and therefore, the optimal contract distorts the inefficient type's output downward. However, the conditions characterizing the optimal level of pollution and pricing rule for the efficient type are not affected by information asymmetry.

Proposition 4. The optimal pollution (output) for the inefficient type is higher (lower) than that of the efficient type. That is $d(\beta)$, is increasing and $q(\beta)$, decreasing in the firm type, or, $\dot{d}(\beta) > 0$ and $\dot{q}(\beta) < 0$.

Proof. The method of proof is similar to that for the no-lobby case (see Proposition 1), which is provided in Appendix A.[12]

Furthermore, in moving from no-lobbying to lobbying, the equilibrium rules governing the pricing/ output regulation for neither of the firm-type are affected. A comparison of (25) with (15) for the efficient type and (26) with (16) for the inefficient type show that, for both types, the equilibrium pricing equations are qualitatively the same. However, as will be shown later, this does not imply that the level of output or price is unchanged due to lobbying. However, the conditions characterizing the stringency of environmental regulation are influenced by lobbying for both the firm types. Hence,

Proposition 5. As the regulator assigns a relatively large weight on the contributions compared to the pure social welfare, i.e. $a > 1$, the environmental lobby induces a lower level of equilibrium pollution for both the types vis-à-vis the case of no lobbying. Thus, the distortion in the inefficient type's permissible level of pollution is smaller vis-à-vis the no-lobbying case.

Proof. This follows from the comparison of the equations (23) and (13) for the efficient type and (24) and (14) for the inefficient type. Comparative statics with respect to parameter a, along the line similar to that in Appendix C yield that both $d\underline{d}/da$ and $d\overline{d}/da$ are negative in sign, implying a lowering of pollution of both the firm types due to lobbying by the environmental group. That $d\underline{q}/d\underline{d} > 0$ and $d\overline{q}/d\overline{d} > 0$ follows from differentiating (25) and (26), and using $V'' < 0$. Given regulator's political inclinations, adopting a more stringent environmental policy for both the types entails capturing higher monetary contributions from the environmental lobby. A more stringent environmental policy induces higher production costs for both the firm types, thus lowering their regulated output.

[12] See Appendix D, however, for the final expressions for $\dot{d}(\beta)$ and $\dot{q}(\beta)$ derived for this case.

Further, since the rent-seeking behavior of the regulator from the environmental lobby tends to partially counter his/ her rent-reduction (accruing to the efficient firm-type) motivation, the regulator will distort the equilibrium allowable pollution of the inefficient type by a lower magnitude in the presence of the environmental lobby vis-à-vis the case of no lobbying. This is captured by the second term in the right hand side of equation (24), which is now smaller than its counterpart in equation (14). Specifically, with $a > 1$, $\frac{\nu}{1-\nu} \cdot \frac{\lambda(1+\mu)}{1+a\mu} \Delta\beta\bar{q} < \frac{\nu}{1-\nu} \lambda\Delta\beta\bar{q}$. The efficient type, would therefore, derive a relatively higher rent in comparison with the baseline case of no-lobbying activity. Thus, at the margin, lower efficiency will be traded off against higher political contributions.

In the next section we consider the case of political lobbying by the monopoly firm. The regulator/ legislator here receives the contributions from the firm while also regulating its activities.

4. Regulation in the Presence of Political Pressure by the Industry Lobby Group

Let the monopolistic firm now constitute the special-interest group or lobby. The lobby is assumed to be functionally specialized, in that, it cares only about its rent/ surplus. A priori the firm would tend to lobby for higher permissible pollution or lower output (that is, higher price) as both of these imply higher rent. The firm receives a net monetary transfer, t, from the regulator. But, it now returns to it political contributions that are related to the specific policy stance of the regulatory agency.

Following Boyer and Laffont (1999), we assume that the firm-lobby offers a fixed proportion of its net rent as contributions to the regulator. (This is similar to the case in the last section where there was an environmental lobby.) Let these contributions be denoted by:

$$\hat{o} = \sigma U ,$$

where U is the rent accruing to the firm and σ is the fixed proportion of the rent that the firm offers to the regulatory agency. We assume that σ is exogenously given. Firm's net rent can be expressed as

$$U = t - \beta(K - d)q - \hat{o} .$$

Substituting for $\hat{o}(.)$, the above will be $U = \frac{1}{1+\sigma}\left[t - \beta(K-d)q\right]$, implying that the contributions operated by the firm-lobby are increasing in the level of allowable pollution and declining in the level of output (since firm's costs are declining in environmental pollution and increasing in output). Consequently, it earns a higher rent and its offers also increase as a fixed proportion of that rent.

Assuming reservation utility to be zero, the IR or the participation constraint of the firm in this case will be

$$U \geq 0 \quad \Rightarrow \frac{1}{1+\sigma}\left[t-\beta\left(K-d\right)q\right] \geq 0 .$$

Unlike the previous section, we do not differentiate between the population of consumers and the environmental group. In this case, the firm-lobby is the only lobby that is modeled. Consequently, as regards the consumers, we are back to the basic case where the net welfare for the consumers is the social value of output, $V(q)$, minus the sum of disutilities they suffer on account of pollution and shadow value of tax paid to the regulator to enable transfers to the firm. Mathematically, it is expressed as $CS = V\left(q\right) - D\left(d\right) - \left(1+\lambda\right)t$.

4.1 Full Information

In this case, the regulator maximizes a weighted sum of social welfare and contributions from the firm/ industry lobby. Social welfare is the consumers' net welfare (including damage from environmental pollution) plus the rent of the firm. Denoting the weights that the regulator assigns to social welfare and to the contributions as α and $\left(1-\alpha\right)$ respectively, his/ her objective function will be

$$\Omega = \alpha W + \left(1-\alpha\right)\hat{o} .$$

In the next step, we normalize the objective function to express it as

$$\Omega' = \frac{\Omega}{\alpha} = W + a\hat{o} ,$$

where the relative weight placed on the contributions is given by a . $a = \frac{1-\alpha}{\alpha}$. As earlier, a higher value of a implies a more politically inclined regulator. The social welfare will be

$$W = CS + U \quad = V\left(q\right) - D\left(d\right) - \left(1+\lambda\right)t + U ,$$

Substituting for W from the above expression and with $\hat{o} = \sigma U$ in Ω', we get

$$\Omega' = V\left(q\right) - D\left(d\right) - \left(1+\lambda\right)t + U + a\sigma U .$$

The regulator maximizes the above objective function subject to the IR constraint, $U \geq 0$.

Putting $U = \frac{1}{1+\sigma}\left[t-\beta\left(K-d\right)q\right]$, the regulator's problem can be written as the following Lagrangean

$$\underset{d,q,t}{\text{Max}}\ V(q) - D(d) - (1+\lambda)t + \frac{1+a\sigma}{1+\sigma}\left[t - \beta(K-d)q\right] + \tilde{\lambda}\cdot\frac{1}{1+\sigma}\left[t - \beta(K-d)q\right],$$

where $\tilde{\lambda}$ is the non-negative Lagrangean multiplier associated with the IR constraint. The first order conditions are:

$$D'(d) = \frac{1+a\sigma}{1+\sigma}\beta q + \tilde{\lambda}\cdot\frac{1}{1+\sigma}\beta q, \tag{27}$$

$$V'(q) = \frac{1+a\sigma}{1+\sigma}\beta(K-d) + \tilde{\lambda}\cdot\frac{1}{1+\sigma}\beta(K-d), \tag{28}$$

$$\tilde{\lambda} = (1+\lambda)(1+\sigma) - (1+a\sigma), \tag{29}$$

$$\tilde{\lambda}\cdot\frac{1}{1+\sigma}\left[t - \beta(K-d)q\right] = 0, \tag{30}$$

$$\tilde{\lambda} \geq 0, \tag{31}$$

$$\frac{1}{1+\sigma}\left[t - \beta(K-d)q\right] \geq 0. \tag{32}$$

The last three conditions pertain to complementarity slackness.

Two situations can emerge here. The first is when the Lagrangean multiplier takes a strictly positive value, that is, $\tilde{\lambda} > 0$. Then, the condition in equation (30) implies that the IR condition in (32) will hold with strict equality or it will bind. In other words, $U = 0$. For $\tilde{\lambda} > 0$, the right hand side of (29) implies the parametric restriction $\frac{1}{\sigma} + 1 > \frac{a-1}{\lambda}$. As we have earlier assumed λ to be small enough, (a parametric restriction warranted by $V(\cdot)$ to be concave), satisfying this inequality requires that σ be small enough, when $a > 1$. Thus, even when the regulator assigns a relatively higher weight on political contributions as compared to pure social welfare (as $a > 1 \Rightarrow (1-\alpha) > \alpha$), this condition entails that he/ she will still push the rent accruing to the firm to zero. This is because the latter offers a very small proportion of its rent to the regulator. In spite of being politically inclined, the regulator receives no contribution in this case since the firm is left with zero rent and the optimal level of contribution will be zero at the political equilibrium.

The other case is that of the multiplier assuming a zero value. Condition (30) then implies that the IR constraint for the firm is not binding or U will be positive. A politically inclined regulator here leaves a positive rent to the firm to maximize the contributions it receives. Hence, in this case,

$\hat{o} = \sigma U$ will be positive. Also, from (29), it requires that the right hand side would be equal to zero. Only if the value of the parameter a satisfies this condition, for a given λ and σ, U will be greater than zero. Thus, under full information, when the regulator receives contributions from the firm, positive rents are permitted to the firm, in spite of these being socially costly, and the regulator receives a positive contribution from the lobby.

4.2 Asymmetric Information

As before, under incomplete information the costs are privately known to the firm, while the regulator is assumed to have the knowledge of the distribution of the cost parameter β. The probability distribution of β is given by $\nu = \Pr\left(\beta = \underline{\beta}\right)$. The regulator maximizes his objective function subject to the ICs and IRs. The ICs for the two types in this case are

$$\frac{1}{1+\sigma}\left[\underline{t} - \underline{\beta}\left(K - \underline{d}\right)\underline{q}\right] \geq \frac{1}{1+\sigma}\left[\bar{t} - \underline{\beta}\left(K - \bar{d}\right)\bar{q}\right],$$

$$\frac{1}{1+\sigma}\left[\bar{t} - \bar{\beta}\left(K - \bar{d}\right)\bar{q}\right] \geq \frac{1}{1+\sigma}\left[\underline{t} - \bar{\beta}\left(K - \underline{d}\right)\underline{q}\right].$$

The corresponding IRs are given by

$$\underline{U} = \frac{1}{1+\sigma}\left[\underline{t} - \underline{\beta}\left(K - \underline{d}\right)\underline{q}\right] \geq 0,$$

$$\bar{U} = \frac{1}{1+\sigma}\left[\bar{t} - \bar{\beta}\left(K - \bar{d}\right)\bar{q}\right] \geq 0.$$

As earlier, it is easy to show that the IC for the efficient type and IR for the inefficient type always guarantee IR for the efficient type. Further, it will be proved later that, at the political optimum, IC for the inefficient type will hold. Thus, the optimization program of the regulator will include the following two constraints only: IC for the efficient type and the IR for the inefficient type. The corresponding Lagrangean expression will be

$$\begin{aligned}\max_{\underline{d},\bar{d},\underline{q},\bar{q},\underline{t},\bar{t}} \; & \nu\left[V\left(\underline{q}\right) - D\left(\underline{d}\right) - (1+\lambda)\underline{t} + \frac{1+a\sigma}{1+\sigma}\left\{\underline{t} - \underline{\beta}\left(K - \underline{d}\right)\underline{q}\right\}\right] \\ & + (1-\nu)\left[V\left(\bar{q}\right) - D\left(\bar{d}\right) - (1+\lambda)\bar{t} + \frac{1+a\sigma}{1+\sigma}\left\{\bar{t} - \bar{\beta}\left(K - \bar{d}\right)\bar{q}\right\}\right] \\ & + \tilde{\lambda}_1 \cdot \frac{1}{1+\sigma}\left[\underline{t} - \underline{\beta}\left(K - \underline{d}\right)\underline{q} - \bar{t} + \underline{\beta}\left(K - \bar{d}\right)\bar{q}\right] + \tilde{\lambda}_2 \cdot \frac{1}{1+\sigma}\left[\bar{t} - \bar{\beta}\left(K - \bar{d}\right)\bar{q}\right]\end{aligned}$$

where $\tilde{\lambda}_1$ and $\tilde{\lambda}_2$ are the Lagrangean multipliers associated with the two constraints respectively. The first order conditions are:

$$D'(\underline{d}) = \frac{1+a\sigma}{1+\sigma}\underline{\beta}\cdot\underline{q} + \frac{\tilde{\lambda}_1}{\nu(1+\sigma)}\underline{\beta}\cdot\underline{q}, \tag{33}$$

$$D'(\bar{d}) = \frac{1+a\sigma}{1+\sigma}\bar{\beta}\cdot\bar{q} - \frac{\tilde{\lambda}_1}{(1-\nu)(1+\sigma)}\underline{\beta}\cdot\bar{q} + \frac{\tilde{\lambda}_2}{(1-\nu)(1+\sigma)}\bar{\beta}\cdot\bar{q}, \tag{34}$$

$$V'(\underline{q}) = \frac{1+a\sigma}{1+\sigma}\underline{\beta}(K-\underline{d}) + \frac{\tilde{\lambda}_1}{\nu(1+\sigma)}\underline{\beta}(K-\underline{d}), \tag{35}$$

$$V'(\bar{q}) = \frac{1+a\sigma}{1+\sigma}\bar{\beta}(K-\bar{d}) - \frac{\tilde{\lambda}_1}{(1-\nu)(1+\sigma)}\underline{\beta}(K-\bar{d}) + \frac{\tilde{\lambda}_2}{(1-\nu)(1+\sigma)}\bar{\beta}(K-\bar{d}), \tag{36}$$

$$\tilde{\lambda}_1 = \nu\left[(1+\lambda)(1+\sigma) - (1+a\sigma)\right], \tag{37}$$

$$\tilde{\lambda}_2 - \tilde{\lambda}_1 = (1-\nu)\left[(1+\lambda)(1+\sigma) - (1+a\sigma)\right], \tag{38}$$

$$\tilde{\lambda}_1 \cdot \frac{1}{1+\sigma}\left[\underline{t} - \underline{\beta}(K-\underline{d})\underline{q} - \bar{t} + \underline{\beta}(K-\bar{d})\bar{q}\right] = 0, \tag{39}$$

$$\tilde{\lambda}_2 \cdot \frac{1}{1+\sigma}\left[\bar{t} - \bar{\beta}(K-\bar{d})\bar{q}\right] = 0, \tag{40}$$

$$\tilde{\lambda}_1 \geq 0, \tag{41}$$

$$\tilde{\lambda}_2 \geq 0, \tag{42}$$

$$\frac{1}{1+\sigma}\left[\underline{t} - \underline{\beta}(K-\underline{d})\underline{q}\right] \geq \frac{1}{1+\sigma}\left[\bar{t} - \underline{\beta}(K-\bar{d})\bar{q}\right], \tag{43}$$

$$\frac{1}{1+\sigma}\left[\bar{t} - \bar{\beta}(K-\bar{d})\bar{q}\right] \geq 0, \tag{44}$$

The conditions in (39) – (44) pertain to complementarity slackness and the multipliers $\tilde{\lambda}_1$ and $\tilde{\lambda}_2$ are nonnegative.

From (37) and (38), we get the relationship that $\frac{\tilde{\lambda}_1}{\nu} = \frac{\tilde{\lambda}_2 - \tilde{\lambda}_1}{1-\nu}$ in equilibrium. Solving this yields $\tilde{\lambda}_1 = \nu\tilde{\lambda}_2$. This relation is satisfied when either $\tilde{\lambda}_1 = \tilde{\lambda}_2 = 0$ or when $\tilde{\lambda}_1 > 0$, $\tilde{\lambda}_2 > 0$, such that $\frac{\tilde{\lambda}_1}{\tilde{\lambda}_2} = \nu$. This implies that if both the multipliers assume a positive value, we get an interior solution, in which case the IC for the efficient type and the IR for the inefficient type will bind at the optimum (from (39) and (40)). Thus, the inefficient type's rent will be pushed down to zero. The efficient type will earn a positive rent and will be able to operate positive contributions to the regulator. On the other hand, if both the multipliers assume a value of zero, conditions (39) and (40) reveal that neither the IC for the efficient type nor the IR for the inefficient type will bind at the optimum. In this case, the regulator will leave a positive rent even to the inefficient type (besides the efficient type), thus reaping a positive political contribution irrespective of the firm type.

Note that substituting for $\tilde{\lambda}$ from (29) in the right hand sides of (27) and (28), the full information equilibrium conditions for either firm type with lobbying will be

$$D'(d) = (1+\lambda)\beta q \,, \tag{45}$$

$$V'(q) = (1+\lambda)\beta(K-d)\,, \tag{46}$$

which are identical to those under full information, but with no lobbying (see equations (2) and (3)).

To check for the effect of asymmetry of information alone on the optimal contract, we substitute for $\tilde{\lambda}_1$ from (37) into the right hand sides of (33) and (35). This yield

$$D'(\underline{d}) = (1+\lambda)\underline{\beta}\cdot\underline{q}\,, \tag{47}$$

$$V'(\underline{q}) = (1+\lambda)\underline{\beta}(K-\underline{d})\,. \tag{48}$$

The equilibrium conditions (47) and (48) are analogous to those in the case of full information (see equations (2) and (3) or (45) and (46) above). Hence, for the efficient type, the regulated level of pollution and output are not deviating from their first-best levels. This is not true in so far as the contract for the inefficient type is concerned, which is distorted in response to incomplete information about firm’s costs. To show this, we substitute for $\tilde{\lambda}_1$ and $\tilde{\lambda}_2$ from (37) and (38) in the right hand sides of (34) and (36) respectively. Further, using the relation $\tilde{\lambda}_1 = \nu\tilde{\lambda}_2$ (as derived earlier), we can express (34) and (36) as

$$D'(\bar{d}) = \frac{1+a\sigma}{1+\sigma}\bar{\beta}\cdot\bar{q} + \frac{(1+\lambda)(1+\sigma)-(1+a\sigma)}{(1-\nu)(1+\sigma)}\left(\bar{\beta}-\nu\underline{\beta}\right)\bar{q}\,, \tag{49}$$

$$V'(\bar{q}) = \frac{1+a\sigma}{1+\sigma}\bar{\beta}\left(K-\bar{d}\right) + \frac{(1+\lambda)(1+\sigma)-(1+a\sigma)}{(1-\nu)(1+\sigma)}\left(\bar{\beta}-\nu\underline{\beta}\right)\left(K-\bar{d}\right). \tag{50}$$

When $\tilde{\lambda}_1 = \tilde{\lambda}_2 = 0$, equations (49) and (50) converge to (45) and (46) respectively, and asymmetry of information has no bearing on the equilibrium pollution and output levels. However, matters are different when $\tilde{\lambda}_1 > 0$ and $\tilde{\lambda}_2 > 0$. It is found that, *Proposition 6:* Optimal pollution, $d(\beta)$, is increasing and output, $q(\beta)$, is decreasing in the firm type. That is, $\dot{d}(\beta) > 0$ and $\dot{q}(\beta) < 0$. This result holds in both the cases, viz. when $\tilde{\lambda}_1 = \tilde{\lambda}_2 = 0$, and when $\tilde{\lambda}_1 > 0$ and $\tilde{\lambda}_2 > 0$.

Proof. Here also, the method of proof is similar to that for the no-lobby case (see Proposition 1) or the environmental lobby case (see Proposition 4), both of which are provided in Appendices A and D respectively.[13]

Proposition 7: Even at the political equilibrium, for $\tilde{\lambda}_1$ and $\tilde{\lambda}_2$ both positive, the optimal regulation under asymmetric information entails an upward adjustment of the pollution and a downward one of the output for the inefficient firm type as compared to that under full information.

Proof. Note the parameter ν in the right hand sides of both (45) and (49). As $\nu \to 0,$ equation (49) collapses into (45) and equation (50) into (46), or that these converge to the first-order conditions under full information. For $0 < \nu < 1$, we have the necessary conditions at the asymmetric equilibrium. Thus, comparative statics with respect to parameter ν help analyze the changes in the optimum pollution and output levels in moving from the full to the incomplete information equilibria. It is found that, for $D'' > 0$, $V'' < 0,$ and both sufficiently large in magnitude, $d\bar{d}/d\nu > 0$ and $d\bar{q}/d\nu < 0.$[14] Thus, asymmetry of information induces a lowering of environmental standard and restriction on regulated output of the inefficient firm.

Intuitively, as far as pollution and output regulation is concerned, either type of firm and the regulator stand to gain from higher allowable pollution and lower output to the extent that firms' production costs are lowered on account of both of these, and the regulator extracts higher rent from the efficient type. However, the efficient firm type reaps a lower rent/ surplus due to higher regulated pollution and lower output for the inefficient firm-type, and this is a source of loss for it. At the asymmetric information equilibrium, the optimal contract for the efficient type is not distorted as compared to the full information equilibrium (this is because its IC binds, although its IR does not), but there is an upward and downward distortion in the pollution and output levels of the inefficient

[13] See Appendix E for a detailed mathematical proof of this proposition for both the cases.
[14] Appendix F contains the mathematical proof.

type respectively, as the rent of the efficient type is declining in the pollution and increasing in the output of the inefficient firm type.

Next, to capture the effect of political lobbying by the firm/ industry alone on the optimal contract, we compare the first-order conditions under incomplete information, but with and without lobbying activity. Accordingly, (47) and (48) are compared with (13) and (15) respectively, for the efficient firm type. As expected, it is revealed that firm politics has no effect on the optimal behavior of the efficient firm. For the inefficient type, the method of proof similar to the one used for Proposition 7 is used to compare the equilibrium pollution and output levels, yielded by the solution to the equilibrium conditions (49) and (50) under lobbying with those obtained by solving (14) and (16) with no-lobbying. It is derived that,

Proposition 8. For $\tilde{\lambda}_1$ and $\tilde{\lambda}_2$ positive and $a > 1$, the presence of the industry lobby induces a lower level of optimal pollution for the inefficient type vis-à-vis the situation of no lobbying (where $a = 1$). The equilibrium price-cost margin for the inefficient type is also lowered vis-à-vis the no lobby case, induced by the regulatory contract, implying a relative upward distortion in the output level.

Proof. From a comparison of equation (49) with (14) and that of (50) with (16), it is evident that as $a \to 1$, the former equations collapse to the latter. Thus, comparative statics with respect to parameter a would provide the direction of change of equilibrium pollution and output levels in making a discrete move from the regulated equilibrium without lobbying to the one where firm-lobbying happens. In view of $D'' > 0$ and $V'' < 0$, and the absolute magnitudes of these being large enough, it is derived that $d\bar{d}/da < 0$ and $d\bar{q}/da > 0$.[15]

Thus, despite being politically inclined, the regulator is induced to lower the optimal pollution and the price-cost margin (thus raising the regulated output) for the inefficient type as compared to the no-lobby case. This might seem counter-intuitive, in that, a politically inclined regulator adopts a more stringent environmental regulation as compared to the benevolent regulator. Intuitively, what happens is as follows. The regulator here is driven by two motives, namely rent extraction from the efficient type (which is also the motive of the benevolent regulator) as this is socially costly, and rent-seeking from both the firm due to his/ her political inclination. By inducing a relatively lower

[15] A detailed proof of this is provided in Appendix G.

optimal pollution and a higher output on the part of the inefficient firm as compared to the benchmark case, he/ she stands to incur a loss in terms of his/ her political motive, as it implies a rise in the production cost (entailing a lower rent and lower political contribution from the firm-lobby) as well as in his/ her efficiency motive as this implies that the efficient type now reaps a greater rent/ surplus. On the other hand, he/ she gains because the social welfare is now higher on account of lower pollution and higher output and partly because the efficient firm would tend to offer the regulator a higher political contribution (which is a fixed proportion of its rent) through greater rent leakage to it. These gains seemingly outweigh the indicated losses.

Finally, it is proved that the optimal regulatory contract is, in fact, incentive compatible for the inefficient type, given the equilibrium conditions as in equations (33)-(38). We can express the IC for the inefficient type as $\overline{U} \geq \frac{1}{1+\sigma}\left[\underline{t}-\overline{\beta}\left(K-\underline{d}\right)\underline{q}\right]=\underline{U}-\frac{1}{1+\sigma}\Delta\beta\left(K-\underline{d}\right)\underline{q}$. In the case of $\tilde{\lambda}_1=\tilde{\lambda}_2=0$, the IC for the efficient type does not bind. Its substitution in the right hand side yields the condition $0>\frac{1}{1+\sigma}\Delta\beta\left[\left(K-\overline{d}\right)\overline{q}-\left(K-\underline{d}\right)\underline{q}\right]$. [16] This is satisfied at the optimum, as $\overline{d}>\underline{d}$, $\overline{q}<\underline{q}$ (from Proposition 6) and both σ and $\Delta\beta$ are positive. On the other hand, when the IC for the efficient type binds at the optimum (both the multipliers take positive values), substituting it with equality on the right hand side of the IC for the inefficient type yields $0\geq\frac{1}{1+\sigma}\Delta\beta\left[\left(K-\overline{d}\right)\overline{q}-\left(K-\underline{d}\right)\underline{q}\right]$. Again from Proposition 6, this is satisfied at the optimum as $\overline{d}>\underline{d}$, $\overline{q}<\underline{q}$, and $\sigma>0$, $\Delta\beta>0$.

5. Conclusions

The paper first characterizes the optimal regulatory contract for the monopoly firm under full information and asymmetric information as the benchmark case. This is subsequently extended to include the influence of an environmental lobby and a firm-lobby respectively. In all the three cases we find that, under informational constraints, both the pricing rule and the incentives for cost reduction (through choice of pollution abatement effort) are distorted for the inefficient or the high cost firm. The directions of these distortions in the contract are identified and found to be the same in

[16] We substitute the IC for the efficient type in the form of $\underline{U}\geq\overline{U}+\frac{1}{1+\sigma}\Delta\beta\left(K-\overline{d}\right)\overline{q}$.

the three cases considered. However, the efficient type's contract is not affected by information asymmetry in all the three cases.

Further, in the presence of the environmental lobby, as expected the politically inclined regulator imposes a more stringent environmental regulation under both full information and incomplete information (for both the firm-types) compared to the benchmark case when there was no lobbying activity. Accordingly, the extent of distortion in the allowable pollution of the inefficient type is less compared to the no lobby case. However, the rule governing the pricing regulation is unaffected by the lobbying activity of the environmental group.

In the presence of the industry lobby, lobbying impacts only the optimal regulatory contract for the inefficient firm-type. Surprisingly, we find that there is now more stringent environmental regulation and a lower price-cost margin allowed to the inefficient type as compared to the first case involving no lobby. This result holds irrespective of whether only the efficient firm type earns a positive rent or when both the firm types are left with a positive rent.

The analysis can be extended in a number of directions. One of them could be to introduce competition among the lobbies (similar to the one in Grossman and Helpman (1994)) in the context of an upcoming election. Obviously, in this case, both the lobbies would simultaneously try to influence the regulator for favorable policy outcomes. The model could also be extended to consider multiple firms who are either price takers or compete in prices in the commodity market, and competition among firm-lobbies when firms' output is differentiated in terms of pollution intensity. In these alternative market situations, the optimal regulatory contract over pollution could throw up some significant policy implications for optimal environmental regulation.

Appendices

Appendix A: *Proof of Proposition 1:* For any β, the following pair of equations define the equilibrium:

$$D'(d)=(1+\lambda)\beta q+\frac{\nu}{1-\nu}\lambda\left(\beta-\underline{\beta}\right)q\,,$$

$$V'(q)=(1+\lambda)\beta(K-d)+\frac{\nu}{1-\nu}\lambda\left(\beta-\underline{\beta}\right)(K-d)\,.$$

Note that as β takes the value $\underline{\beta}$, the two above equations collapse to the first-order conditions (13) and (15), and as $\beta = \overline{\beta}$, they amount to equations (14) and (16). Rearranging terms and letting $\rho_1 = (1+\lambda) + \frac{\nu}{1-\nu}\lambda$, $\rho_2 = \frac{\nu}{1-\nu}\lambda$ yield

$$D'(d) = (\rho_1\beta - \rho_2\underline{\beta})q,$$

$$V'(q) = (\rho_1\beta - \rho_2\underline{\beta})(K-d).$$

We differentiate these two equations with respect to β to determine how the optimal values of d and q change as β changes from $\underline{\beta}$ to $\overline{\beta}$. This yields the following comparative statics:

$$D''(d)\dot{d}(\beta) = \rho_1 q + (\rho_1\beta - \rho_2\underline{\beta})\dot{q}(\beta),$$

$$V''(q)\dot{q}(\beta) = \rho_1(K-d) - (\rho_1\beta - \rho_2\underline{\beta})\dot{d}(\beta).$$ [17]

Solving by substitution, we get the two expressions

$$\dot{d}(\beta) = \frac{[V''(q)q + V'(q)]\rho_1(K-d)^2}{D''(d)V''(q)(K-d)^2 + (V'(q))^2},$$

$$\dot{q}(\beta) = \frac{[D''(d)(K-d) - D'(d)]\rho_1 q^2}{D''(d)V''(q)q^2 + (D'(d))^2}.$$

Note that for any β at the equilibrium, the second-order conditions require that

$$D''(d)V''(q) + \left[(1+\lambda)\beta + \frac{\nu}{1-\nu}\lambda(\beta - \underline{\beta})\right]^2 < 0,$$

using parameterized first-order conditions at the outset, we can write $(1+\lambda)\beta + \frac{\nu}{1-\nu}\lambda(\beta-\underline{\beta}) = \frac{D'(d)}{q} = \frac{V'(q)}{K-d}$. Subsequently, we get the following two conditions for the second-order conditions to go through

$$D''(d)V''(q)(K-d)^2 + (V'(q))^2 < 0,$$

$$D''(d)V''(q)q^2 + (D'(d))^2 < 0.$$

[17] Here, the notation $\dot{d}(\beta)$ denotes $dd/d\beta$ and so on.

These require the benefit function $V(.)$ to be sufficiently concave, or $V'' < 0$, $D''(d) > 0$, and both to be sufficiently large. These are sufficient for $\dot{d}(\beta) > 0$ and $\dot{q}(\beta) < 0$. The cost function, $C(.)$, is not monotonic in β. Differentiating the total cost function with respect to β yields

$$\dot{C}(\beta) = \beta(K-d)\dot{q}(\beta) + (K-d)q - \beta q\dot{d}(\beta).$$

The first term in the right hand side is negative as $\dot{q}(\beta) < 0$ and the third term is also negative as $\dot{d}(\beta) > 0$. But the second term is positive, making the sign of $\dot{C}(\beta)$ indeterminate, and dependent on the values of K and β.

Appendix B: *Proof of Proposition 2:* Since the move from the full information to asymmetric information equilibrium can be captured by the following parameterized pair of first-order conditions:

$$D'(\overline{d}) = (1+\lambda)\overline{\beta}.\overline{q} + \frac{\nu}{1-\nu}\lambda\xi(\overline{\beta} - \underline{\beta})\overline{q},$$

$$V'(\overline{q}) = (1+\lambda)\overline{\beta}(K - \overline{d}) + \frac{\nu}{1-\nu}\lambda\xi(\overline{\beta} - \underline{\beta})(K - \overline{d}).$$

Comparative statics with respect to ξ yields that

$$D''(\overline{d})\frac{d\overline{d}}{d\xi} = \left[(1+\lambda)\overline{\beta} + \frac{\nu}{1-\nu}\lambda\xi\left(\overline{\beta} - \underline{\beta}\right)\right]\frac{d\overline{q}}{d\xi} + \frac{\nu}{1-\nu}\lambda\left(\overline{\beta} - \underline{\beta}\right)\overline{q},$$

$$V''(\overline{q})\frac{d\overline{q}}{d\xi} = -\left[(1+\lambda)\overline{\beta} + \frac{\nu}{1-\nu}\lambda\xi(\overline{\beta} - \underline{\beta})\right]\frac{d\overline{d}}{d\xi} + \frac{\nu}{1-\nu}\lambda(\overline{\beta} - \underline{\beta})(K - \overline{d}).$$

Let $(1+\lambda)\overline{\beta} + \frac{\nu}{1-\nu}\lambda\xi\left(\overline{\beta} - \underline{\beta}\right) = \phi_1$, and $\frac{\nu}{1-\nu}\lambda\left(\overline{\beta} - \underline{\beta}\right) = \phi_2$. The solution to the above system of equations will be

$$\frac{d\overline{d}}{d\xi} = \frac{\phi_1\phi_2(K - \overline{d}) + \phi_2 V''\overline{q}}{D''V'' + \varphi_1^{\,2}},$$

$$\frac{d\overline{q}}{d\xi} = \frac{-\phi_1\phi_2\overline{q} + \phi_2 D''(K - \overline{d})}{D''V'' + \phi_1^{\,2}},$$

which will be unambiguously positive and negative respectively since V'' is assumed to be sufficiently negative and D'' sufficiently positive for the second-order conditions to be satisfied.

Appendix C: *Proof of Proposition 3:* Note that amongst the three first-order conditions, it is only equation (20) which contains parameter a in the right hand side. Comparative statics with respect to it yield:

$$D''(d)dd = \frac{-(1)(1+\lambda)(1+\mu)\mu\beta q}{(1+a\mu)^2}da \Leftrightarrow \frac{dd}{da} = \frac{-(1)(1+\lambda)(1+\mu)\mu\beta q}{(1+a\mu)^2 D''} < 0,$$

in view of $D'' > 0.$ Furthermore, totally differentiating equation (21) we get,

$$V''(q)dq = -(1+\lambda)\beta dd \Rightarrow \frac{dq}{dd} = \frac{-(1+\lambda)\beta}{V''} > 0.$$

Appendix D: *Proof of Proposition 4:* Parameterizing the first-order conditions (given by equations 23-26) with respect to β in order to capture the discrete jump in it from one value to the other at the equilibrium and then differentiating these first-order conditions with respect to the same parameter, we get two similar equations in $\dot{d}(\beta)$ and $\dot{q}(\beta)$ like those in Appendix A. Solving by substitution yields the following

$$\dot{d}(\beta) = \frac{\left[V''(q)q + V'(q)\right]\theta\rho_1(K-d)^2}{D''(d)V''(q)(K-d)^2 + \theta\left(V'(q)\right)^2},$$

$$\dot{q}(\beta) = \frac{\left[D''(d)(K-d) - D'(d)\right]\theta\rho_1 q^2}{D''(d)V''(q)\theta q^2 + \left(D'(d)\right)^2},$$

where $\theta = \frac{1+\mu}{1+a\mu} > 0$. These will be positive and negative respectively, in view of $D'' > 0, V'' < 0,$ and both being sufficiently large in magnitude.

Appendix E: *Proof of Proposition 6: Part A:* First consider the case where both the multipliers assume a zero value. The first order conditions (equations (47)-(48) for the efficient type and equations (49)-(50) for the inefficient type) reduce to

$$D'(\underline{d}) = (1+\lambda)\underline{\beta}\cdot\underline{q},$$

$$D'(\overline{d}) = (1+\lambda)\overline{\beta}\cdot\overline{q},$$

$$V'(\underline{q}) = (1+\lambda)\underline{\beta}(K-\underline{d}),$$

$$V'(\overline{q}) = (1+\lambda)\overline{\beta}(K-\overline{d}).$$

For any β at the equilibrium, this is fully captured by the following pair of equations

$$D'(d) = (1+\lambda)\beta q,$$

$$V'(q) = (1+\lambda)\beta(K-d).$$

Differentiation of these two equations with respect to β gives

$$D''(d)\dot{d}(\beta) = (1+\lambda)\left[q + \beta\dot{q}(\beta)\right],$$

$$V''(q)\dot{q}(\beta) = (1+\lambda)\left[(K-d) - \beta\dot{d}(\beta)\right].$$

Solving by substitution, we get

$$\dot{d}(\beta) = \frac{(1+\lambda)\left[V''(q)q + (1+\lambda)\beta(K-d)\right]}{D''(d)V''(q) + (1+\lambda)^2\beta^2},$$

$$\dot{q}(\beta) = \frac{(1+\lambda)\left[D''(d)(K-d) - (1+\lambda)\beta q\right]}{D''(d)V''(q) + (1+\lambda)^2\beta^2}.$$

Note that the denominators in the above two expressions are nothing but what was implied by the second-order condition of the full information equilibrium conditions under the benchmark case of no lobbying (section 2.1, pp 9). The sign of the denominators is, therefore, negative. Further, since, $D'' > 0, V'' < 0,$ and both sufficiently large in magnitude, all these conditions are sufficient for

$\dot{d}(\beta) > 0$ and $\dot{q}(\beta) < 0$.

Part B: Now consider the case of positive multipliers where both the IC for the good type and the IR for the bad type bind at the optimum. The first-order conditions in (47)-(48) for the efficient firm type (that is, $\beta = \underline{\beta}$) and (49)-(50) for the inefficient firm type (that is, $\beta = \overline{\beta}$) could be captured by the following pair of equations:

$$D'(d) = \frac{1+a\sigma}{1+\sigma}\beta q + \frac{(1+\lambda)(1+\sigma)-(1+a\sigma)}{(1-\nu)(1+\sigma)}\left(\beta - \nu\underline{\beta}\right)q,$$

$$V'(q) = \frac{1+a\sigma}{1+\sigma}\beta(K-d) + \frac{(1+\lambda)(1+\sigma)-(1+a\sigma)}{(1-\nu)(1+\sigma)}\left(\beta - \nu\underline{\beta}\right)(K-d).$$

Let $\gamma_1 = \frac{1+a\sigma}{1+\sigma} + \frac{(1+\lambda)(1+\sigma)-(1+a\sigma)}{(1-\nu)(1+\sigma)} = \frac{(1+\lambda)(1+\sigma)-\nu(1+a\sigma)}{(1-\nu)(1+\sigma)} > 0$, and

$\gamma_2 = \nu.\frac{(1+\lambda)(1+\sigma)-(1+a\sigma)}{(1-\nu)(1+\sigma)}$.

This yields

$$D'(d) = \left(\gamma_1\beta - \gamma_2\underline{\beta}\right)q,$$

$$V'(q)=\left(\gamma_1\beta-\gamma_2\underline{\beta}\right)\left(K-d\right).$$

Comparative statics with respect to β yield

$$D''(d)\dot{d}(\beta)=\gamma_1 q+\left(\gamma_1\beta-\gamma_2\underline{\beta}\right)\dot{q}(\beta),$$

$$V''(q)\dot{q}(\beta)=\gamma_1\left(K-d\right)-\left(\gamma_1\beta-\gamma_2\underline{\beta}\right)\dot{d}(\beta).$$

The solution will be given by expressions similar to those obtained in Appendix A

$$\dot{d}(\beta)=\frac{\left[V''(q)q+V'(q)\right]\gamma_1\left(K-d\right)^2}{D''(d)V''(q)\left(K-d\right)^2+\left(V'(q)\right)^2}>0,$$

$$\dot{q}(\beta)=\frac{\left[D''(d)\left(K-d\right)-D'(d)\right]\gamma_1 q^2}{D''(d)V''(q)q^2+\left(D'(d)\right)^2}<0.$$

The signs are in view of the same regularity conditions that we have used in proving other propositions i.e. V'' is sufficiently negative and D'' sufficiently positive.

Appendix F: *Proof of Proposition 7:* Differentiating equations (49) and (50) with respect to parameter ν yields

$$D''(\bar{d})d\bar{d}=\frac{1+a\sigma}{1+\sigma}\bar{\beta}d\bar{q}+\frac{(1+\lambda)(1+\sigma)-(1+a\sigma)}{(1-\nu)^2(1+\sigma)}\left(\bar{\beta}-\nu\underline{\beta}\right)\bar{q}d\nu-\frac{(1+\lambda)(1+\sigma)-(1+a\sigma)}{(1-\nu)(1+\sigma)}\underline{\beta}\bar{q}d\nu$$
$$+\frac{(1+\lambda)(1+\sigma)-(1+a\sigma)}{(1-\nu)(1+\sigma)}\left(\bar{\beta}-\nu\underline{\beta}\right)d\bar{q};$$
$$V''(\bar{q})d\bar{q}=-\frac{1+a\sigma}{1+\sigma}\bar{\beta}d\bar{d}-\frac{(1+\lambda)(1+\sigma)-(1+a\sigma)}{(1-\nu)(1+\sigma)}\left(\bar{\beta}-\nu\underline{\beta}\right)d\bar{d}$$
$$+\frac{(1+\lambda)(1+\sigma)-(1+a\sigma)}{(1-\nu)^2(1+\sigma)}\left(\bar{\beta}-\nu\underline{\beta}\right)\left(K-\bar{d}\right)d\nu-\frac{(1+\lambda)(1+\sigma)-(1+a\sigma)}{(1-\nu)(1+\sigma)}\underline{\beta}\left(K-\bar{d}\right)d\nu.$$

These simply yield the following system of equations:

$$D''(\bar{d})d\bar{d}=\alpha_1 d\bar{q}+\alpha_2\bar{q}d\nu;$$
$$V''(\bar{q})d\bar{q}=-\alpha_1 d\bar{d}+\alpha_2\left(K-\bar{d}\right)d\nu,$$

where $\alpha_1=\frac{1+a\sigma}{1+\sigma}\bar{\beta}+\frac{(1+\lambda)(1+\sigma)-(1+a\sigma)}{(1-\nu)(1+\sigma)}\left(\bar{\beta}-\nu\underline{\beta}\right)$>0, and

$$\alpha_2=\frac{(1+\lambda)(1+\sigma)-(1+a\sigma)}{(1-\nu)^2(1+\sigma)}\left(\bar{\beta}-\nu\underline{\beta}\right)-\frac{(1+\lambda)(1+\sigma)-(1+a\sigma)}{(1-\nu)(1+\sigma)}\underline{\beta}$$

$$=\frac{(1+\lambda)(1+\sigma)-(1+a\sigma)}{(1-\nu)^2(1+\sigma)}\left(\bar{\beta}-\underline{\beta}\right)>0.$$

Solving through substitution, we get

$$\frac{d\bar{d}}{d\nu} = \frac{\alpha_1\alpha_2\left(K-\bar{d}\right)+\alpha_2 V''\bar{q}}{D''V''+\alpha_1^{\,2}} > 0, \text{ and}$$

$$\frac{d\bar{q}}{d\nu} = \frac{-\alpha_1\alpha_2\bar{q}+\alpha_2 D''\left(K-\bar{d}\right)}{D''V''+\alpha_1^{\,2}} < 0,$$

in view of $D'' > 0, V'' < 0,$ and both sufficiently large in magnitude, and both α_1 and α_2 positive.

Appendix G: *Proof of Proposition 8:* This time, differentiating (49) and (50) with respect to parameter a yields:

$$D''(\bar{d})d\bar{d} = \frac{1+a\sigma}{1+\sigma}\bar{\beta}d\bar{q} + \frac{(1+\lambda)(1+\sigma)-(1+a\sigma)}{(1-\nu)(1+\sigma)}\left(\bar{\beta}-\nu\underline{\beta}\right)d\bar{q} + \frac{\sigma}{1+\sigma}\bar{\beta}\bar{q}da$$
$$-\frac{\sigma}{(1+\sigma)(1-\nu)}\left(\bar{\beta}-\nu\underline{\beta}\right)\bar{q}da;$$
$$V''(\bar{q})d\bar{q} = -\frac{1+a\sigma}{1+\sigma}\bar{\beta}d\bar{d} - \frac{(1+\lambda)(1+\sigma)-(1+a\sigma)}{(1-\nu)(1+\sigma)}\left(\bar{\beta}-\nu\underline{\beta}\right)d\bar{d} + \frac{\sigma}{1+\sigma}\bar{\beta}\left(K-\bar{d}\right)da$$
$$-\frac{\sigma}{(1+\sigma)(1-\nu)}\left(\bar{\beta}-\nu\underline{\beta}\right)\left(K-\bar{d}\right)da.$$

This can be expressed as the following system of equations:

$$D''(\bar{d})d\bar{d} = \delta_1 d\bar{q} + \delta_2\bar{q}da;$$
$$V''(\bar{q})d\bar{q} = -\delta_1 d\bar{d} + \delta_2\left(K-\bar{d}\right)da,$$

where $\delta_1 = \frac{1+a\sigma}{1+\sigma}\bar{\beta} + \frac{(1+\lambda)(1+\sigma)-(1+a\sigma)}{(1-\nu)(1+\sigma)}\left(\bar{\beta}-\nu\underline{\beta}\right) > 0,$ and

$$\delta_2 = \frac{\sigma}{1+\sigma}\bar{\beta} - \frac{\sigma}{(1+\sigma)(1-\nu)}\left(\bar{\beta}-\nu\underline{\beta}\right) = -\frac{\sigma}{1+\sigma}\frac{\nu}{1-\nu}\left(\bar{\beta}-\underline{\beta}\right) < 0.$$

The solution to this equation system yields that

$$\frac{d\bar{d}}{da} = \frac{\delta_1\delta_2\left(K-\bar{d}\right)+\delta_2 V''\bar{q}}{D''V''+\delta_1^{\,2}} < 0,$$
$$\frac{d\bar{q}}{da} = \frac{-\delta_1\delta_2\bar{q}+\delta_2 D''\left(K-\bar{d}\right)}{D''V''+\delta_1^{\,2}} > 0,$$

since $D'' > 0, V'' < 0,$ and both sufficiently large in magnitude, as well as δ_1 is positive and δ_2 negative in sign.

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