Strategic Debt in Vertical Relationships

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Abstract

We study a vertical relationship between two firms, and we show that the extent of the downstream firm's borrowing affects the contract offered by the upstream firm. We establish a negative relationship between the level of debt and the downstream firm's probability of bankrupt. We also show that, unless the interest rate is very high, there exists a conflict of interest between the upstream and the downstream firm: the latter wants to take on more debt than the former would like it to. We interpret this finding as an explanation of the constraint imposed by franchisors on the debt level of their franchisees.

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1 Introduction

Why do many companies explicitly require their potential franchisees a considerable amount of personal financial investment, and, in general, frown upon franchisees taking on large amount of debt? A prime example is McDonald's, who require a minimum of between $75,000 and $258,000 of non-borrowed personal resources to consider an individual for a franchise.[1]

Prima facie, this is an irrational a priori exclusion of potential franchisees, who may be good entrepreneurs, but short of liquid capital. The literature has proposed several explanations, usually based on some form of asymmetry of information. For example, according to Norton (1995), debt can be used as a screening device by franchisors who need to separate good managers from bad ones. This adverse selection explanation can be complemented with a moral hazard one: debt has a disciplining role on management in so far as the associated risk of business failure motivates owner-managers of franchised outlets not to shirk (Jensen, 1986). A further point is made by Williamson (1985). He argues that when quality is non-contractible, franchisees would chisel on any quality level agreed with the franchisor, thereby damaging the image of the chain. The franchisor therefore requires the franchisee to finance a specific investment by personal resources, reserving at the same time the right to terminate the contract. Termination occurs if the franchisor believes that the quality provided falls short of the required threshold. If the franchisee chisels on quality, she loses the investment. If, on the other hand, the franchisee could borrow, then the cost of early termination would be borne by the lender.

In this paper, we offer an additional explanation for the reluctance of franchisors to allow franchisees to take on debt. Our analysis is not restricted to franchising, but applies to any vertical relationship where there exists a continuing relationship between an upstream firm (franchisor, manufacturer) and a downstream firm (franchisee, retailer). We argue that the level of debt affects the vertical relationship between these two firms. Our paper shows that, when the owner of the downstream firm has an informational advantage, then she can take on debt in order to constrain to her advantage the range of contracts which the upstream firm can offer her. This is because

[1] The lower minimum applies only to highly qualified franchisees. See Website `http://www.mcdonalds.com/a_system/factsheet`. Another example is offered by the Starway Corporation's web page `http://www.starway.net.au/ukfrcapital.html`:

> We recommend that you be able to fully fund the purchase of any small business. On the other hand, Starway Corporation understands that few people have enough cash lying around to buy a business. Thus we have no objection to franchisees borrowing a small portion of the capital cost to get started. However, we do suggest that borrowings be kept to a minimum. .... If you intend to fund a franchise with borrowed money, you should discuss the matter with us in person.

the upstream firm, in choosing the contract to offer, needs to take into account that the downstream firm can always opt to go bankrupt, and is more likely to do so the higher its level of debt.

Our main result is Proposition 3. We show that, unless the interest rate is very high, the downstream firm wants to take on as much debt as the market is willing to lend. A conflict of interest arises because the upstream firm would prefer to deal with an unleveraged firm (Proposition 4). It therefore becomes rational for the upstream firm to impose a limit on the extent of the downstream firm's borrowing as a precondition for engaging in a relationship at all.

Our analysis can also be seen as a contribution to the theory of the firm. A common stylised way of modeling the entrepreneur is as a wealth constrained individual who must therefore resort to external funding to finance a business idea (Holmstrom and Tirole, 1989). In this literature, the individual with the specialised knowledge/expertise would not want to resort to debt if she had enough personal resources. We depart from this assumption: in our model, the owner of the downstream firm has enough personal wealth to pay for the entire investment. Nevertheless, she prefers to finance her activity through debt.

We obtain therefore an instance of the strategic use of debt: if allowed, the downstream firm takes on more debt than she needs. She does so in order to manipulate the behavior of another agent to her advantage. This is reminiscent of the principle highlighted in Brander and Lewis (1986). They show that oligopolists artificially increase their debt/equity ratio in order to commit to a more aggressive output strategy. In the Brander and Lewis model, the financial structure of a firm, i.e. its debt to equity ratio, affects the behaviour of its product market competitors; in our paper, the financial structure of the downstream firm affects the behaviour of the upstream firm. In both cases the channels through which this influence occurs are the increase in the probability of bankruptcy brought about by debt, and the limited liability effect, which limits the extent of the losses incurred by the agent who takes on debt.

The paper is organised as follows. In the next section we describe the model. Section 3 studies the contract offered by the upstream firm, while Section 4 illustrates the choice of debt by the downstream firm. Section 5 concludes. Proofs of all the propositions are contained in the appendix.

2 The model

We consider an upstream manufacturer who supplies his product to a downstream retailer. The market demand, Q, is a function of the retail price, p_r: $Q(p_r)$, with $Q'(p_r) < 0$.

The retailer has constant marginal cost, θ. $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$ is a random variable with distribution $F(\theta)$ and density $f(\theta) = F'(\theta)$. The hazard rate is $h(\theta) = \frac{F(\theta)}{f(\theta)}$. We make the standard assumption that $h(\theta)$ is non decreasing:[2]

$$h'(\theta) = \frac{d}{d\theta}\left[\frac{F(\theta)}{f(\theta)}\right] \geq 0. \tag{1}$$

We follow Gal-Or (1991) in assuming that the actual realisation of θ is the retailer's private information. This implies that the presence of the retailer is necessary for any sales to occur. We simplify Gal-Or's model by assuming that the demand function is common knowledge. We extend her approach by adding investment and debt to her analysis. Specifically, we assume that, in order for any sales to take place at all, as well as the retailer's presence, an investment in a relation specific asset is also necessary. The characteristics of the investment make it impossible for the manufacturer to take charge of retailing himself.[3]

Let $I > 0$ be the cost of the sunk investment. The retailer chooses an amount E of her personal wealth, and borrows the remaining $(I - E)$ at the exogeneously given borrowing rate, r_d. This rate is independent of the proportion of the asset's cost which is financed by debt up to a maximum $G_{max} > 0$. This simple relationship between the amount borrowed and the interest rate may reflect the use of rules of thumb, or industry/category specific lending guidelines by the lender. Including interest, the retailer's financial obligation, G, is therefore given by:

$$G = (I - E)(1 + r_d). \tag{2}$$

We assume limited liability, so that the retailer can keep any money not used in the purchase of the asset. This is invested in a riskless project which pays an interest rate r_l, normalised, without loss of generality, to zero. We also assume that there are no bankruptcy costs. In the case of bankruptcy the salvage value of the equipment bought from investment I is assumed, without loss of generality, to be zero.

The sequence of events is described in Figure 1.

3 The manufacturer's problem

The analysis of the contract proposed by the manufacturer to the retailer follows closely Gal-Or (1991).

[2]See Laffont and Tirole (1993, p.67) for a discussion.

[3]In the fast food industry, this investment could be a market research which reveals the type of advertisement necessary in a given location. Another industry where franchise can occur is mining. The investment could be the purchase of a test drill; only the mining engineer (the downstream firm) has the expertise to interpret the results (learn θ); the extraction is then conducted on behalf of the owner of the land (the upstream firm). In these examples it is clear that no production can take place if either the personal expertise or the machinery is not there.

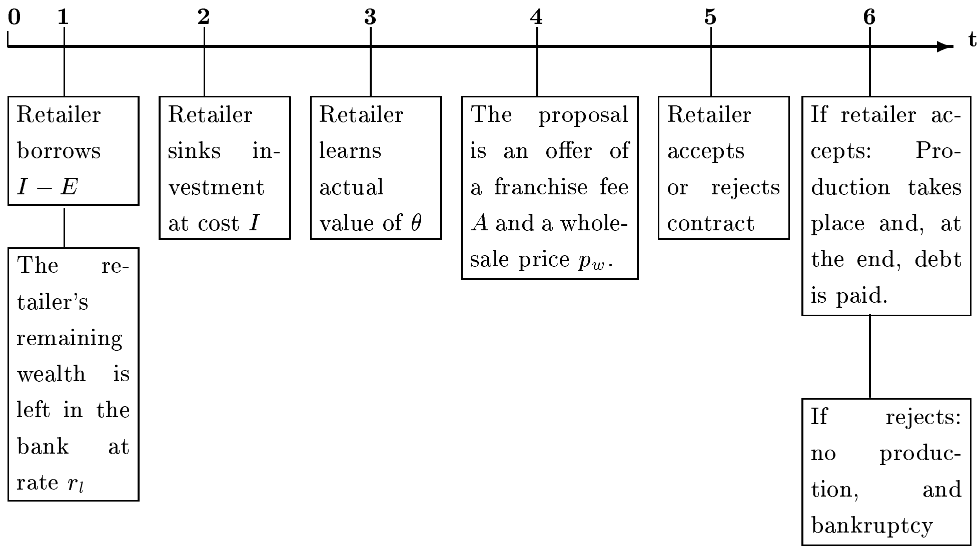

Figure 1: Timing of the game

The manufacturer only decision occurs at date 4. He maximises his expected profit, taking the decisions made at dates $1-3$ as given. The solution technique uses the revelation principle: the manufacturer asks for a report on θ, and commits to a wholesale price $p_w(\theta)$ and a fee $A(\theta)$ as functions of the reported θ. These functions are incentive compatible if the retailer is better off reporting θ truthfully than reporting any different value $\hat{\theta}$. According to the revelation principle, the manufacturer cannot do better than choosing the pair of incentive compatible functions which yield the highest profit. Once θ is known, retailer and manufacturer have aligned objectives with regard to the retail price: it will simply be the monopoly price given θ (this ceases to be the case if the retailer has superior information about demand; see Gal-Or (1991) for details). The retailer is of course free not to sign any contract and go bankrupt. This implies that if the contract is signed, the retailer's overall utility, including repayment of the debt, must be at least equal to his reservation utility, here normalised to zero. Formally, the manufacturer chooses the contract subject to two constraints: the individual rationality constraint (guaranteeing that the retailer makes non-negative profits) and the incentive compatibility constraint. The incentive compatibility constraint for this problem is shown by Gal-Or (1991) to be given by:

$$\dot{\pi_r}(\theta) = -Q\left(p_r(\theta)\right), \tag{3}$$

where $\pi_r(\theta)$ denotes the retailer's ex-post profit as a function of the realised value of θ, after the debt is paid back:

$$\pi_r(\theta) = Q(p_r(\theta))\big(p_r(\theta) - p_w(\theta) - \theta\big) - A(\theta) - G. \tag{4}$$

Note that it is not generally optimal to proceed with retail for any value of θ: for θ sufficiently high, the joint incentive of manufacturer and retailer might be not to undertake production and let the latter go bankrupt. In this case, the retailer loses her own share of the investment, E, and the lender incurs a loss of G. Therefore, unlike Gal-Or, we endogenise the highest value of θ for which production occurs. Formally, the manufacturer solves a free terminal point optimal control problem:

$$\max_{\pi_r(\theta),\theta^*,p_r(\theta)} = \int_{\underline{\theta}}^{\theta^*} \Big\{Q\left(p_r(\theta)\right)\big[(p_r(\theta) - \theta) - \pi_r(\theta) - G\big]\Big\} f(\theta)d\theta \tag{5}$$

$$\text{s.t.} \quad \dot{\pi_r}(\theta) = -Q(p_r(\theta)) \tag{6}$$

$$\text{and} \quad \pi_r(\theta) \geq 0 \ ; \ \pi_r(\theta^*) = 0. \tag{7}$$

In (5) $A(\theta)$ is substituted away using (4). For cost realisations above θ^*, the retailer is bankrupt and the project is abandoned. For states of the world below θ^*, the retailer pays back G and, at the end, production takes place. Finally, the participation constraint (7) is derived from the consideration that it must be $\pi_r(\theta) \geq 0$ for $\theta \leq \theta^*$, and $\pi_r(\theta) < 0$ for $\theta > \theta^*$. Given that $\pi_r(\theta)$ is decreasing in θ (by (6)), these conditions are implied by (7).

Proposition 1 *At the solution of the manufacturer's problem, the wholesale price, $p_w(\theta)$, the franchise fee, $A(\theta)$, the retail price $p_r(\theta)$, and the retailer's profit $\pi_r(\theta)$ are given by the following expressions for $\theta \leq \theta^*$:*

$$Q\left(p_r(\theta)\right) + Q'\left(p_r(\theta)\right)\left[p_r(\theta) - \theta - h(\theta)\right] = 0 \tag{8}$$

$$p_w(\theta) = p_r(\theta) - \theta \tag{9}$$

$$A(\theta) = -\left[\pi_r(\theta) + G\right] \tag{10}$$

$$\pi_r(\theta) = \int_{\theta}^{\theta^*} Q(p_r(\tilde{\theta}))d\tilde{\theta} \tag{11}$$

where θ^* is given by:

$$Q\left(p_r(\theta^*)\right)\left[p_r(\theta^*) - \theta^* - h(\theta^*)\right] = G \tag{12}$$

As (8) shows, the retailer charges a price higher than that of a vertically integrated monopolist, unless $\theta = \underline{\theta}$. This is defined "excessive retail price distortions" by Gal-Or (1991) who discusses it in more detail. We refer the reader to her paper for a full discussion of (8) and (9). According to (9) the wholesale price is set to a level which enables the retailer to cover exactly its retailing cost. This avoids double marginalisation.

As in Gal-Or (1991), the franchise fee is set at a negative level: it is a fixed transfer from the manufacturer to the retailer. When $\theta = \theta^*$, this transfer is equal to the retailer's debt obligation G, to leave $\pi_r(\theta^*) = 0$. For $\theta < \theta^*$, the transfer increases in order to induce truthful revelation of θ.

The cut-off point θ^* is determined by the trade-off between two opposing forces. On the one hand, reducing θ^* reduces the probability that production takes place. On the other hand, it also reduces the rent that must be left to the retailer if production does take place (from (11)). The following proposition shows that this cut-off point, and hence the probability of bankruptcy, is an increasing function of the level of debt G.

Proposition 2 *The cut-off point θ^* and the level of debt G are related as follows:*

$$\frac{d\theta^*}{dG} = -\frac{1}{Q\left(p_r(\theta^*)\right)\left\{1 + h'(\theta^*)\right\}} < 0. \tag{13}$$

4 The choice of debt by the retailer

We are now ready to study the retailer's choice of debt, which is made at date 1, when θ is still unknown. A trade-off is involved in this decision. On the one hand, given the investment specificity, the amount financed by the retailer's personal contribution is lost if the realisation of θ is sufficiently high that bankruptcy is chosen at date 6. Increasing debt reduces this personal contribution and therefore allows the retailer to keep for herself a larger share of her own personal wealth in the event of bankruptcy. On the other hand, from Proposition 2 we know that the greater the value of debt, the larger the probability of bankruptcy and therefore the more likely that the share of personal wealth invested in the project is lost.

The retailer is risk neutral, and chooses G to maximise her expected profit. Her problem therefore is:

$$\max_{G \in [0, G_{max}]} V(G) = \int_{\underline{\theta}}^{\theta^*(G)} \left[\int_{\theta}^{\theta^*(G)} Q(p_r(\tilde{\theta}) d\tilde{\theta} \right] dF(\theta) - \left(I - \frac{G}{1 + r_d} \right), \tag{14}$$

where the dependence of θ^* on G is made explicit. The term the in round brackets in (14) is the portion of the investment paid for by the retailer.

Assumption 1 *For every $\theta \in [\underline{\theta}, \overline{\theta}]$:*

$$h''(\theta) < \frac{1 + h'(\theta)}{h(\theta)}. \tag{15}$$

$h(\theta)$ is defined in (1) as the hazard rate. This assumption holds for most commonly used distribution functions.

Proposition 3 *Let Assumption 1 hold. Then $V(G)$ is convex. Therefore any stationary point is a local minimum, and the maximum must be at one of the extreme points of the domain, either 0 or G_{max}.*

The following Corollary determines a sufficient condition for the retailer to choose the maximum level of debt.

Corollary 1 *If $h'(\theta) > r_d$ for $\theta \in [\theta^*(G_{max}), \overline{\theta}]$, then the retailer's profit is strictly increasing in G.*

Thus, for sufficiently low values of the interest rate, the retailer wants to borrow as much as she can. For higher values of the interest rates, whether the retailer chooses 0 or G_{max} depends on the sign of the difference $V(G_{max}) - V(0)$. The next result gives an expression for this difference.

Corollary 2

$$V(G_{max}) - V(0) = \frac{G_{max}}{1 + r_d} - \int_{\theta^*(G_{max})}^{\overline{\theta}} Q\big(p_r(\theta)\big)\, F(\theta) d\theta. \tag{16}$$

The interest rate negatively affects the choice between no debt and maximum debt: for fixed G_{max}, $V(G_{max}) - V(0)$ is decreasing in r_d. This is natural: a higher cost of debt makes it less likely that debt will be taken on.

The next result illustrates the potential for conflict between the manufacturer and the retailer.

Proposition 4 *The date 1 expected profits of the manufacturer are strictly decreasing in G.*

The intuition is simply that if the retailer decides to borrow, then the manufacturer is compelled to pay a trasfer fee, given by (10), which unambiguously reduces his expected profits.

Taken together, Propositions 3 and 4 highlight a potential conflict of interest between the retailer and the manufacturer. They also illustrate why the latter may wish to impose an exogenous constraint on debt. In the presence of this conflict of interest, and given that both parties are risk-neutral, it makes sense to investigate how their joint profit varies with the retailer's debt.

The relationship between joint profit and debt is not in general unambiguous. It becomes so, however, with the natural assumption that G_{max} is determined by a competitive process among lenders, with value determined by the condition that the expected profit from lending is zero.[4]

[4]For a model with a similar assumption, see Brander and Spencer (1989). Like the assumption of an exogenously fixed (up to G_{max}) interest rate, this might reflect a relatively passive role of the lenders, or their reliance on aggregate statistics for an industry.

Assumption 2 *G_{max} is the solution in G of the zero expected profit condition for lenders:*

$$F\left(\theta^*(G)\right)\, r_d\, G - \left(1 - F\left(\theta^*(G)\right)\right)\, G = 0. \tag{17}$$

Hence:

$$F\left(\theta^*(G_{max})\right) = \frac{1}{1+r_d}. \tag{18}$$

This is the supply curve of loans; naturally, it is an increasing function of r_d. For any strictly positive lending rate r_d, (17) has the interesting implication that $\theta^*(G_{max}) < \overline{\theta}$; that is, the probability of bankruptcy is always positive when the retailer chooses the highest possible level of debt.

In this case we have

Proposition 5 *Let Assumption 2 hold. At date 1, the sum of the expected profit of manufacturer and retailer is a strictly decreasing function of G.*

This result shows that it is in the joint interest of the two firms to commit not to take on debt. Moreover, Proposition 4 shows that the manufacturer, unlike the retailer, has an individual incentive to enforce the rule. When the retailer prefers to take on as much debt as possible, there exists a Pareto improving long term contract. At date 0 in Figure 1, the parties agree that the retailer is not to take on debt. Indeed, this is typical of franchise contracts, and Proposition 5 offers a rationale for this type of conditions. Both parties can be made better off if, before any activity is undertaken, the retailer agrees not to take on any debt. The additional surplus generated in this way could make both parties strictly better off. [5]

We end the paper with a numerical example. This is important because it shows that both the minimum and the maximum value can be chosen in plausible situations.

Let the demand be linear, $Q = a - p_r$ with $a = 3.2$, and let the distribution of θ be exponential, $F(\theta) = \frac{1-e^{-\lambda(\theta-\underline{\theta})}}{1-e^{-\lambda(\overline{\theta}-\underline{\theta})}}$. This has hazard function $h(\theta) = \frac{\lambda e^{\lambda(\theta-\underline{\theta})}-1}{\lambda}$ increasing in θ and satisfying Assumption 1 for $\underline{\theta} = 0.1$, $\overline{\theta} = 3.0$, $\lambda = -5.8$. The simulation results are listed in Table 1.[6]

Table 1 shows that, for low interest rates, the difference in (16) is positive and therefore the retailer prefers G_{max}. An increase in the interest rate leads

[5] While we find that the franchisor would prefer the initial investment to be entirely equity-financed, the literature has highlighted a positive role of debt. Norton (1995) argues that debt can signal the franchisee's ability and also limit the incentive to free-ride. In practice, this implies a minimum proportion of equity financing of less than 1. For instance, McDonald's requires that at least 40% of the total cost of a new restaurant (estimated between $408,600 and $647,000) must be paid from the franchisee's personal resources; the franchisee is then allowed to finance the remainder through a loan.

[6] Details of their derivation are available on request.

Table 1: Numerical values of the cut-off point θ^*, of the debt level G_{max}, of eq. (16), of the probability of bankruptcy, $1 - F(\theta^*)$, for different values of the interest rate r_d.

r_d	θ^*	G_{max} x 1,000	$V(G_{max}) - V(0)$ x 1,000	$1 - F(\theta^*)$
0.01	2.998	.2146	.188246	.0099
0.11	2.982	.5193	.156708	.0991
0.21	2.967	.9136	.104260	.1735
0.31	2.953	1.374	.032776	.2366
0.32	2.952	1.423	.024687	.2424
0.33	2.950	1.472	.016440	.2481
0.34	2.949	1.522	.00803641	.2537
0.35	2.948	1.573	-.000520443	–

to higher borrowing and an increase in the probability of bankruptcy, as indicated in Proposition 2. For $r_d \geq 0.35$, $V(G_{max}) - V(0)$ becomes negative and the retailer, rather than borrowing and facing a high risk of bankruptcy, chooses to finance the investment entirely via personal resources.

5 Conclusion

The paper illustrates how financial arrangements can affect the relationship between an upstream and a downstream firm. In particular, we suggest a reason for a franchisor to impose limits on the franchisees' borrowing. We show that a franchisee may prefer to finance the investment necessary to carry out the franchised operation, with borrowed funds. This unambiguously reduces the franchisor's expected profit and creates a conflict of interest between the parties. This conflict can be solved, with possible beneficial effects for both firms, by imposing an upfront restriction on the franchisee's ability to borrow. Our explanation is thus complementary to existing theories of franchising based on a one-sided moral hazard perspective, where equity financing constitutes a device against quality chiseling by franchisees (Mathewson and Winter, 1985).[7]

A traditional argument for franchising is that franchisors face a binding capital constraint and resort to franchising to overcome it.[8] In our analysis, the franchisor resorts to a downstream firm because the latter has superior skills in retailing. If the franchisee is wealth constrained, and considering

[7]Alternative explanations consider a two-sided moral hazard problem, where both the franchisor and the franchisee need incentives to perform. See Bhattacharyya and Lafontaine (1995), and Lal (1990). Empirical findings in Lafontaine (1992) show that franchising is best explained by a model that assumes moral hazard on the part of both firms.

[8]See Norton(1995) for a critical analysis.

the negative effects of the franchisee's debt on the franchisor's profit, the franchisor can find it beneficial to provide capital to the franchisee. This is often observed in practice.[9]

Appendix

A *Proof of Proposition 1*

The first part is only sketched, since it can be found in Gal-Or (1991). Consider the manufacturer's problem (5)-(7).

The Hamiltonian is:

$$\mathcal{H} = \Big[Q(p_r(\theta))\big(p_r(\theta) - \theta\big) - \pi_r(\theta) - G\Big] f(\theta) - \mu(\theta) Q(p_r(\theta)) \tag{19}$$

where $\mu(\theta)$ is the costate variable. The control and the state variable of the problem are, respectively, $p_r(\theta)$ and $\pi_r(\theta)$. Since there is no uncertainty regarding the demand schedule, choosing $p_r(\theta)$ or $p_w(\theta)$ is equivalent for the manufacturer. Indeed, once the retail price has been worked out, to avoid double marginalisation the manufacturer imposes an wholesale price as identified by (9). Application of the maximum principle yields

$$\mu(\theta) = F(\theta) \tag{20}$$

and hence all the results in the first part of the Proposition.

To prove the second part of the proposition, consider the problem (5)-(7) as a free-terminal point problem (Leonard and Van Long, 1992), whose transversality condition is given by: $\mathcal{H}(\theta^*) = 0$. Recalling that $\pi_r(\theta^*) = 0$ and using (20), θ^* satisfies:

$$\Big[Q(p_r(\theta^*))\big(p_r(\theta^*) - \theta^*\big) - G\Big] f(\theta^*) - F(\theta^*) Q\left(p_r(\theta^*)\right) = 0 \tag{21}$$

which gives (12). **Q.E.D**

B *Proof of Proposition 2*

Total differentiation of (12) yields:

$$\begin{aligned}\Bigg\{\bigg[Q(p_r(\theta)) + Q'(p_r(\theta))\left[p_r(\theta) - \theta - \frac{F(\theta)}{f(\theta)}\right]\bigg] \dot{p}_r(\theta^*) - \\ Q(p_r(\theta^*))\left\{1 + \frac{d}{d\theta^*}\left[\frac{F(\theta^*)}{f(\theta^*)}\right]\right\}\Bigg\} d\theta^* = dG\end{aligned} \tag{22}$$

From (8), the terms multiplying $\dot{p}_r(\theta)$ are equal to zero. Rearranging yields (13). **Q.E.D.**

C *Proof of Proposition 3*

Let $\Omega\left(\theta_2, \theta_1\right) = \int_{\theta_1}^{\theta_2} [Q(p_r(\tilde{\theta}))] d\tilde{\theta}$. Then the maximand in (14) can be written as:

$$V(G) = \int_{\underline{\theta}}^{\theta^*(G)} \Omega\left(\theta^*(G), \theta\right) dF(\theta) - \left(I - \frac{G}{1 + r_d}\right). \tag{23}$$

[9]Lafontaine (1992) reports that 223 out of 1,114 franchisors declared in a survey that they are willing to provide financing to their franchisees.

Differentiate to get:

$$\frac{dV(G)}{dG} = \Omega(\theta^*(G),\theta^*(G))\frac{d\theta^*}{dG}f(\theta^*(G)) + \int_{\underline{\theta}}^{\theta^*(G)}\left[\frac{\partial\Omega(\theta^*(G),\theta)}{\partial\theta^*(G)}\frac{d\theta^*}{dG}\right]f(\theta)d\theta + \frac{1}{1+r_d} \quad (24)$$

Note that the first term on the RHS is equal to zero (as $\Omega(\theta,\theta) = 0$ for every θ) and that $\left[\frac{\partial\Omega(\theta^*,\theta)}{\partial\theta^*(G)}\right] = Q(p_r(\theta^*(G)))$. Use these facts and (13) to write (24) as:

$$\frac{dV(G)}{dG} = -\frac{F(\theta^*(G))}{1+h'(\theta^*(G))} + \frac{1}{1+r_d} \quad (25)$$

Differentiate again:

$$\begin{aligned}\frac{d^2V(G)}{dG^2} &= -\frac{d\theta^*}{dG}\frac{f(\theta^*(G))\{1+h'(\theta^*(G))\} - F(\theta^*(G))\,h''(\theta^*(G))}{\{1+h'(\theta^*(G))\}^2} \\ &= -\frac{d\theta^*}{dG}f(\theta^*(G))\frac{\{1+h'(\theta^*(G)) - h(\theta^*(G))\,h''(\theta^*(G))\}}{\{1+h'(\theta^*(G))\}^2} \quad (26)\end{aligned}$$

This is positive if Assumption 1 holds. **Q.E.D.**

D *Proof of Corollary 1*

Consider (25). At $G = G_{max}$, a sufficient condition for (25) to be positive is that $h'(\theta^*) > r_d$, given that $F(\theta^*(G_{max})) < 1$. When $G = 0$, a similar argument applies, after recalling that $F(\overline{\theta}) = 1$. **Q.E.D.**

E *Proof of Corollary 2*

The retailer's expected profit in the two extreme cases of 0 and G_{max} debt is, respectively:

$$V(0) = \int_{\underline{\theta}}^{\overline{\theta}}\Omega\left(\overline{\theta},\theta\right)\, f(\theta)d\theta \,-\, I \quad (27)$$

$$V(G_{max}) = \int_{\underline{\theta}}^{\theta^*(G_{max})}\Omega\left(\theta^*(G_{max}),\theta\right)\, f(\theta)d\theta \,-\, \left(I - \frac{G_{max}}{1+r_d}\right) \quad (28)$$

Take their difference:

$$V(0) - V(G_{max}) = \int_{\underline{\theta}}^{\theta^*(G_{max})}\left[\Omega\left(\overline{\theta},\theta\right) - \Omega\left(\theta^*(G_{max}),\theta\right)\right] f(\theta)d\theta + \int_{\theta^*(G_{max})}^{\overline{\theta}}\Omega\left(\overline{\theta},\theta\right)\, f(\theta)\,d\theta \,-\, \frac{G_{max}}{1+r_d}. \quad (29)$$

Since $\Omega\left(\overline{\theta},\theta\right) - \Omega\left(\theta^*(G_{max}),\theta\right) = \Omega\left(\overline{\theta},\theta^*(G_{max})\right)$, we can evaluate the first integral in (29) obtaining:

$$V(0) - V(G_{max}) = \Omega\left(\overline{\theta},\theta^*(G_{max})\right)\, F(\theta^*(G_{max}) + \int_{\theta^*(G_{max})}^{\overline{\theta}}\Omega\left(\overline{\theta},\theta\right)\, f(\theta)\,d\theta \,-\, \frac{G_{max}}{1+r_d}. \quad (30)$$

We have: $\left[\dfrac{\partial \Omega(\overline{\theta},\theta)}{\partial \theta}\right] = -Q(p_r(\theta))$. Use integration by parts to unite the second term in (30) as:

$$\int_{\theta^*(G_{max})}^{\overline{\theta}} \Omega\left(\overline{\theta},\theta\right)\, f(\theta)\, d\theta = \Omega\left(\overline{\theta},\theta\right)\, F(\theta)|_{\theta^*(G_{max})}^{\overline{\theta}} + \int_{\theta^*(G_{max})}^{\overline{\theta}} Q\big(p_r(\theta)\big)\, F(\theta) d\theta, \tag{31}$$

and substitute it in (30) to obtain (16). **Q.E.D.**

F Proof of Propositions 4 and 5

Considering eqs.(5), (9), (10) and the retailer's profit (11) the manufacturer's expected profit $\pi_m(G)$ can be expressed as:

$$\begin{aligned}\pi_m(G) &= \int_{\underline{\theta}}^{\theta^*(G)} \left[Q(p_r(\theta))(p_r(\theta)-\theta)-G\right] f(\theta) d\theta \\ &- \int_{\underline{\theta}}^{\theta^*} \int_{\theta}^{\theta^*(G)} \left[Q(p_r(\theta))d\theta\right] f(\theta) d(\theta)\end{aligned} \tag{32}$$

Taking the derivative with respect to G and substituting (12) yields:

$$\frac{d\pi_m(G)}{dG} = -F(\theta^*(G)) < 0 \tag{33}$$

At date 1, joint profits are:

$$\pi_m(G) + V(G) = \int_{\underline{\theta}}^{\theta^*(G)} \left[Q(p_r(\theta))(p_r(\theta)-\theta)-G\right] f(\theta) d\theta - \left(I - \frac{G}{1+r_d}\right). \tag{34}$$

Differentiate, substitute (12) and use (18) to obtain:

$$\frac{d\ (\pi_m(G)+V(G))}{d\,G} = -\frac{F(\theta^*(G))}{1+h'(\theta^*(G))} < 0 \tag{35}$$

Q.E.D.

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