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« Cyclical Mackey Glass Model for Oil Bull Seasonal »

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Cyclical Mackey Glass Model for Oil Bull Seasonal

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Abstract

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In this article, we propose an innovative way for modelling oil bull seasonals taking into account seasonal speculations in oil markets. Since oil prices behave very seasonally during two periods of the year (summer and winter), we propose a modification of Mackey Glass equation by taking into account the rhythm of seasonal frequencies. Using monthly data for WTI oil prices, Seasonal Cyclical Mackey Glass estimates indicate that seasonal interactions between heterogeneous speculators with different expectations may be responsible for pronounced swings in prices in both periods. Moreover, the seasonal frequency $\pi / 3$(referring to a period of 6 months) appears to be persistent over time.

Keywords: Oil bull seasonal, Seasonal speculations, Heterogeneous agents model, Seasonal Cyclical Mackey Glass models.

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1- Introduction

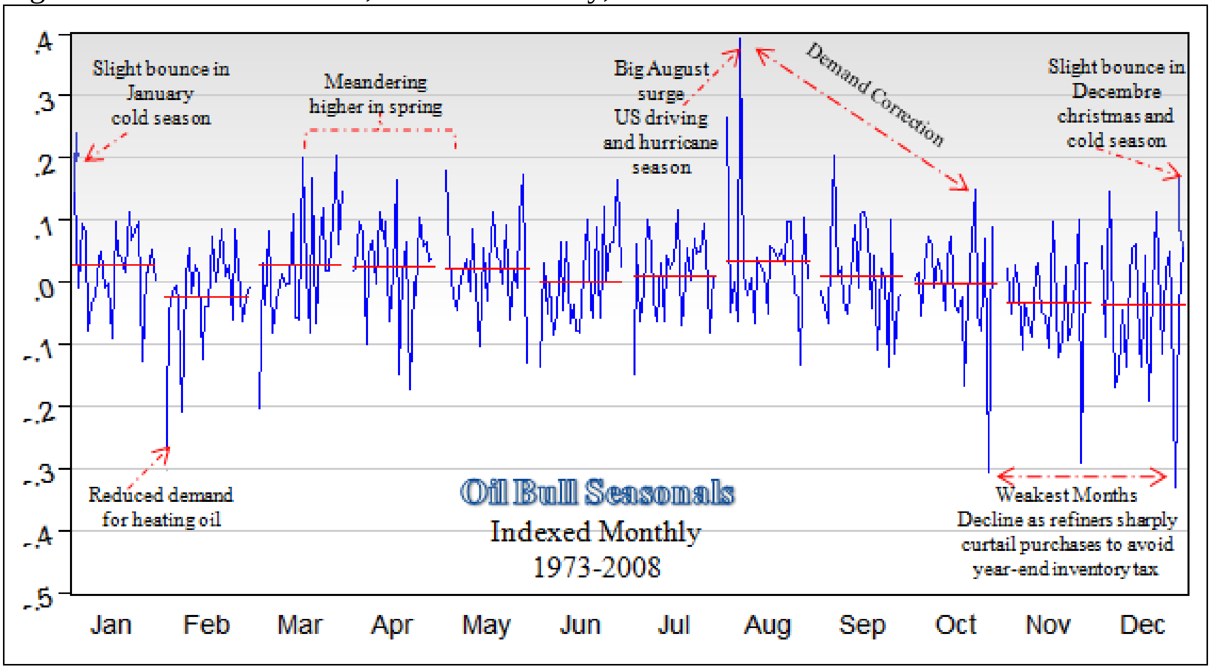
Forecasting crude oil prices is still one of the big challenges encountered by economists and econometricians. Oil prices are clearly characterized by unpredictable and volatile price movements. The supply is inelastic in short run, and the future position of prices depends on the future situation of demand. Therefore, an additional demand for oil, triggers speculation and prices has moved up sooner than they would have otherwise (Collins & al., 2006; Kraugman, 2008; Kaufman & al. 2009 and Fattouh, 2010). It is in this spirit that, and in order to gain a deeper understanding of the underlying market, we have been becoming more interested in seasonals effects which affect oil prices. Since, crude oil is characterized by two particular times of year during which they behave very seasonally, due to the summer driving/hurricane season and the winter heating season.

During the study period 1973-2008, crude oil shows some interesting seasonal highlights (figure 1). Going to each year, the price exhibited strongest strength on average in August. This surge can attribute it to various factors. One is anticipation of the hurricane season in the Gulf and hence possible supply disruptions spawned by hurricanes. Another is the fact that August is often the biggest vacation month which drives very strong gasoline demand. August tends to be the highest-demand month for gasoline, exerting upwards pressure on oil prices. Since a rational bull began, oil has risen. As the summer driving season ends, and the weather gets colder, the demand for all petroleum products wanes. This slowdown in demand is coupled with an increase in supply caused by the sharp curtail purchases of refiners to avoid year-end inventory tax, explain the pullback in prices during October-November period (Winston, 2009).

Then starting from Decembre to January, the crude oil seasonal uptrend increases. For a variety reasons including high heating fuel demand and Christmas travel season, oil prices and oil stocks tend to do well in the winter months. They are a great winter speculation. Giving that demand is tied to temperature, with demand increasing as the temperatures drops. As seasonal weather variations are unpredictable, demand forecasting is almost impossible. Consequently, an additional demand caused by a cold snap, triggers speculation and drives up prices. Thus, a seasonal rational bubble can form.

Since and taking into account the underlying assumption is that there are different types of agents with heterogeneous expectations active in the market, heterogeneous speculators interactions in both supply and demand sides may are responsible of wide swings in oil prices in both periods. Therefore, the uncertainty and the anxiety on the future is a main factor that can play an important role in the large price changes of crude oil and can be an indication of speculative behaviour in the oil market. Thus, it is sufficient that an additional demand caused by the presence of fear factor or a climate hazard, lead to increase the seasonal speculative then a bull speculative is a potential result (Greenspan, 2006)

Figure 1: Oil Bull Seasonals, Indexed Monthly, 1973-2008



Econometrically, modelling oil seasonal speculations by non linear dynamics has aroused considerable interest. The classical seasonal linear models, based on seasonal auto-regressive integrated moving average models (SARIMA), are not strongly fitted. Therefore, treating the seasonal behaviour in presence of non linear structure has become necessarily. Thus, many studies used non linear processes to detect the presence of seasonal behaviours in time series structures. This class of models was introduced by Franses and Ooms (1997) who suggested a Periodic Auto-regressive procedure. Their model highlights the importance of considering seasonal behaviour in the presence of a nonlinear stochastic process. In another hand, Guiming and Getz (2007) used a stochastic approach of basic structural model (BSM), a state-space time series model, which exhibits seasonal and multi-annual variations in abundance. Moreover, in order to take into account complex structures, Kyrtsou and Terraza

(2009) introduced seasonal chaos-stochastic processes that permitted to capture seasonal fluctuations in stock prices. Ferrara and Guégan (2008) proposed a Seasonal Cyclical Long Memory model, which includes generalized long memory processes and seasonal long memory processes.

Despite the fact that bull seasonals is a significant aspect of oil price time series, models cited above do not address to treat this kind of anomaly. Therefore, this paper intends to interrogate the role of market speculation in rising oil prices during both famous periods (winter and summer). Particularly, we will identify the hypothesis that whether the seasonal speculative activities of heterogeneous speculators are responsible of prices swings. To this aim, we propose a modification of the Mackey Glass equation taking into account the rhythm of seasonal frequency (we called Seasonal Cyclical Mackey Glass model). Using this kind of modelling to forecast oil prices appears to be an attractive alternative, due to its unique ability of modelling the seasonal effects in the presence of deterministic behaviour.

This structure of the paper is as follow. Section 2 give a description of our stylized model of the oil market with heterogeneous interacting traders. In section 3, we present the Seasonal Cyclical Mackey-Glass models. Section 4 contains a description of the data that has been used, empirical and the estimation results. Our concluding remarks will end the paper.

2- A stylized model

Our models inspired by the chartist-fundamentalist approach, which has proven to be quite successful in replicating some important stylized fact of oil market (Ellen & Zwinkels, 2010). The underlying assumption of this model is that there are different types of agents with seasonal heterogeneous expectations active in both winter-summer seasons in the market.

The group of speculators is divided between fundamentalists and chartist. The seasonal demand for oil for fundamentalists is based on the difference between the price at time t and the expected price at time t+1 of the season s. s represent winter and summer seasons.

$$D_{t,s}^{F} = a^{F}\left[E_{t,s}^{F}(P_{t+1,s}) - P_{t,s}\right] \quad (1)$$

In which $P_{t,s}$ is the price in period t and season s. a^F represent a positive reaction parameter and E the expectation operator. Fundamentalist seasonal demand will increase as they expect the future price to be higher than the current price in the season and vice versa. The fundamentalist expected seasonal price given by:

$$E^F_{t,s}(P_{t+1,s}) = P_{t,s} + b^F_1 (P_{t,s} - F_{t,s})^+ + b^F_2 (P_{t,s} - F_{t,s})^- \tag{2}$$

In which $F_{t,s}$ is the fundamental price in period t at season s. The equation shows that the price movement expected by fundamentalists is caused by the deviation of the price from the fundamental value in the season. Fundamentalists' seasonals reactions to overvaluation (undervaluation) is captured by $b^F_1 \in [-1,0]\left(b^F_2 \in [-1,0]\right)$ and expected to be negative, since, in the season s, fundamentalists will expect the oil price to decrease (increase) if the current price is above (below) the fundamental value. Whenever, b^F_1 equals b^F_2 , there is a symmetric reaction to overvaluation and undervaluation.

The second group of speculators is called chartists. These speculators apply a very simple form of technical analysis to form their expectations about future prices. The seasonal demand of chartists is linearly conditional on the expected price changes.

$$D^C_{t,s} = a^C \left[E^C_{t,s}(P_{t+1,s}) - P_{t,s}\right] \tag{3}$$

Where a^C denotes a positive reaction parameter. This implies that demand will rise as chartists expect the future price to be higher than the current price in the same season s. chartists seasonals expectations are given by:

$$E^C_{t,s}(P_{t+1,s}) = P_{t,s} + b^C_1 (P_{t,s} - P_{t-1,s})^+ + b^C_2 (P_{t,s} - P_{t-1,s})^- \tag{4}$$

A distinction is made between an upward or downward trend, or past price decrease and increase. Since technical traders expect trend movements to continue in the same direction, we expect both b^C_1 and b^C_2 to be positive. Negative parameters would imply contrarian behavior. If $b^C_1 > b^C_2$, chartists react more to price increase. In the other hand, if $b^C_1 < b^C_2$, chartists are more eager to sell in a downtrend than to buy in an upward trend.

Total market seasonal demand for oil consists of the real demand plus the weighted average of the seasonal demand of technical traders and fundamentalists:

$$D_{t,s}^{M} = D_{t}^{R} + W_{t,s} D_{t,s}^{F} + (1 - W_{t,s}) D_{t,s}^{C} \text{ and } W_{t,s} = \left[1 + \exp\left(-\gamma \left[\frac{A_{t,s}^{F} - A_{t,s}^{C}}{A_{t,s}^{F} + A_{t,s}^{C}} \right] \right) \right]^{-1} \qquad (5)$$

Where $W_{t,s} \in <0,1>$ is the part of fundamentalist in the market, such that $1 - W_{t,s}$ is the fraction of chartist in period t in the same season. Parameter γ is the intensity of choice and represent the extent to which performance of a certain strategy determines whether it is adopted. With $\gamma > 0$, a strategy that is performing better in period t is more broadly applied on time t+1 at season s, and therefore the seasonal demand of that group will weigh more heavily in period t+1. Conversely, if $\gamma = 0$ the reverse situation would take place (DeJong et al., 2009a ; Ellen & Zwinkels, 2010). Price changes, finally, are a function of excess seasonal demand plus a noise term.

$$P_{t+1,s} = P_{t} + \varphi \left[D_{t,s}^{M} - S_{t,s} \right] + \varepsilon_{t} \qquad (6)$$

Where S is the supply of oil is assumed to be a linear function of price. φ is a positive price adjustment parameter governing market frictions and ε_t is supposed to be a random noise term. Therefore, the solution for the oil price can be derived as:

$$\begin{aligned} \Delta P_{t+1,s} = a + bP_{t} &+ W_{t,s} \left(\alpha_{1} \left(P_{t,s} - F_{t,s} \right)^{+} + \alpha_{2} \left(P_{t,s} - F_{t,s} \right)^{-} \right) \\ &+ \left(1 - W_{t,s} \right) \left(\beta_{1} \left(P_{t,s} - P_{t-1,s} \right)^{+} + \beta_{2} \left(P_{t,s} - P_{t-1,s} \right)^{-} \right) + \varepsilon_{t,s} \end{aligned} \qquad (7)$$

From equation (7) we can see that, for a given value of α and β, fundamentalist and chartists traders' stabilizing seasonal impact on the oil price increases nonlinearly with their confidence in fundamental and technical analysis. We now turn to the empirical implementation of the model.

3- The seasonal cyclical Mackey Glass model

The particular systems we chose are the famous Mackey-Glass (1977) non linear time delay differential equation. The model is given by the following equations:

$$\frac{dp}{dt} = \alpha \frac{P_{t-\tau}}{1 + P_{t-\tau}^{c}} - \delta P_{t-1} \qquad \text{Where } c > 0 \qquad (8)$$

We must note that the choice of lags τ and c is crucial since they determine the dimensionally of the system. α and δ are parameters to estimates. Let us now modify the MG equation and take into account the rhythm of seasonal frequency. This rhythm defines the frequency of separate seasons. During each cycle period prices increase to some maximal value and decreases back to its normal situation. The seasonal frequencies are defined as $\omega = \frac{2\pi}{s} = 2\pi f$ where $f = \frac{1}{s}$ and s is the number of observations per year (for example, $s = 1$ for annual data, $s = 12$ for monthly data...etc). Hence, for oil prices, the instantaneous ventilation V is a non negative periodic function. We suppose that it can be modelled as $V = [1 + \sin\omega(t-\tau)]$.

$$P_t = \alpha \frac{P_{t-\tau}}{1 + P_{t-\tau}^c}\left(1 + \sin\omega(t-\tau)\right) - \delta P_{t-1} + \varepsilon_t \tag{9}$$

Where ε_t is *i.i.*d. Much of chaos properties are still valid when noise is added to the system, provided the noise level is not too high (Guégan, 1994). Moreover, the stochastic part added in the Mackey Glass equation takes two cases. In the first case, we add white noise to the Modified Mackey Glass equation (Homoskedastic errors) where $\varepsilon_t \sim \mathrm{N}(0,1)$. When anomalies are Heteroskedastics, the stochastic part added on the Modified Mackey Glass equation follows an ARCH(1) process, where $\varepsilon_t / I_t \sim \mathrm{N}(0, \mathrm{h}_t)$. h_t is the conditional variance (Kyrtsou and Terraza, 2003).

The local asymptotic stability of the equilibrium of equation (9) implies global asymptotic stability, that is, all solutions converge to zero when t tends towards infinity. To formulate a criterion of asymptotical statibility for equation (9), stability of seasonal point can be studied as suggested by Landa and Rosenblum (1995). As a results, $\left(P_{t-\tau} - P^* \equiv 0\right)$ is oscillatory instable if $V^* \leq SP^*$ and

$$\tau > \tau_{cr} = \frac{\arcsin\left[-V^* / SP^*\right]}{\alpha\sqrt{S^2 P^{*2} - V^{*2}}} \tag{10}$$

Where P^* is the singular point of equilibrium, $V^* = V\big|_{P_{t-\tau}=P^*}$, $S = \left(dV / dX_{t-\tau}\right)\big|_{P_{t-\tau}=P^*}$. The period of oscillations close to the stability boundary is approximately equal to 6τ

(approximately one pick per 6 months). We suppose that the frequency of the seasonal rhythm f weakly depends on prices level at some previous moment of time, and that the purpose of this control is to maintain the linearity of prices level. We also assume that the estimated level of prices may not necessarily be in the equilibrium state but may vary around it neighbourhood $\left(P^{*}\right)$. This suggests that we consider small modulations of the frequency of the seasonal rhythm:

$$f = f_0 + \beta\left(P_{t-\tau} - P^{*}\right) \quad \text{Where } 0 \leq |\beta| \leq 1 \tag{11}$$

Where β is the parameter of modulation, and the frequency of normal seasonality rhythm f_0 is equal to $\pi/3$ (referring to a period of 6 months). As shown previously, this small modulation leads, nevertheless, to a nontrivial effect. For $\tau < \tau_{cr}$ and β varying nearly around zero we obtain a periodic effects in time and in level. For $\tau > \tau_{cr}$ and $\beta = 0$ we observe a quasi periodic regime with a basic frequency f_0. For $\tau > \tau_{cr}$ and $\beta > 0$ we observe irregularly seasonal effects in level.

4- Empirical results.

The data consists of the following real monthly spot prices at the New York Mercantile exchange (NYMEX) of light crude oil of West Texas (WTI) from January 1973 to December 2008. We focus, however, on market returns from these spot prices. The data was obtained from the Information Administration Energy (IAE). In order to proceed to an unbiased and unambiguous interpretation of long memory and nonlinearity phenomena, oil prices should first be rendered stationary. ADF applied on oil raw series and showed that the presence of unit root in oil spot prices (table 1). Therefore, we consider first order differencing of raw series, denoted (DLOIL) and defined by $DLOIL_t = \ln(OIL)_t - \ln(OIL)_{t-1}$.

Table 1 presents the descriptive statistics for the oil returns. Using ADF for testing the unit root, we firmly accept stationary of oil series at 5% significance level. We observe that there is excess kurtosis relative to the standard distribution. The distribution is positively skewed. The combination of a significant asymmetry and leptokurtosis indicates that oil prices series is not normally distributed as is suggested by Jarque-Bera statistic. The Engle (1982) test result confirms the presence of Heteroskedasticity and residuals are auto correlated.

Moreover, the fractional integration parameter d estimated by the GPH, The null hypothesis of interest is whether the return series are integrated of order zero ($H_0 : d = 0$), versus the alternative of fractional integration ($H_1 : d \neq 0$). Estimates for the fractional integration parameter d are provided in table 1, along with t-statistics for the null hypothesis $d = 0$. We consider the point estimates by the GPH estimator with an estimation window of $T^{0.8}$. These estimates indicate evidence of long memory in oil spot prices, but with $d_{GPH} > 0$. Positive values of the fractional differencing parameters indicate predictability in variance. The point estimates are characterized by persistent process, suggesting that the variance of the series is dominated by low frequency (slow cycle) and the spectral density tend to infinity when the frequency tends towards zero. The statistic test shows that the movement of oil prices appears as results of an exogenous shock affecting the oil market (Elder and Serletis, 2008).

Table 1: Statistics Summary of DLOIL

	Skew	Kurt	JB	ADF		ARCH (12)	Q(12)	GPH
				Raw	Δ			
DLOIL	1.97	26.8	10513[a]	0.48	-49.25[a]	24.87[a]	27.92[a]	0.274[a]
			(0.00)	(0.81)	(0.00)	(0.01)	(0.00)	(0.02)

[a] Reject the null hypothesis at 5% significance level. The Q(12) statistic represents the Ljung-Box (Q) statistics for autocorrelations in the residuals.

In testing presence of seasonal effects in oil prices, we first estimate autoregressive models for oil series with control for possible seasonal effects, as in:

$$DLOIL_t = \sum_{i=1}^{p} \alpha_i DLOIL_{t-1} + \sum_{j=1}^{12} \beta_j D_{jt} + \varepsilon_t \tag{12}$$

Where D_{jt} represent the 12[th] month-of-the-year dummies. The lag length is selected based on the Akaike criterion. Table 2 reports results from OLS regressions. There is evidence of seasonal effects in oil returns series. We found a significantly positive coefficient in August and negatively in January. This result coincides with those of Hamilton (2006) which showed that the demand of crude oil in both August and January months are the highest in year, thus prices are logically the highest in both months. Moreover, in the context of descriptive tests, we have to test whether oil prices structure contains non linear and the chaos process. But the presence of linear structure may be is responsible for the rejection of chaos. Therefore, we have to eliminate the low frequencies signals from oil prices structure.

Table 2: Seasonality test

Oil	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Est	- 0.035^{w} (0.01)*	-0.021 (0.1)**	0.006 (0.64)	0.020 (0. 08)**	-0.013 (0.35)	0.047 (0.01)*	0.022 (0.87)	0.026^{s} (0.04)*	-0.003 (0.80)	-0.014 (0.34)	-0.02 (0.03)*	-0.033^{w} (0.02)*

*Represent the significance level of 5%. ** Represent the significance level of 10%. ^{W} is the winter effects. ^{S} is the summer effects.

However, we filter our series using the ARFIMA model. Then we apply the non linear statistics test on ARFIMA filtered residuals (RFDLOIL) to investigate the hypothesis of non linear seasonal process after controlling for long memory. Statistics results of ARFIMA (*p*,*d*,*q*) processes are summarized in Table 3. The value of the fractional integration parameter is $\hat{d} = 0.39$ and is accepted at 1% significant level (between ± 0.5). Applying ARCH-LM test on the ARFIMA filtered residuals (RFDLOIL) confirms that the errors are Heteroskedastics but are not auto correlated and also the RFDLOIL series is not normally distributed as is suggested by Jarque-Bera statistic.

Table 3: ARFIMA tests on DLOIL

$M\hat{A}$	$\hat{d}$	$ARCH-LM(12)$	$Q(12)$	J.B
0.10 (0.00)*	0.39 (0.00)*	11.83 (0.00)*	17.68 (0.12)	3687 (0.00)*

**results accepted at 1% significant level.*

Due to the fact that nonlinearity is a necessary (but not sufficient) condition for chaos, the BDS test (Brock and al., 1996) is used to test the null of whiteness against the alternative of non-white linear and non-white nonlinear dependence. It is based on the estimation of the correlation integral, which was introduced in the context of dynamical systems by Grassberger and Procaccia (1983).

Table 4: The BDS test results (RFDLOIL)

ε / σ	BDS Statistic (0.5)	P-Value
m=2	0.05	0.00*
m=3	0.11	0.00*
m=4	0.15	0.00*
m=5	0.17	0.00*
m=6	0.18	0.00*

** The critical value is 1.96 for the 5% significant level.*

Practitioners of BDS test usually consider different embedding dimensions. We use six embedding dimensions for BDS test. We set $\varepsilon = 0.5\sigma$. It is obvious from table 4 that the null of whiteness is rejected according to all computed statistics, and hence the remaining dependence is consistent with nonlinear dynamic explanation. We can conclude that there is evidence of nonlinearity of general form.

To test the chaos, we use Wolf and al. (1985) test to compute the Lyapunov exponents. To that end the notion of Lyapunov exponent is introduced since it is usually taken as an indication of the underlying dynamic system characteristic. In the presence of noise, as it is often the case with real world data sets, the meaning of 'detecting deterministic chaotic dynamics' is ambiguous. Thus, the algorithm developed by Wolf and al. (1985), which is used to estimate the growth rate of the propagation of small perturbations in the initial conditions, appears to be not robust[4]. Therefore, given that there is large amount of exogenous influence perturbing the endogenous dynamics, it is necessary to define Lyapunov exponent in a stochastic context (see Tong 1992). Nychka et al. (1992) defined as $X_{t+1} = F(X_t) + \varepsilon_{t+1}$.

Since the largest Lyapunov exponent λ_1 has often been of main interest in the literature, we mainly focus our analysis on the largest Lyapunov exponent and simply denote it as λ. However, it should be noted that other exponents λ_i for $2 \le i \le d$ also contain some important information related to the stability of the system, including the directions of divergence and contraction of trajectories (see Nychka et al., 1992) and the types of non-chaotic attractors (see Dechert and Gençay, 1992). The presence of a positive exponent is sufficient for diagnosing particular classes of chaos and presents local instability in a given direction. The results obtained of the log-differenced price series are reported in the table 5. The best Lyapunov exponent is that which minimises the SIC criteria. Results show that the minimum SIC value occurs when we use 6 hidden units. In this case, the corresponding Lyapunov exponents are λ_1=0.1239 e-05 and λ_2=-0.1972. For both cases, λ_1 is positive and λ_2 is negative. Consequently, we conclude that there is clear evidence for a mixture of process. The fact that λ_1 is slightly positive could be due to the existence of high dimensional chaos which could be confused with stochastic process[5].

[4] Lyapunov exponents test of Wolf and al. (1985) is very sensitive to the noise level. Thus, we cannot be sure that the test is robust when we have a high noise level in financial series.

[5] Noise can be always be interpreted as a deterministic time evolution in infinite dimension (Ruelle, 1994)

On the other hand, since λ is negative, we cannot conclude that there is a stochastic behaviour; behaviours may be periodic (Mosekilde and Laugesen, 2007). In the case where we have high periodic effects, seasonality can produce high variance, similar to that of stochastic behaviours. In other words, the presence of heteroskedasticity in the series may result from seasonal effects. Therefore, we suppose that we are facing a complex structure composite of mixture of process: a slightly irregular behaviour that is sensitive to small perturbations (chaos) and periodic behaviour.

Table 5: Lyapunov exponents' estimates on ARFIMA filtered residuals

Hiddens	λ_1	λ_2	*SIC*
1	-0.2934 e-05	-1.9076	-12.98
2	0.2798 e-05	-0.9528	-12.94
3	0.1955 e-05	-0.5671	-13.24
4	0.1788 e-05	-0.4190	-13.38
5	0.1388 e-05	-0.2918	-13.47
6	0.1239 e-05	-0.1972	-13.55
7	-0.1299 e-05	-0.1311	-13.52
8	0.1179 e-05	-0.0576	-13.47
9	0.2100 e-05	-0.0202	-13.49
10	-0.2222 e-05	-0.0418	-13.51

From preceding applications, we conclude that the hypothesis of nonlinearity in oil spot prices movements cannot be rejected and is not an *i.i.d*. process. Moreover, it is not clearly determine, according to the Lyapunov exponents' statistics, what exactly is the source of the nonlinear behaviour. The plausible explanation is that there are both chaotic and periodic behaviours in log-differenced oil prices returns. To this hypothesis, we apply Seasonal Cyclical Mackey-Glass models. Using $\tau = 5$ and $c = 2$ [6] with frequency $\pi / 3$ (referring to period of 6 months), the SCMG parameters estimated are significant at 5% level (table 6). Therefore, the model detects important evidences of non linearity is the seasonal bull in oil return. Moreover, the found seasonal solution for $\tau = 5$ $\left(\tau < \tau_{cr}\right)$ and $\beta = 0.0029$ (around zero) seems to be chaotic solution. Furthermore, one can say that it can be quasi periodic. The difficult to distinguish clearly between both processes may be due to high noise level. In consequence, the seasonal bull detected in oil prices at frequency $\pi / 3$ appears to be persistent over time. Finally, tests on SCMG filtered residuals showed that residuals are empty of heteroskedasticity and autocorrelation, despite that of presence of no normality

[6] Selection with SIC criterion.

(JB)[7]. Meaning that, the heteroskedasticity in residuals may due to periodic behaviours in oil prices.

Table 6: The parameters results of SCMG model at frequency $\omega = \pi / 3$

Coefficient	Coefficients	*p-value*	ARCH-LM	Q(12)	JB
$\hat{\alpha}$	0.4016	0.05*	2.56	12.66	124*
$\hat{\delta}$	0.1493	0.00*	(0.24)	(0.39)	(0.01)
$\hat{\beta}$	0.0029	0.06**			

* $\hat{\alpha}$ and $\hat{\delta}$, are accepted at 5% significance level. ** The coefficients are accepted at 10% significant level. Equation (5) applied on SCMG residuals to verify whether residuals structure contain a cyclical effects. Statistics test showed that residuals are empty of cyclical effects.

We describe now the forecasting experience. We apply four models on the RFDLOIL series to compute the root-mean-squared error (RMSE hereafter) of each model. The models are SARIMA, SCLM, SMG-GARCH and SCMG. To have a quick summary of the results, we compute the ratios of the RMSE, by dividing the RMSE from the SCMG model by the one from each model. Thus, a ratio lowers than one indicates a better forecasting performance of the SCMG model (table 7).

Table 7: Ratios of the RMSE for the SCMG model over the RMSE of each model

	SCMG/SARIMA	SCMG/SCLM	SCMG/SMG-GARCH
Ratio of RMSE	0.919	0.992	0.92

The table reports forecast evaluation statistics for a full sample horizon. The sample cover total of 433 forecasts for the horizon considered. The forecasting models are: $SARIMA(3,0,0)(2,1,0)_6$, where the estimates model as follow: $\left(I - B^6\right)\left(I - 0{,}224B - 0.279B^2 - 0.132B^3\right)\left(I - 0.542B^6 - 0.188B^{12}\right)X_t^1 = \varepsilon_t$, $R^2 = 0.147$. The second is Seasonal Cyclical Long Memory model. The parameter estimate of the model associated to the cycle of period six months $\pi / 3$. The estimates model defined as: $\left(I - 0{,}321B + 0.244279B^2\right)^{0.065}\left(I + 0.119B\right)^d X_t^1 = \varepsilon_t$, where $R^2 = 0.23$. Finally, the seasonal MG-GARCH (1,1). We used the Dummy variable from the period of the 15 December to 30 January and from the period of 1 August to 30 september equal to 1 and 0 otherwise. Using $\tau = 1$ and $c = 2$, the model is accepted at 5% significance level. $R^2 = 0.192$.

5- **Conclusion**

In this paper we developed an empirical oil market model to detect the dynamic seasonal cyclical behaviours in oil prices series. The main conclusion obtained from this application

[7] Residuals remained structure is not identified. This may due to an unknown structure or to a misspecification of one of our parameters.

is that oil has more potential to be existing strong seasonally in both December-January and August time of year. Therefore, the movements associated with frequency $\pi/3$ appeared to be persistent over time. Moreover, results suggest that speculative activities are responsible for changes in spot prices in both peaks of year, especially when the speculative trading strategies are influenced by periodic information. Thus, heterogeneous agents' hypothesis and their non linear trading impact influenced by seasonal effects may explain the pronounced swings in oil prices, as witnessed in recent years. As consequence, these results are interesting for this crucial commodity investors, which contain an excellent information that can help fine-tune the timing of entry and points for oil-stocks investors and speculators to maximize gains in this ongoing oil-stock bull, and it is important to be looking at all aspects of the markets. Finally, the SCMG models can be very competitive in terms of forecasting in comparison with classical linear and non linear models.

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