entER
for
nomic Research

Discussion paper

Tilburg University

Center
for
Economic Research

No. 9704

EXCHANGE RATE TARGET ZONES: A NEW APPROACH

By Frank de Jong, Feike C. Drost and Bas J.M. Werker

January 1997

ISSN 0924-7815

Exchange Rate Target Zones: A New Approach

Frank De Jong Feike C. Drost Bas J. M. Werker *

January 17, 1997

Abstract

We propose a new model for an exchange rate target zone that captures most stylized facts from the existing target zone models while remaining analytically tractable. The model is based on a modified two-limit version of the Cox, Ingersoll, and Ross (1985) model. In the model the exchange rate is kept within the band because the variance decreases as the exchange rate approaches the upper or lower limits of the band. We also consider an extension of the model with parity adjustments, which are modeled as Poisson jumps. Estimation of the model is by GMM based on conditional moments. We derive prices of currency options in our model, assuming that realignment jump risk is idiosyncratic. Throughout, we apply the theory to EMS exchange rate data. We show that, after the EMS crisis of 1993, currencies remain in an implicit target zone which is narrower than the officially announced target zones.

JEL codes: F31, G13
Keywords: exchange rate target zones, currency option pricing

Address:
Department of Econometrics
Tilburg University
PO Box 90153
5000 LE Tilburg, The Netherlands
phone: +31-13-4662911
fax: +31-13-4663280

---

*De Jong and Drost, Tilburg University; Werker, Université Libre de Bruxelles and Tilburg University. Part of the work on this paper was performed while the second and third author were visiting the Université de Toulouse I (IDEI/GREMAQ). Joost Driessen and Manfred Bartels provided excellent computational assistance. We thank seminar participants at Tilburg and Toulouse, Jesper Lund, Nour Meddahi, Eric Renault, and Paul Söderlind for useful comments. Special thanks to Antoon Pelsser for showing us how to derive the marginal distribution of the target zone process.

1 Introduction

Following the seminal contribution by Krugman (1991), a lot of effort has been put into the modeling of exchange rates in a target zone.[1] In such a target zone, the exchange rate has a lower and upper bound, which the central banks defend by interventions in the foreign exchange markets. Krugman's important observation is that, due to the forward looking nature of exchange rate determination, these interventions have a stabilizing effect even when exchange rates are well inside the band and intervention is not currently effective. Obviously, for exchange rates in the European Monetary System (EMS) the assumption of a fully credible target zone is violated by historical exchange rate behaviour. Most exchange rates were prone to parity adjustments (realignments). Subsequent target zone models, e.g. Bertola and Caballero (1992), Bertola and Svensson (1993), and Dumas, Jennergren and Näslund (1995a), have therefore incorporated some form of realignment risk.

The empirical analysis of Krugman's model and its extensions has not been without problems. The estimation of these models is not trivial due to the use of a regulated Brownian motion as the driving process in most of these models. The likelihood function for this process is rather complicated (see Ball and Roma (1994)). An alternative is the method of simulated moments, but the experience with these estimators is not very satisfactory (see Smith and Spencer (1991), Lindberg and Söderlind (1994), and De Jong (1994)). Second, the pricing of derivatives, such as currency options, in those target zone models involves the solution of a non-trivial partial differential equation, see Dumas, Jennergren, and Näslund (1995a).

In this paper, we propose a relatively simple model for exchange rates in a target zone that captures most stylized facts from the existing target zone literature while remaining analytically tractable. In particular, estimation of the model and option pricing are simpler than in existing target zone models. The setup of our model deviates from the usual approach in the target zone literature because we do not start from a specification of economic fundamentals. Instead, we specify a stochastic process for the exchange rate itself. The model can be considered as the reduced form of a structural target zone model which involves economic fundamentals.[2] The stochastic process for the exchange rate exhibits mean reversion of the Ornstein-Uhlenbeck type. The instantaneous volatility

[1] For an overview of this literature see Svensson (1992a).

[2] A reduced form approach to target zone modeling is also taken by Vlaar and Palm (1993), Koedijk, Stork and De Vries (1996) and Bekaert and Gray (1996), who all specify their model in discrete time.

depends on the level of the exchange rate and decreases if the exchange rate approaches the boundary of the target zone.[3] Realignment jumps follow a Poisson process with constant intensity which is independent of the position of the exchange rate in the band. The size of the jump is not fixed but may depend on the current level of the exchange rate and the current central parity.

An advantage of our model over existing target zone models is that conditional expectations and variances of the exchange rates take simple analytical forms. Estimation of the model is by GMM based on these conditional moments. An application of the model is to derive estimates of implicit target zones, which are potentially different from officially announced target zones. In particular we estimate implicit target zones for EMS currencies. After the EMS crisis of 1993, the official target zone was widened from 2.25% to 15% for most currencies. Our empirical results show that an implicit target zone of around 6% is obeyed by the major EMS currencies.

The final part of the paper deals with currency option pricing in the target zone model. Here we distinguish two cases, models with and without realignment jumps. In the model without realignment jumps the market is complete and prices can be obtained as the expected payoff under the risk-neutral distribution of the exchange rate. Introducing realignment risk complicates the analysis because markets become incomplete in that case. Consequently, a single no-arbitrage condition is no longer sufficient to derive currency option prices. We solve this problem along the lines proposed by Melenberg and Werker (1996). In the first step of this method, option prices are calculated conditional on the jump process. In the second step the option price is obtained by taking the expectation of the conditional option prices with respect to the true distribution of the jumps.

The plan of this paper is as follows. In Section 2 we present the model and derive several important properties. In Section 3 we consider estimation of the model. We consider the small sample properties of these estimators using Monte Carlo simulations. In Section 4 we present an empirical application to EMS exchange rate data. In Section 5 we demonstrate how to use our model for the pricing of currency options. Finally, Section 6 concludes.

[3]This model can be seen as two-limit version of the square root process, popularized in the financial literature by Cox, Ingersoll, and Ross (1985) in a model for the term structure of interest rates.

2 Model

In our model, we want to capture some well-known stylized facts of exchange rates in a target zone (see Vlaar and Palm (1993) for an overview). First, EMS exchange rates exhibit mean reversion within the band. Stated differently, exchange rates tend towards their central parity. However, there are infrequent adjustments of this central parity due to realignments. These adjustments may cause a long run upward or downward trend in the exchange rate. There is also evidence of strong heteroskedasticity due to a time-varying volatility of the exchange rates. Finally, EMS exchange rates exhibit jumps, not only due to realignments but also within the band. Our model intends to capture these stylized facts. We incorporate reversion towards the central parity, conditional heteroskedasticity and jumps in the model. To introduce the main features of our approach we start this section with a simple model that does not allow for parity adjustments. Realignments jumps will be introduced later.

2.1 A model without realignments

Let S_t be the exchange rate process under consideration and suppose that this exchange rate may not deviate more than a certain percentage, say z, from a central parity μ_t, i.e

$$\mu_t/(1+z) \leq S_t \leq \mu_t(1+z).$$

Put $X_t = \log S_t$, $M_t = \log \mu_t$, and $Z = \log(1+z)$, then the target zone condition on the exchange rate can be rewritten as

$$M_t - Z \leq X_t \leq M_t + Z. \tag{2.1}$$

The assumption that the band is symmetric is made mainly for notational simplicity; it is not difficult to introduce two distinct variables to describe the upper and lower margin of the band, say Z_- and Z_+. The width of the target zone will be non-random in all our models.

In this section we discuss a model where the central parity is constant, $M_t = m$, which corresponds to a fully credible target zone. The exchange rate is assumed to satisfy the following stochastic differential equation,

$$dY_t = -\rho Y_t dt + \sigma\sqrt{Z^2 - Y_t^2} dW_t, \tag{2.2}$$

where W_t is a Wiener process. In this model we have the following parameters and constants: $m, \rho > 0, Z > 0$, and $\sigma > 0$. If the continuous exchange rate process X_t

approaches the boundaries $m \pm Z$, the volatility of the error term $\sigma\sqrt{Z^2 - (X_t - m)^2}dW_t$ decreases to zero, while the autoregressive part $-\rho(X_t - m)dt$ drives the exchange rate back to the central parity. As is clear from the square-root in (2.2), this model is strongly inspired by the model of Cox, Ingersoll, and Ross (1985) and the shifted CIR model of Duffie and Kan (1995). On the other hand, the instantaneous variance is, like in the geometric Brownian motion, a polynomial of degree two. In contrast to the CIR type models with one boundary at the bottom, we have two boundaries here. To keep the exchange rate in the target zone we do not need to specify an explicit intervention process as is usual in the target zone literature.

One of the interesting features of our model is the marginal distribution of the exchange rate. The Krugman (1991) model predicts a U-shaped marginal distribution, which is generally not found empirically. There is ample evidence that the marginal distribution has most mass near the middle of the band [see, e.g., Bertola and Caballero (1992), Flood, Rose, and Mathieson (1991), and Lindberg and Söderlind (1994)]. Let $Y_t = X_t - m$ denote the deviation of the log exchange rate from the fixed central parity. From the Kolmogorov equations it follows that the marginal density of Y_t is given by

$$p(y) = \frac{1}{Z}\frac{\Gamma(\beta + \frac{1}{2})}{\Gamma(\beta)\sqrt{\pi}}(1 - (y/Z)^2)^{\beta-1}, \tag{2.3}$$

where $\beta \equiv \rho/\sigma^2$. This distribution can take various shapes,[4] depending on the sign of β. If $0 < \beta < 1$, the distribution is U-shaped with peaks at the boundaries. For $\beta = 1$, the marginal distribution is uniform. If $\beta > 1$ the pull towards the central parity is sufficiently strong to generate a unimodal, hump-shaped marginal density. This density is empirically more plausible than the marginal distribution in the model of Lindberg and Söderlind (1994). In that model, the marginal distribution has most mass in the middle of the band but also has two peaks at the boundaries.

To be able to estimate the unknown parameters, it would be convenient to have an explicit form of the discrete time model. Efficient econometric analysis of the model must be based on the conditional distribution of the exchange rate given the history of the process. However, the conditional distribution is rather complicated and unknown for the model with jumps. Therefore, we decided to base our econometric analysis on the conditional moments of the process, which have simple analytical forms. Let $h > 0$ denote the time interval between two observations. In Appendix A we derive the

[4]Several possibilities are graphed in Figure 1.

following conditional mean, variance, and skewness[5]

$$E_t(Y_{t+h}) = e^{-\rho h} Y_t, \tag{2.4a}$$

$$Var_t(Y_{t+h}) = \{e^{-\sigma^2 h} - 1\} e^{-2\rho h} Y_t^2 + \{1 - e^{-(\sigma^2+2\rho)h}\} \frac{\sigma^2 Z^2}{\sigma^2 + 2\rho}, \tag{2.4b}$$

$$Sk_t(Y_{t+h}) = \{e^{-3\sigma^2 h} - 1\} e^{-3\rho h} Y_t^3 + \{1 - e^{-(3\sigma^2+2\rho)h}\} e^{-\rho h} Y_t \frac{3\sigma^2 Z^2}{3\sigma^2 + 2\rho}. \tag{2.4c}$$

From these expressions the unconditional moments of Y_t follow immediately:

$$\begin{aligned} E(Y_t) &= 0, \\ Var(Y_t) &= \frac{\sigma^2 Z^2}{\sigma^2 + 2\rho}, \\ Sk(Y_t) &= 0. \end{aligned}$$

The discrete observations of the logarithm of the exchange rate in the target zone model have an AR(1)-ARCH(1) structure. This can be made more explicit by writing the conditional variance as

$$Var_t(Y_{t+h}) = \omega_h + \alpha_h Y_t^2, \tag{2.5}$$

with

$$\omega_h = \{1 - e^{-(\sigma^2+2\rho)h}\} \frac{\sigma^2 Z^2}{\sigma^2 + 2\rho} > 0,$$

and

$$\alpha_h = \{e^{-\sigma^2 h} - 1\} e^{-2\rho h} < 0.$$

Obviously, the volatility decreases if the process approaches one of the boundaries $\pm Z$. The conditional skewness of the exchange rate also depends on the current position of the exchange rate in the band. We will exploit these analytical expressions for the conditional moments to derive estimators for the parameters in the model, see Section 3.

2.2 Models with realignments

The main drawback of the model discussed above is the absence of parity changes. We now generalize the ideas of the previous subsection to a class of models with realignments. We model the central parity as a separate stochastic process, M_t, which exhibits occasional jumps. Following Svensson (1992b), Bertola and Svensson (1993), and Dumas, Jennergeren, and Näslund (1995a) the jumps are modeled as a Poisson process

[5]The conditional skewness, $Sk_t(X_{t+h})$, is defined as the centralized third moment $E_t[(X_{t+h} - E_t(X_{t+h}))^3]$

with constant jump intensity. The size of the jumps is predetermined and depends on the current exchange rate and the current central parity. The model for the exchange rate itself is similar to the model presented in the previous section. The drift of the exchange rate is of the Ornstein-Uhlenbeck type, but with the current central parity as reference point. The instantaneous variance follows the parameterization in (2.2). This leads to the following model[6]

$$dX_t = -\rho(X_{t-} - M_{t-})dt + \sqrt{Z^2 - (X_{t-} - M_{t-})^2}dW_t, \quad (2.6a)$$

$$dM_t = (X_{t-} - M_{t-})dN_t. \quad (2.6b)$$

where N_t is a Poisson process with jump intensity θ. The effect of the realignment jump in this model is to reset the central parity, M_t, to the current value of the exchange rate, X_{t-},

$$M_t = M_{t-} + (X_{t-} - M_{t-})\Delta N_t = \begin{cases} M_{t-} \\ X_{t-} \end{cases} \text{if } \Delta N_t = \begin{cases} 0 \\ 1 \end{cases}.$$

This realignment structure does not allow for jumps outside the current target zone and the exchange rate itself is a continuous variable.[7]

To show that the system (2.6a)-(2.6b) allows a solution (X_t, M_t), let $Y_t = X_t - M_t$ be the exchange-rate-within-the-band, defined as the deviation between the exchange rate X_t and the central parity M_t. The stochastic differential equation for Y_t is given by

$$dY_t = -\rho Y_{t-}dt + \sigma\sqrt{Z^2 - Y_{t-}^2}dW_t - Y_{t-}dN_t. \quad (2.7)$$

This determines a unique solution Y_t. Plugging in this solution into (2.6b) yields M_t and, hence, also X_t. This proves the existence of a unique solution of the system of stochastic differential equations (2.6a)-(2.6b). As before, it is clear that $|Y_t| = |X_t - M_t| \leq Z$, so that the exchange rate X_t remains in the band $[M_t - Z, M_t + Z]$.

The conditional first and second moments (variances and covariances) of the exchange rate X_t and the central parity M_t in model (2.6a)-(2.6b) can be calculated using the methods explained in Appendix A. To simplify the exposition we present the conditional moments of the central parity process M_t and the difference process $Y_t = X_t - M_t$.

$$E_t Y_{t+h} = e^{-h(\rho+\theta)}Y_t, \quad (2.8a)$$

$$E_t M_{t+h} = M_t + \frac{\theta}{\rho+\theta}\{1 - e^{-h(\rho+\theta)}\}Y_t, \quad (2.8b)$$

[6]The expression X_{t-} denotes the exchange rate just before time t.

[7]A similar assumption is made in the model of Dumas, Jennergren, and Näslund (1995a).

$$Var_t Y_{t+h} = \{e^{-h(\sigma^2+2\rho+\theta)} - e^{-2h(\rho+\theta)}\}Y_t^2 + \{1 - e^{-h(\sigma^2+2\rho+\theta)}\}\frac{\sigma^2 Z^2}{\sigma^2+2\rho+\theta}, \quad (2.8c)$$

$$Cov_t(Y_{t+h}, M_{t+h}) = \frac{\theta}{\rho+\theta}\{e^{-2h(\rho+\theta)} - e^{-h(\rho+\theta)}\}Y_t^2, \quad (2.8d)$$

$$Var_t M_{t+h} = \frac{\theta}{\sigma^2+2\rho+\theta}\{1 - e^{-h(\sigma^2+2\rho+\theta)}\}(Y_t^2 - \frac{\sigma^2 Z^2}{\sigma^2+2\rho+\theta}) + \frac{\theta^2}{(\rho+\theta)^2}\{2e^{-h(\rho+\theta)} - e^{-2h(\rho+\theta)} - 1\}Y_t^2 + (\frac{\sigma^2 Z^2}{\sigma^2+2\rho+\theta})\theta h. \quad (2.8e)$$

These formulas show that the difference process Y_t has again the AR-ARCH property, similar to the exchange rate in the fixed parity model. The mean reversion is determined by the sum $\rho + \theta$. The variance of the central parity M_t increases with the horizon h and therefore X_t and M_t are integrated of order 1, using the terminology of Engle and Granger (1987). Moreover, the central parity M_t and the exchange rate X_t are co-integrated. The direction of the drift is determined by the sign of Y_t: if the exchange rate is above the central parity the expected drift is upward, otherwise it is downward.

2.3 Conclusion

To conclude this section, we discuss the merits of the models above. All the formulations capture the stylized facts in exchange rates: decreased volatility near the boundaries and mean reverting processes. Moreover, the conditional moments have an AR-ARCH property: conditional expectations are autoregressive and conditional variances depend on past squared observations. The model without realignments (2.2) is just a starting point and could be used for descriptive purposes for periods when it is known that there are no parity changes. To include the possibility of parity changes we extend (2.2) to the system of stochastic differential equations (2.6a) (2.6b). Of course, one may criticize the simplicity of the realignment process since the probability of jumps is constant over time and the central parity jumps exactly to the prevailing exchange rate at the time of the jump.[8] However, some important properties of EMS exchange rates are captured by this realignment model. Of course, the proof of the pudding is in the eating and in Section 4 we will see how these models perform for several currencies in the EMS.

[8]This assumption is of course empirically easily rejected but is maintained to facilitate the analysis of the model.

3 GMM estimation of the target zone model

Target zone models of exchange rates have proved to be hard to estimate. The original model of Krugman (1991) and most of its extensions (Lindberg and Söderlind (1994), Bertola and Caballero (1992), and Bertola and Svensson (1993), among others) are based on the regulated Brownian motion or a regulated Ornstein-Uhlenbeck process, sometimes augmented with a jump process. This stochastic structure makes it hard to analyze discretely sampled data since the likelihood function and conditional moments do not take simple analytical forms. In response, many authors (Smith and Spencer (1991), Lindberg and Söderlind (1994), and others) have relied on method of simulated moments estimation, which is feasible because the underlying processes are relatively easy to simulate. The experience with this type of estimators is not generally positive, however. The problem is that unconditional expectation and variance of the exchange rate provide relatively little information about the parameters of interest, see De Jong (1994) for an empirical assessment of the performance of these estimators in a target zone model. The best estimator available so far is the Maximum Likelihood estimator of Ball and Roma (1994), which is specific for the model with exogenous mean reversion and a fully credible target zone (originally proposed by Lindberg and Söderlind (1994)). The estimator works well, but is fairly complicated and not easily extended to other target zone models.

3.1 The GMM estimator

In our models, the likelihood function of the exchange rate is not known, except in some special cases. However, we do have exact analytical expressions for the conditional moments of at least second order. The information in these moments can be utilized using the GMM framework. The GMM literature is vast; a paper which focusses especially on conditionally heteroskedastic models is Meddahi and Renault (1996). The general idea is as follows: let m_t denote a vector which is composed of deviations between powers of the innovations, ε_t, and their conditional expectations. By construction, the conditional expectation of this vector is zero: $E_{t-1}m_t(\theta) = 0$. In general, this moment depends on a vector of unknown parameters, θ. To estimate θ, we can exploit moment restrictions of the form

$$E[z_{t-1}m_t(\theta)] = 0, \tag{3.1}$$

where the instrument z_{t-1} is known at $t-1$. It is well known (see, e.g. Godambe (1985) and Newey (1990)) that the optimal instrument given the choice of the moment vector m_t is

$$z_{t-1} = E_{t-1}(\frac{\partial m'}{\partial \theta})Var_{t-1}(m_t)^{-1}. \tag{3.2}$$

In general, including more elements in the moment vector m_t will enhance the efficiency of the GMM estimator, provided that the optimal instrument is used to generate the moment restrictions (3.1).

If the true $Var_{t-1}(m_t)$ is very complicated, the optimal instrument can be difficult to construct. Following Meddahi and Renault (1996), in such a case we can specify some approximation V_{t-1} and use the following instrument

$$z_{t-1} = E_{t-1}(\frac{\partial m'}{\partial \theta})V_{t-1}^{-1}. \tag{3.3}$$

The GMM estimator based on this instrument will be less efficient, but may be more tractable than the estimator based on the optimal instrument (3.2). Alternatively, if an initial consistent estimator is available, we could construct the optimal instrument by non-parametric estimates of $E_{t-1}(\frac{\partial m'}{\partial \theta})$ and $Var_{t-1}(m_t)$, where m_t is evaluated in the initial consistent estimate (see Wefelmeyer (1996)).

Using the optimal instrument (3.2) or the approximation (3.3), the number of moment conditions in equation (3.1) equals the number of parameters, so that the GMM estimator is the solution to the system

$$\sum_{t=1}^{n} z_{t-1}m_t(\hat{\theta}) = 0. \tag{3.4}$$

The limiting distribution of $\hat{\theta}$ follows from

$$\sqrt{n}(\hat{\theta} - \theta_0) \cong -\sqrt{n}[\sum_t z_{t-1}\frac{\partial m_t}{\partial \theta'}]^{-1}\sum_t z_{t-1}m_t. \tag{3.5}$$

The asymptotic variance of $\hat{\theta}$ takes the familiar form $A^{-1}BA^{-1}$, where A and B are the limits of

$$A_n \equiv \frac{1}{n}\sum_t z_{t-1}E_{t-1}(\frac{\partial m_t}{\partial \theta'}), \tag{3.6a}$$

$$B_n \equiv \frac{1}{n}\sum_t z_{t-1}Var_{t-1}(m_t)z'_{t-1}, \tag{3.6b}$$

respectively. Note that in these expressions conditional expectations have been taken. In the special case of the GMM estimator based on the optimal instrument (3.2) the asymptotic variance of $\hat{\theta}$ equals $A^{-1} = B^{-1}$.

In finding the numerical solution to the moment equation (3.4) we have to take into account that the instrument z_{t-1} may depend on θ. The previous analysis goes through if we evaluate $z_{t-1}(\theta)$ in some initial consistent estimate, $\hat{\theta}^{(o)}$. In that case we can use the standard Newton-Raphson algorithm to solve the moment equation (3.4),

$$\hat{\theta}^{(i+1)} = \hat{\theta}^{(i)} - [\sum_t z_{t-1}(\hat{\theta}^{(o)}) \frac{\partial m_t}{\partial \theta'} |_{\hat{\theta}^{(i)}}]^{-1} \sum_t z_{t-1}(\hat{\theta}^{(o)}) m_t(\hat{\theta}^{(i)}) \tag{3.7}$$

An asymptotically equivalent method is to consider z_{t-1} as a function of $\hat{\theta}$, so that $z_{t-1} = z_{t-1}(\hat{\theta})$. The Newton-Raphson iterations then are

$$\hat{\theta}^{(i+1)} = \hat{\theta}^{(i)} - [\sum_t H_t(\hat{\theta}^{(i)})]^{-1} \sum_t z_{t-1}(\hat{\theta}^{(i)}) m_t(\hat{\theta}^{(i)}), \tag{3.8}$$

where $H_t(\theta)$ is the derivative of $z_{t-1}m_t$ with respect to θ. This derivative is rather complicated, but we may replace it by its conditional expectation

$$E_{t-1}H_t(\theta) = z_{t-1}(\theta)E_{t-1}(\frac{\partial m_t}{\partial \theta'}) \tag{3.9}$$

which is typically much easier to calculate. When using the optimal instrument (3.2) or the approximation (3.3), $\sum_t E_{t-1}H_t(\theta)$ will be symmetric and positive definite almost surely.

3.2 Examples

We now discuss some examples of the GMM estimator which are useful for estimating our exchange rate target zone models

EXAMPLE 3.1 Meddahi and Renault (1996) show that the Quasi Maximum Likelihood Estimator (see Gouriéroux, Montfort, and Trognon (1984)) is a special case of the GMM estimator. We briefly describe their analysis in this example. Suppose we have a time series model which specifies the conditional mean and variance of X_t,

$$E_{t-1}X_t = \mu_t(\theta),$$

$$Var_{t-1}(X_t) = h_t(\theta).$$

The QML estimator is the vector θ that maximizes the quasi log-likelihood

$$\ln L = -\frac{1}{2}\sum_{t=1}^{n}[\ln(h_t(\theta)) + \frac{(x_t - \mu_t(\theta))^2}{h_t(\theta)}].$$

The first order condition from which $\hat{\theta}$ is solved is given by

$$-\sum_{t=1}^{n}\left[\frac{1}{2h_t(\theta)}\frac{\partial h_t}{\partial\theta} - \frac{(x_t-\mu_t(\theta))}{h_t(\theta)}\frac{\partial\mu_t}{\partial\theta} - \frac{(x_t-\mu_t(\theta))^2}{2h_t(\theta)^2}\frac{\partial h_t}{\partial\theta}\right] = 0.$$

We now verify that the GMM estimator based on the conditional mean and variance has exactly the same first order conditions. Define $\varepsilon_t = X_t - \mu_t$ and the vector of moments

$$m_t = \begin{pmatrix} \varepsilon_t \\ \varepsilon_t^2 - h_t \end{pmatrix},$$

where the dependence on θ is suppressed in the notation. By construction we have $E_{t-1}m_t = 0$. The derivative of m_t is given by

$$\frac{\partial m_t}{\partial\theta'} = \begin{pmatrix} -\partial\mu_t/\partial\theta' \\ -2\varepsilon_t\partial\mu_t/\partial\theta' - \partial h_t/\partial\theta' \end{pmatrix}.$$

Writing $\mu_{it} = E_{t-1}\varepsilon_t^i$ we obtain

$$E_{t-1}(\frac{\partial m_t}{\partial\theta'}) = -\begin{pmatrix} \partial\mu_t/\partial\theta' \\ \partial h_t/\partial\theta' \end{pmatrix},$$

$$Var_{t-1}(m_t) = \begin{pmatrix} h_t & \mu_{3t} \\ \mu_{3t} & \mu_{4t} - h_t^2 \end{pmatrix}.$$

The optimal instrument therefore depends on the third and fourth conditional moment of X_t. An approximation to $Var_{t-1}(m_t)$ is found by assuming that the conditional distribution of X_t is normal. This yields the simplified covariance matrix

$$V_{t-1} \equiv \begin{pmatrix} h_t & 0 \\ 0 & 2h_t^2 \end{pmatrix}$$

and, hence, the instrument

$$z_{t-1} = E_{t-1}(\frac{\partial m'}{\partial\theta})V_{t-1}^{-1} = -(\frac{\partial\mu_t}{\partial\theta}\frac{1}{h_t}, \frac{\partial h_t}{\partial\theta}\frac{1}{2h_t^2}).$$

The moment equation to be solved therefore is

$$\sum_{t=1}^{n} z_{t-1}m_t(\theta) = \sum_{t=1}^{n}\left[-\frac{\partial\mu_t}{\partial\theta}\frac{\varepsilon_t}{h_t} - \frac{\partial h_t}{\partial\theta}\frac{(\varepsilon_t^2 - h_t)}{2h_t^2}\right] = 0.$$

which is exactly the first order condition of the QML estimator.

EXAMPLE 3.2 In this example we extend the QML estimator by using also the conditional skewness of X_t, defined as $s_t(\theta) \equiv E_{t-1}(X_t - \mu_t(\theta))^3$. As before, define $\varepsilon_t = X_t - \mu_t$ and the vector of moments

$$m_t = \begin{pmatrix} \varepsilon_t \\ \varepsilon_t^2 - h_t \\ \varepsilon_t^3 - s_t \end{pmatrix}.$$

The derivative of m_t with respect to the parameter vector is given by

$$\frac{\partial m_t}{\partial \theta'} = \begin{pmatrix} -\partial\mu_t/\partial\theta' \\ -2\varepsilon_t\partial\mu_t/\partial\theta' - \partial h_t/\partial\theta' \\ -3\varepsilon_t^2\partial\mu_t/\partial\theta' - \partial s_t/\partial\theta' \end{pmatrix},$$

with conditional expectation

$$E_{t-1}(\frac{\partial m_t}{\partial \theta'}) = -\begin{pmatrix} \partial\mu_t/\partial\theta' \\ \partial h_t/\partial\theta' \\ \partial s_t/\partial\theta' + 3h_t\partial\mu_t/\partial\theta' \end{pmatrix}.$$

The conditional variance of m_t is given by (with $\mu_{kt} = E_{t-1}\varepsilon_t^k$)

$$Var_{t-1}(m_t) = \begin{pmatrix} h_t & & \\ s_t & \mu_{4t} - h_t^2 & \\ \mu_{4t} & \mu_{5t} - h_t s_t & \mu_{6t} - s_t^2 \end{pmatrix}.$$

An approximation to this matrix is obtained by calculation higher moments as if the conditional distribution were normal, which yields

$$V_{t-1} = \begin{pmatrix} h_t & 0 & 3h_t^2 \\ 0 & 2h_t^2 & 0 \\ 3h_t^2 & 0 & 15h_t^3 \end{pmatrix},$$

with inverse

$$V_{t-1}^{-1} = \frac{1}{6}\begin{pmatrix} 15h_t^{-1} & 0 & -3h_t^{-2} \\ 0 & 3h_t^{-2} & 0 \\ -3h_t^{-2} & 0 & h_t^{-3} \end{pmatrix}.$$

Using this matrix yields the instrument

$$z_{t-1} = E_{t-1}(\frac{\partial m'}{\partial \theta})V_{t-1}^{-1} = -(\frac{\partial \mu_t}{\partial \theta}\frac{1}{h_t} - \frac{\partial s_t}{\partial \theta}\frac{1}{2h_t^2}, \frac{\partial h_t}{\partial \theta}\frac{1}{2h_t^2}, \frac{\partial s_t}{\partial \theta}\frac{1}{6h_t^3})$$

and one has to solve the moment equation

$$-\sum_{t=1}^{n}[\frac{\partial \mu_t}{\partial \theta}\frac{\varepsilon_t}{h_t} + \frac{\partial h_t}{\partial \theta}\frac{(\varepsilon_t^2 - h_t)}{2h_t^2} + \frac{\partial s_t}{\partial \theta}\frac{(\varepsilon_t^3 - 3\varepsilon_t h_t - s_t)}{6h_t^3}] = 0.$$

The conditional expectation of the matrix of first derivatives turns out take a very simple form,

$$E_{t-1}H_t(\theta) = \frac{\partial \mu_t}{\partial \theta}\frac{\partial \mu_t}{\partial \theta'}\frac{1}{h_t} + \frac{\partial h_t}{\partial \theta}\frac{\partial h_t}{\partial \theta'}\frac{1}{2h_t^2} + \frac{\partial s_t}{\partial \theta}\frac{\partial s_t}{\partial \theta'}\frac{1}{6h_t^3},$$

which is obviously positive definite.

3.3 Simulation experiments

We tested the performance of the GMM estimator in the target zone models on simulated data. In the experiments, we simulate data from the simple model (2.2) without realignments. In our simulations, the midpoint of the band is normalized at zero, $m = 0$, and the bandwidth is equal to the EMS bandwidth, $Z = 2.25$, expressed as a percentage. The values of the parameters ρ and σ^2 are chosen to resemble actual EMS exchange rate data. From the estimates in Section 4 it appears that the choice $(\rho, \sigma^2) = (0.05, 0.02)$ for weekly data is reasonable.[9] However, we also conducted experiments for data with a higher variance, $\sigma^2 = 0.04$. Note that both sets of parameters imply a hump-shaped marginal distribution function.

To simulate from the continuous time process we used a refined Euler discretisation scheme. More specifically, we simulated daily data from repeated drawings from a normal distribution with conditional mean and variance given by the expressions in equations (2.4a)-(2.4b). We sampled one in five simulated values, which corresponds to using weekly observations. Using these simulated time series we estimated the parameters by QML and with the GMM estimator that also uses the conditional skewness, as discussed in Example 3.2. Note that both estimators do not use the optimal instruments based on the true variance-covariance matrix of the moments. Instead, the approximations of this matrix based on the normal distribution are used (see Examples 3.1 and 3.2).

To summarize, the experiments differ in the following aspects:

- The true parameter values: $(\rho, \sigma^2) = (0.01, 0.004)$ or $(\rho, \sigma^2) = (0.01, 0.008)$;
- Whether to treat Z as given or as an additional parameter to estimate;
- The number of simulated observations, 300 or 1200.

All experiments were repeated 100 times.

In the left panels of Table 1 we report descriptive statistics of the parameter estimates in the case where the bandwith Z is fixed. The results are encouraging. The mean of the estimator is quite close to the true value and the standard errors are small. Only in the small sample (300 observations) there seems to be a slight overestimation of ρ; the estimates for σ^2 are about right on average. The GMM standard errors are in line with the Monte Carlo standard deviation of the parameter estimates. The gain in efficiency

[9] The parameter values are given on a weekly basis. Hence, the half-life of the process in weeks is $ln(2)/\rho$, which equals around 14 for $\rho = 0.05$.

by adding the third moment in the estimation is marginal for this model with a fixed bandwidth.

The right panels of Table 1 reports the results for the experiments where the bandwidth Z was treated as an additional unknown parameter. Again, there is a slight upward bias in the estimates of ρ. The estimates of σ^2 are also slightly upward biased and have much bigger standard errors than in the case where Z is fixed. There is now a clear gain in efficiency of the three moment GMM estimator over the QML estimator. Especially the QML estimates of Z in the case of 300 weekly observations have very large standard errors and show a large upward bias. The GMM estimates of Z are much more precise. The GMM estimator for the other parameters (ρ and σ) seems also more efficient than the QML estimator.

The final simulation results concern the model with realignments. Table 2 presents statistics of the GMM estimator that uses the conditional mean and variance of the exchange rate within the band, as well as the conditional mean of the central parity, (2.8a)–(2.8c). The weighting matrix is again based on a normal approximation along the lines of Example 3.2. We report simulation evidence for one set of true values, $(\rho, \sigma^2, \theta) = (0.05, 0.02, 0.02)$, and a sample size of 700 weekly observations. The 3 moment GMM estimator performs rather well, although there is a slight downward bias in θ, which is compensated by an upward bias in ρ. We also experimented with a '5 moment GMM' estimator which uses, in addition to those three moments, the conditional variance of the central parity and the conditional covariance of the exchange rate with the band and the central parity, (2.8a)–(2.8e). The performance of this '5 moment' GMM estimator is worse than the performance of the three moment GMM[10] and is therefore not used in the empirical applications.

4 Empirical results for the EMS

In this section we present empirical results of our target zone model on the exchange rates of EMS currencies to the Deutsche mark. The setup of the section is as follows. First, we present estimates of the target zone model (2.2) with fixed central parity and fully credible bands. Our specification gives the opportunity to treat the bandwidth as a free parameter, so that we can estimate implicit bands. These may be wider or narrower than the officially announced band, indicating a weaker or stronger commitment to the

[10]This may be due to the use of an inefficient weighting matrix, or a result of small sample problems.

exchange rate parity than specified by the official target zone. After that, we show the evidence on the model containing realignment jumps, (2.6a)–(2.6b), which is more suited for the EMS which showed several realignments. The data used are weekly log exchange rates of the Deutsche mark expressed in local currency, multiplied by 100 to allow an interpretation as a percentage deviation from the central parity.

4.1 Estimation results for model without realignments

In Table 3 we report the estimates of the model with a fixed central parity and no jumps, which was presented in equation (2.2). To estimate this model we use data only from the most recent stable period in the EMS, which runs from January 1987 to August 1992 and contains 297 weekly observations.[11] The officially allowed deviation from the central parity in this period was 2.25%. The model is estimated using the GMM estimator presented in Section 3, based on the conditional mean, variance, and skewness given in (2.4a)–(2.4c). The first part of Table 3 reports estimates for a fixed bandwidth parameter $Z = 2.25$. The mean reversion is quite strong for the Dutch guilder, which is not very surprising as the guilder is always very close to the central parity. For the other currencies the mean reversion is less strong but significantly different from zero, and always larger than the estimated variance parameter σ^2. The latter result implies a hump-shaped marginal distribution of the exchange rate. The results for the Danish krone are somewhat different as the estimate of ρ is not significantly different from zero and smaller than the estimated σ^2.

The second part of Table 3 presents estimates over the same period where the bandwidth Z was treated as an additional free parameter. For the Belgian franc and the Danish krone, the implicit band is somewhat wider than the official band, but the difference is not significant. For the French franc the implicit band is significantly narrower than the official target zone. For reasons mentioned earlier, no reasonable estimate for the bandwidth Z of the Dutch guilder could be obtained.

Table 4 extends the sample period to the full period in which currencies participated in the EMS exchange rate mechanism with narrow bands. This covers the period February 1980 to June 1993.[12] The weeks with realignments were omitted from the

[11]The Irish pound and the Italian lira were subject to realignment in this period and are therefore excluded from the analysis. Other currencies, such as the Spanish peseta and the Portuguese escudo, have participated in the EMS for too short a period for a meaningful econometric analysis.

[12]The Italian lira dropped out of the EMS in 1992; the last observation for the lira is therefore 7 July 1992.

estimation, which was achieved by skipping the weeks with a realignment in computing the GMM criterion. The point estimates are of course different from the ones obtained over the shorter period, but qualitatively the conclusions are similar. If the bandwidth is estimated, the estimated mean reversion is (in most cases) somewhat weaker and the estimated variance parameter is smaller. These effects are probably due to the result that the implicit bandwith is wider than the official bandwith. The differences are not statistically significant, however.

Next, we extend the data to cover the period after the EMS crisis in August 1993. The subsequent widening of the bands on most currencies to 15% deviation from the central parity is often viewed as a *de facto* reversion to floating exchange rates, see Obstfeld and Rogoff (1995). On the other hand, Campa and Chang (1996) present evidence that observed prices of options on the Deutsche mark/Italian Lira and the Deutsche mark/French franc exchange rates over this period are consistent with much narrower bands. We try to assess this hypothesis by estimating implicit bands and testing whether these are narrower or wider than the posted ones. Table 5 presents estimates of this implicit bandwidth. The model assumes that before the 1993 realignment the relevant bandwidth is equal to the official bandwidth of 2.25% (6% for the lira). The bandwidth after that realignment is treated as an unknown parameter. The other parameters (ρ and σ^2) are constant over the whole sample period. The results show that the implicit bandwidth for the period after the 1993 realignments, estimated around 6% for most currencies, is narrower than the officially annouced bandwidth. The estimated implicit bandwidth for the Italian lira, which wasn't in the EMS in this period, is much larger than the estimate for the other currencies. The Dutch guilder was virtually unaffected by the realignments. Notice that the estimates of the implicit bandwidth for the French franc and the Italian lira are wider than the minimal bandwith consistent with option prices, as derived in Campa and Chang (1996).

4.2 Estimation results for model with realignments

In Table 6 the estimates of the target zone model with realignments (2.6a)-(2.6b) are reported. We use a GMM estimator based on the conditional mean and variance of the exchange rate within the band ($Y_t = X_t - M_t$) and the conditional mean of the central parity, given in (2.8a)-(2.8c). The most surprising feature of the results are the large estimates for the jump intensity θ. The estimated number of jumps per year is around 2, which is much greater than the actual number of realignments. This means that some large changes within the band could be confused for realignments. The estimates of ρ are typically negative, but the sum of ρ and θ is always positive, which ensures that the exchange rate within the band, Y_t, is mean reverting.

The somewhat disappointing results of the model with realignments lead us to an alternative estimator for the jump intensity, θ. A simple frequency estimator (the actual number of realignments divided by the total number of observations) was calculated. The remaining parameters were estimated by GMM taking the estimated value of θ as given. Standard errors were calculated as if θ were known. The empirical results of this procedure are reported in the right-hand-side panel of Table 6. The estimates for ρ and σ^2 make much more sense now. Compared with the estimates obtained from the model without realignments, the estimated variance parameter σ^2 is typically much smaller, especially if the jump probability is high. Apparently, for several currencies the jump process accounts for a substantial part of the variance in exchange rates. This will have important ramifications for option pricing, as we demonstrate in Section 5.

5 Option pricing

One of the most important reasons to consider continuous time models in finance is the pricing of financial derivatives. Since the seminal work of Black and Scholes (1973) it is well-known that continuous time models often allow for explicit calculation of option prices under a no-arbitrage assumption only. In this section we will derive prices of European-type options written on exchange rates that stay within a target zone. More specifically we will consider the exchange rate processes discussed in Section 2. The first model does not allow for realignments in the central parity. It will turn out that the market described in this case is complete, so that option prices can be calculated under an absense of arbitrage possibilities assumption. Model (2.6a)–(2.6b) does allow for realignments. It is straightforward to see that these realignments cannot be hedged perfectly when only trading the underlying exchange rate (and a riskless asset). Therefore, in this case the market is incomplete. In order to derive option prices we will follow the approach initiated by Hull and White (1987). More specifically we will follow the framework outlined in Melenberg and Werker (1996).

It will turn out that analytic formulas are hard to obtain. Therefore we will calculate option prices using risk-neutral simulation techniques. Moreover it will turn out that the absence of arbitrage opportunities in a target zone model imposes restrictions on the domestic and foreign instantaneous interest rates. The often made assumption of constant interest rates is, for instance, in this case incompatible with the absence of arbitrage possibilities.

5.1 Option pricing in models without realignments

We consider the pricing of European call options in the target zone model without realignments (2.2). To do so (and in fact obtain the price of any derivative claim on the currency) we derive the risk-neutral distribution of the exchange rate. Recall that, with X_t denoting the log exchange rate, we have

$$\mathrm{d}X_t = -\rho(X_t - m)\mathrm{d}t + \sigma\sqrt{Z^2 - (X_t - m)^2}\mathrm{d}W_t.$$

We let r denote the domestic instantaneous interest rate and r^f the foreign interest rate. In principle we could allow both interest rates to be arbitrary functions of X_t. Later it will turn out that a no-arbitrage assumption induces restrictions on the fuctional form of r and r^f.

Proposition 5.1 *Under the above assumptions the risk-neutral distribution of the exchange rate is described by*

$$d\exp(X_t) = \left[r_t - r_t^f\right]\exp(X_t)dt + \sigma\sqrt{Z^2 - (X_t - m)^2}\exp(X_t)d\tilde{W}_t, \tag{5.1}$$

with $\tilde{W}$ *a risk-neutral Brownian motion.*

PROOF: Consider a foreign money market account, whose value (in the foreign currency) satisfies

$$\mathrm{d}V_t^f = r_t^f V_t^f \mathrm{d}t.$$

In the domestic currency the value, $V_t = \exp(X_t)V_t^f$, consequently satisfies

$$\begin{aligned}
\mathrm{d}V_t &= \exp(X_t)\mathrm{d}V_t^f + V_t^f \mathrm{d}\exp(X_t) \\
&= \left[r_t^f - \rho(X_t - m) + \frac{1}{2}\sigma^2(Z^2 - (X_t - m)^2)\right] V_t\mathrm{d}t + \sigma V_t\sqrt{Z^2 - (X_t - m)^2}\mathrm{d}W_t.
\end{aligned}$$

The discounted value $\exp(-\int_0^t r_s\mathrm{d}s)V_t$ therefore follows the SDE

$$\begin{aligned}
\mathrm{d}\exp(-\int_0^t r_s\mathrm{d}s)V_t &= \exp(-\int_0^t r_s\mathrm{d}s)\mathrm{d}V_t - \exp(-\int_0^t r_s\mathrm{d}s)r_tV_t\mathrm{d}t \\
&= \left[r_t^f - r_t - \rho(X_t - m) + \frac{1}{2}\sigma^2(Z^2 - (X_t - m)^2)\right]\exp(-\int_0^t r_s\mathrm{d}s)V_t\mathrm{d}t \\
&\quad + \sigma\exp(-\int_0^t r_s\mathrm{d}s)V_t\sqrt{Z^2 - (X_t - m)^2}\mathrm{d}W_t \qquad (5.2) \\
&= \sigma\exp(-\int_0^t r_s\mathrm{d}s)V_t\sqrt{Z^2 - (X_t - m)^2}\mathrm{d}\tilde{W}_t, \qquad (5.3)
\end{aligned}$$

where equality (5.3) signifies the no-arbitrage assumption, i.e. $\tilde{W}_t$ is a risk-neutral Brownian motion[13]. This in turn implies that

$$\begin{aligned} dV_t &= \exp(\int_0^t r_s ds) d \exp(-\int_0^t r_s ds) V_t + r_t V_t dt \\ &= r_t V_t dt + \sigma V_t \sqrt{Z^2 - (X_t - m)^2} d\tilde{W}_t. \end{aligned}$$

To obtain the risk-neutral distribution of the exchange rate note that $X_t = \log(V_t/V_t^f)$ which implies

$$\begin{aligned} dX_t &= V_t^{-1} dV_t - \frac{1}{2} V_t^{-2} d\langle V_\cdot, V_\cdot \rangle_t - (1/V_t^f) dV_t^f \\ &= \left[r_t - r_t^f - \frac{1}{2}\sigma^2 (Z^2 - (X_t - m)^2) \right] dt + \sigma \sqrt{Z^2 - (X_t - m)^2} d\tilde{W}_t. \end{aligned} \quad (5.4)$$

Using Itô's Lemma to rewrite (5.4) in terms of the exchange rate $\exp(X_t)$ yields the desired result. □

We have shown that, under the no-arbitrage assumption, exchange rates $\exp(X_t)$ satisfy (5.1) where $\tilde{W}$ is a risk-neutral Brownian motion. If $r_t - r_t^f$ would be constant, (5.1) does not have a solution, which implies that arbitrage opportunities should exist. It is not difficult to see that these arbitrage opportunities occur if the exchange rate is near the target zone boundaries. In case, e.g., $r_t - r_t^f > 0$ and the exchange rate is close to the upper boundary, domestic instantaneous risk-free investments yield higher than foreign risk-free returns while the exchange rate is known not to increase by much because it remains within the target zone. Similar reasoning applies in case $r_t - r_t^f < 0$.

The above results imply that $r_t - r_t^f$ must be time-varying. We postulate r_t and r_t^f to be functions of X_t. We specify that the foreign interest rate is constant, $r_t^f = r$ while the interest differential is given by

$$r_t - r_t^f = -\tilde{\rho}(X_t - \tilde{m}), \quad (5.5)$$

where $\tilde{\rho} > 0$ denotes the risk-neutral mean-reversion coefficient and $\tilde{m} \in (m - Z, m + Z)$. The choice $\tilde{\rho} = \rho$ and $\tilde{m} = m$ implies Uncovered Interest Parity, because in that case the interest rate differential equals the expected instantaneous exchange rate depreciation. By (5.4) and (5.5) we obtain

$$d(X_t - m) = -\left[\tilde{\rho}\left((X_t - m) + \delta_m\right) + \frac{1}{2}\sigma^2 (Z^2 - (X_t - m)^2)\right] dt + \sigma \sqrt{Z^2 - (X_t - m)^2} d\tilde{W}_t, \quad (5.6)$$

[13]Formally, we know that discounted values of attainable payoffs are martingales under the risk-neutral distribution. Therefore (5.3) holds for some martingale $\tilde{W}$. Since its quadratic variation must be t, because quadratic variations are invariant under absolutely continuous changes of measure, Levy's characterization theorem implies that $\tilde{W}$ is a Brownian motion.

where $\delta_m = m - \tilde{m}$. Notice that now the log exchange rate is mean reverting and therefore will remain inside the target zone under the risk neutral distribution. Given such parametric assumptions, derivation of prices of European type call options on the exchange rate is straightforward. Consider a European call option with maturity T and excercise price $K \equiv \exp(m + k)$. A value of $k = 0.02$ implies that the excercise price of the option is 2 percent above the long-term value of the exchange rate. Under the no-arbitrage assumption, the domestic option price is given by

$$\tilde{E}_0 \exp(-\int_0^T r_s \mathrm{d}s) \max\{\exp(X_T) - K, 0\},$$

where $\tilde{E}$ signifies that the expectation is taken under the risk-neutral probability measure. Particular option prices are easily obtained using risk-neutral simulation from the stochastic differential equation (5.6).

5.2 Option pricing in models with realignments

In this section we consider option pricing in the model with realignments, (2.6a)–(2.6b). As mentioned above this model describes an incomplete market so that option prices are not solely determined by a no-arbitrage assumption. Consequently we will have to make assumptions on how to deal with the realignment risk. The method we use builds on the work of Melenberg and Werker (1996). Essentially, this method integrates out all possible jump processes. Conditional on a specific jump process, the market is complete and option pricing follows the approach outlined in the previous subsection. Option prices are obtained taking the expectation of these prices (expressed in domestic currency) with respect to the distribution of the jump process.

This approach is different form the approach proposed by Dumas, Jennergen, and Näslund (1995b). Building on the work of Merton (1976) they propose to adjust the drift term under the risk-neutral density for the probability of jumps. The adjustment is such that the jump risk is not priced from the point of view of the domestic investor. An advantage of our approach is that a no-arbitrage relation between call option prices in domestic currency and put option prices in foreign currency is always satisfied (for a proof see Appendix B), which is not the case in the model of Dumas, Jennergen, and Näslund.

The ideas used in this section are quite straightforward. We consider the model

$$\begin{aligned} \mathrm{d}X_t &= -\rho(X_{t-} - M_{t-})dt + \sigma\sqrt{Z^2 - (X_{t-} - M_{t-})^2}dW_t, \\ \mathrm{d}M_t &= (X_{t-} - M_{t-})dN_t. \end{aligned}$$

There are two sources of uncertainty in this model, W and N. Together these define an incomplete market. However, if we condition on N, we are left we only a single source of uncertainty and hence the market is "conditionally complete". This allows for the computation of option prices conditional on the behaviour of N along the same lines as in Section 5.1. To obtain the actual derivative prices we must take the expectation over N of this conditional option pricing formula. This expectation will be taken under the true probability measure. For rigorous proofs of the validity and an interpretation of this procedure we refer to Melenberg and Werker (1996).

Proposition 5.2 *In the model with idiosyncratic realignment risk the risk-neutral distribution of the exchange rate is described by*

$$d\exp(X_t) = \left[r_t - r_t^f\right]\exp(X_t)dt + \sigma\sqrt{Z^2 - (X_t - M_t)^2}\exp(X_t)d\tilde{W}_t, \qquad (5.7a)$$

$$dM_t = (X_{t-} - M_{t-})dN_t. \qquad (5.7b)$$

PROOF: We can completely copy the proof of Proposition 5.1, working conditional on N. This immediately yields (5.7a). Under the stated assumptions the risk-neutral distribution with respect to N simply equals the true one. This yields (5.7b). □

5.3 Numerical examples

In this section we give some numerical examples of our currency option pricing model. We compare the option prices generated by our model with option prices based on the models of Garman and Kohlhagen (1983) and Grabbe (1983). These models are essentially adaptions of the standard Black-Scholes model for the pricing of currency options and assume that the exchange rate follows a geometric Brownian motion and that interest rates are constant. We will show that our model sometimes can generate prices which are very different from the Black-Scholes values.

In the first experiment we calculate prices of European call options on exchange rates with maturity 52 weeks. The initial value of the exchange rate is equal to the central parity. Option prices are calculated from the target zone model without realignment jumps (2.2), the target zone model with realignments, and the Black-Scholes model. For the target zone models, the interest rate differential follows specification (5.5), where M_{t-} replaces $\bar{m}$ in the model with realignments. The numerical results for the target zone models are based on 10000 simulations where an Euler approximation with 1000 steps is used. The Black-Scholes prices are obtained by using the instantaneous standard deviation $\sigma\sqrt{Z^2 - X_0^2}$ as the volatility parameter and a constant interest rate equal to the initial interest rate.[14]

The prices and implied volatilities of call options for various exercise prices are graphed in Figure 2A.[15] The figure clearly shows that, in a credible target zone, the Black-Scholes model overprices currency options relative to our model. The main reason for this over-pricing is that the Black-Scholes model ignores the mean reversion and the decreasing volatility of the exchange rate close to the boundaries of the target zone. The overpricing is most pronounced for deep in-the-money and out-of-the-money options. Option prices in the model with realignments are higher than the prices in the model without realignments. This is not surpirising since the realignments increase the disperion of the exchange rate at maturity, which increases option prices. Despite the rather high probability of a realigment jump, the Black-Scholes model still overprices currency options. This result is specific for this parameter setting, however.

The second experiment is the same as the first, except for the intial value of the exchange rate, which is chosen to be below the central parity, $X_0 = M_0 - 0.01$. The prices and implied volatilities of the model prices are graphed in Figure 2B. Like in the first experiment, the target zone model prices are lower than the Black-Scholes prices. Unlike in the first experiment, option prices in the model with realignments are *lower* than prices in the model without realignments. The explanation for this results is that, due to the low initial value of the exchange rate, downward jumps of the central parity are more likely than upward jumps, which causes a lower expected payoff of the call option.

The third experiment is again the same as the first, but with an initial value of the exchange rate above the central parity, $X_0 = M_0 + 0.01$. The numerical results are graphed in Figure 2C. We first discuss the results for the model without realignments. In contrast with the previous results, the model prices for some options are now larger than the Black-Scholes prices. In the model with realignments, upward jumps are now much more likely than downward jumps because the initial exchange rate is above the central parity. Hence, call prices in the model with realignments are larger than both the Black-Scholes prices and the prices in the model with a credible target zone.

[14]The other parameter settings are as follows: the bandwidth Z equals 0.0225; the scale parameter $\sigma^2 = 0.014$; the parameters of the interest rate differential are $\tilde{\rho} = 0.01$ and $\delta_m = 0$; the foreign interest rate is fixed at $r_t^f = 0.0008$ per week. In the model with realignments the jump intensity is $\theta = 0.045$ and $\theta = 0.09$ on a weekly basis.

[15]In the figures the exercise price is expressed as a fraction of the initial central parity, $\exp(M_0)$

We repeated the preceding analysis for options with a shorter maturity, 13 weeks. The deviations between the target zone model prices and the Black-Scholes prices are now less pronounced, but the same general conclusions can be drawn. Note that the usual results in the literature on volatility smiles show that deviations from the Black-Scholes model are more pronounced for close-to-maturity options, see for example Renault (1995) who investigates option pricing for stochastic volatility models.

To summarize, the target zone model yields call option prices which are very different from Black-Scholes prices. In the model without realignments, the Black-Scholes model overestimates (in most cases) option prices because it ignores and the decreasing volatility of the exchange rate in the target zone. Another difference between the Black-Scholes model and the target zone model is the mean reversion under the risk-neutral distribution induced by the time-varying interest rate differential in (5.5). Realignments tend to increase option prices because of the increased dispersion of the option payoff. On the other hand, if there is a relatively large probability of downward jumps the model with realignments may generate lower option prices than the model without realignments.

6 Conclusions

In this paper we develop new models for exchange rates in a target zone. These models are based on the idea that the volatility of the exchange rate should decrease near the band, while mean-reversion stays in effect. Both models with a fully credible band and models with realignments are considered. Since the likelihood function for discrete time observations of the exchange rates is not known, we develop a GMM estimator based on exact conditional moments. Simulation experiments show that the GMM estimator works quite well for parameter values typically found for EMS currencies. In the empirical part we estimate the model on EMS data. One of the interesting results is that we estimate implicit bandwidths for the period after the 1993 realignment. It turns out that these implicit bands, though wider than the previous 2.25% bandwidth, are narrower than the official target zone which admits 15% deviations to either side of the central parity. The final section of the paper develops a method to price currency options in target zone models with and without realignments. We conclude the paper by calculating option prices in our models. The results show that naive application of the Black-Scholes formula can significantly overprice (if there are no realignments) or underprice (if there is a high probability of devaluation) European type call options.

A potential further application of the model is an assesment of target zone credibility.

Following Bertola and Svensson (1993) and Svensson (1993), one may adjust foreign-domestic interest rate differentials for the expected rate of devaluation within the target zone to get an estimate of the expected rate of realignment. Alternatively, the intensity of jumps in the model could be estimated using observed option prices, for example along the lines of Campa and Chang (1996). Such applications are left for further research.

A Conditional moments

In this appendix we show how to derive the conditional moments of the stochastic process (2.2), which we repeat here for convenience

$$dY_u = -\rho Y_u du + \sigma\sqrt{Z^2 - Y_u^2} dW_u.$$

With Itô's theorem we obtain

$$de^{\rho u} Y_u = e^{\rho u}\sigma\sqrt{Z^2 - Y_u^2} dW_u.$$

By integrating both sides over the interval, $(t, t+h]$,

$$\begin{aligned} Y_{t+h} &= e^{-\rho h} Y_t + e^{-\rho h}\sigma \int_{(t,t+h]} e^{\rho(u-t)}\sqrt{Z^2 - Y_u^2} dW_u \\ &= e^{-\rho h} Y_t + \varepsilon_{t+h}. \end{aligned}$$

Observe that the innovations,

$$\varepsilon_{t+h} = e^{-\rho h}\sigma \int_{(t,t+h]} e^{\rho(u-t)}\sqrt{Z^2 - Y_u^2} dW_u,$$

form a martingale difference sequence. The conditional expectation therefore equals

$$E_t(Y_{t+h}) = e^{-\rho h} Y_t.$$

The conditional variance of the innovations can be derived as follows,

$$\begin{aligned} Var_t Y_{t+h} &= E_t \varepsilon_{t+h}^2 \\ &= e^{-2\rho h}\sigma^2 \int_{(t,t+h]} e^{2\rho(u-t)}\{Z^2 - E_t Y_u^2\} du \\ &= e^{-2\rho h}\sigma^2 \int_{(t,t+h]} e^{2\rho(u-t)}\{Z^2 - e^{-2\rho(u-t)} Y_t^2 - Var_t Y_u\} du. \end{aligned} \tag{A.1}$$

Define $f(h) = e^{2\rho h} Var_t Y_{t+h}$ and obtain from (A.1) the following ordinary differential equation,

$$\frac{d}{dh} f(h) = -\sigma^2 f(h) + \sigma^2\{e^{2\rho h} Z^2 - Y_t^2\},$$

with boundary condition $f(0) = 0$. This equation is solved by

$$\begin{aligned} f(h) &= \{e^{-\sigma^2 h} - 1\} Y_t^2 + \{e^{2\rho h} - e^{-\sigma^2 h}\}\frac{\sigma^2 Z^2}{\sigma^2 + 2\rho} \\ &= \{e^{-\sigma^2 h} - 1\}\{Y_t^2 - \frac{\sigma^2 Z^2}{\sigma^2 + 2\rho}\} + \{e^{2\rho h} - 1\}\frac{\sigma^2 Z^2}{\sigma^2 + 2\rho}. \end{aligned}$$

Thus, the conditional variance is given by

$$\begin{aligned} Var_t Y_{t+h} &= e^{-2\rho h} f(h) \\ &= \{e^{-\sigma^2 h} - 1\} e^{-2\rho h} Y_t^2 + \{1 - e^{-(\sigma^2 + 2\rho)h}\}\frac{\sigma^2 Z^2}{\sigma^2 + 2\rho}. \end{aligned}$$

In a similar way one can derive all higher conditional moments. Using Itô's formula, one easily verifies that the drift for Y_t^p is given by a polynomial in lower powers of Y_t. Analogously to the derivation of the conditional expectation of Y_t, one may derive from the SDE for Y_t^p an ordinary differential equation for $E_t Y_{t+h}^p$. Solving this equation for $p = 3$ yields (2.4c).

The conditional moments of the model with realignment jumps (2.6a)–(2.6b) are derived as follows. The conditional expectations[16] of X_{t+h} and M_{t+h} given information available at time t are obtained by integrating over the interval $(t, t+h]$. This yields

$$\begin{aligned} E_t X_{t+h} &= X_t + E_t \int_{(t,t+h]} dX_u = X_t - \rho \int_{(t,t+h]} (E_t X_u - E_t M_u) du, \\ E_t M_{t+h} &= M_t + E_t \int_{(t,t+h]} dM_u = M_t + \theta \int_{(t,t+h]} (E_t X_u - E_t M_u) du. \end{aligned}$$

Solving the corresponding differential equations,

$$\begin{aligned} \frac{d}{dh} E_t X_{t+h} &= -\rho(E_t X_{t+h} - E_t M_{t+h}), \\ \frac{d}{dh} E_t M_{t+h} &= \theta(E_t X_{t+h} - E_t M_{t+h}), \end{aligned}$$

with boundary conditions $E_t X_t = X_t$ and $E_t M_t = M_t$, one obtains,

$$\begin{aligned} E_t X_{t+h} &= \{\frac{\theta}{\theta+\rho} + \frac{\rho}{\theta+\rho} e^{-(\theta+\rho)h}\} X_t + \frac{\rho}{\theta+\rho}\{1 - e^{-(\theta+\rho)h}\} M_t, \\ E_t M_{t+h} &= \frac{\theta}{\theta+\rho}\{1 - e^{-(\theta+\rho)h}\} X_t + \{\frac{\rho}{\theta+\rho} + \frac{\theta}{\theta+\rho} e^{-(\theta+\rho)h}\} M_t. \end{aligned}$$

Hence, the conditional expectations are weighted averages of the current price and the current reference price. Jointly, they form a VAR(1), compare the situation with the model of subsection 2.1. The martingale implications of these two conditional moments allow for estimation of the mean parameters ρ and θ. Similar but tedious calculations yield the conditional second moments.

B Put Call Parity

In this appendix we show that the option prices in our model satisfy an arbitrage relation between put and call options. Let us consider a call option (denominated in domestic currency) on one unit of foreign currency with excercise price K. The payoff of this option is

$$C_T = \max(0, S_T - K)$$

Let us also consider a put option (denominated in foreign currency) on one unit of domestic currency with excercise price $1/K$, whose payoff is

$$P_T^* = \max(0, \frac{1}{K} - \frac{1}{S_T})$$

[16] When using conditional expectations, denoted by E_t, we condition only on the observable variables $X_h, X_{2h}, \ldots, X_t$ and not on $\{W_s, 0 \leq s \leq t\}$, the complete past of the Brownian motion driving the price process.

It is easy to see that the payoffs of these instruments are related by

$$C_T/S_T = KP_T^* = \max(0, 1 - \frac{K}{S_T})$$

By a no-arbitrage argument the price of these instruments before maturity should be related by a similar equality

$$C_t/S_t = KP_t^* \tag{B.1}$$

This equality is sometimes referred to as an international put call parity.

In the model without realignments the market is complete and the price of the call option in domestic currency is given by (ignoring the discounting)

$$C_t = E^Q(C_T)$$

where Q is the risk-neutral distribution function for payoffs denominated in domestic currency. Similarly, the price of the put option is given by

$$P_t^* = E^{Q^*}(P_T^*)$$

where Q^* is the risk-neutral distribution function for payoffs denominated in foreign currency. In the model without realignments the prices of these instruments satisfy arbitrage relation (B.1).

If there are jumps in the exchange rate the market is incomplete and an additional assumption about the treatment of jump risk should be made. The approach in Melenberg and Werker (1996) first calculates prices conditional on the jump process, I. Let $C_t(I)$ denote the price of a call option conditional on the jump process, then as before

$$C_t(I) = E^Q(C_T \mid I)$$

where Q now is the risk-neutral distribution conditional on I. The price of the put option is calculated in a similar fashion,

$$P_t^*(I) = E^{Q^*}(P_T^* \mid I)$$

Because the market is conditionally complete, the put-call parity holds conditionally

$$C_t(I)/S_t = KP_t^*(I)$$

To calculate the price of the options we propose to take expectations of the conditional prices with respect to the true distribution of the jumps, hence

$$C_t = E^P(C_t(I)) = E^P(E^Q[C_T \mid I])$$

The price of put options is calculated in exactly the same way,

$$P_t^* = E^P(P_t^*(I)) = E^P(E^{Q^*}[P_T^* \mid I])$$

Since the expectations of $C_t(I)$ and $P_t^*(I)$ are taken with respect to the same distribution of I, the put-call parity (B.1) holds in the model with realignments.

An alternative approach is proposed by Dumas, Jennergren and Näslund (1995), who deal with jump risk along the lines of Merton (1976). In that approach the put-call parity (B.1) does not hold if there are jumps and the prices of the options calculated in domestic and foreign currency are different.

References

BALL, C. AND A. ROMA (1994), Target zone modelling and estimation for European Monetary System exchange rates, *Journal of Empirical Finance* **1**, 385-420.

BEKAERT, G. AND S.F. GRAY (1996), Target zones and exchange rates: An empirical investigation, working paper Stanford University.

BERTOLA, G. AND R. CABALLERO (1992), Target zones and realignments, *American Economic Review* **82**, 520-536.

BERTOLA, G. AND L.E.O. SVENSSON (1993), Stochastic devaluation risk and the empirical fit of target zone models, *Review of Economic Studies* **60**, 689-712.

BLACK, F. AND M. SCHOLES (1973), The pricing of options and corporate liabilities, *Journal of Political Economy* **81**, 637-659.

CAMPA, J.M. AND P.H.K. CHANG (1996), Arbitrage-based tests of target-zone credibility: Evidence from ERM cross-rate options, *American Economic Review* **86**, 726-740.

COX, J., J. INGERSOLL AND S. ROSS (1985), A thoery of the term structure of interest rates, *Econometrica* **53**, 385-407.

DE JONG, F. (1994), A univariate analysis of EMS exchange rates in a target zone, *Journal of Applied Econometrics* **9**, 31-45.

DUFFIE, D. AND R. KAN (1995), A yield factor model of interest rates, working paper, Stanford University

DUMAS, B., L.P. JENNERGREN AND B. NÄSLUND (1995a), Realignment risk and currency option pricing in target zones, *European Economic Review* **39**, 1523-1544.

DUMAS, B., L.P. JENNERGREN AND B. NÄSLUND (1995b), Siegel's paradox and the pricing of currency options, *Journal of International Money and Finance* **14**, 213-223.

ENGLE, R.F. AND C.W. GRANGER (1987), Co-integration and error correction: Representaion, estimation and testing, *Econometrica* **55**, 251-276.

FLOOD, R.P., A. ROSE AND D. MATHIESON (1991), An empirical exploration of exchange rate tarhet zones, *Carnegie-Rochester Series on Public Policy* **35**, 7-65.

GARMAN, M.B. AND S.W. KOHLHAGEN (1983), Foreign currency option values, *Journal of International Money and Finance* **2**, 231-237.

GODAMBE, V.P. (1985), The foundations of finite sample estimation in stochastic processes, *Biometrica* **72**, 419-428.

GOURIÉROUX, C., A. MONTFORT AND A. TROGNON (1984), Pseudo Maximum Likelihood methods: theory, *Econometrica* **52**, 681-700.

GRABBE, J.O. (1983), The pricing of call and put options on foreign exchange, *Journal of International Money and Finance* **2**, 239-253.

HULL, J. AND A. WHITE (1987), The pricing of options on assets with stochastic volatility, *Journal of Finance* **42**, 281-300.

KOEDIJK, C., PH. STORK AND C.G. DE VRIES (1996), An EMS target zone model in discrete time, forthcoming in *Journal of Applied Econometrics.*

KRUGMAN, P. (1991), Target zones and exchange rate dynamics, *Quarterly Journal of Economics* **106**, 669-682.

LINDBERG, H. AND P. SÖDERLIND (1994), Target Zone models and Intervention Policy: the Swedish case, *Scandinavian Journal of Economics* **96**, 499-514.

MEDDAHI, N. AND E. RENAULT (1996), Linear statistical inference for ARCH-type processes, working paper Université de Toulouse.

MELENBERG, B. AND B.J.M. WERKER (1996), On the pricing of options in incomplete markets, discussion paper, Tilburg University.

MERTON, R.C. (1976), Option pricing when the underlying stock returns are discontinuous, *Journal of Financial Economics* **3**, 125-144.

NEWEY, W. (1990), Semiparametric efficiency bounds, *Journal of Applied Econometrics* **5**, 99-135.

OBSTFELD, M. AND K. ROGOFF (1995), The Mirage of Fixed Exchange Rates, *Journal of economic perspectives* **9**, 73-96.

RENAULT, E. (1995), Econometric models of option pricing errors, paper presented at the Econometric Society World Meeting, Tokio, August 1995.

SMITH, G.W. AND M.G. SPENCER (1991), Estimation and testing in models of exchange rate target zones and process switching, In: P. Krugman and M. Miller (eds.), *Exchange Rate Targets and Currency Bands*, Cambridge University Press.

SVENSSON, L.E.O. (1992a), An interpretation of recent research on exchange rate target zones, *Journal of Economic Perspectives* **6**, 119-144.

SVENSSON, L.E.O. (1992b), The foreign exchange risk premium in a target zone with devaluation risk, *Journal of International Economics* **33**, 21-40.

SVENSSON, L.E.O. (1993), Assessing target zone credibility: Mean reversion and devaluation expectations in the ERM, 1979-1992, *European Economic Review* **37**, 763-802.

VLAAR, P. AND F. PALM (1993), The message in weekly exchange rates in the European Monetary System: Mean reversion, heteroskedasticty and jumps, *Journal of Business and Economics Statistics* **11**, 351-360.

WEFELMEYER, W. (1996), Quasi-likelihood models and optimal inference, *The Annals of Statistics* **24**, 405-422.

Table 1: Simulation results target zone model without realignments.

Weekly data (300 or 1200 observations) were simulated from process (2.2) using an Euler discretisation scheme with five intermediate steps. The Euler scheme is refined by drawing from a normal distribution with conditional mean and variance given by the expressions in equations (2.4a)-(2.4b). The table reports the average over 100 simulations of the point estimate and the estimated standard error. The Monte Carlo standard deviation of the point estimates is reported in [ ].

$(\rho, \sigma^2, Z) = (0.05, 0.02, 2.25)$

	$\hat{\rho}$	$\hat{\sigma}^2$	$\hat{\rho}$	$\hat{\sigma}^2$	$\hat{Z}$
300 observations:					
mean QML estimate	0.0566	0.0202	0.0556	0.0211	9.4357
mean st. error	(0.0169)	(0.0018)	(0.0246)	(0.0132)	(1.2439)
st. deviation	[0.0175]	[0.0016]	[0.0185]	[0.0062]	[71.7937]
mean GMM estimate	0.0541	0.0202	0.0578	0.0226	2.1828
mean st. error	(0.0160)	(0.0018)	(0.0154)	(0.0046)	(0.1758)
st. deviation	[0.0160]	[0.0018]	[0.0183]	[0.0066]	[0.2283]
1200 observations:					
mean QML estimate	0.0505	0.0201	0.0526	0.0208	2.2524
mean st. error	(0.0076)	(0.0009)	(0.0084)	(0.0036)	(0.1489)
st. deviation	[0.0088]	[0.0009]	[0.0080]	[0.0027]	[0.1213]
mean GMM estimate	0.0504	0.0199	0.0524	0.0208	2.2346
mean st. error	(0.0076)	(0.0009)	(0.0077)	(0.0020)	(0.0734)
st. deviation	[0.0072]	[0.0009]	[0.0080]	[0.0027]	[0.0997]

$(\rho, \sigma^2, Z) = (0.05, 0.04, 2.25)$

	$\hat{\rho}$	$\hat{\sigma}^2$	$\hat{\rho}$	$\hat{\sigma}^2$	$\hat{Z}$
300 observations:					
mean QML estimate	0.0539	0.0402	0.0549	0.0415	2.2411
mean st. error	(0.0134)	(0.0034)	(0.0202)	(0.0163)	(0.2219)
st. deviation	[0.0147]	[0.0038]	[0.0145]	[0.0047]	[0.3254]
mean GMM estimate	0.0519	0.0385	0.0551	0.0415	2.2214
mean st. error	(0.0132)	(0.0037)	(0.0123)	(0.0052)	(0.0430)
st. deviation	[0.0161]	[0.0040]	[0.0153]	[0.0061]	[0.0670]
1200 observations:					
mean QML estimate	0.0517	0.0404	0.0501	0.0401	2.2520
mean st. error	(0.0067)	(0.0018)	(0.0064)	(0.0052)	(0.0743)
st. deviation	[0.0072]	[0.0016]	[0.0065]	[0.0034]	[0.0498]
mean GMM estimate	0.0492	0.0389	0.0526	0.0411	2.2267
mean st. error	(0.0066)	(0.0020)	(0.0061)	(0.0026)	(0.0208)
st. deviation	[0.0073]	[0.0021]	[0.0068]	[0.0030]	[0.0344]

Table 2: Simulation results target zone model with realignments.

Weekly data (700 observations) were simulated from process (2.6a)–(2.6b) using an Euler discretisation scheme with five intermediate steps. The Euler scheme is refined by drawing from a normal distribution with conditional mean and variance given by the expressions in equations (2.8a)–(2.8c). The table reports the average over 100 simulations of the point estimate and the estimated standard error. The Monte Carlo standard deviation of the point estimates is reported in [ ].

$$(\rho, \sigma^2, \theta) = (0.05, 0.02, 0.02)$$

	$\hat{\rho}$	$\hat{\sigma}^2$	$\hat{\theta}$
3 moment GMM:			
mean estimate	0.0611	0.0222	0.0093
mean st. error	(0.0161)	(0.0018)	(0.0073)
st. deviation	[0.0296]	[0.0025]	[0.0118]

Table 3: Estimation results for the fixed parity model

The model is given in equation (2.2). The sample period is January 1987 to August 1992 (297 weekly observations). Estimation by GMM based on conditional mean, variance and skewness (2.4a)–(2.4c).

	$\hat{\rho}$	$\hat{\sigma}^2$	$\hat{\rho}$	$\hat{\sigma}^2$	$\hat{Z}$
Bfr/DM	0.053	0.02058	0.044	0.01384	2.594
	(0.017)	(0.00266)	(0.014)	(0.00644)	(0.456)
Dfl/DM	0.212	0.00235			
	(0.054)	(0.00033)			
Dkr/DM	0.020	0.05178	0.032	0.00903	3.272
	(0.081)	(0.02619)	(0.013)	(0.00757)	(1.222)
Ffr/DM	0.019	0.01150	0.023	0.01631	2.022
	(0.009)	(0.00172)	(0.008)	(0.00331)	(0.058)

Table 4: Estimation results for fixed parity model until last realignment.

The model is given in equation (2.2). The sample period is 1 February 1980 to 25 June 1993 except for the Lira: 1 February 1980 to 10 July 1992 . GMM estimation based on conditional mean, variance and skewness (2.4a)-(2.4c). Weeks with realignments were omitted from the GMM criterion. Due to these exclusions the number of observations is different for each series.

	$\hat{\rho}$	$\hat{\sigma}^2$	$\hat{\rho}$	$\hat{\sigma}^2$	$\hat{Z}$
Bfr/DM	0.075	0.03705	0.010	0.00450	3.420
	(0.028)	(0.00896)	(0.007)	(0.00644)	(1.000)
Dfl/DM	0.052	0.00640			
	(0.020)	(0.00118)			
Dkr/DM	0.061	0.03236	0.034	0.01455	2.678
	(0.014)	(0.00449)	(0.008)	(0.00484)	(0.326)
Ffr/DM	0.117	0.04743	0.031	0.00536	3.750
	(0.059)	(0.01452)	(0.009)	(0.00585)	(1.810)
Irp/DM	0.028	0.02342	0.027	0.00885	2.896
	(0.003)	(0.00345)	(0.008)	(0.00288)	(0.350)
Lire/DM	0.011	0.00387	0.012	0.00317	6.498
	(0.006)	(0.00046)	(0.005)	(0.00081)	(0.601)

Table 5: Estimation results for fixed parity model with implicit bandwith after last realignment.

The model is given by equation (2.2) Sample period: 1 February 1980 to 3 March 1995 (788 weekly observations). Bandwidth is fixed at official value before 25 June 1993 (for the Lira: 10 July 1992); estimation of implicit bandwidth Z_{impl} after this date. GMM estimator based on conditional mean, variance and skewness (2.4a) (2.4c).

	$\hat{\rho}$	$\hat{\sigma}^2$	$\hat{Z}_{impl}$
Bfr/DM	0.076	0.03381	6.918
	(0.028)	(0.00878)	(1.342)
Dfl/DM	0.044	0.00654	1.014
	(0.017)	(0.00123)	(0.087)
Dkr/DM	0.062	0.03009	8.581
	(0.014)	(0.00439)	(1.025)
Ffr/DM	0.117	0.04486	5.596
	(0.056)	(0.01432)	(0.590)
Irp/DM	0.030	0.02325	5.322
	(0.003)	(0.00334)	(0.537)
Lire/DM	0.010	0.00389	24.275
	(0.006)	(0.00046)	(3.200)

Table 6: Estimates of target zone model with realignments

The model estimated is given by equations (2.6a)–(2.6b). The sample period is 1 February 1980 to 25 June 1993 (700 weekly observations), except for the Lira: 1 February 1980 to 10 July 1992 (650 weekly observations). GMM estimation based on conditional moments (2.8a)–(2.8c). In the second panel θ is estimated by a frequency estimator.

	$\hat{\rho}$	$\hat{\sigma}^2$	$\hat{\theta}$	$\hat{\rho}$	$\hat{\sigma}^2$	$\hat{\theta}$
Bfr/DM	-0.020	0.00771	0.046	0.012	0.00941	0.009
	(0.011)	(0.00227)	(0.015)	(0.009)	(0.00305)	
Dfl/DM	-0.023	0.00213	0.072	0.046	0.00561	0.001
	(0.021)	(0.00030)	(0.019)	(0.019)	(0.00086)	
Dkr/DM	-0.005	0.01822	0.051	0.059	0.01616	0.009
	(0.019)	(0.00486)	(0.026)	(0.025)	(0.01337)	
Ffr/DM	-0.025	0.01394	0.065	0.083	0.01456	0.007
	(0.023)	(0.00569)	(0.030)	(0.037)	(0.01475)	
Irp/DM	-0.032	0.01110	0.079	0.075	0.01752	0.010
	(0.022)	(0.00282)	(0.029)	(0.027)	(0.00885)	
Lire/DM	-0.027	0.00390	0.061	0.016	0.00757	0.012
	(0.023)	(0.00119)	(0.028)	(0.012)	(0.00757)	

Figure 1. Marginal distribution of target zone process.

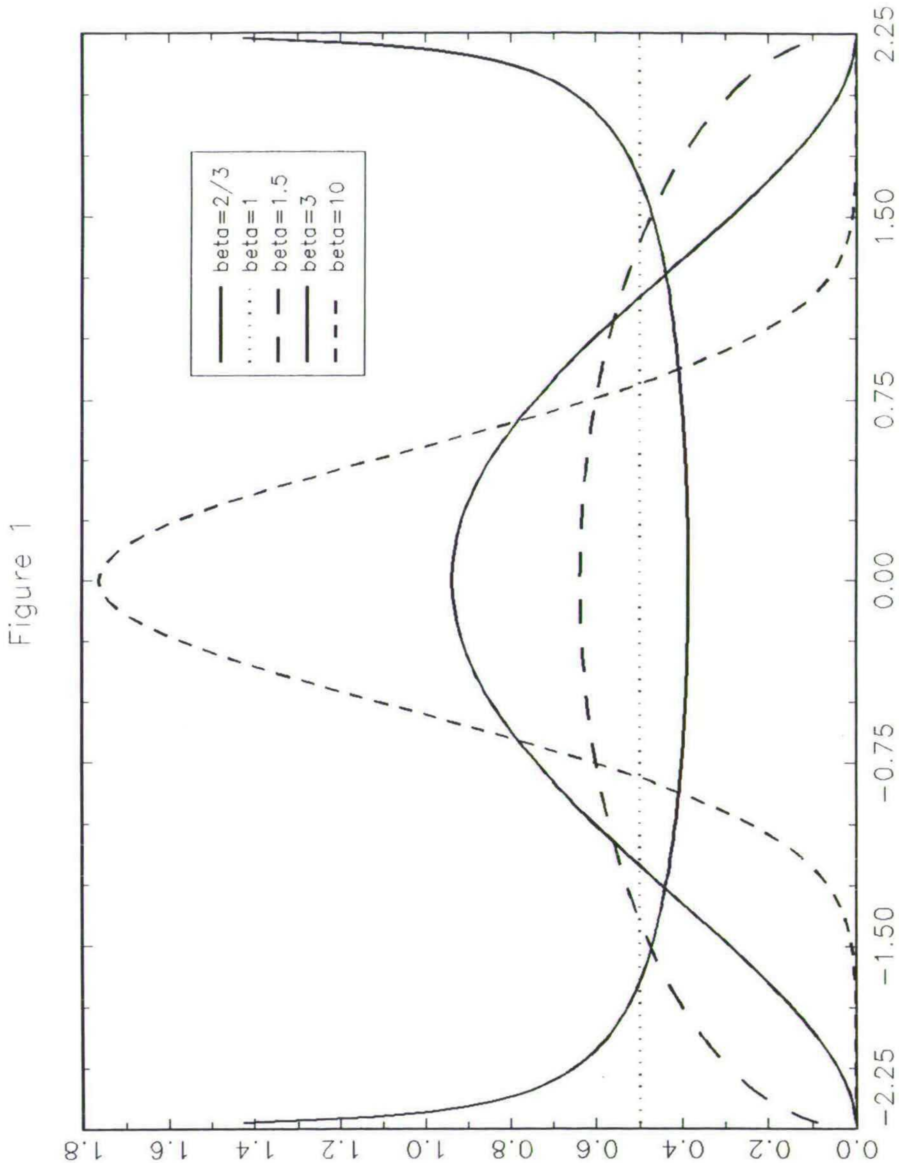


Figure 2A Option prices and implied volatilities. $X_0 = M_0$, $\tau = 52$.

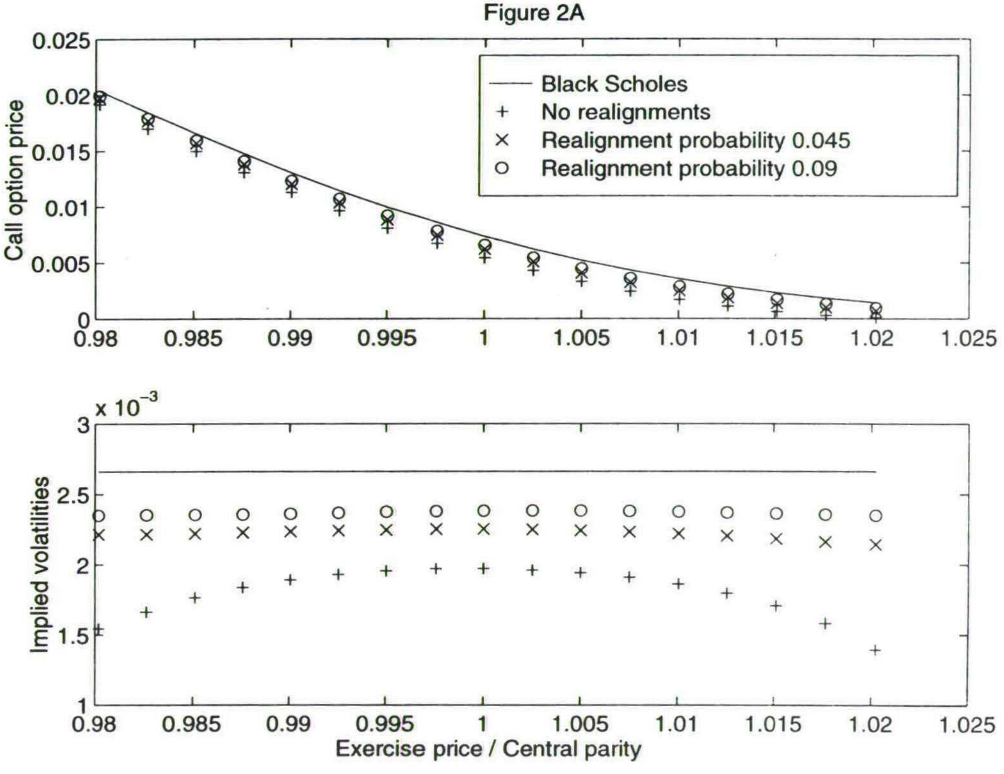


Figure 2B Option prices and implied volatilities. $X_0 = M_0 - 0.01,\ \tau = 52.$

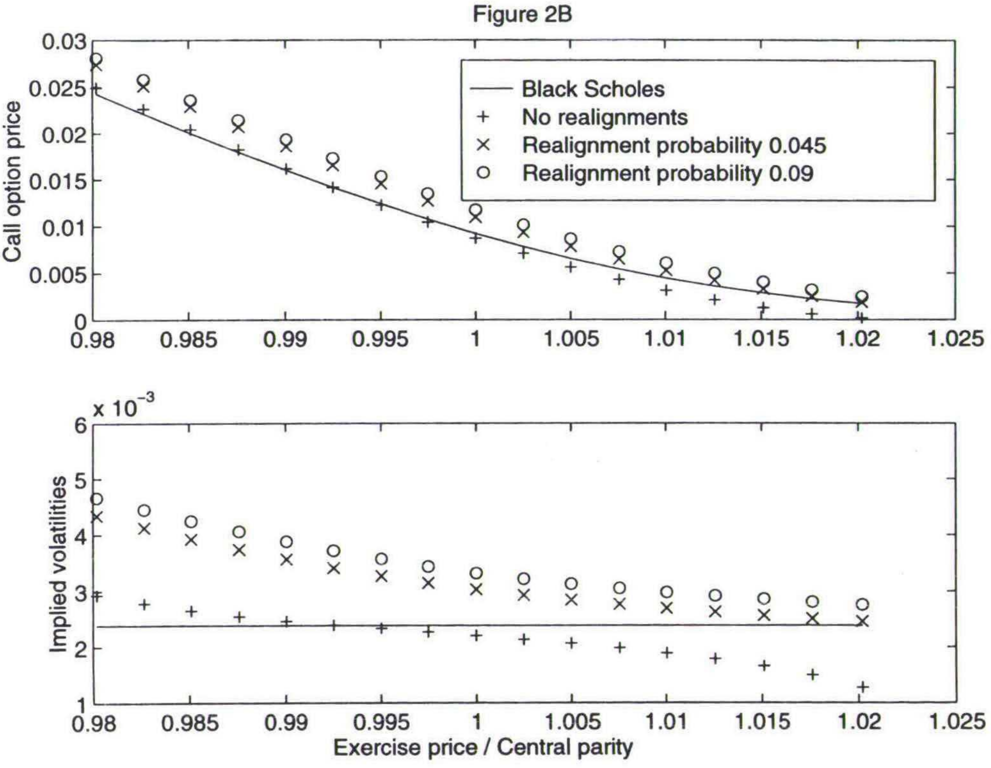
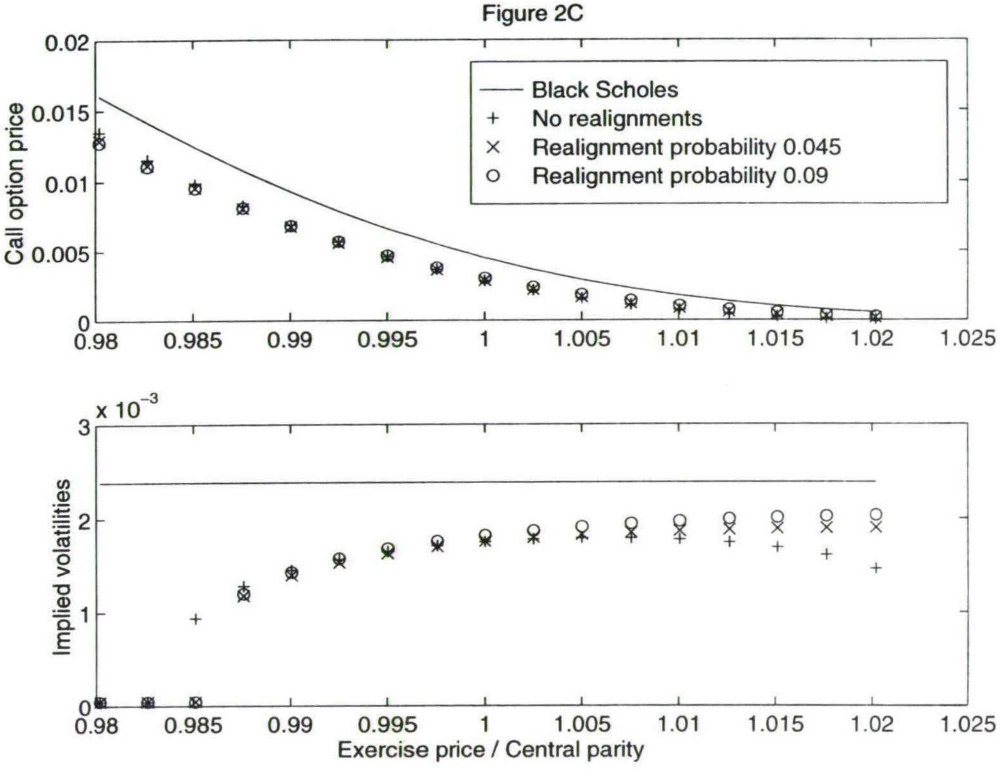


Figure 2C Option prices and implied volatilities. $X_0 = M_0 + 0.01$, $\tau = 52$.



No.	Author(s)	Title
9630	C. Fershtman and A. de Zeeuw	Tradeable Emission Permits in Oligopoly
9631	A. Cukierman	The Economics of Central Banking
9632	A. Cukierman	Targeting Monetary Aggregates and Inflation in Europe
9633	F. de Roon, C. Veld and J. Wei	A Study on the Efficiency of the Market for Dutch Long Term Call Options
9634	B. van Aarle, L. Bovenberg M. Raith	Money, Fiscal Deficits and Government Debt in a Monetary Union
9635	G. van der Laan, D. Talman and Z. Yang	Existence of and Equilibrium in a Competitive Economy with Indivisibilities and Money
9636	I. Brouwer, J. van der Put C. Veld	Contrarian Investment Strategies in a European Context
9637	M. Berg, F. van der Duyn Schouten and J. Jansen	Optimal Service Policies to Remote Customers with Delay-Limits
9638	F.C. Drost and C.A.J. Klaassen	Efficient Estimation in Semiparametric GARCH Models
9639	A.G. de Kok and F.B.S.L.P. Janssen	Demand Management in Multi-Stage Distribution Chain
9640	D.A. Freund, T.J. Kniesner A.T. LoSasso	How Managed Care Affects Medicaid Utilization A Synthetic Difference-in-Differences Zero-Inflated Count Model
9641	M.J. Lee	Instrumental Variable Estimation For Linear Panel Data Models
9642	W. Härdle, E. Mammen and M. Müller	Testing Parametric versus Semiparametric Modelling in Generalized Linear Models
9643	C. Dustmann, N. Rajah and A. van Soest	Part-Time Work, School Success and School Leaving
9644	S.C.W. Eijffinger and M. Hoeberichts	The Trade Off Between Central Bank Independence and Conservativeness
9645	R. Sarin and P. Wakker	A Single-Stage Approach to Anscombe and Aumann's Expected Utility
9646	J.P. Ziliak and T.J. Kniesner	The Importance of Sample Attrition in Life Cycle Labor Supply Estimation
9647	P.M. Kort	Optimal R&D Investments of the Firm

No.	Author(s)	Title
9648	M.P. Berg	Performance Comparisons for Maintained Items
9649	H. Uhlig and Y. Xu	Effort and the Cycle: Cyclical Implications of Efficiency Wages
9650	M. Slikker and A. van den Nouweland	Communication Situations with a Hierarchical Player Partition
9651	H.L.F. de Groot	The Struggle for Rents in a Schumpeterian Economy
9652	R.M. de Jong and J. Davidson	Consistency of Kernel Estimators of heteroscedastic and Autocorrelated Covariance Matrices
9653	J. Suijs, A. De Waegenaere and P. Borm	Stochastic Cooperative Games in Insurance and Reinsurance
9654	A.N. Banerjee and J.R. Magnus	Testing the Sensitivity of OLS when the Variance Matrix is (Partially) Unknown
9655	A. Kalwij	Estimating the Economic Return to Schooling on the basis of Panel Data
9656	M. Lind and F. van Megen	Order Based Cost Allocation Rules
9657	A. van Soest, P. Fontein and Rob Euwals	Earnings Capacity and Labour Market Participation
9658	C. Fernández and M.F.J. Steel	On Bayesian Modelling of Fat Tails and Skewness
9659	R. Sarin and P. Wakker	Revealed Likelihood and Knightian Uncertainty
9660	J.R. Magnus and J. Durbin	A Classical Problem in Linear Regression or How to Estimate the Mean of a Univariate Normal Distribution with Known Variance
9661	U. Gneezy and J. Potters	An Experiment on Risk Taking and Evaluation Periods
9662	H.J. Bierens	Nonparametric Nonlinear Co-Trending Analysis, with an Application to Interest and Inflation in the U.S.
9663	J.P.C. Blanc	Optimization of Periodic Polling Systems with Non-Preemptive, Time-Limited Service
9664	M.J. Lee	A Root-N Consistent Semiparametric Estimator for Fixed Effect Binary Response Panel Data
9665	C. Fernández, J. Osiewalski and M.F.J. Steel	Robust Bayesian Inference on Scale Parameters
9666	X. Han and H. Webers	A Comment on Shaked and Sutton's Model of Vertical Product Differentiation
9667	R. Kollmann	The Exchange Rate in a Dynamic-Optimizing Current Account Model with Nominal Rigidities: A Quantitative Investigation

No.	Author(s)	Title
9668	R.C.H. Cheng and J.P.C. Kleijnen	Improved Design of Queueing Simulation Experiments with Highly Heteroscedastic Responses
9669	E. van Heck and P.M.A. Ribbers	Economic Effects of Electronic Markets
9670	F.Y. Kumah	The Effect of Monetary Policy on Exchange Rates: How to Solve the Puzzles
9671	J. Jansen	On the First Entrance Time Distribution of the M/D/∞ Queue: a Combinatorial Approach
9672	Y.H. Farzin, K.J.M. Huisman and P.M. Kort	Optimal Timing of Technology Adoption
9673	J.R. Magnus and F.J.G.M. Klaassen	Testing Some Common Tennis Hypotheses: Four Years at Wimbledon
9674	J. Fidrmuc	Political Sustainability of Economic Reforms: Dynamics and Analysis of Regional Economic Factors
9675	M. Das and A. van Soest	A Panel Data Model for Subjective Information on Household Income Growth
9676	A.M. Lejour and H.A.A. Verbon	Fiscal Policies and Endogenous Growth in Integrated Capital Markets
9677	B. van Aarle and S.-E. Hougaard Jensen	Output Stabilization in EMU: Is There a Case for an EFTS?
9678	Th.E. Nijman, F.A. de Roon and C.Veld	Pricing Term Structure Risk in Futures Markets
9679	M. Dufwenberg and U. Gneezy	Efficiency, Reciprocity, and Expectations in an Experimental Game
9680	P. Bolton and E.-L. von Thadden	Blocks, Liquidity, and Corporate Control
9681	T. ten Raa and P. Mohnen	The Location of Comparative Advantages on the Basis of Fundamentals only
9682	S. Hochguertel and van Soest	The Relation between Financial and Housing Wealth of Dutch A. Households
9683	F.A. de Roon, Th.E. Nijman and B.J.M. Werker	Testing for Spanning with Futures Contracts and Nontraded Assets: A General Approach
9684	F.Y. Kumah	Common Stochastic Trends in the Current Account
9685	U.Gneezy and M. Das	Experimental Investigation of Perceived Risk in Finite Random Walk Processes

No.	Author(s)	Title
9686	B. von Stengel, A. van den Elzen and D. Talman	Tracing Equilibria in Extensive Games by Complementary Pivoting
9687	S.Tijs and M. Koster	General Aggregation of Demand and Cost Sharing Methods
9688	S.C.W. Eijffinger, H.P. Huizinga and J.J.G. Lemmen	Short-Term and Long-Term Government Debt and Nonresident Interest Withholding Taxes
9689	T. ten Raa and E.N. Wolff	Outsourcing of Services and the Productivity Recovery in U.S. Manufacturing in the 1980s
9690	J. Suijs	A Nucleolus for Stochastic Cooperative Games
9691	C. Seidl and S.Traub	Rational Choice and the Relevance of Irrelevant Alternatives
9692	C. Seidl and S.Traub	Testing Decision Rules for Multiattribute Decision Making
9693	R.M.W.J. Beetsma and H. Jensen	Inflation Targets and Contracts with Uncertain Central Banker Preferences
9694	M. Voorneveld	Equilibria and Approximate Equilibria in Infinite Potential Games
9695	F.B.S.L.P. Janssen and A.G. de Kok	A Two-Supplier Inventory Model
9696	L. Ljungqvist and H. Uhlig	Catching up with the Keynesians
9697	A. Rustichini	Dynamic Programming Solution of Incentive Constrained Problems
9698	G.Gürkan and A.Y. Özge	Sample-Path Optimization of Buffer Allocations in a Tandem Queue - Part I: Theoretical Issues
9699	H. Huizinga	The Dual Role of Money and Optimal Financial Taxes
96100	H. Huizinga	The Taxation Implicit in Two-Tiered Exchange Rate Systems
96101	H. Norde, F. Patrone and S. Tijs	Characterizing Properties of Approximate Solutions for Optimization Problems
96102	M. Berg, A. De Waegenaere and J. Wielhouwer	Optimal Tax Reduction by Depreciation: A Stochastic Model
96103	G. van der Laan, D. Talman and Z. Yang	Existence and Approximation of Robust Stationary Points on Polytopes
96104	H. Huizinga and S.B. Nielsen	The Coordination of Capital Income and Profit Taxation with Cross-Ownership of Firms

No.	Author(s)	Title
96105	H. Degryse	The Total Cost of Trading Belgian Shares: Brussels Versus London
96106	H. Huizinga and S.B. Nielsen	The Political Economy of Capital Income and Profit Taxation in a Small Open Economy
96107	T. Dieckmann	The Evolution of Conventions with Endogenous Interactions
96108	F. de Jong and M.W.M. Donders	Intraday Lead-Lag Relationships Between the Futures-, Options and Stock Market
96109	F. Verboven	Brand Rivalry, Market Segmentation, and the Pricing of Optional Engine Power on Automobiles
96110	D. Granot, H. Hamers and S. Tijs	Weakly Cyclic Graphs and Delivery Games
96111	P. Aghion, P. Bolton and S. Fries	Financial Restructuring in Transition Economies
96112	A. De Waegenaere, R. Kast and A. Lapied	Non-linear Asset Valuation on Markets with Frictions
96113	R. van den Brink and P.H.M. Ruys	The Internal Organization of the Firm and its External Environment
96114	F. Palomino	Conflicting Trading Objectives and Market Efficiency
96115	E. van Damme and S. Hurkens	Endogenous Stackelberg Leadership
96116	E. Canton	Business Cycles in a Two-Sector Model of Endogenous Growth
9701	J.P.J.F. Scheepens	Collusion and Hierarchy in Banking
9702	H.G. Bloemen and E.G.F. Stancanelli	Individual Wealth, Reservation Wages and Transitions into Employment
9703	P.J.J. Herings and V.J. Vannetelbosch	Refinements of Rationalizability for Normal-Form Games
9704	F. de Jong, F.C. Drost and B.J.M. Werker	Exchange Rate Target Zones: A New Approach

P.O. [partly covered by library label] LANDS