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## **Asymmetric Collusion and Merger Policy**

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## Abstract

In their merger control, EU and the US have considered symmetric size distribution (cost structure) of firms to be a factor potentially leading to collusion. We show that forbidding mergers leading to symmetric market structures can induce mergers leading to asymmetric market structures with higher risk of collusion, when firms face indivisible costs of collusion. In particular, we show that if the rule determining the collusive outcome has the property that the large (efficient) firm benefits sufficiently more from collusion when industry asymmetries increase, collusion can become more likely when firms are moderately asymmetric.

*Keywords:* Collusion; Cost Asymmetries; Merger Policy

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# 1 Introduction

Symmetry of firms in an industry is a factor that has attracted attention in merger reviews by antitrust authorities on both sides of the Atlantic. The EU Commission, for instance, notes in the Guidelines on the assessment of horizontal mergers that “[f]irms may find it easier to reach a common understanding on the terms of coordination if they are relatively symmetric”.<sup>1</sup> Similarly, the US Horizontal Merger Guidelines state that “reaching terms of coordination may be facilitated by product or firm homogeneity”.<sup>2</sup> The relevance is, for instance, illustrated by the *Baby-food case*, where the Federal Trade Commission argued that the proposed merger would make the two remaining suppliers’ cost structures more similar, which would allow these rivals to “arrive at a mutually advantageous detente”.<sup>3</sup>

The importance of size differences between firms as a factor influencing collusion has mainly been motivated by two arguments. First, firms with different size (costs) will prefer different collusive prices which will increase the problem of coordinating on the collusive price. Second, given that a collusive price is reached, it has been shown that small (more inefficient) firms gain less from the collusive agreement with respect to their optimal deviation strategies. Moreover, a large firm is proportionally more penalized

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<sup>1</sup>The Guidelines make an explicit reference to the Gencor-Lonrho merger (Case/M 619) and the Nestlé-Perrier merger (Comp/M 190). In the latter case, the Commission found that Nestlé/Perrier and BSN would be jointly dominant in the French market for bottled water and, more specifically, that “[a]fter the merger, there would remain two national suppliers on the market which would have similar capacities and similar market shares (symmetric duopoly) ... Given this equally important stake in the market and their high sales volumes, any aggressive competitive action by one would have a direct and significant impact on the activity of the other supplier and most certainly provoke strong reactions with the result that such actions could considerably harm both suppliers in their profitability without improving their sales volumes. Their reciprocal dependency thus creates a strong common interest and incentive to maximize profits by engaging in anti-competitive parallel behavior. This situation of common interests is further reinforced by the fact that Nestlé and BSN are similar in size and nature, are both active in the wider food industry and already cooperate in some sectors of that industry.”

<sup>2</sup>Horizontal Merger Guidelines, U.S. Department of Justice and the Federal Trade Commission, Issued: April 2, 1992, Available at [http://www.usdoj.gov/atr/public/guidelines/horiz\\_book/hmg1.html](http://www.usdoj.gov/atr/public/guidelines/horiz_book/hmg1.html)

<sup>3</sup>FTC ./. H.J. Heinz Company, Public Version of Commission Reply Brief, January 17, 2001, Downloaded from <http://www.ftc.gov/os/2001/01/heinze.pdf>.

See also discussion in Dick (2003).

in the punishment phase, and therefore has a greater incentive to deviate.<sup>4</sup>

However, this reasoning disregards the possibility that sharing the costs associated with running a cartel is limited or costly. For instance, coordination problems might prevent firms from jointly initiating a price increase, thus leaving the burden on one firm. Moreover, it is likely to be less costly to have only few firms administering and monitoring the price behavior of the colluding firms.<sup>5</sup> Finally, the cost of buying out potential entrants (or mavericks) typically falls on the acquiring firm, permitting other incumbents to free-ride on the investment. In order to focus on this aspect, we assume there to be an indivisible cost associated with collusion, which cannot be shared and will be taken by the largest (most efficient) firm in the industry.

Our first main result is to show that if the rule determining the collusive outcome is such that the large (efficient) firm's gain from collusion increases with industry asymmetries, there are levels of indivisible cost of collusion such that: *(i)* firms do not collude when asymmetries are minimal; *(ii)* firms do collude when asymmetries are moderate; and *(iii)* firms do not collude when asymmetries are large.<sup>6</sup> For collusion to arise, firms have to be asymmetric or else the largest (most efficient) firm would lack the incentive to cover the indivisible cost of collusion. On the other hand, firms cannot be too asymmetric, since a large asymmetry gives the smallest firm in the collusive agreement a strong incentive to deviate from the collusive conduct (the smallest firm is the one with the highest potential of stealing the business of its rivals). Then, we show that this result holds in a differentiated products Bertrand model (henceforth DPB model) for *(i)* a joint profit maximizing cartel, and *(ii)* an equal price increasing cartel.

We then show that if the rule determining the collusive outcome instead has the property that the large (efficient) firm's benefits from collusion do not increase with industry asymmetries, asymmetries between firms hurt collusion possibilities even more when an indivisible cost of collusion is present. For collusion to arise in this case, firms cannot be too asymmetric since the large (efficient) firm will then have no incentives to cover the indivisible cost

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<sup>4</sup>See Compte, Jenny and Rey (2002) and Vasconcelos (2005) and earlier contributions by Verboven (1997) and Rothschild (1999).

<sup>5</sup>Harrington (2006, pp. 49-51) provides some evidence that in real world cartels, the tasks of monitoring sales volumes and auditing sales volumes (to solve problems of misreporting sales volumes) are usually carried out by a single cartel member.

<sup>6</sup>Levenstein and Suslow (2006, p 47) report in their survey article that cost-asymmetries have ambiguous effects on collusive stability.

of collusion. This is shown to hold in a DPB model under a constant market share cartel.<sup>7</sup>

We then use an endogenous merger model, developed by Horn and Persson (2001a), to study the effect of merger policy in this context.<sup>8</sup> Our second main result is that an “anti-symmetry” merger policy (i.e., blocking mergers leading to a symmetric industry structure) can induce firms to choose mergers leading to asymmetric market structures with higher production costs and a lower aggregated producer and consumer surplus. The large (efficient) firm may not find it profitable to bear the fixed cost of coordination after a merger resulting in symmetry and hence, collusion will not be feasible. However, a policy that blocks a merger inducing a symmetric market structure will instead result in a merger that induces an industry structure where firms are moderately asymmetric, creating the most favorable conditions for collusion to arise. Moreover, for some parameter values, the gains from successful collusion which are obtained in this asymmetric industry structure are so high that they will more than compensate for the fact that one of the colluding firms has high production costs. Consequently, by forbidding a merger leading to a symmetric industry structure, competition authorities may end up with a merger that does not only create lower industry cost savings, but also facilitates collusion.

Finally, we endogenize the indivisible cost of collusion by introducing a potential entrant whose entry would lead to a cartel breakdown. When this is the case, a cartel member may prevent entry by conducting a buyout acquisition of the potential entrant. However, this buyout will be costly for the acquirer and could thus be viewed as an endogenous indivisible cost which must be incurred so as to protect the cartel. It follows that the cartel member undertaking this buyout needs to earn sufficiently high profits in the subsequent cartel interaction and thus, the analysis and the mechanisms described above hold.

Strategic acquisitions by a ringleader to protect a cartel seem to have been important in several cases. One prominent example is the *pre-insulated pipe cartel* in Europe in the 1990s, subsequently referred to as the ABB case.<sup>9</sup>

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<sup>7</sup>This rule has been used in several cartels in practice (see Harrington 2006).

<sup>8</sup>The Horn and Persson (2001a) model has, for instance, been applied to study how the pattern of domestic and cross-border mergers depends on trade costs by Horn and Persson (2001b), and the incentive of domestic and cross-border mergers in unionized oligopoly by Lommerud, Sorgard and Straume (2006).

<sup>9</sup>Case No IV/35.691/E-4: Pre-Insulated Pipe Cartel, *Official Journal of the European*

In 1987, just before the merger with ASEA creating ABB, Brown Boveri Company embarked on a strategic program for acquiring district heating pipe producers across Europe. According to the European Commission: “The organization of the cartel represented a strategic plan by ABB to control the district heating industry ... It is abundantly clear that ABB systematically used its economic power and resources as a major multinational company to reinforce the effectiveness of the cartel and to ensure that other undertakings complied with its wishes ... The gravity of the infringement is aggravated in ABB’s case by the following factors: ABB’s role as the ringleader and instigator of the cartel and its bringing pressure on other undertakings to persuade them to enter the cartel.”<sup>10</sup>

Moreover, ABB took the bulk of the costs of running the cartel. For instance, in the situation where Powerpipe, a firm outside the cartel, tried to expand its activities, ABB used large resources trying to eliminate the maverick from the market.<sup>11</sup> This predatory activity was multi-dimensional and costly as indicated by the following quote in §§91-92:

“Numerous passages in ABB strategy documents during this period covered by this Decision refer to plans to force Powerpipe into bankruptcy ... In 1993 ABB embarked on a systematic campaign of luring away key employees of Powerpipe, including its then managing director, by offering them salaries and conditions which were apparently exceptional in the industry.”

The paper continues as follows. The model is spelled out in Section 2. Section 2.1 studies the collusion pattern when firms are asymmetric and one firm faces an indivisible fixed cost of collusion, while Section 2.2 shows that an anti-symmetry merger policy can be counter productive. Section 3 provides an extension of the basic model where the indivisible cost of cartelization is endogenized. Finally, Section 4 concludes.

## 2 The model

We consider a market with initially three firms, denoted 1, 2 and 3. Firms play an infinitely repeated game. In period 0, there is a merger formation

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*Communities*, L 24/1, 30.1.1999.

<sup>10</sup>§169 and §171 in the Decision.

<sup>11</sup>As pointed out by the Commission, “Its systematic orchestration of retaliatory measures against Powerpipe, aimed at its elimination.”

game where a two-firm merger is possible.<sup>12</sup> Then, from period 1 onwards, the firms resulting from the merger formation process play a standard repeated oligopoly game. The game is solved backwards.

## 2.1 Stage 2: The repeated oligopoly game

Pre-merger, firms 1, 2 and 3's constant variable costs are  $c_1 = c > 0$ ,  $c_2 = 0$ , and  $c_3 = 0$ , respectively. Post merger, there will be two firms active in the market, where the merged firm's variable cost  $c_m$  is equal to the lowest variable cost of the participating firms (i.e.  $\min(c_i, c_j) = 0$  for  $i \neq j$ ), and the non-merged firm's variable cost is denoted  $c_n$ . Firm  $i$ 's per-period profit is denoted  $\pi_i^t(x_i^t, x_{-i}^t, c_i, c_{-i})$ , where  $x_i^t$  is firm  $i$ 's price in period  $t$ . The partial derivative of the profit with respect to the firm's own marginal cost is assumed to be negative, i.e.  $\frac{\partial \pi_i^t(x_i^t, x_{-i}^t, c_i, c_{-i})}{\partial c_i} < 0$ , while the partial derivative with respect to a competitor's marginal cost is positive, i.e.  $\frac{\partial \pi_i^t(x_i^t, x_{-i}^t, c_i, c_{-i})}{\partial c_{-i}} > 0$ . Time is denoted by  $t = 0, \dots, \infty$ .

The present value of firm  $i$ 's profit is

$$\Pi_i = \sum_{t=0}^{\infty} \delta^t \cdot \pi_i^t(x_i^t, x_{-i}^t, c_i, c_{-i}), \quad (1)$$

where  $\delta \in (0, 1)$  is the (common) discount factor.

Firms will try to collude, which may or may not be possible in the repeated game. As argued in the introduction, there is some cost of collusion that is indivisible and not easily shared. To highlight this feature of collusion, we make the following assumption.

**Assumption 1** The merged (efficient) firm must incur a fixed indivisible cost  $f$  per period of collusion.

We then assume that firms employ standard grim-trigger strategies (Friedman, 1971) to sustain the collusive agreement, i.e., whenever one firm deviates from the collusive norm, the other firm will play non-cooperatively forever after.

Collusion is said to be sustainable if, for each firm  $i$  ( $i = m, n$ ), the potential short-run gains from cheating are no greater than the present value

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<sup>12</sup>To explain why the merger opportunity arises now and not before is outside the scope of this paper. However, the merger could be triggered by a cost shock, for instance.

of expected future losses that are due to the subsequent punishment. This trade-off is captured by the analysis of firms' incentive compatibility constraints (ICC). The ICC for firms  $m$  and  $n$  are given by:

$$\sum_{t=0}^{\infty} [\pi_m^C(x_m^C, x_n^C, c_m, c_n) - f] \cdot \delta^t \geq \pi_m(x_m^*(x_n^C), x_n^C, c_m, c_n) + \sum_{t=1}^{\infty} \pi_m(x_m^B, x_n^B, c_m, c_n) \delta^t \quad (2)$$

and

$$\sum_{t=0}^{\infty} \pi_n^C(x_n^C, x_m^C, c_n, c_m) \delta^t \geq \pi_n(x_n^*(x_m^C), x_m^C, c_n, c_m) + \sum_{t=1}^{\infty} \pi_n(x_n^B, x_m^B, c_n, c_m) \delta^t, \quad (3)$$

respectively, where  $x_i^*(\bullet)$  represents firm  $i$ 's reaction function, and  $x_i^C$  and  $x_i^B$  denote the value assumed by firm  $i$ 's price at the collusive equilibrium and the Bertrand-Nash equilibrium of the one shot game, respectively.

Rewriting the ICC (2) of the merged entity (which bears the indivisible cost), we can solve for the critical indivisible cost  $\bar{f}$ , which is the greatest indivisible cost that the larger firm can incur and still find it profitable to stay in the cartel:

$$\bar{f}(c_n, \delta) = \pi_m^C(x_m^C, x_n^C, c_m, c_n) - (1 - \delta) \pi_m(x_m^*(x_n^C), x_n^C, c_m, c_n) - \delta \pi_m(x_m^B, x_n^B, c_m, c_n) \quad (4)$$

which is a continuous function in the marginal cost of the non-merged firm ( $c_n$ ) and in the discount factor ( $\delta$ ).

To focus on the effect of the indivisible cost of collusion on collusion possibilities, in the symmetric case we assume:

**Assumption 2** Firms are sufficiently patient, i.e.  $\delta$  is sufficiently high, for collusion to be sustainable when both firms have zero production cost,  $c_n = c_m = 0$ , and the indivisible fixed cost of collusion is zero,  $f = 0$ .

To derive a simple graphical solution to the model, we also assume that:

**Assumption 3** There exists a unique value for  $c_n$  (denoted  $c^*$ ) for which the ICC of the non-merged firm, i.e. (3), binds with equality.



In the following, we will use a Differentiated Product Bertrand (DPB) model to derive specific results. The DPB model is presented in the Appendix. Assumption 3 holds in the DPB model and in other asymmetric collusion models in the literature such as Vasconcelos (2005) and Compte *et al.* (2002).

The equilibrium collusion pattern is illustrated in Figure 1, where the variable cost of the non-merged firm  $c_n$  is depicted on the  $x$ -axis and the indivisible fixed cost of collusion  $f$  is depicted on the  $y$ -axis, and where  $\delta$  is fixed.

**The symmetric market structure.** Let us first address the case where the non-merged firm faces a zero variable production cost (recall that the merged firm is assumed to have a zero variable production cost). ICC-s then depicts the indivisible fixed cost at which the merged firm's ICC (2) binds when the non-merged firm also faces a zero production cost. We refer to this as  $f'$ , where  $f' \equiv f(0, \delta)$ . This curve is horizontal since in this case, no firm faces a positive variable production cost. It follows that collusion is sustainable in the symmetric market if  $f < f'$ .

**The asymmetric market structure.** Let us now turn to the case where the non-merged firm faces a positive production cost,  $c_n > 0$ . When this is the case, ICC-n then depicts the production cost  $c_n$  at which the non-merged firm's ICC (3) binds. This curve is vertical since the non-merged firm is assumed not to pay the per-period fixed indivisible cost. In addition, Assumptions 2 and 3 imply that ICC (3) holds only if cost asymmetries are sufficiently low, i.e., if  $c_n < c^*$ .

Finally, ICC-m depicts the pairs  $(c_n, f)$  for which the merged firm's ICC (2) binds. At  $c_n = 0$ , this curve obviously coincides with the curve ICC-s. The slope of the curve ICC-m is ambiguous, however.

In order to illustrate our first result, we make use of the following assumption, which implies that the ICC-m curve is upward sloping at  $c_n = 0$  and is never below ICC-s:

**Assumption 4a**  $\bar{f}$  is weakly increasing in  $c_n$  and  $\left. \frac{\partial \bar{f}}{\partial c_n} > 0 \right|_{c_n=0}$ .

Assumption 4a is shown to hold in the DPB model under several assumptions on how firms share profits in the collusive phase:<sup>13</sup> (i) the joint profit-maximizing (JPM) cartel, where firms choose prices or quantities to

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<sup>13</sup>See Appendices B and C.

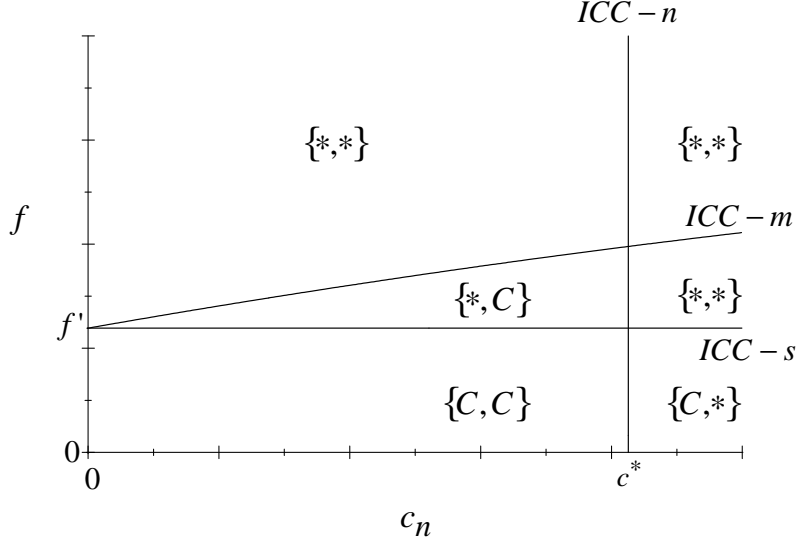


Figure 1: Equilibrium Collusion Pattern: ICC-m upward sloping

maximize joint profits, and (ii) the equal price increase cartel, where each firm increases its price by an equal amount from the non-collusive equilibrium price.

Clearly, there will be collusion in this asymmetric market structure if and only if we are below the ICC-m curve and to the left of the ICC-n curve. So, we can now characterize the equilibrium collusion pattern under Assumptions 1-3 and 4a. This is done in Figure 1, where  $\{C, *\}$  indicates that collusion can be sustained in the symmetric market structure but not in the asymmetric market structure,  $\{*, C\}$  indicates that collusion cannot be sustained in the symmetric market structure but can be sustained in the asymmetric market structure, etc.

We specifically note that there exist parameter values in the DPB model under the joint profit-maximizing rule or the equal price increase rule such that Figure 1 is valid. We derive Figure 1 from the DPB model in Appendices B and C1. Now, using Figure 1 we have the following result:

**Proposition 1** *Under Assumptions 1-3 and 4a, and at a fixed indivisible cost so high that no collusion can be sustained in the symmetric case ( $f > f'$ ),*

there exists a fixed indivisible cost such that (i) firms do not collude for low asymmetries; (ii) firms collude for intermediate asymmetries; and (iii) firms do not collude for large asymmetries.

**Proof.** Let  $f = f' + \epsilon$  (i.e. ICC-s does not hold), where  $\epsilon$  is arbitrarily small and positive and let  $c'$  denote the value of  $c_n$  for which the ICC-m binds with equality at  $f = f' + \epsilon$ . Then, there exist (i)  $c_i < c'$  such that ICC-m does not hold; (ii)  $c_{ii} \in (c', c^*)$  such that the ICC-m and the ICC-n hold; and (iii)  $c_{iii} > c^*$  such that ICC-n does not hold. ■

This result thus shows that there exist levels for the fixed indivisible cost of collusion such that (i) firms do not collude when asymmetries are small; (ii) firms do collude when asymmetries are moderate; and (iii) firms do not collude when asymmetries are large. The intuition is simple. For collusion to arise, firms cannot be too symmetric since the largest (most efficient) firm will then not have the incentive to cover the fixed indivisible cost of collusion.<sup>14</sup> On the other hand, firms cannot be too asymmetric either, for if there are strong asymmetries, the smallest firm in the collusive agreement will have strong incentives to deviate from the collusive conduct and steal the business of its rival.

To illustrate our second result, we now make use of the following assumption, which implies that the ICC-m curve is downward sloping at  $c_n = 0$  and is never above ICC-s:

**Assumption 4b**  $\bar{f}$  is weakly decreasing in  $c_n$  and  $\left. \frac{\partial \bar{f}}{\partial c_n} < 0 \right|_{c_n=0}$ .

Assumption 4b is shown to hold in the DPB model under the constant market shares rule (see Appendix C2), where firms reduce prices keeping market shares constant, i.e. equal to the market shares in the Bertrand-Nash equilibrium. We can now characterize the equilibrium collusion pattern under Assumptions 1-3 and 4b. Using Figure 2, we can then derive the following result:

**Proposition 2** *Under Assumptions 1-3 and 4b, and at such a low fixed indivisible cost that collusion can be sustained in the symmetric case ( $f < f'$ ), there exist levels for the fixed indivisible cost such that (i) firms collude for small asymmetries; and (ii) firms do not collude for large asymmetries.*

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<sup>14</sup>Remember that under the joint profit-maximizing or the equal price increase rule, this firm benefits less from cartel formation when industry is more symmetric.

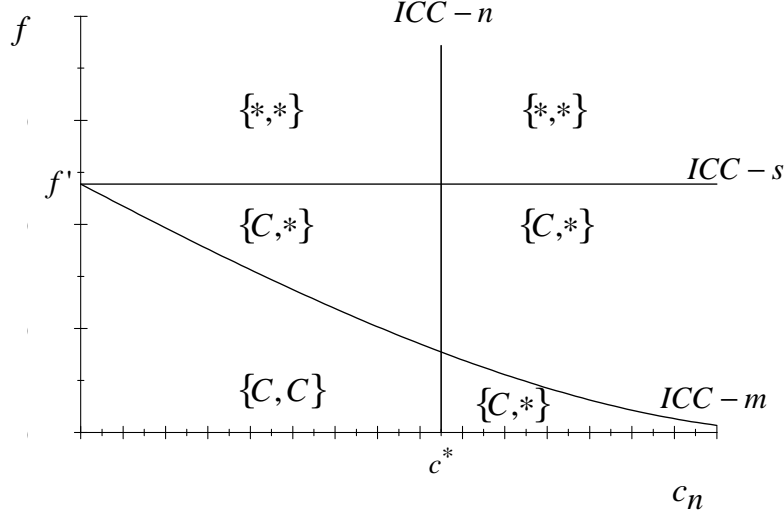


Figure 2: Equilibrium Collusion Pattern: ICC-m downward sloping

**Proof.** Let  $f = f' - \varepsilon$ , where  $\varepsilon$  is arbitrarily small and positive, and let  $\tilde{c}$  denote the value of  $c_n$  for which the ICC-m binds with equality at  $f = f' - \varepsilon$ . Then, there exist (i)  $\tilde{c}_i < \tilde{c}$  such that ICC-m and ICC-n hold; and (ii)  $\tilde{c}_{ii} > \tilde{c}$  such that ICC-m does not hold. ■

This result thus shows that there exist levels for the fixed indivisible cost of collusion such that (i) firms collude when asymmetries are minimal but do not collude when asymmetries are larger. For collusion to arise, firms cannot be too asymmetric, otherwise the largest (most efficient) firm will not have the incentive to cover the fixed indivisible cost of collusion since it benefits less from the cartel being formed under the constant market share rule. Moreover, firms cannot be too asymmetric for another reason. If there are strong asymmetries, the smallest firm in the collusive agreement will have strong incentives to deviate from the collusive conduct and steal the business from its rival.<sup>15</sup>

<sup>15</sup>Note that it can be shown that the non-merged (inefficient) firms will have a lower incentive to cover the fixed cost than the merged (efficient) firm in these examples.

## 2.2 Stage 1: The merger model

In order to determine the merger pattern, we make use of an endogenous merger model developed by Horn and Persson (2001a) where the merger formation is treated as a cooperative game of coalition formation. The merger model has three basic components: (i) a specification of the owners determining whether one ownership structure  $M^i$  dominates another structure; (ii) a criterion for determining when these owners prefer the former structure to the latter; and (iii) a stability (solution) criterion that selects the ownership structures seen as solutions to the merger formation game on basis of all pairwise dominance rankings.

The dominance relation is such that for some market structure  $M^j$  to dominate or block another structure  $M^i$ , all owners involved in forming and breaking up mergers between the two structures in some sense prefer the dominating structure to the other structure. Which owners are then able to influence whether  $M^j$  dominates  $M^i$ ? Owners belonging to identical coalitions in the two structures cannot affect whether  $M^j$  will be formed instead of  $M^i$ , since payments between coalitions are not allowed. But all remaining owners can influence this choice. Owners who are linked this way in a dominance ranking between two structures  $M^i$  and  $M^j$  belong to the same *decisive group of owners with respect to market structures  $M^i$  and  $M^j$* , denoted by  $\mathcal{D}_g^{ij}$ . The formal definition of a decisive group may appear somewhat opaque.<sup>16</sup> However, the concept itself is straightforward and, in “practice”, it is very easy to find the decisive groups. For instance, in a dominance ranking of  $M^D = \{12, 3\}$  and  $M^T = \{1, 2, 3\}$ , owner 3 belongs to identical firms in both structures and hence, is not decisive. The only decisive group with respect to these two structures is hence  $\mathcal{D}_1^{DT} = \{1, 2\}$ . Or, in a dominance ranking of  $M^D = \{12, 3\}$  and  $M^M = \{123\}$ , owners 1, 2 and 3 belong to different firms in the two structures. Hence, all three owners belong to a decisive group. The decisive groups are thus  $\mathcal{D}_1^{DM} = \{1, 2, 3\}$ .

$M^j$  dominates  $M^i$  via a decisive group if and only if the combined profit of the decisive group is larger in  $M^j$  than in  $M^i$ . But, with more than one decisive group, these groups may dominate in opposite directions. It is therefore required that for  $M^j$  to dominate  $M^i$ , written  $M^j \text{ dom } M^i$ , domination holds for *each* decisive group with respect to  $M^i$  and  $M^j$ .

The definition of decisive groups and the *dom* relation describes how to rank any pair of ownership structures. It remains to specify how these

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<sup>16</sup>See Horn and Persson (2001a).

rankings should be employed in order to predict the outcome of the merger formation. To this end, define those structures that are *undominated*, i.e., that are in the core, as *Equilibrium Ownership Structures* (EOS).

To our knowledge, all papers on mergers and cartels have treated the merger decision as exogenous, not determining the equilibrium merger. The focus of this section will be to identify the equilibrium merger under two different merger policies. The first is a *laissez-faire* policy where all two-firm mergers are allowed, but a merger to monopoly is forbidden. The second is an anti-symmetric merger policy, where no mergers leading to symmetric industry structures are allowed.

We can then derive the following Lemma:

**Lemma 1** *Under the laissez-faire policy, the equilibrium merger gives rise to the highest aggregate duopoly profit.*

**Proof.** This follows from Proposition 2 in Horn and Persson (2001a) and from the fact that, in our model, the profit flows both under collusion and non-collusion regimes are time invariant. ■

Recall that we have assumed  $c_1 = c$  and  $c_2 = c_3 = 0$ . So, the marginal costs associated with the possible merged entities are  $c_{12} = c_{13} = c_{23} = 0$ . Let  $M^A$  denote the market structure following a merger between firm 1 and firm 3, referred to as a merger to symmetry, and  $M^B$  denote the market structure following a merger between firms 2 and 3, referred to as a merger to asymmetry. We can then state that a merger to symmetry will take place if and only if the following condition holds:

$$\Pi^{Ah} = \pi_m^{Ah} + \pi_n^{Ah} > \pi_m^{Bk} + \pi_n^{Bk} = \Pi^{Bk}, \quad (5)$$

where  $h, k = *, C$ .

Using condition (5), Figure 1 and the DPB model we can then derive the following result:

**Proposition 3** *An anti-symmetric merger policy (blocking mergers to a symmetric market structure) can induce a merger without cost synergies (i.e. between firm 2 and firm 3), yielding a higher average production cost and a lower aggregated producer and consumer surplus.*

**Proof.** See the derivation of Figure 3 in Appendix B. ■

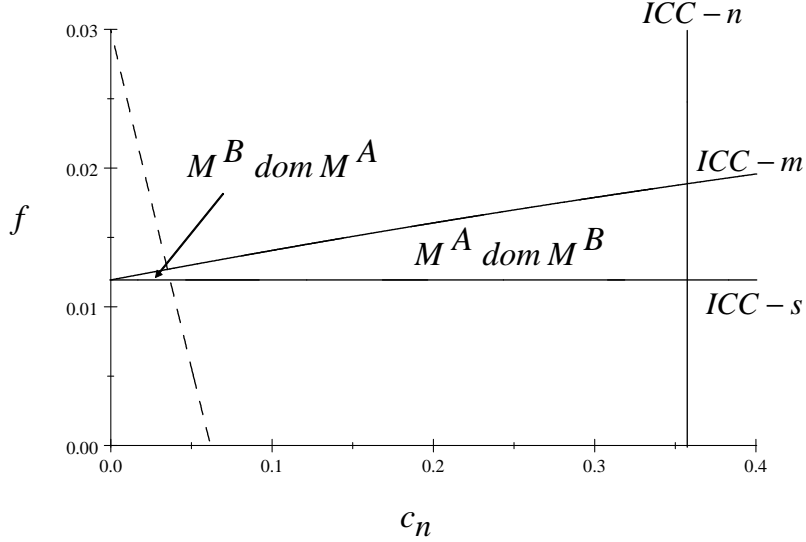


Figure 3: Equilibrium Mergers

This result can be illustrated in Figure 3, which builds on Figure 1 and where the merger condition (5) is included. In what follows, let us focus the attention on the region of Figure 1 where ICC-m and ICC-n hold but ICC-s does not. Then, consider the merger condition (5) which in this case becomes:

$$\Pi^{Ah} = \pi_m^{A*} = \pi_n^{A*} > \pi_m^{BC} + \pi_n^{BC} = \Pi^{Bh}, \quad (6)$$

since in this region, firms will not collude in the symmetric market structure  $M^A$  but will be able to collude in the asymmetric market structure  $M^B$  (see Figure 1). The downward sloping dashed line represents merger condition (6) when it is binding. This dashed line divides the region under consideration into two different subregions. In the right subregion,  $M^A$  dominates  $M^B$ , whereas the opposite holds in the left subregion.

It is important to note at this point that in the whole region under analysis, firms will not be able to collude in case a merger results in a perfectly symmetric market structure. Therefore, the reason why the symmetric market structure  $M^A$  dominates the asymmetric market structure  $M^B$  in the right subregion of the region under analysis is *not* the fact that a merger

to symmetry results in collusion amongst the remaining firms in the market. Instead, what drives this result is that this merger will give rise to very significant savings on industry costs.

So, by adopting an anti-symmetry merger policy, antitrust authorities force firms to opt for the merger leading to the asymmetric market structure,  $M^B$ . This alternative merger will, in the region under analysis, create the most favorable conditions for collusion to arise since asymmetries between the two remaining firms are moderate after the merger (see Proposition 1 and Figure 1). In addition, for some parameter values, the gains from successful collusion which are obtained in this asymmetric industry structure are so high that they will more than compensate for the fact that one of the colluding firms has high production costs. Consequently, by forbidding a merger leading to a symmetric industry structure, competition authorities may end up with a merger that does not only create lower industry cost savings, but also facilitates collusion.

### 3 Endogenous indivisible cost of cartelization

We here provide an example of an endogenous indivisible cost of creating and maintaining a cartel. As described in the Introduction, in 1987, Brown Boveri Company (later ABB) embarked on a strategic program for acquiring district heating pipe producers across Europe. However, this program was considered to be unfairly costly as indicated by the following quote from §28 of the EC Decision: “ABB for its part considers that it was unfairly having to bear all the cost of industry reorganization while other producers obtained a free ride.” Moreover, in reaction to the Powerpipe’s (the cartel outsider) aggressive behavior and entry into new geographical markets, ABB decided to use several predatory strategies to protect the cartel.

To capture this process of creating and maintaining the cartel, consider the following modified model set-up, where we keep the repeated oligopoly game we had before but change period 1’s interaction in the following way. Now, firm 3 is outside the cartel market and contemplates entering it. The cartel might prevent this entry by buying out the potential entrant. To this end, we assume that the efficient firm, firm 2, makes a take-it-or-leave-it offer to firm 3.

There are two scenarios to consider, one with an acquisition and one without an acquisition:



**Case 1: No buyout.** In this case, firm 3 enters the market facing a variable cost  $c_3$ . It is assumed that if entry occurs, collusion is not sustainable and firms compete à la Bertrand in each period, generating profits  $\pi_1^B(c_1, c_2, c_3)$ ,  $\pi_2^B(c_1, c_2, c_3)$ , and  $\pi_3^B(c_1, c_2, c_3)$ , respectively. This assumption of one-shot Bertrand competition could be supported by the fact that collusion is not sustainable with three firms in the market or by firm 3 deciding to denounce the existence of the collusive agreement to the antitrust authorities about the illegal behavior upon entering the market.<sup>17</sup>

**Case 2: A buyout takes place.** To focus on the buyout as an indivisible collusion cost, we assume that a buyout would not be profitable if collusion did not occur post buyout. We can then use the above set-up to determine whether there will be collusion, taking the indivisible buyout cost  $A$  into account. It immediately follows that if a buyout takes place in equilibrium, firm 2 will make an offer equal to firm 3's reservation price, i.e. its profit if it enters. The ICC for firm 2 is then given by:

$$\sum_{t=0}^{\infty} [\pi_2^C(p_2^C, p_1^C, c_1, c_2) - A] \cdot \delta^t > \pi_2(p_2^*(p_1^C), p_1^C, c_1, c_2) + \sum_{t=1}^{\infty} \pi_2(p_2^B, p_1^B, c_1, c_2) \delta^t, \quad (7)$$

where the indivisible acquisition cost  $A = \pi_3^B(c_1, c_2, c_3)$  is the reservation price of firm 3. The ICC for firm 1 is given by

$$\sum_{t=0}^{\infty} \pi_1^C(p_1^C, p_2^C, c_1, c_2) \delta^t > \pi_1(p_1^*(p_2^C), p_2^C, c_1, c_2) + \sum_{t=1}^{\infty} \pi_1(p_1^B, p_2^B, c_1, c_2) \delta^t. \quad (8)$$

To simplify the presentation, we assume the buyout price  $A$  to be independent of the level of asymmetry, i.e.  $A = \pi_3^B(c_1, c_2, \bar{c}_3) = \bar{A}$ . Consequently, it is assumed that if  $c_1$  decreases,  $c_2$  will increase in such a way that the acquisition price will not change. Then, using assumptions A1, A2, A3, and A4a, it directly follows that we can make use of the graphical solution in Figure 1 to describe the equilibrium, where in the horizontal axis we have  $c_1$  (the marginal cost of the outsider to the merger) and in the vertical axis we now represent  $A$  (instead of  $f$ ).

What will then happen if we relax the assumption that the acquisition price is independent of the cost asymmetry? If the buyout price (reservation

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<sup>17</sup>See Friedman and Thisse (1994, p. 272).

price) decreases in asymmetries, collusion becomes relatively more likely under asymmetry, whereas the opposite is true if the buyout price increases in asymmetries.

We then make use of the DPB model to show the existence of an equilibrium where the buyout of a collusion breaking potential entrant will be undertaken if and only if the market structure is medium asymmetric. Thus, we can show that:

**Proposition 4** *There exist parameter values in the DPB model under the joint profit-maximizing rule such that firm 2 buys out firm 3 if and only if collusion occurs post-buyout, which is the case if and only if the market structure is medium asymmetric.*

**Proof.** See Appendix D. ■

## 4 Conclusion

In the existing literature, mergers leading to symmetric size distribution (cost structure) of firms have been shown to increase the risk of collusion, since the asymmetry between firms creates incentives to deviate both in the collusive phase and the punishment phase. Allowing for indivisible costs of collusion, which one firm must bear, we show that forbidding mergers leading to symmetric market structures can induce mergers leading to asymmetric market structures with higher production costs and a higher risk of collusion. In particular, we show that if the rule determining the collusive outcome has the property that the large (efficient) firm benefits sufficiently more from collusion when industry asymmetries increase, collusion can become more likely when firms are asymmetric.

These results thus suggest the importance of studying the environment for post merger collusion in detail, taking into account the role played by a potential ring-leader in the collusion process.

The key to our result is the chosen rule for determining the collusive outcome. If the cartel behavior rule has the property that the gains of the large (efficient) firm from collusion increase with industry asymmetries, our main results are valid. Basically, what is important is that the “advantages” of the large (efficient) firm in the non-cooperative interaction (e.g. its larger market share, lower production costs, larger capital stock, or higher quality products) are used to give leverage in capturing more of the surplus created

in the collusive outcome. This will be the case for some particular rules of how to determine the cartel outcome. As shown above, a rule with this characteristic is the joint profit maximization rule in the DPB model. But this rule has some problematic features, as discussed in Harrington (2004). In particular, it could allocate the gains from collusion very unevenly.

Notice, however, that the constant market share rule, which is one of the most frequently used allocation rules, also has some odd features in our setting: the small (inefficient) firm gains more from collusion than the large (efficient) firm. The reason is that under the constant market share rule, the large (efficient) firm is forced to reduce output much more than the small (inefficient) firm which implies that the “cost” due to the output reduction to a larger extent falls on this large (efficient) firm.<sup>18</sup> This rule is evidently used in practice, but this property suggests that other rules should be used in practice where the large (efficient) firm gains most from collusion. For example, we show that the large (efficient) firm gains most from collusion under an equal price increase rule. However, more research is needed to better understand which rules are used in what context and what are their implications for mergers and cartel policies.

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<sup>18</sup>See Norbäck and Persson (2006) for a formalization of this argument.

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## **A The DPB model**

In this section, we describe the DPB model used to derive specific results.

Consider an infinitely repeated game with two firms, denoted with subscripts  $i = 1, 2$ . Each firm produces a single variety of a good  $X$  and the quantity of variety  $i$  is denoted  $q_i$ . The fixed costs of production are taken to be zero and firm  $i$ 's constant marginal cost is given by  $c_i$ . Let  $c_1 = 0$  and  $c_2 = c > 0$ .

Following Singh and Vives (1984), we assume that a representative consumer maximizes the following utility function:

$$U(q_1, q_2) = a_1 q_1 + a_2 q_2 - \frac{1}{2} (b_1 q_1^2 + b_2 q_2^2 + 2\theta q_1 q_2) + y, \quad (9)$$

where  $y$  is a numeraire 'outside' good. This utility function gives rise to a linear demand structure, where direct demands can be written as:

$$q_1(p_1, p_2) = \alpha_1 - \beta_1 p_1 + \gamma p_2 \quad (10)$$

$$q_2(p_1, p_2) = \alpha_2 - \beta_2 p_2 + \gamma p_1, \quad (11)$$

where  $d \equiv (b_1 b_2 - \theta^2)$ ,  $\alpha_i \equiv (a_i b_j - a_j \theta) / d$ ,  $\beta_i \equiv b_j / d$  for  $i \neq j$ ,  $i = 1, 2$ , and  $\gamma \equiv \theta / d$ .

Firms compete in prices. Every period of time can be divided into the following stages: firms first set prices, consumers then determine the demand for the two varieties of the good and, finally, markets clear.

The profit of firm  $i$  is  $\Pi_i = \sum_{t=0}^{\infty} \delta^t \cdot \pi_i^t(p_1^t, p_2^t)$ , where  $p_i^t$  is the price charged by firm  $i$  in period  $t$  and firm  $i$ 's individual profit in period  $t$  is

$$\pi_i(p_1^t, p_2^t) = q_i(p_1^t, p_2^t) \cdot [p_i^t - c_i]. \quad (12)$$

## A.1 Non-cooperative behavior and joint profit maximization

Consider a single period game. If the firms play noncooperatively, each firm solves the following optimization problem

$$\max_{p_i} q_i(p_1, p_2) \cdot (p_i - c_i). \quad (13)$$

From the first-order conditions (FOCs) of the previous maximization problem, it can be concluded that the (positively sloped) reaction functions of firms 1 and 2 are, respectively, given by:

$$p_1^*(p_2) = \frac{\alpha_1 + \gamma p_2}{2\beta_1}, \quad (14)$$

$$p_2^*(p_1) = \frac{\alpha_2 + \beta_2 c + \gamma p_1}{2\beta_2}. \quad (15)$$

Now, some algebra shows that in the unique Bertrand-Nash equilibrium of the one-shot game, firms' prices and individual profits are:

$$p_1^B = \frac{2\alpha_1\beta_2 + \alpha_2\gamma + \beta_2\gamma c}{4\beta_1\beta_2 - \gamma^2}, \quad (16)$$

$$p_2^B = \frac{2\alpha_2\beta_1 + \alpha_1\gamma + 2\beta_1\beta_2 c}{4\beta_1\beta_2 - \gamma^2}, \quad (17)$$

$$\pi_1(p_1^B, p_2^B) = \beta_1(p_1^B)^2, \quad (18)$$

$$\pi_2(p_1^B, p_2^B) = \beta_2 \left( \frac{2\alpha_2\beta_1 + \alpha_1\gamma + c(\gamma^2 - 2\beta_1\beta_2)}{4\beta_1\beta_2 - \gamma^2} \right)^2. \quad (19)$$

If firms instead decide to maximize their joint profit in the one-shot game, their equilibrium prices result from the following optimization problem:

$$\max_{p_1, p_2} \sum_{i=1}^2 q_i(p_1, p_2) \cdot (p_i - c_i). \quad (20)$$

From the FOCs of the previous maximization problem, and after some rearranging, it is concluded that the cooperative prices of firms 1 and 2 are, respectively, given by:

$$p_1^C = \frac{\alpha_2\gamma + \alpha_1\beta_2}{2(\beta_1\beta_2 - \gamma^2)}, \quad (21)$$

$$p_2^C = \frac{\alpha_2\beta_1 + \alpha_1\gamma + c(\beta_1\beta_2 - \gamma^2)}{2(\beta_1\beta_2 - \gamma^2)}. \quad (22)$$

In addition, firms' collusive equilibrium profits are:

$$\pi_1(p_1^C, p_2^C) = \frac{(\alpha_1 + c\gamma)(\alpha_2\gamma + \alpha_1\beta_2)}{4(\beta_1\beta_2 - \gamma^2)}, \quad (23)$$

$$\pi_2(p_1^C, p_2^C) = \frac{(\alpha_2 - c\beta_2)(\alpha_2\beta_1 + \alpha_1\gamma + c(\gamma^2 - \beta_1\beta_2))}{4(\beta_1\beta_2 - \gamma^2)}. \quad (24)$$

## A.2 Deviation profits

If a given firm is considering deviating in period  $t$ , when firms are supposed to set prices (21) and (22) then, making use of firms' reaction functions (14) and (15), it may be concluded that each firm's optimal deviation price is given by:

$$p_1^*(p_2^C) = \frac{\alpha_1(2\beta_1\beta_2 - \gamma^2) + \gamma\alpha_2\beta_1 - c\gamma(\gamma^2 - \beta_1\beta_2)}{4\beta_1(\beta_1\beta_2 - \gamma^2)}, \quad (25)$$

and

$$p_2^*(p_1^C) = \frac{\alpha_2(2\beta_1\beta_2 - \gamma^2) + \gamma\alpha_1\beta_2 - 2c\beta_2(\gamma^2 - \beta_1\beta_2)}{4\beta_2(\beta_1\beta_2 - \gamma^2)}. \quad (26)$$

Hence, the corresponding deviation profits for firms 1 and 2 are, **respectively**, given by:

$$\pi_1(p_1^*(p_2^C), p_2^C) = \beta_1(p_1^*(p_2^C))^2, \quad (27)$$

$$\pi_2(p_1^C, p_2^*(p_1^C)) = \frac{(\alpha_2(2\beta_1\beta_2 - \gamma^2) + \gamma\alpha_1\beta_2 + 2\beta_2c(\gamma^2 - \beta_1\beta_2))^2}{16\beta_2(\gamma^2 - \beta_1\beta_2)^2}. \quad (28)$$

## B Generating Figures 1 and 3 and proving Propositions 1 and 3

In what follows, let  $\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 1$  and  $\gamma = 1/3$ .

### B.1 Deriving Figure 1

#### B.1.1 Curve ICC-s

Making use of eqs. (4), (18), (23), (27), some algebra shows that:

$$f' = \bar{f}(0, \delta) = \frac{3}{8} - (1 - \delta)\frac{25}{64} - \delta\frac{9}{25}. \quad (29)$$

For a given (and sufficiently high) value of the discount factor  $\delta$ , the previous equation is represented by the horizontal line ICC-s in Figure 1.

### B.1.2 Curve ICC-n

Making use of eqs. (3), (19), (24) and (28), and after some rearranging, it may be concluded that condition (3) becomes:

$$\frac{(1 - c_n)(3 - 2c_n)}{8} \frac{1}{1 - \delta} > \frac{(5 - 4c_n)^2}{64} + \frac{(21 - 17c_n)^2}{1225} \frac{\delta}{1 - \delta}. \quad (30)$$

Now, for a given value of  $\delta$ , the value of  $c_n$  for which the previous condition binds is represented by the vertical line ICC-n in Figure 1.

### B.1.3 Curve ICC-m

Combining the results in eqs. (4), (18), (23), (27), it may be concluded that:

$$\bar{f}(c_n, \delta) = \frac{1}{8}(c_n + 3) - (1 - \delta) \frac{(2c_n + 15)^2}{576} - \delta \frac{9(c_n + 7)^2}{1225}. \quad (31)$$

For a given  $\delta$ , the previous condition is represented by the ICC-m curve in Figure 1.

So, this section proves that there exist parameter values in the DPB model such that Figure 1 is valid, as claimed in Proposition 1.

## B.2 Deriving Figure 3

In Figure 3, we assume that  $\delta = 0.9$ . This being the case and making use of eqs. (29), it is straightforward to show that the equations representing the ICC-s, the ICC-n and the ICC-m curves are, respectively, given by:

$$f' = \bar{f}(0, 0.9) = 0.011938, \quad (32)$$

$$c^* = 0.35742, \quad (33)$$

$$\bar{f}(c_n, 0.9) = \frac{1}{8}(c_n + 3) - 0.1 \frac{(2c_n + 15)^2}{576} - 8.1 \frac{(c_n + 7)^2}{1225}. \quad (34)$$

Consider now the region in Figure 1 where ICC-m and ICC-n hold, but ICC-s does not. In this region, firms will not collude in the symmetric market structure  $M^A$ . Hence, making use of eqs. (18)-(19), it can easily be concluded that in this symmetric market structure, the present discounted value of individual firms' profits and the industry profit is, respectively, given by:

$$\pi_m^{A*} = \pi_n^{A*} = \frac{9}{25} \frac{1}{1 - \delta}, \quad (35)$$



$$\Pi^{A*} = \frac{18}{25} \frac{1}{1-\delta}. \quad (36)$$

On the other hand, and restricting the attention to the same region of parameter values, it is shown that firms will be able to collude in the asymmetric market structure  $M^B$ . Therefore, making use of eqs. (23)-(24), it is concluded that in the asymmetric market structure, the present discounted value of individual firms' profits and the industry profit is, respectively, given by:

$$\pi_m^{BC} = \left( \frac{1}{8} (c_n + 3) - f \right) \frac{1}{1-\delta}, \quad (37)$$

$$\pi_n^{BC} = \frac{1}{8} (1 - c_n) (3 - 2c_n) \frac{1}{1-\delta}, \quad (38)$$

$$\Pi^{BC} = \left( \frac{c_n^2 - 2c_n + 3}{4} - f \right) \frac{1}{1-\delta}. \quad (39)$$

Now, making use of eqs. (36) and (39), it may be concluded that the merger condition  $\Pi^{A*} > \Pi^{BC}$  (see eq. (5)) boils down to:

$$f > \frac{c_n^2 - 2c_n + 3}{4} - \frac{18}{25}. \quad (40)$$

The previous eq. is represented by the dashed line in Figure 3.

Finally, we need to check whether the prices charged by the merged entity and the non-merged firm are higher in market structure  $M^B$  than in market structure  $M^A$ . Regarding the merged entity (firm  $m$ ), making use of eqs. (16) and (21), it is easy to check that it will charge higher prices in case the ex-post merger market structure is  $M^B$ , since:

$$p_m^{A*} = \frac{3}{5} < p_m^{BC} = \frac{3}{4}. \quad (41)$$

As for the non merged entity (firm  $n$ ), making use of eqs. (17) and (22), it is simple to check that it will also set a higher price in market structure  $M^B$  than in market structure  $M^A$ , since:

$$p_n^{A*} = \frac{3}{5} < p_n^{BC} = \frac{1}{4} (2c_n + 3), \quad (42)$$

which is true for any  $c_n > 0$ . Hence, a merger leading to an asymmetric market structure  $M^B$  will, in the region under consideration, give rise to a lower consumer surplus than a merger leading to a completely symmetric industry structure,  $M^A$ . This completes the proof of Proposition 3.

## C Alternative cartel behavior rules

In Appendices *A* and *B*, joint profit maximization was used as a criterion for selecting the collusive outcome. This section studies the properties of two other alternative cartel behavior rules which have been used in practice: the equal price increase rule and the constant market shares rule.

As before, we focus the attention on a specific parametrization of the DPB model where  $\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 1$  and  $\gamma = 1/3$ . This being the case, from eqs. (16), (18) and (19), it may be concluded that the Bertrand-Nash equilibrium profits are given by:

$$\pi_1(p_1^B, p_2^B, c) = \frac{9(7+c)^2}{1225}, \quad (43)$$

$$\pi_2(p_1^B, p_2^B, c) = \frac{(21-17c)^2}{1225}. \quad (44)$$

### C.1 The equal price increase rule

Suppose that along the collusive path, each firm reduces its output by the same amount  $\Delta > 0$  (leading to an equal price increase). Notice that firm  $i$ 's optimal value of  $\Delta$  results from the following maximization problem:<sup>19</sup>

$$\max_{\Delta} \left( \frac{3}{2} - \frac{9}{8}(q_i^B - \Delta) - \frac{3}{8}(q_j^B - \Delta) - c_i \right) (q_i^B - \Delta), \quad (45)$$

where  $q_i^B$  denotes firm  $i$ 's quantity in the Bertrand-Nash equilibrium,  $i, j = 1, 2, i \neq j$ . It can easily be checked that the optimal values of  $\Delta$  for firms 1 and 2 are, respectively, given by:

$$\Delta_1^* = \frac{1}{10} + \frac{1}{70}c, \quad (46)$$

$$\Delta_2^* = \frac{1}{10} - \frac{17}{210}c. \quad (47)$$

So, the two firms in the cartel will have to bargain over the value of  $\Delta \in [\Delta_2^*, \Delta_1^*]$ . Assume that the chosen level of  $\Delta$  will be  $(\Delta_2^* + \Delta_1^*)/2$ . The

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<sup>19</sup>Making use of eqs. (10)-(11), it is straightforward to conclude that, for the specific parameter values we have chosen, firm  $i$ 's inverse demand function is given by  $p_i = 3/2 - 9/8q_i - 3/8q_j$ .

corresponding cooperative prices and profits are then given by:

$$\widehat{p}_1^C = \frac{3}{4} + \frac{1}{28}c, \quad (48)$$

$$\widehat{p}_2^C = \frac{3}{4} + \frac{13}{28}c, \quad (49)$$

$$\widehat{\pi}_1^C = \frac{(21+c)(21+5c)}{1176}, \quad (50)$$

$$\widehat{\pi}_2^C = \frac{(7-5c)(21-19c)}{392}. \quad (51)$$

Now, making use of eqs. (14), (15), (48) and (49), it is shown that the optimal deviation prices for firms 1 and 2 are, respectively, given by:

$$p_1^*(\widehat{p}_2^C) = \frac{5}{8} + \frac{13}{168}c, \quad (52)$$

$$p_2^*(\widehat{p}_1^C) = \frac{5}{8} + \frac{85}{168}c. \quad (53)$$

The corresponding optimal deviation profits are:

$$\widehat{\pi}_1^D = \frac{(105+13c)^2}{28\,224}, \quad (54)$$

$$\widehat{\pi}_2^D = \frac{(105-83c)^2}{28\,224}. \quad (55)$$

So, combining eqs. (4), (43), (50) and (54), it may be concluded that

$$\bar{f}_\Delta(c_n, \delta) = \frac{(21+c_n)(21+5c_n)}{1176} - (1-\delta) \frac{(105+13c_n)^2}{28\,224} - \delta \frac{9(7+c_n)^2}{1225}. \quad (56)$$

Now, it can easily be checked that, for a given value of  $\delta$  (say  $\delta = 0.9$ ),  $\bar{f}_\Delta(c_n, \delta)$  increases in  $c_n$  in the relevant region of parameter values. So, the previous condition can be represented by such an upward sloping ICC-m curve as that presented in Figure 1.

## C.2 The constant market shares rule

Suppose that along the collusive path, firm  $i$ 's output is  $q_i^C = \alpha q_i^B$ , where  $q_i^B$  denotes firm  $i$ 's quantity in the Bertrand-Nash equilibrium,  $i, j = 1, 2, i \neq j$ , and  $\alpha \in (0, 1)$ . When this is the case, each firm's market share along the collusive path coincides with its market share at the Bertrand Nash equilibrium. The problem, however, is to identify the value of  $\alpha$  chosen by the cartel.

Notice that firm  $i$ 's optimal value of  $\alpha$  results from the following maximization problem:

$$\max_{\alpha} \left( \frac{3}{2} - \frac{9}{8}\alpha q_i^B - \frac{3}{8}\alpha q_j^B - c_i \right) \alpha q_i^B. \quad (57)$$

It can easily be checked that the optimal values of  $\alpha$  for firms 1 and 2 are, respectively, given by:

$$\alpha_1^* = \frac{35}{2(21 - 2c)}, \quad (58)$$

$$\alpha_2^* = \frac{35(3 - 2c)}{18(7 - 4c)}. \quad (59)$$

So, the two firms in the cartel will have to bargain over the value of  $\alpha \in [\alpha_2^*, \alpha_1^*]$ . Let the chosen level of  $\alpha$  be  $\tilde{\alpha}_1$ .<sup>20</sup> The corresponding cooperative prices and profits are then given by:

$$\tilde{p}_1^C = \frac{3}{4}, \quad (60)$$

$$\tilde{p}_2^C = \frac{3(21 + 8c)}{4(21 - 2c)}, \quad (61)$$

$$\tilde{\pi}_1^C = \frac{9(7 + c)}{8(21 - 2c)}, \quad (62)$$

$$\tilde{\pi}_2^C = \frac{1}{8} \frac{(21 - 17c)(8c^2 - 60c + 63)}{(21 - 2c)^2}. \quad (63)$$

Now, making use of eqs. (14), (15), (60) and (61), it is shown that optimal deviation prices for firms 1 and 2 are, respectively, given by:

$$p_1^*(\tilde{p}_2^C) = \frac{105}{8(21 - 2c)}, \quad (64)$$

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<sup>20</sup>Since we have assumed the most efficient firm to be the one which must cover indivisible per-period coordination fixed costs, it may be natural to assume that it has all the bargaining power in the choice of  $\alpha$ .

$$p_2^*(p_1) = \frac{5}{8} + \frac{1}{2}c. \quad (65)$$

The corresponding optimal deviation profits are:

$$\tilde{\pi}_1^D = \frac{11\,025}{64(21-2c)^2}, \quad (66)$$

$$\tilde{\pi}_2^D = \frac{(5-4c)^2}{64}. \quad (67)$$

### C.2.1 Deriving Figure 2

**Curve ICC-s** Making use of eqs. (4), (43), (62) and (66), some algebra shows that:

$$f' = \bar{f}(0, \delta) = \frac{9}{8} \frac{7}{21} - (1-\delta) \frac{11\,025}{64(21)^2} - \delta \frac{9(7)^2}{1225}. \quad (68)$$

For a given (and sufficiently high) value of  $\delta$ , the previous equation is represented by the horizontal line ICC-s in Figure 2.

**Curve ICC-n** Making use of eqs. (44), (63) and (67), it may be concluded that condition (3) can be rewritten as:

$$\frac{1}{1-\delta} \frac{1}{8} \frac{(21-17c_n)(8c_n^2-60c_n+63)}{(21-2c_n)^2} > \frac{(5-4c_n)^2}{64} + \frac{(21-17c_n)^2}{1225} \frac{\delta}{1-\delta}. \quad (69)$$

For a given value of  $\delta$ , the value of  $c_n$  for which the previous constraint is binding is represented by the vertical line ICC-n in Figure 2.

**Curve ICC-m** Combining the results in eqs. (4), (43), (62) and (66), it may be concluded that:

$$\bar{f}(c_n, \delta) = \frac{9}{8} \frac{7+c_n}{21-2c_n} - (1-\delta) \frac{11\,025}{64(21-2c_n)^2} - \delta \frac{9(7+c_n)^2}{1225}. \quad (70)$$

It can easily be checked that for a given sufficiently high value of the discount factor  $\delta$ ,  $\bar{f}(c_n, \delta)$  decreases in  $c_n$ . So, the previous condition is represented by the downward sloping curve ICC-m in Figure 2.

## D Proof of Proposition 4

Take the specific parametrization of the DPB model used above, where  $\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 1$  and  $\gamma = 1/3$ .

If the two firms, whose constant marginal costs are  $c_1$  and  $c_2$ , play non-cooperatively, then it is easily shown that their Bertrand-Nash equilibrium profits are given by:

$$\pi_1^B(c_1, c_2) = \frac{1}{1225} (21 - 17c_1 + 3c_2)^2, \quad (71)$$

$$\pi_2^B(c_1, c_2) = \frac{1}{1225} (21 + 3c_1 - 17c_2)^2. \quad (72)$$

If firms instead decide to maximize their joint profit in the one-shot game, their equilibrium profits will be as follows:<sup>21</sup>

$$\pi_1^C(c_1, c_2) = \frac{1}{24} (3c_1 - c_2 - 3) (2c_1 - 3), \quad (73)$$

$$\pi_2^C(c_1, c_2) = \frac{1}{24} (3c_2 - c_1 - 3) (2c_2 - 3). \quad (74)$$

Some algebra also shows that firms' deviation profits are equal to:

$$\pi_1^D(c_1, c_2) = \frac{1}{576} (12c_1 - 2c_2 - 15)^2, \quad (75)$$

$$\pi_2^D(c_1, c_2) = \frac{1}{576} (12c_2 - 2c_1 - 15)^2. \quad (76)$$

Now, making use of eq. (7), it may be concluded that the maximum acquisition cost  $\tilde{A}$  that firm 2 would be willing to pay is the one for which the ICC (7) is binding, i.e.,

$$\tilde{A} = \pi_2^C(p_2^C, p_1^C, c_1, c_2) - (1 - \delta)\pi_2(p_2^*(p_1^C), p_1^C, c_1, c_2) - \delta\pi_2(p_2^B, p_1^B, c_1, c_2), \quad (77)$$

which, making use of eqs. (71)-(76), can, for the specific parametrization of the DPB model we have been using throughout the paper, be rewritten as:

$$\begin{aligned} \tilde{A}(c_1, c_2, \delta) = & \left( \frac{1}{24} (3c_2 - c_1 - 3) (2c_2 - 3) \right) - (1 - \delta) \left( \frac{1}{576} (12c_2 - 2c_1 - 15)^2 \right) \\ & - \delta \frac{1}{1225} (21 + 3c_1 - 17c_2)^2. \end{aligned} \quad (78)$$

<sup>21</sup>If  $c_1 = 0$  and  $c_2 = c$ , eqs. (71) and (71) boil down to eqs. (43) and (44), respectively.

Now, while the previous equation puts forward the threshold value for the acquisition price, the actual acquisition price is  $A = \pi_3^B(c_1, c_2, \bar{c}_3)$ . For the specific parametrization of the DPB model we are using, it is straightforward to show that firm 3's individual (triopoly) profits in a Bertrand-Nash equilibrium are given by:

$$\pi_3^B(c_1, c_2, c_3) = \frac{(13c_3 - 3c_1 - 3c_2 - 21)^2}{784} - F_3 \equiv A(c_1, c_2, \bar{c}_3), \quad (79)$$

where  $F_3 \geq 0$  denotes firm 3's fixed costs of production.

Consider now an example where  $\delta = 0.9$ ,  $c_2 = 0.1$ ,  $c_3 = c_1$  and  $F_3 = 0.4$ . Figure 4 then presents the threshold acquisition price  $\tilde{A}(c_1, c_2 = 0.1, \delta = 0.9)$  (solid upward sloping curve) and the actual acquisition price  $A(c_1, c_2 = 0.1, \bar{c}_3 = c_1)$  (dashed downward sloping curve). So, firm 2 will decide to buy out firm 3 and collude after this buyout in the region where the dashed decreasing curve is below the solid upward sloping curve (so that the actual price paid to acquire firm 3 is lower than the maximum threshold price  $\tilde{A}$  identified above). In addition, Figure 4 also presents a vertical line which represents firm 1's ICC (eq. (8)), which making use of eqs. (71), (73) and (79) and for the specific parametrization of the model we are using in this example, implies that firm 1 will decide to collude if  $c_1 \leq 0.43360$ .

So, clearly, firm 2 will decide to buy out firm 3 and collusion between firms 1 and 2 will take place in the market structure induced by this buyout if costs are medium asymmetric. This completes the proof of Proposition 4.

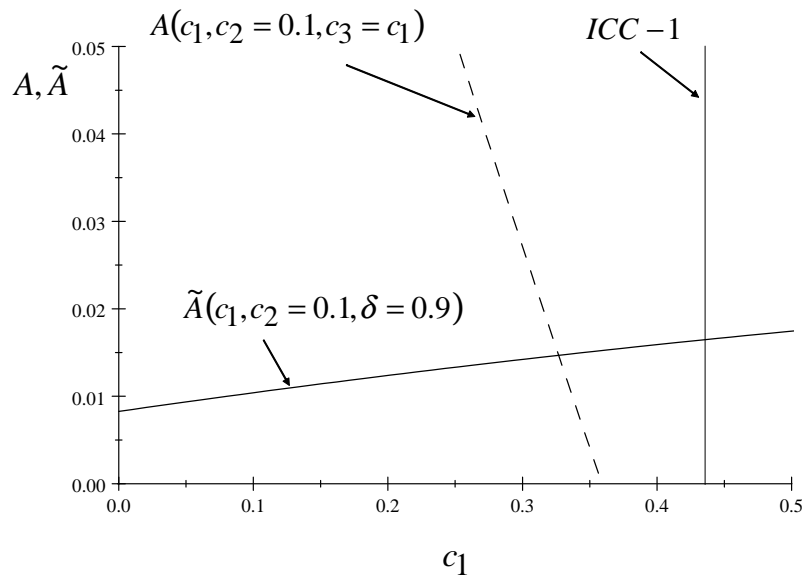


Figure 4: Equilibrium Pattern when Indivisible Costs are Endogenous