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Property Tax and Urban Sprawl
Theory and Implications for U.S. Cities

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Property Tax and Urban Sprawl Theory and Implications for U.S. Cities*

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Abstract: This article attempts a formal analysis of the connection between property tax and urban sprawl in U.S. cities. We develop a theoretical model that includes households (who are also landlords) and land developers in a regional land market. We then test the model empirically based on a national sample of urbanized areas. The results we obtained from both theoretical and empirical analyses indicate that increasing property tax rates reduces the size of urbanized areas.

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1 Introduction

Urban sprawl is characterized by scattered and poorly planned low-density development beyond the edge of urbanized areas. Over the past century, the U.S. cities have expanded and density of land used per person has declined drastically. Here are some facts:

- Nationwide, land consumed for building far outpaces population growth. According to the American Farmland Trust, between 1960 and 1990, the amount of developed land in metro areas more than doubled, while the population grew by less than half. For example, between 1970 and 1990, greater Cleveland lost 11 percent of its population, yet developed land grew by 33 percent; greater Chicago gained 4 percent in population but 46 percent in residential land; Los Angeles' population grew by 45 percent while its developed land increased by 300 percent.
- Census Bureau figures show that in 1920, the average density of urbanized areas (which includes cities, suburbs, and towns) was 6,160 persons per square mile. In 1990, the number had diminished to 2,589.

Urban sprawl is a major concern across the U.S. cities. In general, urban sprawl has a variety of economic, social, and environmental consequences. Sprawling development wastes resources by increasing public expenditures in providing infrastructure and services. Urban sprawl increases travel time and distance. Low-density development reduces the feasibility of mass transit, thus increasing reliance on private automobile usage. This automobile excess increases pollution, congestion, alienation, and the use of scarce energy resources. Sprawl also causes the excessive loss of farmland (for overviews on urban sprawl issues, see Brueckner, 2000, Nechyba and Walsh, 2004, and Glaeser and Kahn, 2004).

Needless to say, urban sprawl has more than one cause. The long-standing debate on land taxation and its virtues (George, 1879; Skaburskis and Tomalty, 1997) reveals that property tax might be on the list of causes of urban sprawl. A property tax can be viewed as a tax levied at equal rates on both the land and capital embodied in structures while, in a pure land tax, the tax on capital (i.e., improvements) is set to zero. Abundant literature – for example, Arnott and MacKinnon (1977), Case and Grant (1991), Oates and Schwab (1997), Mills (1998), and Brueckner and Kim (2003) – provides arguments on how property tax promotes inefficiently and under-used land development. The standard result in this literature is that land is developed less intensively under property taxation than under a pure land tax, leading to a spatial extension of cities. The tax on improvements to land also raises the perceived cost of buildings and the owner can reduce the tax burden by designing projects that use relatively more land in comparison to improvements. This leads to lower than optimum densities and forces the city to spread more than it would had a perfectly neutral tax has been used to finance local services and infrastructure. In summary, the distortions generated by the property tax may have promoted sprawling development patterns.

Despite ample discussion on property and urban sprawl, the net effect of property tax on spatial sizes of city is ambiguous from a closer examination on previous theoretical models. Brueckner and Kim's (2003) is the only theoretical analysis that incorporates a land market to investigate the connection between urban spatial expansion and the property tax.[1] However in their full analysis, the net effect of property tax on spatial sizes of city is ambiguous. A literature review further indicates that there has been no empirical study that carries out a regression equation relating a city's spatial size to a property tax measure and other relevant variables.

The aim of this paper is to develop further the analysis on the net effect of property tax on the spatial size of cities and to test it using U.S. data.

We first develop a theoretical model that investigates the property tax's effects on urban sprawl. We take a log-linear utility function, which allows us to have closed-form solutions and to show that, unambiguously, an increase in property taxes reduces city size and thus urban sprawl. We are also able to derive some cross-effect results, namely the higher the commuting cost and the smaller the city (in terms of population), the higher the negative effect of the property tax on urban sprawl. This shows, for example, that in bigger cities the effect of property taxes on urban sprawl is lower than in small cities. Using this utility function, we then develop further the model by relaxing one of the key assumptions in Brueckner and Kim (2003), the fact that landlords are absent and live outside the city. This assumption, in particular, implies that the rent generated by the land does not appear as income for the urban residents, accruing instead to individuals living elsewhere. This limits the scope of a general equilibrium analysis. We thus relax this assumption and consider a model in which landlords are residents of the city. The model becomes more complicated since landlords are now the city residents and thus new income effects are generated. In fact, as it is standard in a fulled closed city model, the city residents are now assumed to form a government, which rents the land for the city from rural landlords at the agricultural rent. The city government, in turn, subleases the land to city residents at the competitive rent. Even in

[1] Arnott and MacKinnon (1977) is another exception but most of the analysis is solved using numerical simulations.

this more complicated model, we are able to demonstrate that an increase in property taxes reduces urban sprawl, showing how robust is this result.

We then undertake an empirical analysis to test our main theoretical result, namely the negative impact of property taxes on urban sprawl. To identify the impact of property tax on city size, we use instrumental variables because of the simultaneity problem between these two variables. We need an instrument that predicts changes in property tax rates, but is unrelated to changes in city size (after controlling for other relevant factors). An appropriate instrument for the property tax rate is the magnitude of state aid to schools. In this context, the impact of property tax on city size is estimated using two-stage least squares (2SLS), treating the property tax variable as endogenous and the other right-hand-side variables as exogenous. Our empirical results confirm the main prediction of the theoretical model: an increase in property taxes reduces urban sprawl.

2 Theory

We now develop our theoretical model in order to examine the connection between property tax and urban sprawl. For the sake of the presentation, we first expose the Brueckner and Kim (2003)'s model. Then, we develop their model in the case of a log-linear utility function and finally relax the restrictive assumption of absentee landlords, which is not realistic in the context of U.S. cities, to explore the full-closed city model.

2.1 Brueckner and Kim (2003)

Let us present the model of Brueckner and Kim (2003).

City The city is *monocentric*, *closed* and *linear*[2] where the Central Business District (CBD hereafter) is located as the origin (zero). All land is own by *absentee landlords.*

[2]In fact, Brueckner and Kim (2003) assume that the city is circular. The linearity assumption does not affect any of their results.

Firms (land developers) There is a housing industry with has the following production function:[3]

$$Q = H(L, K) \tag{1}$$

where L and K are respectively land and capital (or nonland input). This function is increasing and concave in each of its argument. It is assumed that the housing production function $H(L, K)$ has constant returns to scale, which implies that the production function can be written as:

$$h = h(S) \tag{2}$$

where $S \equiv K/L$ represents the capital per acre of land or *improvments* per acre. S is also referred to as the *structural density* (Brueckner, 1987) and is an index of the height of buildings. The function $h(S) \equiv Q/L$ defined by (2) is the housing output per acre of land, with $h'(S) > 0$ and $h''(S) < 0$.

When there are no taxes, the profit is given by:

$$\Pi = R_H H(L, K) - R\, L - rK$$

or equivalently

$$\pi \equiv \Pi/L = R_H\, h(S) - (R + rS)$$

where π is the profit per acre of land, R_H is the rental price per unit of housing service q, R is the rent per unit of land (land cost per acre) and r the price of capital (or the cost per unit of S).

When θ, the property tax rate, is introduced, each profit-maximizing firm of the housing industry behaves as:[4]

$$\max_S \{\pi = R_H\, h(S) - (1+\theta)(R + rS)\} \qquad \text{at each } x \in [0, x_f] \tag{3}$$

Consumers/Workers Each household contains one person. Each individual chooses z and q that maximize their utility function under their budget constraint, i.e.

$$\max_{z,q} U(z, q) \text{ s.t. } z + R_H\, q = y - t\, x \tag{4}$$

[3] Observe that the housing capital K is assumed to be perfectly malleable. This strongly simplifies the analysis since it implies that producers are able to costlessly adjust both their capital and land inputs, and, as a result, the issue of durability of structures is not analyzed here.

[4] Observe that it does not matter whether the developer or the urban resident pays the property tax θ. The same results would emerge if the residents pay at a rate θ, so that the gross-of-tax rent price is written $R_H(1 + \theta)$. Then, the developer profit will just be $R_H h(S) - (R + rS)$, with no tax term showing up.

where x is the distance to the CBD, z and q are, respectively, the consumption of the composite good (which price is taken as the numeraire) and the lot size (or dwelling size), y, the common income, and t the pecuniary commuting cost per unit of distance. It is assumed that $U(z.q)$ is well-behaved, i.e. it is increasing and strictly concave in each of its argument and smooth (differentiable). The program (4) is equivalent to

$$\max_q U(y - t\,x - R_H\,q, q)$$

which leads to[5]

$$\frac{U_q(z,q)}{U_z(z,q)} = R_H \tag{5}$$

where we use the following notations:

$$U_z \equiv \frac{\partial U}{\partial z}\ ,\ U_q \equiv \frac{\partial U}{\partial q}$$

Equation (5) implicitly defines $q = q(x,y)$. Using the budget constraint,we obtain

$$z(x,y) = y - t\,x - R_H(x)q(x,y)$$

Plugging these two values into the utility function gives the following indirect utility function:

$$U((y - t\,x - R_H(x)q(x,y), q(x,y)) \equiv u \tag{6}$$

where u is the common utility level reached in equilibrium by all residents in the city. Finally, by taking the inverse of this function, we can determine the bid rent of all individuals as

$$R_H = R_H(x,u)$$

It is easy to show that

$$\frac{\partial R_H(x,u)}{\partial x} < 0\ \ ,\ \frac{\partial R_H(x,u)}{\partial u} < 0$$

Plugging this value $R_H(x,u)$ in $q = q(x,y)$, which using (5) defines $q = q(x,u)$ by the following equation:

$$\frac{U_q(z,q)}{U_z(z,q)} = R_H(x,u) \tag{7}$$

Again, it is easy to show that

$$\frac{\partial q(x,u)}{\partial x} > 0\ \ ,\ \frac{\partial q(x,u)}{\partial u} > 0$$

[5] The second order condition is given by

$$U_{zz}R_H^2 + U_{qq} - 2R_H U_{zq}$$

and is assumed to be negative. A sufficient condition is that $U_{zq} > 0$.

Equilibrium Plugging (6) in (3), the land developer's program becomes

$$\max_S \{\pi = R_H(x,u)h(S) - (1+\theta)[R(x) + rS]\} \qquad \text{at each } x \in [0, x_f]$$

First order condition yields:[6]

$$R_H(x,u)h'(S) = (1+\theta)\, r \tag{8}$$

which implicitly gives

$$S = S(x,u,\theta)$$

Again, we have

$$\frac{\partial S}{\partial x} = -\frac{\partial R_H}{\partial x}\frac{h'(S)}{R_H h''(S)} < 0\ ,\ \frac{\partial S}{\partial u} = -\frac{\partial R_H}{\partial u}\frac{h'(S)}{R_H h''(S)} < 0$$

$$\frac{\partial S}{\partial \theta} = \frac{r}{R_H h''(S)} < 0$$

We can now define the *population density* as $D \equiv h\left[S(x,u,\theta)\right]/q(x,u)$, which is the ratio between square feet of floor space per acre of land and square feet of floor space per dwelling (person). This is a different concept than the *structural density* or *improvments* defined by $S(x,u,\theta)$. The two concepts are important to understand the main results of Brueckner and Kim (2003). As noted above, the improvments (i.e. the intensity of land development) are a measure of building height so a higher S means that developers construct higher buildings, containing more housing floor space per acre of land. On the other hand, a higher population density means that either the housing floor space is higher or the dwelling size is lower.

Let us now go back to the analysis. Since $H(.)$ has constant returns to scale, in equilibrium, the housing industry is such that all firms make zero profit at each x, that is

$$R_H(x,u)h(S(x,u,\theta)) - (1+\theta)\left[R(x) + rS(x,u,\theta)\right] = 0$$

which implies

$$R(x,u,\theta) = \frac{R_H(x,u)}{(1+\theta)} h\left[S(x,u,\theta)\right] - rS(x,u,\theta) \tag{9}$$

It is easy to show that

$$\frac{\partial R(x,u,\theta)}{\partial x} = \frac{\partial R_H}{\partial x}\frac{h}{(1+\theta)} < 0\ ,\ \frac{\partial R(x,u,\theta)}{\partial u} = \frac{\partial R_H}{\partial u}\frac{h}{(1+\theta)} < 0$$

[6] The second order condition is always satisfied since

$$R_H(x,u)h''(S) < 0$$

$$\frac{\partial R(x,u,\theta)}{\partial \theta} = -R_H \frac{h}{(1+\theta)^2} < 0 \tag{10}$$

We can now formally define the equilibrium.

Definition 1 *An urban land-use equilibrium in a linear and closed city with absentee landlords is a vector* $(u, x_f,\ R(x))$ *such that:*

$$R(x_f, u, \theta) = R_A \tag{11}$$

$$\int_0^{x_f} \frac{h\left[S\left(x,u,\theta\right)\right]}{q(x,u)} dx = N \tag{12}$$

$$R(x) = \max\left\{R(x_f,u,\theta), R_A\right\} \quad \text{at each } x \in \left(0, x_f^*\right] \tag{13}$$

where $R(x_f, u, \theta)$, $S\left(x, u, \theta\right)$, $q(x, u)$ *are defined by (9), (8), and (7), respectively.*

Equation (11) says that the bid rent of the individuals must be equal to the agricultural land at the city fringe. Equations (12) gives the population constraint. Finally, equation (13) defines the equilibrium land rent as the upper envelope of the equilibrium bid rent curves of all workers' types and the agricultural rent line.

The main result of Brueckner and Kim (2003) can be summarized as follows:

Proposition 1 *The comparative statics of the equilibrium is as follows:*

$$\frac{\partial x_f}{\partial \theta} \gtreqless 0$$

However, for CES preferences,[7] *if the elasticity of substitution* $\sigma \geq 1$, *then* $\partial x_f / \partial \theta < 0$, *while if* $\sigma < 1$, *the sign of* $\partial x_f / \partial \theta$ *is still ambiguous.*

By remembering our discussion about structural versus population density, the intuition of this result is easy to understand. There are two countervailing effects of an increase of a property tax θ on urban sprawl x_f. On the one hand, an increase in θ has a *direct negative* effect on the profit of developers, which accordingly reduce the level of improvements (or structural density). As a result, for a given size of dwellings, buildings are shorter and thus the population density is lower. Because population is fixed (closed city), it has to be that the city increases in size (this is referred to as the *building height*

[7] That is

$$U(z,q) = \left[\alpha z^{-\beta} + (1-\alpha)\, q^{-\beta}\right]^{-1/\beta}$$

where $-1 \leq \beta < +\infty$ and with $\sigma = 1/(1+\beta)$ giving the elasticity of substitution, where $0 \leq \sigma < +\infty$.

effect). On the other hand, an increase in θ has an *indirect negative* effect on households' housing consumption because the tax on land and improvments is partly shifted forward to consumers, which yields a higher price of housing and thus a lower dwelling size. Since smaller dwellings imply an increase in population density D and thus more urban sprawl (this is referred to as the *dwelling size effect*). The net effect is thus ambiguous in the general case. In the CES case, when the consumptions of z (composite good) and q (housing) are highly substituable ($\sigma \geq 1$), the dwelling-size effect becomes more important and the net effect is such that an increase in θ decreases urban sprawl.

2.2 A new specific case

We would like now to go further and, for that, we need to make some assumptions. We will use a log-linear utility function (quasi-linear preferences), which has nice properties, especially in urban economics (see Zenou, 2005).

Consumers/Workers We assume the following utility function

$$U(z, q) = z + \log q \tag{14}$$

First order condition gives

$$q(x) = \frac{1}{R_H(x)} \tag{15}$$

$$z(x, y) = y - t.x - 1 \tag{16}$$

The indirect utility function can thus be written as:

$$u = y - t.x - 1 - \log R_H(x) \tag{17}$$

and the bid rent function is given by:

$$R_H(x, u) = \exp(y - t.x - 1 - u) \tag{18}$$

We have:

$$\begin{aligned} \frac{\partial R_H(x,u)}{\partial x} &= -t \exp(y - t.x - 1 - u) < 0 \\ \frac{\partial R_H(x,u)}{\partial u} &= -\exp(y - t.x - 1 - u) < 0 \end{aligned}$$

Plugging this value $R_H(x, u)$ in $q = q(x)$ gives finally

$$q(x, u) = \frac{1}{\exp(y - t.x - 1 - u)} \tag{19}$$

It is important to observe that, even though the housing consumption q is not directly affected by income y (see (15)),[8] it is indirectly affected by income

[8] This is because of the log-linear nature of the utility function, which is defined in (14).

through the land rent (see (19)). Indeed, when income increases, the bid rent increases (see (18)) since people are richer. As a result, because housing is more costly, they consume less land and thus reduces their dwelling size. Formally,

$$\frac{\partial q(x,u)}{\partial y} = -\frac{1}{[\exp(y - t.x - 1 - u)]^2} < 0$$

We also easily obtain:

$$\begin{aligned}\frac{\partial q(x,u)}{\partial x} &= t\exp\left[1 + u + t.x - y)\right] > 0 \\ \frac{\partial q(x,u)}{\partial u} &= \exp\left[1 + u + t.x - y)\right] > 0\end{aligned}$$

Equilibrium Plugging (6) in (3), the land developer's program becomes:

$$\max_S \left\{\pi = R_H(x,u)h(S) - (1+\theta)\left[R(x) + rS\right]\right\} \qquad \text{at each } x \in [0, x_f]$$

First order condition yields:

$$R_H(x,u)h'(S) = (1+\theta)\, r$$

which is:

$$h'(S) = (1+\theta)\, r \exp(t.x + 1 + u - y)$$

and gives

$$S(x,u,\theta) = h'^{-1}\left[(1+\theta)\, r\exp(t.x + 1 + u - y)\right]$$

The second explicit form that we adopt is the following:[9]

$$h(S) = 2\sqrt{S} \tag{20}$$

which implies that

$$h'(S) = \frac{1}{\sqrt{S}} > 0 \text{ and } h''(S) = -\frac{1}{2}S^{-3/2} < 0$$

Thus we have

$$S(x,u,\theta) = \left[\frac{\exp(y - t.x - 1 - u)}{(1+\theta)\, r}\right]^2$$

and thus

$$h\left[S(x,u,\theta)\right] = 2\sqrt{S(x,u,\theta)} = 2\frac{\exp(y - t.x - 1 - u)}{(1+\theta)\, r}$$

[9]Formulation (20) implies that the housing industry has the a Cobb-Douglas production function, which is defined as follows:

$$Q = H(K,L) = 2K^{1/2}L^{1/2}$$

It is easy to verify that this production is concave and exhibits constant returns to scale.

Again, we have

$$\begin{aligned}
\frac{\partial S}{\partial x} &= -t\frac{2\exp(y-t.x-1-u)}{\left[(1+\theta)\,r\right]^2} < 0 \\
\frac{\partial S}{\partial u} &= -\frac{2\exp\left(y-t.x-1-u\right)}{\left[(1+\theta)\,r\right]^2} < 0
\end{aligned}$$

$$\frac{\partial S}{\partial \theta} = -2r\frac{\exp(y-t.x-1-u)^2}{\left[(1+\theta)\,r\right]^3} < 0$$

Since $H(.)$ has constant returns to scale, in equilibrium, the housing industry is such that all firms make zero profit at each x, that is:

$$R(x,u,\theta) = \frac{R_H(x,u)}{(1+\theta)}h\left[S(x,u,\theta)\right] - rS(x,u,\theta)$$

In our case, we have

$$R(x,u,\theta) = \frac{\left[\exp(y-t.x-1-u)\right]^2}{(1+\theta)^2\,r}$$

and thus

$$\begin{aligned}
\frac{\partial R(x,u,\theta)}{\partial x} &= -t\frac{2\exp(y-t.x-1-u)}{(1+\theta)^2\,r} < 0 \\
\frac{\partial R(x,u,\theta)}{\partial u} &= -\frac{2\exp(y-t.x-1-u)}{(1+\theta)^2\,r} < 0
\end{aligned}$$

$$\frac{\partial R(x,u,\theta)}{\partial \theta} = -2\frac{\left[\exp(y-t.x-1-u)\right]^2}{(1+\theta)^3\,r} < 0$$

Let us use the definition 1 of the equilibrium. Solving the first equation (11) yields

$$t.x_f = y-1-u-\log(1+\theta)-\log\sqrt{rR_A}$$

and the second equation (12) leads to

$$t.x_f = y-1-u-\frac{1}{2}\log\left\{\exp\left[2(y-1-u)\right]-(1+\theta)\,rtN\right\}$$

By combining these two equations, we finally obtain:

$$x_f^* = \frac{1}{t}\log\left[1+\frac{tN}{(1+\theta)\,R_A}\right] \tag{21}$$

$$u^* = y-1-\frac{1}{2}\log\left\{(1+\theta)\,r\left[(1+\theta)\,R_A+tN\right]\right\} \tag{22}$$

Proposition 2 *Assume that the city is closed and the landlords are absent. Then, if the utility function is quasi-linear and defined as in (14) and the production function $h(S)$ is Cobb-Douglas as in (20), we have:*

$$\frac{\partial x_f}{\partial \theta} < 0 \quad , \quad \frac{\partial u}{\partial \theta} < 0$$

Moreover,

$$\frac{\partial^2 x_f}{\partial \theta \partial t} > 0 \quad , \quad \frac{\partial^2 u}{\partial \theta \partial t} > 0$$

$$\frac{\partial^2 x_f}{\partial \theta \partial N} < 0 \quad , \quad \frac{\partial^2 u}{\partial \theta \partial N} > 0$$

The following comments are in order. First, an increase in the property tax unambiguously decreases both urban sprawl and utility. This is because our utility function is not a special case of the CES utility function proposed by Brueckner and Kim (2003) since in their model the elasticity of substitution $\sigma = 1/(1+\beta)$ is a constant that depends only on the parameter β whereas here it is given by: $\sigma = 1 + 1/z$, which, in equilibrium and using (16) is equal to: $\sigma = 1 + 1/(y - t.x - 1)$, and thus depend on distance to jobs. Of course, in our case, $\sigma > 1$, which explains why $\partial x_f / \partial \theta < 0$. Second, we have a new result here that was not present in Brueckner and Kim (2003), which is interesting and may be tested empirically. Indeed, we are able to derive some cross-effect results: the higher the commuting cost and the smaller the city (in terms of population), the higher the negative effect of the property tax on urban sprawl. This shows, for example, that in bigger cities the effect of property taxes on urban sprawl is lower than in small cities.

2.3 The case when landlords are *not* absent: The fully closed city

We would like now to go further by extending this model. In most cities in the world landlords are not absent and thus we would like to relax the assumption of absentee landlords. In the fulled closed city, which is discussed here, urban land is rented from absentee landlords at a price equaling the agricultural rent (for a standard analysis of a fulled closed city, see Pines and Sadka, 1986, and Fujita, 1989, ch. 3).[10]

To be more precise, the city residents are now assumed to form a government, which rents the land for the city from rural landlords at agricultural

[10] As noted by Brueckner and Kim (2003), one could even go further by also including in the income of urban residents the revenue from housing capital. We do not pursue this avenue here because we believe that it is reasonable to assume that housing capital accrues to absentee owners while for land rents it is more to difficult to accept this assumption.

rent R_A. The city government, in turn, subleases the land to city residents at the competitive rent $R(x) \equiv R(x, u, \theta)$ at each location x. We can define the total differential rent (TDR) from the city as:

$$\begin{aligned} TDR &= \int_0^{x_f} [R(x) - R_A]\, dx \\ &= \int_0^{x_f} R(x)dx - x_f R_A \end{aligned} \tag{23}$$

The income of each individual is now given by $y + TDR/N$.[11] As a result, the program of each individual is now given by:

$$\max_{z,q} U(z, q) = z + \log q \text{ s.t. } z + R_H(x)q = y + \frac{TDR}{N} - t\,x \tag{24}$$

This is equivalent to

$$\max_q U\left(y + \frac{TDR}{N} - t\,x - R_H(x)q, q\right)$$

Everything is now the same, the only difference is to replace y by $y + TDR/N$. This complicates the analysis because TDR is endogenous and depends on u and x_f. We easily obtain the following values:

$$R_H(x, u) = \exp\left[y + \frac{TDR}{N} - t.x - 1 - u\right]$$

$$q(x, u) = \exp\left[1 + u + t.x - y - \frac{TDR}{N}\right]$$

$$S(x, u, \theta) = \left[\frac{\exp\left[y + TDR/N - t.x - 1 - u\right]}{(1+\theta)\, r}\right]^2$$

$$R(x, u, \theta) = \frac{\left[\exp(y + TDR/N - t.x - 1 - u)\right]^2}{(1+\theta)^2\, r}$$

where

$$\frac{TDR}{N} = \frac{1}{N}\int_0^{x_f} R(x, u, \theta)dx - x_f \frac{R_A}{N}$$

Lemma 1 *We have*

$$\frac{\partial R(x, u, \theta)}{\partial x} \lesseqgtr 0 \Leftrightarrow x_f \lesseqgtr \frac{N}{2R(x, u, \theta)}$$

$$\frac{\partial R(x, u, \theta)}{\partial u} \lesseqgtr 0 \Leftrightarrow x_f \lesseqgtr \frac{N}{2R(x, u, \theta)}$$

[11] Observe that, as noted above, the utility function (14) implies that the dwelling size q does not *directly* but *indirectly* depend on income y through bid rent. So, the fully closed city model is relevant here since an increase in the property tax θ affects the TDR and thus the income of urban residents, which in turn affects their housing consumption.

$$\frac{\partial R(x,u,\theta)}{\partial \theta} \lesseqgtr 0 \Leftrightarrow x_f \lesseqgtr \frac{N}{2R(x,u,\theta)}$$

In particular, if $x_f < \frac{N}{2R(0,u,\theta)}$*, then* $\frac{\partial R(x,u,\theta)}{\partial x} < 0$, $\frac{\partial R(x,u,\theta)}{\partial u} < 0$ *and* $\frac{\partial R(x,u,\theta)}{\partial \theta} < 0$.

Proof. See the Appendix.

We can now write the equilibrium conditions (11) and (12). The first one is:

$$R(x_f,u,\theta) = R_A$$

which is given by:

$$\left[\exp\left(y + \frac{1}{N}\int_0^{x_f} R(x,u,\theta)dx - x_f\frac{R_A}{N} - t.x_f - 1 - u\right)\right]^2 = R_A\left(1+\theta\right)^2 r$$

After some manipulations, this equation can be written as:

$$\left(t + \frac{R_A}{N}\right)x_f - \frac{1}{N}\int_0^{x_f} R(x,u,\theta)dx = y - 1 - u - \log\left(1+\theta\right) - \log\sqrt{rR_A} \quad (25)$$

The second equilibrium condition (12) is equal to:

$$\int_0^{x_f} \frac{h\left[S\left(x,u,\theta\right)\right]}{q(x,u)}dx = N$$

which is given by

$$\int_0^{x_f} \left[\exp 2(y + TDR/N - t.x - 1 - u)\right]dx = \frac{(1+\theta)\,rN}{2}$$

After some manipulations, this equation can be written as:

$$\begin{aligned} & \exp\left[2\left(y - 1 - u + \frac{1}{N}\int_0^{x_f} R(x,u,\theta)dx - x_f\frac{R_A}{N}\right)\right] \qquad (26) \\ = \; & \exp\left[2\left(y - 1 - u + \frac{1}{N}\int_0^{x_f} R(x,u,\theta)dx - \left(\frac{R_A}{N} + t\right)x_f\right)\right] + (1+\theta)\,rtN \end{aligned}$$

Proposition 3 *Assume that the city is closed and the landlords are the city residents (the full closed city case). Then, if the utility function is quasi-linear and defined as in (14) and the production function* $h(S)$ *is Cobb-Douglas as in (20), we have:*

$$\frac{\partial x_f}{\partial \theta} < 0 \quad , \quad \frac{\partial u}{\partial \theta} < 0$$

Proof. See the Appendix.

In the fully closed city model, increasing property taxes does reduce the city size and thus urban sprawl. This shows that this last result is robust, even when there are general equilibrium effects since, contrary to the absentee landlords' case, here the rent generated by the land appears as income for the urban residents. Indeed, because urban resident are here landowners and because the latter bear a significant portion of the property tax burden, the *building height effect* mentioned earlier is stronger and thus the reduction in dwelling size outweights the decrease in building height. The net effect of increasing a property tax thus unambiguously reduces the size of the city. We thus believe that, as soon as q and S are endogenous (and they should be), this negative effect of θ on urban sprawl is quite strong. It is obtained in the context of a quasi-linear utility function (Proposition 3) for a fully closed city and in the case of a CES utility function for a closed city when the elasticity of substitution is large enough (Proposition 1).[12] It has to be observed that in the extreme case of Leontief preferences where $\sigma = 0$, and resorting to numerical simulations only, Brueckner and Kim (2003) show that an increase in θ may increase city size. This is not very convincing because, in the real-world, we do believe that households do substitute non-spatial good consumption with housing consumptions. Also no formal theoretical result has been obained. The next section, which deals with the test of this model, will shed light on this issue.

3 Data and empirical analysis

3.1 Developing a national sample of effective tax rates

We would like now to test the main result of propositions 2 and 3, i.e. the fact that increasing property taxes reduces urban sprawl. For that, we choose the "urbanized area" as our unit of analysis. Urbanized areas are defined generally as cities with 50,000 or more inhabitants and their surrounding densely settled urban fringe, whether or not incorporated.[13]

We take the following approach to measure an effective tax rate for each

[12] Arnott and MacKinnon (1977) find the same result.

[13] Urbanized areas differ in concept from metropolitan areas. In general, metropolitan areas are defined as cities with 50,000 or more inhabitants, their counties, and surrounding counties that have a high degree of social and economic integration with the core. Metropolitan areas thus include urban population not contiguous to the core as well as rural population. Thus, as suggested by Brueckner and Kim (2003), the urbanized area corresponds to the requirements of the theory in a better way than other census-defined units.

urbanized area. We first collect effective tax rates imposed by different level of taxing jurisdictions – counties, cities, townships, and school districts. These data are collected from the Department of Taxation from various states, Association of County Commissioners, and local governmental units. Many states conduct the tax rate survey to collect effective tax rates from various localities and have made effective tax rates available at websites.[14]

One of main purposes of collecting tax rates by the state is to offer a common standard for the comparison of tax rates among taxing jurisdictions. Therefore those rates are comparable across areas. Usually, the effective tax rates are obtained by adjusting the nominal tax rate with the sales/assessment ratio, which is estimated and determined by the state agencies. For those states without available information from websites, we obtained data on tax rates by contacting local government units to obtain data on effective rates imposed by the counties, cities, townships, and school districts. We do not collect effective tax rates from special districts such as fire, water, sewer, etc. as those tax rates are generally not reported by the state agencies. Since special districts are formed to provide services to the inhabitants of a limited area, we argue that the omission of including tax rates from special districts would not have a significant impact on the result of this study.

We then construct the aggregated effective tax rate for an urbanized area. Specifically, we employ spatial analysis techniques by using GIS. We first obtain data on GIS boundaries of various taxing jurisdictions such as counties, cities, townships, and school districts.[15] Using our collected effective tax rates from these various taxing localities, we then create a weighted average of tax rate for the urbanized area by combining input data from various jurisdictions based on spatial correspondence and association between these layers. For illustration of our approach, Figure 1 presents various boundaries of taxing districts for a hypothetical urbanized area, where it is assumed for simplicity that this urbanized area can be divided into five parts: $P1$ with effective tax rates ($T1$) from county 1 and school district 1; $P2$ with effective tax rates ($T2$) from county 1, city 1 and school district 2; $P3$ with effective tax rates ($T3$) from county 2 and school district 3; $P4$ with effective tax rates ($T4$) from county 2, city 2 and school district 4; and $P5$ with effective tax rates ($T5$) from county 2 and school district 4. Then the weighted average of the effective tax rate for

[14]Examples of these websites include:

North Carolina: http://www.ncacc.org/taxrate.htm;

Illinois: http://www.revenue.state.il.us/Publications/LocalGovernment/00PTAX50.pdf;

New York: http://urban.nyu.edu/research/etr/etr-nyc-1999.pdf.

[15]These data are available from U.S. Census, or can be purchased from GeoCommunity (a GIS data depot).

the urbanized area can be calculated as:

$$\frac{Area_{P1}}{Area_{UA}} \times T1 + \frac{Area_{P2}}{Area_{UA}} \times T2 + \frac{Area_{P3}}{Area_{UA}} \times T3 + \frac{Area_{P4}}{Area_{UA}} \times T4 + \frac{Area_{P5}}{Area_{UA}} \times T5$$

In reality it is more complicated than this scenario since a county or a city can both have multiple school districts or a same school district might belong to different cities. Using GIS overlaying techniques allows us to cope with these complexities.

$[Insert\ Figure\ 1\ here]$

We excluded urbanized areas with a population size larger than five million. Our final sample includes 448 observations. The distribution of effective tax rates by urbanized areas is shown in Figure 2.

$[Insert\ Figure\ 2\ here]$

3.2 Empirical strategy and data

An empirical test based on the above theoretical analysis is extremely useful to facilitate the debate on the relationship with property tax and urban development. The analytical framework is presented graphically in Figure 2. The figure enumerates various interplaying factors in a regional land market that affect city size and urban density.

The figure includes, on its left side, a number of exogenous variables such as population, income, agricultural rent and commuting cost that affect spatial growth of cities. Our theoretical model of section 2, provides a clear explanation of the spatial growth of cities. Given the confluence of an expanding population, rising incomes, and falling commuting costs, it is not surprising that most U.S. cities have expanded rapidly in recent decades. Brueckner and Fansler (1983)'s study tested the validity of this set of exogenous variables.

$[Insert\ Figure\ 3\ here]$

The figure then includes property tax, the main interest of this study. We show that households (who are also landlords) and developers respond to various influences identified in the framework via the regional land market, and this in turn determines spatial city size.

We then perform a regression analysis to examine the effect of property tax on spatial sizes of cities. This analysis allows us to isolate the effects of property tax on city size while controlling for other factors. Specifically, dependent and independent variables and associated measurements are summarized in Table 1. Summary statistics of these variables are presented in Table 2.

[*Insert Tables* 1 and 2 *here*]

The dependent variable, the size of urbanized areas, is derived from retrieving data from the U.S. Census and is measured by the spatial size of the urbanized area in acres in 2000.

The independent variables are derived from retrieving data from the U.S. Census, survey, and secondary data sources and the list includes the following:

- Population: Population is measured by 2000 urbanized area population;
- Income: Income is represented by a measure of average household income, standardized by housing costs across urbanized areas in the U.S.
- Agricultural Rent: Agricultural rent is measured by the 1999 median agricultural land value per acre for the county containing the urbanized area;
- Commuting Cost:[16] Transportation expenditure by government per capita in the urbanized area is used. Higher the expenditure indicates lower commuting costs.
- Property Tax: As mentioned above, we employ overlay techniques in GIS and create a weighted average tax rate for each urbanized area in 1997.

The challenge in estimating a causal impact of property tax on city size is to overcome simultaneity bias. As shown by the theoretical model, high property tax might lead to two countervailing effects, which, *in fine*, will reduce the size of cities. On the other hand, inefficiently expanded cities might increase property tax rates to raise local revenues to provide infrastructures. To address this endogeneity problem, we perform a Hausman endogeneity test. We find that the differences between the IV estimates and OLS estimates are large enough to suggest that the OLS estimates are inconsistent. We then test to see if the reason for the inconsistent estimates is due to the endogeneity of property tax rate. We found that the Hausman statistics is 62.52 (chi-square) and is significant at the 0.000 level. The small $p-$value indicates that there is a significant difference between the IV and OLS coefficients, and the OLS is not consistent. We therefore adopt an instrumental variables approach in

[16] Brueckner and Fansler (1983) used two proxies for commuting cost: percentage of commuters using transit and percentage of households owning one or more automobiles. However Brueckner and Fansler pointed out these two proxies performed poorly in their model examining the economics of urban sprawl. We explored percentage of commuters using transit, percentage of households owning one or more automobiles, and road density (measured by street miles per square mile) as proxies for commuting cost. Even though these proxies are not significant in the regression, our main result (i.e. the effect of property tax on city size) is not altered when using these proxies.

which an instrument is used to predict the property tax θ_i, which is treated as an endogenous variable.

To be more precise, to identify the impact of property tax on city size using *instrumental variables*, we need an instrument that predicts changes in property tax rates, but is unrelated to changes in city size (after controlling for other relevant factors). An appropriate instrument for the property tax rate is the *magnitude of state aid to schools*. In Illinois, for example, state aid to schools is low compared to most other states, which means that property taxes are relatively high in Illinois. Data on state aid to schools are available from the National Center for Education Statistics (NCES). In this context, the impact of property tax on city size is estimated using two-stage least squares (2SLS), treating the property tax variable as endogenous and the other right-hand-side variables as exogenous. The first stage of the 2SLS regression indicates that the instrument chosen is appropriate since it shows that the relation between property tax and the magnitude of state aid to local governments is significant. In the second stage, we insert the predicted values into the city-size equation. In particular, we estimate the following:

$$\theta_{i,t} = \alpha_i X_{i,t+3} + \delta Z_{i,t+3} + \eta_i$$

$$UA_{i,t+3} = \beta_i X_{i,t+3} + \gamma \theta_{i,t} + \varepsilon_i$$

where i indexes the relevant spatial unit (the urbanized area for example) and $t = 1997$, $UA_{i,t+3}$ is the size of the spatial unit i at time $t+3 = 2000$, $X_{i,t+3}$ is a vector of control variables in unit i at time $t+3 = 2000$, $Z_{i,t+3}$ is the appropriate instrument in unit i at time $t+3 = 2000$, and $\theta_{i,t}$ denotes the property tax in i at time $t = 1997$. The error terms ε_i and η_i are normally distributed. The instrument $Z_{i,t+3}$ is correlated with $\theta_{i,t}$ and is uncorrelated with ε_i. Observe that we lag the property tax θ by three years because the effect of θ on the size of an urbanized areas is obviously not instantaneous but takes some time.

3.3 Empirical results and discussion

According to our theoretical model (section 2), the key relationship is between property tax and urban sprawl. As stated above, we run a 2SLS regression. The first stage of the IV procedure amounts to regressing θ_i, the property tax in area i, on Z_i, the magnitude of state aid to schools in area i (our instrument). The results of this first stage regression suggest that our instrument Z_i has a strong predictive power since it enters the equation with a coefficient of -0.30 and a $t-$ratio of 2.738. The negative sign was expected since more state aid to schools in an area implies quite naturally lower poperty taxes in this area. Let us now focus on the second stage. Regression results using OLS and 2SLS with instruments (IV) are respectively presented in columns two and three of Table 3. When the regression is performed without instruments (OLS), so that the simultaeous bias between these two variables is not taking care of, the effect of the property tax on city size is not significant, though negative. When the regression is implemented using the "magnitude of state aid to schools" as an instrument for the property tax rate, we find, on the contrary that an increase in property tax does reduce the city size in the United States. In terms of magnitude, a 10 percent increase in property tax reduces on average the city size and thus the urban sprawl by 730 acres.[17]

[*Insert Table* 3 *here*]

This has important policy implications for the United States. In particular, if urban sprawl is considered to be "harmful" for the welfare of the society, then local governments should increase the property tax. Of course, one has to be extremely precise and careful in the definition of welfare. In order to address this issue, two extensions of our theoretical framework can be considered. First, one has to define in a precise way what is the welfare of the society. The most natural way is to take the weighted sum of all agents in the city, here the workers (who are also landlords) and the firms. Then one can calculate the exact loss of welfare when the city expands. Second, and more importantly, one can determine the *optimal* city property tax. In our current model, the property tax rate is exogenous. We thus need to add a new actor, the city-planner, who will determine the optimal property tax that maximizes the welfare of the society under a city budget constraint. Because the city-planner internalizes the externalities of urban sprawl, this model will be also able to determine the optimal size of the city or equivalently the optimal "sprawl" of a city. A direct consequence of this analysis is that different cities should have different property tax rates and thus different optimal urban sizes. This will enable us to compare the optimal tax given by the model and the one observed in the real world for each urbanized area and thus say which city has too low property tax and thus excessive urban expansion. This is important and we leave it for future research.

[17]For the other variables, in both regressions, we show quite naturally that larger population size, larger income and lower commuting costs (proxied by larger transportation investment) are associated with larger urban sizes.

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APPENDIX

Proof of Lemma 1

The equation that defines $R(x,u,\theta)$ can be written as:

$$\frac{1}{2}\log R(x,u,\theta)-\frac{1}{N}\int_0^{x_f} R(x,u,\theta)dx-y+1+u+x_f\frac{R_A}{N}+t.x+\log\left(1+\theta\right)+\log\sqrt{r}=0 \tag{27}$$

By totally differentiating this equation, we obtain

$$\frac{\partial R(x,u,\theta)}{\partial x}=\frac{t}{\frac{x_f}{N}-\frac{1}{2R}}=\frac{2NRt}{2Rx_f-N}$$

$$\frac{\partial R(x,u,\theta)}{\partial u}=\frac{1}{\frac{x_f}{N}-\frac{1}{2R}}=\frac{2NR}{2Rx_f-N}$$

$$\frac{\partial R(x,u,\theta)}{\partial \theta}=\frac{\frac{1}{(1+\theta)}}{\frac{x_f}{N}-\frac{1}{2R}}=\frac{2NR}{\left(1+\theta\right)\left(2Rx_f-N\right)}$$

∎

Proof of Proposition 3

We can write the two equilibrium conditions (25) and (26) as the following system:

$$\begin{cases} F(x_f,u,\theta)=0 \\ G(x_f,u,\theta)=0 \end{cases}$$

where

$$F(x_f,u,\theta)\equiv\left(t+\frac{R_A}{N}\right)x_f-\frac{1}{N}\int_0^{x_f} R(x,u,\theta)dx+u-y+1+\log\left(1+\theta\right)+\log\sqrt{rR_A}$$

$$\begin{aligned} G(x_f,u,\theta) \equiv\ & \exp\left[2\left(y-1-u+\frac{1}{N}\int_0^{x_f} R(x,u,\theta)dx-x_f\frac{R_A}{N}\right)\right] \\ & -\exp\left[2\left(y-1-u+\frac{1}{N}\int_0^{x_f} R(x,u,\theta)dx-\left(\frac{R_A}{N}+t\right)x_f\right)\right] \\ & -\left(1+\theta\right)rtN \end{aligned}$$

Differentiating these equations yields (using Lemma 1)

$$\begin{aligned} F_{x_f} &= \left(t+\frac{R_A}{N}\right)-\frac{R_A}{N}=t>0 \\ F_u &= 1-\frac{1}{N}\int_0^{x_f}\frac{\partial R(x,u,\theta)}{\partial u}dx \\ &= 1-\frac{2Rx_f}{2Rx_f-N}<0 \\ F_\theta &= \frac{1}{1+\theta}-\frac{1}{N}\int_0^{x_f}\frac{\partial R(x,u,\theta)}{\partial \theta}dx \\ &= \frac{1}{1+\theta}\left[1-\frac{2Rx_f}{2Rx_f-N}\right]<0 \end{aligned} \tag{28}$$

$$
\begin{aligned}
G_{x_f} &= 2\left(\frac{R_A}{N}-\frac{R_A}{N}\right)\exp\left[2\left(y-1-u+\frac{1}{N}\int_0^{x_f}R(x,u,\theta)dx-x_f\frac{R_A}{N}\right)\right] \\
&\quad -2\left(\frac{R_A}{N}-\frac{R_A}{N}-t\right)\exp\left[2\left(y-1-u+\frac{1}{N}\int_0^{x_f}R(x,u,\theta)dx-\left(\frac{R_A}{N}+t\right)x_f\right)\right] \\
&= 2\,t\exp\left[2\left(y-1-u+\frac{1}{N}\int_0^{x_f}R(x,u,\theta)dx-\left(\frac{R_A}{N}+t\right)x_f\right)\right]>0 \\
G_u &= 2\left(\frac{1}{N}\int_0^{x_f}\frac{\partial R(x,u,\theta)}{\partial u}dx-1\right)(1+\theta)\,rtN \qquad (29) \\
&= 2\left(\frac{2Rx_f}{2Rx_f-N}-1\right)(1+\theta)\,rtN>0 \\
G_\theta &= 2\left(\frac{1}{N}\int_0^{x_f}\frac{\partial R(x,u,\theta)}{\partial \theta}dx\right)(1+\theta)\,rtN-rtN \\
&= rtN\left(\frac{4Rx_f}{2Rx_f-N}-1\right)>0
\end{aligned}
$$

In a matrix form we have

$$
\begin{pmatrix} F_{x_f} & F_u \\ G_{x_f} & G_u \end{pmatrix}\begin{pmatrix} \partial x_f/\partial\theta \\ \partial u/\partial\theta \end{pmatrix}=\begin{pmatrix} -F_\theta \\ -G_\theta \end{pmatrix}
$$

By the Cramer's rule, we obtain

$$
\frac{\partial x_f}{\partial \theta}=\frac{\begin{vmatrix} -F_\theta & F_u \\ -G_\theta & G_u \end{vmatrix}}{|A|}=\frac{-G_uF_\theta+G_\theta F_u}{|A|}<0 \qquad (30)
$$

$$
\frac{\partial u}{\partial \theta}=\frac{\begin{vmatrix} F_{x_f} & -F_\theta \\ G_{x_f} & -G_\theta \end{vmatrix}}{|A|}=\frac{-G_\theta F_{x_f}+G_{x_f}F_\theta}{|A|}<0 \qquad (31)
$$

where $|A|=\begin{vmatrix} F_{x_f} & F_u \\ G_{x_f} & G_u \end{vmatrix}=G_uF_{x_f}-G_{x_f}F_u>0.$

To show the first result (30), observe that:

$$
sign\frac{\partial x_f}{\partial \theta}=sign\left[-G_uF_\theta+G_\theta F_u\right]
$$

Using (28) and (29), we have

$$
-G_uF_\theta+G_\theta F_u
$$

$$
\begin{aligned}
&= 2\left(\frac{2Rx_f}{2Rx_f-N}-1\right)(1+\theta)\,rtN\frac{1}{1+\theta}\left(\frac{2Rx_f}{2Rx_f-N}-1\right) \\
&\quad -rtN\left(\frac{4Rx_f}{2Rx_f-N}-1\right)\left(\frac{2Rx_f}{2Rx_f-N}-1\right) \\
&= -rtN\left(\frac{2Rx_f}{2Rx_f-N}-1\right)<0
\end{aligned}
$$

To show the second result (31), it suffices to use (28) and (29). ∎

Figure 1. Urbanized Area, County, City, and School District Boundaries

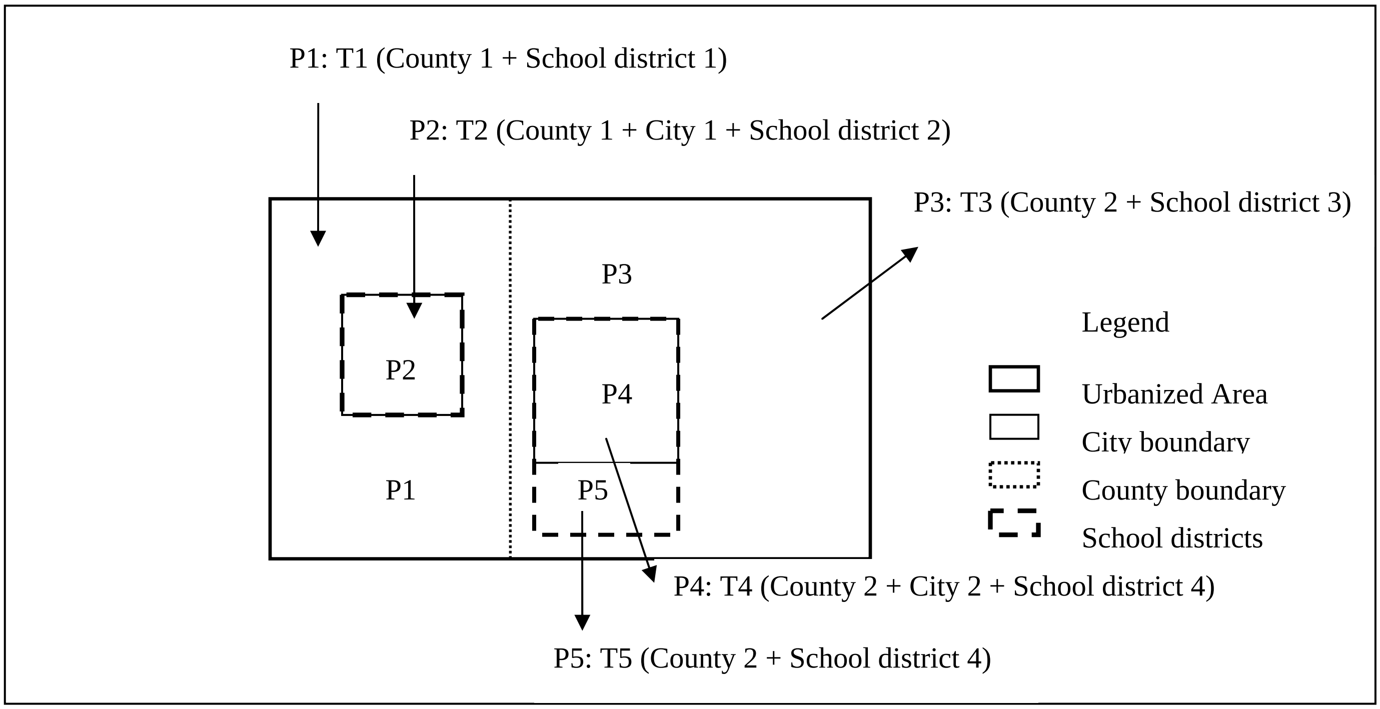

Figure 2. Effective Tax Rates by Urbanized Area

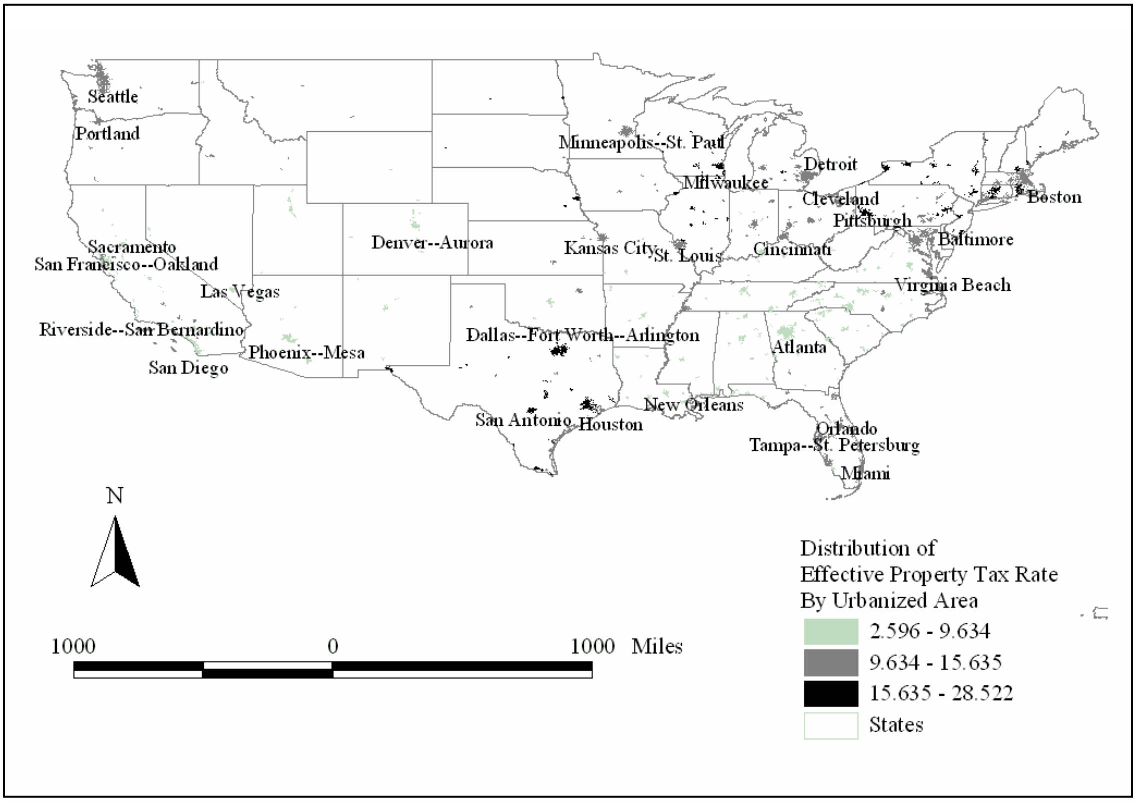

Figure 3. Analytical Framework for Evaluating Spatial Sizes of Cities

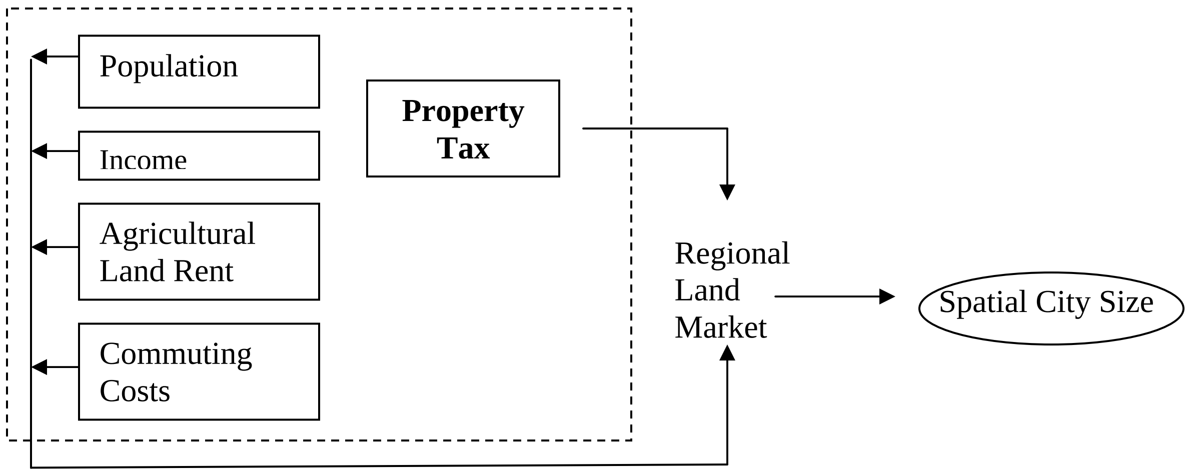

Table 1. Dependent and Independent Variables and Measurements

Variables	Measurements (Data Source)
Dependent Variable	
Urbanized Area	Spatial size of the urbanized area in acres in 2000 (U.S. Census).
Independent Variables	
Population	Urbanized area population in 2000 (U.S. Census).
Income	Average household income in USD in 2000 standardized by housing costs in 2000 (U.S. Census).
Agricultural Land Rent	The median agricultural land value per acre in 1999 for the county containing the urbanized area (U.S. Census).
Transportation expenditure	Transportation expenditure by government in USD per capita in 2000 (U.S. Census)
Property Tax	A weighted average property tax rate for each urbanized area (U.S. Census, Web survey, Secondary Data sources and GIS operation) calculated in 1997.

Table 2. Descriptive Statistics of the Variables

	Minimum	Maximum	Mean	Std. Deviation
Urbanized Area	7,742	1,256,051	90,112	141,797
Population	49,776	4,918,839	333,239	635,474
Income (Standardized by housing costs)	20,633	79,614	40,466	9,409
Agricultural Land Rent	0	224,006	1,418	10,954
Transportation expenditure	0	45,4177	12,274	38,703
Property Tax	2.60	28.52	11.35	5.03

Sample size: 448

Table 3. Regression Results

Endogenous variable: Spatial size of the urbanized area in 2000

	OLS	IV
Constant	37445.621***	38129.737***
	(3.72)	(3.78)
Population	0.160***	0.166***
	(25.80)	(25.12)
Income	0.282***	0.328***
	(4.56)	(4.62)
Agricultural Land Rent	0.985	1.044
	(1.24)	(1.46)
Transportation expenditure	0.544***	0.754***
	(5.56)	(7.94)
Property Tax	-150.214	-73.024**
	(0.372)	(1.94)

Notes: Absolute values of robust t-statistics are in parentheses.
* significant at 10% level
** significant at 5% level
***significant at 1% level.