Working Paper

A New Ridge Regression Causality Test in the Presence of Multicollinearity

by

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Abstract

This paper analyzes and compares the properties of the most commonly applied versions of the Granger causality (GC) test to a new ridge regression GC test (RRGC), in the presence of multicollinearity. The investigation has been carried out using Monte Carlo simulations. A large number of models have been investigated where the number of observations, strength of collinearity, and data generating processes have been varied. For each model we have performed 10000 replications and studied seven different versions of the test. The main conclusion from our study is that the traditional OLS version of the GC test over-rejects the true null hypothesis when there are relatively high (but empirically common levels of) multicollinearity, while it is established that the new RRGC test will remedy or substantially decrease this problem.

Keywords: Granger causality test, multicollinearity, ridge parameters, size and power.

JEL Classification: C32

1.   Introduction

The purpose of this paper is to evaluate the effect of multicollinearity on the most commonly applied tests for causality in the sense of Granger (1969). The dynamic nature of the Granger causality (GC) test implies that it, by pure definition, generally suffers from considerably high degrees of multicollinearity problems, primarily induced by its extensive lag structure. By means of Monte-Carlo simulations, it is demonstrated that multicollinearity causes over-rejections of the true null hypotheses for the traditional GC tests. As a remedy to this problem, a new ridge regression Granger causality (RRGC) test is proposed where ridge regression is used instead of ordinary least squares (OLS) to estimate the parameters in the dynamic regression model. In comparison to the traditional versions of the GC test, our newly proposed RRGC test exhibits superior size properties, which therefore may be considered as the main original contribution of this paper.

The concept of multicollinearity was first introduced by Frisch (1934) in order to denote a situation where the independent variables in the regression model are correlated. Despite the fact that high levels of multicollinearity is a very common problem when estimating dynamic models, no one (at least to the author's knowledge) has yet studied the effects of multicollinearity on the GC test. The main problem associated to multicollinearity is that it leads to instability and large variance of the OLS estimator. This may induce two different effects on the GC test which is also illustrated in the simulation section of this paper. Firstly, it might lead to a slower convergence rate of the tests based on asymptotic results since larger samples are required to obtain stable OLS estimates of the parameters. Secondly, it may cause over-rejections of the true null hypotheses in small and moderately sized samples regardless whether the tests are based on asymptotic distribution or not. Hence, if we apply the traditional GC tests in the presence of multicollinearity we need to obtain very large sample sizes, which often is not available in many areas of economics.

The method of ridge regression first introduced by Hoerl and Kennard, (1970a,b) is nowadays established as an effective and efficient remedial method to deal with the general problems caused by multicollinearity. The main advantage of the ridge regression method is to reduce the variance term of the slope parameters which is demonstrated in some recent papers (see Kibria, 2003; Khalaf and Shukur, 2005; Alkhamisi and Shukur, 2007 and Muniz and Kibria 2009). In view of the fact that the simulation results in this paper identified that

multicollinearity causes severe problems for the traditional GC tests (for empirically relevant sample sizes) a new RRGC test is proposed. This method reduces the parameter instability and the new versions of the test exhibit superior statistical size properties in comparison to the commonly applied GC tests.

The paper is organized as follows: In section 2, we describe the GC test and define the generalized ridge regression estimator. Subsequently, in section 3, the Monte Carlo design is formalized, while in Section 4 we analyze the results obtained from the simulation study. Finally, in Section 5 the conclusions of the paper are summarized.

2.   Methodology

This section describes the testing and estimation methodology.

2.1 Granger causality test

The central idea that is exploited by the GC test is the simple fact that events in the past can cause events to happen today while future events cannot, thus, we utilize the fundamental truth that cause must precedes effect. The GC test for two variables y_t and x_t can be defined as follows. x_t does not Granger cause y_t, if and only if, prediction of y_t based on the universe U of predictors is no better than prediction based on U−$\{x_t\}$, i.e. on the universe with x_t omitted. According to Granger and Newbold (1986) one can test for Granger causality by evaluating a zero restriction in each of the single linear equations in the VAR-model. This basic method is a very common method of testing for Granger causality in empirical works (see e.g. Almasri and Shukur, (2003); and Ramsey and Lampart, 1998) and can be explained by considering the following linear regression model:

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u}, \tag{1}$$

where $\mathbf{y}$ is a $T \times 1$ vector of observations, $\mathbf{X}$ is a $T \times (2p+1)$ matrix of observations of the independent variables, $\boldsymbol{\beta}$ is a $(2p+1) \times 1$ vector of coefficients, p is the number of the lagged variables in the VAR(p) model and $\mathbf{u}$ is a $T \times 1$ vector of residuals. The coefficient vector in expression (1) can be estimated using ordinary least squares (OLS):

$$\hat{\boldsymbol{\beta}} = (\mathbf{X'X})^{-1} \mathbf{X'y} . \tag{2}$$

In order to test for Granger causality the following linear restrictions should be tested:

$$H_0 : \mathbf{R\beta} = \mathbf{r} \quad \text{vs.} \quad H_1 : \mathbf{R\beta} \neq \mathbf{r}. \tag{3}$$

where $\mathbf{R}$ is a fixed $q \times (2p+1)$ matrix and $\mathbf{r}$ is a fixed $q \times 1$ vector of restrictions. To test the restrictions of expression (3) the following Wald (W), Likelihood Ratio (LR), Lagrange Multiplier (LM) and the F-test will be used:

$$W = \frac{T \left(\mathbf{R\hat{\beta}} - \mathbf{r}\right)' \left(\mathbf{R X'X R'}\right)^{-1} \left(\mathbf{R\hat{\beta}} - \mathbf{r}\right)}{s_u} \tag{4}$$

$$LR = T \left(\log\left(\frac{W}{T} + 1 \right) \right) \tag{5}$$

$$LM = \frac{T \left(\mathbf{R\hat{\beta}} - \mathbf{r}\right)' \left(\mathbf{R X'X R'}\right)^{-1} \left(\mathbf{R\hat{\beta}} - \mathbf{r}\right)}{s_r} \tag{6}$$

$$F = \frac{\Delta \left(\mathbf{R\hat{\beta}} - \mathbf{r}\right)' \left(\mathbf{R X'X R'}\right)^{-1} \left(\mathbf{R\hat{\beta}} - \mathbf{r}\right)}{q s_u} \tag{7}$$

where $s_u = \hat{\mathbf{u}}_u' \hat{\mathbf{u}}_u$ and $s_r = \hat{\mathbf{u}}_r' \hat{\mathbf{u}}_r$ are the matrices of cross-products of residuals from the *unrestricted* regression and *restricted* regression (when H_0 is imposed), respectively. The first three tests are all asymptotically $\chi^2(q)$ distributed while the fourth test is distributed as an $F(q, \Delta)$, where $\Delta = T - 2p - 1$. Moreover, a small sample correction of the W, LR and LM (WC, LRC and LMC) tests is made to the first three tests where T is replaced by Δ.

2.2 Ridge regression

The effect of multicollinearity between the explanatory variables is that the matrix of cross-products $\mathbf{X'X}$ is ill-conditioned which leads to instability and large variance of the OLS estimates. If this instability is not reflected by an increase in the covariance matrix then the traditional GC tests is biased. As a substitute and a remedy to the multicollinearity problems induced by the OLS estimator, Hoerl and Kennard (1970a,b) proposed the following ridge regression estimator.

$$\hat{\mathbf{\beta}} = \left(\mathbf{X'X} + k\mathbf{I}\right)^{-1} \mathbf{X'y} , \tag{8}$$

where ($k \geq 0$) is the so called ridge parameter. In order to estimate k, Hoerl and Kennard (1970a) suggested the following expression:

$$\hat{k}_{HK} = \frac{S^2}{\hat{\alpha}^2_{max}},$$

where $S^2 = \overline{\mathbf{y} - \mathbf{X}\hat{\boldsymbol{\beta}}}{}' \overline{\mathbf{y} - \mathbf{X}\hat{\boldsymbol{\beta}}} \big/ n - 2p - 1$ and $\hat{\alpha}^2_{max}$ is defined as the maximum element of $\boldsymbol{\gamma}\hat{\boldsymbol{\beta}}$ where $\boldsymbol{\gamma}$ is the eigenvector of $\mathbf{X'X}$. However, in Alkhamisi and Shukur (2007) it is illustrated that there are many other superior ways of estimating k. The authors found that the following two ridge estimators work particularly well:

$$\hat{k}_{ARITHM} = \frac{1}{p} \sum_{i=1}^{p} \left(\frac{S^2}{\hat{\alpha}_i^2} + \frac{1}{t_i} \right) \text{ and } \hat{k}_{NAS} = \max\left(\frac{S^2}{\hat{\alpha}_i^2} + \frac{1}{t_i} \right),$$

where $\hat{\alpha}^2_{max}$ is defined as the i*th* element of $\boldsymbol{\gamma}\hat{\boldsymbol{\beta}}$. Other alternative potentially successful ridge regression estimators are proposed by Kibria and Muniz (2009):

$$\hat{k}_{KM4} = \left(\prod_{i=1}^{p} \frac{1}{\sqrt{\frac{s^2}{\hat{\alpha}^2_{max}}}} \right)^{\frac{1}{p}} , \ \hat{k}_{KM5} = \left(\prod_{i=1}^{p} \sqrt{\frac{s^2}{\hat{\alpha}^2_{max}}} \right)^{\frac{1}{p}} \text{ and } \ \hat{k}_{KM6} = median\left(\frac{1}{\sqrt{\frac{s^2}{\hat{\alpha}^2_{max}}}} \right).$$

Now, the new RRGC test will be applied using the RR estimators instead of the OLS estimator of $\boldsymbol{\beta}$.

3.   The Monte-Carlo simulation

3.1 The design of the experiment for size calculations

The data used for the Monte Carlo simulation experiment are replicated according to the following data generating processes when the lag length equals two:

$$\begin{cases} y_t = 0.03 + 0.1 y_{t-1} + 0.08 y_{t-2} + + \beta_1 x_{t-1} + \cdots + \beta_p x_{t-p} + \varepsilon_{1t} \\ x_t = 0.02 + \lambda x_{t-1} + \varepsilon_{2t} \end{cases}$$

and the following when the lag length equals four:

$$\begin{cases} y_t = 0.03 + 0.1 y_{t-1} + 0.08 y_{t-2} + 0.06 y_{t-3} + 0.04 y_{t-4} + \beta_1 x_{t-1} + \cdots + \beta_p x_{t-p} + \varepsilon_{1t} \\ x_t = 0.02 + \lambda x_{t-1} + \varepsilon_{2t} \end{cases}.$$

The focus of this paper is to study the effect of the degree of multicollinearity between lags of the x variables of the GC test. As a first step, in order to evaluate whether the degree of multicollinearity has a direct impact on the statistical size of the GC test, and to test whether ridge regression is a remedy to this potential problem, we use the following DGPs:

DGP 1: $\lambda = 0$ DGP 2: $\lambda = 0.8$

DGP 3: $\lambda = 0.95$ DGP 4: $\lambda = 0.99$

It should be stressed that the parameter values are empirically very likely cases in real-world economics and they are encountered in many studies (e.g. Almasri and Shukur (2003) and Hacker et al. (2010)). Another factor that may have an impact on the GC test is the distribution of the error term. In previous research, this is illustrated by for instance Kibria (2003) and Alkhamisi and Shukur (2007) who demonstrated that increase in the variance of a normally distributed error term will enlarge the problem of multicollinearity. The sample size is another relevant factor that is expected to affect the performance of the GC test since the Wald, LR and LM tests are based on an asymptotic distribution that often leads to poor properties in empirically relevant sample sizes. Another important factor in this context is the lag-length specification. It can be expected that estimating more parameters leads to a higher probability of rejecting a true null hypothesis. To demonstrate the effects of increasing the lag lengths we vary the degrees of freedom (net observations after each regression) instead of the numbers of observations since it is well-known that it is the degrees of freedom and not the absolute sample size that matters on the performance of the tests. In Table 1, the fixed and varying factors that constitute the actual Monte Carlo experiment are summarized.

Table 1. Values of factors in the experiment

Factor	Symbol	Design
Number of replicates	N	10 000
Degrees of freedom	df	15, 25, 50, 100
Nominal size	π_0	5%
Lag length	p	2, 4
The distribution of the error term	ε	$N(0,1)$, $N(0,10)$

The size of the Granger causality test is examined by observing the rejection frequency when x does not Granger cause y. Therefore, the β parameters of the linear regression models are set to zero when the statistical sizes of the tests are evaluated. In order to evaluate the empirical statistical size of the tests the following confidence interval is calculated:

$$\pi_0 \pm 2\sqrt{\frac{\pi_0(1-\pi_0)}{N}} . \tag{9}$$

If, based on our simulation experiment, the actual statistical size is within the bounds of this interval the evaluated test is considered as unbiased (at a specified significance level). Throughout this paper we consistently defines biasedness at the 5% level of significance.

3.2 The design of the experiment to calculate the power

When the power is calculated the β parameters in the linear regression models should not equal zero since the time series x_t should actually Granger cause y_t. The number of replicates when calculating the power of the tests equals 1,000 and the chosen parameter values of β are defined in following Table 2:

Table 2: Values of parameter combinations for the power calculation

	β_1	β_2	β_3	β_4
p = 2				
1. very weak causality	0.1	0.05	-	-
2. weak causality	0.2	0.1	-	-
3. strong causality	0.3	0.15	-	-
p = 4				
1. very weak causality	0.1	0.05	0.025	0.025
2. weak causality	0.15	0.1	0.05	0.025
3. strong causality	0.25	0.15	0.075	0.05

4.   Results

In this section the results from the Monte Carlo experiment are presented. All the factors that are varied in the design of the Monte Carlo simulation are expected to have an impact on the performance of the tests. We will especially focus on discussing whether ridge regression can serve as a small-sample correction of the tests based on asymptotic results and to determine whether the new RRGC test is robust to multicollinearity.

The simulation study indicates that applying the RRGC test using the $\hat{k}_{ARITHM}$, $\hat{k}_{NAS}$ and $\hat{k}_{KM5}$ as ridge estimator leads to an immense underestimation of the nominal size. Since it is of no use to present several tables consisting of almost only zeros the result from the statistical size calculation from these estimators are excluded from this paper. Furthermore, none of the traditionally applied GC tests and most of the tests when using ridge regression performs well when the data are collinear. The results from these tests are therefore only presented when analyzing the statistical size of the tests. When we calculate the tests's statistical power, only the F-test when using $\hat{k}_{KM6}$ will be presented since the other tests have extensively biased sizes. Finally there is no effect on the statistical size when the variance of the normal distribution is increased. Therefore, we only present the size when the error term follows a standard normal distribution. However, full results are available from the authors upon request.

4.1   Analysis of the statistical size of the Granger causality test

This section presents the actual sizes of the different Granger causality tests for the different DGPs. The actual sizes of the tests are presented in tables 3-6. The confidence interval in equation (9) is doubled in magnitude in order to emphasize the pattern of well-performing tests more clearly. Therefore, if the actual size of a test exhibits a rejection frequency between 0.0413 and 0.0587 it is considered as unbiased, which is marked out as shaded cells in the following tables.

The multicollinearity effect

The effect of increasing the degree of multicollinearity in the linear regression model is that the actual size of the tests also increases. For example in Table 3 when using the OLS estimation method then the F-test is has unbiased size in the absence of multicollinearity

(DGP 1). However, for the other DGPs the F-test tends to over-reject the null hypotheses. The other tests that are based on asymptotic distributions are often biased even for DGP 1 and this bias increases by the degree of multicollinearity. This increase in bias leads to a slower convergence rate towards the nominal size. For example, the LM test is unbiased for DGP 1 when the sample size equals 50 but when we include multicollinearity in the model the test is not unbiased even when the degrees of freedom increase to 100. Thus, when the data is collinear we need to have very large sample sizes in order to obtain unbiased test statistics if we want to use the OLS to estimate the model. This is true not only for the tests based on asymptotic distributions but also for the F-test. On the other hand, when ridge regression method is applied the effects of increasing the multicollinearity decreases, especially for $\hat{k}_{KM4}$ and $\hat{k}_{KM6}$. For these estimators the bias of the tests based on asymptotic distributions actually decreases as the degree of multicollinearity increases. However, these tests are still severely biased and should, therefore, not be used. Instead, when the explanatory variables are highly correlated we recommend the F-test based on $\hat{k}_{KM6}$ as ridge estimator to test for the Granger causality. For DGP 2, DGP3, and DGP 4 this test is almost always unbiased.

The lag-length effect

As previously mentioned, instead of considering the sample size, the tests' statistical sizes are evaluated with regards to the degrees of freedom for different models with various lag lengths. In this context, using OLS as estimation method, increasing the lag length does not cause any problems for DGP 1 for the F-test. However, the bias increases for all DGPs for the tests based on asymptotic distributions. This is also the case for the small-sample corrected of W, LR and LM tests. For the W test, the over-rejection increases while for the LR and LM tests the under-rejection of the nominal size increases. In addition to the above effects, there is also an interaction effect between increasing the lag length and the degree of multicollinearity. The problem caused by multicollinearity increases as the lag length increases for all estimation methods.

The degrees of freedom effect

When increasing the degrees of freedom, the actual size becomes substantially closer to the nominal size, which is especially true for the tests based on asymptotic distributions. However, even for DGP 1 when using small sample corrections of the W and LM tests the

actual size is always biased when we have access to less than 50 degrees of freedom. The LRC and LRE are then superior options. However, when x_t is purely random then it is better to use the F-test than the tests based on the asymptotic distribution. For all DGPs when the new RRGC test is used, the bias of the tests based on asymptotic distribution slightly decreases but it is still non-ignorable.

Table 3: OLS

p= 2	W	LR	LM	WC	LRC	LMC	F
DGP 1							
15	0.1481	0.1080	0.0681	0.0807	0.0473	0.0172	0.0464
25	0.1090	0.0874	0.0651	0.0718	0.0501	0.0297	0.0504
50	0.0771	0.0656	0.0556	0.0584	0.0491	0.0394	0.0480
100	0.0632	0.0595	0.0546	0.0560	0.0508	0.0458	0.0487
DGP 2							
15	0.1858	0.1419	0.0952	0.1090	0.0691	0.0255	0.0677
25	0.1265	0.1027	0.0769	0.0840	0.0633	0.0399	0.0656
50	0.0874	0.0767	0.0663	0.0690	0.0586	0.0488	0.0654
100	0.0711	0.0656	0.0609	0.0626	0.0575	0.053	0.0640
DGP 3							
15	0.1988	0.1524	0.0963	0.1134	0.0697	0.0274	0.0697
25	0.1385	0.1117	0.0848	0.0932	0.0706	0.0502	0.0706
50	0.0969	0.0861	0.0755	0.0789	0.0684	0.0555	0.0684
100	0.0743	0.0699	0.0637	0.0656	0.0602	0.0552	0.0602
DGP 4							
15	0.1995	0.1538	0.1020	0.1160	0.0716	0.0281	0.0708
25	0.1365	0.1102	0.0831	0.0912	0.0679	0.0447	0.0694
50	0.0966	0.0831	0.0732	0.0764	0.0659	0.0548	0.0697
100	0.0726	0.0681	0.0633	0.0640	0.0598	0.0555	0.0613
p=4	W	LR	LM	WC	LRC	LMC	F
DGP 1							
15	0.3202	0.2168	0.1011	0.1088	0.0355	0.0002	0.0487
25	0.1977	0.1366	0.0783	0.0833	0.0411	0.0111	0.0492
50	0.1046	0.0818	0.0596	0.0615	0.0442	0.028	0.0471
100	0.0767	0.0667	0.0577	0.0583	0.0484	0.0394	0.0505
DGP 2							
15	0.3733	0.2681	0.1278	0.1404	0.0507	0.0026	0.0655
25	0.2338	0.1654	0.0950	0.1021	0.0536	0.0158	0.0625
50	0.1293	0.1033	0.0765	0.0792	0.0549	0.0346	0.0595
100	0.0891	0.0744	0.0613	0.0628	0.0532	0.0442	0.0551
DGP 3							
15	0.3992	0.2909	0.1467	0.1611	0.0615	0.0072	0.0710
25	0.2528	0.1881	0.1149	0.1220	0.0681	0.0233	0.0779
50	0.1451	0.1174	0.0910	0.0921	0.0679	0.0459	0.0745
100	0.0900	0.0778	0.0667	0.0673	0.0590	0.0477	0.0611
DGP 4							
15	0.3935	0.2861	0.1398	0.1527	0.0521	0.0041	0.0708
25	0.2527	0.1881	0.1175	0.1245	0.0691	0.0194	0.0803
50	0.1378	0.1101	0.0812	0.0838	0.0648	0.0396	0.0701
100	0.0880	0.0758	0.0643	0.0651	0.0573	0.0475	0.0587

Shaded cells indicate unbiased results.

Table 4: Ridge parameter estimated using $\hat{k}_{HK}$

p= 2	W	LR	LM	WC	LRC	LMC	F
DGP 1							
15	0.0954	0.0697	0.0348	0.0513	0.0300	0.0083	0.0464
25	0.0587	0.0450	0.0270	0.0356	0.0252	0.0145	0.0454
50	0.0372	0.0317	0.0248	0.0281	0.0246	0.0177	0.0437
100	0.0263	0.0244	0.0216	0.0233	0.0213	0.0183	0.0437
DGP 2							
15	0.1516	0.1138	0.0664	0.0861	0.0510	0.0170	0.0643
25	0.0969	0.0776	0.0545	0.0633	0.0473	0.0270	0.0651
50	0.0602	0.0517	0.0430	0.0464	0.0406	0.0318	0.0580
100	0.0415	0.0376	0.0333	0.0351	0.0326	0.0287	0.0326
DGP 3							
15	0.1819	0.1397	0.0848	0.1036	0.0599	0.0201	0.0599
25	0.1151	0.0947	0.0695	0.0791	0.0573	0.0344	0.0573
50	0.0790	0.0704	0.0601	0.0647	0.0536	0.0438	0.0536
100	0.0629	0.0588	0.0535	0.0551	0.0506	0.046	0.0506
DGP 4							
15	0.1838	0.1403	0.0896	0.1043	0.0660	0.0232	0.0744
25	0.124	0.0982	0.0728	0.0815	0.0608	0.0383	0.0709
50	0.0821	0.0712	0.0619	0.0648	0.0556	0.0477	0.0622
100	0.0671	0.0622	0.0576	0.0591	0.0543	0.0495	0.0607
p=4	W	LR	LM	WC	LRC	LMC	F
DGP 1							
15	0.2441	0.1657	0.0634	0.0862	0.0281	0.0000	0.0374
25	0.1247	0.0884	0.0406	0.0548	0.0258	0.0046	0.0313
50	0.0554	0.0426	0.0278	0.0329	0.0229	0.0119	0.0252
100	0.0326	0.0277	0.0195	0.0226	0.0184	0.0125	0.0190
DGP 2							
15	0.2809	0.2032	0.0769	0.1013	0.0321	0.0000	0.0441
25	0.1892	0.1360	0.0743	0.0849	0.0420	0.0091	0.0510
50	0.0997	0.0786	0.0535	0.0581	0.0420	0.0249	0.0459
100	0.0595	0.0505	0.0412	0.0434	0.0366	0.0282	0.0380
DGP 3							
15	0.3568	0.2528	0.1190	0.1336	0.0459	0.0002	0.0640
25	0.1982	0.1403	0.0816	0.0904	0.0469	0.0110	0.0548
50	0.1115	0.0873	0.0625	0.0671	0.0487	0.0312	0.0525
100	0.0722	0.0623	0.0521	0.0540	0.0454	0.0376	0.0472
DGP 4							
15	0.3800	0.2748	0.1311	0.1474	0.0473	0.0002	0.0643
25	0.2421	0.1713	0.1040	0.1116	0.0588	0.0161	0.0686
50	0.1362	0.1079	0.0818	0.0852	0.0608	0.0387	0.0655
100	0.0871	0.0756	0.0655	0.0664	0.0543	0.0430	0.0572

Shaded cells indicate unbiased results.

Table 5: Ridge parameter estimated using $\hat{k}_{KM4}$

p= 2	W	LR	LM	WC	LRC	LMC	F
DGP 1							
15	0.1450	0.1050	0.0640	0.077	0.045	0.0120	0.0465
25	0.1061	0.0854	0.0624	0.0698	0.0486	0.0296	0.0500
50	0.0775	0.0665	0.0565	0.0601	0.0519	0.0410	0.0530
100	0.0651	0.0610	0.0561	0.0577	0.0528	0.0473	0.0501
DGP 2							
15	0.1724	0.1285	0.0743	0.0944	0.0544	0.0139	0.0583
25	0.1239	0.0975	0.0706	0.0795	0.0585	0.0377	0.0569
50	0.0815	0.0713	0.0610	0.0641	0.0540	0.0450	0.0559
100	0.0693	0.0639	0.0587	0.0606	0.0557	0.0513	0.0575
DGP 3							
15	0.1653	0.1255	0.0690	0.0919	0.0516	0.0125	0.0516
25	0.1248	0.0991	0.0735	0.0824	0.0630	0.0364	0.0630
50	0.0910	0.0787	0.0669	0.0704	0.0603	0.0494	0.0603
100	0.0700	0.0641	0.0596	0.0610	0.0558	0.0513	0.0558
DGP 4							
15	0.1360	0.1004	0.0514	0.0717	0.0415	0.0084	0.0369
25	0.1169	0.0908	0.0646	0.0748	0.0526	0.0302	0.0561
50	0.0854	0.0743	0.0631	0.0675	0.0577	0.0468	0.0592
100	0.0655	0.0602	0.0550	0.0575	0.0519	0.0482	0.0576
p=4	W	LR	LM	WC	LRC	LMC	F
DGP 1							
15	0.3073	0.2112	0.0832	0.1019	0.0284	0.0000	0.0406
25	0.2039	0.1419	0.0760	0.0821	0.0414	0.0085	0.0491
50	0.1117	0.0870	0.0642	0.0664	0.0437	0.0275	0.0485
100	0.0786	0.0689	0.0584	0.0595	0.0494	0.0386	0.0518
DGP 2							
15	0.3483	0.2440	0.0950	0.1219	0.0349	0.0000	0.0481
25	0.223	0.1591	0.0944	0.1010	0.0531	0.0120	0.0640
50	0.1213	0.0961	0.0696	0.0721	0.0504	0.0325	0.0557
100	0.0876	0.0770	0.0636	0.0654	0.0543	0.0444	0.0563
DGP 3							
15	0.3498	0.2413	0.0883	0.1206	0.0335	0.0000	0.0483
25	0.2261	0.1595	0.0887	0.0968	0.0486	0.0108	0.0593
50	0.1226	0.0963	0.0717	0.0744	0.0516	0.0309	0.0547
100	0.0846	0.0744	0.0649	0.0663	0.0554	0.045	0.0574
DGP 4							
15	0.3175	0.2118	0.0646	0.0980	0.0263	0.0000	0.0383
25	0.2256	0.1597	0.0869	0.0982	0.0490	0.0093	0.0575
50	0.1354	0.1061	0.078	0.0814	0.0561	0.0342	0.0617
100	0.0911	0.0779	0.0664	0.0678	0.0585	0.0478	0.0607

Shaded cells indicate unbiased results.

Table 6: Ridge parameter estimated using $\hat{k}_{KM6}$

p= 2	W	LR	LM	WC	LRC	LMC	F
DGP 1							
15	0.1405	0.1048	0.0636	0.0783	0.0441	0.0127	0.0420
25	0.1021	0.0800	0.0591	0.0671	0.0458	0.0264	0.0494
50	0.0734	0.0651	0.0543	0.0582	0.0473	0.0365	0.0487
100	0.0666	0.0622	0.0569	0.0590	0.0536	0.0487	0.0491
DGP 2							
15	0.1791	0.1336	0.0783	0.0974	0.0554	0.0169	0.0584
25	0.1219	0.0978	0.0703	0.0794	0.0557	0.0342	0.0573
50	0.0879	0.0780	0.0680	0.0711	0.0599	0.0486	0.0568
100	0.0677	0.0624	0.0573	0.0592	0.0546	0.0499	0.0519
DGP 3							
15	0.1596	0.1171	0.0622	0.0852	0.0482	0.0125	0.0482
25	0.1249	0.1017	0.0739	0.0848	0.0602	0.0368	0.0572
50	0.0868	0.0765	0.064	0.0683	0.0559	0.0462	0.0559
100	0.0720	0.0669	0.0621	0.0639	0.0594	0.0537	0.0594
DGP 4							
15	0.1349	0.0916	0.0424	0.0644	0.0350	0.0056	0.0389
25	0.1133	0.0897	0.0619	0.0736	0.0525	0.0289	0.0548
50	0.0885	0.0777	0.0664	0.0697	0.0584	0.0467	0.0585
100	0.0742	0.0699	0.0655	0.0666	0.0621	0.0565	0.0567
p=4	W	LR	LM	WC	LRC	LMC	F
DGP 1							
15	0.3061	0.2086	0.073	0.0955	0.0242	0.0000	0.0456
25	0.1909	0.1310	0.0758	0.0819	0.0408	0.0088	0.0483
50	0.1172	0.0904	0.0635	0.0647	0.0437	0.0265	0.0477
100	0.0764	0.0662	0.0551	0.0565	0.0461	0.0392	0.0478
DGP 2							
15	0.3416	0.2339	0.0818	0.1121	0.0337	0.0001	0.0459
25	0.224	0.1609	0.0900	0.0977	0.0481	0.0122	0.0570
50	0.1249	0.0991	0.0716	0.0748	0.0525	0.0317	0.0576
100	0.0811	0.0702	0.0604	0.0613	0.0518	0.0420	0.0548
DGP 3							
15	0.338	0.2272	0.0690	0.1081	0.0270	0.0000	0.0446
25	0.2182	0.1509	0.0815	0.0911	0.0422	0.0092	0.0512
50	0.1179	0.0910	0.0684	0.0708	0.0501	0.0320	0.0543
100	0.0889	0.0760	0.0651	0.0663	0.0551	0.0447	0.0574
DGP 4							
15	0.3018	0.1964	0.0517	0.0908	0.0220	0.0000	0.0329
25	0.2352	0.1677	0.0908	0.1048	0.0518	0.0093	0.0572
50	0.1354	0.1062	0.0784	0.0814	0.0564	0.0349	0.0587
100	0.0871	0.0773	0.0651	0.0664	0.0551	0.0467	0.0579

Shaded cells indicate unbiased results.

4.2 Analysis of the statistical power of the Granger causality test

The analysis of the power of the test is of central importance since a test will be of little use if it does not have enough power to reject a false null hypothesis. However, in the simulation part of this study it is detected that most applied tests in previous research suffer from serious size distortions for DGP 2 to DGP 4. Since it is meaningless to compare the power of biased test to power of unbiased tests, the power functions are only illustrated for tests that generally are unbiased in most of the cases. Thus, the power is only calculated when the parameters of the regression model is estimated using KM6 as ridge estimator together with the F test. In Figure 1 the power of the test when the lag length equals to two is showed and in Figure 2 we display the power when the lag length equals four. The most important factors for the power of the test are the degree of correlation, the sum of the causality parameters, the sample size and the lag length. All of those individual factors have positive impact on the power functions. Thus, the power becomes higher as any of these factors increases. The most remarkable positive effect has the degree of correlation. It is clear from the power functions that the new test is useful in the presence of multicolinearity.
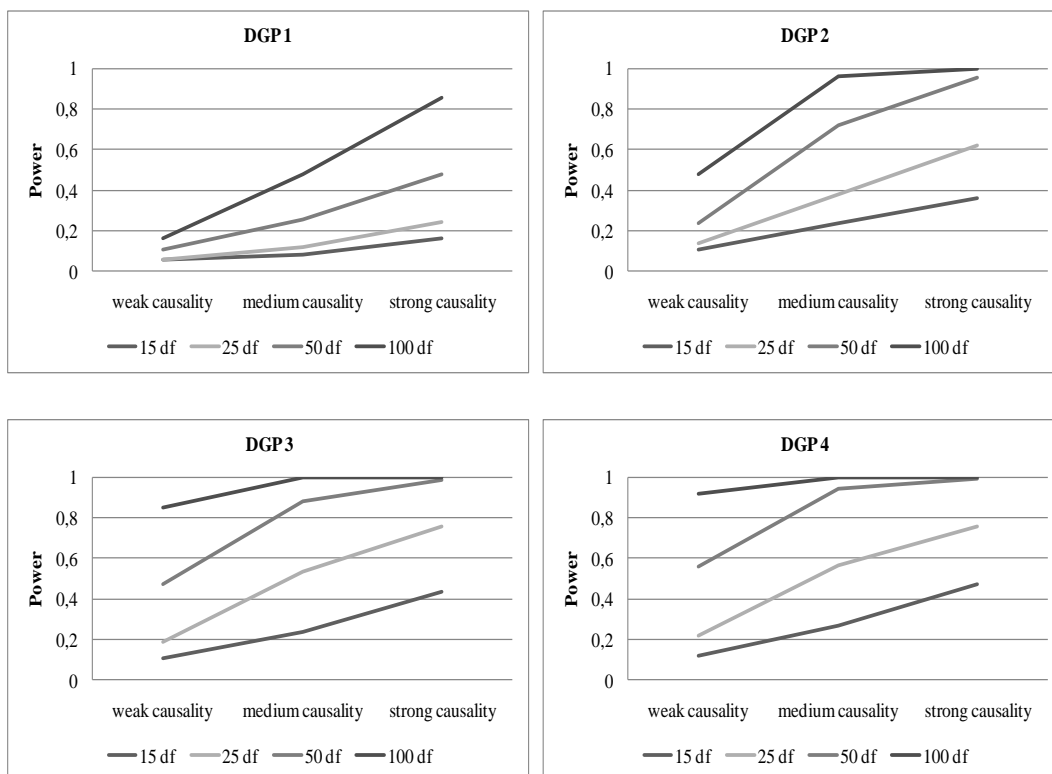

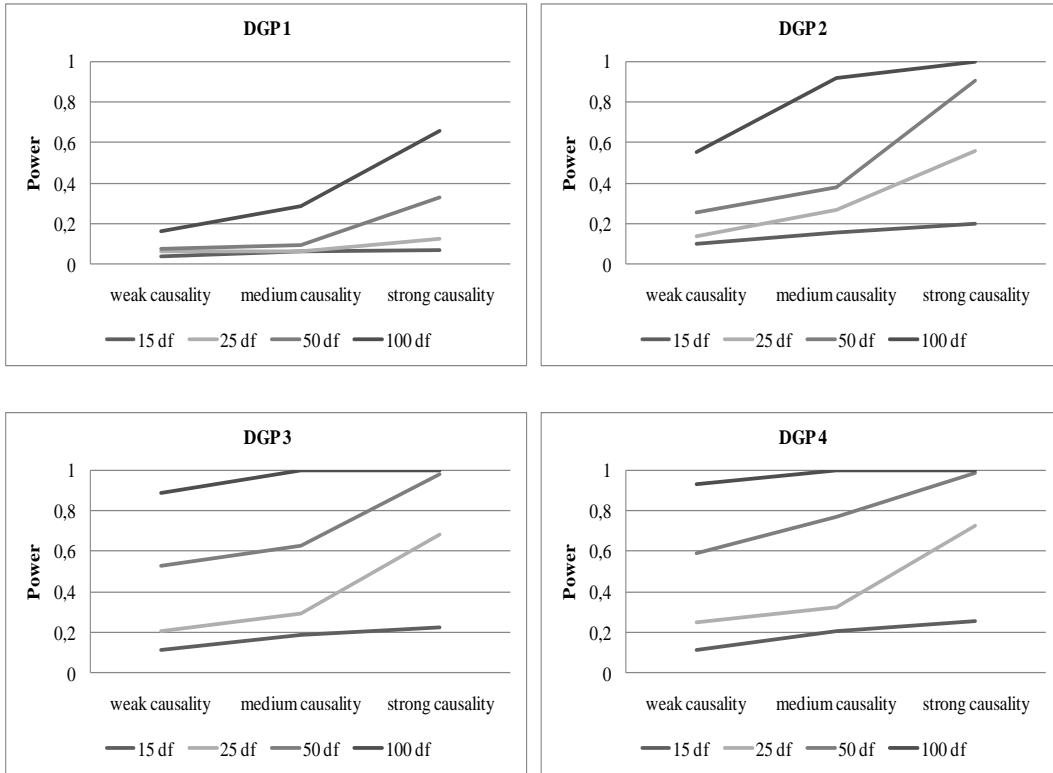**Figure 1:** Power of the F test using KM6 as ridge estimator when the lag length equals 2.

Figure 2: Power of the F test using KM6 as ridge estimator when the lag length equals 4.

5. Conclusions

This paper concludes that the traditional forms of the Granger causality test method over-reject the true null hypothesis in the presence of multicollinearity. A new test named Ridge Regression Granger Causality (RRGC) test is suggested as a remedy to the problem. In order to compare the properties of all the Granger causality tests in this study a simulation experiment is conducted. The factors varied in the Monte Carlos simulation are the sample size, the lag length of the dynamic regression model and the degree of multicollinearity. For every applied DGP the performance of Wald (W), LR, LM, WC, LRC, LMC and the F-test are investigated when the regression model is estimated by OLS in comparison to ridge regression. The result of the analysis confirms that increasing the lag length or the degree of multicollinearity have a negative impact on the statistical size of the Granger causality test while increasing the sample size has a positive impact. The optimal method is to estimate the regression model by the use of KM6 as ridge estimator and by testing for Granger causality using the F-test. Thereafter, the power of the best test is calculated. The main factors that have an impact on the power of the test are the sum of the causality parameters, the sample

size the lag length, and the degree of multicollinearity. A high value for these factors leads to higher power of the test. The main conclusion and essentially unique contribution of this paper is that multicollinearity causes over-rejections of the true null hypotheses for the traditional GC test and that the RRGC test can be used instead of traditional GC methods to gain control of the over-rejection of the null hypotheses in the presence of multicollinearity.

References

Alkhamisi, M. and Shukur, G. (2007). A Monte-Carlo simulation of Recent Ridge Parameters. *Communications in Statistics- Theory and Methods* 36:535-547.

Almasri, A. and Shukur, G. (2003). An Illustration of the Causality Relation between Government Spending and Revenue Using Wavelet Analysis on Finnish Data. *Journal of Applied Statistics* 30:571–584.

Anderson, T. W. (1984). *An Introduction to Multivariate Statistical Analysis*. Second Edition, Wiley:New York.

Frisch, R. (1934). Statistical Confluence Analysis by Means of Complete Regression Systems", Publication 5 (Oslo: University Institute of Economics, 1934).

Granger, C. W. J. (1969). Investigating Causal Relations by Econometric models and Cross-Spectral Methods. *Econometrica* 37:24-36.

Granger, C. W. J. & Newbold, P. (1986). *Forecasting Economic Time Series*. Second edition, New York: Academic Press.

Hoerl, A. E. and Kennard R. W. (1970a). Ridge regression: biased estimation for non-orthogonal problems. *Technometrics* 12:55-67.

Hoerl, A. E. and Kennard, R. W. (1970b). Ridge Regression: Application to Non-Orthogonal Problems. *Technometrics* 12:69-82.

Khalaf, G. and Shukur, G. (2005). Choosing ridge parameters for regression problems. *Communications in Statistics- Theory and Methods* 34:1177-1182.

Kibria B.M.G. (2003). Performance of some new ridge regression estimators. *Communications in Statistics- Theory and Methods* 32:419-435.

Muniz, G. and Kibria, B. M. G. (2009). On some ridge regression estimators: An Empirical Comparisons. *Communications in Statistics-Simulation and Computation* 38:621-630.

Hacker, R. S., Kim, H. and Månsson, K., (2010). An Investigation of the Causal Relations between Exchange Rates and Interest Rate Differentials Using Wavelets, No 215, Working Paper Series in Economics and Institutions of Innovation, Royal Institute of Technology, CESIS - Centre of Excellence for Science and Innovation Studies.

Ramsey, J. B. and Lampart, C. (1998). Decomposition of Economic Relationships by Timescale Using Wavelets. *Macroeconomic Dynamics*, 2:49-71.