

Piecework versus merit pay: a mean field game approach to academic behavior

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Abstract. This paper applies the mean field game approach pioneered by Lasry and Lions (2007) to the analysis of the researchers' academic productivity. It provides a theoretical motivation for the stability of the universally observed Lotka's law. It shows that a remuneration scheme taking into account the researchers rank with respect to the academic resume can induce a larger number of researchers to overtake a minimal production standard. It thus appears as superior to piecework remuneration.

Keywords : Mean Field Game, Academic production, incentives, Lotka's law.

JEL Classification : C72, I23, O31

1 Introduction

Over the last twenty years, academic publications tend to become the generalized gauge of the academic performance. Would assessment be directly grounded on the number of publications, on the number of citations or on the impact factor of the journals, all bibliometric criteria give a dramatic role to the characteristics of scientific productions in the evaluation of individual researchers (Groot and Garcia-Valderrama, 2006, Bence and Oppenheim 2004). In return these publications play an increasing role in setting individual salaries and promotions (Hamermesh et al 1982).¹

A vast strand of research deals with the influence of the academic resume on the researchers' careers and rewards. For instance, Stephan (1996) emphasizes that any publication contributes to the researcher's social recognition which is a main part of the academic reward. Diamond (1986) shows that the marginal worth of a citation may vary between 50 and 1300 dollars. In a same way, Swidler and Goldreyer (1998) state that the present value of the first top finance journal article is between 19 493 and 33 754 dollars. In France, since 2008, the jury of the "Concours d'Aggregation pour le recrutement des professeurs d'universite" in charge of the recruitment of professors

¹Boyes et al. (1984) emphasizes that research output is more important for promotion than in-house activities like teaching or administrative tasks



of economics is explicitly using a bibliometric evaluation rule based on the number of publications and the quality of their support (as revealed by the CNRS ranking) to assess applicants (Levy Garboua 2008, Ellison 2010).

Despite these major changes in the governance of the research system, the distribution of the scientific production among the academic population appears as an unchanging rule of the academic production pattern. Almost one century ago, Lotka (1926), already highlighted the inverse relationship between the number of scientific publications in an academic resume and the number of authors that achieve such a CV. Dealing with the productivity of researchers in chemistry and physics, the initial work of Lotka revealed a statistical regularity whereby the majority of authors (60%) publish a single contribution during their career while a small number contributes an essential part of the total number of scientific contributions².

Lotka's law is regarded as one of the three pillars of contemporary scientometrics³. Its validity has been the subject of numerous empirical verifications. An illustration of this law is given by Le Coadic (2005) to characterize the production of French researchers in chemistry. It is also verified in Economics (Cox and Chung, 1991), finance (Chung and Cox 1990), medicine (Kawamura et al 1999)... The robustness of this law is also attested by similar studies using other indexes: Combes and Linnemer (2001) focus on the number of articles weighted by the quality of journals and Courtault et al (2010, 2011) considered the distribution of the h index (Hirsch 2005). Both found that these indexes are distributed according to Lotka's law.⁴

In these settings, the aim of the present paper is to show that Lotka's distribution may be a persistent equilibrium distribution of the scientific production in a game where the publication choices of researchers are made under alternative regimes of incentives. In this purpose, we develop a highly stylized mean field game in the line of Guéant, Lasry and Lions (2010) to

²Lotka's law states that the number of authors making n publications is about $1/n^a$ of those making one publication, where a is approximately equal to two. For example, the number of researchers who have ten contributions represents one percent of the number of researchers who have one publication only.

³the two other are Bradford's law (on a specific topic, a limited number of journals publish a large number of contributions and a large number of journals publish a limited number of items) and Zipf's law (if in a given text, words are ordered according to their frequency, the product rank by frequency is approximately constant). It is easy to show that all these laws are particular cases of Pareto's law, i.e.: roughly 80% of the effects comes from 20% of the causes (Cf. Egghe and Rousseau 1990, Newman 2005).

⁴This robustness of similar distribution has been shown in a large number of fields in economics and finance (income and wealth, the size of cities and firms, stock market returns, trading volume, international trade etc...), Cf. Gabaix (2009)

consider the optimal decision of a rational academic researcher interacting with a continuum of other researchers.

Introduced by Lasry and Lions (2007), the Mean Field Games Theory allows the study of strategic behavior when many agents are in a situation of interaction. When the number of players is important (mainly when there is a continuum of agents), one can hardly assume that a given player is able to take into account all the other players interactions and to compute the Nash equilibrium of a game. In this case, the Mean Field Game Theory adopts the methodological approach of statistical physics in the modelization of the interaction of a great number of particles. Faced to this insurmountable computational problem, physicists consider each particle as being influenced by a mean field exerted by all other particles while simultaneously taking into account the influence of each particle on the mean field. In order to formalize the behavior of a great number of rational agents, the Mean Field Game Theory assumes each agent to be influenced by the mean field made of the distribution of other players' behavior and considers the consequences of each individual decision on this mean field. In a mean field game, an agent cannot directly influence other agents but each agent contributes to the formation of a mean field which influences in turn the behavior of each individual.

Developed in continuous time, this approach involves methods of stochastic calculus and dynamic optimization. In a Mean Field Game, the dynamics of the system is governed by two equations: a backward Hamilton-Jacobi-Bellman equation describing the optimal behavior of agents given the distribution of the other players and a forward Kolmogorov equation which takes into account the influence of each player on the mean field. The fixed point of those two equations gives the Nash equilibrium of the game : Lasry and Lions proved that the coupled system, i.e. the backward HJB equation and the forward Kolmogorov equation can be obtained as the asymptotic limit of Nash equilibrium of N players as N goes to infinity.

It is important to note that the approach of mean field games is different from what is done in standard games theory : the specific partial differential equations system (forward/backward structure) studied in MFG theory is new and does not appear in the literature before MFG, see for instance Khan and Sun (2002) or Aumann (1964). Moreover, one have to mention that the forward/backward structure is a real difference between MFG theory and econophysics where the anticipations process of agents is not sufficiently taken into account.

In the paper at hand, we consider infinite living researchers who decide at each point of time the level of their academic production taking into

account the tradeoff between the cost of this production and the reward that it provides. The equilibrium of such a game is computed for alternatives compensation mechanisms. In particular, the game allows to show that the Paretian distribution is a stable solution under merit pay. Finally, some comparative statics allows to show that the fraction of academic researchers having a publication score above a given threshold is an increasing function of monetary incentive and a decreasing function of time preference.

In the line of Pareto (1896) and Allais (1974) who believed that economic policy could not influence the distribution of income and wealth (supposed to be independent even of the economic system), we found that economic policy cannot change the distribution of the scientific production. However, policy may influence both the speed at which this distribution moves and its dispersion and proves to have an impact on the overall research output.

Section 2 presents the mean field game approach of academic behavior both with and without externalities. Section 2.2, takes explicitly into account the influence of ranking on the researchers' remuneration. The analysis of the effects of this externality in the reward system is the main innovation of the paper. Section 3 interprets the economic meaningfulness of the results. A last section concludes the paper.

2 A mean field game approach to academic behavior)

Let a_t denote the instantaneous scientific production of a researcher at date t and x_t be the cumulative measure of this production at the same date.⁵ The relation between a_t and x_t is given by :

$$dx_t = a_t dt$$

Let there be a continuum of researchers. Define $m(t, x)$ as the density of the distribution of x_t at date t . By assumption, the support of the distribution at date 0 is $[1, \infty)$, where we consider that the worst researcher at date 0 is a newly mint PhD student with only his PhD Thesis. At date t , the optimization problem of the researcher is given by :

$$U(t, x) = \max_{(a_s)} \left\{ \int_t^{+\infty} f(x_s, a_s, m(s, x_s)) e^{-r(s-t)} ds / x_t = x \right\} \quad (1)$$

⁵ x_t and a_t could represent the number of papers weighted by the quality of the scientific reviews or any other bibliometric index.

where $f(x_s, a_s, m(s, x_s))$ is the researcher's instantaneous utility function and r takes into account the researcher's time preference. Below, we will consider several specific expressions for the function $f(., ., .)$.

If we suppose, for instance, that m is known the solution of this optimization problem is given by the Hamilton-Jacobi-Belmann (HJB) equation⁶ :

$$\frac{\partial U}{\partial t}(t, x) + \max_a \left\{ f(x, a, m(t, x)) + a \frac{\partial U}{\partial x} \right\} - rU = 0 \quad (2)$$

The control is given by $a = \operatorname{argmax}(f(x, a) + a \frac{\partial U}{\partial x})$. Heuristically one can obtain this equation using the dynamic programming principle of Bellman :

$$U(t, x) = \max_{(a_s)} \left\{ f(x, a_t, m(t, x))dt + e^{-r dt} U(t + dt, x + dx_t) / x_t = x \right\} \quad (3)$$

At time $t=0$, each researcher is endowed with an initial academic production. Let m_0 be the initial density of x_0 , the scientific production of the researchers at time 0. $m(t, \cdot)$ is the solution of Kolmogorov equation:

$$\frac{\partial m}{\partial t}(t, x) + \frac{\partial}{\partial x}(a(t, x)m(t, x)) = 0 \quad (4)$$

For the reader convenience, we give the formal derivation of this equation which is also useful for the stochastic case. Let Φ be a smooth function with compact support,

$$\int m(t + dt, x)\Phi(x)dx = \int m(t, x)\Phi(x)dx + \int \frac{\partial m}{\partial t}(t, x)\Phi(x)dt dx$$

But $m(t, x)$ is the density of the distribution of x_t at date t , so

$$\begin{aligned} \int m(t + dt, x)\Phi(x)dx &= \mathbb{E}(\Phi(x_{t+dt})) = \mathbb{E}(\Phi(x_t) + \Phi'(x_t)dx_t) \\ &= \mathbb{E}(\Phi(x_t) + \Phi'(x_t)a_t dt) = \int m(t, x)(\Phi(x) + \Phi'(x)a_t dt)dx \\ &= \int (m(t, x)\Phi(x) - \frac{\partial}{\partial x}(a(t, x)m(t, x))\Phi(x)dt)dx \end{aligned}$$

Hence

$$\int \frac{\partial m}{\partial t}(t, x)\Phi(x)dt dx = - \int \frac{\partial}{\partial x}(a(t, x)m(t, x))\Phi(x)dt dx$$

and this identity implies the Kolmogorov equation (Φ is a test function).

In order to get explicit solutions we have now to consider explicit forms for $f(., .)$.

⁶See for example Fleming and Soner (2005) or Brock (1987)

2.1 A model without externalities

In their research activities, researchers have to consider the trade-off between the cost of carrying out their research and the specific reward of total and instantaneous scientific production (Swidler and Goldrayer 1998). This trade off between costs and remuneration may be formalized as :

$$f(x, a) = \alpha x + \beta a - a^2, \quad (5)$$

where β is the instantaneous reward that a researcher gets from an additional publication. This reward incorporates both a monetary premium (as in some scientific institutions such as business schools which pay researchers for each new publication) and a psychological reward given by the mere fact of being published by a selective journal (Stephan 1996).

α represents the reward that a researcher gets given his academic resume. Once again this incorporates both monetary and social rewards. The cumulative measure of scientific production is essential in determining promotions and raises for scientific researchers (Diamond 1986, Stephan 1996). Beside, the more an academic researcher publishes, the greater his readership and the higher will be the influence of his work on the academic field.

The last term in the previous equation measures the research cost associated with an instantaneous production a . This cost is assumed to be increasing at an increasing rate. For the sake of simplicity, we consider a quadratic cost function.

In this section, we ignore the researchers interactions. The externalities generated by the ranking of the academic researchers within in a field of research will be dealt with Mean Field Game in subsection 2.

Although we do not need to use the MFG equations for this model without externalities, for the reader convenience, we revisit the solution of the model via the MFG "tools". For a traditional exposition using calculus of variation, see the appendix.

The control is given by $a = \operatorname{argmax}(f(x, a) + a \frac{\partial U}{\partial x}) = \frac{1}{2}(\frac{\partial U}{\partial x} + \beta)$. Replacing in (2), we find

$$\begin{aligned} \frac{\partial U}{\partial t}(t, x) + f(x, \frac{1}{2}(\frac{\partial U}{\partial x} + \beta)) + \frac{1}{2}(\frac{\partial U}{\partial x} + \beta) \frac{\partial U}{\partial x} - rU &= 0 \\ \frac{\partial U}{\partial t}(t, x) + \alpha x + \frac{\beta^2}{4} - rU + \frac{\beta}{2} \frac{\partial U}{\partial x} + \frac{(\frac{\partial U}{\partial x})^2}{4} &= 0 \end{aligned} \quad (6)$$

It is usual, in regards with the terms of this equation, to look for a solution with the following form $U(t, x) = b(t)x^2 + c(t)x + g(t)$.

Replacing in (6) and by identifying the coefficients of the polynomial gives :

$$b'(t) = -b^2(t) + rb(t) \tag{7}$$

$$c'(t) = -b(t)c(t) - \beta b(t) + rc(t) - \alpha \tag{8}$$

$$g'(t) = -c^2(t)/4 - \beta/2c(t) + rg(t) - \beta^2/4 \tag{9}$$

For the solution to be admissible (see the integrand in the maximization criterion), one must have :

$$x(t) = x_0 + \frac{\alpha + \beta r}{2r}t \tag{10}$$

and $a(t) = \frac{\alpha + \beta r}{2r}$.

Moreover, we know that $m(t, x)$ is solution of :

$$\partial_t m(t, x) + a(t)\partial_x m(t, x) = 0 \tag{11}$$

$$m(0, x) = m_0(x) \tag{12}$$

Even this is the simplest transport equation we give the following details for the reader convenience. First, one can see that $m(t, x(t)) = cte = m(0, x(0))$, indeed

$$\frac{dm(t, x(t))}{dt} = \frac{\partial m}{\partial t} + x'(t)\frac{\partial m}{\partial x} = \frac{\partial m}{\partial t} + a(t)\frac{\partial m}{\partial x} = 0$$

then (expressing $x(0)$ as a function of $x(=x(t))$) we find

$$m(t, x) = m_0\left(x - \frac{\alpha/r + \beta}{2}t\right) \tag{13}$$

In this first model, the instantaneous production of each researchers is constant ($da/dt = 0$). Each researcher exhibits the same instantaneous scientific production ($\partial a/\partial x = 0$) and any initial distribution (and in particular the Pareto distribution) will be preserved over time while "traveling" (like a travelling-wave) with the following velocity : $\frac{\alpha/r + \beta}{2}$. At time t , $m(t, x)$ is thus the translation of m_0 by $\frac{\alpha/r + \beta}{2}t$.

To illustrate graphically this phenomena let us fix the parameters $\alpha = r = 0,05$ and $\beta = 0$ and take for $m(0, x)$ a bell-type curve distribution, we obtain the following graphics (Fig. 1) for the densities at $t = 0$ and $t = 20$, i.e. $m(0, x)$ and $m(20, x)$:

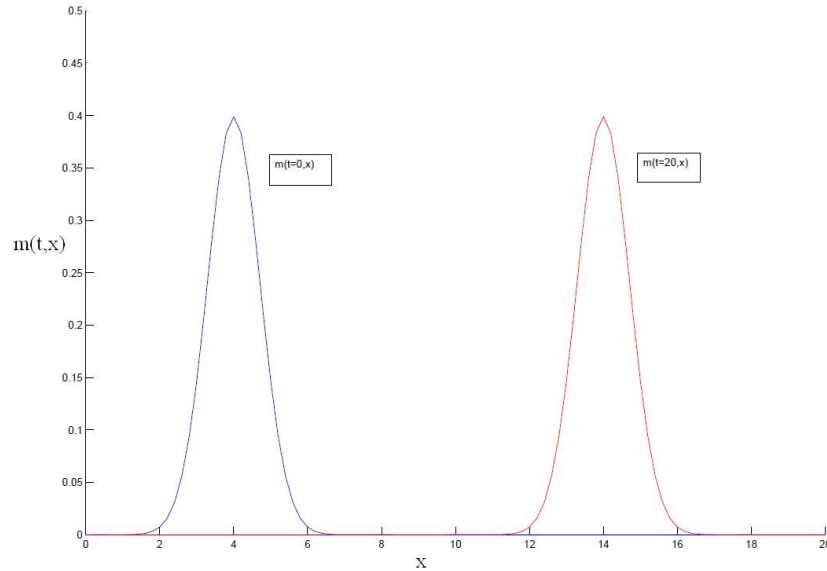


Fig. 1

The speed of the wave describes the intensity of the publication activity of a generation since it compares the stock of publications of the same pool of researchers at two different dates. It is an increasing function of α and β and a decreasing function of the researchers' time preference. Indeed, the higher this time preference is, the lower is the discounted value of net benefit of a publication and so the lower is the quantity of publications.

Recall that β is the measure of the direct utility of an extra publication, in this model $\frac{\alpha}{r}$ is the indirect utility of this extra publication through its impact on the academic resume. As an additional publication increases the stock of publications at each future date, its discounted value is $\frac{1}{r}$.

2.2 A model with ranking

This section explicitly introduces the role of ranking in the academic reward system. The reward of the academic work is rarely strictly tight to the absolute volume of publication. Instead, most of the remuneration system rest on the comparative number of works. The objective of any researcher is to be in the highest possible place in the overall ranking of his peers.

Following Merton (1968, 1969) the goal of a scientist is to be the first to obtain a particular result, the proof of the priority being given by publication. The reward to priority is the recognition awarded by the scientific

community to being first. This phenomenon of giving rewards to priority is particularly illustrated with Nobel prizes which are given to persons having obtained an outstanding result in the first place, however, this is not limited to Nobel Prizes, but is the rule of the promotion game for every researchers. Moreover, the competition between researchers is not limited only to the publication race. Obviously, the process of promotion is the result of a tournament in which each academic resume is assessed with respect to others.

Beside, peer recognition within a field is related to one's ranking in the academia. For instance, REPEC provides every month a ranking of the registered authors within one country or a wider geographic country. Newspapers also give periodically ranking of academic institutions or even of scientific researchers. Scientific journal publish similar results (Dubois et al 2010, Courtault et al 2011, Combe et Linemer 2001, Bosquet et al 2010).

To implement the influence of the relative situation of a researcher in a scientific community, we assume now that the utility function depends on the ranking of a researcher. This brings an externality into the model which can be fruitfully studied with Mean Field Games.

One way to choose this dependence is to take the remuneration inversely proportional to m .⁷ Indeed, the number of people having achieved a particular ranking is smaller the higher the ranking. Instead of using the cumulative distribution function as an index of ranking, it is therefore equivalent and it will prove easier to use the inverse of the distribution function.

Hence we propose the following model :

$$\max_{(a_t)} \left\{ \int_0^{+\infty} \left(\alpha \frac{1}{m(t, x(t))} + \beta \dot{x}(t) - \dot{x}^2(t) \right) e^{-rt} dt / x(0) = x_0 \right\} \quad (14)$$

where $dx_t = a_t dt$.

We introduce the value function :

$$U(t, x) = \max_{(a_s)} \left\{ \int_t^{+\infty} \left(\alpha \frac{1}{m(s, x(s))} + \beta \dot{x}(s) - \dot{x}^2(s) \right) e^{-rs} ds / x(t) = x \right\} \quad (15)$$

The Partial Differential Equations system is given by :

$$\frac{\partial U}{\partial t}(t, x) + \max_a \left\{ \frac{\alpha}{m(t, x)} + \beta a(t) - a^2(t) + a \frac{\partial U}{\partial x} \right\} - rU = 0 \quad (16)$$

⁷ $1/m$ can be used as a ranking since m is decreasing as we will show that the equilibrium distribution is a Pareto distribution.

$$\frac{\partial m}{\partial t}(t, x) + \frac{\partial}{\partial x}(am) = 0 \quad (17)$$

$$m(0, x) = m_0(x) \quad (18)$$

The control is given by $a = \operatorname{argmax} \left\{ \frac{\alpha}{m(t, x(t))} + \beta a(t) - a^2(t) + a \partial_x U \right\} = \frac{\partial_x U + \beta}{2}$. Replace this expression in the equations (16) - (17) - (18) we get :

$$\partial_t U(t, x) + \frac{\alpha}{m(t, x)} + \frac{1}{4}(\beta + \partial_x U)^2 - rU = 0 \quad (19)$$

$$\partial_t m(t, x) + \partial_x \left(\frac{\partial_x U + \beta}{2} m \right) = 0 \quad (20)$$

$$m(0, x) = m_0(x) \quad (21)$$

We recall that we are interested by the "persistence" of a Pareto density in this system, so as explained in the introduction we take $m(0, x) = m_0(x) = \frac{1}{x^2} 1_{x>1}$.

Theorem 2.1. *For an initial distribution of researcher given by $m(0, x) = m_0(x) = \frac{1}{x^2} 1_{x>1}$, the system (19) - (20) admits as solution :*

$$m(t, x) = \frac{\frac{\alpha}{r}t + 1}{(x - \frac{\beta}{2}t)^2} 1_{x > \beta/2t + \frac{\alpha}{r}t + 1} \quad (22)$$

$$U(t, x) = \frac{\frac{\alpha}{r}(x - \frac{\beta}{2}t)^2}{\frac{\alpha}{r}t + 1} + \frac{\beta^2}{4r} \quad (23)$$

the control is given by :

$$a(t, x) = \frac{\frac{\alpha}{r}(x - \frac{\beta}{2}t)}{\frac{\alpha}{r}t + 1} + \frac{\beta}{2} \quad (24)$$

and for the researcher who starts with x_0 , we find :

$$x(t) = \left(\frac{\alpha}{r}t + 1\right)x_0 + \frac{\beta}{2}t. \quad (25)$$

Proof. Suppose that we are looking for an $x(t)$ in the form $x(t) = h(t)x_0 + v(t)$, then $a(t, x) = h'(t)x_0 + v'(t)$ and the Kolmogorov equation :

$$\partial_t m(t, x) + \frac{h'(t)}{h(t)}m + a(t, x)\partial_x m(t, x) = 0 \quad (26)$$

$$h(t)\partial_t m(t, x) + h'(t)m + a(t, x)h(t)\partial_x m(t, x) = 0$$

then for $m(0, x) = \frac{1}{x^2}1_{x>1}$, we find $m(t, x) = \frac{1}{h(t)}m(0, (x - v(t))/h(t)) = \frac{h(t)}{(x-v(t))^2}1_{x>v(t)+h(t)}$. Replacing this expression in (19) gives :

$$\partial_t U(t, x) + \frac{\alpha(x - v(t))^2}{h(t)}1_{x>v(t)+h(t)} + \frac{1}{4}(\beta + \partial_x U)^2 - rU = 0 \quad (27)$$

One can now check that $U(t, x) = \frac{\alpha(x - \frac{\beta}{2}t)^2}{\frac{\alpha}{r}t + 1} + \frac{\beta^2}{4r}$ is solution of (27) for $v(t) = \frac{\beta}{2}t$ and $h(t) = \frac{\alpha}{r}t + 1$. Hence

$$x(t) = (\frac{\alpha}{r}t + 1)x_0 + \frac{\beta}{2}t$$

and the control

$$a(t, x) = \frac{\frac{\alpha}{r}(x - \frac{\beta}{2}t)}{\frac{\alpha}{r}t + 1} + \frac{\beta}{2},$$

or in other terms : $a(t) = a(t, x(t)) = \frac{\frac{\alpha}{r}(x(t) - \frac{\beta}{2}t)}{\frac{\alpha}{r}t + 1} + \frac{\beta}{2} = \frac{\alpha}{r}x_0 + \frac{\beta}{2}$. □

Note that the solution given in Theorem 2.1 is a static Pareto's Optimum. Indeed, any two researchers with initial positions given by x_0 and \tilde{x}_0 have identical marginal rates of substitution :

$$\frac{\partial_x U(t_1, x(t_1))}{\partial_x U(t_2, x(t_2))} = \frac{\frac{\frac{\alpha}{r}(x(t_1) - \frac{\beta}{2}t_1)}{\frac{\alpha}{r}t_1 + 1}}{\frac{\frac{\alpha}{r}(x(t_2) - \frac{\beta}{2}t_2)}{\frac{\alpha}{r}t_2 + 1}} = \frac{2\frac{\alpha}{r}x_0}{2\frac{\alpha}{r}x_0} = \frac{2\frac{\alpha}{r}\tilde{x}_0}{2\frac{\alpha}{r}\tilde{x}_0} = 1 \quad (28)$$

3 Economic interpretation : Impact of the remuneration scheme on the density profile

This section studies the impact of the remuneration scheme on the distribution of the academic production over the continuum of researchers.

Fig. 2 presents the densities $m(0, x)$ and $m(5, x)$ for the parameters value $\alpha = r = 0,05$ and $\beta = 0$:

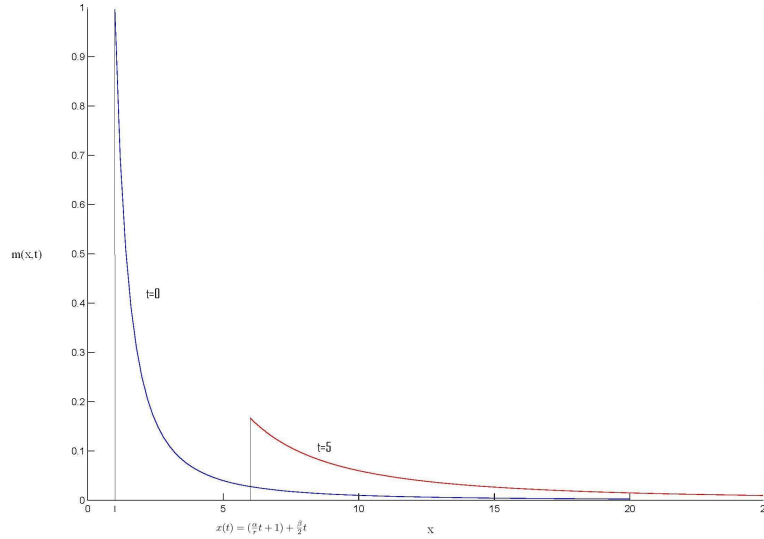


Fig. 2

Recall that at date 0, the worst researcher has an academic resume of 1, at date t , his scientific production will be: $x(t) = (\frac{\alpha}{r}t + 1) + \frac{\beta}{2}t$.

Fig. 2 illustrates two important features of the distribution over time : first, for all dates, the distribution of academic resumes is always Pareto although the law deforms over time. Second, the instantaneous scientific production of any researcher is constant over time according to $a(t) = \frac{\alpha}{r}x_0 + \frac{\beta}{2}$. this speed is an increasing function of the initial CV of the researcher. In order to maintain his initial position, a researcher is induced to produce accordingly. Over time, the gap between scientific CV of researchers is growing.

Moreover, the score of publication $x(t)$ and the instantaneous scientific production $a(t)$ are both increasing functions of α and β , and a decreasing functions of time preference. However, the consequences of a change in these parameters are contrasted. An increase in β has the same positive influence for every researcher, be they bad or good, whereas an increase in α will foster relatively more the academic production of the best scientists (25). Similarly, the influence of r is mediated through the initial position.

In order to measure the relative influence on the dispersion of the scientific production, we introduce the following measure:

$$F(t, x) = \int_{x + \frac{\beta}{2}t + \frac{\alpha}{r}t + 1}^{+\infty} m(t, y) dy \quad (29)$$

$$= \int_{x + \frac{\beta}{2}t + \frac{\alpha}{r}t + 1}^{+\infty} \frac{\frac{\alpha}{r}t + 1}{(y - \frac{\beta}{2}t)^2} 1_{y > \frac{\beta}{2}t + \frac{\alpha}{r}t + 1} dy = \frac{\frac{\alpha}{r}t + 1}{x + \frac{\alpha}{r}t + 1} \quad (30)$$

which represents the number of researchers who exhibit an academic resume over and above the minimum production $\frac{\beta}{2}t + \frac{\alpha}{r}t + 1$ plus an arbitrary level x of scientific production.⁸ We can interpret this measure as the fraction of outstanding researchers.

One interesting aspect of $F(t, x)$ is that it shows that only the parameter α influences positively the number of people having an academic resume of at least x . Instantaneous remuneration β has no influence on this measure. Although it has an influence on the speed of the wave and hence on the total productivity of the whole community, the fraction of outstanding researchers can be increased by a rise in α .

4 conclusion

In this paper, we use the Mean Field Game approach to study the impact of different types of remunerations over scientific production. One of the main result of the paper is that of the motivation of Lotka's law. Indeed we showed that Pareto's distribution is persistent. Once we start with a Pareto distribution for the academic resume, this distribution stays Pareto. However, the characteristics of this distribution evolves over time. This result is achieved through competition among the researchers who aim at maintaining their rank within the distribution. One interesting result of the paper is that remuneration of ranking with respect to the academic resume influences both the speed at which the overall distribution moves over time, in the same way as piecework remuneration do, and more importantly boosts the productivity of outstanding researchers. In order to achieve these results, the model had to make some simplifying assumptions. First, we consider that researchers have an infinite horizon, a more appropriate model should consider overlapping generations. It is difficult to predict what will be the exact influence of this assumption on the equilibrium distribution. However,

⁸Taking the number of researchers exhibiting an academic resume above x only would not make sense as for any given level x there is a time t for which the proportion of people exhibiting an academic resume above x is equal to one

we might guess that the speed at which the equilibrium distribution moves would slow down as at each period. The older generation of researchers who have in averaged the most important academic resume is replaced by a generation of junior researchers having weaker academic achievement. The ripple effect exerted on the speed of the distribution is more likely to be reduced. Second our approach oversimplifies the remuneration scheme. For instance we do not take into account the ratchet effect implied by some academic procedure such as tenure. Once tenure is acquired, a drop of the academic production has little impact over remuneration. Besides, we did not consider either the influence of ranking with respect to the instantaneous academic production which would allow us to take into account remuneration schemes of the type of the french "Prime d'Excellence Scientifique". Finally, scientific production is not the only output of academic researchers, the scientific method requires, in fact, that many researchers can devote a significant part of their research time not to make fundamental discoveries, but to validate the innovative research and to reveal, through their citations, the best ideas produced.

Despite these limitations our paper is interesting for at least two reasons. It provides first a theoretical foundation to the empirical stability of Lotka's law. Above this, in a period where governments intend to foster applied and academic research and aim at improving the efficiency of their research institutions, this paper brings some insight on the effect of various incentive schemes on scientific production. The Mean Field Game approach developped here allows to analyze the consequences of competition between researchers and to underlines the positive effects of the race for the best ranks both on the volume of scientific production and on the visibility of the best researchers.

5 Appendix : Resolution of the model without externalities using variational calculus

It should be noted though that this model can be easily solved using calculus of variations. Replace $a(t)$ by $\dot{x}(t)$ in the optimization problem (1), it becomes :

$$\max_{(x_t)} \left\{ \int_0^{+\infty} (\alpha x + \beta \dot{x}(t) - \dot{x}^2(t)) e^{-rt} dt / x(0) = x_0 \right\}$$

which can be written as $\max_{(x_t)} \left\{ \int_0^{+\infty} J(t, x(t), \dot{x}(t)) dt / x(0) = x_0 \right\}$ where $J(t, x, v) = (\alpha x + \beta v - v^2) e^{-rt}$. The Euler-Lagrange equation for this prob-

lem:

$$\frac{d}{dt}\left(\frac{\partial J}{\partial v}\right) = \frac{\partial J}{\partial x}$$

becomes $\frac{d}{dt}((\beta - 2\dot{x}(t))e^{-rt}) = \alpha e^{-rt}$, $-2\ddot{x}(t) - r(\beta - 2\dot{x}(t)) = \alpha$, i.e.

$$-2\ddot{x}(t) + r2\dot{x}(t) = \alpha + r\beta$$

The homogeneous equation admits a general solution $Ae^{rt} + B$. Given the right hand side of the equation and the fact that we are searching admissible solution, one must have $A = 0$, which gives $x(t) = x_0 + \frac{\alpha + r\beta}{2r}t$.

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