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Mortgage Amortization and Amplification

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Abstract

Mortgages characterized by negative or low early amortization schedules amplify the macroeconomic effects of a housing risk shock. We analyze the role of mortgage amortization in a two-sector DSGE model with housing risk and endogenous default. Mortgage loan contracts extend to two periods and have adjustable rates. The fraction of principal to be repaid in the first period can vary. As the fraction of principal to be paid in the first period falls, steady-state mortgages and leverage increase and the impact of a housing risk shock on consumption and output is amplified. Borrowers prefer negative amortization. If free to choose the amortization schedule, borrowers would repay most of the principal in the last period of the contract. Low early repayments of principal allow borrowers to hold on to their housing stock and postpone default to the second period having incurred small sunk costs.

Keywords: Housing; Mortgage default; Mortgage risk

JEL Codes: E32, E44, G01, R31

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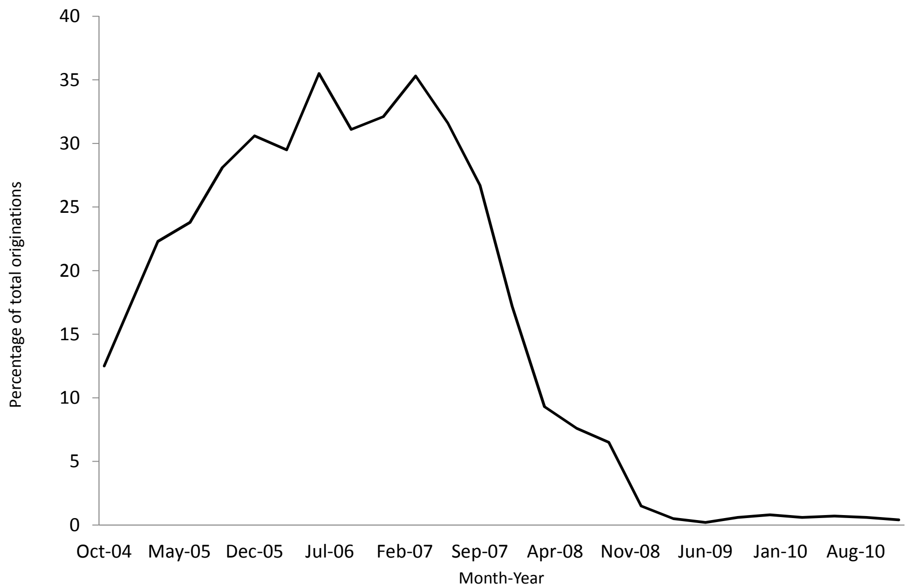


Source: Inside Mortgage Finance Publications

Figure 1: Nonstandard Mortgages

1 Introduction

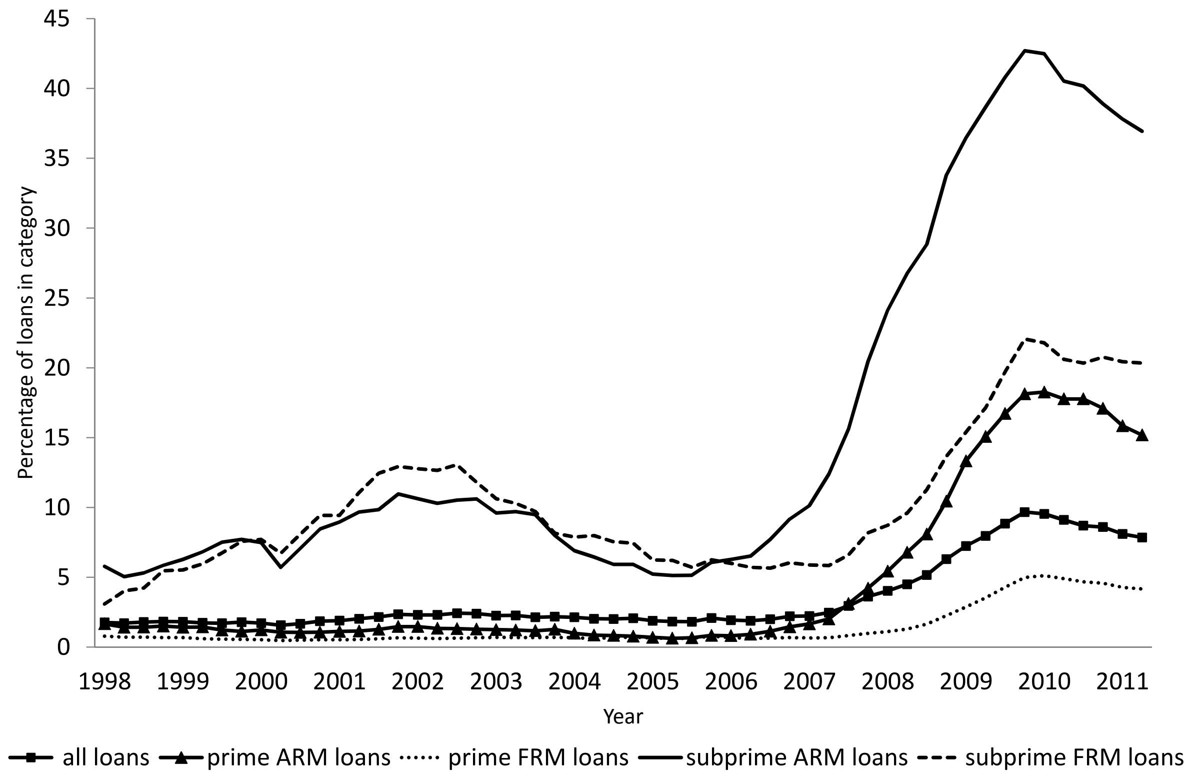
The recent financial crisis and the ensuing Great Recession have their roots in the bursting of the housing bubble in the United States. Academic and policy discussions have pointed to a number of likely contributions to the housing bubble, among which changes in methods of housing finance. Bernanke (2010) points specifically to several changes in the mortgage market. First, a significant increase in the percentage of new mortgage applications for Adjustable-Rate Mortgage (ARM) products. Second, the appearance of exotic mortgage products such as interest-only ARMs, 40-year amortization ARMs, negative amortization ARMs, and pay-option ARMs. These nonstandard mortgage products share a feature: the reduction in the initial monthly payments relative to conventional Fixed-Rate Mortgage (FRM) contracts. Bernanke (2010) shows that initial monthly payments could be as low as 14% of a comparable fixed-rate mortgage payment for a negative amortization ARM and even lower for a pay-option ARM.

The percentage of ARMs originated with various exotic features and extended to non-prime borrowers increased rapidly from 2002 to 2006. Figure 1 shows the percentage of exotic mortgage products (including interest-only ARMs, pay-option ARMs and 40-year balloon mortgages) as a percentage of total originations from 2004Q4 to 2010Q4. The incidence of nonstandard mortgages among total originations tripled in just nine quarters, going from 12.5% in 2004Q4 to 35.3% in 2007Q1. Originations increased sharply over the period 2000 to 2006. Outstanding home mortgage liabilities of households and nonprofit organizations increased by 3.9 trillion USDs over that period, going from 46% to 73% of GDP. Using micro-level data Miam and Sufi (2009) document that subprime areas experienced rapid growth in mortgage credit from 2002 to 2005 despite a decline in relative or even absolute income growth. At the same time, the expansion in mortgage credit to subprime areas is closely correlated with the increase in securitization of subprime mortgages. Overall these findings suggest that, by allowing low initial monthly payments, nonstandard mortgage contracts expanded the supply of mortgages and possibly played an important role in the building of the housing bubble.

There is also evidence that nontraditional mortgage contracts were at the heart of the mortgage default crisis. Figure 2 reports seriously delinquent mortgages, which are mortgages more than 90 days past due or in foreclosure, over the period 1998Q1 to 2011Q2. Several mortgage categories are reported: subprime ARM and FRM loans; prime ARM and FRM loans; and all loans.[1] For each category, delinquent mortgages are reported as percentage of loans in the category. First, the default rate on all loans increased from 2% to 10% during the mortgage default crisis. Second, the rate of default of ARM loans started rising a full year earlier and it increased twice as much as that of FRM loans, both for subprime and prime loans. Delinquency rates for subprime ARM loans increased ninefold between 2005Q4 and 2009Q4 while delinquency rates for subprime FRM loans increased fourfold over the same time period. As for prime loans, delinquency rates for ARM loans increased 23 times over the period 2005Q4 and 2009Q4 while those for FRM loans increased fourfold over the same period. Third, the default rate of prime and subprime loans were substantially different even before the mortgage default crisis started. The delinquency rate for prime loans was well below 1% before 2007 while the delinquency rate for subprime loans was around 5.5%. The evidence suggests that ARMs contributed more heavily to the mortgage default crisis than FRMs. Since nonstandard mortgage contracts as defined above[2] are ARM contracts and since these contracts became widely used before the

[1] All loans include prime ARM and FRM loans, subprime ARM and FRM loans, FHA (Federal Housing Administration) and VA (Veteran Administration) loans.

[2] With the possible exception of 40-years balloon mortgages, which are short-term (3 or 5 years) fixed-rate loans with fixed monthly payments based upon a 40-year fully amortizing schedule and a lump-sum payment at the end of its term.



Source: National Delinquency Survey, Mortgage Bankers Association

Figure 2: Seriously Delinquent Mortgages

mortgage default crisis, we interpret this evidence as suggesting that exotic mortgages played a key role in the onset of the mortgage default crisis.

If nontraditional mortgage contracts are a likely key explanation of the housing bubble, they may have also exacerbated the ensuing mortgage default crisis and deepened the subsequent recession. As house prices fell, the holders of exotic mortgages who had made low initial payments had all the incentives to walk away from their loans. The goal of our paper is to analyze how mortgage amortization affects the incentive to default and whether low early amortization amplifies the macroeconomic effects of a default crisis. To address these issues we consider ARMs and focus on the amortization schedule. In our model loan contracts extend to two periods and they specify the fraction of principal that must be repaid in the first period of the contract. By varying this fraction from zero to one we encompass a continuum of amortization schedules. When the fraction of principal to be repaid in the first period is close to one, we have high early amortization schedules. As the fraction of principal to be repaid in the first period falls to zero, we have low early or even negative amortization schedules – the latter occurring when the first-period payment does not even cover interest costs on the entire mortgage.

We build a two-sector DSGE model with housing. There are two households that differ in terms of their discount factor. Savers have a higher discount factor and lend to Borrowers, who have a lower discount factor. Household preferences are defined over non-durable consumption, housing services and hours worked. Borrowers pledge their homes as collateral for mortgages. We assume that loan contracts are nonrecourse in our model, as it is the case in a number of U.S. states. This means that lender's recovery in case of default is strictly limited to the collateral. Every period Borrowers experience an idiosyncratic housing shock that is private information. Borrowers that experience low realizations of the idiosyncratic shock default on their debts; non-defaulting Borrowers pay an adjustable rate on their mortgages. Savers pay a monitoring cost and seize the houses of defaulting Borrowers. The spread between the adjustable mortgage rate and the rate on risk-free loans is the external finance premium paid by Borrowers.

The first part of our analysis takes the amortization schedule as given. As the fraction of principal to be repaid in the first period falls, mortgages and leverage increase. Intuitively, early repayments act as sunk costs. Low early repayments effectively induce Borrowers not to default in the first period, take advantage of the housing stock for an additional period, and then default in the second period incurring low sunk costs. Borrowers have an incentive to become more leveraged and increase their housing stock. This strategic incentive to postpone default reduces but does not eliminate default in the first period of the contract and raises sharply default in the second period of the contract. The overall effect is a sizeable increase of the average default rate for low early amortization mortgages. Intuitively, it is more appealing to default before a large repayment toward the end of the contract rather than before a small one.

To analyze the effects of the mortgage default crisis, we assume that housing risk is time-variant and analyze the dynamic response to a housing risk shock, namely an unanticipated temporary increase in the standard deviation of the idiosyncratic housing risk. This risk shock aims to capture (in an admittedly stylized manner) the increased credit risk caused by the growth of subprime mortgage lending. The subprime share of total outstanding mortgages increased from 2% in the first quarter of 2000 to 14% in the last quarter of 2006; this growth should then be compounded with the growth in mortgage lending discussed earlier. Mortgage default data from 1995 to 2011 suggests that subprime mortgage loans are six times more likely to be defaulted on than prime mortgage loans; the equivalent figure for the period 1995 to 2006 is ten.

A housing risk shock raises default rates, external finance premia and generates a credit crunch. Borrowers experience a significant worsening of their financial situation, which forces them to de-leverage and cut both non-durable consumption and housing investment. Low early or even negative amortization intensifies the negative effects a housing risk shock on aggregate consumption and output. This amplification mechanism is driven by the strategic incentive to postpone default, which generates steady-state and dynamic effects. Low early repayments come hand in hand with higher steady-state leverage ratios and housing demand. When Borrowers need to de-leverage, they cut consumption and housing investment more aggressively, thereby depressing aggregate demand more. Since Borrowers postpone default, housing demand takes longer to rebound. The average default rate is highest with low early amortization. Larger housing demand reductions generate steeper declines in house prices, which give way to a second-round of dynamic effects, as lower house prices bring more Borrowers into default.

The second part of our paper analyzes the case where Borrowers choose the amortization schedule, namely the fraction of principal to be repaid in the first period. In our framework impatient agents are borrowing constrained so they have well-defined preferences over the repayment schedule. If free to choose, Borrowers would repay 99% of the principal in the last period of the contract. In our model this corresponds to a negative amortization schedule. Borrowers prefer to postpone the bulk of principal repayment to the end of the contract so as to retain the possibility of defaulting having already made small repayments and therefore incurring small sunk costs.

2 Related Literature

A growing literature embeds durable goods in an otherwise standard New Keynesian model. Barsky, House and Kimball (2007) show that price stickiness of durable goods plays a key role in the transmission mechanism of monetary policy. More precisely, if prices of durable goods are sticky, the model behaves as if most prices are sticky. On the other hand, if prices of durable goods are flexible, the model behaves as if most prices are flexible. When durable prices are flexible, the durable goods sector contracts in response to a monetary expansion, thereby offsetting the expansion in the non-durable goods sector and leaving GDP unchanged. Hence, their model predicts negative sectoral co-movement following a monetary policy shock. Erceg and Levin (2006) use VAR evidence to document positive sectoral co-movement as well as higher sensitivity of the durable good sector (relative to the non-durable one) to the nominal interest rate in response to a monetary shock. To match this evidence with the model impulse responses, Erceg and Levin (2006) assume wage stickiness and the same degree of price stickiness in the durable and non-durable sector. Carlstrom and Fuerst (2006) argue that price stickiness in the durable sector is counterfactual and suggest to add adjustment costs à la Topel and Rosen (1988) in the durable sector. Monacelli (2009) and Sterk (2010) analyze whether introducing credit market frictions can help to solve the co-movement puzzle. In Monacelli (2009) durable goods are used as collateral and sectoral outputs co-move in response to a monetary shock provided the durable sector displays some degree of price stickiness. Sterk (2010), on the other hand, finds that credit market frictions make it more difficult to generate sectoral co-movement following a monetary policy shock. Even though we do not focus on monetary policy shocks, our two-sector model with housing and non-durable goods incorporates the necessary features suggested by this literature to generate co-movement in response to a monetary shock.

Another strand of literature incorporates financial frictions à la Kiyotaki and Moore (1997) into a model with housing, non-durable sticky prices, and two households with different discount factors. To ensure the existence of an equilibrium, Iacoviello (2005) assumes an exogenous borrowing constraint according to which impatient agents can borrow a fraction of the expected discounted future value of their houses. Iacoviello and Neri (2010) build and estimate a DSGE model with housing. Loans are always fully repaid and there is no default on mortgages in these papers. Forlati and Lambertini (2011) follow Iacoviello (2005) to build a model with two household groups and housing as a durable good used as collateral but allow for idiosyncratic risk in housing investment. Their household problem is akin to that of entrepreneurs in Carlstrom and Fuerst (1997) and Bernanke, Gertler and Gilchrist (1999) and endogenous default on mortgages arises in equilibrium. Forlati and Lambertini (2011) show that an unanticipated increase in idiosyncratic housing risk generates a recession. Aoki, Proudman and Vlieghe (2004) also introduce a financial accelerator á la Bernanke and Gertler in a model with housing. In their model, unlike Forlati and Lambertini (2011), risk-neutral home owners buy houses and rent them to risk-averse consumers and their focus on the transmission of a monetary policy shock. Our model draws from Forlati and Lambertini (2011) in featuring idiosyncratic housing risk and endogenous default on mortgages; to capture sectorial co-movement in response to monetary policy shocks, it also allows for sticky wages and adjustment costs à la Topel and Rosen (1988) in the housing sector.

The novel feature of our framework is that we allow for an amortization schedule and analyze its role in the transmission of a housing risk shock. More precisely, we extend the mortgage contract in Forlati and Lambertini (2011) to two periods and encompass different amortization schedules. A related paper is Calza, Monacelli and Stracca (2011), which studies how the transmission mechanism of monetary policy is affected by the structure of housing finance. They present VAR evidence that monetary shocks are amplified in countries characterized by more flexible or developed mortgage markets. To rationalize these findings they build a DSGE model with durable and non-durable goods and an exogenous borrowing constraint. They allow for one- and two-period contracts, where the latter features a fixed interest rate and equal repayments in the first and second period, and show that consumption falls more with one-period contracts following a monetary policy tightening. Our framework differs from Calza et al. (2011) in a number of ways, as it features endogenous default and adjustable mortgage rates. More importantly, we allow for and focus on the role of different amortization schedules.

Some recent papers examine the effects of risk shocks. For example, Christiano, Motto and Rostagno (2009) augment a standard monetary DSGE model to include financial markets and a financial accelerator and fit the model to European and U.S. data. They analyze an increase in the standard deviation of idiosyncratic risk in loans to entrepreneurs. In our setting, idiosyncratic risk is in mortgage loans. Iacoviello (2010) introduces the banking sector in a model with housing and studies an exogenous shock to how much borrowers repay. This repayment shock is exogenous and different from default because borrowers do not lose their houses following a negative repayment shock.

3 The Model

Our starting point is a model with patient and impatient households that consume non-durable goods and housing services and work. There is idiosyncratic housing risk so that endogenous default on mortgages arises in equilibrium, as in Forlati and Lambertini (2011). Here we extend the contract to two periods and allow for different amortization schedules. More precisely, the mortgage contract specifies the loan amount and the fraction x of the principal to be repaid (plus interests) at the end of the first period. The remaining fraction $1-x$ of the principal (plus interests) is repaid at the end of the second period. By varying the parameter x we encompass different amortization scenarios. x close to zero represents the case where small repayments are done in the first period while most of principal plus interests is repaid at the end of the second period. All schedules with x below the interest rate imply negative amortization. $x = 0.5$ implies a linear amortization schedule; x close to 1 is an extreme case of high early amortization where the entire repayment is done at the end of the first period. The first part of our analysis takes x as given; the last part of the analysis makes Borrowers choose x optimally.

3.1 Households

The economy is populated by a continuum of households distributed over the $[0,1]$ interval. A fraction ψ of identical households has discount factor β while the remaining fraction $1-\psi$ has discount factor $\gamma > \beta$. We are going to refer to the households with the lower discount factor as Borrowers, as these households value current consumption relatively more than the other agents and therefore want to borrow. We are going to refer to households with the higher discount factor as Savers.

Borrowers

Borrowers have a lifetime utility function given by

$$\sum_{t=0}^{\infty} \beta^t E_0 \left\{ U\left(X_t, N_{C,t}, N_{H,t}\right) \right\}, \quad 0 < \beta < 1, \tag{1}$$

where $N_{C,t}$ is hours worked in the non-durable sector, $N_{H,t}$ is hours worked in the housing sector, and X_t is an index of non-durable and durable consumption services defined as

$$X_t \equiv \left[(1-\alpha)^{\frac{1}{\eta}} C_t^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} H_{t+1}^{T\;\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \tag{2}$$

where C_t denotes consumption of non-durable goods, H_{t+1}^T denotes consumption of housing services, α is the share of housing in the consumption index and $\eta \geq 0$ is the elasticity of substitution between housing and non-durable services. The Borrower household's housing stock at the end of period t is the sum of the housing stocks (net of depreciation) purchased in the last two periods and connected to the outstanding mortgage contracts:

$$H_{t+1}^T \equiv \frac{1}{2} \left\{ H_{t+1} + H_t(1-\delta)[1 - G_t(\bar{\omega}_{1,t})] \right\}, \tag{3}$$

where T stands for total, H_{t+1} is the housing stock purchased in period t, H_t is the housing stock purchased in period $t-1$ minus the fraction lost to default in period t, $G_t(\bar{\omega}_{1,t})$, and net of depreciation δ. We will derive explicitly the term $G_t(\bar{\omega}_{1,t})$ later. We assume that housing services are equal to the housing stock. Assuming that services are a fraction of the stock does not affect the results qualitatively.

We assume the following period utility function:

$$U(X_t, N_t) \equiv \ln X_t - \frac{\nu}{1+\varphi} N_t^{1+\varphi} = \ln X_t - \frac{\nu}{1+\varphi} \left[N_{C,t}^{1+\xi} + N_{H,t}^{1+\xi} \right]^{\frac{1+\varphi}{1+\xi}}, \qquad \varphi, \xi \geq 0. \tag{4}$$

Our specification for the disutility of labor follows Iacoviello and Neri (2010) in assuming that hours in the non-durable and housing sector are imperfect substitutes, as consistent with the evidence by Horvath (2000). For $\xi = 0$ hours in the non-durable and housing sector are perfect substitutes. On the other hand, positive values of ξ result in wages not being equalized in the two sectors and the substitution of hours across sectors in response to wage differentials being reduced. The parameter φ is the inverse of the Frisch labor supply elasticity.

The Mortgage Contract

The Borrower household consists of many members that are divided into two ex-ante identical groups. These two groups alternate in being assigned the resources to purchase new houses and finalize current mortgage contracts. More precisely, at t the household decides total housing investment H_{t+1} and the state-contingent mortgage rates to be paid on the mortgages. The household then assigns equal resources to all members of group t, which is the group that purchases H_{t+1}. The $i-$th member purchases the housing stock H^i_{t+1}, where $\int_i H^i_{t+1} di = H_{t+1}$, finalizes the mortgage contract connected to the housing stock H^i_{t+1} following the instructions of the household, and manages his housing stock. The mortgage contract lasts two periods during which the member cannot purchase new houses or finalize new or additional contracts. The mortgage contract specifies the total loan amount, L_{t+1}, and the fraction to be repaid at the end of the first period, x. We assume $x \in (0, 1)$. Hence, the mortgage contract generates two loans:

$$L_{1,t+1} \equiv xL_{t+1}, \quad L_{2,t+1} \equiv (1 - x)L_{t+1}. \tag{5}$$

$L_{1,t+1}$ is repaid (gross of interests) at the beginning of period $t + 1$ while $L_{2,t+1}$ is repaid (gross of interests) at the beginning of period $t + 2$. Group t will re-enter the mortgage and housing market in period $t + 2$, once the current mortgage contract has run its course, and at that point it will become group $t + 2$. Group $t + 1$ purchases houses and finalizes mortgage contracts in period $t + 1$, and so on and so forth. Our model features an overlapping mortgage structure as illustrated in Table 1. Each row of Table 1 refers to a group, each column refers to a period, and the cell at the intersection represents the action taken by the row-specific group in the column-specific period. Positive signs are mortgage contracts finalized; negative signs are denote repayments. Every period is characterized by two repayments, a first- and a second-period repayment by different groups, and the completion of a new mortgage contract.

In what follows we describe in detail the decisions of group t.

The Incentive Compatibility Constraint

Housing investment is risky. After the mortgage contract is finalized, each member experiences each period an idiosyncratic shock that affects his housing value. In period $t + 1$ the group-t, i-th household member experiences an idiosyncratic shock $\omega^i_{1,t+1}$ such that the housing value

Groups \ Period	**t**	**t + 1**	**t + 2**
group $t+1$		L_{t+2}	$-xL_{t+2}$
group t	L_{t+1}	$-xL_{t+1}$	$-(1-x)L_{t+1}$
group $t-1$	$-xL_t$	$-(1-x)L_t$	
group $t-2$	$-(1-x)L_{t-1}$		

Table 1: Overlapping Mortgage Structure

in period $t+1$ is $\omega^i_{1,t+1}P_{H,t+1}(1-\delta)H^i_{t+1}$, where $P_{H,t+1}$ is the nominal house price in period $t+1$; provided default has not taken place in period $t+1$, in period $t+2$ he experiences an idiosyncratic shock $\omega^i_{2,t+2}$ such that the housing value in period $t+2$ is $\omega^i_{2,t+2}P_{H,t+2}(1-\delta)^2H^i_{t+1}$. For simplicity we assume that, at the end of the period, the housing stock and mortgages left after default are equally redistributed among all members in the household. We label the shock ω_j because it affects the housing value $j=1$ or 2 periods after the mortgage has been finalized. Idiosyncratic housing risk captures the fact that housing prices display geographical variation even in the absence of aggregate shocks. Alternatively, one can think of idiosyncratic effects to the housing stock such as damages and (un-modeled) home improvements or idiosyncratic housing preference shocks. The random variables $\omega^i_{j,t+1}$ are i.i.d. across members of the same group and log-normally distributed with a cumulative distribution function $F_{t+1}(\omega^i_j)$, which obeys standard regularity conditions and we assume to be same for ω_1 and ω_2.[3] The mean and variance of $\ln\omega^i_{j,t+1}$ are chosen so that $E_t(\omega^i_{j,t+1})=1$ at all times for both $j=1,2$. This implies that while there is idiosyncratic risk at the household-member level, there is no risk at either the group or the household level and $E_t(\omega^i_{j,t+1}H^i_{t+1})=H_{t+1}$. We are going to assume that housing investment riskiness can change over time, namely that the standard deviation $\sigma_{\omega,t}$ of $\ln\omega^i_{j,t+1}$ is subject to an exogenous shock and displays time variation. The random variable $\omega^i_{j,t+1}$ is observed by the $i-$th member and by the household but can only be observed by lenders after paying a monitoring cost.

After the idiosyncratic shock is realized, the household member decides whether to pay the due installment on his mortgage or to default. Intuitively, loans connected to housing stocks

[3]The c.d.f. is continuous, at least once-differentiable, and it satisfies

$$\frac{\partial\omega_j h(\omega_j)}{\partial\omega_j}>0,\quad j=1,2$$

where $h(\omega)$ is the hazard rate.

that experience high realizations of the idiosyncratic shock are repaid while loans connected to housing stocks with low realizations are defaulted on. It is easier to start from the repayment decision in the last period of the contract. Let $\bar{\omega}_{2,t+2}$ be the threshold value of the idiosyncratic shock for which the member of group t is willing to pay the second installment of his mortgage. The incentive compatibility constraint in period $t+2$ is

$$\bar{\omega}_{2,t+2}(1-\delta)^2 P_{H,t+2} H_{t+1}[1-G_{t+1}(\bar{\omega}_{1,t+1})] = (1+R_{Z2,t+2}) L_{2,t+1}[1-F_{t+1}(\bar{\omega}_{1,t+1})]. \quad (6)$$

The right-hand side of (6) is the payment that must be made in period $t+2$. $R_{Z2,t+2}$ is the two-period state-contingent adjustable rate that non-defaulting Borrowers pay on the two-period loans that have survived default in period $t+1$, $L_{2,t+1}[1-F_{t+1}(\bar{\omega}_{1,t+1})]$. The left-hand side of (6) is the housing value of the marginal member, namely the member who experiences $\bar{\omega}_{2,t+2}$ and is indifferent between paying and defaulting. This housing value is the housing stock H_{t+1} purchased in period t, net of depreciation, net of the housing stock lost to default in period $t+1$, $G_{t+1}(\bar{\omega}_{1,t+1})$, and evaluated at current house price $P_{H,t+2}$. We explicitly derive and explain the term $G_{t+1}(\bar{\omega}_{1,t+1})$ later. Loans connected to $\omega^i_{2,t+2} \in [\bar{\omega}_{2,t+2}, \infty]$ are repaid. On the other hand, loans connected to $\omega^i_{2,t+2} \in [0, \bar{\omega}_{2,t+2})$ are *underwater* mortgages, namely mortgages for which the current value of the house is lower than the loan associated to it. These members have negative equity in their houses and, as a result, they default on these loans. Lenders pay a monitoring cost to assess and seize the collateral connected to the defaulted loan. It is the presence of monitoring that induces Borrowers to truthfully reveal their idiosyncratic shock and justifies the incentive compatibility constraint (6).[4] The household members that default on their mortgages lose their housing stocks.

Consider now the first repayment decision of group t. The mortgage contract requires that the fraction x of $L_{t+1}, L_{1,t+1}$ is repayed in period $t+1$. The incentive compatibility constraint in period $t+1$ is

$$\bar{\omega}_{1,t+1}(1-\delta) P_{H,t+1} H_{t+1} = (1+R_{Z1,t+1}) L_{1,t+1} + E_{t+1}\left\{Q_{t+1,t+2}[1-F_{t+2}(\bar{\omega}_{2,t+2})](1+R_{Z2,t+2}) L_{2,t+1}\right\}. \quad (7)$$

The right-hand side is the present expected value of current and future mortgage payments for

[4] See the seminal work of Townsend (1979).

the members of group t. These include the current repayment of the one-period loan $L_{1,t+1}$ at the state-contingent adjustable rate $R_{Z1,t+1}$ and the present value (using the Borrower's discount factor $Q_{t+1,t+2}$)[5] of the second repayment, taking into account the probability of defaulting in period $t+2$. The left-hand side of (7) is the value of the house for the member that experiences the idiosyncratic shock $\bar{\omega}_{1,t+1}$. This is the marginal member who is indifferent between repaying the first installment of his mortgage and defaulting. All mortgages connected to $\omega^i_{1,t+1} \in [0, \bar{\omega}_{1,t+1})$ are underwater and defaulted on. Loans connected to $\omega^i_{1,t+1} \in [\bar{\omega}_{1,t+1}, \infty)$ are repaid. Defaulting household members lose their housing stocks. If a household member defaults on $L_{1,t+1}$, the entire mortgage is terminated; the housing stock is seized by lenders and the remaining part of the mortgage is revoked.

So far we have described the repayment decision process for the Borrower household members of group t. The same decision process holds for the Borrower household members of group $t+1$, who purchase houses H_{t+2} and finalize total mortgages L_{t+2}, broken down in one-period mortgages $L_{1,t+2}$ and two-period mortgages $L_{2,t+2}$.

A few comments on our assumptions are in order at this point. Mortgages are nonrecourse in our model. This means that mortgages are secured by the pledge of collateral (the house) and the lender's recovery is strictly limited to the collateral. Defaulting Borrowers are not personally liable for the difference between the loan and the collateral value. This is a natural assumption in our model because housing is the only asset held by Borrowers. In addition to this, nonrecourse debt is broadly applicable to most U.S. states, especially those that experienced soaring mortgage delinquencies, and the focus of our paper is on the United States.

In Bernanke et al. (1999) the monitoring cost is equal to a fraction of the realized gross payoff to the defaulting firm's capital. We follow Bernanke et al. (1999) and assume that the monitoring cost in our model is equal to the fraction μ of the housing value. This assumption has two important implications. The first implication is that the foreclosure cost is proportional to the value of the house under foreclosure. The second implication is that mortgage default causes a decline in the housing stock and services, which are destroyed due to monitoring. This second implication generates a (rather unrealistic) rebound in housing demand. For this reason our analysis will show housing demand net of monitoring costs.[6]

[5]The Borrower's discount factor is derived in Appendix A.

[6]We could alternatively assume that monitoring costs are rebated back to households in a lump-sum fashion.

Regarding the defaulting household members, we follow the literature on matching and assume there is perfect insurance among such members so that consumption of non-durable goods and housing services are ex-post equal across all members of the Borrower household. Hence, Borrower household members are ex-post identical.

We can now aggregate the two groups and formulate the budget constraint at period t for the Borrower household:

$$P_{C,t}C_t+P_{H,t}H_{t+1}+[1-F_t(\bar{\omega}_{1,t})](1+R_{Z1,t})L_{1,t}+[1-F_{t-1}(\bar{\omega}_{1,t-1})][1-F_t(\bar{\omega}_{2,t})](1+R_{Z2,t})L_{2,t-1}=$$

$$L_{1,t+1}+L_{2,t+1}+W_{C,t}N_{C,t}+W_{H,t}N_{H,t}+(1-\delta)^2\left[1-G_{t-1}(\bar{\omega}_{1,t-1})\right]\left[1-G_t(\bar{\omega}_{2,t})\right]P_{H,t}H_{t-1}. \quad (8)$$

On the left-hand side of the budget constraint we find the use of resources, which includes the purchase of consumption goods with price $P_{C,t}$, housing, and the repayments of loans. $1-F_t(\bar{\omega}_{1,t})$ is the fraction of one-period loans $L_{1,t}$ taken in period $t-1$ that is repaid to lenders and $R_{Z1,t}$ is the state-contingent interest rate paid on such loans by non-defaulting Borrowers. Similarly, $[1-F_{t-1}(\bar{\omega}_{1,t-1})][1-F_t(\bar{\omega}_{2,t})]$ is the fraction of two-period loans $L_{2,t-1}$ taken in period $t-2$ that is repaid to lenders; these loans have survived default both in $t-1$ and t. On the right-hand side of the budget constraint we find resources. $W_{C,t}$ is the nominal wage in the consumption good sector and $W_{H,t}$ is the nominal wage in the housing sector. Borrower's resources include the new loans $L_{1,t+1}, L_{2,t+1}$, which are the loans finalized by generation-t Borrowers to be repaid in period $t+1$ and $t+2$, respectively, as well as the final housing stock of generation $t-2$ members, who re-enter the housing and mortgage markets as generation t. This final housing stock is equal to the original purchase H_{t-1} net of depreciation and net of the fractions $G_{t-1}(\bar{\omega}_{1,t-1})$ and $G_t(\bar{\omega}_{2,t})$ lost to default in period $t-1$ and t, respectively. We explicitly derive these terms later. The housing stock of group $t-1$ does not appear in the period t budget constraint because this generation does not purchase houses in period t.

The Participation Constraints

Our mortgage contract generates one- and two-period loans that guarantee lenders a pre-determined rate of return. As in Bernanke et al. (1999), the idea is that Savers have access to alternative assets that pay a risk-free rate return that pins down the return on mortgages.

Savers make one-period loans $L_{1,t+1}$ and two-period loans $L_{2,t+1}$ to Borrowers and demand the gross rates of return $1 + R_{L1,t}$ and $1 + R_{L2,t}$, respectively. These rates of return are non-state contingent and pre-determined at t; they are also gross of a time-invariant credit spread due to the use of real resources for financial intermediation. Hence, the time t participation constraint of lenders for one-period loans is given by:

$$(1 + R_{L1,t})L_{1,t+1} = \int_0^{\bar{\omega}_{1,t+1}} \omega_{1,t+1}(1-\mu)(1-\delta)P_{H,t+1}H_{t+1}f_{t+1}(\omega_1)d\omega_1 \tag{9}$$

$$+ \int_{\bar{\omega}_{1,t+1}}^{\infty} (1 + R_{Z1,t+1})L_{1,t+1}f_{t+1}(\omega_1)d\omega_1,$$

where $f_{t+1}(\omega_1)$ is the probability density function of ω_1.The return on one-period loans is equal to the housing stock net of monitoring costs and depreciation of defaulting Borrower members (the first term on the right-hand side of (9)) and the repayment by non-defaulting members (the second term on the right-hand side of (9)). After idiosyncratic and aggregate shocks have realized, the threshold value $\bar{\omega}_{1,t+1}$ and the state-contingent mortgage rate $R_{Z1,t+1}$ are determined so as to satisfy the participation constraint above. Hence, our mortgage contract is characterized by adjustable mortgage rates. The participation constraint holds state by state and not in expected terms. An aggregate state that raises $\bar{\omega}_{1,t+1}$ and thereby default generates an increase in the adjustable rate $R_{Z1,t+1}$ paid by non-defaulting members in order to satisfy the participation constraint (9) in that state.

The time t participation constraint of lenders for two-period loans is given by

$$(1 + R_{L2,t})L_{2,t+1} = \int_0^{\bar{\omega}_{2,t+2}} \omega_{2,t+2}(1-\mu)(1-\delta)^2[1 - G_{t+1}(\bar{\omega}_{1,t+1})]P_{H,t+2}H_{t+1}f_{t+2}(\omega_2)d\omega_2$$

$$+ \int_{\bar{\omega}_{2,t+2}}^{\infty} (1 + R_{Z2,t+2})[1 - F_{t+1}(\bar{\omega}_{1,t+1})]L_{2,t+1}f_{t+2}(\omega_2)d\omega_2. \tag{10}$$

As before, the pre-determined rate of return on two-period loans comes from seizing the housing stock of defaulting members and repayment by non-defaulting ones. Both the housing stock of defaulting members and the loans repaid by non-defaulting ones are suitably adjusted for the default that occurred in period $t + 1$.

Let

$$G_{t+1}(\bar{\omega}_{j,t+1}) \equiv \int_0^{\bar{\omega}_{j,t+1}} \omega_{j,t+1} f_{t+1}(\omega_j) d\omega_j, \qquad j = 1,2 \tag{11}$$

be the expected value of the idiosyncratic shock conditional on the shock being less than or equal to the threshold value $\bar{\omega}_{j,t+1}$, multiplied by the probability of default, and let

$$\Gamma_{t+1}(\bar{\omega}_{j,t+1}) \equiv \bar{\omega}_{j,t+1} \int_{\bar{\omega}_{j,t+1}}^{\infty} f_{t+1}(\omega_j) d\omega_j + G_{t+1}(\bar{\omega}_{j,t+1}), \qquad j = 1,2. \tag{12}$$

Using these definitions and the participation constraints (9) and (10), the Borrower budget constraint in real terms can be written as

$$C_t + p_{H,t}H_{t+1} + \frac{l_{1,t}}{\pi_{C,t}}(1+R_{L1,t-1}) + \frac{l_{2,t-1}}{\pi_{C,t}\pi_{C,t-1}}(1+R_{L2,t-2}) = l_{1,t+1} + l_{2,t+1} \tag{13}$$

$$+p_{H,t}H_t(1-\delta)(1-\mu)G_t(\bar{\omega}_{1,t}) + p_{H,t}H_{t-1}(1-\delta)^2[1-G_{t-1}(\bar{\omega}_{1,t-1})][1-\mu G_t(\bar{\omega}_{2,t})] + w_{C,t}N_{C,t} + w_{H,t}N_{H,t},$$

where $p_{H,t}$ is the relative price of houses in terms of non-durable consumption at t, $\pi_{C,t}$ is non-durable-good inflation and $w_{C,t}, w_{H,t}$ are real wages in the C and H sector, respectively, in terms of $P_{C,t}$. $l_{1,t+1} \equiv L_{1,t+1}/P_{C,t}$ are real one-period loans finalized at t, $l_{2,t+1} \equiv L_{2,t+1}/P_{C,t}$ are real two-period loans finalized at t, etc.

Making use of definitions (11), (12) and the incentive compatibility constraints (7) and (6), the participation constraint at t on the two-period loans can be written in real terms as follows

$$(1+R_{L2,t})\frac{l_{2,t+1}}{\pi_{C,t+1}\pi_{C,t+2}} = p_{H,t+2}H_{t+1}(1-\delta)^2[1-G_{t+1}(\bar{\omega}_{1,t+1})][\Gamma_{t+2}(\bar{\omega}_{2,t+2}) - \mu G_{t+2}(\bar{\omega}_{2,t+2})]. \tag{14}$$

The participation constraint on one-period loans in real terms is

$$(1+R_{L1,t})\frac{l_{1,t+1}}{\pi_{C,t+1}} = p_{H,t+1}H_{t+1}(1-\delta)[\Gamma_{t+1}(\bar{\omega}_{1,t+1}) - \mu G_{t+1}(\bar{\omega}_{1,t+1})] \tag{15}$$

$$-H_{t+1}(1-\delta)^2[1-G_{t+1}(\bar{\omega}_{1,t+1})]E_{t+1}\left\{Q_{t+1,t+2}\, p_{H,t+2}\,\pi_{C,t+2}[\Gamma_{t+2}(\bar{\omega}_{2,t+2}) - G_{t+2}(\bar{\omega}_{2,t+2})]\right\}.$$

We define the loan-to-value (LTV henceforth) ratio as

$$LTV \equiv \frac{(1+R_{L1,t})l_{1,t+1} + E_{t+1}Q_{t+1,t+2}(1+R_{L2,t})l_{2,t+1}}{\pi_{C,t+1}p_{h,t+1}H_{t+1}(1-\delta)}. \tag{16}$$

The LTV ratio measures the total mortgage (principal plus interests) as a fraction of the net housing value. Models with exogenous borrowing constraints typically feature a constant LTV ratio. In our model the LTV ratio varies endogenously.

Wage Determination

We model imperfectly competitive labor markets that generate a wage-inflation Philips curve as in Schmitt-Grohe and Uribe (2007). Labor decisions are taken by two unions in each household, one for each sector, which monopolistically supply labor to a continuum of labor markets indexed by $k \in [0,1]$ in each sector $j = H, C$. Here we focus on the unions that supply Borrower's labor to the sectors; the unions that supply Saver's labor to the sectors take decisions in a similar manner. The union that supplies Borrower's labor to sector j decides the nominal wage to charge in each labor market k in j and it is assumed to satisfy demand, namely

$$N_{j,t}(k) = \left(\frac{w_{j,t}(k)}{w_{j,t}}\right)^{-\varepsilon_w} N_{j,t}^d, \tag{17}$$

where, at time t, $w_{j,t}(k)$ denotes the real wage charged by the union in labor market k in sector j, $w_{j,t}$ is the index of real wages prevailing in sector j, $N_{j,t}^d$ is the aggregate demand for Borrowers' labor by firms in sector j, ε_w is the elasticity of substitution across labor types, and $N_{j,t}(k)$ is the supply of labor in market k of sector j. This demand is formally derived later in section 3.2. The union takes the aggregate demand $N_{j,t}^d$ and the wage index $w_{j,t}$ as given when it decides the wage to charge in labor market k, $W_{j,t}(k)$. Notice that the union decides the nominal wage, even though our model is written in real terms. In addition, the total number of hours supplied in sector j by Borrowers must be equal to the sum of the hours supplied in each market k:

$$\psi N_{j,t} = \psi \int_0^1 N_{j,t}(k)\, dk = N_{j,t}^d \int_0^1 \left(\frac{w_{j,t}(k)}{w_{j,t}}\right)^{-\varepsilon_w} dk. \tag{18}$$

This constraint is taken into account by the household in its maximization problem – see Appendix A. A similar constraint holds for the hours supplied by Savers.

We introduce wage stickiness in the model by assuming that each union can optimally set wages only in a fraction $\varrho_j \in (0,1), j = C, H$, of randomly chosen labor markets. In these labor markets, the union can freely set the nominal wage $W_{j,t}(k)$; we assume no wage indexation so that, in the other labor markets, wages remain equal to those of the last period. For simplicity, we assume the same degree of wage stickiness in the two sectors so that $\varrho_C = \varrho_H = \varrho$.

The first part of our analysis takes the fraction of one-period loans out of total loans, x, as a parameter exogenously given, which can be interpreted as a fixed feature of the mortgage contract. This implies that the Borrower can choose total real loans l_{t+1} but not its composition. In other words, the Borrower cannot choose the amortization schedule for his mortgage. Borrowers maximize (1) subject to the budget constraint (13), the participation constraints (14) and (15), the loan structure (5), and the labor market constraint (18) for sector C and its counterpart for sector H with respect to the variables $C_t, H_{t+1}, N_{C,t}, N_{H,t}, l_{t+1}, \bar{\omega}_{1,t+1}, \bar{\omega}_{2,t+2}, W_{C,t}(k), W_{H,t}(k)$. The respective first-order conditions are spelled out in Appendix A.

Savers

We denote Savers' variables with a ˜. Savers maximize lifetime utility

$$\max \sum_{t=0}^{\infty} \gamma^t E_0 \left\{ U(\widetilde{X}_t, \widetilde{N}_{C,t}, \widetilde{N}_{H,t}) \right\}, \quad 0 < \beta < \gamma < 1, \tag{19}$$

where $\widetilde{X}_t$ is defined similarly to (2). We assume that α, the weight of housing in the consumption index, and the utility function of Savers are identical to those of Borrowers. Savers maximize lifetime utility subject to the sequence of real budget constraints:

$$\begin{aligned} \widetilde{C}_t + p_{H,t}\widetilde{H}_{t+1} + p_{A_l,t}A_{l,t+1} + \widetilde{l}_{1,t+1} + \widetilde{l}_{2,t+1} &= (1-\delta)p_{H,t}\widetilde{H}_t + (p_{A_l,t} + r_{A_l,t})\, A_{l,t} \\ +(1+R_{D1,t-1})\frac{\widetilde{l}_{1,t}}{\pi_{C,t}} + (1+R_{D2,t-1})\frac{\widetilde{l}_{2,t-1}}{\pi_{C,t}\pi_{C,t-1}} &+ \widetilde{w}_{C,t}\widetilde{N}_{C,t} + \widetilde{w}_{H,t}\widetilde{N}_{H,t} + \widetilde{\Delta}_t, \end{aligned} \tag{20}$$

where $A_{l,t+1}$ is the stock of land owned by Savers, $p_{A_l,t}$ denotes the real land price and $r_{A_l,t} \equiv \frac{R_{A_l,t}}{P_{C,t}}$, where $R_{A_l,t}$ is the rental price at which land is rented to the intermediated good producers of the housing sector. We assume that mortgage loan intermediation requires real resources that generate a time-invariant credit spread, namely a spread between the interest rate received by Savers and that paid by Borrowers. More precisely, $R_{D1,t-1}$ is the interest rate on Saver's one-period loans, which is equal to $1 + R_{D1,t-1} = \frac{1+R_{L1,t-1}}{1+\Theta}$, where $\Theta \geq 0$ is the constant spread. Similarly $R_{D2,t-1}$ is the interest rate on Savers' two-period loans which is equal to $1 + R_{D2,t-1} =$

$\frac{1+R_{L2,t-1}}{1+\Theta}$. See Appendix A for details. Finally, $\widetilde{\Delta}_t$ denote profits in the intermediate goods sector, which are taken as given by Savers.

As for Borrowers, Savers' labor decisions are taken by two unions, one for each sector, which monopolistically supply labor to a continuum of labor markets. Each union chooses optimally nominal wages in a fraction $\widetilde{\varrho}$ of randomly chosen labor markets; in the other markets wages remain unchanged. For simplicity, we assume that the degree of wage stickiness in the two sectors are equal, hence $\widetilde{\varrho}_C = \widetilde{\varrho}_H = \widetilde{\varrho}$. The maximization problem faced by Savers' unions is identical to the problem faced by Borrowers' unions and we do not repeat it here.

Savers maximize (19) subject to the budget constraint (20) with respect to $\widetilde{C}_t, \widetilde{H}_{t+1}, \widetilde{N}_{C,t}, \widetilde{N}_{H,t}$, $\widetilde{l}_{t+1}$, $A_{l,t+1}, \widetilde{W}_{C,t}(k), \widetilde{W}_{H,t}(k)$. The first-order conditions are summarized in Appendix A.

3.2 Firms and Technology

Both the non-durable C and the housing H sector have intermediate and final good producers.

Final Good Producers

Final good producers are perfectly competitive and produce $Y_{j,t}$, $j = C, H$. The technology in the $j-$th final good sector is given by

$$Y_{j,t} = \left(\int_0^1 Y_{j,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{21}$$

where $\varepsilon > 1$ is the elasticity of substitution among intermediate goods. Standard profit maximization implies that the demand for intermediate good i is given by

$$Y_{j,t}(i) = \left(\frac{P_{j,t}(i)}{P_{j,t}} \right)^{-\varepsilon} Y_{j,t}, \ \forall i \tag{22}$$

where the price index is

$$P_{j,t} = \left(\int_0^1 P_{j,t}(i)^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}.$$

Intermediate Good Sectors

There are two intermediate good sectors, one in each sector $j = C, H$, and in each intermediate sector there is a continuum of firms, each producing a differentiated good $i \in [0, 1]$. These firms are monopolistically competitive. Each firm uses all labor inputs supplied in the economy to produce its own good. The production function (and therefore the labor demand by intermediate firm i in sector j) is:

$$\left[\zeta^{\frac{1}{\varsigma}} N_{j,t}(i)^{\frac{\varsigma-1}{\varsigma}} + (1-\zeta)^{\frac{1}{\varsigma}} \widetilde{N}_{j,t}(i)^{\frac{\varsigma-1}{\varsigma}}\right]^{\frac{\varsigma}{\varsigma-1}}$$

where

$$N_{j,t}(i) = \left(\int_0^1 N_{j,t}(k,i)^{\frac{\varepsilon_w - 1}{\varepsilon_w}} dk\right)^{\frac{\varepsilon_w}{\varepsilon_w - 1}}, \qquad \widetilde{N}_{j,t}(i) = \left(\int_0^1 \widetilde{N}_{j,t}(k,i)^{\frac{\varepsilon_w - 1}{\varepsilon_w}} dk\right)^{\frac{\varepsilon_w}{\varepsilon_w - 1}}.$$

$N_{j,t}(i)$ and $\widetilde{N}_{j,t}(i)$ are bundles of labor inputs supplied respectively by Borrowers and Savers. $\zeta \in (0, 1)$ is the labor share of Borrowers in the firm's labor demand and $\varsigma > 0$ is the elasticity of substitution across labor bundles. When ς goes to infinity, labor inputs become perfect substitutes. For simplicity these two parameters are assumed to be equal across sectors. Intermediate good firms in the non-durable sector adjust their prices according to a Calvo-type mechanism. Hence, in any given period, an intermediate good firm in sector C may reset its price with probability $1 - \theta_C$. Conversely, the prices in the housing sector are fully flexible. We also assume firm-level adjustment costs in the housing sector.

Non-Durable Sector

Intermediate good firm i in the C sector produces according to the following production function:

$$Y_{C,t}(i) = A_{C,t}\left[\zeta^{\frac{1}{\varsigma}} N_{C,t}(i)^{\frac{\varsigma-1}{\varsigma}} + (1-\zeta)^{\frac{1}{\varsigma}} \widetilde{N}_{C,t}(i)^{\frac{\varsigma-1}{\varsigma}}\right]^{\frac{\varsigma}{\varsigma-1}}, \quad i \in \{C\}, \tag{23}$$

where $A_{C,t}$ is the stochastic level of technology in sector C. In period t firm i chooses labor and, if given the possibility, it re-optimizes its nominal price $P^*_{C,t}(i)$ so as to maximize the expected discount sum of nominal profits over the period during which its price remains unchanged. The maximization problem as well as the first-order conditions relative to $N_{C,t+k|t}(i)$, $\widetilde{N}_{C,t+k|t}(i)$ and

$P^{*}_{C,t}(i)$ are spelled out in detail in Appendix A. In our model marginal costs are a CES index of wages net of productivity; since wages are equal across firms in the sector, marginal costs are also equal across firms.

Housing Sector

Intermediate good i in sector H produces according to the following production function:

$$Y_{H,t}(i) = A_{H,t} A_{l,t}^{1-\kappa}(i) \left[\zeta^{\frac{1}{\varsigma}} N_{H,t}(i)^{\frac{\varsigma-1}{\varsigma}} + (1-\zeta)^{\frac{1}{\varsigma}} \widetilde{N}_{H,t}(i)^{\frac{\varsigma-1}{\varsigma}} \right]^{\frac{\varsigma\kappa}{\varsigma-1}}, \quad i \in \{H\} \tag{24}$$

where $A_{H,t}$ is the stochastic level of technology in sector H, $A_{l,t}(i)$ is the stock of land used as input in production, and κ is the labor share in housing production. We follow Topel and Rosen (1988) and assume there are firm-level adjustment costs in the housing sector for intermediate good producers. As for prices in the housing sector, we assume they are fully flexible. In fact Iacoviello and Neri (2010) estimate a DSGE housing model of the United States and find a degree of price stickiness in the housing sector equal to zero. Hence, each period firm i chooses labor $N_{H,t}(i)$, $\widetilde{N}_{H,t}(i)$, the amount of land to rent $A_{l,t}(i)$, and the price $p_{H,t}(i)$ so as to maximize the expected discounted value of current and future profits net of adjustment costs. The first-order conditions for the maximization problem of the firms are reported in Appendix A.

3.3 Monetary Policy

We assume that monetary policy follows a Taylor-type rule for the one-period nominal interest rate:

$$\frac{1+R_{L1,t}}{1+R_{L1}} = A_{M,t} \left[\pi_{C,t}^{\phi_\pi} \right]^{1-\phi_r} \left[\frac{1+R_{L1,t-1}}{1+R_L} \right]^{\phi_r}, \quad \phi_\pi > 1, \ \phi_r < 1, \tag{25}$$

where R_{L1} is the steady-state nominal interest rate, ϕ_π is the coefficient on the inflation target, ϕ_r is the coefficient on the lagged interest rate, and $A_{M,t}$ is a monetary policy shock. In our benchmark calibration monetary policy targets inflation in the non-durable goods sector and implements interest-rate smoothing, which mimics the zero lower bound and the impossibility for the central bank to further reduce the policy rate.

3.4 Market Clearing

Equilibrium in the non-durable goods market requires that production of the final non-durable good equals aggregate demand:

$$Y_{C,t} = \psi C_t + (1-\psi)\widetilde{C}_t + \Theta l_t, \tag{26}$$

where the last term on the right-hand side are the resources used for financial intermediation. Similarly, equilibrium in the housing market requires

$$Y_{H,t} = \psi \left\{ H_{t+1} - (1-\delta)(1-\mu)G_t(\bar{\omega}_{1,t})H_t - (1-\delta)^2[1-\mu G_t(\bar{\omega}_{2,t})][1-G_{t-1}(\bar{\omega}_{1,t-1})]H_{t-1} \right\}$$

$$+(1-\psi)\left[\widetilde{H}_{t+1} - (1-\delta)\widetilde{H}_t\right] + g(Y_{H,t} - Y_{H,t-1}). \tag{27}$$

Output in the housing sector net of monitoring costs is equal to

$$Y^N_{H,t} = Y_{H,t} - \psi\mu\left[(1-\delta)G_t(\bar{\omega}_{1,t}) + (1-\delta)^2 G_t(\bar{\omega}_{2,t})[1-G_{t-1}(\bar{\omega}_{1,t-1})]\right] H_t. \tag{28}$$

Equilibrium in the labor market requires

$$\psi N_{j,t} = \psi \int_0^1 N_{j,t}(k)\, dk = N^d_{j,t} \int_0^1 \left(\frac{w_{j,t}(k)}{w_{j,t}}\right)^{-\varepsilon_w} dk \qquad j = C, H \tag{29}$$

$$(1-\psi)\widetilde{N}_{j,t} = \psi \int_0^1 \widetilde{N}_{j,t}(k)\, dk = \widetilde{N}^d_{j,t} \int_0^1 \left(\frac{\widetilde{w}_{j,t}(k)}{\widetilde{w}_{j,t}}\right)^{-\varepsilon_w} dk \qquad j = C, H \tag{30}$$

where $N^d_{j,t} = \int_0^1 N_{j,t}(i)di$ and $\widetilde{N}^d_{j,t} = \int_0^1 \widetilde{N}_{j,t}(i)di$. Equilibrium in the credit market is achieved if

$$\psi l_t(1+\Theta) = (1-\psi)\widetilde{l}_t. \tag{31}$$

Land is in fixed supply

$$\int_0^1 A_{l,t}(i)di = A_{l,t} = \bar{A}_l. \tag{32}$$

We define total output as

$$Y_t = Y_{C,t} + p_{H,t}Y_{H,t}. \tag{33}$$

Notice that our measurement of total output reflects variations in the relative price of housing.

National account statistics, on the other hand, measure GDP at constant relative prices.

3.5 Exogenous Shocks

We take technology as deterministic and set $A_{C,t} = A_{H,t} = 1$ for all t. As for the idiosyncratic risk in the housing sector, we follow Bernanke et al. (1999) and assume that $\omega_{j,t}$ is distributed log-normally:

$$\ln \omega_{j,t} \sim N(-\frac{\sigma_{\omega,t}^2}{2}, \sigma_{\omega,t}^2), \qquad j = 1, 2. \tag{34}$$

In words, idiosyncratic shocks in the first and second period of the mortgage are different but drawn from the same distribution. As stated earlier, the mean of the distribution is chosen so that $E_t(\omega_{j,t+1}) = 1$ for $j = 1, 2$. We believe that housing investment risk increased with the entrance in the mortgage market of subprime debtors. Our model captures this increase in riskiness as an exogenous time variation in the standard deviation of idiosyncratic housing investment risk: we assume that the standard deviation of $\ln \omega_{j,t}$ follows a first-order autoregressive process:

$$\ln \frac{\sigma_{\omega,t}}{\sigma_\omega} = \rho_\sigma \ln \frac{\sigma_{\omega,t-1}}{\sigma_\omega} + \varepsilon_{\sigma_{\omega,t}}, \tag{35}$$

where $\varepsilon_{\sigma_{\omega,t}}$ is an i.i.d. shock with mean zero and finite standard deviation σ_{σ_ω} and ρ_σ is the serial correlation coefficient.

Private agents know these exogenous processes and use them to form correct expectations.

4 Steady-State Analysis

4.1 Benchmark Calibration

Table 2 summarizes the parameter values for our benchmark calibration. We follow Monacelli (2009) in choosing the rate of depreciation for housing and the elasticity of substitution between non-durable goods and housing services. We choose an annual depreciation rate for housing of 4 percentage points, implying $\delta = 0.01$. The elasticity of substitution between non-durable consumption and housing is $\eta = 1$, which implies a Cobb-Douglas specification for the composite consumption index X_t. The Saver's discount factor γ is set equal to 0.99, which pins down the steady-state deposit interest rate at $R_D = 0.0076$ on a quarterly basis. This implies an annual risk-free interest rate equal of 3.06 percentage points. The average 6-Month Certificate of deposit rate net of inflation in the personal consumption expenditure chain-price index over the period 1964Q4 to 2011Q3 is 2.4 percentage points. The Borrowers' discount factor β is set equal to 0.95. The difference between Savers' and Borrower's discount factors plays a role in determining the tightness of the participation constraints and thereby the steady-state mortgage interest rates, R_{Z1} and R_{Z2}.

U.S. private fixed investment in structures, residential and nonresidential, has been on average 5 percent of GDP from 1960 to 2009, while during the period 2000 to 2007 it averaged 8 percent of GDP. We set the parameter α that measures the share of housing in the consumption bundle equal to 0.16, so that the housing sector represents 8 percent of total output at the steady state. The inverse of the Frisch elasticity of labor supply φ is set equal to one, as in Barsky et al. (2007) and as typical in the macro literature. As for the parameter ξ that measures the degree of substitutability between hours worked in the two sectors, we set it equal to 0.871. This is the appropriate weighted average of the ξ for Borrowers and Savers estimated by Iacoviello and Neri (2010).

We assume that housing prices are fully flexible. For non-durable goods, θ_C is set equal to 0.67 to imply that firms in the non-durable sector change their prices on average every nine months. We set $\varrho = \tilde{\varrho}$, the Calvo probability for wages in the C and H sector, equal to 0.73. This implies that, on average, wages are changed less often than prices in the non-durable and housing sectors. For monetary policy we set $\phi_\pi = 1.5$, as standard in the literature. For the benchmark calibration we set $\phi_r = 0.9$ because interest rate inertia mimics the zero lower bound, which was reached in 2009Q1. We assume that the Borrower and Saver groups have equal size so that $\psi = 0.5$.

For technology, we follow Calza et al. (2011) and set the elasticity of substitution among intermediate goods ε equal to 7.5 in each sector. Labor inputs are imperfect substitutes in production and the elasticity of substitution across Borrower's and Saver's labor is $\varsigma = 3$. We assume that the share of Borrower's labor in the production function ζ is equal to 0.5. We follow Altig, Christiano, Eichembaum and Linde (2011) and assume a steady-state markup of wages over the marginal rate of substitution between leisure and consumption of 5 percent and we set $\varepsilon_w = 21$. We believe that housing risk shocks are persistent but there is no previous work we can rely on. Christiano et al. (2009) estimate the persistence of the idiosyncratic productivity

Parameter	Value	Description
γ	0.9925	Discount factor of Savers
β	0.95	Discount factor of Borrowers
ψ	0.5	Relative size of Borrower group
δ	0.01	Rate of depreciation for housing
ε	7.5	Elasticity of substitution for intermediate goods
ε_w	21	Elasticity of substitution for labor services
ς	3	Elasticity of substitution across labor inputs
ζ	0.5	Share of Borrower labor in the production function
ξ	0.871	Elasticity of substitution across labor types
α	0.16	Share of housing in consumption bundle
ν	2.5	Disutility from work
η	1	Elasticity of substitution between C and H goods
φ	1	Inverse of elasticity of labor supply
θ_C	0.67	Calvo probability in C
ϱ	0.73	Calvo probability wages in C and H
ϕ_π	1.5	Taylor-rule coefficient on inflation
ϕ_r	0.9	Taylor-rule coefficient on past nominal interest rate
σ_ω	0.1	Standard deviation of idiosyncratic shocks
ρ_σ	0.9	Serial correlation of risk shocks
μ	0.2283	Monitoring cost
$\bar{A}_l$	0.5	Land fixed supply
κ	0.9	Share of labor in the housing production function
Θ	0.0085	Financial intermediation cost
χ	0.5	Adjustment cost in housing production

Table 2: Benchmark Calibration

Variable	
x	0.5
Loan-to-Value Ratio	78
Leverage Ratio	63.42
Default Rate 1†	4
Default Rate 2†	0
Average Default Rate †	2
Mortgage Interest Rate 1 †	4.4
Mortgage Interest Rate 2 †	6.8
External Finance Premium 1†	3.0
External Finance Premium 2†	3.8
Average External Finance Premium†	3.4
Credit Spread †	3.4

Percentage points
†Annual
Note: The Leverage Ratio is calculated as $l/(l + w_C N_C + w_H N_H)$

Table 3: Steady State under the Benchmark Calibration and $x = 0.5$

shock for the United States to be 0.85. We set $\rho_\sigma = 0.9$.

Regarding the mortgage market, we need to specify values for the parameters x, σ_ω and μ; at the same time we want to match the pre-crisis delinquency rate and the LTV ratio. Since we do not have data on the average amortization schedule, we set $x = 0.5$ and consider mortgage loans with an equal principal repayment in every period for our benchmark scenario. The U.S. LTV ratio was equal to 77 percentage points on 2006Q4 and its average value between 1973Q1 and 2010Q4 was 76 percentage points.[7] According to the National Delinquency Survey of the Mortgage Banker Association, seriously delinquent mortgages are all mortgages more than 90 days past due or in foreclosure. U.S. seriously delinquent mortgages averaged 2.3 percent of total mortgages between 1979Q1 and 2010Q4 and they represented 2.2 percent of total mortgages in 2006Q4. In our model, the LTV ratio and the delinquency rate are non-linear functions of σ_ω and μ. Higher monitoring costs reduce loans and thereby the LTV ratio and the default rate; higher idiosyncratic volatility lowers mortgage loans and the LTV ratio and raises the default rate. We choose the standard deviation of idiosyncratic housing price shocks σ_ω to be equal to 0.1 at the steady state. Given the chosen value for σ_ω, we set monitoring costs μ equal to 0.2283 to find a steady-state LTV ratio between 75.5 and 78.5 percentage points, depending on the value of the parameter x, which matches the pre-crisis LTV ratio of 77 percentage points. Table 3 reports the steady-state value of mortgage market variables under the benchmark calibration. For $x = 0.5$ the LTV ratio is 78. The mortgage market parameters generate an average default rate of 2 percent under the benchmark calibration, which is the weighted average of default rates on one- and two-period loans.

We set the parameter Θ that measures the cost of financial intermediation equal to 0.0085. Given the parameters μ and σ_ω of the mortgage market, this cost together with the difference in discount factors between Savers and Borrowers determines the adjustable mortgage rates. In our model the adjustable mortgage rate on one-period loans is 4.4 percentage points and the mortgage rate on two-period loans is 6.8 percentage points; the average adjustable mortgage rate with $x = 0.5$ is therefore 5.6 percentage points. Between 1980 and 2009 the Adjustable Rate Mortgage Index[8] has averaged 8.7 percentage points, which implies an average real rate of

[7]Source: Terms on Conventional Single-Family Mortgages, Monthly National Averages, All Homes, Federal Housing Finance Agency.

[8]Data from the Finance Board's Monthly Survey of Rates and Terms on Conventional Single-Family Nonfarm Mortgage Loans and reported by the Federal Housing Finance Agency.

5.3 percentage points. The parameter Θ also generates the spread between the lending and the deposit rate. In the data, the spread measured as the difference between the bank prime loan rate and the 6-Month certificate of deposit was equal to 3 percentage points in 2006Q4 and it averaged 2 percentage points between 1964Q3 to 2011Q3. Our model produces a credit spread of 3.4 percentage points. The average external finance premium is 3.4 percentage points. The leverage ratio for Borrowers at the steady state is calculated as

$$\text{Leverage Ratio} = \frac{l}{l + w_C N_C + w_H N_H},$$

which measures the fraction of total expenses financed by total loans, namely consumption of C and H plus loan repayment over loans. The leverage ratio captures the dependence of Borrowers from external funding.

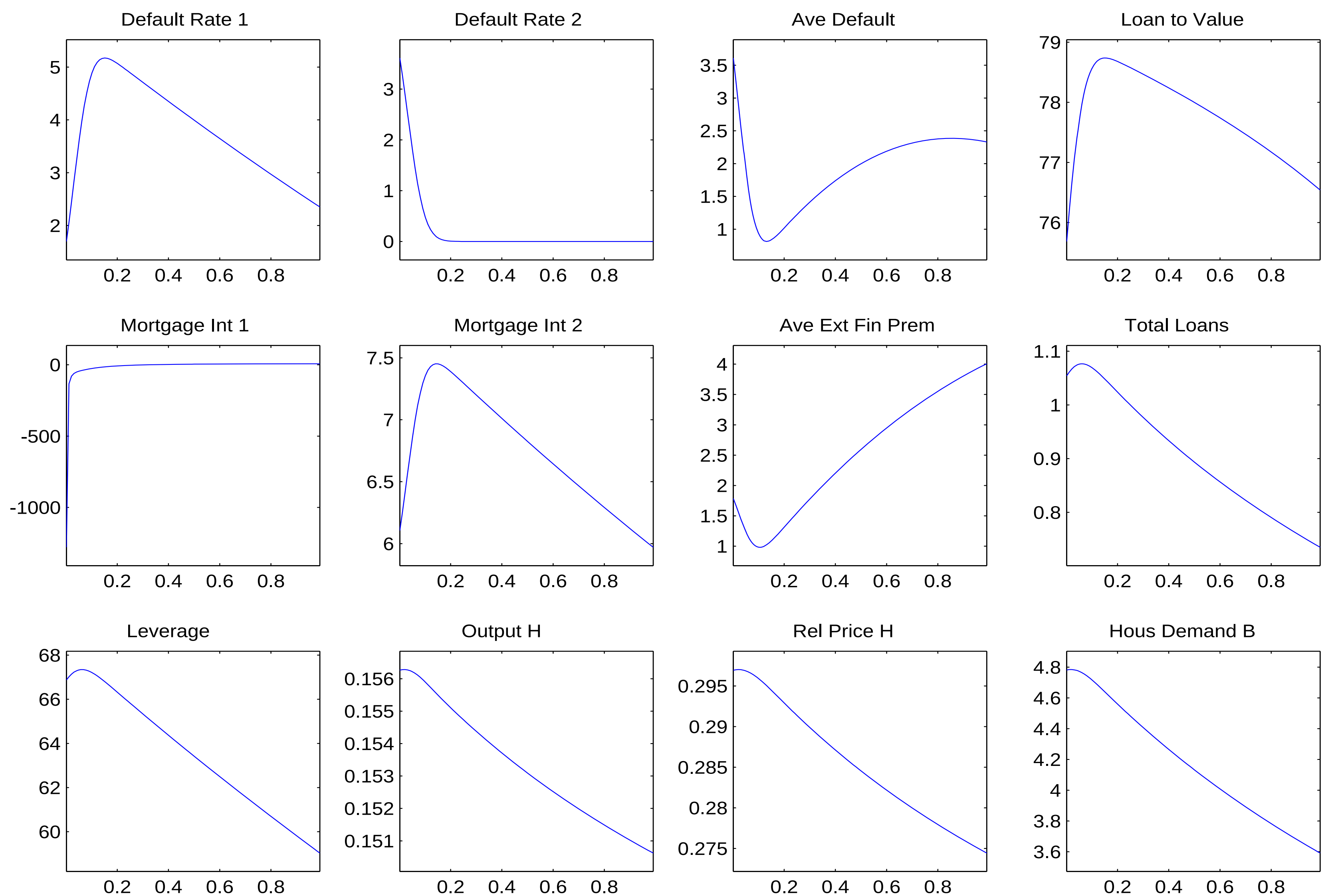
4.2 Comparative Statics

The parameter x is the fraction of the total mortgage L that must be repaid in the first period. This parameter therefore measures how much amortization is front-loaded. *High* values of x correspond to high early or front-loaded amortization as most of the principal is repaid early on in the term. On the other hand, $x = 0$ implies that the entire mortgage, principal plus interests, is repaid at the end of the second period with no intermediate repayment or amortization. In general, we are going to refer to low values of x as the case of *low* early amortization in the sense that a small fraction of the mortgage is amortized early in the term. Notice that low early amortization may include negative amortization in our model, which occurs when the first-period repayment is less than the accrued interest on the total principal. On the other hand,

Figure 3 shows the steady state of our model as x varies in the interval $(0, 1)$ under our benchmark calibration. The financial variables exhibit non-monotonic behavior. Borrower's incentives depend on whether default on two-period loans is zero or positive in equilibrium. Consider the case of low early amortization, namely x small. For low values of x the Borrower has an incentive not to default in the first period, hold on to the housing stock for an additional period (and get utility out of it) and default in the second period. When the first-period repayment is low, the value of postponing default, while getting housing services anyway, is high. This explains why the first-period default rate starts low and increases with x, while the second-period default rate starts high and falls with x. When the first-period repayment is low, the value of the housing stock seized as collateral is high enough to support low or even negative first-period adjustable mortgage rates, i.e. the non-defaulting members may even pay less than their principal. The default rate on first-period loans is *high* relative to the repayment because the decision to default depends on the present value of all liabilities, namely current and future payments due. On the other hand, the adjustable mortgage rate guarantees the pre-determined rate of return to Savers.

As x continue rising, the incentive to postpone default to the second period gradually disappears. As the first-period repayment becomes larger, default on one-period loans increases while default on two-period loans falls. Repaid one-period loans act as a sunk cost on second-period decisions. The adjustable mortgage rate on one-period loans increases quickly with the first-period repayment; the adjustable mortgage rate on two-period loans also increases, driven by an increase in the volume of such loans. The average external finance premium follows the behavior of the average default rate. Beyond a certain value of x the Borrower finds housing investment less attractive, as he cannot hold on to the house for two periods by paying little in the first installment and defaulting on the second one. Hence, total loans and Borrower housing demand peak at a low value of x and then fall.

The incentive described above continues as long as the default rate in the second period is positive. For x above a critical value, Borrower has no incentive to default in the second period after having repaid in the first. Further increases in x reduce the default rate on one-period loans as well as the total amount of lending. Larger first-period installments require higher adjustable mortgage rates R_{Z1} to be paid by non-defaulting Borrowers, which in turn reduce the default rate on the first installment. At this point the Borrower can better smooth borrowing costs by reducing his leverage. The mortgage rate on two-period loans and the LTV ratio fall, which effectively reduce Borrower's demand for housing and loans. Borrowing-constrained agents are not indifferent about the repayment schedule and a larger anticipated repayment (a higher x) renders the mortgage less appealing. Borrower's housing demand drives output in the housing sector and house prices, which are high for low values of x and fall afterwards. Our steady-state analysis suggests that non-traditional mortgage products that reduce initial mortgage repayments, such as negative-amortization or interest-rate-only mortgages, increase aggregate



Default rates, external finance premium and mortgage interest rate are annual and in percentage points. The loan-to-value and leverage ratios are in percentage points.

Figure 3: Steady State as x varies in the interval $(0, 1)$

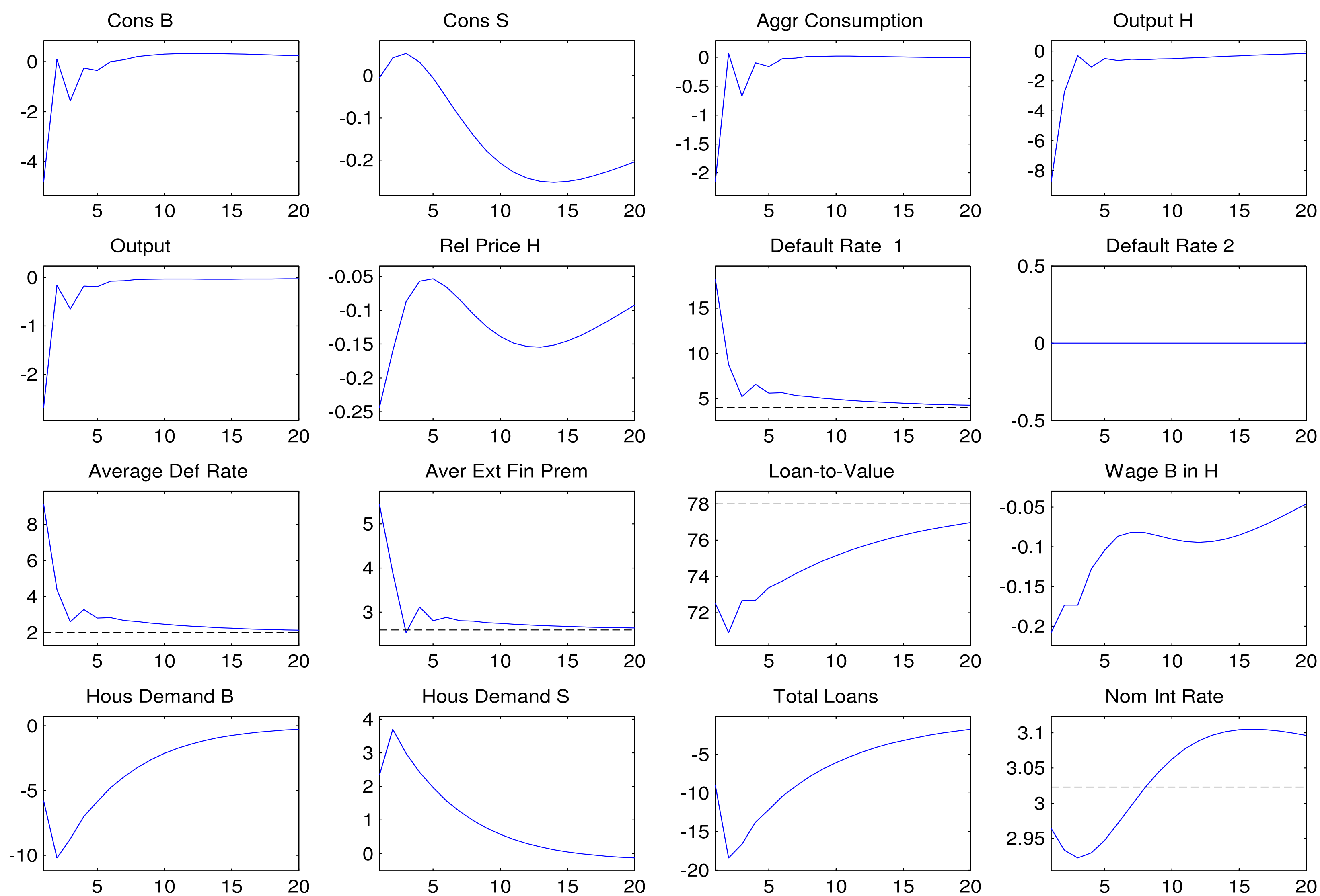
housing demand, housing prices and leverage.

5 Dynamic Simulation

5.1 Mortgage Default Crisis

We analyze the dynamic response of our model to an unanticipated increase in $\sigma_{\omega,t}$, the standard deviation of the distribution of idiosyncratic housing investment risk ω_1 and ω_2. This increase in risk wants to capture the situation in which loans are made on the basis of an expected distribution for idiosyncratic risk, but the actual distribution turns out to be characterized by a higher standard deviation. Hence, the riskiness of mortgages changes over time and these changes are persistent. A mean-preserving increase in $\sigma_{\omega,t}$ implies an increase in the skewness of the distribution of $\omega_{j,t}$. Since the log-normal distribution does not take negative values, the lower tail of the distribution becomes thicker. Thus, for the same value of $\bar{\omega}_{j,t}$, a higher standard deviation implies a higher cumulative distribution function and therefore a higher default rate on mortgages.

Figure 4 shows the impulse responses of our model to a 50% increase in σ_{ω}. Default and interest rates are annual and in percentage points. The size of the shock is chosen so as to raise the average default rate on impact to around 10%, as the delinquency rate on U.S. real estate loans reached 10.44% in the first quarter of 2010. In this example $x = 0.5$. Hence, equilibrium default on two-period loans is equal to zero. The shift in the distribution of $\omega_{1,t}$ and $\omega_{2,t}$ increases the rate of default on one-period loans while leaves default on two-period loans equal to zero. The financial conditions of Borrowers worsen significantly. First, Borrower household members who default experience a housing loss while those who repay face on average higher adjustable mortgage rates and external finance premia. In fact, the adjustable mortgage rate on two-period loans (not shown in the graph) increases to compensate Savers for the two-period loans that will not be repaid because of default on the first period installment. As a result, the average external finance premium goes up. Second, the LTV ratio falls, thereby limiting the capacity to borrow out of the existing housing stock. Third, Borrowers experience an increase in real debt via the Fisher effect stemming from deflation (not shown in the graph). Fourth, wages fall in both sectors (not shown in the graphs), which in turn trigger a reduction in hours



Note: Default rates, external finance premium and nominal interest rate are annual and in percentage points. The loan-to-value ratio is in percentage points. B = Borrower; S = Saver

Figure 4: Impulse Responses to a 50% Increase in σ_ω: $x = 0.5$

and wage income. As a result of all these effects, Borrowers de-leverage and reduce consumption of non-durable goods as well as their housing demand. Lower housing demand by Borrowers causes a fall in housing prices. Savers are consumption smoothers and respond to lower house prices and lower interest rates by raising non-durable consumption and housing demand.

Wages fall in both sectors due to reduced demand and in spite of wage rigidity. Wage differential arise in equilibrium both across groups and across sectors because of imperfect substitutability of hours. The housing sector experiences a contraction driven by Borrower's house downsizing, no matter whether output in the H sector is measured gross or net of monitoring costs. The C sector also experiences a sharp recession stemming from Borrower's reduction in consumption. Total output in the economy falls.

5.2 Amplification

Figure 5 compares the responses of our model to a housing risk shock for three different values of x: $0.99, 0.5, 0.01$. With $x = 0.99$ the mortgage is repaid almost completely in the first period. We will refer to this case as high early amortization. $x = 0.5$ represents the case of equal amortization among the two periods; $x = 0.01$ is the case where the mortgage is almost entirely repaid at the end of its term. We will refer to this case as low early amortization. Figure 5 shows that reducing x and therefore reducing early amortization generates amplification. Moving from $x = 0.99$ to $x = 0.01$ makes the fall in output deeper and more persistent; the housing sector experiences a much deeper and sharper recession with low early amortization.

Once again Borrower's incentives depend on whether default on the second installment is zero or positive in equilibrium. First we compare the cases where $x = 0.5$ and $x = 0.99$. When a sizable fraction of the mortgage is repaid early on, it is extremely unlikely to experience negative equity in the house in the second period so that default on the second installment is zero. An unanticipated housing risk shock raises default on the first installment of the mortgage, which in turn raises Borrower losses due to confiscation by lenders of the housing stock connected to the defaulted loans. This negative wealth effect is higher when $x = 0.5$ because mortgages are less appealing with higher first-period repayments. In fact, Borrowers' steady-state housing stock and leverage are higher when $x = 0.5$ relative to the case where $x = 0.99$. Following a risk shock, Borrowers must cut consumption, housing and loans by more in the economy

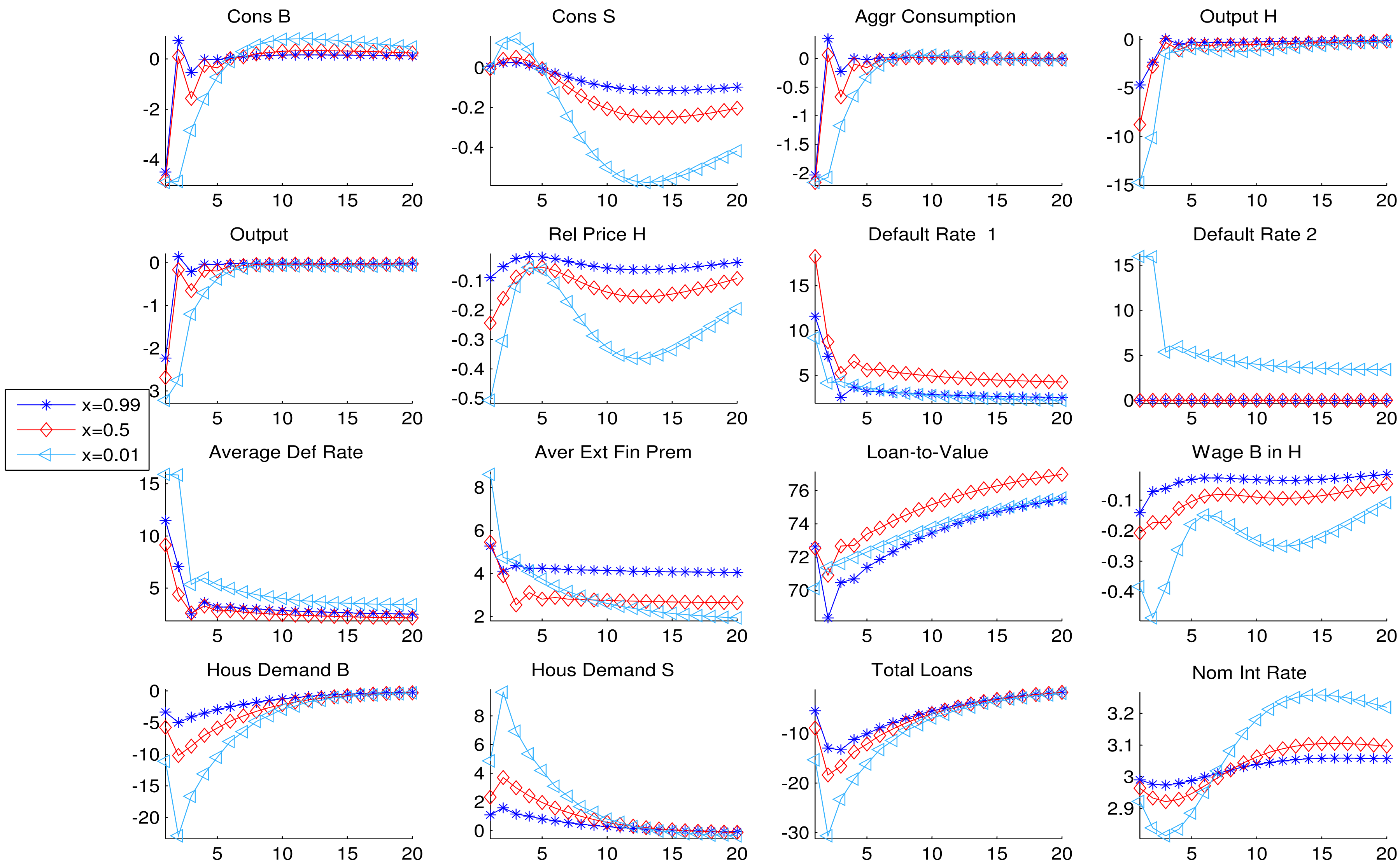


Figure 5: Impulse Responses to a 50% Increase in σ_ω: A Comparison

with higher leverage. The deeper fall in the demand of non-durable goods reduces real wages more and explains the larger drop in housing prices. These first-round effects generate further contraction and amplification via their downward pressure on house prices. Lower house prices exacerbate default on one-period loans as more Borrower members experience negative equity in the house, thereby further worsening their financial conditions and the economic downturn.

Consider now the impulse responses to a housing risk shock when $x = 0.01$. The default rate on two-period loans is positive in equilibrium. Beside the stronger negative wealth effect due to higher leverage, an additional factor contributes to generate amplification: the strategic incentive to postpone default. Borrowers prefer to default on the second installment of the mortgage so as to keep the housing stock for an additional period. As a result, the default rate on one-period loans responds by less with low early amortization relative to the case where $x = 0.5$ or $x = 0.99$ while default on two-period loans increases substantially and remains high until outstanding contracts are replaced by new ones. The strategic incentive to postpone default combined with the presence of multi-period contracts generates *persistence* in the average default rate, output in the housing sector, aggregate consumption of nondurable goods and total output. We believe that mortgage contracts extending to more periods would make the effects of our risk shock even more persistent. Lower default on the first installment leaves the Borrower household with a larger housing stock and reduces on impact his demand for new houses,[9] which further depresses housing prices and delays the rebound in the housing sector. The steeper fall in house prices generates second-round effects by triggering more Borrower members to have negative equity in their houses and consequently to default.

5.3 Optimal Amortization

So far the parameter x has been taken as given. Borrowers could increase or decrease their total mortgages keeping the ratio of one- and two-period loans constant. This section relaxes this assumption and allows Borrowers to choose optimally the amortization schedule, namely to choose $L_{1,t+1}$ and $L_{2,t+1}$. In terms of the model, Borrowers now have a separate first-order condition for each of these variables.

For our benchmark calibration Borrowers choose $x^{opt} = 0.012$. In words, the one-period loan

[9] To see why this is the case recall that $H_{t+1}^{T} = H_{t+1} + (1-\delta)(1-G(\bar{\omega}_{1,t}))H_t$.

represents 1.2% of the total mortgage and early amortization is very low. Positive default rates arise both on first- and second-period installments. Hence, if free to choose, Borrowers prefer low early or even negative amortization – in fact, $x = 0.012$ implies negative amortization in our model. Not surprisingly, Borrowers prefer to have small first-period installments and to repay most of the loan at the end of the term. This allows them to postpone default to the end of the contract and take advantage of the housing stock for an additional period. Low early amortization schedules imply low first-period repayments and thereby low sunk costs. Interestingly, the Borrower does not choose $x = 0$, namely he does not choose to eliminate first-period payments altogether.

The responses of our model under optimal amortization are similar to the case where $x = 0.1$ shown in Figure 5 and we do not repeat them here.

6 Conclusions

We build a housing model with mortgages that last two periods and endogenous default. The mortgage contract in our model is admittedly simple (and simpler than the contracts found in reality), but rich enough to capture the strategic incentives of mortgage contracts characterized by low early amortization. When principal repayment is *very* low in the early periods of the contract, the borrower has an incentive to hold on to the housing stock and postpone default. This leads to higher leverage at the steady state and to deeper contractions in response to a housing risk shock. We believe that both motives played an important role in the U.S. mortgage default crisis of 2008-09.

Our model can be a starting point for policy analysis. The degree of early amortization plays an important role in individual decision to default and thereby in the transmission of shocks. In our theoretical framework the cost of default is entirely borne by borrowers. It would be interesting to analyze the case where lenders participate in such losses. Coupled with a meaningful financial sector, this extension could provide a framework for analyzing the effects of the mortgage default crisis on financial intermediation.

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A Model Equations

A.1 Borrowers

The first-order conditions relative to $C_t, H_{t+1}, N_{C,t}, N_{H,t}, l_{t+1}, \bar{\omega}_{1,t+1}, \bar{\omega}_{2,t+2}$, given x, are:

$$U_{C,t} - \lambda_{BC,t} = 0, \tag{36}$$

$$\begin{aligned} U_{H,t} - \lambda_{BC,t}p_{H,t} + \beta(1-\delta)E_t\{U_{H,t+1} + (1-\mu)G_{t+1}(\bar{\omega}_{1,t+1})p_{H,t+1}\lambda_{BC,t+1} + \lambda_{PC,t+1}p_{H,t+1} \\ \times[\Gamma_{t+1}(\bar{\omega}_{1,t+1}) - \mu G_{t+1}(\bar{\omega}_{1,t+1})]\} + \beta^2(1-\delta)^2 E_t p_{H,t+2}\{\lambda_{BC,t+2}[1-G_{t+1}(\bar{\omega}_{1,t+1})] \\ [1-\mu G_{t+1}(\bar{\omega}_{2,t+2})] + \lambda_{PC,t+2}[\Gamma_{t+2}(\bar{\omega}_{2,t+2}) - \mu G_{t+2}(\bar{\omega}_{2,t+2})]\} = 0, \end{aligned} \tag{37}$$

$$U_{N_j,t} + \lambda_{BC,t}\frac{w_{j,t}}{\mu_{j,t}} = 0, \quad j = C, H, \tag{38}$$

$$\lambda_{BC,t} - \beta(1+R_{L1,t})xE_t\left[\frac{\lambda_{PC1,t+1} + \lambda_{BC,t+1}}{\pi_{C,t+1}}\right] - \beta^2(1+R_{L2,t})(1-x)\left[\frac{\lambda_{PC2,t+2} + \lambda_{BC,t+2}}{\pi_{C,t+1}\pi_{C,t+2}}\right] = 0, \tag{39}$$

$$\begin{aligned} -\frac{U_{H,t+1}}{\beta(1-\delta)} + E_t p_{H,t+1}\left\{\lambda_{BC,t+1}(1-\mu) + \lambda_{PC1,t+1}\left[\frac{\Gamma'_{t+1}(\bar{\omega}_{1,t+1})}{G'_{t+1}(\bar{\omega}_{1,t+1})} - \mu\right]\right\} + E_t p_{H,t+2}\beta(1-\delta) \\ \left\{\lambda_{BC,t+2}[1-\mu G_{t+2}(\bar{\omega}_{2,t+2})] + \left[\pi_{C,t+2}\frac{Q_{t+1,t+2}}{\beta}\lambda_{PC1,t+1}\right][\Gamma_{t+2}(\bar{\omega}_{2,t+2}) - \mu G_{t+2}(\bar{\omega}_{2,t+2})]\right\} = 0 \end{aligned} \tag{40}$$

$$-\lambda_{BC,t+2}\mu G'_{t+2}(\bar{\omega}_{2,t+2}) + [\lambda_{PC2,t+2} - \lambda_{PC1,t+1}\pi_{C,t+2}Q_{t+1,t+2}]\left[\Gamma'_{t+2}(\bar{\omega}_{2,t+2}) - \mu G'_{t+2}(\bar{\omega}_{2,t+2})\right] = 0, \tag{41}$$

where $\lambda_{BC,t}$ is the Lagrangian multiplier on Borrowers budget constraint (13), $\lambda_{PC1,t+1}$ is the Lagrangian multiplier on the participation constraint (15), $\lambda_{PC2,t+2}$ is the Lagrangian multiplier on the participation constraint (14), $\mu_{j,t}, j = C, H$ is the Lagrangian multiplier on the labor market constraint (18) in sector j. Notice that the first-order conditions with respect to $\bar{\omega}_{1,t+1}$ and $\bar{\omega}_{2,t+2}$ are state by state and not in expected terms.

To find the first-order conditions relative to $W_{C,t}(k), W_{H,t}(k)$, we rewrite the relevant part of the Lagrangian associated with the optimization problem of Borrowers as follows

$$\mathcal{L}_t^w = E_t \sum_{s=0}^{\infty} (\varrho\beta)^s \lambda_{BC,t+s} \left\{ \left[N_{C,t+s}^d w_{C,t}(k) \left(\frac{w_{C,t}(k)}{w_{C,t+s}} \right)^{-\varepsilon_w} + N_{H,t+s}^d w_{H,t}(k) \left(\frac{w_{H,t}(k)}{w_{H,t+s}} \right)^{-\varepsilon_w} \right] + \right.$$
$$\left. \frac{w_{C,t+s}}{\mu_{C,t+s}^w} \left[N_{C,t+s} - N_{C,t+s}^d \left(\frac{w_{C,t}(k)}{w_{C,t+s}} \right)^{-\varepsilon_w} \right] + \frac{w_{H,t+s}}{\mu_{H,t+s}^w} \left[N_{H,t+s} - N_{H,t+s}^d \left(\frac{w_{H,t}(k)}{w_{H,t+s}} \right)^{-\varepsilon_w} \right] \right\}. \tag{42}$$

The first-order condition relative to $w_{C,t}(k)$ is

$$E_t \sum_{s=0}^{\infty} (\varrho\beta)^s \lambda_{BC,t+s} N_{C,t+s}^d \left(\frac{w_{C,t}(k)}{w_{C,t+s}} \right)^{-\varepsilon_w} \prod_{z=0}^{s} \pi_{C,t+z}^{\varepsilon_w} \left[-\left(\frac{\varepsilon_w - 1}{\varepsilon_w} \right) \frac{w_{C,t}(k)}{\prod_{z=0}^{s} \pi_{C,t+z}} + \frac{w_{C,t+s}}{\mu_{C,t+s}^w} \right] = 0,$$

and the first-order condition relative to $w_{H,t}(k)$ is

$$E_t \sum_{s=0}^{\infty} (\varrho\beta)^s \lambda_{BC,t+s} N_{H,t+s}^d \left(\frac{w_{H,t}(k)}{w_{H,t+s}} \right)^{-\varepsilon_w} \prod_{z=0}^{s} \pi_{C,t+z}^{\varepsilon_w} \left[-\left(\frac{\varepsilon_w - 1}{\varepsilon_w} \right) \frac{w_{H,t}(k)}{\prod_{z=0}^{s} \pi_{C,t+z}} + \frac{w_{H,t+s}}{\mu_{H,t+s}^w} \right] = 0.$$

To write these equations in a recursive manner we define

$$f_{1j,t} = \frac{\varepsilon_w - 1}{\varepsilon_w} w_{j,t}(k) E_t \sum_{s=0}^{\infty} \lambda_{BC,t+s} N_{j,t+s}^d \left(\frac{w_{j,t}(k)}{w_{j,t+s}} \right)^{-\varepsilon_w} \prod_{k=0}^{s} \pi_{C,t+k}^{\varepsilon_w - 1}, \quad j = C, H,$$

$$f_{2j,t} = -(w_{j,t}(k))^{-\varepsilon_w} E_t \sum_{s=0}^{\infty} \lambda_{BC,t+s} N_{j,t+s}^d w_{j,t+s}^{\varepsilon_w} \frac{w_{j,t+s}}{\mu_{j,t+s}} \prod_{k=0}^{s} \pi_{C,t+k}^{\varepsilon_w}, \quad j = C, H.$$

Making use of (38) and simplifying, the wage-setting equations simplify to

$$f_{1j,t} = \frac{\varepsilon_w - 1}{\varepsilon_w} w_{j,t}(k) N_{j,t}^d \left(\frac{w_{j,t}(k)}{w_{j,t}} \right)^{-\varepsilon_w} + \varrho\beta E_t \pi_{C,t+1}^{\varepsilon_w - 1} \left(\frac{w_{j,t+1}(k)}{w_{j,t}(k)} \right)^{\varepsilon_w - 1} f_{1j,t+1}, \quad j = C, H, \tag{43}$$

$$f_{2j,t} = -U_{N_j,t} \left(\frac{w_{j,t}(k)}{w_{j,t}} \right)^{-\varepsilon_w} N_{j,t}^d + \varrho\beta E_t \pi_{C,t+1}^{\varepsilon_w} \left(\frac{w_{j,t+1}(k)}{w_{j,t}(k)} \right)^{\varepsilon_w} f_{2j,t+1}, \quad j = C, H, \tag{44}$$

$$f_{1j,t} = f_{2j,t}, \quad j = C, H. \tag{45}$$

The Borrowers' stochastic discount factor between period t and $t+1$ is given by

$$Q_{t,t+1} \equiv \frac{1}{1 + R_{L1,t}}, \tag{46}$$

which corresponds to what the Borrower should pay if he could borrow with a risk-free one-period bond.

A.2 Savers

The first-order conditions with respect to $\widetilde{C}_t, \widetilde{H}_{t+1}, \widetilde{N}_{C,t}, \widetilde{N}_{H,t}, \widetilde{l}_{t+1}, A_{l,t+1}, \widetilde{w}_{C,t}(k), \widetilde{w}_{H,t}(k)$ are:

$$U_{\widetilde{C},t} - \widetilde{\lambda}_{BC,t} = 0, \tag{47}$$

$$U_{\widetilde{H},t+1} - \widetilde{\lambda}_{BC,t} p_{H,t} + \gamma(1-\delta) E_t \left[\widetilde{\lambda}_{BC,t+1} p_{H,t+1} \right] = 0, \tag{48}$$

$$U_{\widetilde{N}_j,t} + \widetilde{\lambda}_{BC,t} \widetilde{w}_{j,t} = 0, \quad j = C, H, \tag{49}$$

$$-\widetilde{\lambda}_{BC,t} + \gamma x (1 + R_{D1,t}) E_t \left[\frac{\widetilde{\lambda}_{BC,t+1}}{\pi_{C,t+1}} \right] + \gamma^2 (1-x)(1 + R_{D2,t}) E_t \left[\frac{\widetilde{\lambda}_{BC,t+2}}{\pi_{C,t+1} \pi_{C,t+2}} \right] = 0, \tag{50}$$

$$-p_{A_l,t} \widetilde{\lambda}_{BC,t} + E_t \left[(p_{A_l,t+1} + r_{A_l,t+1}) \widetilde{\lambda}_{BC,t+1} \right] = 0, \tag{51}$$

where $\widetilde{\lambda}_{BC,t}$ is the Lagrangian multiplier on Savers budget constraint (20). The first-order conditions relative to $\widetilde{w}_{C,t}(k), \widetilde{w}_{H,t}(k)$ are given by

$$\widetilde{f}_{1j,t} = \frac{\varepsilon_w - 1}{\varepsilon_w} \widetilde{w}_{j,t}(k) \widetilde{N}^d_{j,t} \left(\frac{\widetilde{w}_{j,t}(k)}{\widetilde{w}_{j,t}} \right)^{-\varepsilon_w} + \widetilde{\varrho} \gamma E_t \pi^{\varepsilon_w - 1}_{C,t+1} \left(\frac{\widetilde{w}_{j,t+1}(k)}{\widetilde{w}_{j,t}(k)} \right)^{\varepsilon_w - 1} \widetilde{f}_{1j,t+1}, \quad j = C, H, \tag{52}$$

$$\widetilde{f}_{2j,t} = -\widetilde{U}_{\widetilde{N}_j,t} \left(\frac{\widetilde{w}_{j,t}(k)}{\widetilde{w}_{j,t}} \right)^{-\varepsilon_w} \widetilde{N}^d_{j,t} + \widetilde{\varrho} \gamma E_t \pi^{\varepsilon_w}_{C,t+1} \left(\frac{\widetilde{w}_{j,t+1}(k)}{\widetilde{w}_{j,t}(k)} \right)^{\varepsilon_w} \widetilde{f}_{2j,t+1}, \quad j = C, H, \tag{53}$$

$$\widetilde{f}_{1j,t} = \widetilde{f}_{2j,t}, \quad j = C, H. \tag{54}$$

We follow Curdia and Woodford (2010) and assume that real non-durable resources are used in the process of originating loans. More precisely, total loans $l_t + \Theta l_t$ are required in order to originate a quantity $\tilde{l}_t = l_t$, where $\Theta > 0$. Perfect competition in the loan market then implies that $(1 + R_{Di,t}) = (1 + R_{Li,t})/(1 + \Theta)$, for $i = 1, 2$ and $\forall t$.

A.3 Intermediate Firms in the Non-Durable Sector

The maximization problem for firm i is given by

$$E_t \left\{ \sum_{k=0}^{\infty} \theta_C^k \Lambda_{t,t+k} \left[P_{C,t}^*(i) Y_{C,t+k|t}(i) - W_{C,t+k} N_{C,t+k|t}(i) - \widetilde{W}_{C,t+k} \widetilde{N}_{C,t+k|t}(i) \right.\right. \tag{55}$$

$$\left.\left. + mc_{C,t+k|t}(i) P_{C,t+k} \left[A_{C,t+k} \left[\zeta^{\frac{1}{\varsigma}} N_{C,t+k|t}(i)^{\frac{\varsigma-1}{\varsigma}} + (1-\zeta)^{\frac{1}{\varsigma}} \widetilde{N}_{C,t+k|t}(i)^{\frac{\varsigma-1}{\varsigma}} \right]^{\frac{\varsigma}{\varsigma-1}} - Y_{C,t+k|t}(i) \right] \right] \right\}$$

where $Y_{C,t+k|t}(i)$ denotes output in period $t+k$ for a firm i that last changed its price in period t. A similar interpretation applies to $N_{C,t+k|t}(i)$ and $\widetilde{N}_{C,t+k|t}(i)$. $mc_{C,t+k|t}(i)$ is the real marginal cost of a firm i that last changed its price in period t.

In (55) the demand and the stochastic discount factor are respectively given by

$$Y_{C,t+k|t}(i) = \left(\frac{P_{C,t}^*(i)}{P_{C,t+k}} \right)^{-\varepsilon_C} Y_{C,t+k}, \qquad \Lambda_{t,t+k} \equiv \frac{P_{C,t}}{P_{C,t+k}} \frac{\gamma^k \widetilde{\lambda}_{BC,t+k}}{\widetilde{\lambda}_{BC,t}}.$$

The first-order conditions relative to $N_{C,t+k|t}(i)$ and $\widetilde{N}_{C,t+k|t}(i)$ are

$$-W_{C,t+k} + mc_{C,t+k|t}(i) P_{C,t+k} A_{C,t+k}^{1-\frac{1}{\varsigma}} Y_{C,t+k|t}(i)^{\frac{1}{\varsigma}} \zeta^{\frac{1}{\varsigma}} N_{C,t+k|t}(i)^{-\frac{1}{\varsigma}} = 0, \tag{56}$$

$$-\widetilde{W}_{C,t+k} + mc_{C,t+k|t}(i) P_{C,t+k} A_{C,t+k}^{1-\frac{1}{\varsigma}} Y_{C,t+k|t}(i)^{\frac{1}{\varsigma}} (1-\zeta)^{\frac{1}{\varsigma}} \widetilde{N}_{C,t+k|t}(i)^{-\frac{1}{\varsigma}} = 0, \tag{57}$$

which state that the nominal marginal cost equals the ratio of the nominal wage to the marginal product of each type of labor input. By rearranging (56) and (57) we obtain:

$$mc_{C,t+k|t}(i) = \frac{1}{A_{C,t+k} P_{C,t+k}} \left[\zeta W_{C,t+k}^{1-\varsigma} + (1-\zeta) \widetilde{W}_{C,t+k}^{1-\varsigma} \right]^{\frac{1}{1-\varsigma}}. \tag{58}$$

According to (58), $mc_{C,t+k|t}(i) = mc_{C,t+k}$. Marginal costs are equal across firms because wages are the same across all firms.

The first-order condition relative to the price is given by

$$E_t \left\{ \sum_{k=0}^{\infty} \theta_C^k \Lambda_{t,t+k} \left[\left(\frac{P_{C,t}^*(i)}{P_{C,t+k}} \right)^{(-\varepsilon_C - 1)} Y_{C,t+k} \left(\frac{P_{C,t}^*(i)}{P_{C,t+k}} - \frac{\varepsilon_C}{\varepsilon_C - 1} mc_{C,t+k} \right) \right] \right\} = 0. \tag{59}$$

Finally, it can be shown[10] that, under Calvo price setting, the optimal price set by re-optimizing firms is linked to the aggregate price behavior by the following condition:

$$\left(\frac{P^*_{C,t}}{P_{C,t}}\right)^{(1-\varepsilon_C)} = \frac{1-\theta_C \pi_{C,t}^{\varepsilon_C - 1}}{1-\theta_C}, \tag{60}$$

where $\pi_{C,t}$ denotes gross inflation in sector C.

A.4 Intermediate Firms in the Housing Sector

$$E_t \sum_{k=0}^{\infty} \frac{\gamma^t \widetilde{\lambda}_{BC,t}}{\gamma^k \widetilde{\lambda}_{BC,t+k}} \left\{ p_{H,t+k}(i) Y_{H,t+k}(i) - p_{H,t+k} g\left(Y_{H,t+k}(i) - Y_{H,t+k-1}(i)\right) - w_{H,t+k} N_{H,t+k}(i) \right.$$
$$-\widetilde{w}_{H,t+k} \widetilde{N}_{H,t+k}(i) - r_{A_l,t+k} A_{l,t+k}(i) + mc_{H,t+k}(i) p_{H,t+k} \left[A_{H,t} A^{1-\kappa}_{l,t+k}(i) \left[\zeta^{\frac{1}{\varsigma}} N_{H,t+k}(i)^{\frac{\varsigma-1}{\varsigma}} \right.\right.$$
$$\left.\left.\left. + (1-\zeta)^{\frac{1}{\varsigma}} \widetilde{N}_{H,t+k}(i)^{\frac{\varsigma-1}{\varsigma}} \right]^{\frac{\varsigma\kappa}{\varsigma-1}} - Y_{H,t+k}(i) \right] \right\},$$

where $g(Y_{H,t}(i) - Y_{H,t-1}(i))$ are firm-level adjustment costs as in Topel and Rosen (1988) such that $g(0) = g'(0) = 0$ and $g''(0) = \chi > 0$. $R_{A_l,t}$ is the nominal rental price of land; the demand and the stochastic discount factor are respectively given by

$$Y_{H,t}(i) = \left(\frac{p_{H,t}(i)}{p_{H,t}}\right)^{-\varepsilon_H} Y_{H,t}$$

The first-order conditions with respect to $N_{H,t}(i)$, $\widetilde{N}_{H,t}(i)$ and $A_{l,t}$ are

$$-w_{H,t} + mc_{H,t}(i) p_{H,t} \kappa Y_{H,t}(i) \left[\zeta^{\frac{1}{\varsigma}} N_{H,t}(i)^{\frac{\varsigma-1}{\varsigma}} + (1-\zeta)^{\frac{1}{\varsigma}} \widetilde{N}_{H,t}(i)^{\frac{\varsigma-1}{\varsigma}} \right]^{-1} \zeta^{\frac{1}{\varsigma}} N_{H,t}(i)^{-\frac{1}{\varsigma}} = 0, \tag{61}$$

$$-\widetilde{w}_{H,t} + mc_{H,t}(i) p_{H,t} \kappa Y_{H,t}(i) \left[\zeta^{\frac{1}{\varsigma}} N_{H,t}(i)^{\frac{\varsigma-1}{\varsigma}} + (1-\zeta)^{\frac{1}{\varsigma}} \widetilde{N}_{H,t}(i)^{\frac{\varsigma-1}{\varsigma}} \right]^{-1} (1-\zeta)^{\frac{1}{\varsigma}} \widetilde{N}_{H,t}(i)^{-\frac{1}{\varsigma}} = 0, \tag{62}$$

$$-r_{A_l,t} + mc_{H,t}(i) p_{H,t} (1-\kappa) Y_{H,t}(i) A_{l,t}^{-1} = 0, \tag{63}$$

By using (61), (62) and (63) we obtain:

$$mc_{H,t}(i) = \frac{1}{A_{H,t} p_{H,t} \kappa^{\kappa} (1-\kappa)^{(1-\kappa)}} r^{1-\kappa}_{A_l,t} \left[\zeta w^{1-\varsigma}_{H,t} + (1-\zeta) \widetilde{w}^{1-\varsigma}_{H,t} \right]^{\frac{\kappa}{1-\varsigma}}. \tag{64}$$

[10] For a formal proof see for instance Woodford (2003).

Condition (64) states that the marginal cost of producing one more unit in the H sector is equal across firms and depends on Dixit-Stiglitz aggregator of the input prices, i.e. the land rental rate, Borrower and Saver wages.

The first-order condition relative to the price is given by

$$\frac{p_{H,t}(i)}{p_{H,t}} - \frac{\varepsilon_H}{\varepsilon_H - 1} \left\{ mc_{H,t} - g' \left(\left(\frac{p_{H,t}(i)}{p_{H,t}} \right)^{-\varepsilon} Y_{H,t} - \left(\frac{p_{H,t-1}(i)}{p_{H,t-1}} \right)^{-\varepsilon} Y_{H,t-1} \right) \right.$$
$$\left. + E_t \left[\frac{\gamma \lambda_{BC,t+1}}{\lambda_{BC,t}} \frac{p_{H,t+1}}{p_{H,t}} g' \left(\left(\frac{p_{H,t+1}(i)}{p_{H,t+1}} \right)^{-\varepsilon} Y_{H,t+1} - \left(\frac{p_{H,t}(i)}{p_{H,t}} \right)^{-\varepsilon} Y_{H,t} \right) \right] \right\} = 0. \quad (65)$$

According to condition (65), in the absence of adjustment costs firms set prices as a mark-up over the their marginal cost. With adjustment costs, the price is set as a mark-up over the marginal cost gross of the effect of a price change on current and future adjustment costs.