Income Risk, Saving and Taxation: Will Precautionary Saving Survive?

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Abstract

Former theoretical and empirical studies find that precautionary savings are reduced in the presence of social security systems. The saving motive, however, does not change: individuals respond to increasing income risk by increasing their savings. Although this still holds for common tax and transfer systems, we show that this is not a feature of all tax and transfers systems. In contrast to former studies, we focus on the impact of the variability of future income (higher degree risk).

Keywords: Income risk, saving behavior, social security, unemployment benefit, taxation.

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1 Introduction

Since the late 1950s, economists argue that economic agents prefer smooth consumption paths over their life-cycle and therefore aim to shift consumption between periods via saving. In the presence of uncertain future income, individuals additionally aim to insure themselves against future labor income risk [see Leland (1968) or Levhari and Srinivasan (1969)]. Risk averse individuals have to conceive a strategy to attenuate the realization of low incomes. In a laissez-faire economy, there is no other option than to save income for future periods to avoid very low levels of income. This is referred to as “precautionary savings”. In a welfare state, different aspects of the tax and transfer system (marginal tax rates, level of unemployment insurance, etc.) are likely to substitute individual savings.

The impact of unemployment insurance on precautionary savings has been part of the economic literature since the late 1970s [e.g., Flemming (1978)]. From a theoretical point of view, more recent studies identify that unemployment insurance crowds out precautionary savings [e.g. Hubbard et al. (1995), Engen and Gruber (2001)]. Based on their theoretical model, Engen and Gruber (2001) evaluate the model’s predictions empirically. They use differences in unemployment insurance systems across US states to estimate the impact of unemployment insurance on savings. Their results suggest that a reduction of unemployment benefits leads to an increase in the savings of individuals. Furthermore, the crowding-out effect of unemployment insurance on savings is greater the higher the unemployment risk.

Although the results of former theoretical and empirical studies show that precautionary savings are reduced by social security systems, the saving motive itself remains intact. Savings still increase with labor income risk. However, this increase is either implicitly or explicitly based on a shift in expected total income. Following the nomenclature of Eeckhoudt and Schlesinger (2008), we define this shift as an increase in first degree income risk. More general, an increasing income risk is defined as any change in the distribution of future labor
income. If the mean is kept constant, i.e. only higher degrees of income risk are considered, this will be referred to as a change of the variability of labor income. Whenever the focus is on the variability of income only, the question arises whether the presence of taxes and transfers not only crowds out, but even overrides the precautionary savings motive.

In this paper we address the impact of a tax and transfer system theoretically by using a stochastic dominance approach. In contrast to mean-variance analysis this method eases the comparative statics [Rothschild and Stiglitz (1971)], allowing us to obtain easily interpretable results. We introduce a tax and transfer system into the risk-savings model developed by Gunning (2010). Within this model we analyze the effect of the tax and transfer system on the response of savings to a change in income risk. The results depend on the design of the tax and transfer system. There is a wide range of designs that will not override the savings motive. Only in some cases does the specific design of the tax and transfer system have the potential to induce decreasing savings when the variability of future labor income rises.

The remainder of this paper is organized as follows. In section 2 we briefly summarize the theoretical model by Gunning (2010). Section 3 describes our extension to this model, presents the results and discusses our findings with focus on the institutional framework. Section 4 concludes.

2 Gunning’s Model

The risk-savings model by Gunning (2010) distinguishes four types of risk: wealth risk, asset risk, capital income risk and labor income risk. The model is designed to analyze the effect of an increasing risk in any of these four types of risks on the saving behavior of economic

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1 In the literature both approaches are used to analyze risk and uncertainty for various economic issues. For a comparison see e.g. Yitzhaki (1982).
agents. For the aim of our paper, it is sufficient to summarize the basic framework for labor income risk only.

In the model an economic agent maximizes lifetime utility \( U \) over two periods. The utility function of the agent is characterized by constant relative risk aversion (CRRA), is concave and time-additive. The first-period income is certain and the second-period labor income is stochastic. In the first period, the agent consumes a fraction of his given financial wealth \( w \) and transfers the rest into the second period by saving an amount \( k \) \((c_1 = w - k)\).\(^2\) The agent maximizes expected lifetime utility with respect to savings \( k \):

\[
\max_k U = u(w - k) + \beta Eu(c_2). \tag{1}
\]

The instantaneous utility \( u(.) \) in the first period is determined by first-period consumption. Expected utility in the second period is discounted by the time preference parameter \( \beta \) and depends on second-period consumption \((c_2)\). Assuming secure capital assets (no depreciation), second-period consumption can be written as

\[
c_2 = f(y, k, r, s) = y - (1 - s) y + (1 + r) k = sy + (1 + r) k. \tag{2}
\]

Second-period consumption is determined by the mean labor income \( y \), savings \( k \), the interest rate \( r \), and an exogenous stochastic income shock \( s \) which is drawn from a known distribution. The shock has an expected value of unity and is only defined for positive values \((s \geq 0, E[s] = 1)\). Savings and the interest from savings increase second-period consumption. Maximization of lifetime utility gives the well-known Euler condition:

\[
u'(c_1) = \beta E \left[ u'(c_2) \frac{\partial c_2}{\partial k} \right]. \tag{3}
\]

With the help of equation (3) one can investigate the impact of an increasing risk on savings.

\(^2\) Debt is excluded, as the borrowing constraint is binding, so that no debt exists at the end of the second period [Gunning (2010), Engen and Gruber (2001) and Eeckhoudt and Schlesinger (2008)]. The random second-period income does not necessarily guarantee sufficient funds to pay first-period debt.
From the right-hand side of equation (3), the term in brackets will be defined as a function of \( s \), \( \phi(s) \equiv u'(c_2) \frac{\partial c_2}{\partial k} \). Then, from Jensen’s inequality, it can be shown that if \( \phi(s) \) is strictly convex in \( s \), a mean-preserving spread in the distribution of \( s \) will increase savings [Rothschild and Stiglitz (1971)]. Figure 1 illustrates that mechanism in an example with a simplified two-point distribution of \( s \) \((E[s] = 1)\).

**Figure 1: Effect of an increase in second-order risk**

![Figure 1](image)

Source: Authors’ illustration.

Figure 1 depicts marginal second-period utility of savings \( \phi(s) \) as a function of second-period consumption. Higher second-period consumption is associated with a lower marginal utility of saving, thus, the function must be decreasing. The points \( A_1 \) and \( A_2 \) depict an arbitrary two-point distribution with probability mass of one half in each point. Thus, expected

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3 Consider an individual who aims to maximize her expected utility,

\[ E[U] = \int U(\theta, \alpha) dF(\theta). \]

The variable \( \theta \) is random and \( \alpha \) is the control variable the individual chooses to maximize utility. The optimal \( \alpha \) must satisfy \( E[U_\alpha(\theta, \alpha)] = 0 \). If \( U \) is monotonically decreasing in \( \alpha \) and \( U_\alpha(\theta, \alpha) \) is a convex function of \( \theta \), an increase in risk (increase in the variability of \( \theta \)) will raise \( E[U_\alpha(\theta, \alpha)] \). Hence, \( \alpha_{opt} \) will increase as well.
consumption will be in the middle of the dashed line connecting $A_1$ and $A_2$. A mean-preserving spread that leaves the probability masses unchanged now shifts the points outwards to $B_1$ and $B_2$, respectively. The dashed line that connects these points shifts upwards. The vertical dotted line marks expected second-period consumption, the horizontal ones expected marginal utility. Thus, with convex marginal utility (as shown in Figure 1) a mean-preserving spread in the distribution of $s$ increases c. p. the expected marginal utility of consumption. This, in turn, leads to an increase in the right-hand side of equation (3). In order for the Euler condition to hold, the savings $k$ need to be adjusted. An increase in $k$ will increase the left-hand side and decrease the right-hand side decreased. Thus, in order for the Euler condition to be met again, savings must rise.

Gunning (2010) shows that for the case of labor income risk, $\phi(s)$ is always strictly convex in the shock $s$. Thus, increasing variability of labor income (an increase in higher degree risk) always increases savings. This result complies with the empirical findings mentioned above.

3 Extending Gunning’s Model

Taxes and transfers scale second-period consumption, since they do not affect income ordering. Thus, when introducing taxes and transfers to Gunning’s model, one might assume that the result of increasing savings in the presence of higher risk continues to hold. Therefore a mean-preserving spread in the distribution of $s$ should still cause an increase in second-period income risk. To examine in detail the impact of a tax and transfer system on the savings decision we modify Gunning’s (2010) model by adding a tax term $T$ to second-period consumption.\(^4\) Moreover, we set the average second-period labor income $y$ equal to unity

\(^4\) Taxes and transfers cannot be transferred between both periods. Thus, first-period taxes and transfers can be neglected and the financial wealth of the first period is given as a net value.
which enables us to interpret the stochastic parameter \( s \) as the stochastic gross labor income. 

Second-period consumption defined in equation (2) is thus changed to

\[
c_2 = s + (1 - r)k - T(s, (1 + r)k). \tag{4}
\]

The tax \( T \) is a function of second-period gross labor income \( s \) and the interest-bearing savings \((1 + r)k\). The expected tax burden faced by an individual is assumed to be positive and not affected by a spread in the labor income distribution.\(^5\) Depending on the savings and the realization of \( s \), the individual tax burden might become negative. Hence, equation (4) also includes the possibility of low income households receiving transfers. This kind of modeling provides several advantages. First, it allows the derivation of an expression for the marginal utility of savings, \( \phi(s) \). This expression can be used to evaluate the second order characteristics without further mathematical complexity. Second, the definition of the tax and transfer system is very flexible. It easily captures many different regimes. Finally, the general definition of the tax term eases the formal analysis and discussion.

One drawback of our notion of a negative tax is that it describes only top-up benefits. Including a case in which a worker does not work as soon as the second-period labor income falls below a certain threshold would require a non-continuous consumption function. This would prevent a global characterization of the convexity.

As in Gunning (2010), precautionary savings increase with an increase in variability, whenever the function \( \phi(s) \) is convex in \( s \). The modification of second-period consumption to a nonlinear function of \( s \) does not affect the mathematical implications [see Rothschild and Stiglitz (1970, 1971)]. The second derivative of \( \phi(s) \) with respect to \( s \) is given by:

\(^5\) Under this assumption, a mean-preserving spread in \( s \) leaves the mean of second-period consumption unchanged. Thus, we can analyze the impact of an increasing variability of future labor income by excluding first-order risk (mean variation). When all public transfers are included, the expectation of the net tax-payer is zero. A necessary requirement for this assumption is an immediate adaption of the tax system. While not altering the regime, levels have to be adjusted, such that agents in some higher income quantiles received transfers before the spread, but have to pay taxes after the spread.
In equation (5), \( u', u'' \) and \( u''' \) denote the first, second and third derivatives of the second-period utility function with respect to second-period consumption. The variables \( c_k \) and \( c_s \) denote the first derivative of second-period consumption with respect to savings \( k \) and gross wage \( s \), respectively. Furthermore, \( c_{ss} \) denotes the second derivative of second-period consumption with respect to gross wage \( s \). The cross derivative of second-period consumption with respect to savings and gross wage is given by \( c_{ks} \). Finally, \( c_{kss} \) represents the derivative of \( c_{ks} \) with respect to the gross wage.

Strict convexity of \( \phi(s) \) is given if and only if equation (5) is positive \( (\phi''(s) > 0) \). Rearranging the inequality \( \phi''(s) > 0 \) and substituting gives the following proposition.

**Proposition.** The precautionary savings motive survives (i.e. a mean-preserving spread in the distribution of gross labor income will increase savings) in the presence of a tax and transfer system if and only if

\[
-\frac{u'''}{u''} > -\frac{t_{ss}}{(1-T_s)^2} - 2\frac{t_{sk}}{(1-T_k)(1-T_s)} + \frac{1}{(-\frac{u''}{u'})^2(1-T_k)(1-T_s)^2} \tag{6}
\]

for all possible realizations of gross labor income \( s \) and a given choice of savings \( k \).

**Proof.** If inequality (6) holds then it follows immediately that the marginal utility of savings \( \phi(s) \) is convex in gross labor income \( s \). Thus savings must increase in response to an increase in variability of gross labor income [see Rothschild and Stiglitz (1970, 1971)].
Using a CRRA utility function, $u''$ is negative, while the third derivative is positive. Hence, the left-hand side (LHS) of inequality (6) is positive. The LHS of inequality (6) represents the index of absolute prudence [see Kimball (1990)]. Higher prudence implies that an individual saves more for a given income risk. Mazzocco (2004) points out that the coefficient of absolute prudence measures the convexity of marginal utility [Mazzocco (2004) p. 1171].

The right-hand side (RHS) of inequality (6) consists of three terms describing the properties of the tax and transfer system. The variable $T_s$ denotes the marginal labor income tax rate and $T_{ss}$ represents the progression of labor income tax.\(^6\) Analogously, $T_k$ is the marginal capital (savings) tax rate and $T_{sk}$ describes how an additional unit of capital affects the marginal labor income tax rate (and vice versa).\(^7\) Finally, $T_{ssk}$ describes how the labor income tax progression / degression changes if savings change. The third term on the RHS of inequality (6) contains the inverse of the measure of absolute risk aversion $\left(-\frac{u''}{u'}\right)$. The higher this index, the higher is the risk aversion. As the first derivative of the utility function is positive, the inverse of the measure of absolute risk aversion is positive as well.

As the LHS of condition (6) is positive, the inequality is always satisfied if the RHS is negative. However, in the case of $\text{RHS} > 0$, it is possible that $\text{RHS} > \text{LHS}$ for at least some $s$. This implies that the effect of an increasing risk on savings is ambiguous.

With the help of inequality (6), we can now derive specifications of the tax and transfer system that may override the precautionary savings motive. The level of taxes or transfers does not enter inequality (6) explicitly and therefore cannot induce concavity of $\phi(s)$ on its own. However, it enters inequality (6) via its impact on consumption and therefore via the first, second and third derivatives of the utility function.

\(^6\) If $T_{ss}$ is negative, labor income tax is degressive.

\(^7\) The capital tax can also be rearranged to a capital income tax.
For simplicity, we take the level of the tax or transfer as given (the LHS of (6) remains constant), which allows us to focus on the RHS only. Additionally, in order to restrict the discussion to plausible regimes, we consider only marginal tax rates for capital and for labor income that are positive and below unity. All denominators on the RHS of (6) are thus positive and less than unity. The higher these tax rates are (closer to unity), the larger is the impact of the remaining characteristics of the tax system.

Income tax progression i.e. $T_{ss} > 0$ will never jeopardize the inequality in (6) alone. Income tax degression, however, may induce concavity of $\phi(s)$ for some realizations of labor income and therefore may invert the precautionary savings motive. Formally, a negative $T_{ss}$ makes the first term in (6), $\frac{T_{ss}}{(1-T_0)^2}$, positive. Figure 2 illustrates such a function that is partially concave.

**Figure 2: Partially concave marginal utility**

Source: Authors’ illustration.
The solid line depicts a possible marginal utility curve under degressive income taxation. Assuming all other parameters are fixed, the function $\phi(s)$ is now concave for a certain domain of $s$. This implies that the effect of an increase in risk on savings is not necessarily positive. This is easily understood if the same exercise as in Figure 1 is repeated. The points $A_1$ and $A_2$ represent an arbitrary two point distribution with probability one half each. A mean-preserving spread shifts these points outwards to $B_1$ and $B_2$, respectively. The expected second-period consumption and marginal utility is at the half of the dashed lines. The dashed lines now intersect, with the expected marginal utility in period 2 being lower after the spread. For this simple example, an increase in expected marginal utility, even if some realizations are in the domain of $s$ in which $\phi(s)$ is convex, cannot be guaranteed.

An analogous effect may be observed if the marginal capital tax negatively depends on gross labor income ($T_{ks} < 0$), i.e. capital taxation increases as gross labor income declines. The same holds true for a marginal labor income tax if the marginal labor income tax depends negatively on the stock of private assets ($T_{sk} < 0$; capital-income-tax-responsiveness). In both cases the second term of the RHS of inequality (6), $-2 \frac{T_{sk}}{(1-T_k)(1-T_s)}$, becomes positive. Finally, if the progression / regression of labor income taxation increases with higher capital holdings, i.e. $T_{ssk} > 0$, precautionary savings may be eliminated as well. Intuitively, whenever higher savings increase the labor income tax rate, the effect of risk on saving must be ambiguous in some settings. The positive effect of precautionary savings in a high risk environment is mitigated by the negative effect of savings on the tax burden. If the negative impact on the tax

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8 An example for this case is the German labor market legislation and labor income taxation for individuals who earn wages below the top-up benefit threshold. The top-up benefit is means-tested, thus the stock of private assets (savings) affects the size of the top-up benefit. Assume that an individual of this group is endowed with a low stock of savings and realizes a low second-period income. Thus, the overall second-period consumption is close to the publicly guaranteed minimum consumption level. For very low incomes, any additional labor income or private asset reduces the transfer almost one to one. A person can only get a top-up benefit if the private savings do not exceed a specific amount, which implies a negative $T_{ks}$. 
burden outweighs the insurance function of savings, the precautionary savings motive may be lost.

However, there is no regime for which inequality (6) is reversed for all s. This would imply the strict concavity of $\phi(s)$ over the whole domain of $s$, which is excluded by $\phi(s) > 0$ and $\phi'(s) < 0$ for all $s$. In contrast there are regimes in which inequality (6) holds over the whole domain of $s$. For each combination of a flat or progressive labor income tax ($T_{ss} \geq 0$) and a capital tax rate that is independent of or increasing in gross labor income ($T_{ks} \geq 0$), $\phi(s)$ is convex in the stochastic parameter $s$ for all possible values of $s$.\(^9\) Thus, under tax and transfer regimes common in many industrialized countries, a mean-preserving spread in future labor income induces an increase in savings, implying that these tax and transfer systems do not undermine the precautionary savings motive.

### 4 Conclusion

This paper considers the response of precautionary saving to changes in labor income risk in the presence of a tax and transfer system. Previous theoretical research has established that a public transfer, which guarantees a minimum consumption (income), has an unambiguously negative impact on precautionary savings, but leaves the precautionary savings motive itself untouched. However, these papers focus on an increase in first degree risk (i.e. a change of the mean income). We examine whether their result also holds for a rise in higher degree risk (i.e. in the variability of labor income). Our analysis is based on the risk-savings model presented by Gunning (2010) which we augment by a tax and transfer term capable of flexibly capturing different regimes. Our results suggest that in most cases the precautionary savings motive remains intact. There are, however, tax regimes that may eliminate the precautionary savings motive.

\(^9\) This holds true if the labor income tax progression is independent of, or increasing in, the savings; $T_{ks} \leq 0$.\(^{11}\)
savings motive. At the margin, the following features enhance the probability of such a result: labor income tax degression, a less favorable interaction of labor income and marginal capital tax (capital-income-tax-responsiveness) as well as an increasing income tax progression / degression with higher savings (responsiveness to progression). The level of taxes and transfers only matters by shifting the relative importance of these characteristics. The most common real-world tax and transfer systems do not exhibit most of those features and hence the precautionary savings motive survives.
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