# A Two-Country NATREX Model for the Euro/Dollar

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## Abstract

This paper develops a NATREX (NATural Real EXchange rate) model for two large economies, the Eurozone and the United States, which are fully specified and allowed to interact. After description of the theoretical framework grounding on dynamic disequilibrium modelling approach in continuous time, we implement empirical analysis. First, we estimate the model in its structural form as a simultaneous nonlinear differential equation system for the 1975-2003 period. Second, we simulate the Euro/USD NATREX series in- and out-of-sample by using parameters estimates. The simulated equilibrium real exchange rate enables us to determine a benchmark against which the dynamics of the actual real exchange rate can be measured.

JEL Code: F31, F36, F47.

Keywords: NATREX, equilibrium exchange rate, Euro/USD, structural approach, continuous time econometrics, misalignment.

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# 1 Introduction

Since the introduction of the Euro as common currency for 11 member states of the EU, the Euro/USD exchange rate has surprised most observers for its highly unexpected dynamics. The study of this anomalous behavior has given rise to a growing literature that looks for theoretically coherent explanations. Despite the large number of studies on the issue, however, the state of the art does not seem to have reached any satisfactory conclusion: the dynamics of the Euro/USD from 1999 until nowadays remains mostly puzzling to the economic theory. Yet, understanding what drives the Euro/USD real exchange rate developments is crucial for both theoretical and policy implications. In this paper, rather than trying to explain the *actual* real exchange rate ($RER$), we study the determinants of the real *equilibrium* exchange rate ($REER$) of the Euro/USD in order to provide a yardstick against which the development of the actual real exchange rate is gauged.

Among the several approaches suggested by the literature on the real equilibrium exchange rate (monetary model, $FEER$ - Fundamental Equilibrium Exchange Rate, $DEER$ - Desirable Equilibrium Exchange Rate, $BEER$ - Behavioral Equilibrium Exchange Rate, $PEER$ - Permanent Equilibrium Exchange Rate, *NATREX* - NATural Real EXchange rate)[1], in this paper we adopt the *NATREX* approach due to Stein (1990). It is based on a specific theoretical dynamic stock-flow model to derive the equilibrium real exchange rate. The equilibrium concept ensures simultaneously internal (economy is at capacity output) and external (long term accounts of balance of payments) equilibrium, and reflects the behavior of the fundamental variables behind investment and saving decisions in the absence of cyclical factors, speculative capital movements and movements in international reserves.

Several previous studies have already adopted the *NATREX* approach to explain the medium-long term equilibrium dynamics of the real exchange rate in a number of industrial economies: US (Stein, 1995), Australia (Lim and Stein, 1995), Germany (Stein and Sauernheimer, 1996), France (Stein and Paladino, 1999), Italy (Gandolfo and Felettigh, 1998; Federici and Gandolfo, 2002), Belgium (Verrue and Colpaert, 1998), China (Holger et al., 2001), Hungary (Karadi, 2003) and the Eurozone (Detken et al., 2002, Duval, 2002, Stein, 2001).

However, in the previous literature, the *NATREX* approach has been always applied to a small country framework where the "rest of the world" is treated as given. Yet, the recognition of the interdependence of the world economy requires to extend the framework and endogenize the "rest of the world" in a two-country context. To the best of our knowledge, this work is the first to build a two-country *NATREX* model where the two economies are fully specified and allowed to interact. The model is specified and estimated in continuous

[1] For a survey see MacDonald and Stein (1999).

time employing quarterly data for the Eurozone and the US in the 1975-2003 period.

The remainder of the paper is organized as follows. Section 2 briefly introduces the theory of the *NATREX*. Section 3 presents our theoretical model, whereas section 4 illustrates the *NATREX* derivation. Section 5 presents the estimation results and the model's predictive performance. Section 6 studies the deviation of the actual real exchange rate dynamics from the *NATREX* simulated series. Section 7 draws some concluding remarks.

# 2 *NATREX* Approach

Before turning to the specification of the theoretical model, in this section we summarize the main features of the *NATREX* approach, while referring the reader to Allen (1995) and Stein (1995, 2001, 2006) for a more complete treatment.

The *NATREX* is a moving equilibrium real exchange rate representing the trajectory of the *medium-to-long run* equilibrium. The (medium-term) *equilibrium value* is a sustainable rate that ensures: (a) *internal equilibrium,* which requires the economy to be at capacity output, and (b) *external equilibrium,* which implies equilibrium in the balance of payments in the absence of cyclical factors, speculative capital movements and movements in international reserves. In the long term, the ratio of net foreign debt over GDP is at its steady state level, and domestic and foreign long term real interest rates are equal.

In the absence of short term cyclical and speculative factors ($H$), long term capital inflows equal excess national (private plus public) investment over saving, so that real market equilibrium and external equilibrium conditions coincide:

$$I - S = -CA = 0 \tag{1}$$

where saving ($S$), investment ($I$) and current account ($CA$) respond to endogenous fundamentals ($A$), such as the existing stocks of capital, wealth and net foreign debt, and exogenous fundamentals ($Z$) such as thrift and productivity. Denoting by $e_{R,t} = e_{R,t}(Z, A, H)$ the actual rate, by $e_{NAT} = e_{NAT}(Z, A)$ the *NATREX* and by $e_R^* = e_R^*(Z)$ the long run rate, the actual real exchange rate can be written as:

$$\underbrace{e_{R,t}}_{\text{actual rate}} = \underbrace{[e_{R,t}(Z,A,H) - e_{NAT}(Z,A)]}_{\text{transitory short term factors}} + \underbrace{[e_{NAT}(Z,A) - e_R^*(Z)]}_{\text{trajectory to the steady state}} + \underbrace{e_R^*(Z)}_{\text{steady state rate}}$$

In this approach individuals know that they cannot perfectly foresee the evolution of the fundamentals and do not possess the perfect knowledge of the structural equations of the system. In a context of intertemporal optimization over infinite horizon, rational agents use efficiently all the available information and make their intertemporal decisions relying on a sub-optimal feedback control (SOFC) rule (Infante and Stein, 1973; Stein, 1995). Since agents are not endowed with perfect foresights, the optimal trajectory of the economy remains unknown. The SOFC, which is a closed loop control that only requires current measurements of a variable, ensures that the economy converges to the unknown (and possibly changing) optimal trajectory.

Total (private plus public) consumption and investment functions are modelled independently and are derived through dynamic programming techniques with feedback control. The model can be solved for its medium run and long run (steady state) solutions (see section 4). Any perturbation on the real fundamentals of the system pushes the equilibrium $RER$ on a new medium-to-long-run trajectory. Since cyclical, transitory and speculative factors are considered noise, averaging out at zero in the long run, the actual $RER$ converges to the equilibrium trajectory.

# 3 Theoretical Model

We consider two large economies, the Eurozone ($EU$) and the United States ($US$), which are modelled symmetrically. The model is defined by dynamic equations for fundamental variables in each country plus the national account identities. The equations specify the dynamics for, respectively: net social (private plus public) investment, internal social (private plus public) consumption, trade balance (goods and services), interest rate, technology and capital stock. Variables adjust with a certain lag ($1/\alpha$) to their desired (partial equilibrium) level, according to the dynamic disequilibrium modelling approach in continuous time. In what follows variables are real, "D" denotes the operator $d/dt$, and the hat "$\hat{}$" stands for "desired". Furthermore, the nominal exchange rate ($e$) for the Euro/USD is defined as the number of Euro per one USD. The real exchange rate is denoted by $e_R$; an increase in the index means a loss in competitiveness of the US.

**Investment**

Saving and investment decisions are made independently by individual agents. This is equivalent to saying that families choose saving and consumption, while firms decide over investment and production. Net investment in fixed capital ($I_K$) is given by the sum of private and public investment. It adjusts to its partial equilibrium level with a mean time lag ($1/\alpha_1$) due to adjustment costs:

$$\mathrm{D}I_K^{US} = \alpha_1^{US}(\hat{I}_K^{US} - I_K^{US}) \quad (2)$$

$$\mathrm{D}I_K^{EU} = \alpha_1^{EU}(\hat{I}_K^{EU} - I_K^{EU}). \quad (3)$$

The desired investment in the two countries is:

$$\hat{I}_K^{US} = f_1^{US}[(MPK^{US} - R^{US})] \tag{4}$$

$$\hat{I}_K^{EU} = f_1^{EU}[(MPK^{EU} - R^{EU})] \tag{5}$$

$$\text{where sgn } f_1 [...] = \text{sgn} [...] \, , \, f_1^{'} > 0$$

$MPK$ is productivity of capital and $R$ is real interest rate. To model the investment function for $\hat{I}_K$, we follow Infante and Stein (1973) and Stein (1995). As explained in section 2, the SOFC rule based on current measurements of the marginal product of capital ensures that the economy will converge toward the optimal trajectory. It predicts that the optimal rate of investment responds positively to the difference between the productivity of capital and the real interest rate as illustrated in equations (4) and (5).

**Consumption**

Social (public plus private) internal consumption ($C$) adjusts to its partial equilibrium level with a mean time lag $(1/\alpha_2)$:

$$\text{D}C^{US} = \alpha_2^{US}(\hat{C}^{US} - C^{US}) \tag{6}$$

$$\text{D}C^{EU} = \alpha_2^{EU}(\hat{C}^{EU} - C^{EU}). \tag{7}$$

According to Stein and Sauernheimer (1996), the appropriate optimization process entails that:

$$\hat{C}^{US} = f_2^{US}(Y^{US}, F^{US}) \tag{8}$$

$$\hat{C}^{EU} = f_2^{EU}(Y^{EU}, F^{EU}) \tag{9}$$

where $Y$ denotes domestic output and $F$ net foreign debt. The desired consumption function is derived as follows. Private and public agents optimize their utility under an intertemporal budget constraint. According to standard optimization theory, consumption is proportional to current output. When foreign debt grows above the level considered sustainable, the government employs a restrictive fiscal policy by decreasing current expenditure. This gives that partial optimal consumption is a positive function of domestic output and a negative function of foreign debt.

**Trade**

According to the *NATREX* theory, the partial adjustment process also holds for the balance of trade ($BT$):

$$\text{D}BT^{US} = \alpha_3^{US}(\widehat{BT}^{US} - BT^{US}) \tag{10}$$

$$\text{D}BT^{EU} = \alpha_3^{EU}(\widehat{BT}^{EU} - BT^{EU}) \tag{11}$$

where $BT = X - M$, $X$ denotes exports and $M$ imports. The partial equilibrium values are:

$$\widehat{BT}^{US} = f_3^{US}(Y^{EU}, Y^{US}, e_R) \tag{12}$$
$$\widehat{BT}^{EU} = f_3^{EU}(Y^{US}, Y^{EU}, e_R) \tag{13}$$

and are obtained considering that, according to the standard assumptions in international economics, real exports are affected positively by the other country's output, whereas imports respond positively to the home country's real output. Furthermore, given our definition for the real Euro/USD exchange rate, an increase in $e_R$ means an appreciation of the USD vis-à-vis the Euro and, thus, leads to a decrease (increase) of the US (EU) exports and an increase (decrease) of the US (EU) imports.

In a two-country world, the following condition must hold:

$$\frac{1}{e}BT^{EU} + BT^{US} = 0. \tag{14}$$

As it is easily verified, given (14), one between (11) and (10) is redundant[2].

**Real Interest Rates**

The dynamic equations for the real interest rate ($R$) respectively in the Eurozone and in the US are given by[3]:

$$\mathrm{D}R^{US} = \alpha_4^{US}(\hat{R}^{US} - R^{US}) \tag{15}$$
$$\mathrm{D}R^{EU} = \alpha_4^{EU}(\hat{R}^{EU} - R^{EU}) \tag{16}$$

where:

$$\hat{R}^{US} = R^{EU} + \rho^{US} \tag{17}$$
$$\hat{R}^{EU} = R^{US} + \rho^{EU} \tag{18}$$

and $\rho$ is the risk premium.

[2] Taking the first derivative of equation (14), we have:.

$$\begin{aligned} D(\frac{1}{e}BT^{EU} + BT^{US}) &= (-\frac{1}{e^2}BT^{EU})De + \frac{1}{e}DBT^{EU} + DBT^{US} \\ &= -\frac{1}{e}BT^{EU}\frac{De}{e} + \frac{1}{e}DBT^{EU} + DBT^{US} = 0 \end{aligned}$$

whence, by substitution from (14), we get

$$BT^{US}\frac{De}{e} + \frac{1}{e}DBT^{EU} + DBT^{US} = 0$$

or

$$\frac{1}{e}DBT^{EU} = -BT^{US}\frac{De}{e} - DBT^{US}$$

which shows that, given $DBT^{US}$, $BT^{US}$ and $\frac{De}{e}$, the quantity $\frac{1}{e}DBT^{EU}$ is completely determined. Thus (11) can be omitted from estimation.

[3] For the long run characteristics of the system composed of (15) and (16) see the mathematical appendix A.1.

The real interest rate parity (*RIP*) theory states that if investors make their decisions in real terms, then portfolio equilibrium in an open economy requires equality between expected rates of return in real terms, possibly with a risk premium. In this model, investors face a long time horizon, deal with both direct and portfolio investment, and trade domestic as well as foreign assets. Rational investors keep trading, and let interest differentials adjust, until they become indifferent between domestic and foreign assets, i.e. the *RIP* condition with risk premium holds ((17) and (18) respectively for the US and the Eurozone). This condition, however, is not valid instantaneously, but rather is achieved with a certain time lag ($1/\alpha_4$), due to market imperfections and to the corresponding sluggishness in the re-equilibrating process, as described by (15) and (16).

Eaton and Gersovitz (1981) establish that, because of the moral hazard associated with sovereign risk, the risk premium of international lending varies positively with the stock of debt held by the given country. Moreover, Sachs (1984), Sachs and Cohen (1982), and Cooper and Sachs (1985) suggest that the cost of servicing the debt may be curbed by means of growth-oriented policies and policies that enhance the country's foreign exchange earning capacity. It follows that the risk premiums may be expressed as a positive function of the ratio between foreign debt and some measure of the earning capacity of the economy, such as the capital stock (van der Ploeg, 1996; Bhandari, Hague, and Turnovsky, 1990), that is:

$$\rho^{US} = f_4^{EU}(F^{US}/K^{US}) \quad (19)$$

$$\rho^{EU} = f_4^{EU}(F^{EU}/K^{EU}). \quad (20)$$

In the long run, two further conditions must hold. Indeed in a two-large economy world, a situation where one country is continuously characterized by a positive, while the other by a negative stock of foreign debt is not sustainable in the long run. Therefore, the long run equilibrium requires that:

$$F^{EU} = F^{US} = 0. \quad (21)$$

Given equations (19) and (20), it also follows that:

$$\rho^{EU} = \rho^{US} = 0. \quad (22)$$

**Output**

In the two economies, output adjusts with a lag ($1/\alpha_5$) to the excess demand:

$$\mathrm{D}Y^{US} = \alpha_5^{US}\left(Y_D^{US} - Y_S^{US}\right) \quad (23)$$

$$\mathrm{D}Y^{EU} = \alpha_5^{EU}\left(Y_D^{EU} - Y_S^{EU}\right) \quad (24)$$

where $Y_D$ and $Y_S$ are respectively aggregate demand and aggregate supply, and by national account identity it turns out that $Y_D = C + I + (X - M)$. In the empirical analysis we allow demand and supply adjustment speeds to differ ($\alpha_5$ and $\alpha_6$).

In this paper we follow Federici and Gandolfo (2002) and introduce endogenous growth. The production function is modelled in an "$AK$" fashion[4], that is output, respectively in the Eurozone and in the US, is given by:

$$Y_S^{US} = A^{US} K^{US} \quad (25)$$

$$Y_S^{EU} = A^{EU} K^{EU} \quad (26)$$

where $A$ is a positive constant which reflects the technological level. We assume that $A$ adjusts with a lag $(1/\alpha_7)$ to its partial equilibrium level, thus the corresponding dynamic equations in the two economies are described by:

$$\mathrm{D}A^{US} = \alpha_7^{US} \left( \hat{A}^{US} - A^{US} \right) \quad (27)$$

$$\mathrm{D}A^{EU} = \alpha_7^{EU} \left( \hat{A}^{EU} - A^{EU} \right) \quad (28)$$

In our model $\hat{A}$ is a function of the stock of accumulated knowledge, $\Omega$, which has a positive but decreasing effect. In addition, $A(\Omega)$ has an upper limit $\bar{A}$, since it is implausible to think that the productivity of capital can go to infinity. Therefore:

$$\hat{A}^{US} = A^{US}(\Omega^{US}) \quad (29)$$

$$\hat{A}^{EU} = A^{EU}(\Omega^{EU}) \quad (30)$$

$$\text{with } A' > 0, \ A'' < 0, \ A \leq \bar{A}$$

where $\Omega$ may be in turn expressed as a function of the accumulated R&D expenditure ($I_{R\&D}$):

$$\Omega^{US} = \gamma^{US} \int_{-\infty}^{t} I_{R\&D}^{US}(s)\, ds \text{ or } \dot{\Omega} = \gamma^{US} I_{R\&D}^{US} \quad (31)$$

$$\Omega^{EU} = \gamma^{EU} \int_{-\infty}^{t} I_{R\&D}^{EU}(s)\, ds \text{ or } \dot{\Omega} = \gamma^{EU} I_{R\&D}^{EU} \quad (32)$$

Total investment ($I$) is divided between investment in fixed capital ($I_K$) and investment in R&D ($I_{R\&D}$):

$$I^{US} = I_K^{US} + I_{R\&D}^{US} \quad (33)$$

$$I^{EU} = I_K^{EU} + I_{R\&D}^{EU} \quad (34)$$

[4] This function has come to be known in the recent literature as the "$AK$" production function (Barro and Sala-i-Martin, 1995), but its use in growth theory has a long tradition: e.g. Harrod (1939) and Domar (1946); Klump and Streissler (2000) show that the von Neumann production function can be reduced to the $AK$ type. More sophisticated forms (including other factors of production) could be considered, but on the basis of the parsimony principle we decided to start with the simplest possible form.

R&D investment enhances the marginal productivity of capital (by increasing the stock of accumulated knowledge) but with a certain lag ($\gamma$). The lag is crucial: more investment in R&D means less increase in the capital stock and hence smaller growth immediately; it also means higher productivity and thus higher growth later.

We need now to determine the optimal allocation of $I$ between $I_K$ and $I_{R\&D}$. Given that investment is private+public, we can think of the choice being determined by a maximizing policy maker, whereby the potential growth rate of output is maximized. The analysis that follows holds for both economies, so the superscripts are omitted for notational simplicity. Since $Y = AK$, it follows that $\dot{Y}/Y = \dot{A}/A + \dot{K}/K$, hence the maximization problem is:

$$\max_{\{I_K, I_{R\&D}\}} \left[\frac{\dot{A}}{A} + \frac{\dot{K}}{K}\right] = \max_{\{I_K, I_{R\&D}\}} \left[\frac{A'\dot{\Omega}}{A} + \frac{I_K}{K}\right]$$
$$\text{sub } I_K + I_{R\&D} - I = 0. \tag{35}$$

Performing the constrained optimization (see mathematical appendix A.2), we obtain:

$$I_{R\&D} = \frac{1}{H}I, \text{ and} \tag{36}$$
$$I_K = \frac{H-1}{H}I \tag{37}$$

where $I = f_1(A - R)$, and $H = \gamma^2 K^2 \left[\left(\frac{A'}{A}\right)^2 - \frac{A''}{A}\right] + \gamma^2$ ($> 0$). Equations (36) and (37) enable us to define respectively investment in R&D and investment in fixed capital as (variable) fractions of total investment.

By definition, the changes in the capital stock, neglecting the depreciation, are:

$$\mathrm{D}K^{US} = I_K^{US} \tag{38}$$
$$\mathrm{D}K^{EU} = I_K^{EU} \tag{39}$$

**External Debt Constraint**

Finally, in the two countries external debt constraints hold:

$$\mathrm{D}F^{US} = -\left(X^{US} - M^{US}\right) - NFI^{US} \tag{40}$$
$$\mathrm{D}F^{EU} = -\left(X^{EU} - M^{EU}\right) - NFI^{EU} \tag{41}$$

where $NFI$ is net factor income from abroad.

Some definitional equations complete the model. They are valid for both economies and, therefore, superscripts denoting the economy are omitted. The current account ($CA$) is given by:

$$CA = BT + NFI$$

where, neglecting net labour income from abroad, it turns out that $NFI = -RF$, being $RF$ the net interest payments. The stock of foreign debt is in turn defined as:

$$F(t) = F_0 - \int_0^t CA(\tau)\, d\tau \tag{42}$$

which derives from the balance-of-payments accounting identity:

$$CA + \mathrm{D}F = 0, \tag{43}$$

where $\mathrm{D}F$ is the change in the stock of foreign debt. Therefore we have[5]

$$\mathrm{D}F = -CA. \tag{44}$$

If we integrate (44) and assume that the arbitrary constant of integration is $F_0$, we obtain (42).

We have also studied stability property of the model. The relevant analysis is reported in mathematical appendix A.3.

# 4 Derivation of the *NATREX*

In the *NATREX* equilibrium, the current account is in equilibrium given output growth at capacity ($Y_P$). This requires $CA = 0$ and $Y_P = Y_S$ for each economy. In addition, long run equilibrium requires absence of risk premium, what implies in turn $\hat{R} = R = \overline{R}$ for each economy, where $\overline{R}$ is the same for both countries and can be calculated endogenously (see mathematical appendix A.1). Since, on the other hand, the production function implies $MPK = A$, this shows the crucial role of $A$ and $\overline{R}$: it is the difference between them that enhances or hinders investment according to the investment equations (4) and (5). Investment will be positive if $A > \bar{R}$. This will cause capital growth, and hence output growth.

To derive the *NATREX*, we observe that current account equilibrium, $CA^{EU} = CA^{US} = 0$, implies $F^{EU} = F_0^{EU}$ ($F^{US} = F_0^{US}$). However, in the long run equilibrium the stock of net foreign assets must also be zero. Hence:

$$F^{EU} = F^{US} = 0.$$

Furthermore, in equilibrium, $BT^{EU} = \widehat{BT}^{EU}$ ($BT^{US} = \widehat{BT}^{US}$). It follows that: $CA^{EU} = \widehat{BT}^{EU} = 0$ ($CA^{US} = \widehat{BT}^{US} = 0$), and so $e_R$ can be determined from the implicit function

[5] For the sake of clarity, we recall that, according to the accounting principles of the balance of payments (IMF, 1993, p. 7), a decrease in foreign liabilities or an increase in foreign assets ($\mathrm{D}F > 0$) should be recorded as a positive figure (credit), and, conversely, an increase in foreign liabilities or a decrease in foreign assets ($\mathrm{D}F < 0$) should be recorded as a negative figure (debt).

$\widehat{BT}^{EU} = f_3^{EU}\left(Y_P^{EU}, Y_P^{US}, e_R\right) = 0$ (or $\widehat{BT}^{US} = f_3^{US}\left(Y_P^{US}, Y_P^{EU}, e_R\right) = 0$). If the appropriate invertibility conditions on the Jacobian are satisfied, we have

$$e_{NAT} = \phi\left(Y_P^{EU}, Y_P^{US}\right), \tag{45}$$

where the subscript *NAT* stands for *NATREX*. The same can be derived from $\widehat{BT}^{US} = f_3^{US}\left(Y_P^{US}, Y_P^{EU}, e_R\right) = 0$,

$$e_{NAT} = \psi\left(Y_P^{US}, Y_P^{EU}\right), \tag{46}$$

where, of course, equation (45) and (46) must have the same value[6].

This concerns the long run equilibrium growth path. In the medium run, however, the requirements of no risk premium ($R = \overline{R}$) and no capital flows, are too stringent. A more plausible alternative is to allow for $R \neq \overline{R}$ and non-zero capital flows, while keeping the basic requirements of *NATREX*, namely $CA = 0$ and $Y_S = Y_P$. Therefore, we have $\widehat{BT}^{US} = f_3^{US}\left(Y_S^{US}, Y_S^{EU}, e_R\right) - R^{US}F^{US} = -\frac{1}{e}\widehat{BT}^{EU}$ $(= -\left[f_3^{EU}\left(Y_S^{EU}, Y_S^{US}, e_R\right) - R^{EU}F^{EU}\right])$. In this formulation the *NATREX* turns out to be

$$e_{NAT} = \chi(Y_P^{US}, Y_P^{EU}, R^{US}F^{US}). \tag{47}$$

This is the *medium run NATREX*.

# 5 Estimation

## 5.1 Model Specification

In this section we present the model in its exact specification for empirical estimation. Consistently with the relevant literature, among the various possible functional forms for the behavioral equations presented in section (3), we have chosen the log-linear one (see also Detken et al., 2002; and Gandolfo and Federici, 2002). All coefficients are written in such a way that they are supposed to be positive unless otherwise stated. In what follows variables are expressed in domestic currency and converted in real terms using the GDP deflator (base year = 2000). For empirical purposes, let us rewrite the model as follows:

**Behavioral equations:**

$$\mathrm{D}\ln k^{US} = \alpha_1^{US}(\ln \hat{k}^{US} - \ln k^{US}) \tag{48}$$

$$\mathrm{D}\ln k^{EU} = \alpha_1^{EU}(\ln \hat{k}^{EU} - \ln k^{EU}) \tag{49}$$

where $k$ is capital stock growth rate and is defined as: $\ln k = DK/K = I_K/K$. $I_K$ is net fixed capital formation given by $I_K = I_K - \delta K = \dot{K}$ and $\ln \hat{k} = \gamma_1\left(\ln \hat{A} - \ln R\right)$. As

[6] It also turns out that the system is conditionally stable (see mathematical appendix A.3).

illustrated in the theoretical model (equations (29)-(37)), productivity is a function of the optimal allocation of $I$ between $I_K$ and $I_{R\&D}$ (more investment in R&D implies smaller increase in the capital stock and smaller growth immediately, but higher productivity and thus higher growth later). Since both investment in R&D and investment in fixed capital can be expressed as fractions of total investment[7], the productivity parameter $\hat{A}$ can be defined as $\gamma A_0(\Omega)$, where $\Omega = I_{R\&D}/I_K$ and $A_0$ is a constant term[8].

$$\mathrm{D}\ln C^{US} = \alpha_2^{US}(\ln \hat{C}^{US} - \ln C^{US}) \tag{50}$$

$$\mathrm{D}\ln C^{EU} = \alpha_2^{EU}(\ln \hat{C}^{EU} - \ln C^{EU}) \tag{51}$$

where $\ln \hat{C} = \gamma_2 - \gamma_3 F/K + \gamma_4 \ln Y$.

$$\mathrm{D}\ln X^{US} = \alpha_{3X}^{US}(\ln \hat{X}^{US} - \ln X^{US}) \tag{52}$$

$$\mathrm{D}\ln M^{US} = \alpha_{3M}^{US}\left(\ln \hat{M}^{US} - \ln M^{US}\right) \tag{53}$$

where $\ln \hat{X} = \gamma_5 + \beta_1 \ln Y^* - \beta_2 \ln e_R + \beta_3 \ln Y^{ROW}$ and $\ln \hat{M} = \gamma_6 + \beta_4 \ln Y + \beta_5 \ln e_R$[9]. $Y^{ROW}$ is a weighted average of Japan's and UK's GDPs. Given that in the real world exports and imports may have different adjustment speeds, they have been separately taken, rather than directly considering their balance.

$$\mathrm{D}\ln R^{US} = \alpha_4(\ln \hat{R}^{US} - \ln R^{US}) \tag{54}$$

$$\mathrm{D}\ln R^{EU} = \alpha_4(\ln \hat{R}^{EU} - \ln R^{EU}) \tag{55}$$

where $\ln \hat{R} = \ln \overline{R} + \gamma_7 F/K$ and $\overline{R}$ is the world real interest rate.

$$\mathrm{D}\ln Y^{US} = \alpha_5^{US} \ln Y_D^{US} - \alpha_6^{US} \ln Y_S^{US} \tag{56}$$

$$\mathrm{D}\ln Y^{EU} = \alpha_5^{EU} \ln Y_D^{EU} - \alpha_6^{EU} \ln Y_S^{EU} \tag{57}$$

where $Y_S = AK$ and $Y_D = C + I + BT$. Note that we allow for a difference in demand and supply adjustment speeds.

**Definitional equations:**

$$\mathrm{D}K^{US} = k^{US} K^{US} \tag{58}$$

$$\mathrm{D}K^{EU} = k^{EU} K^{EU} \tag{59}$$

[7] Given $I_{R\&D} = (1/H)I$ and $I_K = [(H-1)/H]I$, we can also write $I_{R\&D} = [1/(H-1)]I_K$.

[8] The unavailability of $\bar{A}$ (cf. (29)-(30)) has led us to drop the adjustment equations (27) and (28) in estimation. This also explains why the parameter $\alpha_7$ is not present in estimation.

[9] The trade balance for the EU is obtained as residual given equation (61).

where $k = I_K/K$.

$$DF^{US} = -BT^{US} - NFI^{US} \tag{60}$$

**External equilibrium equations:**

$$\frac{1}{e}BT^{EU} + BT^{US} - ET = 0 \tag{61}$$

$$\frac{1}{e}F^{EU} + F^{US} - EF = 0 \tag{62}$$

where $ET$ and $EF$ are residual terms that capture the (exogenous) rest of the world effect.

## 5.2 Estimation Results

The model (48)-(62) is a nonlinear differential equation system that can be estimated in continuous time (on continuous time econometrics see Gandolfo, 1981; Bergstrom, 1984; Wymer, 1972, 1993, 1997). We use quasi-FIML nonlinear continuous time estimator developed by Wymer (1993) and implemented in the computer program ESCONA in the WYSEA package (Wymer, 2004)[10]. Our sample period ranges from 1975:1 to 2003:4, at quarterly frequency (for details see data mathematical appendix B).

**Table 1: Observed and estimated values of the endogenous variables and corresponding estimation residuals**

| | *Endogenous variable y(t)* | | | | *Error in estimated y(t)* | |
|---|---|---|---|---|---|---|
| | *Observed* | | *Estimated* | | | |
| | *Mean* | *St. Dev.* | *Mean* | *St. Dev.* | *Mean* | *St. Dev.* |
| $k^{US}$ | 0.017129 | 0.008954 | 0.017112 | 0.008855 | 0.000017 | 0.000904 |
| $C^{US}$ | 7.252867 | 0.269132 | 7.252616 | 0.269696 | 0.000251 | 0.006770 |
| $X^{US}$ | 4.926065 | 0.459010 | 4.925476 | 0.458343 | 0.000598 | 0.027582 |
| $M^{US}$ | 5.124038 | 0.510957 | 5.123774 | 0.508310 | 0.000264 | 0.031955 |
| $R^{US}$ | 0.011003 | 0.005878 | 0.010296 | 0.005487 | 0.000707 | 0.004065 |
| $Y^{US}$ | 7.356703 | 0.238264 | 7.356309 | 0.238208 | 0.000394 | 0.011158 |
| $k^{EU}$ | 0.006347 | 0.001107 | 0.006347 | 0.001069 | 0.000000 | 0.000334 |
| $C^{EU}$ | 6.917433 | 0.181526 | 6.917658 | 0.181761 | -0.000225 | 0.005660 |
| $R^{EU}$ | 0.009247 | 0.006356 | 0.009666 | 0.005572 | -0.000419 | 0.004119 |
| $Y^{EU}$ | 7.024321 | 0.188383 | 7.024355 | 0.188547 | -0.000035 | 0.007629 |
| $K^{US}$ | 9.338232 | 0.520234 | 9.338150 | 0.520446 | 0.000082 | 0.000959 |
| $K^{EU}$ | 9.739168 | 0.202255 | 9.739151 | 0.202295 | 0.000017 | 0.000260 |
| $F^{US}$ | -448.874251 | 761.150769 | -463.364950 | 778.531391 | 14.490699 | 52.030909 |
| $F^{EU}$ | -81.938745 | 166.451446 | -68.809023 | 156.135988 | -13.129722 | 48.431794 |
| $BT^{EU}$ | 6.114438 | 20.648412 | 6.006889 | 20.271129 | 0.107548 | 4.876214 |

Note: All variables are in log. terms but $F^{US}$, $F^{EU}$, and $BT^{EU}$ which are in nat. numbers.

[10] The software used is available from the authors upon request to replicate the estimation results.

Table 1 shows means and standard deviations of observed and estimated endogenous variables and the corresponding estimation errors for respectively the US and the Eurozone.

The estimated parameters, their asymptotic standard errors and the t-ratios[11] are reported in table 2. We remind to the reader that in the theoretical model all coefficients are written in such a way that they are expected to be positive.

As one can observe the estimated parameters have the expected sign in 29 out of 33 cases (constant terms are logarithms) and mostly statistically significant (in 28 out of 33 cases). Furthermore, the parameters that have different sign from that theoretically predicted ($\gamma_3^{US}$, $\gamma_7^{US}$, $\gamma_3^{EU}$, and $\gamma_8^{EU}$) are not statistically significant at the 5% significance level in three out of four cases.

The parameters $\alpha$ are adjustment speeds, thus their reciprocals, $1/\alpha$, can be interpreted as mean time lags, namely the time required for about 63% of the discrepancy between the actual and the desired value of the variable to be eliminated by the adjustment process incorporated into the partial adjustment equation (Gandolfo, 1981).

We notice that in our model estimated values for adjustment speeds are always positive and statistically significant at the 1% level but in one case ($\alpha_7^{US}$). Corresponding mean time lags are reported in column 5 of table 2.

Let us start commenting the investment equation. In both countries, firms are predicted to adjust their capital stock to the desired level quite slowly ($1/\alpha_1^{US} = 25.028157$ and $1/\alpha_1^{EU} = 18.904306$). This can be explained considering that the accumulation process takes long time, then investment levels adjust accordingly.

With regard to the consumption equations, there is substantial difference in the time lags between the two economies. As one can notice, the mean time lag in the EU is $1/\alpha_2^{EU} = 8.813601$ corresponding to about two years, whereas in the US it turns out to be $1/\alpha_2^{US} = 603.864734$ that seems to suggest a very slow adjustment process. The latter result may be commented in the view of the *Haken's* (1983a, 1983b) *slaving principle* (cf. Gandolfo, 1997). In the presence of slow and fast variables, the evolution of a dynamic system is driven by the slow ones. Consequently, slow variables are called *order* or *slaving* variables, while the fast ones are referred to as *slaved* variables.

In the output function two adjustment speeds are involved, for each country, respectively $\alpha_5^{US}$ and $\alpha_6^{US}$ for the US and $\alpha_5^{EU}$ and $\alpha_6^{EU}$ for the EU. On this regard, we observe that supply adjustment speeds are remarkably low ($\alpha_6^{US}$ and $\alpha_6^{EU}$), while demand adjustment speeds are, on the contrary, quite high ($\alpha_5^{US}$ and $\alpha_5^{EU}$). In our model, thus, according to the mentioned Haken's principle, the US consumption and supply can be considered the

[11] The $t$-ratio is a shorthand notation for the ratio between the parameter point estimate and its asymptotic standard error. It is not implied that the $t$-ratio follows a Student's $t$ distribution. The $t$-ratio has an asymptotic normal distribution (critical values are 1.96 and 2.58 at respectively 5% and 1% significance level).

order variables that govern the adjustment process.

Exports and imports of goods and services in the US have very similar mean time lags ($1/\alpha_{3X}^{US} = 6.573369$ and $1/\alpha_{3M}^{US} = 6.578558$) which suggest that exporting and importing firms are able to align quantities to their respective partial equilibrium values in about one year and half.

**Table 2: Estimation results**

| *Parameter* | *Point estimate* | *Standard error* | *t-ratio* | *Mean time lag* |
|---|---|---|---|---|
| $\alpha_1^{US}$ | 0.039955*** | 0.010499 | 3.81 | 25.028157 |
| $\alpha_2^{US}$ | 0.001656*** | 0.000252 | 6.56 | 603.864734 |
| $\alpha_{3X}^{US}$ | 0.152129*** | 0.017567 | 8.66 | 6.573369 |
| $\alpha_{3M}^{US}$ | 0.152009*** | 0.006185 | 24.58 | 6.578558 |
| $\alpha_4^{US}$ | 0.552013*** | 0.017761 | 31.08 | 1.811552 |
| $\alpha_5^{US}$ | 0.666008*** | 0.119892 | 5.56 | 1.501483 |
| $\alpha_6^{US}$ | 0.004257 | 0.002236 | 1.90 | 234.907212 |
| $\alpha_1^{EU}$ | 0.052898*** | 0.021199 | 2.50 | 18.904306 |
| $\alpha_2^{EU}$ | 0.113461*** | 0.029860 | 3.80 | 8.813601 |
| $\alpha_4^{EU}$ | 1.002440*** | 0.007474 | 137.07 | 0.997566 |
| $\alpha_5^{EU}$ | 0.291293*** | 0.029516 | 9.87 | 3.432970 |
| $\alpha_6^{EU}$ | 0.025502*** | 0.003635 | 7.02 | 39.212611 |
| $\beta_1^{US}$ | 2.267895*** | 0.106974 | 21.20 | |
| $\beta_2^{US}$ | 0.507177*** | 0.039335 | 12.89 | |
| $\beta_3^{US}$ | 0.166537*** | 0.000354 | 470.73 | |
| $\beta_4^{US}$ | 2.062268*** | 0.035839 | 57.54 | |
| $\beta_5^{US}$ | 0.296498*** | 0.038845 | 7.63 | |
| $\gamma_1^{US}$ | 0.327140** | 0.136907 | 2.39 | |
| $\gamma_2^{US}$ | -5.940837*** | 0.898550 | 6.61 | |
| $\gamma_3^{US}$ | -0.486899 | 0.283770 | 1.72 | |
| $\gamma_4^{US}$ | 2.451920*** | 0.750315 | 16.05 | |
| $\gamma_5^{US}$ | -12.041727*** | 0.269305 | 36.76 | |
| $\gamma_6^{US}$ | -9.900159*** | 0.163299 | 15.01 | |
| $\gamma_7^{US}$ | -0.019603** | 0.009857 | 1.99 | |
| $\gamma_8^{US}$ | 6.760411*** | 1.990457 | 3.40 | |
| $\gamma_1^{EU}$ | 0.058303*** | 0.006363 | 9.16 | |
| $\gamma_2^{EU}$ | 0.453626*** | 0.104238 | 4.35 | |
| $\gamma_3^{EU}$ | -0.117985 | 0.824933 | 0.14 | |
| $\gamma_4^{EU}$ | 0.927589*** | 0.015979 | 58.05 | |
| $\gamma_7^{EU}$ | 0.082554 | 0.052737 | 1.57 | |
| $\gamma_8^{EU}$ | -0.749150 | 0.424595 | 1.76 | |
| $A_0^{US}$ | -4.133756*** | 0.059225 | 69.80 | |
| $A_0^{EU}$ | -2.151590*** | 0.044819 | 48.00 | |

Notes: Max log-likelihood value=0.6145248E+04; Gradient norm=0.25675E-02.
Statistical significance: **=5%, ***=1%.

Finally, the interest rate equations are characterized by very high adjustment speeds. Indeed $1/\alpha_4^{EU} = 0.997566$ indicates that the European real interest rate takes less than three months in converging to its partial equilibrium value, whereas $1/\alpha_4^{US} = 1.811552$ suggests that the adjustment time lag in the US is less than six months. This result is reasonable since we expect the monetary authorities to be able to reach their targets in a short time lag.

The $\beta$-coefficients only appear in the US trade equations and may be interpreted as elasticities. They always have the expected sign and are highly statistically significant at any conventional level. $\beta_2^{US}$ and $\beta_5^{US}$ are respectively export and import elasticities with respect to the exchange rate. We observe that the former is larger than the latter. This indicates that a weaker USD will induce a greater expenditure switching by foreign consumers than by US consumers (see also Goldberg and Dillon, 2007). Moreover, $\beta_2^{US} + \beta_5^{US} = 0.803675$ suggests that the Marshall-Lerner condition is not satisfied[12]. Our finding is consistent with Chinn (2007) which obtains total exchange rate elasticities over the period 1975-2006 equal to $0.648 + 0.167 = 0.815$. This is not surprising if one considers that, typically, USD devaluations have not led to substantial trade balance adjustment. An example is offered in the recent periods by the fact that from 2000 onwards the USD has been progressively depreciated while the American current account deficit has steadily and remarkably grown (for the relation between USD exchange rate movements and US current account behavior, see Edwards, 2005). $\beta_1^{US}$ is export elasticity with respect to EU income, while $\beta_4^{US}$ is import elasticity with respect to US income. They are quite large and highly statistically significant, what is reasonable being the US trade mainly responsive to national GDP and world demand, rather than to exchange rate policies. Finally, the export elasticity with respect to the rest of the world income ($\beta_3^{US}$) is much smaller. This result was expected being the estimated model a two-country framework where the rest of the world is not explicitly modelled and is proxied by Japan's and the UK's GDPs only.

We lastly turn to the $\gamma$-coefficients. We comment only those that do not conform to our expectations. The negative values for $\gamma_3^{US}$ and $\gamma_3^{EU}$ would suggest that total consumption positively react (notice that $-\gamma_3^{US} > 0$ and $-\gamma_3^{EU} > 0$) to an increasing stock of foreign debt. This result would be contradicting our theoretical predictions. However, we also observe that those coefficients are very small and not statistically significant at the 5% level. Therefore we conclude that consumption is not statistically affected by foreign debt and the component driven by national income is predominant, both in the Eurozone ($\gamma_4^{EU} = 0.927589$) and, especially, in the US ($\gamma_4^{US} = 2.451920$). $\gamma_7^{US} < 0$ is also contrary to our

[12] The estimates of trade elasticity obtained by the empirical literature vary remarkably, depending on the methodology adopted and the sample used. See for instance Goldstein and Khan (1985), Hooper et al. (2000), and IMF (2007).

theoretical model's prediction. It means that the US real interest rate rises when foreign debt (thus the risk premium of international lending) increases. We evaluate this data evidence considering that, in a two-country monetary policy game, the US would be the leader country in setting the interest rate, disregarding the country risk. So the effect of the risk premium of foreign lending is not as important as in other countries. Finally, a negative value for $\gamma_8^{EU}$, appearing in the EU production function, is also against our expectations. This coefficient however does not turn out statistically significant, thus it does not deserve further comments.

### 5.3 Predictive Performance

Table 3 reports the in-sample root mean square errors (*RMSE*) of static and dynamic forecasts of the endogenous variables. The former are calculated by using the actual values of the predetermined variables, whereas the latter are obtained using, for the lagged endogenous variables, the values forecast by the model in the appropriate period (with the exception on the first period in the sample for which the observed value is taken). Generally dynamic forecasts are poorer than static ones because errors cumulate. Since all reported endogenous variables are expressed in logarithmic terms, the *RMSE* provide the average error as a proportion of the actual level of the corresponding variable.

As table 3 shows, errors produced by static forecasts are always below 5%. The largest errors are those associated to imports and exports. This may be due to the specifications and data aggregation problems concerning the corresponding equations. Errors generated by dynamic forecasts are systematically larger, but are always below 6% with the only

**Table 3: Ex-post root mean square of errors in forecast**

| | *RMSE of single-period forecasts* | *RMSE of dynamic forecasts* |
|---|---|---|
| $k^{US}$ | 0.001550 | 0.002356 |
| $C^{US}$ | 0.008048 | 0.043944 |
| $X^{US}$ | 0.038335 | 0.092566 |
| $M^{US}$ | 0.045394 | 0.117184 |
| $R^{US}$ | 0.006074 | 0.004070 |
| $Y^{US}$ | 0.016840 | 0.049186 |
| $k^{EU}$ | 0.000429 | 0.000818 |
| $C^{EU}$ | 0.006415 | 0.029281 |
| $R^{EU}$ | 0.006168 | 0.004281 |
| $Y^{EU}$ | 0.008791 | 0.039390 |
| $K^{US}$ | 0.001822 | 0.056888 |
| $K^{EU}$ | 0.000445 | 0.013982 |

Note: All variables are in logarithmic terms.

exceptions of import and export *RMSE* that are around 10% of the variable.

## 6 *NATREX*: Facts and Issues

In this section we derive the *NATREX* series by simulation of the estimated model with the appropriate modifications[13]: current account balance equal to zero and output at capacity level (see section 4). It is here worth pointing out that the *NATREX* determination does not purport to track the actual real exchange rate but, on the contrary, to measure the long term equilibrium exchange rate, i.e. the benchmark against which the misalignment of the observed exchange rate can be gauged. We remind that according to our definition of the Euro/USD real exchange rate, an increase in the index means a loss in competitiveness of the US, therefore a rise denotes a real appreciation of the USD. It follows that, when the observed exchange rate (*RER*) is higher (lower) than the *NATREX*, the USD is overvalued (undervalued). Accordingly, we compute an index of relative misalignment defined as $(RER - NATREX)/NATREX$.

Figure 1(a) reports the observed Euro/USD real exchange rate and its *NATREX* value, as well as the corresponding misalignment measure. The equilibrium real exchange rate is obtained by in-sample (1975Q1-2003Q4) simulation according to equation (47).

In order to discuss the deviation of the Euro/USD exchange rate from the equilibrium series, we analyze the evolution of both the actual exchange rate and the current account. Thus, in figure 2 we also report the US current account-GDP ratio over the sample period. We distinguish the following five periods:

1975Q1-1980Q4: This period follows the advent of the floating rates and it is characterized by an undervaluation of the USD. During this phase, the US current account alternates between small surpluses and small deficits.

1981Q1-1989Q3: The USD results to be overvalued. The misalignment index reaches its highest value (0.47) at the end of 1984 corresponding to the peak of the USD strengthening occurred during the Reagan years[14]. During this period the US current account balance turns from surplus into deficit in 1981 and continues to decrease steadily and remarkably until the third quarter of 1986. Reacting to the strong USD and the substantial current account deficit, in September 1985 the G5 countries sign the "Plaza Accord", providing concerted and coordinated interventions in the foreign exchange market in order to induce a gradual USD depreciation. As expected, starting from the mid-1985 the USD real exchange rate experiences a rapidly depreciating trend, and the current account deficit shrinks

[13] We have corrected the *NATREX* simulation by including the portion of the estimation error in the import and export equations imputable to the rest of the world component, which the $Y^{ROW}$ proxy was not able to capture.

[14] For a discussion on the "dazzling dollar" in the first half of the eighties see Frankel (1985).

accordingly. In February 1987, the G6 countries (G5 and Canada) convey through the "Louvre Accord" their commitment "...to cooperate closely to foster stability of exchange rate around current levels" (Louvre Accord, 1987). Consequently, starting from 1987 until the end of the sub-period, the misalignment index is quite stable and fluctuates between 0 and 0.2.

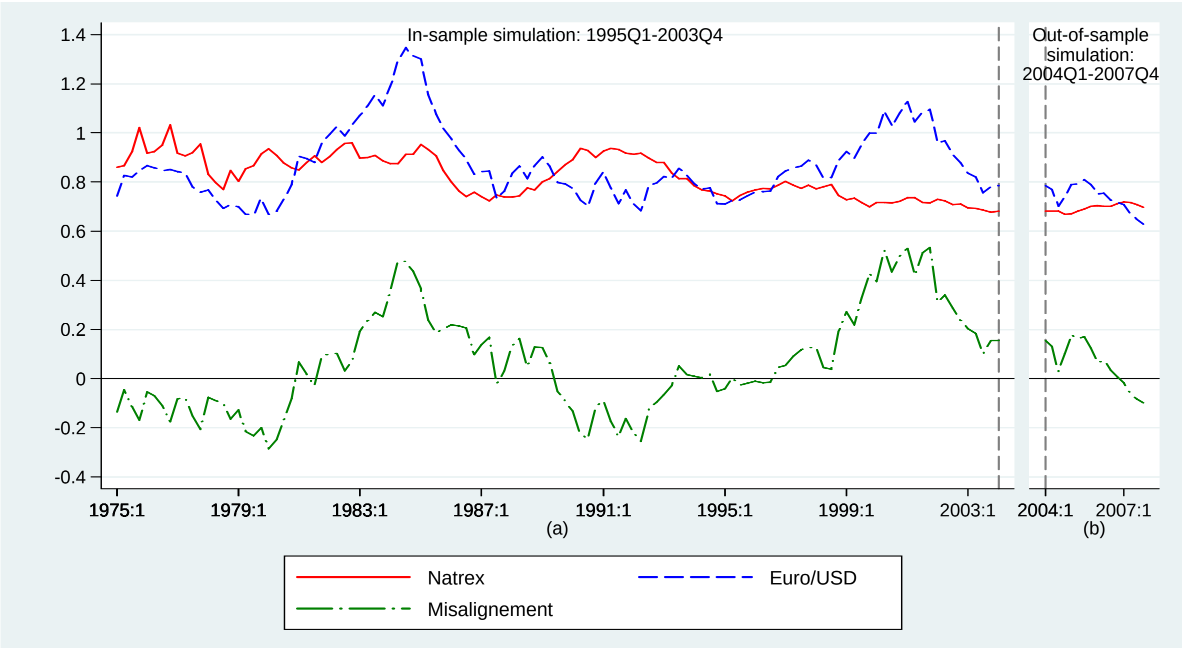


Figure 1: Actual real Euro/USD and NATREX

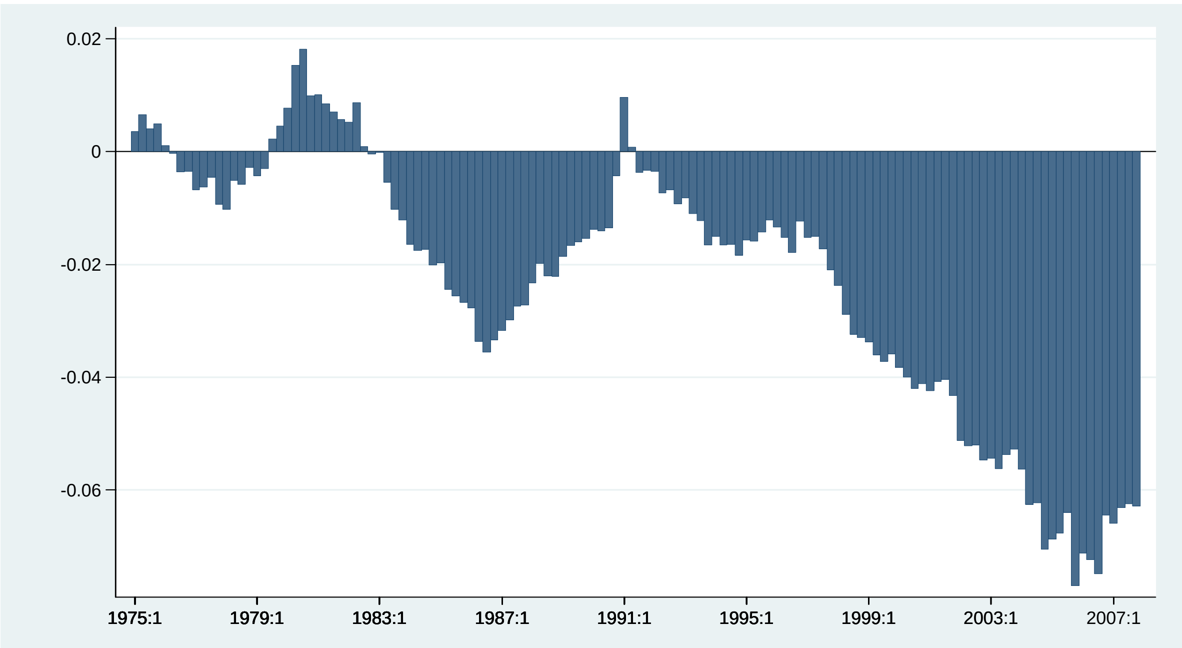


Figure 2: US current account to GDP

1989Q4-1993Q2: The USD turns out to be undervalued, although the misalignment measure remains within a narrow interval. During this period the current account balance improves steadily until the first quarter of 1991 when the US experiences its first surplus since long time. From this point onward the exchange rate depreciates, while the current account starts to decrease again.

1993Q3-1998Q4: In this phase the actual real exchange rate volatility declines and the equilibrium Euro/USD gets very close to the current one (despite the substantial current account deficit).

1999Q1-2003Q4: Finally the USD remains overvalued since the launch of the Euro as single currency of the Eurozone until the end of 2003. During this time interval the USD appreciates up to 2000, while it begins to depreciate about the second quarter of 2002. The current account deficit widens substantially over the entire period exceeding the 5% of the GDP at the end of 2003.

As also mentioned in the introduction, the behavior of the Euro/USD after its launch represents a puzzle to most analysts and market participants. Indeed, the encouraging European growth performance in the second half of the nineties (the annual growth rate shifted from 0.5% in 1995 to almost 3% in 1997) and the successful completion of the European Monetary Union had created expectations for an appreciating trend of the Euro. As well known, they have been deceived. What explains the weakness of the Euro? Was it driven by the fundamentals of the European economies? On May the 8th and again on September the 8th 2000, a *communiqués* of the Ministers of Finance of the Eurozone reassured the citizens that it was not, and that the Euro behavior did not reflect the European macroeconomic performance. This feeling is corroborated by our *NATREX* simulation.

While it is not possible to provide a single explanation on the historical pattern of the Euro/USD real exchange rate, a combination of different interpretations may help to understand why the European single currency was so undervalued at the beginning and in the three subsequent years (CESifo, 2002; Shams, 2005). First, on the portfolio side, an important role was played by the excess supply of the Euro denominated assets. Indeed, the greater confidence in the US economy led the Euro area residents to prefer holding foreign currency denominated assets than Euro denominated ones. This attitude was reinforced by the adverse perceptions of the Euro area policy makers (Koen et al., 2001). Second, the USD overvaluation could be explained by the macroeconomic performance of the US: high productivity growth, the "new economy" fever, more flexible product and labor markets (Lewis, 2007; Alquist and Chinn, 2002; Corsetti and Pesenti, 1999). A third approach focuses on interest rates differentials and relative rates of return of US versus Euroland assets which moved capital flows out of the EU to the US (Bailey et al. 2001). Forth, it can be emphasized the role of fundamental variables: growth rates, inflation differentials, and current account patterns (De Grauwe, 2000, De Grauwe and Grimaldi, 2005). Finally, the Euro's weakness can be seen as the consequence of the initial policy-mix implemented by European fiscal and monetary authorities (Cohen and Loisel, 2001). The behavior of the Euro/USD has inverted its trend in 2002, when the European currency has started to appreciate. This phenomenon may be due to the fact that, after 2002, the Euro became a full-fledged money challenging the international currency position of the USD (Shams, 2005)[15].

Figure 1(b) shows the out-of-sample *NATREX* simulation. We use the in-sample estimated parameters to compute the projected values up to 2007Q4 employing updated actual series of the variables entering equation (47). As one can notice, our model signals an overvaluation of the USD until the end of 2006, when the observed actual Euro/USD series crosses the equilibrium one. Thereafter the Euro/USD real exchange rate behavior keeps a downward direction, implying an (increasing) undervaluation of the USD. Although this finding should be interpreted with some caution, it is consistent with recent studies (cf. CESifo, 2008) which suggest that the Euro/USD is likely to have achieved and even overshot the level that would induce a correction process of the US external imbalances.

# 7 Concluding Remarks

In this paper we build a two-country model for the US and the Euro-area in order to examine the determinants and the dynamics of the Euro/USD equilibrium exchange rate. We adopt the *NATREX* approach (Stein, 1990), which is based on a specific theoretical dynamic stock-flow model to derive the equilibrium real exchange rate. The equilibrium concept ensures simultaneously internal and external equilibrium, and reflects the behavior of the fundamental variables behind investment and saving decisions in the absence of cyclical factors, speculative capital movements and movements in international reserves. The theoretical model is estimated in its structural form as a simultaneous nonlinear differential equation system for the 1975-2003 period. Our results suggest that the model fits well the data, and the estimated parameters are mostly consistent with our theoretical predictions. We have thus carried out a in-sample simulation of the estimated model to determine the real equilibrium exchange rate. The comparison between the *NATREX* and the actual real exchange rate leads us to single out two periods of strong overvaluation of the USD, in the mid-eighties and after the launch of the Euro. Our findings are supported by the literature

[15] For some interesting contributions on the international role of the Euro see the Journal of Policy Modeling (2002) 24(4): 301-410.

on the historical behavior of the Euro/USD. In addition, we have performed out-of sample simulation until the forth quarter of 2007. The results indicate that the current strong depreciation of the US dollar has led the Euro/USD exchange rate to achieve and even overshoot the equilibrium value at the end of 2006.

# A Mathematical Appendix

## A.1 Real Interest Rates

In the long run, recalling that the risk premiums are zero, we have:

$$\mathrm{D}R^{US} = \alpha_4^{US}(\hat{R}^{US} - R^{US}) = \alpha_4^{US}(R^{EU} - R^{US}) \tag{A.1}$$

$$\mathrm{D}R^{EU} = \alpha_4^{EU}(\hat{R}^{EU} - R^{EU}) = \alpha_4^{EU}(R^{US} - R^{EU}) \tag{A.2}$$

the characteristic equation of the system is (for simplicity in this appendix, $\alpha_4^{US}$ and $\alpha_4^{EU}$ are written omitting the "4" subscript):

$$\begin{vmatrix} -\alpha^{US} - \lambda & \alpha^{US} \\ \alpha^{EU} & -\alpha^{EU} - \lambda \end{vmatrix} = \lambda(\alpha^{US} + \alpha^{EU} + \lambda) = 0 \tag{A.3}$$

whereby $\lambda_1 = 0, \lambda_2 = -(\alpha^{US} + \alpha^{EU})$. The solution is:

$$R^{US} = A_1 + A_2 e^{-(\alpha^{US}+\alpha^{EU})t}, \tag{A.4}$$

$$R^{EU} = A_1 \frac{\lambda_1 - a_{11}}{a_{12}} + A_2 \frac{\lambda_2 - a_{11}}{a_{12}} e^{-(\alpha^{US}+\alpha^{EU})t} \tag{A.5}$$

where $a_{11} = -\alpha^{US}, a_{12} = \alpha^{US}$. It follows that:

$$R^{EU} = A_1 + \frac{\lambda_2 + \alpha^{US}}{\alpha^{US}} A_2 e^{-(\alpha^{US}+\alpha^{EU})t} = A_1 + A_2 \frac{-\alpha^{EU}}{\alpha^{US}} e^{-(\alpha^{US}+\alpha^{EU})t}. \tag{A.6}$$

Given the initial conditions $R^{US} = R_0^{US}$, $R^{EU} = R_0^{EU}$ with $t = 0$, to obtain the arbitrary constants, $A_1$, $A_2$, we have:

$$R_0^{US} = A_1 + A_2, \tag{A.7}$$

$$R_0^{EU} = A_1 + \frac{-\alpha^{EU}}{\alpha^{US}} A_2 \tag{A.8}$$

whose solution gives:

$$A_1 = \frac{\frac{-\alpha^{EU}}{\alpha^{US}} R_0^{US} - R_0^{EU}}{\frac{-\alpha^{EU}}{\alpha^{US}} - 1}, \; A_2 = \frac{R_0^{EU} - R_0^{US}}{\frac{-\alpha^{EU}}{\alpha^{US}} - 1}. \tag{A.9}$$

In the long run, the real interest rates of the two countries converge to $A_1$, that is:

$$A_1 = \frac{\frac{-\alpha^{EU}}{\alpha^{US}} R_0^{US} - R_0^{EU}}{\frac{-\alpha^{EU}}{\alpha^{US}} - 1} = \frac{\alpha^{EU} R_0^{US} + \alpha^{US} R_0^{EU}}{\alpha^{US} + \alpha^{EU}} = \frac{\frac{1}{\alpha^{US}} R_0^{US} + \frac{1}{\alpha^{EU}} R_0^{EU}}{\frac{1}{\alpha^{US}} + \frac{1}{\alpha^{EU}}}. \tag{A.10}$$

We observe that in the long run the real interest rates converge to the same constant value (denoted by $\overline{R}$ in the text), which is the weighted average of the initial real interest rates and the weights are given by the corresponding mean time lags.

## A.2 Investment in R&D

The US and the Eurozone economies are modelled symmetrically so that the following description holds for both economies and the superscripts are omitted for notational simplicity. The maximization problem for the determination of the optimal allocation of $I$ between $I_K$ and $I_{R\&D}$ is:

$$\begin{aligned} \max_{\{I_K, I_{R\&D}\}} \left[\frac{\dot{A}}{A} + \frac{\dot{K}}{K}\right] &= \max_{\{I_K, I_{R\&D}\}} \left[\frac{A'\dot{\Omega}}{A} + \frac{I_K}{K}\right] \\ &= \max_{\{I_K, I_{R\&D}\}} \left[\gamma \frac{A' I_{R\&D}}{A} + \frac{I_K}{K}\right] \\ \text{sub } I_K + I_{R\&D} - I &= 0. \end{aligned} \tag{A.11}$$

From the Lagrangian

$$L = \left[\gamma \frac{A' I_{R\&D}}{A} + \frac{I_K}{K}\right] + \lambda(I - I_K - I_{R\&D}) \tag{A.12}$$

we obtain the first-order conditions

$$\frac{\partial L}{\partial I_{R\&D}} = \gamma \frac{A'}{A} - \lambda = 0, \tag{A.13}$$

$$\frac{\partial L}{\partial I_K} = \frac{1}{K} - \lambda = 0, \tag{A.14}$$

whence

$$\gamma \frac{A'}{A} = \frac{1}{K}, \tag{A.15}$$

which states that the proportional increase in the marginal productivity of capital due to R&D expenditure (which measures the marginal benefit, in terms of output growth, of a unit of expenditure devoted to R&D), should always be equal to the reciprocal of the capital stock (which measures the marginal benefit of a unit of expenditure devoted to fixed investment).

Equation (A.15) must hold at every instant of time, determining the desired magnitudes $\hat{I}_K$ and $\hat{I}_{R\&D}$. Differentiating with respect to time we have

$$\gamma \frac{(A''\dot{\Omega})A - A'(A'\dot{\Omega})}{A^2} = -\frac{\dot{K}}{K^2}, \tag{A.16}$$

hence

$$\gamma \frac{A''\gamma I_{R\&D} A - (A')^2 \gamma I_{R\&D}}{A^2} = -\frac{I_K}{K^2} = -\frac{I - I_{R\&D}}{K^2} = \frac{I_{R\&D}}{K^2} - \frac{I}{K^2}. \tag{A.17}$$

Collecting the terms containing $I_{R\&D}$ we obtain

$$\gamma^2 \left[\frac{A''}{A} - \left(\frac{A'}{A}\right)^2 - \frac{1}{\gamma^2 K^2}\right] I_{R\&D} = -\frac{I}{K^2} \tag{A.18}$$

and multiplying through by $-K^2$ we get

$$\gamma^2 K^2 \left[ \left( \frac{A'}{A} \right)^2 - \frac{A''}{A} + \frac{1}{K^2} \right] I_{R\&D} = I, \tag{A.19}$$

where the expression in square brackets is positive, since $A'' < 0$. Defining:

$$H = \gamma^2 K^2 \left[ \left( \frac{A'}{A} \right)^2 - \frac{A''}{A} + \frac{1}{K^2} \right] = \gamma^2 K^2 \left[ \left( \frac{A'}{A} \right)^2 - \frac{A''}{A} \right] + \gamma^2 \tag{A.20}$$

we finally have

$$I_{R\&D} = \frac{1}{H} I, \quad I_K = \frac{H-1}{H} I \tag{A.21}$$

where $I = f_1(A - R)$.

### A.3 Qualitative Analysis

The dynamic structure of the theoretical model around the growth equilibrium (where $Y = Y_D = Y_S = AK$) is summarized below:

$$\begin{array}{ll} \mathrm{D}I = \alpha_1(\hat{I} - I) & \hat{I} = f_1[A - \bar{R}] \\ \mathrm{D}C = \alpha_2(\hat{C} - C) & \hat{C} = c\hat{Y} = cAK \\ \mathrm{D}BT = \alpha_3(\widehat{BT} - BT) & \widehat{BT} = 0 \\ \mathrm{D}R = \alpha_4 \left( \hat{R} - R \right) & \hat{R} = R^* \ (^* \text{ denotes the other country}) \\ \mathrm{D}A = \alpha_7(\hat{A} - A) & \hat{A} = \bar{A} \\ \mathrm{D}K = I_K = \kappa I & \kappa = 1 - \dfrac{1}{H} \end{array} \tag{63}$$

where $H = \gamma^2 K^2 \left[ \left( \frac{A'}{A} \right)^2 - \frac{A''}{A} \right] + \gamma^2$.

System (63) is a system of 12 differential equations (6 for each economy). Fortunately its dimension can be greatly reduced by the following considerations:

1) The third equations - thanks to the fact that the desired value of $BT$ is zero - form an independent subset that can be solved independently, and is easily seen to be stable, since the two independent equations have a characteristic root equal to $-\alpha_3 < 0$.

2) The fourth equations constitute a subsystem that can be solved independently (see appendix A.1) and turns out to be stable, with roots that are all real negative.

Thus we are left with the following system:

$$\begin{array}{ll} \mathrm{D}I = \alpha_1(\hat{I} - I) & \hat{I} = f_1[A - \bar{R}] \\ \mathrm{D}C = \alpha_2(\hat{C} - C) & \hat{C} = c\widehat{Y} = cAK \\ \mathrm{D}A = \alpha_7(\hat{A} - A) & \hat{A} = \bar{A} \\ \mathrm{D}K = I_K = \kappa I. & \end{array} \tag{64}$$

This system is dichotomous, because it gives rise to two four-equation systems - one for the Eurozone and the other for the US - that can be solved independently. Consider the matrix of the linear approximation to each system:

$$\begin{bmatrix} -\alpha_1 & 0 & \alpha_1 (f_1')_0 & 0 \\ 0 & -\alpha_2 & \alpha_2 c(K)_0 & \alpha_2 c\bar{A} \\ 0 & 0 & -\alpha_7 & 0 \\ (\kappa)_0 & 0 & (\kappa_A)_0 & (\kappa_K)_0 \end{bmatrix}, \tag{65}$$

where $\kappa_A = \dfrac{1}{H^2}\dfrac{\partial H}{\partial A}$, $\kappa_K = \dfrac{1}{H^2}\dfrac{\partial H}{\partial K}$ and $(...)_0$ denotes that the variable is evaluated at the equilibrium point. Furthermore, it can be checked[16] that $\dfrac{\partial H}{\partial A} < 0, \dfrac{\partial H}{\partial K} > 0$.

The characteristic equation is

$$\begin{vmatrix} -\alpha_1 - \lambda & 0 & \alpha_1 (f_1')_0 & 0 \\ 0 & -\alpha_2 - \lambda & \alpha_2 c(K)_0 & \alpha_2 c\bar{A} \\ 0 & 0 & -\alpha_7 - \lambda & 0 \\ (\kappa)_0 & 0 & (\kappa_A)_0 & (\kappa_K)_0 - \lambda \end{vmatrix} = 0, \tag{66}$$

which gives

$$(-\alpha_2 - \lambda)(-\alpha_7 - \lambda)(-\alpha_1 - \lambda)\left[(\kappa_K)_0 - \lambda\right] = 0. \tag{67}$$

Hence the characteristic roots are $\lambda_1 = -\alpha_2$, $\lambda_2 = -\alpha_7$, $\lambda_3 = -\alpha_1$, $\lambda_4 = (\kappa_K)_0$.

In conclusion, the system under consideration has three real negative roots and one real positive root. Thus we are in the standard case of saddle-point stability. More precisely, given a first-order differential system in normal form with distinct characteristic roots, partly stable and partly unstable (a *conditionally stable* system), we can always make the system stable provided that we can choose as many initial conditions as there are unstable roots (Gandolfo, 1997, Chap. 18, Sect. 18.2.2.3, Theorem 18.3). In our model, the presence of the government in the consumption and investment equations ensures that it is possible to choose one initial condition so as to make the system stable. Stability will be monotonic, since all stable roots are real.

# B Data Appendix

**Country sample:** Euro-area (Belgium, Germany, Spain, France, Ireland, Italy, Luxembourg, Netherlands, Austria, Portugal, Finland); the US; Rest of World (Japan and UK). **Time sample:** 1975:Q1-2003:Q4. **Units:** Billions of national currency. **Base year:** 2000.

[16] $\dfrac{\partial H}{\partial A} = \gamma^2 K^2 \left[2\left(\dfrac{A'}{A}\right)\left(-\dfrac{A'}{A^2}\right) + \dfrac{A''}{A^2}\right] < 0,$
$\dfrac{\partial H}{\partial K} = 2\gamma^2 K \left[\left(\dfrac{A'}{A}\right)^2 - \dfrac{A''}{A}\right] > 0.$

**Variables:** $C$ (Total Consumption): Total Consumption is defined as the sum of total government consumption and total private consumption. Data source for the Euro-area is the Area Wide Model (AWM, hereafter) (ECB, 2004; Fagan et. al, 2001), while data source for the US is International Financial Statistics - IMF [IFS, hereafter] (2006).

$K$ (Capital Stock): Capital stock for the Euro-area is provided by AWM (2004). Data on gross fixed capital formation are from IFS (2006). Depreciation (consumption of capital) is obtained as $\delta K = I - I^N$, where $I^N$ is net investment. Data on net and gross investment, and the data point of capital stock referred to 2000 are from Kamps (2004). Original data are annual, made quarterly by authors' calculations.

$BT$ (Trade Balance): The trade balance is given by $BT = X - M$, where $X$ is exports of goods and services and $M$ is imports of goods and services. Data source for the US is IFS (2006).

$R$ (Short Term Real Interest Rate): Interest rate for the Euro-area is 3-month interest rate and is provided by the AWM (2004). Interest rate for the US (Treasury Bill Rate); it is the weighted average yield on multiple-price auctions of 13-week treasury bills (IFS, 2006).

$\overline{R}$ (World Real Interest Rate): The world real interest rate is calculated as the weighted average of the real interest rates of the Eurozone, Japan, the US and the UK, the weights being the relative net domestic product of each country relatively to the sum of the net domestic product of the four economies.

$Y$ (Net domestic product): Net domestic product is obtained from the AWM (2004) for the Eurozone and from IFS (2006) for the US.

$Y_P$ (Potential output): Potential output is obtained from the AWM (2004) for the Eurozone and from OECD (2006) for the US.

$F$ (Net foreign debt): For each of the two economies considered[17] net foreign assets are provided by Lane and Milesi-Ferretti (2006).

$NFI$ (Net Factor Income from Abroad): Data source for the Euro-area is AWM (2004), while for the US it is IFS (2006).

$e_R$ (Real Bilateral Exchange Rate): A rise in the index means an increase in the value of the USD against the Euro/ECU, then it denotes a loss of competitiveness. Data are observed at the end of period. Data source is Eurostat - Economy and Finance (2007).

$e$ (Nominal Bilateral Exchange Rate): The Euro/USD nominal exchange rate is defined as number of Euro (ECU until 31 December 1998) per one USD. Data source is Eurostat - Economy and Finance (2007).

$\Omega$ (Share of $R\&D$ Investment): This series is the share of $R\&D$ investment in total gross fixed capital formation and is obtained as $\Omega = I_{R\&D}/I_K$. Data on $R\&D$ investment for

[17] Luxemburg is excluded from the Euro-area because missing in the data source.

the two economies considered are taken from Eurostat - Science and Technology (2007). Original data are annual, made quarterly by authors' calculations.

$Y^{ROW}$ (Net Domestic Product of the Rest of the World): Net domestic product of the rest of the world is computed as the geometric average of the net domestic products of Japan and the UK (weights are relative GDPs).

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