

# Adjustment Costs, Inventories and Output

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## Abstract

This paper analyzes the optimal adjustment strategy of an inventory-holding firm facing price- and quantity-adjustment costs in an inflationary environment. The model nests both the original menu-cost model that allows production to be costlessly adjusted, and the later model that includes price- and quantity-adjustment costs, but rules out inventory holdings. It is shown that the firm's optimal adjustment strategy may involve stockouts. At low inflation rates, output is inversely related to the inflation rate, and the length of time demand is satisfied increases with the demand elasticity but decreases with the storage cost and the real interest rate.

JEL Code: D21, D24, L23.

Keywords: menu costs, quantity-adjustment costs, inventories, stockouts, output, inflation.

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# 1 Introduction

Fixed price-adjustment costs, often called menu costs, play a central role in many explanations of how monetary changes are transmitted and have real effects. Owing to these costs, it may be optimal for a monopolistic firm to keep its current nominal price unchanged, even though this price differs from the static profit-maximizing price. In fact, the theoretical literature has demonstrated that in the face of seemingly minor price-adjustment costs, a fully anticipated inflation may increase average output and welfare if consumers are well informed about prices (Danziger, 1988), but may lead to costly search and decrease welfare if they are not (Benabou, 1988).<sup>1</sup>

There is direct empirical evidence that fixed price-adjustment costs are non-trivial (Levy et al., 1997; Zbaracki et al., 2004).<sup>2</sup> However, recognizing the existence of fixed price-adjustment costs does not mean that other adjustment costs can be ignored. One particular shortcoming of most of the theoretical literature is the implicit assumption that while it is costly to adjust the nominal price, it is costless to adjust the production. However, such asymmetry between price- and quantity-adjustment costs is not well founded. For instance, Bresnahan and Ramey (1994) document the existence of fixed costs of quantity adjustment which may be due to the loss of organizational capital and other internal adjustment costs incurred when inputs are rearranged in order to accommodate output changes (Baily et al., 2001; Jovanovic and Rousseau, 2001). There is also ample evidence of the existence of fixed costs of adjusting labor (Davis and Haltiwanger, 1992; Hamermesh, 1989; Caballero et al., 1997; Abowd and Kramarz, 2003), and of adjusting capital (Doms and Dunne, 1998; Cooper et al., 1999; Nilsen and Schiantarelli, 2003).

The presence of quantity-adjustment costs in addition to price-adjustment costs significantly affects the response to a fully anticipated inflation. Specifically, if the quantity-

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<sup>1</sup> The effects of monetary shocks are studied in Akerlof and Yellen (1985); Mankiw (1985); Blanchard and Kiyotaki (1987); and Ball and Romer (1990). Recent contributions consider, among other things, models in which idiosyncratic shocks impinge on price-adjustment costs (Dotsey et al., 1999), and on productivity (Danziger, 1999; Golosov and Lucas, 2007).

<sup>2</sup> It is also well established that the nominal prices of various goods are kept unchanged for substantial periods, even at high inflation rates when the erosion of real prices can be considerable (Cecchetti, 1986; Danziger, 1987; Kashyap, 1995; Fisher and Konieczny, 2006). Needless to say, nominal price stickiness need not originate from fixed price-adjustment costs, but may be caused by other factors, such as signalling and strategic interactions.

adjustment cost at least equals the price-adjustment cost, Danziger (2001) has shown that average output and welfare are lower with a moderate inflation than with full price stability, and inflation is therefore harmful. Furthermore, Danziger and Kreiner (2002) have shown that quantity-adjustment costs amplify the effects of price-adjustment costs.<sup>3</sup> However, while these later papers provide a more satisfactory modelling of the adjustment costs facing a typical firm than the earlier literature, they themselves are incomplete since they abstract from inventory holdings by assuming that any unsold output is destroyed. Unless the output is completely perishable, inventories are an inevitable consequence of quantity-adjustment costs. A more realistic assumption would be that firms may hold unsold output as inventory for later sale, with the pricing and production decisions naturally taking this into account.

The purpose of this paper is therefore to analyze the behavior of a firm that produces a storable good and faces fixed price- and quantity-adjustment costs in an environment with a constant inflation rate. Keeping goods in inventory may be costly. If the quantity-adjustment cost at least equals the price-adjustment cost, the optimal strategy entails that the firm keeps its nominal price unchanged in periods of equal length and production at a permanent level. At the beginning of a period, the real price is so high that there is unsold output and the firm accumulates inventory. Later in the period, the real price has fallen enough that demand exceeds current production and the firm runs down the inventory.

A firm's inventory may be depleted before the end of a period with a constant nominal price. Stockouts occur at both low and high inflation rates.<sup>4</sup> In the case of low inflation rates, the period with a constant nominal price is then so long that it would be too costly to produce and store enough at the beginning of the period to have sufficient inventory available to satisfy all the demand at the end. Furthermore, production will be shown to decrease with the inflation rate, and the length of time demand is satisfied to increase with the demand elasticity and decrease with the storage cost and the real interest rate.

The paper will make clear that the optimality conditions are different from the case

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<sup>3</sup> Andersen (1995) shows that a small shock may leave output unchanged, while Andersen and Toulemonde (2004) show that in order for a shock to affect output, it must be of intermediate size while price-adjustment costs must be large relative to quantity-adjustment costs.

<sup>4</sup> Consistent with the model, Bills (2005) find that the average stockout rate is about 9% for consumer durables.

without quantity-adjustment costs as well as from the no-inventory case with quantity-adjustment costs. For instance, it is necessary to determine when it is optimal to stock out, and the discounted marginal real revenue from an increase in production (which equals the discounted marginal real cost at the optimal production) includes the real revenue from the later sale of the additional inventory produced in the first part of the period.

The model in this paper nests both the original menu-cost model that allows production to be costlessly adjusted, thereby obviating the need for inventory, and the later model that includes price- and quantity-adjustment costs, but rules out inventory holdings by assumption. Thus, the present model not only remedies the limitations of earlier models, but in addition contains a unifying framework for analyzing the consequences of a firm's optimal pricing and output strategy under all the various alternative assumptions about the cost of adjusting production and the possibility of holding inventory.<sup>5</sup>

The paper also provides a numerical illustration of the output effect of allowing for inventory holdings. Based on realistic parameter values, it is shown that production is considerably higher with inventories than in their absence. This is true even at very high levels of inventory-carrying costs. Accordingly, inventories may play an important role in reducing the output loss from inflation when quantity adjustments are costly.

## 2 The Firm

Consider a firm with an elastic demand function  $D(z_t)$ , where  $z_t$  is the real price of the product at time  $t$ . The real production cost is  $k > 0$  per unit of output. There is a constant rate of inflation  $\mu > 0$ . Price and quantity adjustments involve a fixed cost, which prevents the firm from adjusting its nominal price and production continuously. The fixed cost of quantity adjustment at least equals the fixed cost of price adjustment. This implies, as will be shown below, that the firm's production will remain unchanged at a permanent level while its nominal price will be kept constant in periods of equal length. At the end of each period the nominal price is increased so that all periods start with the same initial real

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<sup>5</sup> On the other hand, to make the model manageable, we follow the literature in assuming that the demand for the firm's product is stationary, which effectively rules out that consumers store the good or substitute intertemporally in consumption, and that the real production cost is constant over time, i.e., that the firm's input prices are flexible.

price. Consequently, when the real price is relatively high at the beginning of a period, the firm produces more than it can sell and accumulates inventory. But when the real price is relatively low later in the period, demand exceeds production and the inventory helps the firm to postpone and possibly eliminate stockouts.

To describe the firm's adjustment strategy, let  $S$  denote the initial real price of a good in a period with a constant nominal price,  $T$  the duration of a period, and  $Y$  the production. Accordingly, if the nominal price is adjusted at time zero, it will remain constant in  $[0, T]$  and the next adjustment of the nominal price will take place at time  $T$ . Inflation reduces the real price of the good to  $z_\tau = Se^{-\mu\tau}$  after  $\tau$  of the period has elapsed such that the terminal real price is  $s \equiv Se^{-\mu T}$ . Since the real price decreases within a period with a constant nominal price, the initial real price and the production will not be set so low that there is excess demand at the beginning of a period. Neither will the initial real price and the production be set so high, and the period so short, that the firm will hold a positive inventory at the end of the period. If demand exceeds the current production, the firm satisfies demand as long as there is an inventory from which sales can be made. Accordingly, if  $\delta$  is the depreciation rate of stored goods, there exists a time  $T_I \in [0, T]$  that is defined by

$$\begin{aligned} \int_0^{T_I} e^{-\delta(T_I-\tau)}[Y - D(Se^{-\mu\tau})]d\tau &= 0 \\ \Leftrightarrow \int_0^{T_I} e^{\delta\tau}[Y - D(Se^{-\mu\tau})]d\tau &= 0, \end{aligned} \quad (1)$$

and at which time the inventory is exhausted. It is assumed that  $\delta \in (-r, \infty)$ , where  $r > 0$  is the real interest rate.<sup>6</sup> If  $T_I < T$ , then demand is satisfied in the first part of the period when  $\tau \in [0, T_I)$ , while stockouts occur in the second part of the period when  $\tau \in [T_I, T)$ . If  $T_I = T$ , then demand is satisfied during the entire period.

When the firm accumulates inventory, it pays the production cost concurrently while receiving the revenue from sales only later. Accordingly, the inventory-carrying cost consists of the storage cost due to the depreciation of goods held in inventory and the real interest on the production cost incurred for later sales.

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<sup>6</sup> It is natural to think of  $\delta$  as being non-negative, but allowing for negative  $\delta$ 's makes the framework more general. In particular, it will be shown later that the extreme case of  $\delta = -r$  is equivalent to a model with price-adjustment costs and no quantity-adjustment costs.

### 3 The Profit Function

Let  $z_I \equiv Se^{-\mu T_I}$  denote the real price corresponding to  $T_I$ . Demand is satisfied for all real prices above  $z_I$ , but is not otherwise. Since  $T_I < T$  implies that  $z_I > s$ , it follows that if  $s < z_I$ , the firm sells  $D(z_t)$  when  $z_t \in (z_I, S]$ , but only sells the current production  $Y$  (leaving  $D(z_t) - Y$  of demand unsatisfied) when  $z_t \in (s, z_I]$ . In contrast, if  $s = z_I$ , the firm always sells  $D(z_t)$ . Accordingly, the firm's instantaneous real profit is

$$\begin{cases} z_t D(z_t) - kY & \text{if } z_t \in (z_I, S], \\ (z_t - k)Y & \text{if } z_t \in (s, z_I]. \end{cases}$$

In Figure 1, the horizontal axis measures the real price and the vertical axis measures the instantaneous real profit. In the absence of fixed costs of quantity adjustment, production would be continuously adjusted to satisfy demand. The instantaneous real profit, stemming solely from the current production, would be  $(z_t - k)D(z_t)$ , which is shown by the fully drawn grey curve. In the presence of a fixed cost of quantity adjustment, however, production remains unchanged at a permanent level. For a given  $S$  and  $Y$ , the instantaneous real profit is shown by the fully drawn black curve.<sup>7</sup> Letting  $z_Y$  denote the real price at which demand equals production, i.e.,  $D(z_Y) = Y$ , it follows that  $z_Y \in (z_I, S)$  and that the instantaneous real profit at  $z_Y$  is the same as it would be if production could be continuously adjusted.

At the beginning of a period with a constant nominal price, the real price is so high that  $z_t \in (z_Y, S]$ , and the demand is less than production so that the firm accumulates inventory. Hence, the instantaneous real profit is below that obtained in the absence of a fixed cost of quantity adjustment. Later in the period, the real price falls to  $z_t \in (z_I, z_Y)$ . The demand exceeds production but the firm satisfies demand by selling from its inventory. The instantaneous real profit is above that obtained in the absence of a fixed cost of quantity adjustment and increases with time.

If  $s < z_I$ , there is a final part of the period where the real price falls below  $z_I$  to  $z_t \in (s, z_I]$ . The firm has then depleted its inventory and can only sell the current production.

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<sup>7</sup> The figure assumes that the production exceeds the initial demand so that  $z_I < S$ . This is clearly satisfied if the firm's strategy does not involve stockouts, and footnote 8 below verifies that it is also satisfied if the firm's strategy does involve stockouts.

Accordingly, the instantaneous real profit drops discontinuously at  $z_I$  as the firm stocks out. In this range of prices, therefore, the instantaneous real profit is linear in the real price and below what it would be in the absence of a fixed cost of quantity adjustment (as long as the real price exceeds  $k$ ), and it vanishes as the real price reaches  $k$ . On the other hand, if  $s = z_I$ , then the real price never falls below  $z_I$ . The firm always satisfies all demand, since the nominal price is increased at the moment the inventory gets depleted.

The instantaneous real profit at the beginning of the period when the firm has not started to sell from inventory, i.e., when  $z_t \in [z_Y, S]$ , is the same as it would be if there were no possibility of holding inventory. The instantaneous real profit is, however, higher when the firm sells from inventory, i.e., when  $z_t \in (z_I, z_Y)$ , since the instantaneous real profit would then be only  $(z_t - k)Y$  if there were no possibility of holding inventory. This is shown by the dashed grey line in Figure 1. If  $s < z_I$ , then there is a final part of the period when the firm only sells its current production, i.e., when  $z_t \in (s, z_I]$ , and the instantaneous real profit is then the same as it would be if there were no possibility of holding inventory.

## 4 The Optimal Adjustment Strategy

At the time of a price adjustment, the firm's discounted real profits from the period with a constant nominal price are

$$S \int_0^{T_I} e^{-(\mu+r)\tau} D(Se^{-\mu\tau}) d\tau + SY \int_{T_I}^T e^{-(\mu+r)\tau} d\tau - kY \int_0^T e^{-r\tau} d\tau - c,$$

where  $c > 0$  is the fixed real cost of a price adjustment. The first term is the discounted real revenue from the part of the period when the firm satisfies demand and the instantaneous real revenue is  $Se^{-\mu\tau} D(Se^{-\mu\tau})$ . The second term is the discounted real revenue from the part of the period when the firm stocks out and the instantaneous real revenue is  $SYe^{-\mu\tau}$ . If  $T_I = T$ , the second term vanishes since the firm never stocks out. The third term is the discounted real production cost for the entire period.

The total discounted real profits from the current and all future periods with a constant nominal price are

$$V \equiv \frac{1}{1 - e^{-rT}} \left[ S \int_0^{T_I} e^{-(\mu+r)\tau} D(Se^{-\mu\tau}) d\tau + SY \int_{T_I}^T e^{-(\mu+r)\tau} d\tau - kY \int_0^T e^{-r\tau} d\tau - c \right]. \quad (2)$$



Assuming that the firm's production will remain unchanged at a permanent level, the firm's optimal adjustment strategy consists of a choice of  $S$ ,  $T$ , and  $Y$  that maximizes  $V$ , given the definition of  $T_I$  in the inventory constraint (1) and that  $T_I \leq T$ .<sup>8</sup> To verify that the firm will in fact never want to change its production given that the fixed cost of quantity adjustment at least equals  $c$ , suppose that the firm starts with the optimal  $S$  and  $Y$  at time  $t = 0$ . Adjusting production before time  $T$  (alone or concomitantly with an adjustment of the nominal price) is at least as expensive as making the real price equal to  $S$  by adjusting only the nominal price. However, the latter strictly dominates since the firm would start the new period with the optimal  $S$  and  $Y$ . It follows then that it will never be optimal for the firm to adjust its production.

In the following, we first analyze the case in which an optimal adjustment strategy involves stockouts, and next, the case in which it does not.

#### 4.1 The Optimal Adjustment Strategy with Stockouts

An optimal adjustment strategy that involves stockouts,  $T_I < T$ , (derived in Appendix A) satisfies

$$SD(S) + z_I[Y - D(S)]e^{-(r+\delta)T_I} - sY - rc = 0, \quad (3)$$

$$(s - k)Y - rV = 0, \quad (4)$$

$$z_I e^{-rT_I} \int_0^{T_I} e^{-\delta\tau} d\tau + S \int_{T_I}^T e^{-(\mu+r)\tau} d\tau - k \int_0^T e^{-r\tau} d\tau = 0. \quad (5)$$

The production at the time of price adjustment increases the firm's real revenue by the immediate sale of  $D(S)$  at the initial real price  $S$ , and the later sale of the remaining production net of depreciation, i.e.,  $[Y - D(S)]e^{-\delta T_I}$ , at the real price  $z_I$  at time  $T_I$  when the inventories would otherwise have been exhausted. Had the nominal price not been adjusted, the same production would instead have increased the real revenue by the immediate sale of  $Y$  at the terminal real price  $s$ . Condition (3) shows that the initial real price is determined such that the discounted instantaneous real revenue from the sale of the production at the time of price adjustment equals what the instantaneous real revenue would be without a price adjustment plus  $rc$ , which is the real interest saved by postponing the cost of price adjustment.

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<sup>8</sup> It is assumed that  $V \geq 0$  so that the firm will remain in business.

According to condition (4), the length of a period with a constant nominal price is such that the instantaneous terminal real profit equals the real interest on  $V$ . The condition entails that at the time of price adjustment the value of the optimal strategy is the same as if the firm would earn the instantaneous terminal real profit at all times. It also entails that the terminal real price at least equals the real production cost,  $s \geq k$ .

If production increases, then all of the additional output in the first part of the period until time  $T_I$ , i.e.,  $\int_0^{T_I} e^{-\delta\tau} d\tau$  net of depreciation, would be sold at the real price  $z_I$  at time  $T_I$ , while the additional output at any time in the second part of the period from time  $T_I$  until time  $T$  would be sold at the contemporaneous real price. Hence, the discounted marginal real revenue from the increased production in  $[0, T_I)$  is  $z_I e^{-rT_I} \int_0^{T_I} e^{-\delta\tau} d\tau$ , and from the increased production in  $[T_I, T)$  is  $S \int_{T_I}^T e^{-(\mu+r)\tau} d\tau$ . Condition (5) therefore expresses that the level of production is such that, for the entire period, the discounted marginal real revenue equals the discounted marginal real cost, i.e.,  $k \int_0^T e^{-r\tau} d\tau$ .<sup>9</sup>

## 4.2 The Optimal Adjustment Strategy without Stockouts

An optimal adjustment strategy that does not involve stockouts,  $T_I = T$ , (derived in Appendix A) satisfies

$$SD(S) + k_T[Y - D(S)]e^{-(r+\delta)T} - sD(s) + k_T[D(s) - Y] - rc = 0, \quad (6)$$

$$sD(s) - kY - k_T[D(s) - Y] - rV = 0, \quad (7)$$

where  $k_T \equiv k \int_0^T e^{r\tau} d\tau / \int_0^T e^{-\delta\tau} d\tau$  is the real cost of having an additional unit available at the end of the period.

Of its production at the time of price adjustment, the firm immediately sells  $D(S)$ , from which it obtains the instantaneous real revenue  $SD(S)$ . The firm also inventories  $Y - D(S)$ , allowing it to reduce the level of production throughout the period and thereby save  $k_T[Y - D(S)]e^{-(r+\delta)T}$  in discounted real production costs in the period. If the nominal price had been kept unchanged, the firm would have had to sell  $D(s)$  to satisfy all demand, and it would have obtained the instantaneous real revenue  $sD(s)$ . Since selling  $D(s)$  would have required the availability of  $D(s) - Y$  of inventoried output from the previous period,

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<sup>9</sup> Condition (5) confirms that  $z_I < S$ , since  $s > k$  implies that the left-hand side of the condition would always be positive for  $T_I = 0$ .

production would have had to be higher in that period, and an additional  $k_T[Y - D(s)]$  in discounted real production costs would have been incurred in that period. According to condition (6), the initial real price is determined so that the gain from a price adjustment equals the gain from keeping the nominal price unchanged plus the real interest saved by delaying the price adjustment.

The instantaneous real profit attributable to sales at the terminal real price is the difference between the instantaneous real revenue from the sale of  $D(s)$  and the total discounted cost of producing enough so that  $D(s)$  can be sold. Condition (7) indicates that a price adjustment takes place when the instantaneous real profit attributable to sales at the terminal real price equals the real interest on  $V$ .

### 4.3 When Are Stockouts Optimal?

In order to determine whether the optimal adjustment strategy involves stockouts, note that if  $T_I < T$  and there are stockouts, then the discounted marginal real revenue from an increase in production decreases in  $T_I$  (for a given  $S$  and  $T$ ).<sup>10</sup> The reason is that the firm has to wait until  $T_I$  before it can benefit from selling the last of its inventory and start to benefit from the higher production when there is excess demand. It follows then that if  $T$  is substituted for  $T_I$ , the left-hand side of condition (5) is negative. That is

$$se^{-rT} \int_0^T e^{-\delta\tau} d\tau - k \int_0^T e^{-r\tau} d\tau < 0,$$

or, equivalently,  $s < k_T$ .

Accordingly, if the terminal real price is less than the real cost of having an additional unit available at the end of a period,  $s < k_T$ , it does not pay for the firm to produce so much that it avoids stocking out at the end of the period, and  $T_I < T$  is optimal. But if the terminal real price at least equals the real cost of the additional unit,  $s \geq k_T$ , it is profitable for the firm to produce enough to always satisfy demand, and  $T_I = T$  is optimal.<sup>11</sup>

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<sup>10</sup> The partial derivative of the left-hand side of condition (5) with respect to  $T_I$  is

$$-(\mu + r + \delta)z_I e^{-rT_I} \int_0^{T_I} e^{-\delta\tau} d\tau < 0.$$

<sup>11</sup> If  $T_I = T$  and  $s = k_T$ , conditions (3)-(4) are identical to conditions (6)-(7). If  $s > k_T$ , condition (7)

## 5 The Optimal Adjustment Strategy at Low Inflation

We first establish that the firm stocks out and how its production depends on the inflation rate.

*Theorem 1:* At low inflation rates  $T_I < T$  and  $dY/d\mu < 0$ .

*Proof:* See Appendix B.

At low inflation rates the nominal price is kept unchanged for so long that the entire inventory is sold and stockouts occur at the end of a period with a constant nominal price. The logic is that at low inflation rates the nominal price is kept constant for long periods, and hence the firm would have to keep inventory for so long that it would be unprofitable to satisfy demand at the end of the period. Furthermore, production decreases with the inflation rate, since discounting puts more weight on the real profits at the beginning of a period than on the real profits at the end. At low inflation rates, therefore, it is particularly important for the firm to increase its instantaneous real profits at the beginning of a period, which requires reducing production and the accumulation of inventory. Consequently, the firm produces less than it would under full price stability.<sup>12</sup>

We next establish how the length of time demand is satisfied depends on the demand elasticity at the monopoly real price (denoted by  $\epsilon$ ), the depreciation rate, and the interest rate.

*Theorem 2:* At low inflation rates  $dT_I/d\epsilon > 0$ ;  $dT_I/d\delta < 0$ ; and  $dT_I/dr < 0$ .

*Proof:* See Appendix C.

The length of time demand is satisfied increases with the demand elasticity at the monopoly real price when the inflation rate is low, the reason being that at low inflation rates the discounted marginal real revenue decreases with the length of time demand is satisfied and increases with the initial real price (which is close to the monopoly real price), while the discounted marginal real cost is independent of both the length of time demand

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shows that the instantaneous real profit from the terminal production is less than the real interest on  $V$ ; that is,  $(s - k)Y < rV$ .

<sup>12</sup> In contrast, in a model with price-adjustment costs only, a firm, on average, produces more than under full price stability. The reason is that since the initial real price is close to the monopoly real price and the periods with a constant nominal price are very lengthy, the real price is below the monopoly real price most of the time. Hence, the output is above the static monopoly output most of the time (Danziger, 1988).

is satisfied and the initial real price. Accordingly, the length of time demand is satisfied is positively related to the monopoly real price. Since the monopoly real price increases with the demand elasticity, a higher demand elasticity is associated with a longer length of time demand is satisfied.

On the other hand, the length of time demand is satisfied decreases with the storage cost and the real interest rate. This is not surprising as the cost of carrying inventory increases with the storage cost and the real interest rate, and in order to economize on this cost, the inventory, and hence the length of time demand is satisfied, decreases with the storage cost and the real interest rate.<sup>13</sup>

## 6 The Optimal Adjustment Strategy at High Inflation

To establish that stockouts occur at high inflation rates, observe that there exists a high inflation rate for which  $V = 0$ . Condition (4) reveals that at this high inflation rate the terminal real price  $s$  equals the production cost  $k$ , and hence  $s < k_T$ .<sup>14</sup> Since the optimal strategy is continuous in  $\mu$ , it follows that  $T_I$  must be less than  $T$  at sufficiently high inflation rates, which implies that inventories are exhausted and stockouts occur in the last part of a period with a constant nominal price. The intuition is that there is only a minor gap between the real price and the production cost towards the end of a period with a constant nominal price. Given the inventory-carrying cost, it is not profitable for the firm to keep goods in inventory for so long that demand is satisfied until the end of the period.<sup>15</sup>

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<sup>13</sup> A comprehensive analysis of uncertainty is beyond the scope of the present paper, but it is not too complicated to ascertain how a single unexpected productivity shock would affect output. Thus, if a productivity shock occurs, then condition (5), which requires that the level of production equalizes the discounted marginal real revenue and the discounted marginal real cost in the entire period, would no longer be satisfied. If the shock is small, it is not worth it for the firm to incur the fixed quantity-adjustment cost in order to change the production level. Since conditions (3) and (4) are independent of  $k$ , a small productivity shock would therefore also not cause the firm to change the time of the next price adjustment, even if this is still far into the future. However, a large positive or negative shock would cause the firm to change its production (perhaps not immediately, but no later than the next time price is adjusted), and move the time of the next price adjustment. In particular, the firm might want to immediately adjust both its production and its price.

<sup>14</sup> In order for condition (7) to hold for  $V = 0$ , it would have to be the case that  $s < k_T$ , which is inconsistent with  $T_I = T$ .

<sup>15</sup> The theoretical conclusion that stockouts are prevalent with high inflation rates is supported by the empirical evidence that shortages are positively correlated with the inflation rate (Lamont, 1997).

## 7 Generality of the Model

Although the model assumes that  $\delta \in (-r, \infty)$ , the optimality conditions are also valid in the extreme cases where  $\delta$  equals  $-r$  or  $\infty$ , as we show in the following.

If  $\delta = -r$ , stored goods grow at a rate equal to the real interest rate, so there is no inventory-carrying cost. The discounted production cost is invariant with respect to the time the goods are produced, and it is immaterial when production takes place. Hence,  $k_T = k$  so that  $s > k_T$  and it is always optimal to satisfy demand to the end of a period. That is,  $T_I = T$  for all inflation rates. Since  $\int_0^T e^{-r\tau} D(Se^{-\mu\tau}) d\tau = Y \int_0^T e^{-r\tau} d\tau$ , the case of  $\delta = -r$  is, in terms of the firm's sales and total discounted real profits, equivalent to the absence of quantity-adjustment costs and production being continuously adjusted. Accordingly, conditions (6)-(7) reduce to

$$\begin{aligned} (S - k)D(S) - (s - k)D(s) - rc &= 0, \\ (s - k)D(s) - rV &= 0, \end{aligned}$$

which are the optimality conditions in the presence of only price-adjustment costs (Sheshinski and Weiss, 1977).

On the other hand, if  $\delta = \infty$ , the output is completely perishable so that unsold goods are wasted. The case is equivalent to not allowing the firm to hold inventory. Since  $k_T = \infty$  and therefore  $s < k_T$ , it is always optimal to stock out at the end of a period. That is,  $T_I < T$  for all inflation rates. The firm's real revenue from the production at the time of price adjustment is then due only to the immediate sale of  $D(S)$ ; since there is no revenue from the unsold  $Y - D(S)$ , the second term in condition (3) vanishes. Similarly, if production increases, the increase in real revenue would be due only to the sale of the additional output in the second part of the period; since there is no revenue from the additional unsold output in the first part of the period, the first term in condition (5) vanishes. Conditions (3)-(5) therefore reduce to

$$\begin{aligned} SD(S) - sY - rc &= 0, \\ (s - k)Y - rV &= 0, \\ S \int_{T_I}^T e^{-(\mu+r)\tau} d\tau - k \int_0^T e^{-r\tau} d\tau &= 0, \end{aligned}$$

which are the optimality conditions for a non-storable good in the presence of both price- and quantity-adjustment costs (Danziger, 2001).

The present model therefore captures not only the typical inventory-holding circumstances, but nests both the original menu-cost model in which a firm has no need to hold inventory because it can costlessly change production, and the later model in which production changes are costly and a firm cannot hold inventory. Thus, our model provides a unifying framework for analyzing the optimal adjustment strategy and the effects of inflation in both of these extreme cases which have been considered in the previous models, as well as in the more realistic, intermediate situations in which a firm faces both price- and quantity-adjustment costs and can hold inventory.

## 8 Variable Production

It is also possible to generalize the model by relaxing the assumption that the fixed quantity-adjustment cost is so high that production will remain constant, and instead assume that production could be expanded in the short run at a weakly increasing marginal real cost. Although it would complicate the analysis, the basic results would continue to hold. In particular, inventories would still play an important role in shaping the firm's optimal adjustment strategy, although their impact would be mitigated since increased demand could be satisfied not only by drawing down inventories, but also by increasing production. Thus, there would still exist a time  $T_I$  at which inventories are exhausted. However, since production can now be increased, demand may be satisfied also if  $\tau \in [T_I, T)$ , and there would exist a time  $T_J \in (T_I, T]$  such that if  $T_J < T$ , demand is satisfied by increased production if  $\tau \in [T_I, T_J)$  and stockouts occur when  $\tau \in [T_J, T)$ , while if  $T_J = T$ , demand is always satisfied. The formal analysis would require defining a time-varying  $Y_t$  as the firm's production at time  $t$ , and  $C(Y_t)$  as the total real production cost when the output is  $Y_t$ . One would then substitute  $Y_t$  for  $Y$  in the inventory constraint (1) and make the appropriate changes in the expression for the total discounted real profits (2) before deriving the optimal adjustment strategy, which would now include determining production changes within periods with a constant nominal price.

A particularly simple case would be if the firm chooses a capacity  $Y$  such that the total real cost is  $kY$  as long as production does not exceed the capacity, and the marginal real

cost is  $k^* > k$  if production exceeds  $Y$ . This could, for instance, reflect that the firm cannot reduce its current work force and that it is necessary to pay overtime or activate less efficient machinery when production is temporarily increased above capacity. The firm's discounted real profits can be obtained from expression (2) by substituting  $T_J$  for  $T_I$  and adding the discounted real profits due to the production at the marginal real cost  $k^*$  between  $T_I$  and  $T_J$ , and they are

$$\frac{1}{1 - e^{-rT}} \left( S \int_0^{T_J} e^{-(\mu+r)\tau} D(Se^{-\mu\tau}) d\tau + SY \int_{T_J}^T e^{-(\mu+r)\tau} d\tau - kY \int_0^T e^{-r\tau} d\tau + \int_{T_I}^{T_J} \left\{ (Se^{-\mu\tau} - k^*) e^{-r\tau} [D(Se^{-\mu\tau}) - Y] \right\} d\tau - c \right).$$

If  $k^* < k_T$  so that  $k^*$  is not so high that the firm will never produce more than  $Y$  (making the option to increase production irrelevant), then  $T_I$  is determined so that  $k \int_0^{T_I} e^{r\tau} d\tau / \int_0^{T_I} e^{-\delta\tau} d\tau$ , which is the real cost of increasing the capacity to make an additional unit available at  $T_I$ , equals  $k^*$ . The firm will produce  $Y$  until its inventories have been exhausted, after which it will satisfy all demand by increasing production as long as the real price is at least  $k^*$ . Since  $Se^{-\mu\tau} = k^* \Leftrightarrow \tau = (1/\mu) \ln(S/k^*)$ , it follows that  $T_J = (1/\mu) \ln(S/k^*)$  if  $s < k^*$  and  $T_J = T$  if  $s \geq k^*$ . Consequently, the firm will stock out before the end of the period if the terminal real price is less than  $k^*$ , but not if the terminal real price is at least  $k^*$ .

## 9 A Numerical Illustration

To assess the importance of inventories for the inflation-output relationship, suppose the demand function is  $D(z_t) = z_t^{-10}$ , the unit production cost is  $k = 1$ , the real interest rate is  $r = 3\%$ , and the cost of price adjustment is  $c = 9^9/10^{11}$ .<sup>16</sup>

Table 1 lists the percentage loss of production (relative to the static monopoly output) due to inflation both when there is no storage cost and at various levels of storage cost.<sup>17</sup>

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<sup>16</sup> The demand function implies a markup (ratio of price to marginal cost) of 10/9, which is consistent with estimated markups that are typically between 1.1 and 1.2 (Rotemberg and Woodford, 1995; Basu and Fernald, 1997). The cost of price adjustment is about 1% of the static monopoly revenue, and thus between 0.7% as found by Levy et al. (1997) and 1.22% as found by Zbaracki et al. (2004). Sensitivity analysis (not reported here) shows that broadly similar results would hold with other constant-elasticity demand functions and unit production costs.



The firm's optimal strategy always involves accumulation of inventory at the beginning of a period with a constant nominal price. For the combinations of inflation rates and storage costs marked by an asterisk, the accumulated inventory is not large enough to satisfy demand until the end of the period with the result that the firm eventually stocks out. Thus, the table verifies that the firm stocks out at low inflation rates. Since Section 6 has proved that at high inflation rates the firm also stocks out at the end of the period, the table shows that at intermediate values of the inflation rates the inventory may be sufficient to satisfy demand until the end of the period.

The output loss always increases with inflation. A higher storage cost, which increases the cost of carrying inventory, is associated with a lower production at all levels of inflation. However, the output loss is much reduced compared to what it would be if the firm could not hold inventory (shown in the penultimate column of Table 1). For instance, if there is no storage cost, the loss is only about a third or less of what the loss would be if there were no possibility of holding inventory. At a realistic storage cost of  $\delta = 20\%$  and inflation rates of 5% or higher, the loss is less than half of what it would be if the firm could not hold inventory. Even if the storage cost is  $\delta = 50\%$  and the inflation rate is 10% or higher, the loss is only a little more than half of what it would be if the firm could not hold inventory.

On the other hand, the output loss is higher than what the loss of average output would be with price-adjustment costs only and production continuously adjusted to satisfy demand (shown in the last column of Table 1). Indeed, in this case, at an inflation rate of 1%, there would be a minor gain of average output.<sup>18</sup>

## 10 Conclusion

This paper has analyzed the optimal adjustment strategy of a firm faced with fixed price- and quantity-adjustment costs in an inflationary environment. An important innovation

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<sup>17</sup> Estimated inventory-carrying costs, which include the real interest, are on average 25% (Stock and Lambert, 2001).

<sup>18</sup> See footnote 11. Since sales and consumption with quantity-adjustment costs and the cost of storage given by  $\delta = -r$  are the same as with price-adjustment costs only, the last column also shows the loss/gain of average consumption (which is different from output) if there were only price-adjustment costs. The gain of average consumption is made possible by the fact that average sales exceed production if  $\delta < 0$ . As shown in Theorem 1, the production and the inflation rate are inversely related at low inflation rates also if  $\delta < 0$ .

of the model is that the firm may keep its unsold production in inventory for later sales. This makes the framework more realistic than previous models that have abstracted from inventories by assuming that any unsold output is immediately destroyed. In fact, the model is quite general since it nests both the original menu-cost model without costs of quantity adjustment and the later model that includes costs of quantity adjustment but makes no allowance for inventories.

The single-firm model could be embedded in a general-equilibrium model of monopolistic competition with many firms, assuming a uniform distribution of the lengths of time since the firms' last price adjustments. It would then be simple to verify that the behavior of the aggregate output would be similar to that of the partial-equilibrium output of a single firm. Welfare would, therefore, be lower with a slow rate of inflation than at a stable price level due to the reduction in production and the cost of carrying inventory.

## Appendix A

### Derivation of the Optimal Adjustment Strategy

First, to determine the optimal adjustment strategy if stockouts are optimal, assume that  $T_I < T$  and differentiate  $V$  totally with respect to  $S$ ,  $T$ , and  $Y$ :

$$\frac{\partial V}{\partial S} = \frac{1}{1 - e^{-rT}} \left\{ \int_0^{T_I} e^{-(\mu+r)\tau} D(Se^{-\mu\tau}) d\tau + S \int_0^{T_I} e^{-(2\mu+r)\tau} D'(Se^{-\mu\tau}) d\tau \right. \\ \left. + Y \int_{T_I}^T e^{-(\mu+r)\tau} d\tau + \frac{e^{-(r+\mu)T_I}}{\mu} [D(z_I) - D(S)e^{-\delta T_I} - Y(1 - e^{-\delta T_I})] \right\} = 0,$$

$$\frac{\partial V}{\partial T} = \frac{1}{e^{rT} - 1} [(s - k)Y - rV] = 0,$$

$$\frac{\partial V}{\partial Y} = \frac{1}{1 - e^{-rT}} \left[ z_I e^{-rT_I} \int_0^{T_I} e^{-\delta\tau} d\tau + S \int_{T_I}^T e^{-(\mu+r)\tau} d\tau - k \int_0^T e^{-r\tau} d\tau \right] = 0,$$

using that the definition of  $T_I$  in eq. (1) implies

$$\frac{\partial T_I}{\partial S} = \frac{D(z_I) - D(S)e^{-\delta T_I} - Y(1 - e^{-\delta T_I})}{\mu S [D(z_I) - Y]}, \quad (\text{A1})$$

$$\frac{\partial T_I}{\partial Y} = \frac{\int_0^{T_I} e^{-\delta\tau} d\tau}{D(z_I) - Y}. \quad (\text{A2})$$

Partially integrating the second integral in  $\partial V/\partial S$ ,

$$S \int_0^{T_I} e^{-(2\mu+r)\tau} D'(Se^{-\mu\tau}) d\tau = \frac{D(S) - e^{-(\mu+r)T_I} D(z_I)}{\mu} - \left( \frac{r}{\mu} + 1 \right) \int_0^{T_I} e^{-(r+\mu)\tau} D(Se^{-\mu\tau}) d\tau.$$

Consequently,

$$\frac{\partial V}{\partial S} = \frac{1}{(1 - e^{-rT})\mu S} \left\{ SD(S)[1 - e^{-(\mu+r+\delta)T_I}] - rS \int_0^{T_I} e^{-(\mu+r)\tau} D(Se^{-\mu\tau}) d\tau \right. \\ \left. + Y\mu S \int_{T_I}^T e^{-(\mu+r)\tau} d\tau - z_I e^{-rT_I} (1 - e^{-\delta T_I}) Y \right\} \\ = \frac{1}{(1 - e^{-rT})\mu S} \left\{ SD(S)[1 - e^{-(\mu+r+\delta)T_I}] - rS \left[ \int_0^{T_I} e^{-(\mu+r)\tau} D(Se^{-\mu\tau}) d\tau \right. \right. \\ \left. \left. + Y \int_{T_I}^T e^{-(\mu+r)\tau} d\tau \right] + YS [e^{-(\mu+r)T_I} - e^{-(\mu+r)T} - e^{-(\mu+r)T_I} (1 - e^{-\delta T_I})] \right\},$$

which by the definition of  $V$  becomes

$$\begin{aligned}\frac{\partial V}{\partial S} &= \frac{1}{(1 - e^{-rT})\mu S} \left\{ SD(S)[1 - e^{-(\mu+r+\delta)T_I}] - (rV + kY)(1 - e^{-rT}) - rc \right. \\ &\quad \left. + YS \left[ e^{-(\mu+r)T_I} - e^{-(\mu+r)T} - e^{-(\mu+r)T_I}(1 - e^{-\delta T_I}) \right] \right\} \\ &= \frac{1}{(1 - e^{-rT})\mu S} \left\{ SD(S) + z_I[Y - D(S)]e^{-(r+\delta)T_I} - sY - rc \right. \\ &\quad \left. + [(s - k)Y - rV](1 - e^{-rT}) \right\}.\end{aligned}$$

An optimal adjustment strategy with stockouts therefore satisfies

$$\begin{aligned}SD(S) + z_I[Y - D(S)]e^{-(r+\delta)T_I} - sY - rc &= 0, \\ (s - k)Y - rV &= 0, \\ z_I e^{-rT_I} \int_0^{T_I} e^{-\delta\tau} d\tau + S \int_{T_I}^T e^{-(\mu+r)\tau} d\tau - k \int_0^T e^{-r\tau} d\tau &= 0.\end{aligned}$$

Next, to determine the optimal adjustment strategy if stockouts are not optimal, assume that  $T_I = T$  and differentiate  $V$  totally with respect to  $S$  and  $T$ :

$$\begin{aligned}\frac{\partial V}{\partial S} &= \frac{1}{1 - e^{-rT}} \left( \int_0^T e^{-(\mu+r)\tau} D(Se^{-\mu\tau}) d\tau + S \int_0^T e^{-(2\mu+r)\tau} D'(Se^{-\mu\tau}) d\tau \right. \\ &\quad \left. - k_T \left\{ \frac{[D(S) - Y]e^{-\delta T} + Y - D(s)}{\mu S} \right\} e^{-rT} \right) = 0, \\ \frac{\partial V}{\partial T} &= \frac{1}{e^{rT} - 1} \{ sD(s) - kY - k_T[D(s) - Y] - rV \} = 0,\end{aligned}$$

using that eq. (1) for  $T_I = T$  implies

$$\begin{aligned}\frac{\partial Y}{\partial S} &= \frac{\int_0^T e^{(\delta-\mu)\tau} D'(Se^{-\mu\tau}) d\tau}{\int_0^T e^{\delta\tau} d\tau} \\ &= \frac{[D(S) - Y]e^{-\delta T} + Y - D(s)}{\mu S \int_0^T e^{-\delta\tau} d\tau},\end{aligned}$$

by partial integration, and

$$\frac{\partial Y}{\partial T} = \frac{D(s) - Y}{\int_0^T e^{-\delta\tau} d\tau}.$$

Partially integrating the second integral in  $\partial V/\partial S$ ,

$$S \int_0^T e^{-(2\mu+r)\tau} D'(Se^{-\mu\tau}) d\tau = \frac{D(S) - e^{-(\mu+r)T} D(s)}{\mu} - \left( \frac{r}{\mu} + 1 \right) \int_0^T e^{-(r+\mu)\tau} D(Se^{-\mu\tau}) d\tau.$$

Accordingly, by the definition of  $V$ , one obtains

$$\begin{aligned}
\frac{\partial V}{\partial S} &= \frac{1}{(1 - e^{-rT})\mu S} \left\{ SD(S) - sD(s)e^{-rT} - rV(1 - e^{-rT}) - kY(1 - e^{-rT}) - rc \right. \\
&\quad \left. - k_T[D(S) - Y]e^{-(r+\delta)T} - k_T[Y - D(s)]e^{-rT} \right\} \\
&= \frac{1}{(1 - e^{-rT})\mu S} \left( SD(S) + k_T[Y - D(S)]e^{-(r+\delta)T} - sD(s) + k_T[D(s) - Y] - rc \right. \\
&\quad \left. + \{sD(s) - kY - k_T[D(s) - Y] - rV\}(1 - e^{-rT}) \right).
\end{aligned}$$

An optimal adjustment strategy without stockouts therefore satisfies

$$\begin{aligned}
SD(S) + k_T[Y - D(S)]e^{-(r+\delta)T} - sD(s) + k_T[D(s) - Y] - rc &= 0, \\
sD(s) - kY - k_T[D(s) - Y] - rV &= 0.
\end{aligned}$$

## Appendix B

### Proof of Theorem 1

*Proof that  $T_I < T$  :* If the inflation rate converges to zero, the initial real price and the production converge to the static monopoly real price  $Z$  and output  $D(Z)$ , where  $Z$  is defined by  $Z + D(Z)/D'(Z) = k$ . The length of a period with a constant nominal price diverges to infinity and the total discounted real profits converge to  $(Z - k)D(Z)/r - c$ . Accordingly, condition (4) shows that  $(\lim_{\mu \rightarrow 0} s - k)D(Z) - (Z - k)D(Z) + rc = 0$  so that the terminal real price converges to  $\lim_{\mu \rightarrow 0} s = Z - rc/D(Z)$ , which is finite. The marginal real cost of a unit, discounted to the end of a period, diverges to infinity. Thus, in the limit the terminal real price is less than the marginal real cost of a unit discounted to the end of a period,  $Z - rc/D(Z) < k_\infty$ . It follows that  $\hat{T}_I \equiv \lim_{\mu \rightarrow 0} T_I$  is finite (if  $\hat{T}_I$  were infinite, then the discounted marginal real revenue would be zero and less than the discounted marginal real cost  $k/r$ ). Since the optimal strategy is continuous in  $\mu$ , it must be the case that  $s < k_T \Rightarrow T_I < T$  at low inflation rates.

*Proof that  $dY/d\mu < 0$  :* It is first established that production decreases with the cost of price adjustment at small inflation rates (where  $T_I < T$ ). Differentiate conditions (3)-(5)

totally with respect to  $c$ ,

$$\frac{dY}{dc} = -\frac{A}{B},$$

where

$$\begin{aligned} A \equiv & (\mu + r + \delta)e^{-\mu T_I} \left[ \frac{D(z_I) - D(S)e^{-\delta T_I} - Y(1 - e^{-\delta T_I})}{D(z_I) - Y} \right] \left[ \frac{Y - D(S)}{\mu} e^{-\delta T_I} (s - k) \right. \\ & \left. - sY \int_0^{T_I} e^{-\delta \tau} d\tau \right] + \mu k e^{r T_I - \mu T} Y \int_0^T e^{-r \tau} d\tau \\ & + (s - k) \left\{ Y e^{r T_I - (\mu + r) T} - [D(S) + S D'(S)] \left[ e^{r T_I} - e^{-(\mu + \delta) T_I} \right] - Y e^{-(\mu + \delta) T_I} \right\}, \end{aligned}$$

using eq. (A1), and  $B < 0$  from the second-order condition for a maximum. Accordingly,  $dY/dc$  has the same sign as  $A$ . At small inflation rates, therefore,  $dY/dc < 0$  if  $\lim_{\mu \rightarrow 0} A < 0$ .

Now,

$$\begin{aligned} \lim_{\mu \rightarrow 0} A = & (r + \delta) \left\{ \lim_{\mu \rightarrow 0} \left[ \frac{D(z_I) - D(S)e^{-\delta T_I} - Y(1 - e^{-\delta T_I})}{D(z_I) - Y} \right] \right\} \\ & \left( \left\{ \lim_{\mu \rightarrow 0} \left[ \frac{Y - D(S)}{\mu} \right] \right\} e^{-\delta \hat{T}_I} (\hat{s} - k) - \hat{s} D(Z) \int_0^{\hat{T}_I} e^{-\delta \tau} d\tau \right) \\ & - (\hat{s} - k) \left[ D(Z) e^{r \hat{T}_I} + Z D'(Z) (e^{r \hat{T}_I} - e^{-\delta \hat{T}_I}) \right]. \end{aligned}$$

Let  $T_Y$  be the time at which demand equals production, and  $\hat{T}_Y$  the limit of  $T_Y$  as the inflation rate approaches zero. That is,  $T_Y \equiv (1/\mu) \ln(S/z_Y)$  and  $\hat{T}_Y \equiv \lim_{\mu \rightarrow 0} T_Y$ . Expand  $D(Se^{-\mu T_I})$  and  $Y = D(Se^{-\mu T_Y})$  at  $\mu = 0$  to obtain

$$\begin{aligned} & \lim_{\mu \rightarrow 0} \left[ \frac{D(z_I) - D(S)e^{-\delta T_I} - Y(1 - e^{-\delta T_I})}{D(z_I) - Y} \right] \\ = & \lim_{\mu \rightarrow 0} \left[ \frac{D(Se^{-\mu T_I}) - D(S)e^{-\delta T_I} - D(Se^{-\mu T_Y})(1 - e^{-\delta T_I})}{D(Se^{-\mu T_I}) - D(Se^{-\mu T_Y})} \right] \\ = & \lim_{\mu \rightarrow 0} \left[ \frac{-D'(Se^{-\mu T_I})Se^{-\mu T_I} \mu T_I + D'(Se^{-\mu T_I})Se^{-\mu T_Y} \mu T_Y (1 - e^{-\delta T_I})}{-D'(Se^{-\mu T_I})Se^{-\mu T_I} \mu T_I + D'(Se^{-\mu T_Y})Se^{-\mu T_Y} \mu T_Y} \right] \\ = & \frac{\hat{T}_I - \hat{T}_Y (1 - e^{-\delta \hat{T}_I})}{\hat{T}_I - \hat{T}_Y}. \end{aligned}$$

Similarly,

$$\lim_{\mu \rightarrow 0} \left[ \frac{Y - D(S)}{\mu} \right]$$

$$\begin{aligned}
&= \lim_{\mu \rightarrow 0} \left[ \frac{D(Se^{-\mu T_Y}) - D(S)}{\mu} \right] \\
&= \lim_{\mu \rightarrow 0} \left[ \frac{-D'(Se^{-\mu T_Y})Se^{-\mu T_Y} \mu T_Y}{\mu} \right] \\
&= -ZD'(Z)\hat{T}_Y.
\end{aligned}$$

Condition (5) shows that  $\hat{T}_I$  is given by

$$Ze^{-r\hat{T}_I} \left( r \int_0^{\hat{T}_I} e^{-\delta\tau} d\tau + 1 \right) - k = 0,$$

which by use of the definition of  $Z$  becomes

$$D(Z)e^{r\hat{T}_I} + ZD'(Z) \left( e^{r\hat{T}_I} - e^{-\delta\hat{T}_I} \right) = (r + \delta)ZD'(Z) \int_0^{\hat{T}_I} e^{-\delta\tau} d\tau. \quad (\text{B1})$$

Consequently,

$$\begin{aligned}
\lim_{\mu \rightarrow 0} A &= -r \left[ \frac{\hat{T}_I - \hat{T}_Y(1 - e^{-\delta\hat{T}_I})}{\hat{T}_I - \hat{T}_Y} \right] \left[ ZD'(Z)\hat{T}_Y e^{-\delta\hat{T}_I}(\hat{s} - k) + \hat{s}D(Z) \int_0^{\hat{T}_I} e^{-\delta\tau} d\tau \right] \\
&\quad - (\hat{s} - k)(r + \delta)ZD'(Z) \int_0^{\hat{T}_I} e^{-\delta\tau} d\tau \\
&= -\frac{r\hat{T}_I^2 [ZD'(Z)(\hat{s} - k) + \hat{s}D(Z)]}{\hat{T}_I - \hat{T}_Y} \\
&= -\frac{r\hat{T}_I^2 D(Z)k(Z - \hat{s})}{(\hat{T}_I - \hat{T}_Y)(Z - k)} \\
&< 0,
\end{aligned}$$

where the last equality uses that  $D'(Z) = -D(Z)/(Z - k) \Rightarrow ZD'(Z)(\hat{s} - k) + \hat{s}D(Z) = kD(Z)(Z - \hat{s})/(Z - k)$ . It can be concluded that  $dY/dc < 0$  at small inflation rates. Since production converges to  $D(Z)$  as the inflation rate approaches zero, production is less than the static monopoly output at small inflation rates,  $Y < D(Z)$ . For this to be the case, production must decrease with the inflation rate at small inflation rates, i.e.,  $dY/d\mu < 0$ .

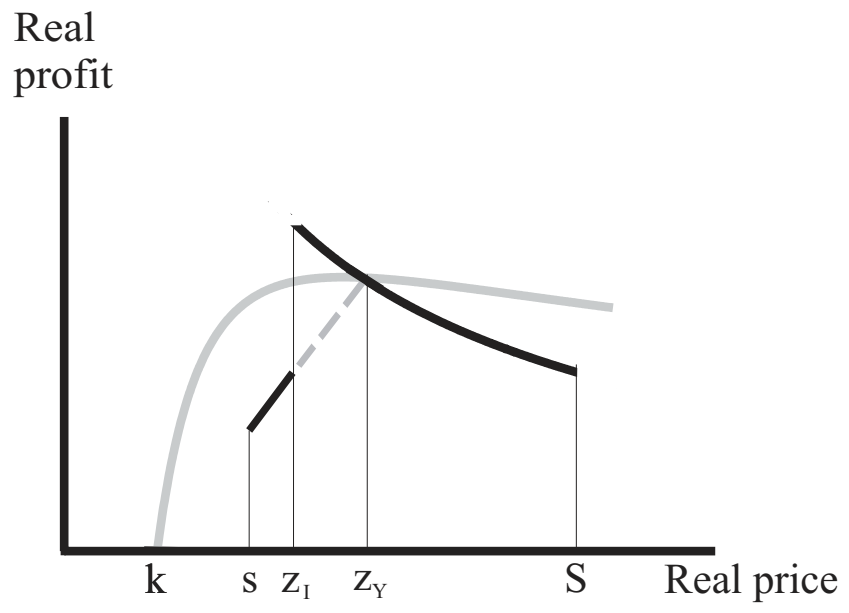


FIGURE 1



TABLE 1 – PERCENTAGE LOSS/GAIN OF OUTPUT

Inflation rate	Price- and quantity-adjustment cost					
	No storage cost	Inventory storage cost			No inventory	Price-adjustment cost only
		5	20	50		
1	– 2.31 *	– 2.80 *	– 4.27 *	– 5.14 *	– 6.71 *	0.10
5	–4.34	–5.07	– 6.88 *	– 9.59 *	– 15.02 *	–2.12
10	–5.99	–6.80	–8.49	–12.22	– 21.26 *	–4.01
15	–7.39	–8.21	–11.02	–14.03	– 26.08 *	–5.55
20	–8.65	–9.47	–11.35	–15.50	– 30.16 *	–6.93

*Notes:* All numbers are in per cent.

An asterisk indicates that firms eventually stock out.

## References

- Abowd, J. M. and F. Kramarz, 2003, The costs of hiring and separations, *Labour Economics* 10, 499-530.
- Akerlof, G. A. and J. L. Yellen, 1985, A near-rational model of the business cycle with wage and price inertia, *Quarterly Journal of Economics* 100 (Supplement), 823-838.
- Andersen, T. M., 1995, Adjustment costs and price and quantity adjustment, *Economics Letters* 47, 343-349.
- Andersen, T. M. and E. Toulemonde, 2004, Adapting prices or quantities in the presence of adjustment costs, *Journal of Money, Credit, and Banking* 36, 177-196.
- Baily, M. N., Bartelsman E. J. and J. Haltiwanger, 2001, Labor productivity: Structural change and cyclical dynamics, *Review of Economics and Statistics* 83, 420-433.
- Ball, L. and D. Romer, 1990, Real rigidities and the non-neutrality of money, *Review of Economic Studies* 57, 183-203.
- Basu, S. and J. G. Fernald, 1997, Returns to scale in U.S. production: Estimates and implications, *Journal of Political Economy* 105, 249-283.
- Benabou, R., 1988, Search, price setting and inflation, *Review of Economic Studies* 55, 353-376.
- Bils, M., 2005, Deducing markup cyclical behavior from stockout behavior, Working Paper.
- Blanchard, O. J. and N. Kiyotaki, 1987, Monopolistic competition and the effects of aggregate demand, *American Economic Review* 77, 647-666.
- Bresnahan, T. F. and V. A. Ramey, 1994, Output fluctuations at the plant level, *Quarterly Journal of Economics* 109, 593-624.
- Caballero, R. J., Engel, E. M. R. A. and J. C. Haltiwanger, 1997, Aggregate employment dynamics: Building from microeconomic evidence, *American Economic Review* 87, 115-137.
- Cecchetti, S. G., 1986, The frequency of price adjustment: A study of the newsstand prices of magazines, *Journal of Econometrics* 31, 255-274.
- Cooper, R., Haltiwanger, J. and L. Power, 1999, Machine replacement and the business cycle: Lumps and bumps, *American Economic Review* 89, 921-946.
- Danziger, L., 1987, Inflation, fixed cost of price adjustment, and the measurement of relative price variability: Theory and evidence, *American Economic Review* 77, 704-713.

- Danziger, L., 1988, Costs of price adjustment and the welfare economics of inflation and disinflation, *American Economic Review* 78, 633-646.
- Danziger, L., 1999, A dynamic economy with costly price adjustments, *American Economic Review* 89, 878-901.
- Danziger, L., 2001, Output and welfare effects of inflation with costly price and quantity adjustments, *American Economic Review* 91, 1608-1620.
- Danziger, L. and C. T. Kreiner, 2002, Fixed production capacity, menu cost and the output-inflation relationship, *Economica* 69, 433-444.
- Davis, S. J. and J. Haltiwanger, 1992, Gross job creation, gross job destruction, and employment, *Quarterly Journal of Economics* 107, 819–863.
- Doms, M. and T. Dunne, 1998, Capital adjustment patterns in manufacturing firms, *Review of Economic Dynamics* 1, 409-429.
- Dotsey, M., King, R. G. and A. L. Wolman, 1999, State-dependent pricing and the general equilibrium dynamics of money and output, *Quarterly Journal of Economics* 114, 655-690.
- Fisher, T. C. G. and J. D. Konieczny, 2006, Inflation and costly price adjustment: A study of Canadian newspaper prices, *Journal of Money, Credit, and Banking* 38, 615-634.
- Golosov, M. and R. E. Lucas Jr., 2007, Menu costs and Phillips curves, *Journal of Political Economy* 115, 171–199.
- Hamermesh, D. S., 1989, Labor demand and the structure of adjustment costs, *American Economic Review* 79, 674-689.
- Jovanovic, B. and P. L. Rousseau, 2001, Vintage organization capital, NBER Working Paper No. 8166.
- Kashyap, A. K., 1995, Sticky prices: New evidence from retail catalogues, *Quarterly Journal of Economics* 110, 245-274.
- Lamont, O., 1997, Do “shortages” cause inflation?, in: Romer, C. D. and D. H. Romer, eds., *Reducing Inflation: Motivation and Strategy* (University of Chicago Press, Chicago and London), 281-300.
- Levy, D., Bergen, M., Dutta, S. and R. Venable, 1997, On the magnitude of menu costs: Direct evidence from large U. S. supermarket chains, *Quarterly Journal of Economics* 112, 791-825.

- Mankiw, N. G., 1985, Small menu costs and large business cycles: A macroeconomic model of monopoly, *Quarterly Journal of Economics* 100, 529-537.
- Mankiw, N. G. and R. Reis, 2002, Sticky information versus sticky prices: A proposal to replace the new Keynesian Phillips curve, *Quarterly Journal of Economics* 117, 1295-1328.
- Nilsen, Ø. A. and F. Schiantarelli, 2003, Zeros and lumps in investment: Empirical evidence on irreversibilities and nonconvexities, *Review of Economics and Statistics* 85, 1021-1037.
- Rotemberg, J. J., 2005, Customer anger at price increases, changes in the frequency of price adjustment and monetary policy, *Journal of Monetary Economics* 52, 829-852.
- Rotemberg, J. J. and M. Woodford, 1995, Dynamic general equilibrium models with imperfectly competitive product markets, in: T. F. Cooley, ed., *Frontiers of business cycle research* (Princeton University Press, Princeton), 243-293.
- Sheshinski, E. and Y. Weiss, 1977, Inflation and costs of price adjustment, *Review of Economic Studies* 44, 287-303.
- Stock, J. R. and D. M. Lambert, 2001, *Strategic Logistics Management* (Irwin, Homewood, Illinois).
- Zbaracki, M. J., Ritson, M., Levy, D., Dutta, S. and M. Bergen, 2004, Managerial and customer costs of price adjustment: Direct evidence from industrial markets, *Review of Economics and Statistics* 86, 514-533.

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