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Finite Sample Properties of Moran's I Test for Spatial Autocorrelation in Probit and Tobit Models - Empirical Evidence

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Finite Sample Properties of Moran's I Test for Spatial Autocorrelation in Probit and Tobit Models - Empirical Evidence*

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Abstract

In this paper, we investigate the finite sample properties of Moran's I test statistic for spatial autocorrelation in limited dependent variable models suggested by Kelejian and Prucha (2001). We analyze the socio-economic determinants of the availability of dialysis equipment in 5,507 Brazilian municipalities in 2009 by means of a probit and tobit specification. We assess the extent to which evidence of spatial autocorrelation can be remedied by the inclusion of spatial fixed effects. We find spatial autocorrelation in both model specifications. For the probit model, a spatial fixed effects approach removes evidence of spatial autocorrelation. However, this is not the case for the tobit specification. We further fill a void in the theoretical literature by investigating the finite sample properties of these test statistics in a series of Monte Carlo simulations, using data sets ranging from 49 to 15,625 observations. We find that the tests are unbiased and have considerable power for even medium-sized sample sizes. Under the null hypothesis of no spatial autocorrelation, their empirical distribution cannot be distinguished from the asymptotic normal distribution, empirically confirming the theoretical results of Kelejian and Prucha (2001), although the sample size required to achieve this result is larger in the tobit case than in the probit case.

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1 Introduction

In many studies in empirical spatial economics, the dependent variable of interest may only be observed in binary form, or some type of censuring may be at work, so that an actual value for the dependent variable is only available for a subset of the observations. Common examples are data on land use choices, election results, the availability of infrastructure investment or natural resources, adoption of public policies, diffusion of innovations, etc. When the observations on these limited dependent variables consist of a cross-section, spatial dependence and spatial heterogeneity can affect estimation and inference (Anselin, 1988).

Spatial effects in models with limited dependent variables are addressed by means of so-called spatial latent variable models (for overviews, see, e.g., Anselin, 2001, 2002; Fleming, 2004). A theoretical foundation for spatial dependence in discrete choice models is presented in Brock and Durlauf (2001). In practice, the spatial probit and spatial tobit specifications in particular have been considered in the spatial econometric literature. Early discussions can be found in Case (1992), McMillen (1992, 1995) and Beron and Vijverberg (2004), among others. Several specialized estimation methods have been suggested for these models, including the EM algorithm (McMillen, 1992), the generalized method of moments, or GMM (Pinkse and Slade, 1998; Fleming, 2004; Pinkse et al., 2006), Bayesian estimators (Bolduc et al., 1997; LeSage, 2000; Holloway et al., 2002; Smith and LeSage, 2004), and simulation estimators (Beron et al., 2003; Beron and Vijverberg, 2004). Representative empirical applications include the study of the adoption of rice hybrids (Holloway et al., 2002), environmental strategies (Murdoch et al., 2003), and the determinants of land use and land development change (Chakir and Parent, 2009; Wang and Kockelman, 2009).

In contrast to the attention to model specification and estimation, it was not until the article by Kelejian and Prucha (2001) that a general framework was developed to test for the presence of spatial autocorrelation in limited dependent variable models. As special cases, Kelejian and Prucha (2001) derived a Moran's I test statistic based on the generalized residuals of the probit and tobit model and demonstrated that its distribution under the null was asymptotically standard normal. In practice, estimating a spatial latent variable model may not always be necessary, but it remains important to assess whether or not spatial autocorrelation may be present. Unlike the case in the standard linear regression model, even the presence of spatial autocorrelation in the error terms will induce heteroskedasticity and hence make the classic probit and tobit estimators inconsistent (see, e.g., Fleming, 2004).

In this paper, we further consider the finite sample properties of Kelejian and Prucha's Moran's I test statistic. We take a two-pronged approach. First, we consider an empirical example in which we model the socio-economic determinants of the availability of dialysis equipment in a cross-section of 5,507 Brazilian municipalities in 2009. We are particularly interested in the extent to which the so-called "inverse care law" of Hart (1971) is reflected in the Brazilian context. According to this law, the availability of medical care will tend to vary inversely with the need of the population served. As a result, "the more favoured people are, socially and economically, the better their health" (Marmot et al., 2010, p. 3). In the Brazilian context, this is particularly relevant, since the 1988 Brazilian Constitution contained provisions for the establishment of the Unified Health System (*Sistema Único de Saúde* - SUS), aimed at reducing inequalities in the supply of health services, by providing adequate access to these services at no cost to the population at the point of delivery. This policy obtained an important spatial component after the Operational Norm of Health Assistance (NOAS/SUS 2001) was instituted in 2001 to organize the health care system at the regional level, avoid inefficiencies and tackle inequalities by consolidating the overall health management at the state level. We estimate both a probit and a tobit specification and test for the presence of spatial autocorrelation using Kelejian and Prucha's Moran's I. We also assess the extent to which the evidence of spatial autocorrelation may be removed by the inclusion of state-level spatial fixed effects (see, e.g., Anselin and Arribas-Bel, 2011).

Our second approach moves beyond a specific empirical example and investigates the properties of the test statistic in a series of Monte Carlo simulation experiments. This is motivated by the general dearth of such evidence in the literature. The only study to date that provides some limited results in a number of simulated settings is Novo (2001), but his work only pertains to the probit model. He assessed the size and power of a number of test statistics in a spatial probit model for sample sizes up to $N = 225$. However, since the experiments are based on only 2,000 replications, the precision of the results is somewhat limited. To our knowledge, there are no results to date on the properties of the test for the tobit model.

We propose to extend the finite sample evidence by considering both probit and tobit specifications. We simulate an extensive set of sample sizes, ranging from 49 to 15,625 observations to assess the rejection frequency of the test statistic both under the null as well as under alternatives of spatial error and spatial lag dependence. In addition, we compare the empirical distribution of the test statistic under the null to its theoretical expectation.

In the remainder of the paper, we first outline the formal specifications of the spatial probit and spatial tobit models and describe the test statistics. In Section 3, we proceed with the empirical study of the availability of dialysis equipment in Brazilian municipalities. Section 4 contains the Monte Carlo study. We close with some concluding comments.

2 Spatial Probit and Spatial Tobit

2.1 Model Specification

The point of departure is the generic linear latent variable model (e.g., Greene, 2002):

$$\mathbf{y}^* = \mathbf{X}'\beta + \mathbf{u}, \tag{1}$$

in which $\mathbf{y}^*$ is a $n \times 1$ vector containing values for the unobserved (latent) dependent variable, $\mathbf{X}'\beta$ is an index function, with $\mathbf{X}$ as a $k \times n$ matrix of observations on the explanatory variables and β as a $k \times 1$ vector of coefficients. Finally, $\mathbf{u}$ is a $n \times 1$ vector of random errors assumed to be normally distributed.

Spatial effects can be introduced into this specification in the usual fashion, either as a spatial lag or as a spatial error model (Anselin, 1988, 2002). However, it is important to keep in mind that these specifications pertain to the latent variable model and not to the observed data. The spatial lag specification is thus:

$$\mathbf{y}^* = \rho \mathbf{W} \mathbf{y}^* + \mathbf{X}'\beta + \mathbf{u},$$

and the spatial error specification consists of Equation (1), with a spatial autoregressive process for the error term:

$$\mathbf{u} = \lambda \mathbf{W} \mathbf{u} + \epsilon,$$

where ρ and λ are the respective spatial autoregressive parameters, $\mathbf{W}$ is the familiar $n \times n$ spatial weights matrix and ϵ is a $n \times 1$ vector of *i.i.d.* normally distributed disturbance terms.

Before proceeding to the actual probit and tobit specifications, it is useful to consider the reduced forms of the lag and error models. For the spatial lag model, this becomes:

$$\mathbf{y}^* = (\mathbf{I} - \rho \mathbf{W})^{-1} \mathbf{X}'\beta + (\mathbf{I} - \rho \mathbf{W})^{-1} \mathbf{u}.$$

Consequently, the latent variable "observed" at a given location is not only a function of the explanatory variables at that location, but also of the explanatory variables in neighboring locations, corresponding to a spatial multiplier effect (Anselin, 2003). More importantly, the error distribution is no longer *i.i.d.*, but obtains a multivariate structure encompassing both spatial autocorrelation and heteroskedasticity.

Similarly, the reduced form for the latent spatial error model is:

$$\mathbf{y}^* = \mathbf{X}'\beta + (\mathbf{I} - \lambda \mathbf{W})^{-1} \mathbf{u}.$$

Again, this yields a non-independent heteroskedastic error distribution. As mentioned above, a consequence of the heteroskedasticity induced by the spatial autoregressive error process is that classic estimators for the probit or tobit specifications will be biased, unlike what holds in the standard linear regression case.

In the probit specification, the value for the latent dependent variable is not observed, but only the presence or absence is given, in the form of a binary dependent variable for those observations where the latent variable exceeds a threshold (typically taken to be 0):

$$y_i = \begin{cases} 1 & \text{if } y_i^* > 0 \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

This leads to the familiar probit model specification:

$$P[y_i = 1|x_i] = P[y_i^* > 0|x_i] = P[u_i < \ x_i\beta|\ x_i] \tag{3}$$

However, unlike what holds in the non-spatial model, the marginal condition on the error term u_i in the spatial probit model does not pertain to an independently distributed normal random variate, but instead is determined by the marginal distribution of a multivariate spatially correlated and heteroskedastic normal density (see, e.g., Beron and Vijverberg, 2004).

In the tobit specification, the latent dependent variables is actually observed, but only for those observations where a threshold value is exceeded:

$$y_i = \begin{cases} y_i^* & \text{if } y_i^* > 0 \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

Under the assumption of normality, the resulting censoring yields a model for the observed y_i as (e.g., Amemiya, 1985) :

$$y_i = \sigma\Phi(\frac{x_i\beta}{\sigma})\left[\frac{x_i\beta}{\sigma} + \frac{\phi(x_i\beta/\sigma)}{\Phi(x_i\beta/\sigma)}\right] + u_i, \tag{5}$$

where σ^2 is the variance of the error term, and Φ and ϕ are respectively the cumulative density function and the probability density function of the standard normal distribution. In the spatial case, the error term will again no longer be *i.i.d.*, but the marginal of a multivariate spatially correlated and heteroskedastic normal distribution.

2.2 Moran's I Test Statistic

The Moran's I test statistic proposed by Kelejian and Prucha (2001) is based on the generalized residuals of the probit and tobit models. The general form of the statistic is:

$$I^* = \frac{\hat{u}_i'\mathbf{W}\hat{u}_i}{\sqrt{tr(\mathbf{W\Sigma W\Sigma} + \mathbf{W}'\mathbf{\Sigma W\Sigma})}} \xrightarrow{d} N(0,1), \tag{6}$$

in which $\hat{u}_i$ is the generalized residual, $\mathbf{\Sigma}$ is a diagonal matrix containing $\hat{\sigma}_i^2$ and $\mathbf{W}$ is the familiar $n \times n$ spatial weights matrix.

In the probit model, the estimate of the generalized residual is:

$$u_i^{(\text{probit})} = y_i - \Phi_{(x_i\beta)}, \tag{7}$$

whereas in the tobit model, the corresponding estimate is:

$$u_i^{(\text{tobit})} = y_i - \sigma\Phi_{(x_i\beta/\sigma)}\left(\frac{x_i\beta}{\sigma} + \frac{\phi_{(x_i\beta/\sigma)}}{\Phi_{(x_i\beta/\sigma)}}\right), \tag{8}$$

using the same notation as above. The individual elements σ_i^2 of the matrix $\mathbf{\Sigma}$ are, respectively, for the probit model:

$$\sigma_i^{2(\text{probit})} = \Phi_{(x_i\beta)}\left(1 - \Phi_{(x_i\beta)}\right), \tag{9}$$

and for the tobit model:

$$\sigma_i^{2(\text{tobit})} = \Phi_{(x_i\beta/\sigma)}\left[(x_i\beta)^2 + \sigma^2\right] + x_i\beta\sigma\phi_{(x_i\beta/\sigma)} - \left(x_i\beta\Phi_{(x_i\beta/\sigma)} + \sigma\phi_{(x_i\beta/\sigma)}\right)^2. \tag{10}$$

With these estimates in hand, we can carry out the specification tests.

3 Provision of Dialysis Equipment in Brazilian Municipalities

3.1 Data and Model Specification

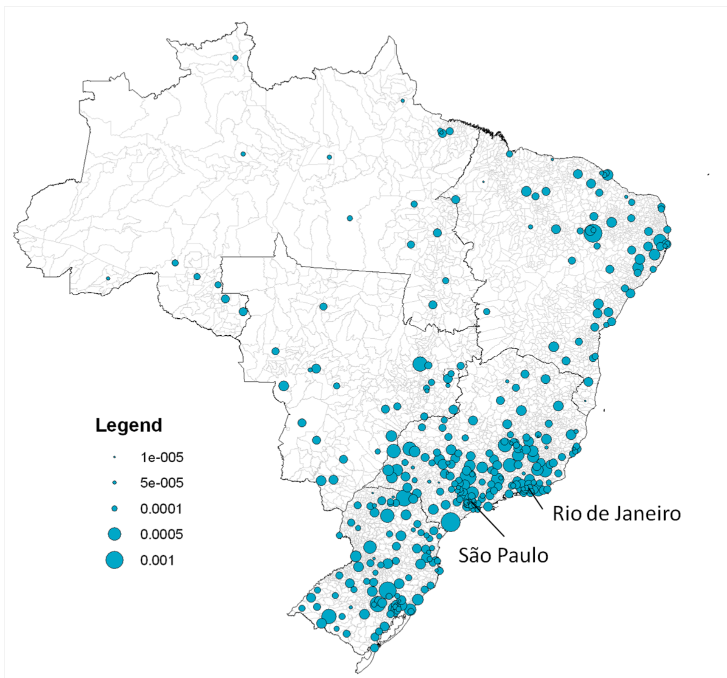
The institutional context for our analysis is the Unified Health System in Brazil (*Sistema Único de Saúde* - SUS), which was established to promote the decentralization of health provision at the regional level, both in terms of the management as well as the funding of health services. It aims to adjust the model of assistance to the real medical needs of the population by bringing the solution of medical problems to the regions where they occur. The SUS establishes that the access to health services is to be guaranteed to all citizens, with full coverage of medical needs and equal treatment to people with equal needs, i.e., horizontal equity (*Lei Orgânica da Saúde* 8.080/1990).

The Operational Norm of Health Assistance (NOAS/SUS 2001) established a Regionalization Guiding Plan that proposes to organize the health care system at the regional level. This norm aims to identify the roles of the municipalities in the state health system and to tackle inequalities in the provision of services. In order to do so, it defines a set of actions to be taken by all municipalities regarding basic health care and supports the creation of regional units, able to fulfill the medical needs of a larger population according to its geographical location, under the coordination of the state.

In contrast to these established policy goals, the actual provision of dialysis equipment in Brazil is highly spatially concentrated, as illustrated in Figure 1. The number of municipalities in Brazil changed from 5507 to 5564 between the years 2000 and 2009. To assure compatibility across data from these two periods, we aggregate the municipalities to consider their borders as they were in 2000. Therefore, our analysis is based on 5507 municipalities. Focusing only on presence-absence of dialysis services, we find a joint count statistic of 214, significant at $p = 0.0001$, confirming the spatial clustering suggested by visual inspection of the map. In 2009, only 373 out of 5,507 Brazilian municipalities had working dialysis equipment at their health care centers. This represents 6.8% of the municipalities. On average, the number of dialysis units was 44.54, ranging from 1 to 1,631 per municipality, with a standard deviation of 110.70.

We consider the availability of dialysis equipment at the municipal level using two different specification. First, we use a probit model to explain the presence of equipment in a given location, irrespective of how many units are available. Second, using a tobit model, we explain the number of dialysis units in each municipality.

Figure 1: Spatial distribution of dialysis equipment, normalized by population



The explanatory variables in both specifications are the same. They are based on the health economics literature which provides several suggestions for socio-economic variables to be included. Common measures of socio-economic status employed in the analysis of the provision of health care are average schooling, the level of income and/or poverty, accommodation attributes, among others (Hart, 1971; Black et al., 1999; Campbell et al., 2001; Ross and Mirowsky, 2001; Egerter et al., 2005; Kirby and Kaneda, 2005, 2006; Mercer and Watt, 2007; Marmot et al., 2010). Education is related to health in many ways, such as general and health-related knowledge (Ross and Wu, 1995; Reynolds and Ross, 1998; Ross and Mirowsky, 1999). Income is one of the most commonly used measures of socio-economic status (Egerter et al., 2005), and is related to access to health services mainly in the ability to afford paid health care services and transportation costs. More than the simple measurement of income, the intensity of poverty and the distribution of the income are also important in the determination of health (Andersen et al., 2002). In turn, household and neighborhood characteristics affect the living environment and are related to access to health care (Thomas et al., 1990; Kirby and Kaneda, 2005, 2006).

Table 1 lists some descriptive statistics for the variables we considered in this paper. These include the number of schooling years for adults 25 years or older, income per capita, poverty intensity, indigence intensity, and population. We included the latter to control for size. Poverty intensity is the difference between the average municipal income of the share of population below poverty line and the line itself. Similarly, indigence intensity is the difference between the average municipal income of the share of population below 1/4 of the minimum wage (in 2000) and this value. The larger these figures, the lower the average municipal income is relative to these benchmarks. In addition, we considered spatial fixed effects in the form of 26 indicator variables, one for each state/federal district.

Except for the population in 2009 (obtained from IBGE), all data were extracted from the Demographic Census of 2000.

Table 1: Socio-economic indicators

Variable	**N**	**Mean**	**Std. dev.**	**Min.**	**Max.**
Schooling years, adults 25 years old or older	5507	4.04 (5.71)	1.29 (1.87)	0.81	9.65
Income per capita (R$/month)	5507	170.81 (297.78)	96.42 (177.24)	28.38	954.65
Poverty intensity	5507	47.09 (46.33)	10.73 (7.91)	15.76	83.03
Indigence intensity	5507	49.79 (55.74)	10.57 (10.78)	0.02	88.35
Population size 2009[1] (x1000)	5507	34.69	202.83	0.84	11,038

Source: Demographic Census 2000/IBGE, except for [1] Population estimatives 2009/IBGE.
Notes: Statistics weighted by population in parenthesis.

3.2 Empirical Results

We first consider the results for the probit specification. Table 2 shows the results for the marginal effects of the estimated coefficients of explanatory variables in the probit. All signs of significant coefficients are as expected according to the inverse care law. The original specification, Model 1, achieves a high share of observations correctly predicted: 97%. In sum, the coefficients indicate that municipalities with better socioeconomic variables are more likely to have dialysis equipment, corroborating the arguments proposed by Hart (1971).

Table 2: Probit model estimation results

Variables	Model 1			Model 2		
	Coef.	S.E.	p-value	Coef.	S.E.	p-value
Constant	-18.0203	0.9314	0	-16.9697	0.821	0
schooling	0.3272	0.0953	0.0001	0.3381	0.0845	0.0006
	(0.0143)	(0.0069)		(0.0131)	(0.0075)	
income per capita	0.0026	0.0011	0.0157	0.0025	0.001	0.0208
	(0.0001)	(0.0001)		(0.0001)	(0.0001)	
indigence intensity	-0.0333	0.0077	0	-0.0357	0.0067	0
	(-0.0015)	(0.0006)		(-0.0013)	(0.0007)	
poverty intensity	0.0157	0.0128	0.3536	0.0104	0.0112	0.2227
	(0.0004)	(0.0005)		(0.0006)	(0.0006)	
Ln(population)	1.4969	0.0902	0	1.39	0.0786	0
	(0.0586)	(0.0227)		(0.0599)	(0.0284)	
state dummies	No			Yes		
Log-Likelihood:	-421.99			-397.57		
LR test:	1884.54		0	1933.39		0
% correctly predicted:	96.77			97.19		
$\Phi(\overline{x}'\widehat{\beta})$:	0.0004			0.0002		
Moran's I:	2.443		0.0146	1.278		0.2012

Note: Marginal effects in parenthesis.

The Moran's I test on the generalized residuals of the probit model, using a queen contiguity matrix for the municipalities yields a value of 2.44, highly significant at $p = 0.01$. However, after including the state spatial fixed effects in Model 2, there is no longer any evidence of spatial autocorrelation, since Moran's I of 1.28 is not significant ($p = 0.20$). One possible explanation for this fact is that states adopt different policies regarding the provision of health equipment. When these idiosyncrasies are not taken into account, the estimated residuals of municipalities that share borders are positively correlated. However, with the spatial fixed effects included in the model specification, the state-specific policies are effectively accounted for and there is no longer any evidence of remaining spatial autocorrelation.

We next move beyond simple presence/absence and consider the role of socio-economic explanatory variables in determining the magnitude of dialysis equipment in municipalities, using a tobit specification. Again, we consider two specifications, one without and one with state spatial fixed effects. The results are shown in Table 3.

The results suggest that the amount of dialysis equipment is positively re-

Table 3: Tobit estimation results

Variables	Model 1			Model 2		
	Coef.	S.E.	p-value	Coef.	S.E.	p-value
Constant	-1096.994	45.1464	0	-1117.225	49.3547	0
schooling	6.4604	6.1805	0.296	3.6436	6.7276	0.588
	(0.4553)	(0.4355)		(0.2491)	(0.46)	
income per capita	0.2227	0.0591	0	0.209	0.0628	0.001
	(0.0157)	(0.0042)		(0.0143)	(0.0043)	
indigence intensity	-2.1346	0.4638	0	-2.1468	0.5474	0
	(-0.1504)	(0.0327)		(-0.1468)	(0.0374)	
poverty intensity	-0.0135	0.7397	0.985	0.3999	0.8454	0.636
	(-0.001)	(0.0521)		(0.0273)	(0.0578)	
Ln(population)	93.6282	4.748	0	98.0774	5.4109	0
	(6.598)	(0.3346)		(6.7055)	(0.3699)	
state dummies	No			Yes		
/sigma	85.2949	2.9845		84.521	2.9593	
Log-Likelihood:	-2387.14			-2372.6		
LR test:	1763.39		0	1792.45		0
Pseudo R^2:	0.2697			0.2742		
Moran's I:	-6.4522		0	-6.4449		0

Note: Marginal effects conditional on being uncensored in parenthesis.

lated to income per capita and population size, in a way that an increase of about R$ 63.7/month in income per capita would result in an additional dialysis equipment. Similarly, and an additional 100% of population corresponds to 7 additional equipments. On the other hand, indigence intensity is negatively related to the amount of dialysis equipment: an additional R$ 6.65 between the average income of those below the indigence line and the line itself is related to less one unit of dialysis equipment.

The Moran's I statistic, again computed using first order queen contiguity, indicates significant negative spatial autocorrelation in the model's residual term. This spatial dependence may be due to the omission of a spatial lag of the dependent variable in case the amount of equipments in a given municipality depends on the amount already available in the neighboring area. The more equipment available in the surrounding area, the less might be needed in a given municipality. Another possible source of spatial dependence is autocorrelation of the error term. An example would be the case of measurement errors due to misspecification of the geographical area of analysis. If a geographical unit broader than the municipality is considered when the decisions regarding the provision of dialyzers are made, to consider a wrong areal unit may induce the correlation of the residuals and render the model estimates inconsistent. In contrast to the probit model, the inclusion of state spatial fixed effects does not remove the evidence of spatial autocorrelation. Moran's I remains virtually unchanged at -6.44 and highly significant.

To assess the effects of the spatial autocorrelation on the model coefficients, we carried out an estimation of both the spatial lag and the spatial error tobit model using the Bayesian estimator included in the spatial econometrics toolbox of LeSage et al. (LeSage and Pace, 2009). We only report the results for the lag specification, since the spatial autoregressive term in the error model turned out not to be significant. This illustrates the power of the Moran's I test against both spatial error and spatial lag alternatives.

Table 4 contains the estimated coefficients, asymptotic standard errors and p-values.[1] Consistent with the value for Moran's I, the estimate for the spatial autoregressive coefficient has a negative sign. However the parameter value itself is small, in both the model without ($\hat{\rho} = -0.0943$) and with spatial fixed effects ($\hat{\rho} = -0.1154$). This suggest a rather weak effect of "free-riding" by municipalities surrounded by neighbors with a higher degree of service provision. The effect of including the spatial lag term on the estimates of the other variables is weak. The magnitude of the coefficients changes slightly, most visibly for population (the coefficient for schooling also changes, but it is not significant in either model). The sign for poverty intensity changes from negative to positive, but this coefficient is not significant in either case. The effect on the coefficients from the inclusion of the state fixed effects is similar. Interestingly, the magnitude of the spatial autoregressive coefficient is larger in the specification with the spatial fixed effects, suggesting that their inclusion does not eliminate evidence of spatial interaction.

Table 4: Spatial tobit estimation results - lag model

Variables	**Model 1**			**Model 2**		
	Coef.	S.E.	p-value	Coef.	S.E.	p-value
Constant	-1079.2526	39.2976	0	-1135.541	53.457	0
schooling	8.6959	5.1996	0.0944	3.9499	5.664	0.4856
income per capita	0.2191	0.0481	0	0.205	0.0578	0.0004
indigence intensity	-1.6358	0.4459	0.0002	-1.8049	0.5132	0.0004
poverty intensity	0.7489	0.7509	0.3186	1.0463	0.819	0.2014
Ln(population)	85.2345	4.6414	0	94.5912	4.9428	0
ρ	-0.0943	0.0243	0.0001	-0.1154	0.0253	0
state dummies	No			Yes		

4 Monte Carlo Simulation Experiments

4.1 Design of the Experiments

In the second phase of our empirical investigation, we consider the properties of the test in a number of simulated data sets. We limit the analysis to regular lattice structures and increase the size of the data set from a 7×7 grid ($N = 49$) to a 125×125 grid ($N = 15,625$). In all, we consider six different sample sizes: $N = \{49, 100, 225, 625, 2500, 15625\}$. We vary the value for the spatial autoregressive parameters λ and ρ over the set {-0.8, -0.5, -0.3, -0.1, -0.01, 0.0, 0.01, 0.1, 0.3, 0.5, 0.8}. Each experiment consists of 10,000 replications performed using PySAL, a python library for spatial analysis developed by the GeoDa Center

[1] The p-values are computed from the estimate and the associated standard error using a standard normal approximation. The p-values reported by the spatial econometrics toolbox use a different convention and are not comparable to the results of our non-spatial models.

for Geospatial Analysis and Computation (Rey and Anselin, 2007). A nominal Type I error of 0.05 is used throughout, which leads to an associated sample standard deviation in each simulation run of $\sqrt{0.05 \times 0.95/10000} = 0.0022$. In total, the combination of parameter values and sample sizes yields 528 separate experimental settings.

The experiments are based on simulating values for the unobserved latent variable y_i^*. This is subsequently turned into an "observed" value of 1 or 0 (for the probit model) or y_i or 0 (for the tobit model) depending on whether $y_i^* > 0$.

Under the null hypothesis of spatial randomness, the model specification is the standard linear latent variable model. We implement this including a constant term and one explanatory variable:

$$\mathbf{y}^* = \iota + 0.5\mathbf{x} + \varepsilon \tag{11}$$

in which ι is a $n \times 1$ vector of ones, $\mathbf{x}$ is a non-stochastic $n \times 1$ regressor vector and $\varepsilon \sim N(0, 1)$.

We consider two cases for the explanatory vector $\mathbf{x}$. In one, it takes values uniformly distributed over the interval [-5,1), such that $\overline{x} = -2$ and $\overline{y}^* = 0$. As a result, with the parameter values used in Equation (11), the sample is balanced, or, $Pr(y \neq 0|x) \approx 0.5$. In the second case, the values of $\mathbf{x}$ are uniformly distributed over the interval [-6,0). In this case, the sample is unbalanced and dominated by zero values, $Pr(y \neq 0|x) \approx 0.36$.[2]

The models under the alternative are obtained in the usual fashion, by applying the reduced form for the appropriate spatial model to the base specification. For the spatial error model, this yields:

$$\mathbf{y}^* = \iota + 0.5\mathbf{x} + (I - \lambda\mathbf{W})^{-1}\varepsilon. \tag{12}$$

For the spatial lag model, the specification is:

$$\mathbf{y}^* = (I - \rho\mathbf{W})^{-1}(\iota + 0.5\mathbf{x} + \varepsilon). \tag{13}$$

4.2 Test Statistics Under the Null Hypothesis

We first consider the rejection frequency under the null hypothesis in the probit model, using $p = 0.05$. The results are given in Table 5. In the case of a balanced sample (column 2) all the rejection frequencies fall within one standard deviation of 0.05. In the unbalanced sample (column 5), convergence to the true size is slightly less rapid, with rejection frequencies slightly outside one standard deviation above 0.05 for $N = 100$ and outside two standard deviations above 0.05 for $N = 225$. For $N > 225$ all rejection frequencies are well within one standard deviation of the correct size. For reference, we also report the average estimate for β and the associated MSE. Except for the smallest values, there is no bias and the MSE decreases with the sample size, as is to be expected.

[2] All the experiments were also performed for $\mathbf{x}$ uniformly distributed over [-4,2), resulting in a sample dominated by non-zero values ($Pr(y \neq 0|x) \approx 0.64$). The results were very similar to those for a balanced sample. Given space restrictions, they were omitted from the paper.

Table 5: Moran's I size, Probit model – 10,000 replications

N	Balanced sample			Unbalanced sample		
	KP Moran's I	$\overline{\beta}$	MSE β	KP Moran's I	$\overline{\beta}$	MSE β
49	0.0507	0.5502	0.0428	0.0513	0.5453	0.0425
100	0.0484	0.5188	0.0104	0.0537	0.5198	0.0113
225	0.0500	0.5061	0.0042	0.0546	0.5079	0.0045
625	0.0483	0.5031	0.0014	0.0479	0.5028	0.0015
2500	0.0482	0.5005	0.0003	0.0499	0.5008	0.0004
15625	0.0501	0.5001	0.0001	0.0497	0.5000	0.0001

Table 6: Moran's I size, Tobit model – 10,000 replications

N	Balanced sample			Unbalanced sample		
	KP Moran's I	$\overline{\beta}$	MSE β	KP Moran's I	$\overline{\beta}$	MSE β
49	0.0485	0.4986	0.0161	0.0313	0.5150	0.0251
100	0.0471	0.5057	0.0082	0.0408	0.5028	0.0093
225	0.0459	0.4996	0.0029	0.0437	0.5000	0.0043
625	0.0511	0.4998	0.0010	0.0440	0.5006	0.0014
2500	0.0480	0.5003	0.0002	0.0535	0.4999	0.0003
15625	0.0471	0.5000	0.0000	0.0515	0.4999	0.0001

The results for the tobit model, reported in Table 6, show a less rapid convergence to the correct size relative to the probit results. In the balanced sample, the rejection level is within two standard deviations of the true value for all sample sizes, but within one standard deviation only for $N = 49$, $N = 625$ and $N = 2,500$, but not for the largest sample size. In the unbalanced sample, the results are less satisfactory, showing under-rejection of the null hypothesis for all $N < 2,500$. The rejection frequency is within two standard deviations for $N = 2,500$ and within one standard deviation for $N = 15,625$, suggesting that larger samples are needed before the test approaches its correct size in the unbalanced case. The estimates for β are largely unbiased and the associated MSE decreases with the sample size, as expected.

We next turn to the extent to which the distribution of the test statistic under the null approaches the asymptotic standard normal. Figures 2 and 3 compare the empirical frequency distributions for the test statistics to the standard normal. Visually, the distributions appear to be very close, especially for the probit model. However, a formal assessment by means of a test for normality (Bera and Jarque, 1981) suggests a more nuanced picture. This statistic focuses in particular on the third and fourth moments of the distribution, detecting deviations from the null in terms of asymmetry or heavy tails.

As shown in Table 7, whereas the null hypothesis of a standard normal distribution cannot be rejected for any of the sample sizes in the probit model, this is not the case for the tobit specification. For the balanced sample, the null is not rejected for any but the smallest sample ($N = 49$). However, in the unbalanced case, this is only obtained for the largest sample ($N = 15,625$). In other words, it seems that the underlying data structure matters for the speed at which the asymptotic distribution is obtained for the Moran's I test in the

Figure 2: Empirical Frequency Distribution (H_0) – Probit model

(a) Balanced sample (b) Unbalanced sample

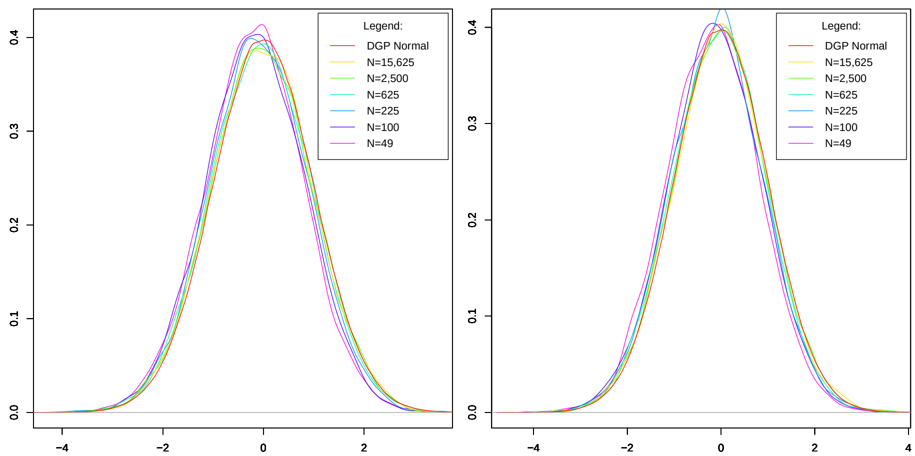


Figure 3: Empirical Frequency Distribution (H_0) – Tobit model

(a) Balanced sample (b) Unbalanced sample

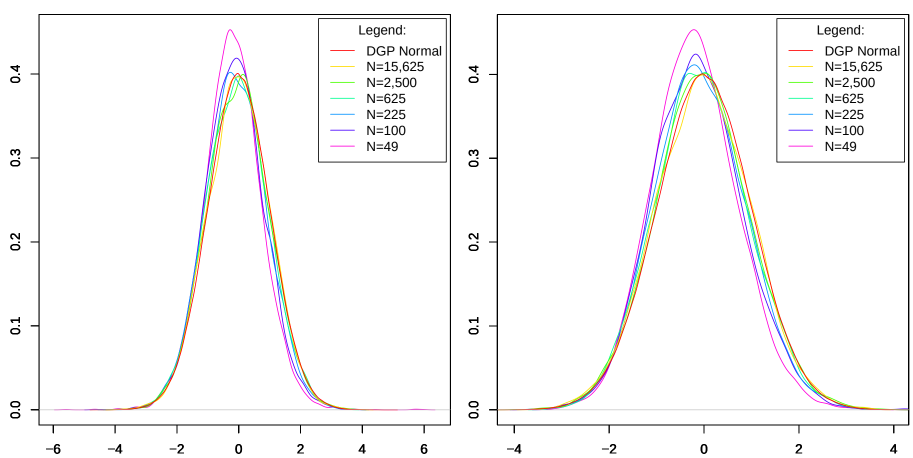


Table 7: Bera-Jarque normality test

N	Probit model		Tobit model	
	Balanced sample	Unbalanced sample	Balanced sample	Unbalanced sample
49	2.1278	0.9588	10.6953	39.5968
	(0.3451)	(0.6192)	(0.0048)	(0.0000)
100	0.7182	2.6172	2.5082	34.7276
	(0.6983)	(0.2702)	(0.2853)	(0.0000)
225	2.2647	3.2929	3.5344	18.7963
	(0.3223)	(0.1927)	(0.1708)	(0.0001)
625	0.2522	3.4651	1.6670	15.0708
	(0.8815)	(0.1768)	(0.4345)	(0.0005)
2500	4.0970	1.4885	1.3107	11.5911
	(0.1289)	(0.4751)	(0.5192)	(0.0030)
15625	1.2113	0.7708	1.0425	0.3048
	(0.5457)	(0.6802)	(0.5938)	(0.8587)

Note: p-values in parenthesis.

tobit model. Much larger sample sizes are required for the unbalanced case relative to the balanced case.

4.3 Power of the Tests

Probit Model

The rejection frequencies for the probit model under the alternative of a spatial error model are reported in Table 8, while those for the spatial lag alternative are given in Table 9.

For the spatial error alternative, the test has considerable power even in medium-sized data sets. For example, for $N = 625$ a rejection frequency of about 80% is achieved for an absolute value of $\lambda = 0.3$. For the largest data set, 99% of rejection is achieved even for $\lambda = 0.1$. The results are very similar for the unbalanced samples, with marginally lower rejection frequencies. Also, the rejection frequencies are similar for positive and negative values of the spatial autoregressive coefficient.

Table 8 also includes the value for the mean estimate of β and the associated MSE. These results clearly illustrate how the estimate becomes biased (relative to the true value of $\beta = 0.5$) with increasing values of the spatial autoregressive coefficient, in both balanced and unbalanced samples. As mentioned, this contrasts with the result in the standard linear regression model, where the estimate for β remains unbiased in the presence of spatial error autocorrelation. In addition, the results in Table 8 show the growing imprecision of the estimate as the spatial autoregressive coefficient increases.

The results for the spatial lag alternative (Table 9) are very similar, although there is a slightly more pronounced difference between the balanced and the unbalanced case. This demonstrated the power of the Moran's I test against both lag and error alternatives. As in the error case, a rejection frequency of close to 80% is obtained for $N = 625$ for ρ as low as 0.03 in the balanced setup.

The rejection frequencies are systematically smaller in the unbalanced case, but only slightly so (e.g., 0.786 relative to 0.814 for $\rho = 0.3$ in $N = 625$).

As in the spatial error case, the estimates for β become increasingly more biased with larger values of ρ. However, somewhat counter intuitively, this is more pronounced in the large samples than in the smaller samples. Also, the MSE gets larger with larger absolute values of ρ although this is most pronounced for $|\rho| = 0.8$, especially in the smaller samples.

Tobit model

The rejection frequencies for the tobit model under the alternative of a spatial error model are reported in Table 10, while those for the spatial lag alternative are given in Table 11.

The results are similar to those for the probit model, although there are also some pronounced differences. Most important is the lesser power for the unbalanced case relative to the balanced design. For example, for the spatial error alternative, a rejection frequency of 96% is obtained in the balanced case for $\lambda = 0.3$ and $N = 625$, which is higher than in the probit case. However, the matching rejection frequency in the unbalanced setup is 82%, more than 10% less. Nevertheless, for sample sizes of 2,500 and higher, there is near uniform rejection, even for small values of λ.

In contrast to the probit case, there is much less effect of the spatial error autocorrelation on the estimate of β. The effect seems to be primarily on the precision of the estimate, and mostly in the smaller data sets. For $N \geq 225$, the MSE only increases for the largest value of λ. This pattern is similar between the balanced and unbalanced case.

As shown in Table 11, the rejection frequencies are similar for the spatial lag alternative, again demonstrating the power of this test against both forms of spatial dependence. Also, the balanced samples show slightly higher power than the unbalanced samples. In contrast to the error case, the effect of spatial correlation on the estimate of β is pronounced, resulting in an increased bias with higher values of ρ. In addition, the MSE increases as well, although in the largest samples, this is only the case for $\rho = 0.8$.

5 Concluding Remarks

In this paper, we highlight the importance of the Moran's I test statistic developed by Kelejian and Prucha (2001) for the probit and tobit models. We provide evidence of the relevance of the test in empirical practice, by means of an illustrative study of the adoption of dialysis equipment in Brazilian municipalities. Whereas in a probit specification, the evidence of spatial autocorrelation could be remedied by including spatial fixed effects, this was not the case for the tobit model. In the latter case, a spatial lag tobit model turned out to be necessary, suggesting a negative spillover effect between municipalities. This could be interpreted as a form of free riding by municipalities surrounded by neighbors

Table 8: Rejection Frequency – Probit Model, Spatial Error

N	λ	Balanced sample			Unbalanced sample		
		KP Moran's I	$\overline{\beta}$	MSE β	KP Moran's I	$\overline{\beta}$	MSE β
49	-0.8	0.903	0.325	0.053	0.872	0.324	0.053
	-0.5	0.4	0.475	0.029	0.351	0.477	0.028
	-0.3	0.173	0.525	0.034	0.149	0.527	0.034
	-0.1	0.07	0.545	0.037	0.067	0.551	0.036
	-0.01	0.051	0.549	0.039	0.051	0.55	0.035
	0.01	0.054	0.55	0.04	0.051	0.553	0.038
	0.1	0.046	0.548	0.039	0.051	0.552	0.035
	0.3	0.088	0.531	0.035	0.081	0.531	0.031
	0.5	0.246	0.494	0.031	0.2	0.499	0.031
	0.8	0.713	0.372	0.057	0.674	0.368	0.045
100	-0.8	0.99	0.314	0.042	0.987	0.318	0.041
	-0.5	0.595	0.453	0.011	0.588	0.454	0.012
	-0.3	0.246	0.494	0.01	0.235	0.499	0.01
	-0.1	0.08	0.516	0.01	0.075	0.517	0.011
	-0.01	0.052	0.518	0.01	0.053	0.52	0.011
	0.01	0.049	0.515	0.01	0.048	0.52	0.012
	0.1	0.055	0.515	0.01	0.056	0.517	0.011
	0.3	0.158	0.498	0.01	0.161	0.503	0.011
	0.5	0.474	0.459	0.012	0.452	0.465	0.011
	0.8	0.959	0.337	0.037	0.96	0.334	0.037
225	-0.8	1	0.321	0.036	1	0.322	0.035
	-0.5	0.887	0.449	0.007	0.84	0.448	0.007
	-0.3	0.443	0.488	0.004	0.384	0.488	0.004
	-0.1	0.093	0.505	0.004	0.093	0.506	0.004
	-0.01	0.053	0.506	0.004	0.049	0.509	0.005
	0.01	0.047	0.508	0.004	0.046	0.509	0.004
	0.1	0.071	0.504	0.004	0.071	0.506	0.004
	0.3	0.372	0.488	0.004	0.33	0.488	0.005
	0.5	0.845	0.451	0.006	0.787	0.452	0.006
	0.8	1	0.329	0.033	1	0.33	0.033
625	-0.8	1	0.326	0.032	1	0.327	0.031
	-0.5	0.998	0.447	0.004	0.998	0.448	0.004
	-0.3	0.805	0.483	0.002	0.772	0.484	0.002
	-0.1	0.16	0.5	0.001	0.144	0.501	0.002
	-0.01	0.053	0.502	0.001	0.047	0.503	0.002
	0.01	0.049	0.502	0.001	0.05	0.503	0.002
	0.1	0.138	0.5	0.001	0.126	0.501	0.002
	0.3	0.779	0.485	0.002	0.752	0.486	0.002
	0.5	0.999	0.448	0.004	0.996	0.449	0.004
	0.8	1	0.328	0.031	1	0.33	0.03
2500	-0.8	1	0.329	0.03	1	0.329	0.03
	-0.5	1	0.447	0.003	1	0.447	0.003
	-0.3	1	0.483	0.001	1	0.483	0.001
	-0.1	0.433	0.499	0	0.4	0.499	0
	-0.01	0.052	0.501	0	0.056	0.501	0
	0.01	0.051	0.501	0	0.057	0.501	0
	0.1	0.414	0.499	0	0.377	0.499	0
	0.3	1	0.483	0.001	0.999	0.483	0.001
	0.5	1	0.447	0.003	1	0.448	0.003
	0.8	1	0.329	0.029	1	0.33	0.029
15625	-0.8	1	0.331	0.029	1	0.331	0.029
	-0.5	1	0.447	0.003	1	0.448	0.003
	-0.3	1	0.482	0	1	0.483	0
	-0.1	0.993	0.498	0	0.988	0.498	0
	-0.01	0.073	0.5	0	0.071	0.5	0
	0.01	0.074	0.5	0	0.07	0.5	0
	0.1	0.992	0.498	0	0.985	0.498	0
	0.3	1	0.482	0	1	0.483	0
	0.5	1	0.447	0.003	1	0.448	0.003
	0.8	1	0.331	0.028	1	0.331	0.029

Table 9: Rejection Frequency – Probit Model, Spatial Lag

N	λ	**Balanced sample**			**Unbalanced sample**		
		KP Moran's I	$\overline{\beta}$	MSE β	KP Moran's I	$\overline{\beta}$	MSE β
49	-0.8	0.89	0.418	0.029	0.846	0.45	0.023
	-0.5	0.396	0.533	0.033	0.348	0.542	0.029
	-0.3	0.176	0.551	0.039	0.152	0.56	0.038
	-0.1	0.065	0.555	0.04	0.064	0.555	0.036
	-0.01	0.049	0.549	0.038	0.05	0.546	0.032
	0.01	0.047	0.552	0.038	0.048	0.548	0.035
	0.1	0.042	0.542	0.038	0.048	0.547	0.035
	0.3	0.098	0.522	0.033	0.091	0.527	0.034
	0.5	0.264	0.488	0.027	0.235	0.492	0.031
	0.8	0.777	0.383	0.033	0.589	0.54	6.073
100	-0.8	0.986	0.362	0.025	0.991	0.358	0.027
	-0.5	0.606	0.462	0.009	0.583	0.471	0.01
	-0.3	0.248	0.494	0.009	0.245	0.504	0.01
	-0.1	0.075	0.513	0.01	0.074	0.519	0.011
	-0.01	0.049	0.517	0.011	0.052	0.519	0.011
	0.01	0.049	0.517	0.011	0.052	0.52	0.012
	0.1	0.052	0.517	0.01	0.051	0.519	0.011
	0.3	0.172	0.516	0.011	0.16	0.507	0.011
	0.5	0.499	0.5	0.01	0.449	0.482	0.012
	0.8	0.969	0.427	0.013	0.901	0.41	0.109
225	-0.8	1	0.386	0.016	1	0.361	0.022
	-0.5	0.895	0.477	0.004	0.867	0.459	0.005
	-0.3	0.448	0.499	0.004	0.402	0.488	0.004
	-0.1	0.098	0.507	0.004	0.097	0.505	0.004
	-0.01	0.051	0.506	0.004	0.05	0.508	0.005
	0.01	0.046	0.507	0.004	0.049	0.509	0.005
	0.1	0.077	0.502	0.004	0.064	0.51	0.005
	0.3	0.377	0.49	0.004	0.32	0.506	0.005
	0.5	0.858	0.461	0.005	0.757	0.484	0.005
	0.8	1	0.366	0.021	0.97	0.393	0.023
625	-0.8	1	0.389	0.013	1	0.395	0.012
	-0.5	0.999	0.473	0.002	0.998	0.48	0.002
	-0.3	0.814	0.493	0.001	0.786	0.5	0.001
	-0.1	0.159	0.503	0.001	0.152	0.505	0.002
	-0.01	0.052	0.502	0.001	0.055	0.503	0.002
	0.01	0.052	0.503	0.001	0.048	0.502	0.002
	0.1	0.138	0.501	0.001	0.136	0.5	0.002
	0.3	0.762	0.49	0.001	0.74	0.484	0.002
	0.5	0.998	0.467	0.002	0.994	0.455	0.004
	0.8	1	0.381	0.015	1	0.362	0.021
2500	-0.8	1	0.38	0.015	1	0.372	0.017
	-0.5	1	0.466	0.001	1	0.462	0.002
	-0.3	1	0.489	0	1	0.487	0.001
	-0.1	0.444	0.499	0	0.396	0.499	0
	-0.01	0.06	0.501	0	0.053	0.501	0
	0.01	0.052	0.5	0	0.055	0.501	0
	0.1	0.421	0.5	0	0.365	0.5	0
	0.3	1	0.49	0	0.999	0.492	0
	0.5	1	0.468	0.001	1	0.47	0.001
	0.8	1	0.383	0.014	1	0.386	0.014
15625	-0.8	1	0.382	0.014	1	0.382	0.014
	-0.5	1	0.467	0.001	1	0.467	0.001
	-0.3	1	0.489	0	1	0.489	0
	-0.1	0.993	0.499	0	0.988	0.499	0
	-0.01	0.074	0.5	0	0.069	0.5	0
	0.01	0.072	0.5	0	0.067	0.5	0
	0.1	0.992	0.499	0	0.985	0.499	0
	0.3	1	0.49	0	1	0.49	0
	0.5	1	0.468	0.001	1	0.468	0.001
	0.8	1	0.384	0.014	1	0.383	0.014

Table 10: Rejection Frequency – Tobit Model, Spatial Error

N	λ	**Balanced sample**			**Unbalanced sample**		
		KP Moran's I	$\overline{\beta}$	MSE β	KP Moran's I	$\overline{\beta}$	MSE β
49	-0.8	0.83	0.451	0.13	0.773	0.47	0.05
	-0.5	0.378	0.494	0.028	0.289	0.512	0.028
	-0.3	0.161	0.499	0.018	0.116	0.517	0.032
	-0.1	0.067	0.501	0.016	0.049	0.513	0.024
	-0.01	0.056	0.501	0.016	0.038	0.515	0.024
	0.01	0.05	0.503	0.148	0.036	0.517	0.024
	0.1	0.05	0.518	3.119	0.038	0.515	0.024
	0.3	0.117	0.498	0.016	0.096	0.518	0.199
	0.5	0.293	0.495	0.021	0.252	0.51	0.027
	0.8	0.732	0.464	0.042	0.702	0.48	0.052
100	-0.8	0.991	0.466	0.016	0.949	0.48	0.027
	-0.5	0.735	0.501	0.009	0.458	0.5	0.01
	-0.3	0.318	0.502	0.007	0.176	0.501	0.009
	-0.1	0.081	0.505	0.007	0.058	0.503	0.009
	-0.01	0.049	0.504	0.007	0.04	0.502	0.009
	0.01	0.046	0.504	0.007	0.046	0.502	0.009
	0.1	0.06	0.505	0.007	0.05	0.503	0.01
	0.3	0.258	0.504	0.008	0.158	0.502	0.01
	0.5	0.66	0.503	0.01	0.452	0.5	0.011
	0.8	0.976	0.475	0.014	0.929	0.48	0.016
225	-0.8	1	0.488	0.006	1	0.495	0.006
	-0.5	0.953	0.5	0.003	0.877	0.501	0.005
	-0.3	0.554	0.499	0.003	0.41	0.501	0.005
	-0.1	0.111	0.501	0.003	0.088	0.502	0.004
	-0.01	0.052	0.501	0.003	0.05	0.501	0.004
	0.01	0.05	0.5	0.003	0.047	0.502	0.004
	0.1	0.093	0.501	0.003	0.082	0.501	0.004
	0.3	0.516	0.501	0.003	0.411	0.501	0.004
	0.5	0.945	0.5	0.003	0.868	0.501	0.005
	0.8	1	0.493	0.006	0.999	0.495	0.008
625	-0.8	1	0.493	0.002	1	0.499	0.002
	-0.5	1	0.5	0.001	0.999	0.5	0.002
	-0.3	0.96	0.5	0.001	0.816	0.5	0.001
	-0.1	0.233	0.5	0.001	0.155	0.501	0.001
	-0.01	0.048	0.5	0.001	0.05	0.5	0.001
	0.01	0.051	0.5	0.001	0.045	0.5	0.001
	0.1	0.21	0.5	0.001	0.148	0.5	0.001
	0.3	0.954	0.5	0.001	0.815	0.5	0.001
	0.5	1	0.499	0.001	0.999	0.501	0.001
	0.8	1	0.495	0.002	1	0.499	0.002
2500	-0.8	1	0.498	0.001	1	0.501	0.001
	-0.5	1	0.5	0	1	0.5	0
	-0.3	1	0.5	0	1	0.5	0
	-0.1	0.649	0.5	0	0.439	0.5	0
	-0.01	0.059	0.5	0	0.058	0.5	0
	0.01	0.056	0.5	0	0.054	0.5	0
	0.1	0.624	0.5	0	0.444	0.5	0
	0.3	1	0.501	0.008	1	0.5	0
	0.5	1	0.501	0.004	1	0.5	0
	0.8	1	0.495	0.001	1	0.505	0.133
15625	-0.8	1	0.5	0	1	0.5	0
	-0.5	1	0.5	0	1	0.5	0
	-0.3	1	0.5	0	1	0.5	0
	-0.1	1	0.5	0	0.994	0.5	0
	-0.01	0.092	0.5	0	0.074	0.5	0
	0.01	0.085	0.5	0	0.075	0.5	0
	0.1	1	0.5	0	0.995	0.5	0
	0.3	1	0.5	0	1	0.5	0
	0.5	1	0.503	0.067	1	0.5	0
	0.8	1	0.5	0	1	0.5	0

Table 11: Rejection Frequency – Tobit Model, Spatial Lag

N	ρ	Balanced sample			Unbalanced sample		
		KP Moran's I	$\overline{\beta}$	MSE β	KP Moran's I	$\overline{\beta}$	MSE β
49	-0.8	0.736	0.604	0.317	0.724	0.539	0.036
	-0.5	0.298	0.533	0.02	0.256	0.539	0.028
	-0.3	0.133	0.518	0.315	0.113	0.521	0.025
	-0.1	0.063	0.506	0.062	0.045	0.516	0.025
	-0.01	0.054	0.498	0.014	0.04	0.514	0.024
	0.01	0.051	0.514	1.844	0.038	0.515	0.025
	0.1	0.054	0.501	0.014	0.046	0.513	0.025
	0.3	0.126	0.522	0.031	0.115	0.527	0.029
	0.5	0.333	0.547	0.027	0.265	0.548	0.038
	0.8	0.745	0.595	0.075	0.477	0.597	0.139
100	-0.8	0.846	0.713	5.422	0.926	0.584	1.302
	-0.5	0.552	0.572	0.02	0.427	0.533	0.01
	-0.3	0.264	0.531	0.016	0.163	0.508	0.008
	-0.1	0.075	0.511	0.007	0.054	0.502	0.009
	-0.01	0.047	0.505	0.007	0.043	0.502	0.009
	0.01	0.043	0.505	0.017	0.045	0.502	0.009
	0.1	0.065	0.504	0.015	0.056	0.507	0.01
	0.3	0.301	0.513	0.015	0.168	0.528	0.014
	0.5	0.729	0.538	0.012	0.385	0.568	0.027
	0.8	0.988	0.58	2.265	0.501	0.744	0.766
225	-0.8	0.999	0.648	0.032	1	0.584	0.016
	-0.5	0.931	0.541	0.005	0.867	0.531	0.005
	-0.3	0.51	0.515	0.003	0.401	0.509	0.004
	-0.1	0.096	0.501	0.003	0.082	0.502	0.004
	-0.01	0.051	0.5	0.003	0.047	0.501	0.004
	0.01	0.051	0.501	0.003	0.046	0.502	0.004
	0.1	0.089	0.501	0.003	0.084	0.504	0.004
	0.3	0.552	0.513	0.003	0.401	0.519	0.006
	0.5	0.956	0.541	0.005	0.798	0.548	0.01
	0.8	1	0.636	0.653	0.947	0.653	0.054
625	-0.8	1	0.653	0.027	1	0.594	0.025
	-0.5	1	0.539	0.003	1	0.543	0.003
	-0.3	0.948	0.514	0.001	0.807	0.514	0.002
	-0.1	0.222	0.501	0.001	0.144	0.502	0.001
	-0.01	0.046	0.5	0.001	0.047	0.5	0.001
	0.01	0.053	0.5	0.001	0.055	0.5	0.001
	0.1	0.223	0.501	0.001	0.156	0.501	0.001
	0.3	0.964	0.511	0.001	0.79	0.509	0.002
	0.5	1	0.534	0.002	0.994	0.53	0.003
	0.8	1	0.655	0.039	0.999	0.612	0.022
2500	-0.8	1	0.664	0.029	1	0.626	0.023
	-0.5	1	0.542	0.002	1	0.542	0.002
	-0.3	1	0.516	0.022	1	0.515	0.001
	-0.1	0.618	0.502	0	0.442	0.502	0
	-0.01	0.054	0.503	0.072	0.051	0.5	0
	0.01	0.059	0.5	0	0.051	0.5	0
	0.1	0.631	0.501	0	0.436	0.5	0
	0.3	1	0.511	0.001	1	0.509	0.001
	0.5	1	0.535	0.002	1	0.532	0.002
	0.8	1	0.641	0.021	1	0.625	0.018
15625	-0.8	1	0.634	0.019	1	0.627	0.019
	-0.5	1	0.535	0.001	1	0.536	0.001
	-0.3	1	0.512	0.001	1	0.511	0
	-0.1	1	0.501	0	0.994	0.501	0
	-0.01	0.089	0.5	0	0.076	0.5	0
	0.01	0.085	0.5	0	0.075	0.5	0
	0.1	1	0.501	0	0.994	0.502	0
	0.3	1	0.512	0	1	0.513	0
	0.5	1	0.537	0.001	1	0.539	0.002
	0.8	1	0.63	0.02	1	0.638	0.019

with higher provision of health services. Alternatively, it could highlight the use of a larger than municipal spatial scale in health planning, in the sense that location in a given municipality would provide services for that municipality, but also for the surrounding ones. As a consequence, the latter would seem to be underserved when studied at the municipal scale. This remains to be further investigated.

We also provide the first extensive results on the finite sample properties of the Kelejian-Prucha tests in terms of size and power in a wide ranging set of Monte Carlo simulation experiments. Both tests approach the correct size even for medium-sized data sets, although the results for the probit case require slightly smaller sizes than the tobit case. Both tests also approach their asymptotic normal distribution under the null. However, this approximation is much better for the probit case. In the tobit case, especially for unbalanced samples, the asymptotic approximation is only obtained for the largest data set size of $N = 15,625$. This effect of the degree of balancing on the properties of the test is something that requires further investigation, since it is not suggested by the theoretical results.

Both tests have good power, even for small values of the spatial autoregressive parameter in medium-sized data sets ($N = 625$). In the tobit case, there is again an effect of the unbalanced design, where the power is less than for the balanced case. Importantly, the test has essentially the same power against a spatial error and a spatial lag alternative. On the one hand, this is a very useful result for a misspecification test. On the other hand, it is less useful in guiding a specification search since it suggests that upon rejection of the null both alternatives need to be estimated.

So far, the Kelejian-Prucha test has seen little adoption in the applied econometrics literature. We hope that with this paper, we were able to demonstrate the usefulness for the test in empirical practice. In order to further the adoption of the tests, they have been included in the current development version of the PySAL software and will be part of a future official release.

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