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EDUCATIONAL STANDARDS IN PRIVATE AND PUBLIC SCHOOLS

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September 2007

# Educational standards in private and public schools* 

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#### Abstract

When school quality increases with the educational standard set by schools, education before college needs not be a hierarchy with private schools offering better quality than public schools. An alternative configuration, with public schools offering a higher educational standard than private schools, is also possible, in spite of the fact that tuition levied by private schools is strictly positive. In our model, private schools can offer a lower educational standard at a positive price because they attract students with a relatively high cost of effort, who would find the high standards of public schools excessively demanding. With the key parameters calibrated for the US and Italy, our model predicts that majority voting in the US supports a system with high quality private schools and low quality public schools, as assumed by Epple and Romano, 1998. An equilibrium with low quality private schools is supported instead in Italy.

JEL codes: J24, H42


Key words: private schools, public schools, majority voting
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## 1 Introduction

Do private schools always provide better service than public schools? The answer is apparently straightforward: since private schools charge a positive price (tuition), they can only attract students by providing better service than public schools, which are funded by the taxpayer (see De Fraja, 2004). Yet quality is not the only service that private schools can provide. In a recent scandal, Italian prosecutors have found that some private schools in the country used to sell high school diplomas at a price. The so called "Diploma no problem" organization provided "good service" to its customers: answers were supplied in advance for written and oral exams, and attendance records were fixed. The national exam for the leaving high school certificate was also by-passed by having customers take the exams in places where the outcome was assured (The Economist, June 12th, 2004, p.31).

In this admittedly extreme example, private schools can charge a fee by allowing customers to grab the degree with little effort: the service offered is not quality but leisure. Less extreme is the evidence discussed by Bertola and Checchi, 2002, who find that Italian public high schools are associated on average to better performance, followed by religious private schools and lay private schools. They interpret this as evidence that private schools in Italy appear to focus more on the recovery of less brilliant students than on across the board high quality education. Italy is not an isolated case. Vandenberghe and Robin, 2004, look at the results of standardized tests in maths, reading and science reported in the 2000 OECD Program for International Student Assessment survey and find that public schools can outperform private schools in France and Austria. De Fraja, 2004, reports evidence on the UK by Marks and coauthors, who find that there is considerable variation in the quality of UK religious - and private - schools: some are very good but other are very poor ${ }^{1}$.

[^1]Across the Atlantic, several studies have investigated the relative effectiveness and quality of private and public schools, with mixed results. On the one hand, Evans and Schwab, 1995, and Neal, 1997, find evidence that private Catholic schools increase student achievement with respect to public schools, especially for minorities. On the other hand, Figlio and Stone, 1999, assess the effect of religious and non-religious US private schools on educational outcomes and find that, in general, only the former increase individual outputs relative to public schools. In a review of this empirical literature, McEwan, 2000, concludes that private Catholic secondary schools in the US have consistent effects on improving college attendance and high school graduation, but almost no effect on individual achievement in standardized tests. Broadly, this evidence suggests that private schools are heterogeneous, with some offering poorer academic quality and some others offering better quality than public schools. Why do households pay to sent their offspring to school of lower academic quality? Figlio and Stone, 1999, argue that parents who enrol their offspring in private schools may care for other outcomes, such as discipline, extracurricular activities, religious matters and the opportunity to interact with a certain peer group.

In spite of the evidence, the theoretical literature - to our knowledge - does not entertain the possibility that private schools can be of lower academic quality than public schools. Recent exceptions are Oliveira, 2006, and McMillan, 2004. Oliveira studies a market with two universities, which compete for students by setting admission standards. McMillan has a model where rent-seeking public schools find it optimal to reduce productivity when a voucher is introduced.

An important paper in this area is Epple and Romano, 1998, who model the education market as a stratified hierarchy of school qualities, with private schools doing systematically better than public schools. Schools in their model are clubs of students who differ in their academic ability, and school quality is simply the average quality of enrolled pupils. The essential reason for the existence of a hierarchy with public schools dominated by private institutions is that the latter must be of higher peer quality than the former, otherwise no
student would be willing to pay to attend a private school. In their model, state schools act as residual repositories, taking in all those students who do not enrol in private schools.

In this paper we question the assumptions that private schools offer only quality for a price and that public schools act as residual repositories. First, private schools can charge a positive price for leisure, access to networks or for religious education ${ }^{2}$. Second, the view that public schools are of the poorest quality is both not always consistent with the stylized facts and not grounded in a policy decision rule, be it the maximization of a social welfare function or a political equilibrium based on majority voting.

Until recently, the literature on school quality has been dominated by the education production function approach, with its emphasis on school resources. This important approach has not produced yet a consensus on the importance of resources for school quality (see Hanushek. 2002, for a review). Following recent research, we believe that a key factor affecting school quality not explored enough by the literature is the set of incentives facing students ${ }^{3}$. Student time, ability and effort are important inputs in education, which can be affected by adequate incentives, including educational standards (Costrell, 1994). When a school increases its standard it raises its quality for two reasons: first, most students respond by working harder, and learning more. Second, since the cost of effort declines with ability, higher standards attract better students.

As in Costrell, 1994, we define the educational standard as the productivity level to graduate. The higher the level, the higher the standard. Depending on the country, the selected standard is enforced by a combination of curriculum choice, tests and grading standards ${ }^{4}$. In this paper, we treat educational stan-

[^2]dards as incentives which affect the quality of public and private schools because they promote individual effort and the self-selection of students by ability.

We consider a simplified market for education with only a public and a private school and a sequential structure. In the first stage of the sequential game, the government decides the educational standard of the public school, which charges no admission fees. The decision criterion used by the government is majority voting. We believe that this is an appealing and intuitive criterion for the US system of primary and secondary education (see Fernandez and Rogerson, 1995), where individuals vote on the level of education provision in their district and on the associated local property taxation. We are aware, however, that there are other possible decision rules, such as welfare maximization (see De Fraja, 2002). Therefore, in the paper we also compare the outcome of majority voting with the choice by a social planner who maximizes a utilitarian welfare function.

In the second stage, a private school enters the market and chooses both the positive tuition fee and its own educational standard, which could be above or below the standard set by the public school. The private school maximizes profits by taking into account that its choice of price and standard affects the demand for its services. Profit maximization is an assumption used in most of this literature, see for instance Stiglitz, 1974, and Epple and Romano, 1998. By restricting entry to a single private school, we focus on the relative quality of public and private schools at the cost of overlooking the heterogeneity of private schools. We feel that the treatment of this heterogeneity is important but would require a separate paper.

An equilibrium in this economy is the combination of the public and private school educational standards and the nonnegative tuition fee set by the private school, which satisfy both majority voting and profit maximization. We show that there are two possible equilibria: a) the private school sets a higher ed-
assessment in the US is not measured as elsewhere against an absolute, external standard, which makes it difficult to convey valuable information on individual ability to the labour market. Betts, 1998, argues in favour of higher educational standards as a key element in high school reform in the US.
ucational standard than the public school; b) the private school sets a lower educational standard than the public school.

While case a) is consistent with Epple and Romano's story, case b) is not as it produces a hierarchy with private schools providing lower quality than public schools. We estimate the key parameters of the model by using empirical evidence from the US and Italy and find that majority voting selects "low quality public school - high quality private school" in the former country and "low quality private school - high quality public school" in the latter country. Interestingly, the choice made by majority voting turns out to be the same taken by a social planner who maximizes the welfare of all the households in the economy. We interpret the difference in standards between public and private schools in these two countries as two different equilibria in our model of educational standards.

Our results have interesting policy implications. A lot of policy debate, and a non-negligible amount of policy practice, such as school vouchers, is based on the assumption that private schools are better from an educational viewpoint. If instead private schools turn out to be better because they provide some other non-educational service, then a substantial part of policy justification for vouchers falls by the wayside.

The paper is organized as follows. Section 2 introduces the model. Section 3 considers both the outcome of majority voting and the one produced by a utilitarian social planner. An application to the US and Italy is discussed in Section 4. Conclusions follow.

## 2 The Model

We consider a sequential Hotelling-type model where education before college is provided by a public and a private school. The timing of the model is as follows: first, the government decides the educational standard of the public school $s_{G} \in[1,2]$; second, a private school decides entry, tuition $p$ and its educational standard $s_{P} \in[1,2]$; third, a continuum of households, with unitary
mass, enrols their offspring in one of the available schools, after observing the educational standard of each school and the tuition price set by the private school.

Learning the more sophisticated maths and science implied by a higher standard requires both higher effort by students and adequate facilities such as labs and libraries. Notice that higher standards are not synonymous of more school resources: while the former require better facilities, the latter without appropriate incentives can fail to improve school quality ${ }^{5}$. Let $k \in[0,1]$ be the unit cost of setting the standard, independently of whether the school is public or private ${ }^{6}$. An increase in the standard requires higher total costs. The costs borne by the public school, $k s_{G}$, are funded by a proportional tax $\tau$ paid by all households, independently of whether they send their offspring to the public school or not. The private school funds its costs by setting a tuition fee $p$.

Attaining the standard requires that students spend individual effort. Since effort is costly, students either spend the minimum level needed to attain the standard or pay no effort at all (see Costrell, 1994). The individual cost of effort depends on innate ability. Since individuals differ in innate ability, not all the pupils in this economy attain the standard and complete secondary school.

The private school decides whether to enter in the market for education and chooses the standard $s_{P}$ and the price $p$ to maximize (expected) profits. Its profit function is

$$
\begin{equation*}
\pi=p D-k s_{P} \tag{1}
\end{equation*}
$$

where $D$ is expected demand, which results from the aggregation of individual enrolment choices. Once the educational standard has been set, the marginal costs of supplying school services are assumed to be zero ${ }^{7}$.

An equilibrium

$$
\left\{s_{G},\left[s_{P}\left(s_{G}\right), p\left(s_{G}\right)\right], D\left(s_{G}, s_{P}, p\right)\right\}
$$

[^3]is such that:

1) $D\left(s_{G}, s_{P}, p\right)$ is the demand function obtained by aggregating individual optimal choices, given $s_{G}, s_{P}, p$;
2) $\left[s_{P}\left(s_{G}\right), p\left(s_{G}\right)\right]$ is the vector of optimal responses to $s_{G}$ by the private school, which takes into account the function $D\left(s_{G}, s_{P}, p\right)$;
3) $s_{G}$ is determined by majority voting involving all households, given the functions $\left[s_{P}\left(s_{G}\right), p\left(s_{G}\right)\right]$ and $D\left(s_{G}, s_{P}, p\right)$. Alternatively, point 3) can be replaced by
$\left.3^{\prime}\right) s_{G}$ is determined by the government to maximize welfare, given the functions $\left[s_{P}\left(s_{G}\right), p\left(s_{G}\right)\right]$ and $D\left(s_{G}, s_{P}, p\right)$.

The sequential structure of the model implies that we can characterize the equilibrium by using backward induction. We start with the last stage of the game, the enrolment decision of households, which in turn determines the demand for the services of the public and private school.

### 2.1 Household choice

Following De Fraja, 2002, each household in this economy consists of a mother and a daughter. Let $y$ be the $\log$ endowed income of the mother and $w$ the $\log$ earnings of the daughter. We exclude liquidity constraints by assuming that each household can freely borrow against the future income of the daughter $w$.

This assumption simplifies drastically the algebra and is not wholly unrealistic. Carneiro and Heckman, 2002, find that between 4 and 8 percent of children in the US are constrained in their college investment decisions. Given that public providers are much more important before college than at college, this percentage is likely to be significantly lower for decisions concerning primary and secondary education ${ }^{8}$. We discuss verbally the consequences for our model of introducing liquidity constraints in Section 4 of the paper.

The discount factor is equal to 1. Daughters and households are heterogeneous and these differences are described by the pair $(\theta, y)$, where $\theta$ is the $\log$

[^4]of the reciprocal of the daughter's ability - or minus log ability, lower for higher ability. We assume that $\theta$ and $y$ are jointly normally distributed, with marginal distributions $\theta \sim N\left(0, \sigma^{2}\right)$ and $y \sim N\left(\mu_{y}, \psi^{2}\right)$. Therefore, both ability $A$ and income $Y$ are lognormally distributed. For future use, we define $\mu$ as the mean of $Y$.

In the real world, students can drop out of school and fail to graduate ${ }^{9}$. In the simplified economy described in this paper, daughters either enrol in school and attain the standard or do not enrol at all, which is equivalent to spending zero effort.

We posit that household utility $U$ is

$$
\begin{equation*}
U=y-\tau-p+w(e, s, \theta, y)-\theta s e \tag{2}
\end{equation*}
$$

where $\log Y(1-\tau) \simeq y-\tau, s=s_{G}, s_{P}, e$ is a dummy equal to one in the event of enrolment and school completion ${ }^{10}$ and to zero otherwise, $p$ is the tuition fee - zero in the public school and positive in the private school - and $\theta s$ is the effort cost of attaining the educational standard $s$ : the higher the standard, the higher the effort required to attain it, but the effort cost is lower for higher innate $\log$ ability ${ }^{11}$. When the daughter fails to enrol, her cost of effort is zero and her reservation utility is $U_{R}=y-\tau-p+w(0, s, \theta, y)$.

The assumption that household utility is concave in income and earnings but linear in the tuition fee and in the cost of effort is in line with the existing literature in the area - see De Fraja, 2002 ${ }^{12}$ - and greatly simplifies the algebra, making the model tractable. The linearity of utility with respect to the tuition fee implies that the enrolment decision - conditional on the educational standards and the tuition price - depends exclusively on individual ability and

[^5]is independent of endowed $\log$ income $y$, consistently with our assumption of no liquidity constraints.

A key finding of the empirical labor economics literature since Mincer - see for instance Murnane, Willett and Levy, 1995, Bowles, Gintis and Osborne, 2001 and Hanushek and Kimko, 2000, Dearden, Meghir and Ferri, 2002 - is that earnings are a log-linear function of individual characteristics, including school quantity and quality. Drawing from this literature, we specify the daughter's earnings as

$$
\begin{equation*}
w(e, s, \theta, y)=\left(\lambda_{0}+\lambda_{1} s\right) e+\lambda_{2} \theta+\lambda_{3} y+\lambda_{4} X \tag{3}
\end{equation*}
$$

where the constant term $\lambda_{0} \in[0,1]$ captures the gains associated to the attained school degree, $\lambda_{1} \in[0,1]$ is the labor market return to the educational standard $s-\lambda_{2}$ and $\lambda_{3}$ are the returns to log individual ability and log family income and $X$ is a vector of residual individual characteristics. The empirical earnings function (3) suggests that the labor market recognizes both the quantity and the quality of education - the former corresponding to the school degree and the latter to the educational standard. Individuals who fail to enrol in school lose $\lambda_{0}+\lambda_{1} s$ in terms of expected log earnings but save the effort costs of attaining the standard $s$.

Conditional on enrolment, households choose either the private or the public school, depending on the tuition fee, individual abilities and educational standards. There are two regimes $-s_{P}>s_{G}$ and $s_{G}>s_{P}$. Since household preferences can vary with the relative ranking of educational standards, we must consider each regime in turn.

### 2.1.1 Regime 1: $s_{P}>s_{G}$

The choice of the private school occurs when $U_{P i} \geq U_{G i}$ - where the subscripts $P$ and $G$ refer to the private and public school respectively - and $U_{P i} \geq$ $U_{R i}$. Using (2) and (3) these conditions can be written as

$$
\begin{equation*}
\theta \leq \lambda_{1}-\frac{p}{s_{P}-s_{G}} ; \quad \theta \leq \lambda_{1}+\frac{\lambda_{0}-p}{s_{P}} \tag{4}
\end{equation*}
$$

Since $s_{P}>s_{G}$, the second condition is redundant, because all the households which prefer the private to the public school also prefer the public school to dropping out. Hence, the demand for private education is

$$
\begin{equation*}
D_{H}=\Phi\left(\frac{\lambda_{1}-\frac{p}{s_{P}-s_{G}}}{\sigma}\right) \tag{5}
\end{equation*}
$$

where $\Phi$ is the standard normal distribution. More able students - with lower $\theta$ - have lower costs of effort and enrol in the private school, where the educational standard is higher.

A second order Taylor expansion of (5) around $\Phi(0)$ yields

$$
\begin{equation*}
\Phi\left(\frac{\lambda_{1}-\frac{p}{s_{P}-s_{G}}}{\sigma}\right)=\Phi(0)+\frac{\phi(0)}{\sigma}\left(\lambda_{1}-\frac{p}{s_{P}-s_{G}}\right) \tag{6}
\end{equation*}
$$

where $\phi$ is the normal density. Since the normal distribution is symmetric around the mean, $\phi^{\prime}(0)=0$ and the second order term vanishes. Moreover, $\Phi(0)=\frac{1}{2}$.

### 2.1.2 Regime 2: $s_{P}<s_{G}$

If $s_{P}<s_{G}$, the private school is selected if

$$
\begin{equation*}
\theta \geq \lambda_{1}+\frac{p}{s_{G}-s_{P}} ; \quad \theta \leq \lambda_{1}+\frac{\lambda_{0}-p}{s_{P}} \tag{7}
\end{equation*}
$$

and the demand for private education is

$$
\begin{equation*}
D_{L}=\Phi\left(\frac{\lambda_{1}+\frac{\lambda_{0}-p}{s_{P}}}{\sigma}\right)-\Phi\left(\frac{\lambda_{1}+\frac{p}{s_{G}-s_{P}}}{\sigma}\right) \tag{8}
\end{equation*}
$$

Using a second order Taylor expansion, we can re-write $D_{L}$ as

$$
\begin{equation*}
D_{L}=\frac{\phi(0)}{\sigma}\left[\frac{\lambda_{0}-p}{s_{P}}-\frac{p}{s_{G}-s_{P}}\right] \tag{9}
\end{equation*}
$$

This demand is non-negative if

$$
\begin{equation*}
p<\lambda_{0} \frac{s_{G}-s_{P}}{s_{G}} \tag{10}
\end{equation*}
$$

Therefore, the higher the (percentage) difference in the educational standard between the public and the private school the higher the tuition fee that the private school can set and still attract a positive demand for its services.

Conditions (4) and (7) show that the private school has no incentive to set the standard at the same level of the public standard, because with a positive price it would attract no student.

### 2.2 The private school

Since the distribution of pupils between schools depends on whether the private school selects a standard higher or lower than $s_{G}$, we need to distinguish two separate cases, $s_{P}>s_{G}$ and $s_{P}<s_{G}$. Consider first the case $s_{P}>s_{G}$ and let the profits of the private school be $\pi_{H}=p D_{H}-k s_{P}$. For a given $s_{G}$, profit maximization with respect to $p$ yields

$$
\begin{equation*}
p=\left(s_{P}-s_{G}\right)\left[\frac{\sigma}{4 \phi(0)}+\frac{\lambda_{1}}{2}\right] \tag{11}
\end{equation*}
$$

which implies that the demand for private school services (5) is positive for any educational standard $s_{P}>s_{G}$. The optimal price increases in the standard $s_{P}$ and decreases in the standard set by the public school. Using (11) in the first order condition for the optimal standard yields

$$
\frac{\partial \pi_{H}}{\partial s_{P}}=\frac{\phi(0)}{\sigma}\left(\frac{\sigma}{4 \phi(0)}+\frac{\lambda_{1}}{2}\right)^{2}-k
$$

Since the right hand side of this expression is independent of $s_{P}$, the optimal standard is a corner solution, and the regime $s_{P}>s_{G}$ requires $k<$ $\frac{\phi(0)}{\sigma}\left(\frac{\sigma}{4 \phi(0)}+\frac{\lambda_{1}}{2}\right)^{2}$. When this condition is verified, the optimal educational standard is $s_{P}=2$. Upon substitution, optimal profits are ${ }^{13}$

$$
\begin{equation*}
\pi_{H}=\frac{\left(2-s_{G}\right)}{16 \phi(0) \sigma}\left(\sigma+2 \phi(0) \lambda_{1}\right)^{2}-2 k \tag{12}
\end{equation*}
$$

[^6]Next, consider the case $s_{P}<s_{G}$ and let the profits of the private school be $\pi_{L}=p D_{L}-k s_{P}$. Profit maximization yields

$$
\begin{equation*}
p=\frac{\lambda_{0}\left(s_{G}-s_{P}\right)}{2 s_{G}} \tag{13}
\end{equation*}
$$

and

$$
\frac{\partial \pi_{L}}{\partial s_{P}}=-p \frac{\phi(0)}{\sigma}\left(\frac{\lambda_{0}-p}{s_{P}^{2}}+\frac{p}{\left(s_{G}-s_{P}\right)^{2}}\right)-k<0
$$

since $\left(\frac{\lambda_{0}-p}{s_{P}^{2}}+\frac{p}{\left(s_{G}-s_{P}\right)^{2}}\right)>0$. Therefore the optimal standard is $s_{P}=1$. The optimal profits of the private school in this regime are

$$
\begin{equation*}
\pi_{L}=\frac{\lambda_{0}^{2} \phi(0)\left(s_{G}-1\right)}{4 \sigma s_{G}}-k \tag{14}
\end{equation*}
$$

### 2.2.1 The choice of the standard above or below $s_{G}$

We now determine how the private school chooses between regime $s_{P}<s_{G}$ and regime $s_{P}>s_{G}$. Noticing that, at the optimal pricing policy, the profit functions (12) and (14) depend on the standard set by the government for public schools, $s_{G}$, we establish the following proposition:

Proposition 1 Assume that $k \leqslant K$. Then there exists a unique value $s_{G}^{*}$ such that the private school optimally chooses $s_{P}=1$ for any $s_{G}>s_{G}^{*}$ and $s_{P}=2$ for any $s_{G}<s_{G}^{*}$. In either case the school makes positive profits ${ }^{14}$.

## Proof. See Appendix

The proposition characterizes the private school's best reply function and shows that, if the private school enters in the education market, it chooses either a higher or a lower educational standard than the public school. If it chooses a higher standard, it sets it to the maximum feasible value. If it chooses

[^7]a lower standard, it sets it to the minimum feasible value ${ }^{15}$. In spite of the fact that households always pay positive tuition fees for private education, only the former case corresponds to the hierarchical model of Epple-Romano, 1998, where the public school is of lower quality than the private school.

The choice of the standard by the private school depends crucially on the standard selected by the public school. Independently of the selected regime, private tuition is a function of the difference between the public and the private standard. Suppose that the government sets a low standard for the public school. In this case, the private school can charge a high price by choosing a high educational standard. As the standard in the public school increases, however, the relative convenience that the private school has of setting a high standard declines, and after a given threshold $-s_{G}^{*}$ - the private school finds it more profitable to switch to a low standard. By so doing, it can increases both tuition and profits ${ }^{16}$.

The type of equilibrium which prevails depends critically on government choice. The government chooses the educational standard of the public school by taking into account the subsequent entry by the private school. We turn to this decision in the next section of the paper.

## 3 The choice of the standard for the public school

We assume that the public sector budget, which consists only of educational expenditures and income taxes, is always balanced. This is equivalent to requiring

$$
\begin{equation*}
\int \tau Y d F(Y)=\tau \mu=k s_{G} \tag{15}
\end{equation*}
$$

where $F(Y)$ is the log-normal cumulative distribution of Y , which yields, upon integration

$$
\begin{equation*}
\tau=\frac{k s_{G}}{\mu} \tag{16}
\end{equation*}
$$

[^8]Therefore, a higher educational standard increases the proportional tax rate paid by all the households in this economy.

We posit that the choice of the public school standard $s_{G}$ is based on majority voting - as in Stiglitz, 1974, and Fernandez and Rogerson, 1995 - and describe the outcome of the voting as follows: first, we select the value of $s_{G}$ preferred by the majority of households in each of the two regimes - the private school with lower and higher standard than the public school. Second, we compare preferred outcomes across regimes and choose the one favored by the majority of voters. Last, we contrast the outcome of majority voting with the one produced by a social planner who maximizes a utilitarian social welfare function.

### 3.1 Regime 1: high quality public school $\left(s_{G} \geq s_{G}^{*}\right)$

In the regime $s_{G} \geq s_{G}^{*}$ the private school chooses $s_{P}=1$ and $p=\frac{\lambda_{0}\left(s_{G}-1\right)}{2 s_{G}}$. We ask how utilities $U_{P i}$ and $U_{G i}$ vary with the public standard $s_{G}$. It turns out that

$$
\frac{\partial U_{P i}}{\partial s_{G}}=-\frac{k}{\mu}-\frac{\lambda_{0}}{2 s_{G}^{2}}<0
$$

for students enrolled in the private school. Since this derivative is negative, households with students in private schools unambiguously prefer the lowest value of $s_{G}$ in the regime, $s_{G}^{*}$. On the other hand

$$
\frac{\partial U_{G i}}{\partial s_{G}}\left\{\begin{array}{lll}
>0 & \text { if } & \theta_{i}<\lambda_{1}-\frac{k}{\mu} \\
<0 & \text { if } & \theta_{i}>\lambda_{1}-\frac{k}{\mu}
\end{array}\right.
$$

for students enrolled in the public school. More able students, for whom the derivative is positive, profit from a higher public standard because of their relatively low cost of effort, and vote for $s_{G}=2$, and less able individuals, for whom the derivative is negative, gain from a lower standard, and vote for $s_{G}=s_{G}^{*}$. The marginal student with ability $\theta_{i}=\lambda_{1}-\frac{k}{\mu}$ is indifferent to the level of the standard, but we assume hereafter that she votes with the group having $\theta_{i}<\lambda_{1}-\frac{k}{\mu}$. We establish the following

Lemma 1 If the group of individuals with $\theta_{i} \leq \lambda_{1}-\frac{k}{\mu}$ is the majority, it chooses $s_{G}=2$. If the group with $\theta_{i}>\lambda_{1}-\frac{k}{\mu}$ is the majority, it chooses $s_{G}=s_{G}^{*}$.

Proof. See the Appendix

### 3.2. Regime 2: low quality public school $\left(s_{G} \leq s_{G}^{*}\right)$

In the regime $s_{G} \leq s_{G}^{*}$, the private school selects $s_{P}=2$ and $p=\left(\frac{\sigma}{4 \phi(0)}+\frac{\lambda_{1}}{2}\right)(2-$ $s_{G}$ ). Thus, we get

$$
\frac{\partial U_{P i}}{\partial s_{G}}=-\frac{k}{\mu}+\left(\frac{\sigma}{4 \phi(0)}+\frac{\lambda_{1}}{2}\right)
$$

for individuals enrolled in the private school, and

$$
\frac{\partial U_{G i}}{\partial s_{G}}\left\{\begin{array}{llll}
>0 & \text { if } & \theta_{i}<\lambda_{1}-\frac{k}{\mu} & \rightarrow \\
s_{G}=s_{G}^{*} \\
<0 & \text { if } & \theta_{i}>\lambda_{1}-\frac{k}{\mu} & \rightarrow \quad \\
s_{G}=1
\end{array}\right.
$$

for individuals in the public school. Again, the marginal individual is indifferent to the standard.

Lemma 2 If $k \leqslant K$, then $\frac{k}{\mu}<\left(\frac{\sigma}{4 \phi(0)}+\frac{\lambda_{1}}{2}\right)$.
Proof. See the Appendix
This lemma implies that $\frac{\partial U_{P i}}{\partial s_{G}}$ is always positive if Proposition 1 holds, as we assume. Then, $s_{G}=s_{G}^{*}$, because the cost of setting up a higher standard in the public school - relative to average income - is lower than the expected return. We have

Lemma 3 If the group of individuals with $\theta_{i} \leq \lambda_{1}-\frac{k}{\mu}$ is the majority, it chooses $s_{G}=s_{G}^{*}$. If the group with $\theta_{i}>\lambda_{1}-\frac{k}{\mu}$ is the majority, it chooses $s_{G}=1$.

Proof. See the Appendix

### 3.3 The choice between regimes

We use the results in the previous sub-sections to compare regimes and establish the following

Proposition 2 If the group with $\theta_{i} \leq \lambda_{1}-\frac{k}{\mu}$ is the majority, it chooses the regime $s_{G} \geq s_{G}^{*}$ and $s_{G}=2$. If the majority is with the group $\theta>\lambda_{1}-\frac{k}{\mu}$, it votes for $s_{G} \leq s_{G}^{*}$ and $s_{G}=1$.

Proof. See the Appendix

We conclude that the group with $\theta_{i} \leq \lambda_{1}-\frac{k}{\mu}$ is the majority when

$$
\begin{equation*}
\Phi\left(\frac{\lambda_{1}-\frac{k}{\mu}}{\sigma}\right)>\frac{1}{2} \tag{17}
\end{equation*}
$$

i.e. when the size of the group is larger than $50 \%$. This is equivalent to the following condition

$$
\lambda_{1}-\frac{k}{\mu}>0
$$

which is satisfied when the difference between the marginal benefit of the educational standard, $\lambda_{1}$, and the marginal cost, $k$, - relative to average household income - is higher than 0 .

Notice that the majority voting condition must be consistent with the condition $k \leqslant K$ required for Proposition 1 to hold. When $\lambda_{1}-\frac{k}{\mu}>0$ we need to check that $k<\min \left[\mu \lambda_{1}, K\right]$. Similarly, when $\lambda_{1}-\frac{k}{\mu}<0$ we must verify that $\mu \lambda_{1}<k \leqslant K$, otherwise the voted equilibrium where the public school offers a lower standard than the private school cannot exist.

In words, Proposition 2 tells us that, when the benefit of increasing the standard is relatively high, not only the very able but also the households with daughters of intermediate ability can profit enough from the higher standard to compensate the cost of attaining it. Therefore the outcome of the vote is $s_{G}=2$. If the benefit declines, however, fewer voters will find it sufficient to compensate the effort required by a high standard, and eventually the majority will shift to $s_{G}=1$.

### 3.4 The social planner

In the "political economy" approach, households vote on the quality of education provision - measured by the educational standard - and take into account that a higher public quality needs to be financed with higher income taxes. In this sub-section we briefly characterize the market for education when the government acts as a social planner, and contrast the results with the findings obtained using the "political equilibrium" approach. We assume that the welfare function used by the government is utilitarian and consists of the simple
aggregation of the utilities of all the households in the economy. Moreover, we limit our attention to the case when the private school exists in either regime. It turns out that the social welfare function is always convex in $s_{G}{ }^{17}$. Therefore, the social optimum is a corner solution in both regimes. In the next section, we apply the model to Italy and the US and compare the outcomes of majority voting and of social welfare maximization in these two countries.

## 4 An Application to Italy and the US

We apply the model to Italy and the US. The earnings function (3) postulates that individual earnings increase both in the quantity of education and in the level of the educational standard. Following Card, 1999, and Brunello, 2002, the monetary return to a year of secondary education is estimated to be equal to $11 \%$ for the US and to $8.8 \%$ for Italy.

While there is substantial evidence on the labor market effects of years of education, much less is known on the effects of a higher educational standard. In the only empirical study for the US we are aware of, Betts and Grogger, 2000, estimate the effects of a higher grading standard on the earnings of young workers using the High School and Beyond survey. According to their definition, a school's grading standard is a measure of how stringently it grades its students. They find that a one percent increase in the grading standard increases earnings by $0.0147^{18}$, a small effect. In order to obtain from this an estimate of $\lambda_{1}$, we assume that the estimated elasticity from Betts and Grogger is equal to the elasticity associated to the standard $s$, which varies between 1 and 2 , and obtain $\frac{\partial w}{\partial s}=\frac{0.0147}{s}$, which we evaluate at average $s=1.5$. We get $\lambda_{1}=0.0098$.

Compared to the US, Italian high schools are organized into an academic (licei classici and licei scientifici) and a vocational track. The latter track can be further divided into vocational schools (istituti professionali) and technical schools (istituti tecnici), with the former having lower educational standards

[^9]than the latter. While vocational schools can last from 3 to 5 years, technical schools and the schools in the academic track usually last 5 years.

The following evidence supports our view that the academic track in Italy has higher educational standards than the vocational track. First, students enrolled in the academic track exert higher effort. Based on the data collected by the Programme for International Student Assessment survey (OECD, 2004), the average number of hours per week spent doing homework is equal to 13 in the academic track and to 7 in the vocational track. Second, the former track attracts the best performing students from lower secondary education. According to the Italian Survey on the School and Work Experience of 1998 High School Graduates (IHSG), close to 33 percent of the students enrolled in the academic track completed with high marks their lower secondary education, compared to only 13 percent in the vocational track ${ }^{19}$. Third, the average standardized test score in maths, reading and problem solving of students aged 15 - who have just started their academic or technical track - is significantly higher in the former (average score: 542) than in the latter (average score: $477)^{20}$. Last but perhaps most important, when we use IHSG data and regress individual graduation marks at the end of high school on age, gender, parental background dummies, the marks attained in junior high school and a dummy equal to 1 if the student has graduated from the academic track and to 0 if she has graduated from the vocational track, we find that the latter dummy attracts a negative and statistically significant coefficient, pointing to a higher standard in the academic track ${ }^{21}$.

We use data from the National Survey on the Income and Wealth of Italian Households (SHIW), carried out by the Bank of Italy on a bi-annual basis, which include information on earnings and school curriculum for a nationally

[^10]representative sample of Italians, and restrict our attention to the sub-sample of individuals aged between 20 and 30, who have attained at most a 5 -years high school diploma. The age restriction is motivated by comparability, as Betts and Grogger focus on entry-level wages.

We generate a discrete indicator of educational standards, equal to 1 if the individual graduated from a vocational school, to 2 is she graduated from a technical school and to 3 for graduation from the academic track, and specify an empirical earnings function which is as close as possible to the one used by Betts and Grogger for the US. After pooling the data for the period 1995 to 2004, we estimate that the elasticity of earnings to the selected measure of educational standards is equal to $0.072^{22}$.

We also use an alternative dataset, the Italian survey on the School and Work Experience of 1998 High School Graduates, carried out by the national statistical office in 2001, three years after graduation. Compared to the SHIW data, these data have the advantage of including also the high school graduates who are attending college, as in Betts and Grogger, and the disadvantage that they do not distinguish between 3-years and 5-years vocational schools. We restrict our attention to individuals aged 24, close enough to the age group considered by Betts and Grogger (25 to 27). It turns out the estimated elasticity of earnings to the educational standard is equal to 0.055 , smaller that the value obtained from the SHIW data but still substantially higher than the elasticity found in the US data ${ }^{23}$.

We prudentially choose the smaller estimate and assume, as for the US, that the estimated elasticity is equal to the elasticity associated to the standard $s_{i}$. This implies that for Italy $\lambda_{1}=0.0371$, about four times as large as the US estimates. The substantially lower return to the educational standard experienced

[^11]by the US is in line with Bishop's analysis of US high schools. Bishop, 1995, points out that not only student effort is poorly rewarded at school, but also is poorly signalled to the external labor market, because of the limited use of external statewide achievement examinations, which are more common in Europe and available in Italy. As a consequence, the US labor market "..fails to reward effort and achievement in high school" (p.18).

Next, we estimate the variance of the distribution of log ability. We use as a measure of ability the scores obtained by more than 5 thousand high school American students and more than 10 thousand Italian students in the maths, reading, science and problem solving tests reported by the OECD 2003 PISA Study (see OECD, 2004). After taking a simple average of these scores for each individual, we compute the coefficient of variation associated to the empirical distribution, which is equal to 0.1847 for the US and to 0.1677 for Italy. Since ability in our model is lognormally distributed, we impose that its coefficient of variation be equal to the empirical measure. Therefore, we have $\frac{\sqrt{e^{\sigma^{2}\left(e^{\left.\sigma^{2}-1\right)}\right.}}}{e^{\frac{1}{2} \sigma^{2}}}=$ 0.1847 for the US, which yields $\sigma=0.183$, and $\frac{\sqrt{e^{\sigma^{2}}\left(e^{\left.\sigma^{2}-1\right)}\right.}}{e^{\frac{1}{2} \sigma^{2}}}=0.1689$ for Italy, which yields $\sigma=0.168$.

We notice that the number of public schools in the real world is much higher than 1. If $M$ is the number of public schools, the budget constraint should be written more realistically as $\frac{M k}{\mu}=\frac{\tau}{s_{G}}$, which corresponds to (16) when $\mu$ is opportunely redefined as $\frac{\mu}{M}$. The cost of setting the standard should include the educational expenditures for teachers, labs, libraries and other facilities, because more or better teachers and facilities are required to enforce a higher standard. These considerations suggest that a reasonable measure of $\tau$ is the share of public educational expenditure for primary and secondary education on GDP. According to the OECD, 2004b, this share in 2000 was equal to 3.5 percent for the US and to 3.2 percent for Italy. We provide an appropriate scale to these numbers by dividing them by the average standard $s_{G}=1.5$ and obtain estimates of $\frac{\tau}{s_{G}}$ equal to 0.023 for the US and to 0.021 for Italy.

Average income $\mu$ in 2000 converted in international dollars using PPP was
equal to 34360 dollars in the US and to 20170 dollars in Italy (source: The World Bank). The number of public schools in the US in the same year was equal to 93273 , which compares to 28133 for Italy ${ }^{24}$. Furthermore, we assume that the variance of log income is as estimated by Baudourian, McDonald and Turley, 2002, and equal to 0.961 for the US and to 0.67 for Italy. Finally, we set the correlation between ability and income $\rho=0.018$, using the estimates of the relationship between test scores and parental income contained in Blau, $1999^{25}$. By applying the formulas for the mean of a lognormal distribution, we obtain $\mu_{y}=9.964$ for the US and $\mu_{y}=9.577$ for Italy.

Replacing the calibrated parameters in condition (17), we obtain

$$
\Phi\left(\frac{0.0098-0.023}{0.183}\right)=0.47<\frac{1}{2}
$$

for the US and

$$
\Phi\left(\frac{0.0371-0.021}{0.168}\right)=0.54>\frac{1}{2}
$$

for Italy.
Moreover, we can check that $k=\frac{\tau}{s_{G}} \frac{\mu}{M}$ is equal to 0.0086 in the US and to 0.0153 in Italy. The former verifies $\frac{\mu}{M} \lambda_{1}<k<K$, as $0.0036<0.0086<0.0117$, while the latter verifies $k<\min \left[\frac{\mu}{M} \lambda_{1}, K\right]$, as $0.0153<\min [0.0266,0.0154] .{ }^{26}$ Therefore, each voting majority is consistent with the condition required for Proposition 1 to hold.

Since the returns to a higher standard are low relative to its costs, our calibration of the US public education system suggests that the majority of voters in this country should favor a low standard in public schools and a high standard in private schools, consistently with the ordering of schools by quality suggested by Epple and Romano, 1998. The opposite occurs in Italy, where the returns to a higher standard dominate the costs. In this country, the majority votes in favor of an equilibrium where public schools are of higher quality than private schools.

[^12]We conclude that these two countries represent two different equilibria of our model of educational standards. We hasten to stress, however, that the size of these majorities is likely to be sensitive to measurement errors in the calibration of the key parameters. Given the paucity of empirical evidence, especially for the US, we believe that additional empirical work in the area is needed before reaching more solid conclusions.

Based on our numerical solutions, students with lower ability in the US either drop out or enrol in the public school. Since they have a relatively high cost of effort, and the return to the standard is relatively low, they favor a low quality public school. The upper part of the ability distribution instead enrols in the private school, where the educational standard is much higher. Therefore, we reproduce the stratification by ability emphasized by Epple and Romano, 1998, in their stylized model of the American schooling system. Rather than assuming, as they do, that public schools act as residual repositories, we obtain the relatively low quality of public schools as the outcome of a voting equilibrium, while allowing private schools to be, in principle, of lower quality than public schools.

Is this equilibrium altered by the presence of liquidity constraints? If these constraints are effective, they must reduce enrolment in the private school in favor of the public school. Since the utility derived from the public school is increasing in the standard for individuals with high ability, these individuals should vote for a high public school standard - exactly as before, given that $\frac{\partial U_{P i}}{\partial s_{G}}>0$ in the regime $s_{G} \leq s_{G}^{*}$. With no change in the majority, the lowest public school standard still prevails.

In the Italian equilibrium, the students with lower ability enrol in private schools, and favor a low standard there. Since individuals with intermediate or higher ability enrol in public schools, there is natural pressure for a higher standard in these schools. Again, liquidity constrained individuals who could not enrol in private schools will end up in public schools. These individuals will vote for a low public school standard, independently of whether they go to a private or a public school. However, since the majority remains in favor of a
high public school standard, the voting equilibrium is unaltered.
Finally, we compare the values of the calibrated social welfare function across the two regimes. Consider first the US. The local maximum is when $s_{G}=2$ in the regime $s_{G} \geq s_{G}^{*}$ and when $s_{G}=1$ in the regime $s_{G} \leq s_{G}^{*}$. By comparing local maxima, we find that the former is higher. Therefore, $s_{G}=1$ is the educational standard which attains the global maximum. Next we look at Italy. By comparing local maxima, we find that $s_{G}=2$ is the educational standard which yields the highest welfare. We conclude that - given the assigned values to the parameters - the choice of the public school standard by majority voting is consistent in both countries with welfare maximization.

## 5 Conclusions

When school quality increases with the educational standard set by schools and attaining the standard requires costly effort, the market for education before college needs not be a hierarchy with private schools offering better quality than public schools. An alternative configuration, with public schools offering a higher educational standard than private schools, can also exist, in spite of the fact that tuition levied by private schools is strictly positive. In the model presented in this paper, private schools can offer a lower educational standard at a positive price because they attract students with a relatively high cost of effort, who would find the high standards of the public school excessively demanding. Clearly, costly effort is only one possible factor driving this result. Alternatives include the fact that private schools provide access to labor market networks, which allow to locate better jobs more easily because of the connections they afford, or that they are "snob" goods, which are consumed because of the reputation they offer (see Corneo and Jeanne, 1997), even if quality is lower than in the public school. In either case, the intuition remains the same: by offering services that are not strictly related to quality, private schools can charge a positive price, offer lower quality than public schools and still make positive profits.

When the educational standard of the public school is chosen by majority voting, we show that the choice between a configuration with high quality public schools and a configuration with high quality private schools depends on the marginal return to the educational standard relative to the marginal cost of setting up the standard. We calibrate the model by using micro-econometric evidence from the US and Italy and find that, based on the calibrated parameters, majority voting in the former country produces a system with high quality private schools and low quality public schools, as assumed by Epple and Romano, 1998. This system is also the one chosen by a social planner who maximizes household welfare using a utilitarian welfare function. In the latter country, another majority voting equilibrium prevails, with public schools2 setting higher educational standards that private schools. Therefore, Italy and the US can be seen as two different equilibria of a model of educational standards.

We believe that the model discussed in this paper has two important policy implications. First, high school reforms that improve educational standards and introduces curriculum based external exams, as suggested by Bishop, 1998, and Betts, 1998, may improve the returns to educational standards in the US. If such improvement is large enough, our model suggests that the system actually in place could shift away from an equilibrium with low quality public schools. Second, policies such as school vouchers requires that private schools are better from an educational viewpoint. If these schools turn out instead to be of lower educational quality than public schools, as in the Italian equilibrium, a key element of the policy justification for vouchers is likely to fall.

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## 6 Appendix

## Proof of Proposition 1

Let $\pi_{H}=\pi_{H}\left(s_{P}=2, s_{G}\right)$ and $\pi_{L}=\pi_{L}\left(s_{P}=1, s_{G}\right)$. Next note that $\frac{\partial \pi_{H}}{\partial s_{G}}<0$ and $\frac{\partial \pi_{L}}{\partial s_{G}}>0$. Denote $\Delta=\pi_{L}-\pi_{H}$ : clearly $\frac{\partial \Delta}{\partial s_{G}}>0$. Moreover let $s_{G}^{*}$ be defined in such a way that $\Delta=0$, and suppose that $s_{G}^{*} \in[1,2]$. Then for $s_{G}<s_{G}^{*}$ we have that $\Delta<0$ and the preferred alternative is $s_{P}=2$. On the other hand, for all $s_{G}>s_{G}^{*}$ we have $\Delta>0$, and the preferred alternative is $s_{P}=1$.

To guarantee entry by the private school, at least one profitable option $\left(s_{P}=\right.$ 1 or $s_{P}=2$ ) must be available. A sufficient condition for this to happen is that $\pi_{H}$ valued at $s_{G}^{*}$ is nonnegative, i.e. $\pi_{H}\left(s_{P}=2, s_{G}^{*}\right) \geqslant 0$ : if so, then $\pi_{H}>0$ for all $s_{G}<s_{G}^{*}$ and $\pi_{L}>0$ for $s_{G} \geq s_{G}^{*}$. More explicitly, replace $s_{G}^{*}$ in $\pi_{H}$ to get

$$
\begin{equation*}
\pi_{H}\left(s_{P}=2, s_{G}^{*}\right)=-\frac{3}{2} k+\frac{A}{2}+B-\frac{\sqrt{(k+A-2 B)^{2}+4 A B}}{2} \tag{18}
\end{equation*}
$$

where

$$
\begin{equation*}
A=\frac{\lambda_{0}^{2} \phi(0)}{4 \sigma}>0 \quad B=\frac{\left(\sigma+2 \phi(0) \lambda_{1}\right)^{2}}{16 \phi(0) \sigma}>0 \tag{19}
\end{equation*}
$$

and $\frac{\partial \pi_{H}\left(s_{P}=2, s_{G}^{*}\right)}{\partial k}=-\frac{3}{2}-\frac{1}{2} \frac{k+A-2 B}{\sqrt{(k+A-2 B)^{2}+4 A B}}<0$ as $\left|\frac{k+A-2 B}{\sqrt{(k+A-2 B)^{2}+4 A B}}\right|<1$.
Let $K$ be the value of $k$ such that $\pi_{H}\left(s_{P}=2, s_{G}^{*}\right)=0$, i.e.

$$
\begin{equation*}
K=\frac{A+B}{2}-\frac{\sqrt{A^{2}+B^{2}}}{2}>0 \tag{20}
\end{equation*}
$$

Then, $s_{G}^{*} \in[1,2]$ whenever $k \leqslant K$, for any $A$ and $B$. Since $\pi_{H}\left(s_{P}=2, s_{G}^{*}\right)>$ 0 as $k<K$, the private school enters the market and sets $s_{P}=1$ if $s_{G}>s_{G}^{*}$ and $s_{P}=2$ if $s_{G}<s_{G}^{*}$.

## Proof of Lemma 1

Since the reciprocal of individual $\log$ ability $\theta$ is distributed in the interval $[-\infty, \infty]$, we need to examine in some detail how households vote in regime $s_{G}>s_{G}^{*}$.

- individuals with $\theta_{i} \in\left[-\infty, \lambda_{1}+\frac{p}{s_{G}-s_{P}}=\lambda_{1}+\frac{\lambda_{0}}{2 s_{G}}\right]$ choose to enrol in the public school. Therefore, those with $\theta_{i}<\lambda_{1}-\frac{k}{\mu}$ vote for $s_{G}=2$, and those with $\lambda_{1}-\frac{k}{\mu}<\theta_{i}<\lambda_{1}+\frac{\lambda_{0}}{2 s_{G}}$ vote for $s_{G}=s_{G}^{*}$. The marginal individual with $\theta_{i}=\lambda_{1}-\frac{k}{\mu}$ is indifferent.
- Individuals with $\theta_{i} \in\left[\lambda_{1}+\frac{\lambda_{0}}{2 s_{G}}, \lambda_{1}+\frac{\lambda_{0}}{s_{G}}\right]$ prefer the private school and vote for $s_{G}=s_{G}^{*}$.
- Individuals with $\theta \in\left[\lambda_{1}+\frac{\lambda_{0}}{s_{G}}, \infty\right]$ do not participate. Since their utility $U_{R}$ is decreasing in $s_{G}$, they vote for $s_{G}=s_{G}^{*}$.


## Proof of Lemma 2

First note that $\left(\frac{\sigma}{4 \phi(0)}+\frac{\lambda_{1}}{2}\right)=\sqrt{B} \sqrt{\frac{\sigma}{\phi(0)}}$ where $B$ is defined as in the proof of Proposition 1. We need to prove that $K-\mu\left(\frac{\sigma}{4 \phi(0)}+\frac{\lambda_{1}}{2}\right) \leq 0$, but we can prove a weaker condition. Replace $\sqrt{A^{2}+B^{2}}$ in the definition of $K$ with $A+B-\sqrt{2 A B}$, a quantity strictly lower than $\sqrt{A^{2}+B^{2}}$. Then we can write

$$
\begin{equation*}
\sqrt{2 A B}-2 \mu \sqrt{B} \sqrt{\frac{\sigma}{\phi(0)}}<0 \tag{21}
\end{equation*}
$$

which simplifies to

$$
\begin{equation*}
\frac{\lambda_{0} \sqrt{\phi(0)}}{2 \sqrt{\sigma}}-\sqrt{2} \mu \sqrt{\frac{\sigma}{\phi(0)}}<0 \quad \rightarrow \quad \lambda_{0}<\frac{2 \sqrt{2}}{\phi(0)} \sigma \mu \sim 7 \sigma \mu \tag{22}
\end{equation*}
$$

The last inequality if verified for all $\lambda_{0} \in[0,1]$ given reasonable values of $\sigma$ and $\mu$ (see the application in Section 5).

## Proof of Lemma 3

In regime $s_{G}<s_{G}^{*}$, voting occurs as follows:

- individuals with $\theta_{i} \in\left[-\infty, \lambda_{1}-\left(\frac{\sigma}{4 \phi(0)}+\frac{\lambda_{1}}{2}\right)\right]$ choose the private school and vote $s_{G}=s_{G}^{*}$ given that $\frac{k}{\mu}<\left(\frac{\sigma}{4 \phi(0)}+\frac{\lambda_{1}}{2}\right)$.


Figure 1: School choice and voting

- Individuals with $\theta_{i} \in\left[\lambda_{1}-\left(\frac{\sigma}{4 \phi(0)}+\frac{\lambda_{1}}{2}\right), \lambda_{1}+\frac{\lambda_{0}}{s_{G}}\right]$ choose the public school. Those with $\theta_{i}<\lambda_{1}-\frac{k}{\mu}$ vote $s_{G}=s_{G}^{*}$, and those with $\theta_{i}>\lambda_{1}-\frac{k}{\mu}$ vote $s_{G}=1$. Notice that given $\frac{k}{\mu}<\left(\frac{\sigma}{4 \phi(0)}+\frac{\lambda_{1}}{2}\right)$ holds, $\lambda_{1}-\frac{k}{\mu}$ lies within the range $\left[\lambda_{1}-\left(\frac{\sigma}{4 \phi(0)}+\frac{\lambda_{1}}{2}\right), \lambda_{1}+\frac{\lambda_{0}}{s_{G}}\right]$
- Finally, individuals with $\theta_{i} \in\left[\lambda_{1}+\frac{\lambda_{0}}{s_{G}}, \infty\right]$ do non participate to upper secondary school and vote for the minimum value of $s_{G}$, i.e. $s_{G}=1$.

Therefore, all individuals with $\theta_{i}<\lambda_{1}-\frac{k}{\mu}$ vote for $s_{G}=s_{G}^{*}$ while all the others vote for $s_{G}=1$

## Proof of Proposition 2

Let the group of students with $\theta_{i} \leq \lambda_{1}-\frac{k}{\mu}$ in Figure 1 be the majority. Then, the voting outcome is $s_{G}=2$ if $s_{G} \geq s_{G}^{*}$ and $s_{G}=s_{G}^{*}$ if $s_{G} \leq s_{G}^{*}$. When
$s_{G} \geq s_{G}^{*}$, the students belonging to the group go to the public school; when $s_{G} \leq s_{G}^{*}$, they go to the private school if $\theta_{i}<\lambda_{1}-\left(\frac{\sigma}{4 \phi(0)}+\frac{\lambda_{1}}{2}\right)$ and to the public school if $\lambda_{1}-\left(\frac{\sigma}{4 \phi(0)}+\frac{\lambda_{1}}{2}\right)<\theta_{i}<\lambda_{1}-\frac{k}{\mu}$. Consider the group going to the public school in either regime and compare their utilities. The regime $s_{G} \geq s_{G}^{*}$ is preferred because

$$
U_{G}\left(s_{G}=2\right)-U_{G}\left(s_{G}=s_{G}^{*}\right)=2\left(\lambda_{1}-\theta_{i}-\frac{k}{\mu}\right)-s_{G}^{*}\left(\lambda_{1}-\theta_{i}-\frac{k}{\mu}\right)>0
$$

Next take the group going to the public school in regime $s_{G} \geq s_{G}^{*}$ and to the private school in the other regime. The former regime is preferred because

$$
U_{G}\left(s_{G}=2\right)-U_{P}\left(s_{G}=s_{G}^{*}\right)=\left(2-s_{G}^{*}\right)\left[-\frac{k}{\mu}+\left(\frac{\sigma}{4 \phi(0)}+\frac{\lambda_{1}}{2}\right)\right]>0
$$

Therefore, when the group with $\theta_{i} \leq \lambda_{1}-\frac{k}{\mu}$ is the majority, the regime $s_{G} \geq s_{G}^{*}$ and the public school standard $s_{G}=2$ are selected.

Turning to the voting behavior of the group with $\theta_{i}>\lambda_{1}-\frac{k}{\mu}$, suppose it is the majority. Then the voting outcome is $s_{G}=s_{G}^{*}$ in the regime $s_{G} \geq s_{G}^{*}$ and $s_{G}=1$ in the regime $s_{G} \leq s_{G}^{*}$. Under the former regime $\left(s_{G} \geq s_{G}^{*}\right)$, students go to the public school if $\lambda_{1}-\frac{k}{\mu}<\theta_{i}<\lambda_{1}+\frac{\lambda_{0}}{2 s_{G}^{*}}$ and to the private school if $\lambda_{1}+\frac{\lambda_{0}}{2 s_{G}^{*}}<\theta_{i}<\lambda_{1}+\frac{\lambda_{0}\left(s_{G}^{*}+1\right)}{s_{G}^{*}}$. Finally, students with $\theta_{i}>\lambda_{1}+\frac{\lambda_{0}\left(s_{G}^{*}+1\right)}{s_{G}^{*}}$ do not enrol in either school. Under the latter regime $\left(s_{G} \leq s_{G}^{*}\right)$, students endowed with $\lambda_{1}-\frac{k}{\mu}<\theta_{i}<\lambda_{1}+\lambda_{0}$ go to the public school and students with $\theta_{i}>\lambda_{1}+\lambda_{0}$ do not enrol in any secondary school. Notice that the definition of thresholds already incorporates the majority voting outcome in the regime.

Consider first the students going to the public school in both regimes, i.e. those with $\lambda_{1}-\frac{k}{\mu}<\theta_{i}<\lambda_{1}+\frac{\lambda_{0}}{2 s_{G}^{*}}$. The regime $s_{G} \leq s_{G}^{*}$ (and the choice $s_{G}=1$ ) is preferred because

$$
U_{G}\left(s_{G}=s_{G}^{*}\right)-U_{G}\left(s_{G}=1\right)=\left(s_{G}^{*}-1\right)\left(-\frac{k}{\mu}+\lambda_{1}-\theta_{i}\right)<0
$$

Take now those with $\lambda_{1}+\frac{\lambda_{0}}{2 s_{G}^{*}}<\theta_{i}<\lambda_{1}+\frac{\lambda_{0}\left(s_{G}^{*}+1\right)}{2 s_{G}^{*}}$ who go to the private school in the first regime and to the public the second. By comparing their payoffs, we obtain

$$
U_{P}\left(s_{G}=s_{G}^{*}\right)-U_{G}\left(s_{G}=1\right)=-\frac{k}{\mu}-\frac{\lambda_{0}}{2 s_{G}^{*}}<0
$$

which implies that their preferred choice is $s_{G}=1$.
Next, consider those with $\lambda_{1}+\frac{\lambda_{0}\left(s_{G}^{*}+1\right)}{2 s_{G}^{*}}<\theta<\lambda_{1}+\lambda_{0}$. While in the first regime they do not enrol in any school, in the second regime they choose the public school. Comparing voting outcomes we obtain

$$
U_{N}\left(s_{G}=s_{G}^{*}\right)-U_{G}\left(s_{G}=1\right)=-\frac{k}{\mu}\left(s_{G}^{*}-1\right)-\left(\lambda_{0}+\lambda_{1}-\theta\right)<0
$$

and the preferred outcome is $s_{G}=1$.
Finally, students endowed with $\theta>\lambda_{1}+\lambda_{0}$ do not enrol in secondary school and only care about reducing their tax burden, which is attained by voting on the lowest available standard $s_{G}=1$.Therefore, when the group with $\theta_{i}>\lambda_{1}-\frac{k}{\mu}$ is the majority, its preferred alternative is unanimously $s_{G}=1$.


[^0]:    *We are grateful to the Editor in charge and three anonymous referees, to Sebastiano Bavetta, Francesca Gambarotto, Maria De Paola, David Figlio, Hideshi Itoh, Luciano Greco, Eric Hanushek, Antonio Nicolò, Anna Sanz de Galdeano and to the audiences in Padova, Siena, Udine, Uppsala and Tokyo (Hitotsubashi) for comments and suggestions. This paper was completed while the first author was visiting CIRJE at Tokyo University, which provided excellent hospitality. The usual disclaimer applies.
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[^1]:    ${ }^{1} \mathrm{He}$ also quotes evidence by Feinstein and Symons, 1999, who find that attendance of private schools does not affect on average individual performance in the UK. Hanushek, 2002, argues that "..it seems natural to believe that Catholic schools also exhibit wide variation in performance, although none of the existing analyses document either the magnitude or the potential causes of such differences." (p.74)

[^2]:    ${ }^{2}$ Cohen Zada and Justman, 2001, study the role of religion in private education.
    ${ }^{3}$ See for instance Betts, 1997, 1998, Brunello and Ishikawa, 1999, Figlio and Lucas, 2004, and De Fraja and Landeras, 2006. Betts argues that ..."..the education production function, which has dominated the school quality literature for the past 25 years, treats students and school resources as symmetric inputs in a neoclassical production function. It thus neglects the fact that the most important 'inputs' - students - are better characterized as economic agents with their own objectives.." (1997, p.2)
    ${ }^{4}$ John Bishop, 1995, 1997, attributes the relatively poor performance of American high school students in international tests to the fact that US high schools tend on average not to reward effort and learning as much as Japanese and European schools do. Moreover,

[^3]:    ${ }^{5}$ See Minter Hoxby, 1996, for a similar point in the context of teachers' unions.
    ${ }^{6}$ We intend to show that schools can end up with different standards even in the presence of a common technology. Needless to say, this differentiation is further emphasized if we introduce heterogeneous unit costs.
    ${ }^{7}$ Positive marginal costs complicate the algebra without providing further insights.

[^4]:    ${ }^{8}$ See also Cameron and Taber, 2004.

[^5]:    ${ }^{9}$ The dropout rate of young Americans - aged 16 to 24 - from high school was $10.9 \%$ in 2000.
    ${ }^{10}$ Secondary school completion is a ticket for college education. While we do not directly consider college education, we do so indirectly, because graduation from high quality primary and secondary schools is expected to increase enrolment in a high quality college.
    ${ }^{11}$ Linear costs of effort when $s \in[1,2]$ generate corner solutions for the educational standard. Since we are mainly interested in the relative standard of the private and public school, this is a convenient simplification.
    ${ }^{12}$ In De Fraja the household utility function is concave in the mother's consumption and linear in the daughter's income.

[^6]:    ${ }^{13}$ A detailed description of the relationship between profits and the educational standard of the public school can be found in the discussion paper version of this paper. See Brunello and Rocco, 2005.

[^7]:    ${ }^{14}$ In Brunello and Rocco, 2005, we relax the condition $k \leqslant K$ and show that the private school can refrain from entry for some values of $s_{G}$.

[^8]:    ${ }^{15}$ The choice of the extreme values is dictated by the assumption that the costs of attaining the standard in the individual utility function are linear in the standard.
    ${ }^{16}$ At the optimal price and private standard, the demand for private schools is a constant, and so is the total cost of setting the standard. Therefore, profits vary only with private tuition $p$.

[^9]:    ${ }^{17}$ Further details are available in Brunello and Rocco, 2005.
    ${ }^{18}$ This elasticity is computed by multiplying the estimated coefficient reported in the first column of their Table $4(0.0053)$ by the sample mean value of the grading standard, equal to 2.78.

[^10]:    ${ }^{19}$ This difference is broadly confirmed by Gasperoni, 1996, who uses a different survey of high school students.
    ${ }^{20}$ Source: Programme for International Student Assessment (PISA), OECD (2003)
    ${ }^{21}$ We estimate that - conditional on the controls listed in the text - the average graduation mark in the academic track (which include licei classici and licei scientifici) is 4.85 percent lower than in the vocational track (which includes istituti tecnici e professionali). Details are availailable from the authors upon request.

[^11]:    ${ }^{22}$ The estimated coefficient associated to the indicator of educational standards is 0.037 (standard error: 0.018).The elasticity reported in the text is obtained by multiplying this coefficient by 1.93 , the sample average value of the standard in this dataset. The detailed estimates are available from the authors upon request.
    ${ }^{23}$ The estimated coefficient associated to the indicator of educational standards is 0.038 (standard error: 0.012).The elasticity reported in the text is obtained by multiplying this coefficient by 1.44 , the sample average value of the standard in this dataset. The detailed estimates are available from the authors upon request.

[^12]:    ${ }^{24}$ Sources: US data from the National Center of Educational Statistics. Italian data refer to 2003 and are from the Italian Ministry of Education
    ${ }^{25}$ Given the lack of similar estimates for Italy, we assume that $\rho$ is the same across the two countries.
    ${ }^{26} \mathrm{~A}$ parallel condition is a fortiori verified for Italy when $\lambda_{1}=0.0481$.

