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Increasing life expectancy and optimal retirement: does population aging necessarily undermine economic prosperity?

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Abstract

In this paper we analyze the effects of changes in longevity and the pace of technological progress on interest rates, savings behaviour and optimal retirement decisions. In so doing we embed the dynamic optimization problem of choosing a life-cycle consumption path and the retirement age into a general equilibrium setting. Thereby we assume that technology evolves exogenously and the production side of the economy can be described by means of a neoclassical production function. Our results show that (i) the aggregate capital to consumption ratio increases and interest rates decrease in response to increases in longevity; (ii) the response of the optimal retirement age to increases in longevity is ambiguous. However, for reasonable parameter values the optimal retirement age increases in longevity; (iii) the aggregate capital to consumption ratio decreases and interest rates increase in response to faster technological progress; (iv) the response of the optimal retirement age to faster technological progress is ambiguous. However, for reasonable parameter values the optimal retirement age increases in the pace of technological improvements.

JEL classification: I15, J11, J26, O11
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1 Introduction

Over the last decades, increases in life expectancy brought tremendous welfare gains by allowing people to live longer, spending more time as healthy retirees and even to increase lifetime consumption. According to Bloom et al. (2007b), life expectancy for the whole world rose from 30 to 65 over the past 150 years and there seems to be no tendency for it to level off. Furthermore, Fries (1980), Mathers et al. (2001) and Mor (2005) suggest a compression of morbidity in the sense that the number of years an individual can expect to be healthy rises even faster than average life expectancy. While the individual gains of these demographic developments are out of question, population aging and its potentially negative economic consequences have become a very important topic not only in academic research but also in the public debate. As examples for the latter, there have even been two special issues on the economic consequences of aging societies in the magazine "The Economist" over the last two years (The Economist, 2009, 2011), which emphasizes the importance of the topic. Altogether the most important concerns are: if people live longer — especially at older ages — they spend more time as retirees, which threatens the sustainability of social security systems (cf. Gruber and Wise, 1998; Gertler, 1999; Bloom et al., 2007a); when the cohort of the baby-boomers retires, the support ratio declines such that fewer workers have to *produce* the goods and services that are *consumed* by all the people living in an economy. This is often referred to as the "accounting effect" of population aging (see for example Gruescu, 2007; Bloom et al., 2008, 2010a); a workforce that grows older can exert negative economic influences if older workers are less productive than younger ones (see Skirbekk, 2008, for an overview); population aging directly affects long-run economic growth perspectives via its impact upon a societies' desire to invest in research and development (see Prettner and Prskawetz, 2010, for an overview).

In this paper we focus on a closely related aspect namely the private optimal responses to demographic change, which are often referred to as its "behavioral effects". For example, if individuals expect to live longer, they will change, among other things, their savings behavior and their retirement decisions. While the former and its associated repercussions on economic growth have already been analyzed intensively in the dynamic general equilibrium literature (see for example Reinhart, 1999; Futagami and Nakajima, 2001; Petrucci, 2002; Heijdra and Ligthart, 2006; Heijdra and Mierau, 2011, and references therein), there are only a few papers analyzing the latter (see for example Bloom et al., 2007a,b; Kuhn et al., 2010), and we are only aware of one contribution that builds *both* aspects into a small open economy framework (Heijdra and Romp, 2009). Since Heijdra and Romp (2009) feature a very rich demographic structure, they have to assume that the interest rate evolves exogenously. This is justified for a small open economy but in large or closed economies it is important to take endogenous interest rates into consideration. The reason is that the interest rate responds to changes in aggregate savings such that there are important feedback effects between the behavior of consumers and the reaction of firms. These feedback effects exert additional influences on the optimal retirement age which cannot be captured within a partial equilibrium setting or within a framework relying on exogenously evolving interest rates. By contrast, taking endogenous interest rates into account allows us to sketch out the general equilibrium interdependences between savings, interest rates, technological progress, optimal individual retirement decisions and longevity.

The model we use builds upon the life-cycle behavior of individuals described in Bloom et al. (2007a) and Bloom et al. (2007b) which we implement as the demand side into a neoclassical growth model with exogenous technological progress. In so doing we endogenize wages and interest rates allowing for additional channels through which changes in exogenous variables can exert an influence on the choice of the optimal retirement age. However, this complicates our modeling framework substantially such that we abstract from the very detailed demographic structure that is featured by Heijdra and Romp (2009).

We find that an increase in longevity raises aggregate savings as compared to aggregate consumption and thereby reduces the interest rate. The overall effect of higher life expectancy on the optimal retirement age is ambiguous. On the one hand there are the *direct* effects that first, higher life expectancy reduces disincentives to work and second, there are the prospects of increases in lifetime consumption associated with working longer. On the other hand there are two opposing *indirect* effects on the choice of an optimal retirement age which are due to the associated reduction in interest rates. First, individuals would have to work *longer* to compensate for the fall in capital income but second, the effects of compound interest on lifetime labor income are weakened such that individuals could be tempted to enjoy leisure *earlier*. If the latter indirect effect is very strong, then it could potentially offset the positive indirect and direct effects of longevity on the retirement age.

Furthermore, we find that faster technological progress leads to a decrease in aggregate capital as compared to aggregate consumption which increases the interest rate. In this case the overall effect on the optimal retirement age is again ambiguous. While faster technological progress and hence faster economic growth means higher lifetime income and thus increases the demand for leisure such that people would like to retire *earlier*, the increase in wages at older ages raises incentives to retire *later*. In addition, the described opposing effects of changes in interest rates work and the overall effect of an increase in the pace of technological progress on the optimal retirement age crucially depends on the relative strength of all the positive and negative effects.

Finally, we are able to characterize parameter restrictions under which increases in longevity and the pace of technological progress positively impact upon the optimal retirement age and we show that these parameter restrictions tend to be fulfilled in industrialized countries. This implies that people would like to retire later in response to increases in life expectancy as long as public pension schemes do not provide excessive incentives for early retirement. This result is in line with Bloom et al. (2007a) and Heijdra and Romp (2009).

The paper is organized as follows: In section 2 we present the theoretical model. First we solve the individual dynamic optimization problem of choosing a consumption-savings path and a retirement age. Then we derive laws of motion for aggregate capital and aggregate savings by integrating over all cohorts alive at a certain point in time. Section 3 contains our analyses with respect to the impacts of changing longevity and changing technological progress on interest rates, aggregate savings and the optimal retirement age. Finally, in section 4 we conclude an sketch out some possible interventions for policy-makers.

2 The model

To analyze the interrelations between longevity, aggregate savings, optimal retirement and economic growth we merge three strands of the literature. First we derive individually optimal consumption-savings and retirement decisions based upon Bloom et al. (2007a). We then aggregate over individuals to derive expressions for optimal economy-wide consumption expenditure growth and economy-wide capital accumulation by relying on the overlapping generations literature of Blanchard (1985). Finally, we close the model by considering a neoclassical production side of the economy in the vein of Solow (1956), Cass (1965) and Koopmans (1965), where final output is produced with capital and labor and we allow for exogenously evolving technological progress.

2.1 The individual consumption-savings-retirement decision

Following Bloom et al. (2007a), individuals born at time t_0 maximize their discounted stream of lifetime utility U

$$U = \int_{t_0}^{\infty} e^{-(\delta+\lambda)} \left[u(c) - \chi\nu(z,t)\right] dt, \tag{1}$$

where δ is the discount rate, λ represents the mortality rate, $u(c)$ refers to instantaneous utility gained by consuming the amount c of the consumption good (which we take as the numéraire), $-\nu(z,t)$ describes instantaneous disutility of labor given life expectancy z at time t and χ is an indicator function with value 1 when working and zero when retired. We supress time arguments whenever it is possible. Individuals choose the amount of optimal consumption over time and their retirement age, i.e., the date when they switch between $\chi = 1$ and $\chi = 0$. Note that the discount rate is augmented by the mortality rate because, as compared to an infinite horizon setting, people who face the risk of death are less likely to postpone consumption into the future. The wealth constraint of individuals reads

$$\dot{k} = \chi w + (\lambda + r)k - c, \tag{2}$$

where k denotes an individual's capital stock — which we assume to be the only savings vehicle and hence it represents an individual's wealth —

and w is wage income. Basically this specification tells us that individuals like to consume more over their life course because it increases utility. In order to be able doing so they have to earn a wage income, while working effort is associated with disutility. Consequently, individuals not only have to deal with the optimal intertemporal consumption-savings decision but also with the trade-off between consumption and leisure. Furthermore, we make use of the assumption introduced by Yaari (1965) that there exists a perfect and fair life insurance company at which individuals can insure themselves against the risk of dying with positive assets. This life insurance company pays individuals a higher than the market rate of return on their capital holdings and in exchange it gets all the wealth of an individual who dies. As a consequence, the life insurance company redistributes capital of individuals who died among those who survived.

In order to get analytical solutions, we assume that the utility function is logarithmic, i.e., $u(c) = \log(c)$ and that disutility of work increases exponentially at the mortality rate, i.e., at a certain instant t we have that $\nu(z,t) = de^{\lambda(t-t_0)}$ whereby d is a scaling constant measuring the unwillingness of individuals to work. The first assumption leads to the following individual consumption Euler equation

$$\dot{c} = (r - \delta)c$$

stating that — similar to the standard neoclassical growth model with infinite lifetime horizons (cf. Ramsey, 1928; Cass, 1965; Koopmans, 1965) — consumption expenditure growth is positive if and only if the interest rate exceeds the rate of pure time preference. This will be the case if the financial sector is willing to pay an interest rate that overcompensates individuals for sacrificing consumption today in order to get consumption in the future. As shown by Yaari (1965), in case of full and fair life insurance coverage, the higher discounting of individuals due to the risk of death is exactly offset by the higher interest rate paid by the life insurance company. Furthermore, if individuals should be willing to work at time t, the instantaneous marginal utility of doing so must not be less than the instantaneous marginal utility of leisure, i.e., the negative disutility of work, and we have

$$\chi = 1 \quad \Leftrightarrow \quad u'(c)w \geq \nu(z,t). \tag{3}$$

Intuitively this equation states that individuals will work as long as the additional utility of working longer in terms of the associated higher consumption is able to compensate them for their disutility of sacrificing leisure time.

Noting that lifetime consumption expenditures have to be equal to lifetime income and denoting the retirement date of an individual by T, the individual lifetime budget constraint can be written as

$$\int_{t_0}^{\infty} e^{-(\lambda+r)(t-t_0)} c(t_0,t)dt \quad = \quad \int_{t_0}^{T} e^{-(\lambda+r)(t-t_0)} w(t_0,t)dt. \tag{4}$$

Integrating and using $c(t_0,t) = c(t_0,t_0)e^{(r-\delta)(t-t_0)}$, which follows from the individual Euler equation, and $w(t_0,t) = w(t_0,t_0)e^{g(t-t_0)}$, which follows from denoting wage growth by g, we arrive at an expression for the fraction of consumption expenditures at birth to wages at birth depending on the date of retirement T

$$\frac{c(t_0,t_0)}{w(t_0,t_0)} \quad = \quad \frac{\lambda+\delta}{\lambda+r-g}\left[1-e^{-(\lambda+r-g)(T-t_0)}\right]. \tag{5}$$

Bloom et al. (2007a) mention that in their partial equilibrium setting $g < r+\lambda$ has to hold for the model to make sense. In a general equilibrium setting with overlapping generations, we will see that this condition is automatically fulfilled for all death rates along a balanced growth path. Next, we denote the optimal retirement date by T^* such that the optimal retirement age is given by $R^* = T^* - t_0$ which can be implicitly expressed as a function of the fraction of wages at birth to consumption expenditures at birth:

$$\frac{c(t_0,t_0)}{w(t_0,t_0)} \quad = \quad \frac{e^{(g+\delta-r-\lambda)R^*}}{d}. \tag{6}$$

Intuitively, this expression tells us that if individuals tend to consume more in relation to initial income, i.e., they save less, then they have to retire later. To put it differently, individuals can "buy" early retirement by saving more. Next we put equations (5) and (6) together which finally yields

$$(\lambda+\delta)d \quad = \quad (\lambda+r-g)e^{(g+\delta-r-\lambda)R^*} + d(\lambda+\delta)e^{(g-r-\lambda)R^*} \tag{7}$$

being an implicit relationship between the optimal retirement age and the mortality rate, the discount rate, the measure for the unwillingness to work, the pace of wage growth and the interest rate. In contrast to Bloom et al. (2007a) and Heijdra and Romp (2009), the interest rate will be endogenous in equilibrium.

2.2 Aggregation over cohorts

There is not only one single representative individual in our model economy. Instead, we have to integrate over all cohorts that are alive at a certain instant t to come up with expressions for aggregate consumption expenditure growth and aggregate capital accumulation. Denoting the aggregate capital stock by K and aggregate consumption expenditures by C leads us to the following aggregation rules (see for example Heijdra and van der Ploeg, 2002)

$$K(t) \equiv \int_{-\infty}^{t} k(t_0, t)N(t_0, t)dt_0, \tag{8}$$

$$C(t) \equiv \int_{-\infty}^{t} c(t_0, t)N(t_0, t)dt_0, \tag{9}$$

where $N(t_0, t)$ denotes the size of the cohort born at time t_0 at date t and $k(t_0, t)$ and $c(t_0, t)$ are their capital holdings and consumption levels, respectively. In order to simplify exposition and in line with Blanchard (1985), we assume that the birth rate equals the death rate such that the flow of newborns is $N(t, t) = \lambda N(t)$, where $N(t) = \int_{-\infty}^{t} N(t_0, t)dt_0 \equiv N$ represents the total population size. Note that each cohort is of size $\lambda N e^{\lambda(t_0 - t)}$ at a certain date $t > t_0$. Taking into account these demographic structures and carrying out the calculations in appendix B leads to the following law of motion for the aggregate capital stock and to the following aggregate Euler equation

$$\dot{K} = rK - C + \Xi W, \tag{10}$$

$$\frac{\dot{C}}{C} = r - \delta - \lambda(\delta + \lambda)\frac{K}{C}, \tag{11}$$

where W refers to aggregate wage income if the whole living population would work, while Ξ denotes the fraction of the population who are still supplying their skills on the labor market, i.e., who are not yet retired. We see that in contrast to the law of motion for individual capital, the mortality rate does not show up on the aggregate level. The reason is that the life insurance company only redistributes wealth of people who died among those who survived, while it does not create or destroy any capital. Furthermore, we see that aggregate consumption expenditure growth falls short of individual consumption expenditure growth because at each instant a fraction of older and therefore wealthier people die and they are replaced by poorer newborns who cannot afford that much consumption. This continually ongoing process slows down aggregate consumption expenditure growth as compared to individual consumption expenditure growth (cf. Heijdra and van der Ploeg, 2002).

One of the properties of a balanced growth path is that the growth rate of wages corresponds to the growth rates of aggregate consumption and aggregate capital, i.e., we have that $\dot{C}/C = \dot{K}/K = \dot{W}/W$. Then, as a consequence of equation (11) and as already mentioned in subsection 2.1, the condition $g < r + \lambda$ is always fulfilled because

$$g = r - \delta - \lambda(\delta + \lambda)\frac{K}{C} < r + \lambda$$

and the net present value of lifetime income is finite.

Finally, in order to come up with analytical solutions, we rewrite the economy-wide wealth constraint as $\dot{K} = Y - C$ with Y being gross domestic product (GDP). This equation states that everything that is produced is either spent on consumption or invested in the form of capital goods. Altogether this means that the following three equations fully describe the consumption side of our model economy

$$\dot{K} = Y - C, \qquad (12)$$

$$\frac{\dot{C}}{C} = r - \delta - \lambda(\delta + \lambda)\frac{K}{C}, \qquad (13)$$

$$(\lambda + \delta)d = (\lambda + r - g)e^{(g+\delta-r-\lambda)R^*} + d(\lambda + \delta)e^{(g-r-\lambda)R^*}, \qquad (14)$$

where the first equation is the economy-wide resource constraint, the second equation is the aggregate Euler equation and the third equation is the implicit relation of the optimal retirement age to the interest rate, the exogenously given preference parameters and the mortality rate. In the following, we will denote the relation between aggregate consumption expenditures and the aggregate capital stock C/K — also being a measure of aggregate savings — by ξ. Furthermore, we will close the model by assuming that the firm sector can be described by a neoclassical production function and therefore conforms to the workhorse neoclassical growth models of Solow (1956), Cass (1965) and Koopmans (1965). This allows us to analyze the interrelations between longevity and economic growth on the one hand, and the retirement age and aggregate savings on the other hand within a general equilibrium setting.[1]

2.3 The production side of the economy

In order to describe the firm sector of our model economy, we rely on the neoclassical growth literature (cf. Solow, 1956; Cass, 1965; Koopmans, 1965) and allow for exogenous technological progress. The aggregate production function can be written as

$$Y = K^{\alpha}(A\Xi N)^{1-\alpha}, \tag{15}$$

where A is the technological frontier of the economy growing at rate $0 < g = \dot{A}/A$ and $0 < \alpha < 1$ is the capital share in aggregate production. Note that the growth rate of technology is the same as those of wages, the reason being that wage growth along a balanced growth path is determined by labor augmenting technological improvements. Assuming perfect competition in factor markets, the interest rate can be written as

$$r = \frac{\partial Y}{\partial K} = \alpha K^{\alpha-1}(A\Xi N)^{1-\alpha} \tag{16}$$

and consequently we have that

$$\frac{Y}{K} = \frac{r}{\alpha}.$$

[1] We also considered two other ways of describing the production side of the economy. The first followed the learning-by-doing endogenous growth literature (see for example Arrow, 1962; Frankel, 1962; Romer, 1986) and the second followed the literature of endogenous technological progress (see for example Romer, 1990). However, the expressions and interrelations were too involved to come up with analytical results in these cases.

Putting all the things together, we can rewrite the system describing our model economy as

$$
\begin{aligned}
g &= \frac{r}{\alpha} - \xi, && (17)\\
g &= r - \delta - \lambda(\delta+\lambda)\frac{1}{\xi}, && (18)\\
(\lambda+\delta)d &= (\lambda + r - g)e^{(g+\delta-r-\lambda)R^*} + d(\lambda+\delta)e^{(g-r-\lambda)R^*}, && (19)
\end{aligned}
$$

where the endogenous variables are r, R^* and ξ. We are now interested in changes of these endogenous variables in response to changes in mortality and changes in the pace of technological progress.

3 Consequences of changing mortality and the pace of technological progress

The system defined by equations (17)-(19) can be solved explicitly for r and ξ. However, we have to resort to implicit comparative statics (cf. Gandolfo, 2010, pp. 325-338) in order to analyze the dependence of the optimal retirement age on the parameters we are interested in. First, we will consider the effects of changes in longevity on the interest rate, the capital to consumption ratio and the optimal retirement age and then we will proceed to the impact of the pace of technological progress on the same set of endogenous variables.

3.1 The effects of changing mortality

First we analyze the response of an economies' consumption to capital ratio and its interest rate to decreases in mortality. Solving the system defined by equations (17)-(19) for r and ξ yields[2]

$$
\begin{aligned}
r &= \frac{1}{2}\left(\alpha g + g + \delta + \sqrt{(-\alpha g + g + \delta)^2 + 4\alpha\lambda(\delta+\lambda)}\right), && (20)\\
\xi &= \frac{g + \delta - \alpha g + \sqrt{(-\alpha g + g + \delta)^2 + 4\alpha\lambda(\delta+\lambda)}}{2\alpha}. && (21)
\end{aligned}
$$

Now we are able to state the following proposition.

[2]We solved the system in Mathematica 6.0. This leads to two solution pairs, one of which can be ruled out because it involves negative values of ξ.

Proposition 1. *An increase in longevity raises aggregate savings as compared to aggregate consumption and lowers the interest rate.*

Proof. Taking the derivative of the two expressions for r and ξ with respect to mortality yields

$$\frac{\partial r}{\partial \lambda} = \frac{\alpha(\delta + 2\lambda)}{\sqrt{(-\alpha g + g + \delta)^2 + 4\alpha\lambda(\delta + \lambda)}}, \tag{22}$$

$$\frac{\partial \xi}{\partial \lambda} = \frac{\delta + 2\lambda}{\sqrt{(-\alpha g + g + \delta)^2 + 4\alpha\lambda(\delta + \lambda)}}. \tag{23}$$

These two expressions are unambiguously positive and since an increase in longevity is represented by a decrease in mortality λ, the above proposition holds. □

The intuition for this finding is that as longevity increases, individuals perform consumption smoothing and choose to save more when they are young in order to be able to sustain a certain consumption expenditure level during their prolonged period of retirement. This leads to a higher aggregate capital stock and because of decreasing returns to capital to a lower interest rate.

Next, we analyze the response of household's retirement decisions to decreases in mortality. This leads us to the following proposition.

Proposition 2. *An increase in longevity has an ambiguous effect on the optimal retirement age. If the retirement age is already high, increases in longevity raise the optimal retirement age further.*

Proof. The Jacobian of the system defined by equations (17)-(19) reads

$$\Delta = \begin{pmatrix} \frac{1}{\alpha} & -1 & 0 \\ 1 & \frac{\lambda(\delta+\lambda)}{\xi^2} & 0 \\ A_1 & 0 & A_2 \end{pmatrix}$$

with

$$\begin{aligned} A_1 &= e^{(g+\delta-r-\lambda)R^*} - R^*(\lambda + r - g)e^{(g+\delta-r-\lambda)R^*} - R^* d(\lambda+\delta)e^{(g-r-\lambda)R^*}, \\ A_2 &= (g + \delta - r - \lambda)(\lambda + r - g)e^{(g+\delta-r-\lambda)R^*} \\ &\quad +(g - r - \lambda)d(\lambda + \delta)e^{(g-r-\lambda)R^*}. \end{aligned}$$

The determinant of the Jacobian is

$$|\Delta| = A_2 \left[1 + \frac{\lambda(\delta+\lambda)}{\alpha\xi^2} \right].$$

Furthermore, since we are interested in the effects of changing mortality on the retirement age, we substitute the third column of the Jacobian by the partial derivatives of the system with respect to mortality such that

$$\Delta_{R^*,\lambda} = \begin{pmatrix} \frac{1}{\alpha} & -1 & 0 \\ 1 & \frac{\lambda(\delta+\lambda)}{\xi^2} & -\frac{2\lambda+\delta}{\xi} \\ A_1 & 0 & A_3 \end{pmatrix},$$

where

$$\begin{aligned} A_3 \;&=\; e^{(g+\delta-r-\lambda)R^*} - R^*(\lambda+r-g)e^{(g+\delta-r-\lambda)R^*} \\ &\quad + de^{(g-r-\lambda)R^*} - R^* d(\lambda+\delta)e^{(g-r-\lambda)R^*} - d. \end{aligned}$$

The determinant of this matrix is

$$|\Delta_{R^*,\lambda}| = A_3 \left[1 + \frac{\lambda(\delta+\lambda)}{\alpha\xi^2} \right] + \frac{2\lambda+\delta}{\xi} A_1.$$

Altogether we can then analyze the effects of changes in longevity on the optimal retirement age. We have the following result

$$\frac{\partial R^*}{\partial \lambda} \;=\; -\frac{|\Delta_{R^*,\lambda}|}{|\Delta|} = -\frac{A_3 \left[1 + \frac{\lambda(\delta+\lambda)}{\alpha\xi^2}\right] + \frac{2\lambda+\delta}{\xi} A_1}{A_2 \left[1 + \frac{\lambda(\delta+\lambda)}{\alpha\xi^2}\right]}.$$

Recall from the aggregate Euler equation that the relations $r > g$ and $g < g + \delta < r + \delta$ have to hold in equilibrium. Therefore we know that $A_2 < 0$ and $A_1 > A_3$. Consequently, $\partial R^*/\partial\lambda < 0$ will hold for sure if $A_1 < 0$. The last inequality in turn is always fulfilled if

$$R^*(\lambda + r - g) > 1. \tag{24}$$

In case of a retirement age of zero, this inequality is violated, while in case of a retirement age corresponding to life-expectancy $1/\lambda$, the inequality is fulfilled. Since it is more likely that this inequality holds if the optimal retirement age is high, the proposition holds. □

The intuition behind this result is the following. We have the *direct* effect of increases in longevity on an individual's optimal retirement decision apparent from equation (7). Dividing this equation by $(\lambda + \delta)$ and taking the derivative of the right hand side (RHS) with respect to mortality yields

$$\begin{aligned} \frac{\partial RHS}{\partial \lambda} &= (\lambda + r - g)\left[\frac{g+\delta-r}{(\lambda+\delta)^2}e^{(g+\delta-r-\lambda)R^*} - R^*\frac{\lambda+r-g}{\lambda+g}e^{(g+\delta-r-\lambda)R^*}\right] \\ & \quad -R^* e^{(g-r-\lambda)R^*} \end{aligned} \tag{25}$$

which is negative for sure because we know from the aggregate Euler equation that $g + \delta - r < 0$. This means that in order to fulfill equation (7) for given d, individuals would want to work longer in case that life expectancy increases. However, there is also an *indirect* effect due to the decrease in the interest rate induced by the higher aggregate capital stock (see proposition 1). This decrease has two opposing effects: on the one hand individuals would want to work longer in order to compensate for the implied loss of lifetime capital income. On the other hand, the compensation of working longer decreases because the effect of compound interest is weakened. If the retirement age is high (and the period of retirement to be financed out of savings is shorter), the former effect dominates. By contrast, if the retirement age is low (and the period of retirement to be financed out of savings is longer), the latter effect is more likely to prevail. For a very low retirement age there is the theoretical possibility that the negative indirect effect due to decreases in the interest rate even overcompensates for the positive direct and indirect effects.

Finally, we summarize our findings regarding a demographic transition from high fertility and mortality to low fertility and mortality in the next proposition.

Proposition 3. *A demographic transition from high fertility and mortality to low fertility and mortality is associated with an increase in wages*

Proof. We know that

$$\frac{Y}{K} = \frac{r}{\alpha}$$

and

$$Y = K^{\alpha}(AL)^{1-\alpha}$$

with $L = \Xi N$. It follows that

$$\frac{r}{\alpha} = \left(\frac{AL}{K}\right)^{1-\alpha} \tag{26}$$

such that the aggregate labor to capital ratio L/K and the interest rate move in line. Furthermore, we know that wages in the economy can be written as

$$w = \left(\frac{K}{L}\right)^{\alpha} A. \tag{27}$$

Since A and α are exogenously given, a decrease in L/K is asscociated with increasing wages. Consequently, due to proposition 1, interest rates decrease following a demographic transition, while the aggregate capital-labor ratio and wages rise. □

The intuitive explanation is that a demographic shift from high fertility and mortality to low fertility and mortality induces a decrease in labor supply because individuals adjust their retirement age less than proportionally. Furthermore, they have to save more in order to sustain consumption during the prolonged period of retirement. Both effects raise the aggregate capital to labor ratio, leading to higher wages. These findings are in line with the results of Lee and Mason (2010) and are complementary to the effects of the demographic devidend as described in Bloom et al. (2003) and Bloom et al. (2010b).

3.2 The effect of changing the pace of technological progress

Next, we analyze the response of the aggregate savings to consumption ratio and the interest rate to increases in the pace of exogenous technological progress governing the growth rate of the economy. In this case we state the following proposition.

Proposition 4. *An increase in economic growth raises aggregate consumption as compared to aggregate savings and raises the interest rate.*

Proof. Taking the derivative of the expressions for r and ξ with respect to economic growth yields

$$\frac{\partial r}{\partial g} = \frac{1}{2}\left(1+\alpha+\frac{(1-\alpha)(g+\delta-\alpha g)}{\sqrt{(g+\delta-\alpha g)^2+4\alpha\lambda(\delta+\lambda)}}\right), \tag{28}$$

$$\frac{\partial \xi}{\partial g} = \frac{1-\alpha}{2\alpha}+\frac{(1-\alpha)(g+\delta-\alpha g)}{2\alpha\sqrt{(g+\delta-\alpha g)^2+4\alpha\lambda(\delta+\lambda)}}. \tag{29}$$

These two expressions are unambiguously positive, therefore the proposition holds. □

The intuition is that with increasing economic growth, individuals can expect higher future earnings and therefore they do not need to save that much to sustain the same level of consumption. Due to this decrease in savings, the capital stock is lower and hence the marginal product of capital, i.e., the interest rate, is higher.

Finally, we analyze the response of household's retirement decisions to faster technological progress and hence economic growth. In this case we state the following proposition.

Proposition 5. *An increase in economic growth has an ambiguous effect on the optimal retirement age. If the retirement age is already high, increases in economic growth raise the optimal retirement age further.*

Proof. We are interested in the effects of changing economic growth on the optimal retirement age and therefore we substitute the third column of the Jacobian Δ by the partial derivatives of the system with respect to economic growth

$$\Delta_{R^*,g} = \begin{pmatrix} \frac{1}{\alpha} & -1 & -1 \\ 1 & \frac{\lambda(\delta+\lambda)}{\xi^2} & -1 \\ A_1 & 0 & A_4 \end{pmatrix},$$

where

$$A_4 = -e^{(g+\delta-r-\lambda)R^*} + R^*(\lambda+r-g)e^{(g+\delta-r-\lambda)R^*} + R^* d(\lambda+\delta)e^{(g-r-\lambda)R^*}.$$

The determinant of this matrix is

$$|\Delta_{R^*,g}| = A_4\left[1+\frac{\lambda(\delta+\lambda)}{\alpha\xi^2}\right] + A_1\left[1+\frac{\lambda(\delta+\lambda)}{\xi^2}\right].$$

Altogether we can then analyze the effects of changes in economic growth on the retirement age which leads to the following result

$$\frac{\partial R^*}{\partial g} = -\frac{\frac{\lambda(\delta+\lambda)}{\xi^2}\left(\frac{A_4}{\alpha}+A_1\right)}{A_2\left[1+\frac{\lambda(\delta+\lambda)}{\alpha\xi^2}\right]}. \tag{30}$$

Recall again from the aggregate Euler equation that the relations $r > g$ and $g < g + \delta < r + \delta$ have to hold in equilibrium. Therefore we know that $A_2 < 0$ and the whole expression will be positive if

$$\frac{A_4}{\alpha} + A_1 > 0. \tag{31}$$

Since $\alpha < 1$ this inequality is fulfilled for sure if

$$R^*(\lambda + r - g) > 1.$$

In case of a retirement age of zero this inequality is violated, while in case of a retirement age corresponding to life-expectancy $1/\lambda$, the inequality is fulfilled. Since it is more likely that this inequality holds if the optimal retirement age is high, the proposition holds. □

The intuition for this result is a little bit different from the one for proposition 2. Again we have the *direct* effect of increases in growth on an individual's optimal retirement decision apparent from equation (7). Dividing this equation by $(\lambda + \delta)$ and taking the derivative of the RHS with respect to economic growth leads to

$$\frac{\partial RHS}{\partial g} = -e^{(g+\delta-r-\lambda)R^*} + \frac{\lambda+r-g}{\lambda+\delta}R^*e^{(g+\delta-r-\lambda)R^*} + R^*d(\lambda+\delta)e^{(g-r-\lambda)R^*} \tag{32}$$

which has an ambiguous sign because there are now two opposing *direct* effects of increasing economic growth on the optimal retirement decision. On the one hand, faster growth leads to higher lifetime income which increases demand for consumption and leisure since both are normal goods. Higher demand for leisure implies that people retire *earlier*. On the other hand, as a consequence of faster economic growth, individuals will have a higher income at the age when they would have decided to retire in case of unchanged growth. This effect leads them to *postpone* the retirement age (see equation

(3)) and hence already the overall impact of the *direct* effect is ambiguous. In addition, however, there is also the *indirect* effect due to the increase in the interest rate induced by *lower*[3] aggregate savings (see proposition 4). This associated increase in the interest rate has again two opposing effects. On the one hand, individual's lifetime income increases due to the higher interest rate which again increases demand for leisure and hence *reduces* the optimal retirement age. On the other hand, working at older ages has a larger positive impact on lifetime income because it leads compound interest to exert its influence longer. If the retirement age is high (and the period of retirement to be financed out of savings is shorter), the former effect dominates. By contrast, if the retirement age is low (and the period of retirement to be financed out of savings is longer), the latter effect is more likely to prevail. Consequently, considering the *direct* and the *indirect* effect together, we have that for low levels of the retirement age, the optimal response to faster economic growth is to decrease the retirement age further, while the converse holds true for high levels of the retirement age.

3.3 Numerical assessment

In order to check the validity of the condition described in equation (24), we consider the G8 countries over the years 1990-2009 and obtain the average real interest rate, the average economic growth rate and the mortality rate implied by life expectancy at birth in the year 2009. Then we calculate the threshold retirement age $\bar{R}$ at which equation (24) is fulfilled with equality as

$$\bar{R} = \frac{1}{\lambda + r - g}.$$

The results are depicted in table 1 with the data being obtained from World Bank (2012). The average interest rates for Germany and Italy were calculated over the years 1990-2002 and 1990-2004, respectively.

We see that in all the countries listed, the implied threshold retirement age is much lower than the actual retirement age (see for example OECD, 2009, for an overview). This implies that for the G8 countries we can be sure that — according to our model — increases in life expectancy and increases in economic growth raise the individually optimal retirement age. In reality,

[3] Note that the indirect effect works exactly the other way round as compared to the results in proposition 2.

Table 1: Implied $\bar{R}$ for the G8 countries

	Canada	France	Germany	Italy
r	0.0423	0.0596	0.0878	0.0556
g	0.0227	0.0165	0.0158	0.0098
λ	0.0123	0.0123	0.0125	0.0123
$\bar{R}$	31.3480	18.0505	11.8343	17.2117
	Japan	Russia	UK	USA
r	0.0330	0.0667	0.0301	0.0455
g	0.0110	0.0280	0.0199	0.0252
λ	0.0121	0.0145	0.0125	0.0127
$\bar{R}$	29.3255	18.7970	44.0529	30.3030

however, there often exists a mandatory retirement age and/or financial and non-financial incentives to retire early — often even earlier than at the mandatory retirement age (Blondal and Scarpetta, 1997; Gruber and Wise, 1998). Therefore, from a policy perspective, it could prove useful to remove incentives for early retirement and to link the mandatory retirement age — at least to a certain extent — to life expectancy (see also Bloom et al., 2007a).

Another aspect worth mentioning is related to the discussion regarding the negative impact of demographic change on economic prosperity. It is often argued that an increase in life expectancy decreases the size of the labor force relative to the number of retirees which lowers overall living standards because *fewer* people have to share a *larger* burden. We can — up to a certain point — address this issue within our considerations. From the aggregate production function, equation (15), it follows that

$$y = k^{\alpha} A^{1-\alpha} \lambda R, \tag{33}$$

where y is per capita GDP, k is the effective capital labor ratio and we used that $\Xi = \lambda R$. Now we see that *ceteris paribus* the direct effect of decreasing mortality is that the fraction of people in retirement increases relative to the fraction of workers and hence per capita GDP decreases. This is the accounting effect of population aging (cf. Bloom et al., 2010a). However, there are also behavioral changes going on namely that the optimal retirement age and the aggregate capital to consumption ratio (which is an indicator for savings), increase. Consequently, the behavioral changes will tend to increase R and k in the above equation and therefore act as mitigating forces for the negative accounting effect of population aging.

4 Conclusions

We implemented a model of optimal individual retirement and optimal individual life-cycle consumption into a general equilibrium framework, where the interest rate and the aggregate consumption to capital ratio are endogenously determined. We have seen that the individual responses to changes in longevity and economic growth are to change the savings behavior *and* to change the desired retirement age. In contrast to Bloom et al. (2007a) and Heijdra and Romp (2009) there are feedback effects of the individual savings decision on the equilibrium interest rate and therefore we get an additional channel through which changes in longevity and changes in economic growth impact upon the optimal retirement age.

In particular, we find that an increase in longevity raises the aggregate capital to consumption ratio and therefore decreases the interest rate. The overall effect of increases in longevity on the optimal retirement age is ambiguous but for reasonable parameter values — implied by data for the G8 countries — the optimal retirement age increases as a consequence of increasing longevity. Furthermore, we also showed that the aggregate capital to consumption ratio and the interest rate rise after the growth rate of the economy increases. In this case the overall effect of faster growth on the optimal retirement age is ambiguous but again for the parameter values associated with the G8 countries we can be sure that — within the confines of our modeling framework — the optimal retirement age increases in economic growth.

In addition, our simplified theoretical framework is able to describe two behavioral changes in response to population aging, namely, an increase in savings and an increase in the retirement age. These two behavioral changes represent important forces for compensating some of the negative impacts due to the accounting effect of population aging.

From a policy perspective we can conclude that for reasonable parameter values, an increase in longevity should be accompanied by increases in the mandatory retirement age and/or by removing incentives for early retirement. This policy recommendation also holds in the partial equilibrium framework of Bloom et al. (2007a) and in the small open economy framework of Heijdra and Romp (2009). Both of these contributions show that individuals prefer to work longer when life expectancy increases and interest rates stay constant. On the aggregate level, the mechanism that we outlined has the additional effect of increasing per capita GDP and therefore mitigating some negative economic impacts attributable to the accounting effect of population aging.

We hope that our analysis is able to shed some light on the interrelations between savings and retirement decisions. However, our framework is very stylized and a multitude of possible ways to make the model more realistic remain for further research. The most promising ones are in our opinion to introduce capital market imperfections, alternative social security systems and performing simulation studies for an economy whose production side is described in an alternative manner.

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Appendix

A Optimal consumption and retirement

The control variables of the individual optimization problem are c and χ and we have the following current value Hamiltonian

$$H = u(c) - \chi\nu(z,t) + \phi\left[\chi w + (\lambda + r)k - c\right]. \tag{34}$$

The first oder conditions (FOCs) are

$$
\begin{aligned}
u'(c) &= \phi, & (35)\\
-\nu(z,t) &\geq -\phi w \text{ for } \chi = 1, & (36)\\
-\nu(z,t) &\leq -\phi w \text{ for } \chi = 0, & (37)\\
(\lambda + r)\phi &= (\delta + \lambda)\phi - \dot{\phi}. & (38)
\end{aligned}
$$

From the first FOC we get $u''(c)\dot{c} = \dot{\phi}$ such that

$$(r-\delta)\frac{u'(c)}{-u''(c)} = \dot{c}. \tag{39}$$

Finally, we have

$$
\begin{aligned}
-\nu(z,t) &\geq -u'(c)w \text{ for } \chi = 1\\
\chi &= 1 \Leftrightarrow u'(c)w \geq \nu(z,t). & (40)
\end{aligned}
$$

The lifetime budget constraint of an individual can be written as

$$
\begin{aligned}
\int_{t_0}^{\infty} e^{-(\lambda+r)(t-t_0)} c(t_0,t)dt, &= \int_{t_o}^{T} e^{-(\lambda+r)(t-t_0)} w(t_0,t)dt,\\
\int_{t_0}^{\infty} e^{-(\lambda+r)(t-t_0)} c(t_0,t_0)e^{(r-\delta)(t-t_0)}dt &= \int_{t_0}^{T} e^{-(\lambda+r)(t-t_0)} w(t_0,t_0)e^{g(t-t_0)}dt,\\
\int_{t_0}^{\infty} e^{-(\lambda+\delta)(t-t_0)} c(t_0,t_0)dt &= \int_{t_0}^{T} e^{-(\lambda+r-g)(t-t_0)} w(t_0,t_0)dt,\\
c(t_0,t_0)\left[\frac{e^{-(\lambda+\delta)(t-t_0)}}{-(\lambda+\delta)}\right]_{t_0}^{\infty} &= w(t_0,t_0)\left[\frac{e^{-(\lambda+r-g)(t-t_0)}}{-(\lambda+r-g)}\right]_{t_0}^{T},\\
\frac{c(t_0,t_0)}{\lambda+\delta} &= \frac{w(t_0,t_0)}{\lambda+r-g}\left[1-e^{-(\lambda+r-g)(T-t_0)}\right],\\
\frac{c(t_0,t_0)}{w(t_0,t_0)} &= \frac{\lambda+\delta}{\lambda+r-g}\left[1-e^{-(\lambda+r-g)(T-t_0)}\right].
\end{aligned}
\tag{41}
$$

Now we denote the optimal retirement time by T^* such that the optimal retirement age is given by $R^* = T^* - t_0$ and we arrive at the following implicit expression for R^*

$$\begin{aligned}
\nu(R^*) &= u'(c(R^*))w(R^*), \\
de^{\lambda R^*} &= \frac{w(t_0,t_0)e^{gR^*}}{c(t_0,t_0)e^{(r-\delta)R^*}}, \\
\frac{c(t_0,t_0)}{w(t_0,t_0)} &= \frac{e^{(g+\delta-r-\lambda)R^*}}{d}. \qquad (42)
\end{aligned}$$

Next we can put equations (41) and (42) together which yields

$$\begin{aligned}
\frac{e^{(g+\delta-r-\lambda)R^*}}{d} &= \frac{\lambda+\delta}{\lambda+r-g}\left(1-e^{-(\lambda+r-g)R^*}\right), \\
\frac{\lambda+\delta}{\lambda+r-g} &= \frac{e^{(g+\delta-r-\lambda)R^*}}{d} + \frac{(\lambda+\delta)e^{-(\lambda+r-g)R^*}}{\lambda+r-g}, \\
\frac{\lambda+\delta}{\lambda+r-g} &= \frac{(\lambda+r-g)e^{(g+\delta-r-\lambda)R^*} + d(\lambda+\delta)e^{-(\lambda+r-g)R^*}}{d(\lambda+r-g)}, \\
(\lambda+\delta)d &= (\lambda+r-g)e^{(g+\delta-r-\lambda)R^*} + d(\lambda+\delta)e^{(g-r-\lambda)R^*}
\end{aligned}$$

being an implicit function for the optimal retirement age R^* (see also Bloom et al., 2007a).

B Aggregating over cohorts

By taking into account our demographic structure, we can rewrite the aggregation rules as

$$C(t) \equiv \lambda N \int_{-\infty}^{t} c(t_0,t)e^{\lambda(t_0-t)}dt_0, \qquad (43)$$

$$K(t) \equiv \lambda N \int_{-\infty}^{t} k(t_0,t)e^{\lambda(t_0-t)}dt_0, \qquad (44)$$

such that differentiating equations (43) and (44) with respect to time yields

$$\begin{aligned}
\dot{C}(t) &= \lambda N\left[\int_{-\infty}^{t}\dot{c}(t_0,t)e^{\lambda(t_0-t)}dt_0 - \lambda\int_{-\infty}^{t}c(t_0,t)e^{\lambda(t_0-t)}dt_0\right] + \lambda N c(t,t) - 0 \\
&= \lambda N c(t,t) - \lambda C(t) + \lambda N\int_{-\infty}^{t}\dot{c}(t_0,t)e^{-\lambda(t-t_0)}dt_0 \qquad (45) \\
\dot{K}(t) &= \lambda N\left[\int_{-\infty}^{t}\dot{k}(t_0,t)e^{\lambda(t_0-t)}dt_0 - \lambda\int_{-\infty}^{t}k(t_0,t)e^{\lambda(t_0-t)}dt_0\right] + \lambda N k(t,t) - 0 \\
&= \lambda N \underbrace{k(t,t)}_{=0} - \lambda K(t) + \lambda N\int_{-\infty}^{t}\dot{k}(t_0,t)e^{-\lambda(t-t_0)}dt_0. \qquad (46)
\end{aligned}$$

Note that newborns do not own any capital, i.e., $k(t,t) = 0$, because there are no bequests. From equation (2) it follows that

$$\begin{aligned}
\dot{K}(t) &= -\lambda K(t) + \lambda N\int_{-\infty}^{t}\left[\chi w + (\lambda + r)k(t_0,t) - c(t_0,t)\right]e^{-\lambda(t-t_0)}dt_0, \\
&= -\lambda K(t) + (\lambda + r)\lambda N\int_{-\infty}^{t}k(t_0,t)e^{-\lambda(t-t_0)}dt_0 \\
&\quad -\lambda N\int_{-\infty}^{t}c(t_0,t)e^{-\lambda(t-t_0)}dt_0 + N\Xi\left(\frac{\lambda w e^{-\lambda(t-t_0)}}{\lambda}\right)_{-\infty}^{t}, \\
&= -\lambda K(t) + (\lambda + r)K(t) - C(t) + \Xi W(t), \\
&= rK(t) - C(t) + \Xi W(t)
\end{aligned}$$

which is the aggregate law of motion for capital, whereby W refers to aggregate wage income if the whole living population would work and Ξ denotes the fraction of the population N who are still supplying their skills on the labor market, i.e., who are not yet retired.

Reformulating an agents optimization problem subject to its lifetime budget restriction, stating that the present value of lifetime consumption expenditures has to be equal to the present value of lifetime wage income plus initial assets, yields the optimization problem

$$\begin{aligned}
\max_{c(t_0,\tau)} \quad & U = \int_{t}^{\infty} e^{(\delta+\lambda)(t-\tau)}\left[\log(c(t_0,\tau)) - \chi\nu(z,t)\right]d\tau \\
s.t. \quad & k(t_0,t) + \int_{t}^{T} w(\tau)e^{-D^A(t,\tau)}d\tau = \int_{t}^{\infty} c(t_0,\tau)e^{-D^A(t,\tau)}d\tau,
\end{aligned} \qquad (47)$$

where the discount factor is $D^A(\tau,t) = \int_t^\tau(\delta + \lambda)ds$. The FOC with respect to consumption is

$$\frac{1}{c(t_0,\tau)}e^{(\delta+\lambda)(t-\tau)} = \lambda(t)e^{-D^A(t,\tau)}.$$

In period $(\tau = t)$ we have

$$c(t_0,t) = \frac{1}{\lambda(t)}.$$

Therefore we can write

$$\begin{aligned}
\frac{1}{c(t_0,\tau)}e^{(\delta+\lambda)(t-\tau)} &= \frac{1}{c(t_0,t)}e^{-D^A(t,\tau)}, \\
c(t_0,t)e^{(\delta+\lambda)(t-\tau)} &= c(t_0,\tau)e^{-D^A(t,\tau)}.
\end{aligned}$$

Integrating and using equation (47) yields

$$\begin{aligned}
\int_t^\infty c(t_0,t)e^{(\delta+\lambda)(t-\tau)}d\tau &= \int_t^\infty c(t_0,\tau)e^{-D^A(t,\tau)}d\tau, \\
\frac{c(t_0,t)}{\delta+\lambda}\left[-e^{(\delta+\lambda)(t-\tau)}\right]_t^\infty &= k(t_0,t) + \underbrace{\int_t^T w(\tau)e^{-D^A(t,\tau)}d\tau}_{h(t)}, \\
\Rightarrow c(t_0,t) &= (\delta+\lambda)\left[k(t_0,t)+h(t)\right], \qquad (48)
\end{aligned}$$

where h refers to human wealth, i.e., wage income, of individuals. Human wealth does not depend on the date of birth because productivity is age independent. The above calculations show that optimal consumption in the planning period is proportional to total wealth with a marginal propensity to consume of $\delta + \lambda$. Aggregate consumption evolves according to

$$\begin{aligned}
C(t) &\equiv \lambda N \int_{-\infty}^t c(t_0,t)e^{\mu(t_0-t)}dt_0 \\
&= \lambda N \int_{-\infty}^t e^{\mu(t_0-t)}(\delta+\lambda)\left[k(t_0,t)+h(t)\right]dt_0 \\
&= (\delta+\lambda)\left[K(t)+H(t)\right]. \qquad (49)
\end{aligned}$$

Note that this implies that aggregate human wealth is defined as $H(t) = \lambda N \int_{-\infty}^t e^{\mu(t_0-t)}h(t)dt_0 = Nh(t)$. Newborns do not own capital because there are no bequests. Therefore

$$c(t,t) = (\delta+\lambda)h(t) \qquad (50)$$

holds for each newborn individual. Putting equations (39), (45), (49) and (50) together yields

$$\begin{aligned}
\dot{C}(t) &= \lambda(\delta+\lambda)H(t) - \lambda(\delta+\lambda)\left[K(t)+H(t)\right] + \\
&\quad \lambda N \int_{-\infty}^{t} (r-\delta)c(t_0,t)e^{-\lambda(t-t_0)}dt_0 \\
&= \lambda(\delta+\lambda)H(t) - \lambda(\delta+\lambda)\left[K(t)+H(t)\right] + (r-\delta)C(t) \\
\Rightarrow \frac{\dot{C}(t)}{C(t)} &= r-\delta+\frac{\lambda(\delta+\lambda)H(t) - \lambda(\delta+\lambda)\left[K(t)+H(t)\right]}{C(t)} \\
&= r-\delta-\lambda(\delta+\lambda)\frac{K(t)}{C(t)}
\end{aligned}$$

which is the aggregate Euler equation.

C Comparative statics for the neoclassical model

We define

$$I := \frac{r}{\alpha} - \xi - g = 0, \tag{51}$$

$$II := r-\delta-\lambda(\delta+\lambda)\frac{1}{\xi} - g = 0, \tag{52}$$

$$III := (\lambda+r-g)e^{(g+\delta-r-\lambda)R^*} + d(\lambda+\delta)e^{(g-r-\lambda)R^*} - (\lambda+\delta)d = 0. \tag{53}$$

Then we have the following partial derivatives

$$
\begin{aligned}
I_r &= \frac{1}{\alpha}, \\
I_\xi &= -1, \\
I_{R^*} &= 0, \\
I_\lambda &= 0, \\
I_g &= -1, \\
II_r &= 1, \\
II_\xi &= \frac{\lambda(\delta+\lambda)}{\xi^2}, \\
II_{R^*} &= 0, \\
II_\lambda &= -\frac{2\lambda+\delta}{\xi}, \\
II_g &= -1, \\
III_r &= e^{(g+\delta-r-\lambda)R^*} - R^*(\lambda+r-g)e^{(g+\delta-r-\lambda)R^*} \\
&\quad -R^*d(\lambda+\delta)e^{(g-r-\lambda)R^*}, \\
III_\xi &= 0, \\
III_{R^*} &= (g+\delta-r-\lambda)(\lambda+r-g)e^{(g+\delta-r-\lambda)R^*} \\
&\quad +(g-r-\lambda)d(\lambda+\delta)e^{(g-r-\lambda)R^*}, \\
III_\lambda &= e^{(g+\delta-r-\lambda)R^*} - R^*(\lambda+r-g)e^{(g+\delta-r-\lambda)R^*} \\
&\quad +de^{(g-r-\lambda)R^*} - R^*d(\lambda+\delta)e^{(g-r-\lambda)R^*} - d, \\
III_g &= -e^{(g+\delta-r-\lambda)R^*} + R^*(\lambda+r-g)e^{(g+\delta-r-\lambda)R^*} \\
&\quad +R^*d(\lambda+\delta)e^{(g-r-\lambda)R^*},
\end{aligned}
$$

such that the Jacobian of the system reads

$$
\Delta = \begin{pmatrix} \frac{1}{\alpha} & -1 & 0 \\ 1 & \frac{\lambda(\delta+\lambda)}{\xi^2} & 0 \\ A_1 & 0 & A_2 \end{pmatrix}
$$

with

$$
\begin{aligned}
A_1 &= e^{(g+\delta-r-\lambda)R^*} - R^*(\lambda+r-g)e^{(g+\delta-r-\lambda)R^*} \\
&\quad -R^*d(\lambda+\delta)e^{(g-r-\lambda)R^*}, \\
A_2 &= (g+\delta-r-\lambda)(\lambda+r-g)e^{(g+\delta-r-\lambda)R^*} \\
&\quad +(g-r-\lambda)d(\lambda+\delta)e^{(g-r-\lambda)R^*}.
\end{aligned}
$$

The determinant of the Jacobian is

$$|\Delta| = A_2 \left[1 + \frac{\lambda(\delta+\lambda)}{\alpha\xi^2}\right]$$

which, by the correspondence principle, ought to be negative. From equation (52) it follows that $A_2 < 0$ and consequently, Δ has the desired sign. Furthermore, if we are interested in the effects of changing mortality on the retirement age, we have that

$$\Delta_{R^*,\lambda} = \begin{pmatrix} \frac{1}{\alpha} & -1 & 0 \\ 1 & \frac{\lambda(\delta+\lambda)}{\xi^2} & -\frac{2\lambda+\delta}{\xi} \\ A_1 & 0 & A_3 \end{pmatrix}$$

where we substituted the third column of the Jacobian by the partial derivatives of the system with respect to mortality and we have that

$$\begin{aligned} A_3 &= e^{(g+\delta-r-\lambda)R^*} - R^*(\lambda+r-g)e^{(g+\delta-r-\lambda)R^*} \\ &\quad + de^{(g-r-\lambda)R^*} - R^* d(\lambda+\delta)e^{(g-r-\lambda)R^*} - d. \end{aligned}$$

The determinant of this matrix is

$$|\Delta_{R^*,\lambda}| = A_3 \left[1 + \frac{\lambda(\delta+\lambda)}{\alpha\xi^2}\right] + \frac{2\lambda+\delta}{\xi} A_1.$$

Altogether we have the following result

$$\frac{\partial R^*}{\partial \lambda} = -\frac{|\Delta_{R^*,\lambda}|}{|\Delta|} = -\frac{A_3\left[1 + \frac{\lambda(\delta+\lambda)}{\alpha\xi^2}\right] + \frac{2\lambda+\delta}{\xi}A_1}{A_2\left[1 + \frac{\lambda(\delta+\lambda)}{\alpha\xi^2}\right]}.$$

Next, if we are interested in the effects of changing economic growth on the retirement age, we have that

$$\Delta_{R^*,g} = \begin{pmatrix} \frac{1}{\alpha} & -1 & -1 \\ 1 & \frac{\lambda(\delta+\lambda)}{\xi^2} & -1 \\ A_1 & 0 & A_4 \end{pmatrix},$$

where we substituted the third column of the Jacobian by the partial derivatives of the system with respect to economic growth and we have that

$$A_4 = -e^{(g+\delta-r-\lambda)R^*} + R^*(\lambda + r - g)e^{(g+\delta-r-\lambda)R^*} + R^* d(\lambda+\delta)e^{(g-r-\lambda)R^*}.$$

The determinant of this matrix is

$$|\Delta_{R^*,g}| = A_4\left[1 + \frac{\lambda(\delta+\lambda)}{\alpha\xi^2}\right] + A_1\left[1 + \frac{\lambda(\delta+\lambda)}{\xi^2}\right].$$

Altogether we have the following result

$$\begin{aligned}\frac{\partial R^*}{\partial g} &= -\frac{|\Delta_{R^*,g}|}{|\Delta|} = -\frac{A_4\left[1+\frac{\lambda(\delta+\lambda)}{\alpha\xi^2}\right] + A_1\left[1+\frac{\lambda(\delta+\lambda)}{\xi^2}\right]}{A_2\left[1+\frac{\lambda(\delta+\lambda)}{\alpha\xi^2}\right]} \\ &= -\frac{\frac{\lambda(\delta+\lambda)}{\xi^2}\left(\frac{A_4}{\alpha} + A_1\right)}{A_2\left[1+\frac{\lambda(\delta+\lambda)}{\alpha\xi^2}\right]}. \end{aligned} \tag{54}$$

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