AJAE Appendix: *Optimal Investment in Transportation Infrastructure When Middlemen Have Market Power: A Developing-Country Analysis*

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Derivation of the Price Schedule (Stage 2 Equilibrium)

We start by deriving the supply function $Q_a(P_a, P_b; s, \gamma)$ representing the quantity of farm product supplied to trader a under prices P_a and P_b. This enables us to derive the reaction function $P_a^*(P_b; s, \gamma)$ (which by symmetry gives us $P_b^*(P_a; s, \gamma)$). The intersection of the two reaction curves represents the equilibrium prices $(\bar{P}_a, \bar{P}_b)$. In what follows, we assume that $s \in [0, 1)$.

The Supply Function

Note first that, given our assumption about the reservation utility of farmers, P_a and P_b must be greater than the assessment s in order to attract suppliers. Further, it is never optimal for, say, trader a, to offer a price strictly greater than $P_b + \gamma$, because the entire market supplies trader a for $P_a = \gamma + P_b$. Similarly, it is not optimal for trader a to offer a price strictly lower than $P_b - \gamma$, because then all farmers would prefer to supply to trader b. Therefore, in deriving $Q_a(P_a, P_b; s, \gamma)$, we will take for granted that $P_a \geq s$, $P_b \geq s$, $P_a \leq \gamma + P_b$ and $P_a \geq P_b - \gamma$.

Second, note that either there exists a farmer who is indifferent between selling to trader a or trader b, or such as farmer does not exist. Suppose first that the indifferent farmer exists. He must be located at a point $Y \in [0, 1]$ satisfying $P_a - s - \gamma Y = P_b - s - \gamma(1 - Y)$, that is, $Y = \frac{P_a - P_b + \gamma}{2\gamma}$. In addition, the profit of this indifferent farmer must be nonnegative (which implies, given the structure of transportation costs, that the profit of all other farmers is nonnegative). Hence, we have $P_a - s - \gamma Y \geq 0$, or $\frac{P_b + P_a - \gamma}{2} - s \geq 0$. The conditions $\frac{P_a - P_b + \gamma}{2\gamma} \in [0, 1]$ and $\frac{P_b + P_a - \gamma}{2} - s \geq 0$ imply that $P_a \leq \gamma + P_b$ and that $P_a \geq \max(P_b - \gamma, \gamma + 2s - P_b)$. The supply to trader a is then $Q_a = Y$.

Suppose now that an indifferent farmer does not exist. Because we have ruled out cases where either trader captures the entire market, the case of no indifferent producer can occur only when farmers located in the middle of the interval produce only for subsistence use. This is the case if the farmer located at the market boundary of trader a (i.e., at point

$X = \frac{P_a - s}{\gamma}$) would not supply his product to trader b, that is, $P_b - s - \gamma(1 - \frac{P_a - s}{\gamma}) < 0$, i.e.,

$P_a < 2s + \gamma - P_b$. The supply to trader a is then $Q_a = X$.

Summarizing the above results, we can write the supply function to trader a as:

$$\text{(A-1)} \quad Q_a(P_a, P_b; s, \gamma) = \begin{cases} \frac{P_a - s}{\gamma} & \text{if} \quad s \leq P_a < \gamma + 2s - P_b \\ \frac{P_a - P_b + \gamma}{2\gamma} & \text{if} \quad \max(P_b - \gamma, \gamma + 2s - P_b) \leq P_a \leq \gamma + P_b \end{cases} .$$

Note that $\gamma + 2s - P_b \geq P_b - \gamma \Leftrightarrow \gamma + 2s - P_b \geq s \Leftrightarrow P_b \leq s + \gamma$. Therefore, if $P_b > s + \gamma$,

$Q_a = \frac{P_a - P_b + \gamma}{2\gamma}$ for all relevant values of P_a.

The Reaction Functions

Trader a maximizes profit given price P_b by solving

$$\text{(A-2)} \quad \max_{P_a} \Pi_a(P_a, P_b; s, \gamma) = (1 - P_a)Q_a(P_a, P_b; s, \gamma).$$

We will first examine the case $s \leq P_b \leq s + \gamma$, and then the case $P_b > s + \gamma$.

1. $s \leq P_b \leq s + \gamma$

   In this case, Q_a has two different expressions on the two subintervals $s \leq P_a \leq \gamma + 2s - P_b$ and $\gamma + 2s - P_b \leq P_a \leq \gamma + P_b$. We thus need to solve problem (A-2) separately on each subinterval and then compare the obtained maximized values, in order to find the solution on the entire domain.

   We first solve

   $$\text{(A-3)} \quad \max_{s \leq P_a \leq \gamma + 2s - P_b} \Pi_a(P_a, P_b; s, \gamma) = (1 - P_a)\left(\frac{P_a - s}{\gamma}\right),$$

   which yields the following solution:

   - if $P_b \leq \gamma + \frac{3}{2}s - \frac{1}{2}$, then $P_a = \frac{1+s}{2}$ and $\Pi_a = \frac{(1-s)^2}{4\gamma}$;

   - if $P_b > \gamma + \frac{3}{2}s - \frac{1}{2}$, then $P_a = \gamma + 2s - P_b$ and $\Pi_a = \frac{(1 - \gamma - 2s + P_b)(\gamma + s - P_b)}{\gamma}$.

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We then solve

$$(\text{A-4}) \quad \max_{\gamma+2s-P_b \leq P_a \leq \gamma+P_b} \Pi_a(P_a, P_b; s, \gamma) = (1-P_a)\left(\frac{P_a - P_b + \gamma}{2\gamma}\right),$$

which yields the following solution:

- if $P_b \geq \max(\gamma + \frac{4}{3}s - \frac{1}{3}, 1 - 3\gamma)$, then $P_a = \frac{P_b - \gamma + 1}{2}$ and $\Pi_a = \frac{(1 - P_b + \gamma)^2}{8\gamma}$;

- if $P_b < \gamma + \frac{4}{3}s - \frac{1}{3}$, then $P_a = \gamma + 2s - P_b$ and $\Pi_a = \frac{(1 - \gamma - 2s + P_b)(\gamma + s - P_b)}{\gamma}$;

- if $P_b < 1 - 3\gamma$, then $P_a = \gamma + P_b$ and $\Pi_a = 1 - \gamma - P_b$.

Note that the three conditions above are mutually exclusive.[1]

We now need to compare the optimized values of Π_a on the two subintervals. Suppose first that $P_b \leq \gamma + \frac{3}{2}s - \frac{1}{2}$. The maximized value of Π_a on $[s, \gamma + 2s - P_b]$ is $\frac{(1-s)^2}{4\gamma}$. Since $P_b \leq \gamma + \frac{3}{2}s - \frac{1}{2}$ implies that $P_b < \gamma + \frac{4}{3}s - \frac{1}{3}$ for $s < 1$, the maximized value of Π_a on $[\gamma + 2s - P_b, \gamma + P_b]$ is $\frac{(1 - \gamma - 2s + P_b)(\gamma + s - P_b)}{\gamma}$. It is then easy to show that $\frac{(1-s)^2}{4\gamma} \geq \frac{(1 - \gamma - 2s + P_b)(\gamma + s - P_b)}{\gamma}$, so that the optimal response of trader a for this range of P_b is $P_a^* = \frac{1+s}{2}$.[2]

Now suppose that $P_b > \gamma + \frac{3}{2}s - \frac{1}{2}$. The maximized value of Π_a on $[s, \gamma + 2s - P_b]$ is $\frac{(1 - \gamma - 2s + P_b)(\gamma + s - P_b)}{\gamma}$, corresponding to price $P_a = \gamma + 2s - P_b$. We need to compare this value to those obtained on the subinterval $[\gamma + 2s - P_b, \gamma + P_b]$ for each of the subcases identified above. The comparisons lead to the following results:

- if $P_b \geq \max(\gamma + \frac{4}{3}s - \frac{1}{3}, 1 - 3\gamma)$, $P_a^* = \frac{P_b - \gamma + 1}{2}$;

- if $P_b < \gamma + \frac{4}{3}s - \frac{1}{3}$, $P_a^* = \gamma + 2s - P_b$;

- if $P_b < 1 - 3\gamma$, $P_a^* = \gamma + P_b$.

2. $P_b > s + \gamma$

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[1]To see why $P_b < \gamma + \frac{4}{3}s - \frac{1}{3}$ and $P_b < 1 - 3\gamma$ are mutually exclusive, note that $P_b < \gamma + \frac{4}{3}s - \frac{1}{3}$ is equivalent to $\gamma > P_b - s + \frac{1-s}{3}$, which by $P_b \geq s$ implies that $\gamma > \frac{1 - P_b}{3}$, a condition incompatible with $P_b < 1 - 3\gamma$.

[2]All derivations are available from the authors upon request.

In this case, the only relevant interval for P_a is $[P_b - \gamma, \gamma + P_b]$, where $Q_a = \frac{P_a - P_b + \gamma}{2\gamma}$.
The solution to problem (A-2) in this case is as follows.

- If $1 - 3\gamma \leq P_b \leq \gamma + 1$, then $P_a^* = \frac{P_b - \gamma + 1}{2}$.

- If $P_b > \gamma + 1$, then $P_a^* = P_b - \gamma$. However, it is never optimal for trader b to offer a price P_b greater than 1, because then he would make a negative margin. Therefore, we can ignore this case.

- if $P_b < 1 - 3\gamma$, then $P_a^* = \gamma + P_b$.

To summarize, the reaction function $P_a^*(P_b; s, \gamma)$ has the following form.

- If $s \leq P_b \leq s + \gamma$:

  - if $P_b \leq \gamma + \frac{3}{2}s - \frac{1}{2}$, then $P_a^* = \frac{1+s}{2}$;

  - if $P_b > \gamma + \frac{3}{2}s - \frac{1}{2}$, then

    * if $P_b \geq \max(\gamma + \frac{4}{3}s - \frac{1}{3}, 1 - 3\gamma)$, $P_a^* = \frac{P_b - \gamma + 1}{2}$;

    * if $P_b < \gamma + \frac{4}{3}s - \frac{1}{3}$, $P_a^* = \gamma + 2s - P_b$;

    * if $P_b < 1 - 3\gamma$, $P_a^* = \gamma + P_b$.

- If $P_b > s + \gamma$:

  - if $P_b \geq 1 - 3\gamma$, then $P_a^* = \frac{P_b - \gamma + 1}{2}$;

  - if $P_b < 1 - 3\gamma$, then $P_a^* = \gamma + P_b$.

By the symmetry of traders, $P_b^*(P_a; s, \gamma)$ has the same form as $P_a^*(P_b; s, \gamma)$.

The Equilibrium Prices

Equilibrium prices $(\bar{P}_a, \bar{P}_b)$ are obtained as the intersection of the reaction curves $P_a^*(P_b; s, \gamma)$ and $P_b^*(P_a; s, \gamma)$. As before, we assume that $0 \leq s < 1$. We also know that $s \leq \bar{P}_i \leq 1$, $i = a, b$. The reaction curves have different shapes according to the relative values of γ and $1 - s$. All possible scenarios are illustrated in figures A-1 to A-8, where the values $\gamma + \frac{4}{3}s - \frac{1}{3}$ and $\gamma + \frac{3}{2}s - \frac{1}{2}$ have been replaced by the symbols $\gamma_{\frac{1}{3}}$ and $\gamma_{\frac{1}{2}}$, respectively, for clarity of presentation. The results can be summarized as follows.

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1. If $0 < \gamma \leq \frac{2}{3}(1-s)$, then $\bar{P}_a = \bar{P}_b = \bar{P} = 1 - \gamma$. The market is covered and the indifferent farmer, who receives a positive profit, is located at $\bar{Y} = \frac{1}{2}$ (figures A-1, A-2 and A-3).

2. If $\frac{2}{3}(1-s) < \gamma \leq 1 - s$:

   - if $\frac{2}{3}(1-s) < \gamma \leq \frac{5}{6}(1-s)$, then $\bar{P}_a, \bar{P}_b \in [\frac{1}{3}(1+2s), \gamma + \frac{4}{3}s - \frac{1}{3}]$ and $\bar{P}_a + \bar{P}_b = 2s + \gamma$ (figure A-4);

   - if $\frac{5}{6}(1-s) < \gamma \leq 1 - s$, then $\bar{P}_a, \bar{P}_b \in [\gamma + \frac{3}{2}s - \frac{1}{2}, \frac{1+s}{2}]$ and $\bar{P}_a + \bar{P}_b = 2s + \gamma$ (figure A-5).

   In both subcases, the market is covered and the indifferent farmer, who has zero profit, is located at $\bar{Y} = \frac{\bar{P}_a - \bar{P}_b + \gamma}{2\gamma} = \frac{\bar{P}_a - s}{\gamma}$. For the subcase $\frac{2}{3}(1-s) < \gamma \leq \frac{5}{6}(1-s)$, the indifferent farmer is located inside the segment $[\frac{1-s}{3\gamma}, 1 - \frac{1-s}{3\gamma}]$. For the subcase $\frac{5}{6}(1-s) < \gamma \leq 1 - s$, he is located inside the segment $[1 - \frac{1-s}{2\gamma}, \frac{1-s}{2\gamma}]$. Those two segments are widest for $\gamma = \frac{5}{6}(1-s)$ where they are both equal to $[\frac{2}{5}, \frac{3}{5}]$.

3. If $\gamma > 1 - s$, then $\bar{P}_a = \bar{P}_b = \bar{P} = \frac{1+s}{2}$. Only farmers located near the endpoints sell their production, and the market radius of each trader is $\bar{X} = \frac{1-s}{2\gamma}$ (figures A-6, A-7 and A-8).

Investment Regimes When the Initial Market is Monopsonistic

To make the analysis tractable, we make the following assumptions.

Assumption A-1 *The transportation improvement technology is described by the function* $\Gamma(\sigma) = \frac{T}{e^{\alpha\sigma}}$, *where* σ *represents the investment per unit distance,* $T > 1$ *represents the pre-investment level of transportation costs per unit distance, and* α *is a positive parameter that gives the constant rate at which transportation costs can be decreased with an investment of \$1 per unit distance.*

Assumption A-2 *The authority spends the assessment revenue to improve transportation only on the portion of the market area that engages in commercial production in stage 2 equilibrium.*

As argued in the article, the functional form in assumption A-1 has several desirable features. It implies diminishing returns to per-unit distance investments, involves parameters that have a direct interpretation, and allows us to simulate a broad variety of improvement technologies while imposing minimal constraints on the technology.

Assumption A-2 reflects an efficient use of assessment funds by the revenue-generating authority, compared to a scenario where, e.g., the entirety of the production region would be improved even in cases where the market is only partially covered. It also implies that when the market is partially covered in stage 2 equilibrium, the minimum transportation cost achievable with an assessment of s dollars is exactly $\gamma = \Gamma(s)$, because the investment per unit of distance on the portion of the production area that is subject to improvement is exactly equal to the nominal assessment, given the unit supply assumption.[3] Therefore, besides its intuitive appeal, assumption A-2 ensures that the transportation cost γ that can

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[3]If transportation improvements take the form of investments in equipment, such as trucks, assumption A-2 ensures that the authority only purchases the volume of equipment needed to transport the output of farms selling their product to the external market.

be achieved with a per-unit assessment of s dollars only depends on s, and not on the participation rate of farmers in stage 2 equilibrium.[4]

Figures A-9 to A-12 depict the four possible investment scenarios when starting from monopsonistic competition, and illustrate the following proposition.

Proposition A-1 *For each $T > 1$, there exists positive numbers $\alpha_1(T)$, $\alpha_2(T)$ and $\alpha_3(T)$ such that*

- $\hat{s} = 0$ *if* $\alpha \leq \alpha_1(T)$;

- $\hat{s} > 0$ *and* $\frac{\hat{\gamma}}{1-\hat{s}} > 1$ *if* $\alpha_1(T) < \alpha < \alpha_2(T)$;

- $\hat{s} > 0$ *and* $\frac{\hat{\gamma}}{1-\hat{s}} = 1$ *if* $\alpha_2(T) \leq \alpha < \alpha_3(T)$;

- $\hat{s} > 0$ *and* $\frac{\hat{\gamma}}{1-\hat{s}} < \frac{2}{3}$ *if* $\alpha \geq \alpha_3(T)$.

Proposition A-1 implies that for any given initial transport cost, the potential of positive assessments to alter the nature of competition between traders directly depends on the efficiency parameter α. For low values of α, producers are better-off not investing in transportation. For slightly higher values of α, they have an incentive to invest in transportation, although not to the extent where the market is completely covered. For α in the intermediate range $[\alpha_2, \alpha_3)$, producers will invest in transportation so that the final state of competition is borderline monopsony/weak duopsony, so the market will be covered. However, due to the non-monotonicity of the profit schedule, it is never optimal to move inside the weak duopsony region. In this range of α, although marginal increases in α do not change the nature of competition between traders, farm profits are still monotonically increasing in α. Finally, for highly efficient technologies ($\alpha \geq \alpha_3$), optimal investments in transportation enable farmers to move to the strict duopsony regime.

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[4]Assumption A-2 is innocuous if the initial transportation cost lies in the weak or strict duopsony range, since in stage 2 equilibrium the market will always be covered.
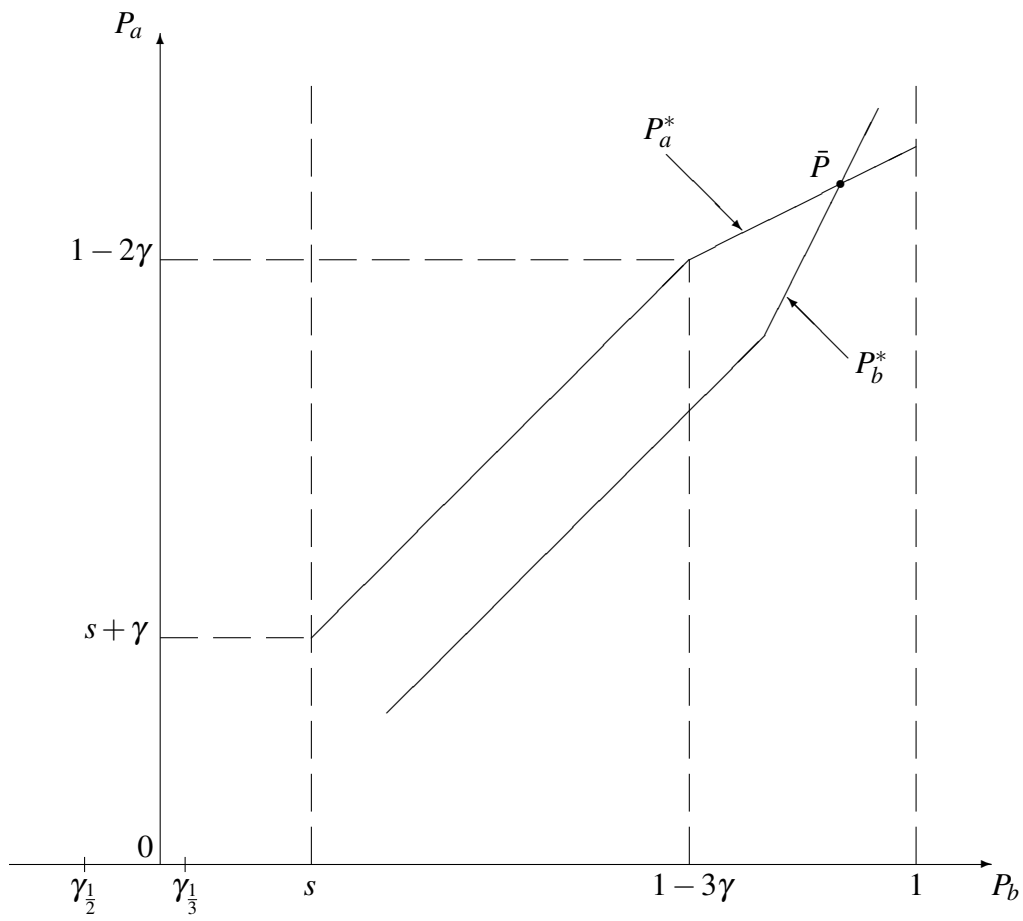
Figure A-1. Equilibrium prices when $0 < \gamma \leq \frac{1}{3}(1-s)$**. The solution is** $\bar{P}_a = \bar{P}_b = \bar{P} = 1 - \gamma$**.**
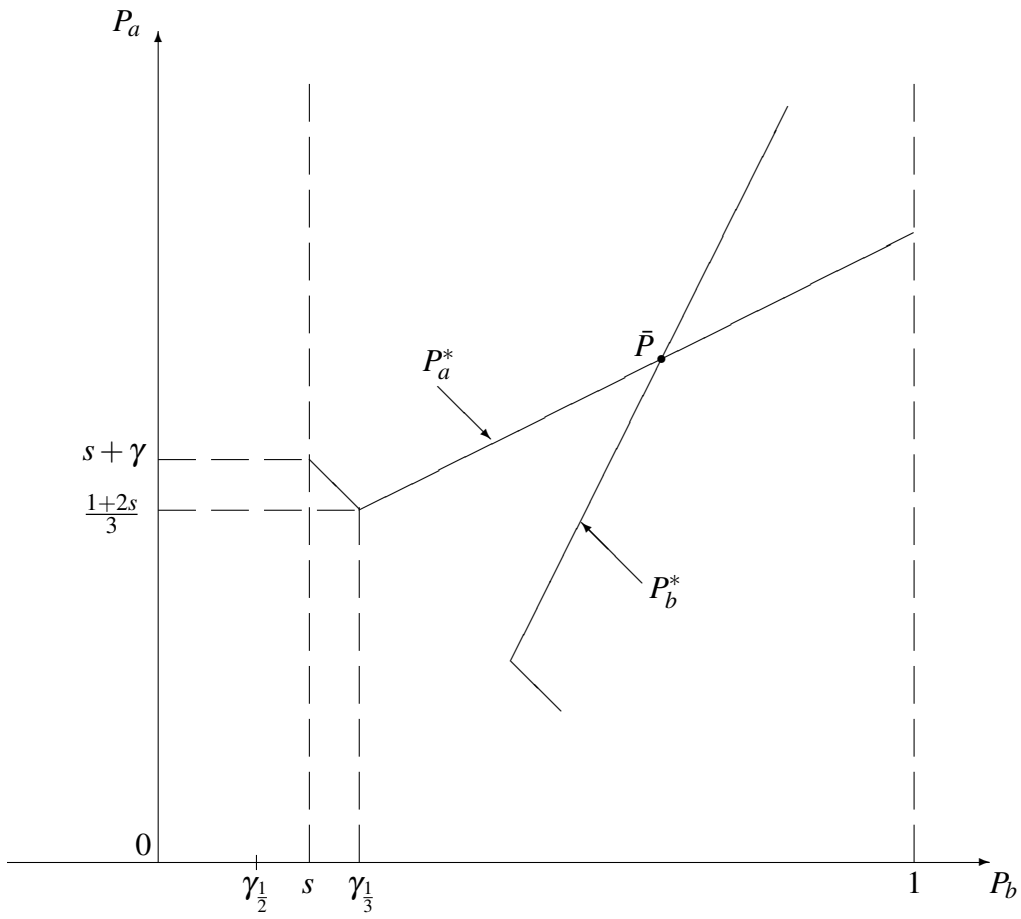
Figure A-2. Equilibrium prices when $\frac{1}{3}(1-s) < \gamma \leq \frac{1}{2}(1-s)$**. The solution is** $\bar{P}_a = \bar{P}_b = \bar{P} = 1 - \gamma$**.**
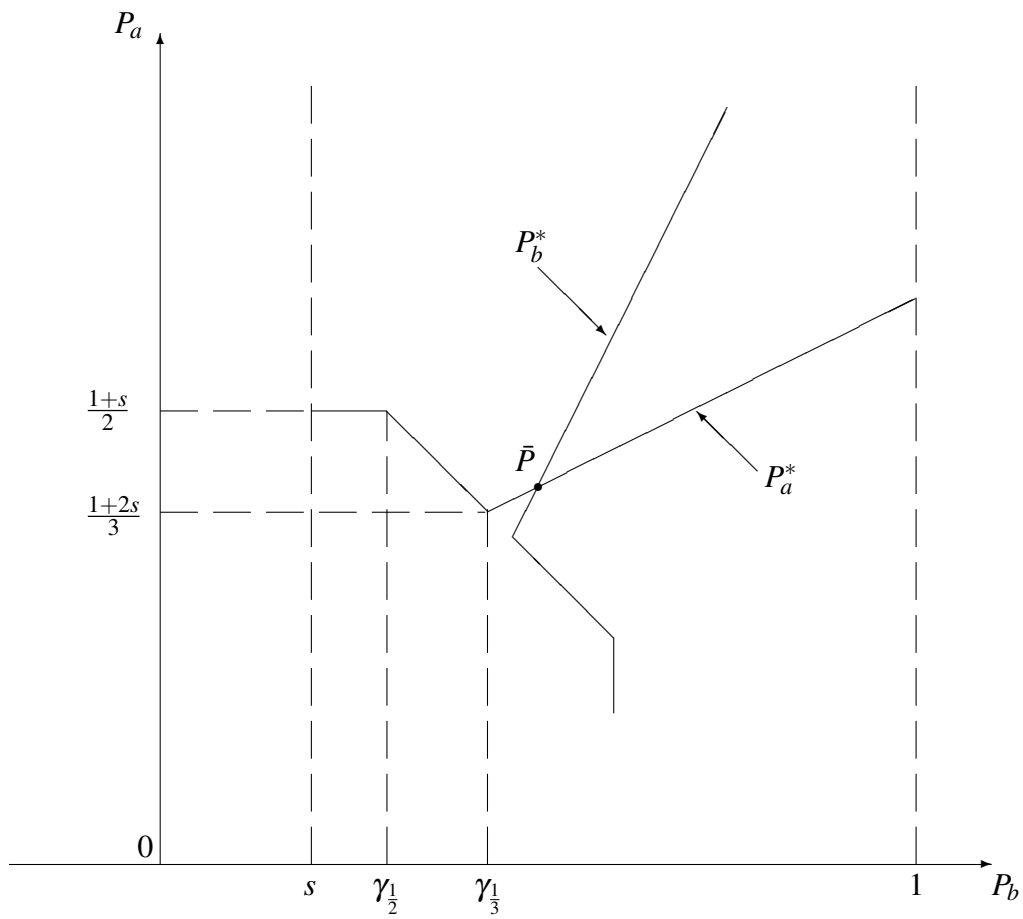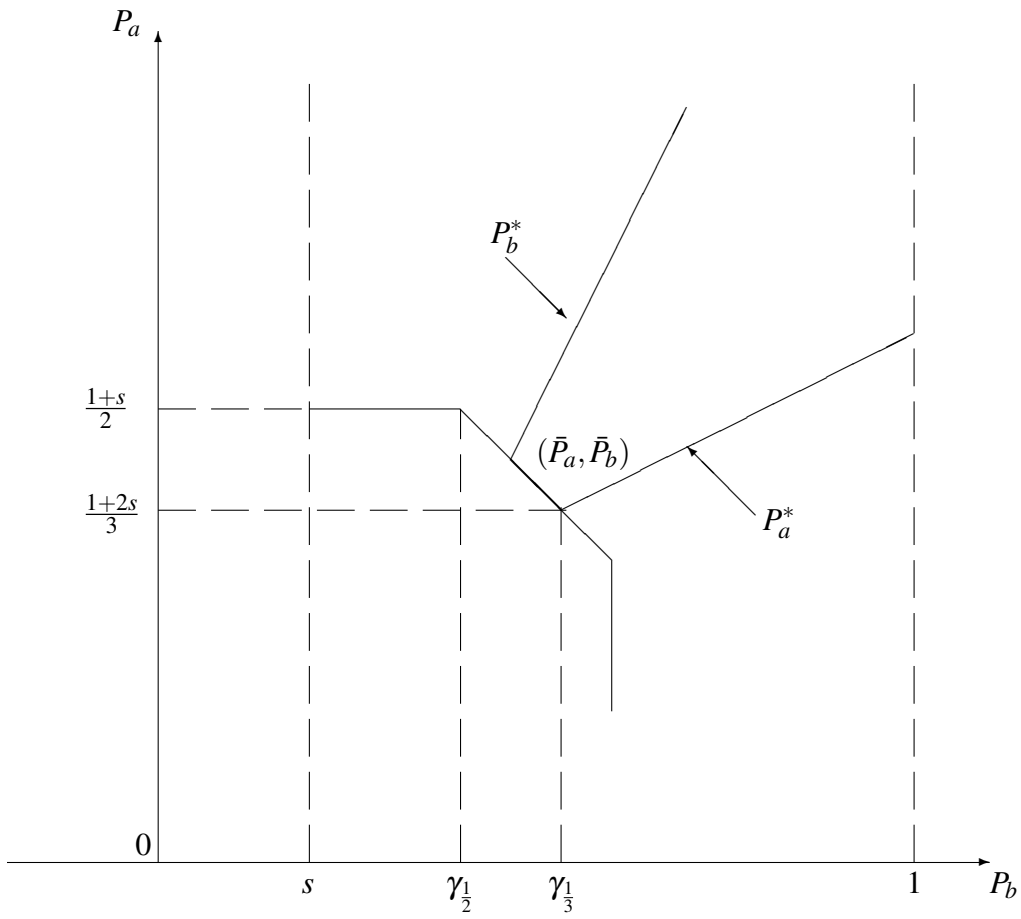
Figure A-3. Equilibrium prices when $\frac{1}{2}(1-s) < \gamma \leq \frac{2}{3}(1-s)$**. The solution is** $\bar{P}_a = \bar{P}_b = \bar{P} = 1 - \gamma$**.**

Figure A-4. Equilibrium prices when $\frac{2}{3}(1-s) < \gamma \le \frac{5}{6}(1-s)$**. The solution is** $\bar{P}_a, \bar{P}_b \in [\frac{1+2s}{3}, \gamma + \frac{4}{3}s - \frac{1}{3}]$**, with** $\bar{P}_a + \bar{P}_b = 2s + \gamma$**.**
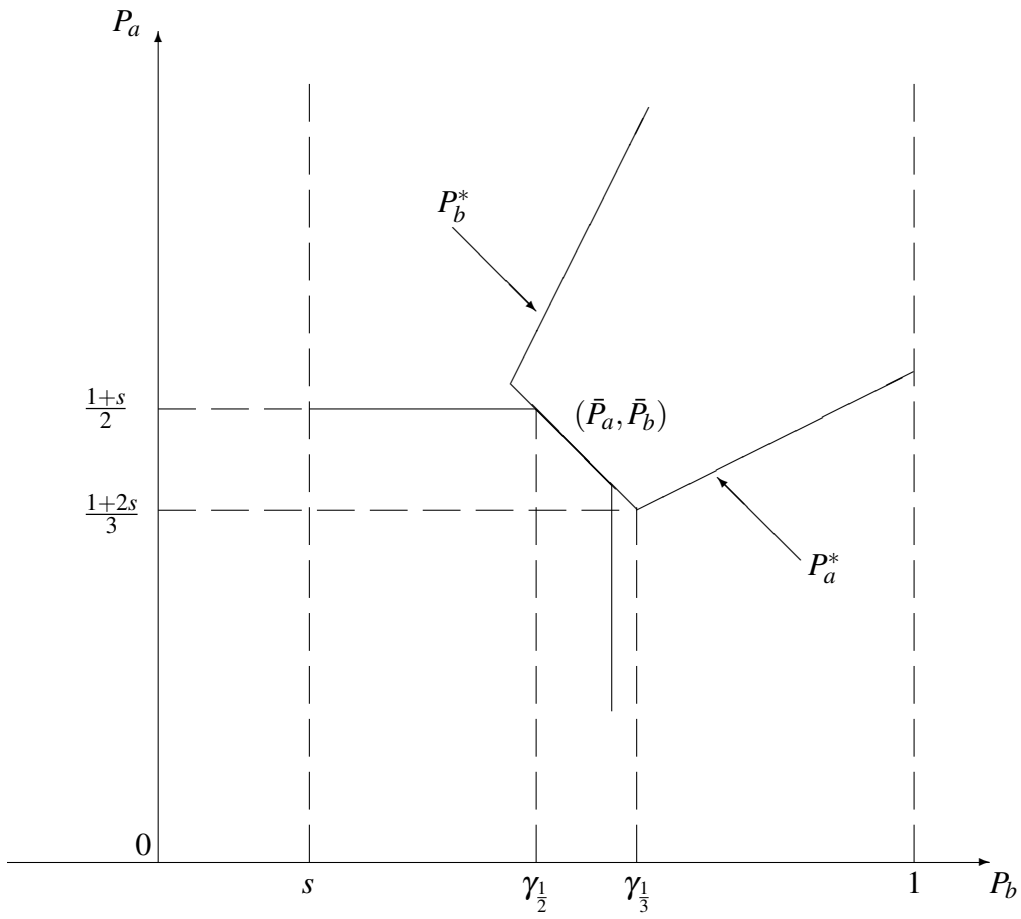
Figure A-5. Equilibrium prices when $\frac{5}{6}(1-s) < \gamma \le 1-s$. The solution is $\bar{P}_a, \bar{P}_b \in [\gamma + \frac{3}{2}s - \frac{1}{2}, \frac{1+s}{2}]$, with $\bar{P}_a + \bar{P}_b = 2s + \gamma$.

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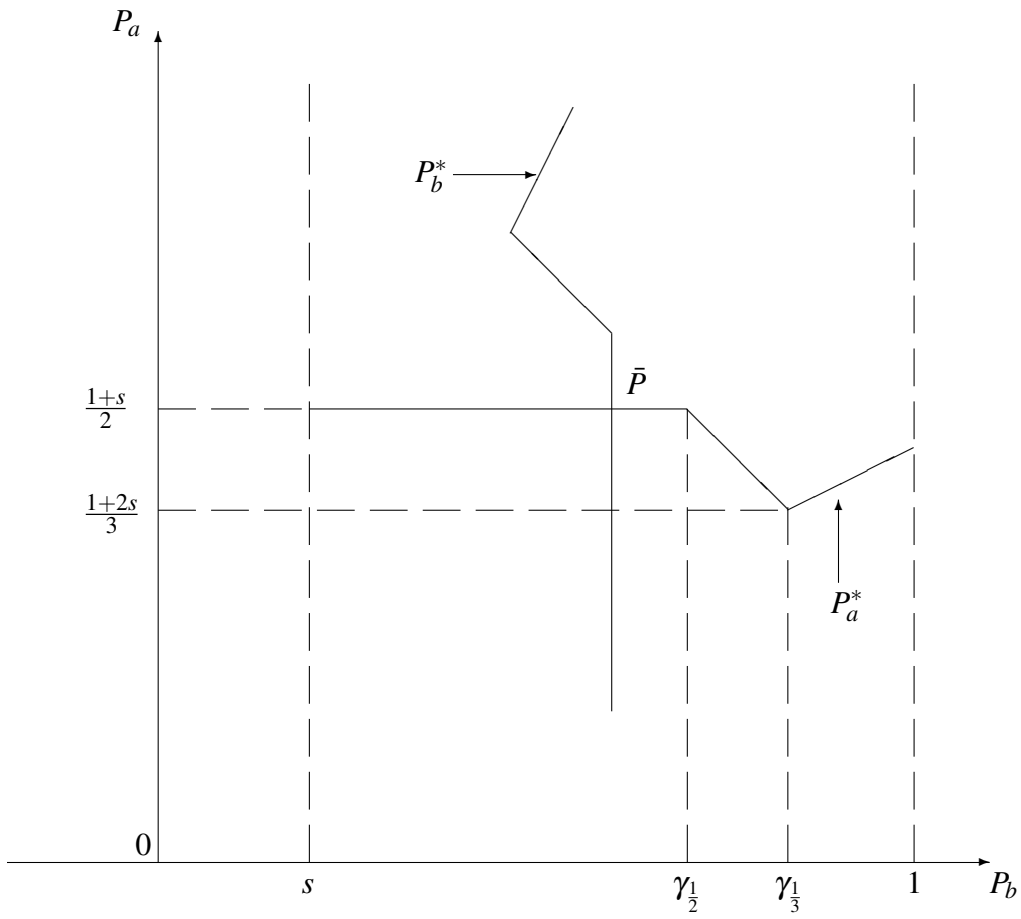
Figure A-6. Equilibrium prices when $1 - s < \gamma \le \frac{4}{3}(1-s)$**. The solution is** $\bar{P} = \frac{1+s}{2}$**.**
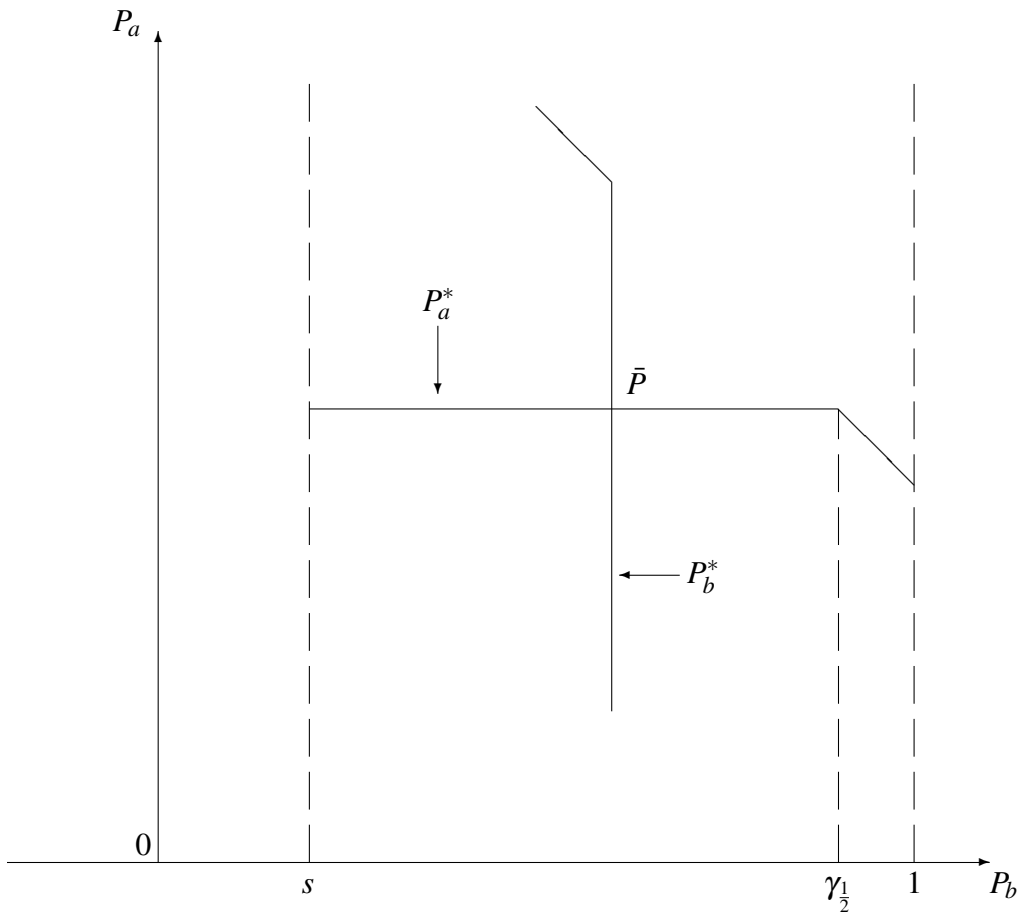
Figure A-7. Equilibrium prices when $\frac{4}{3}(1-s) < \gamma \leq \frac{3}{2}(1-s)$**. The solution is** $\bar{P} = \frac{1+s}{2}$**.**

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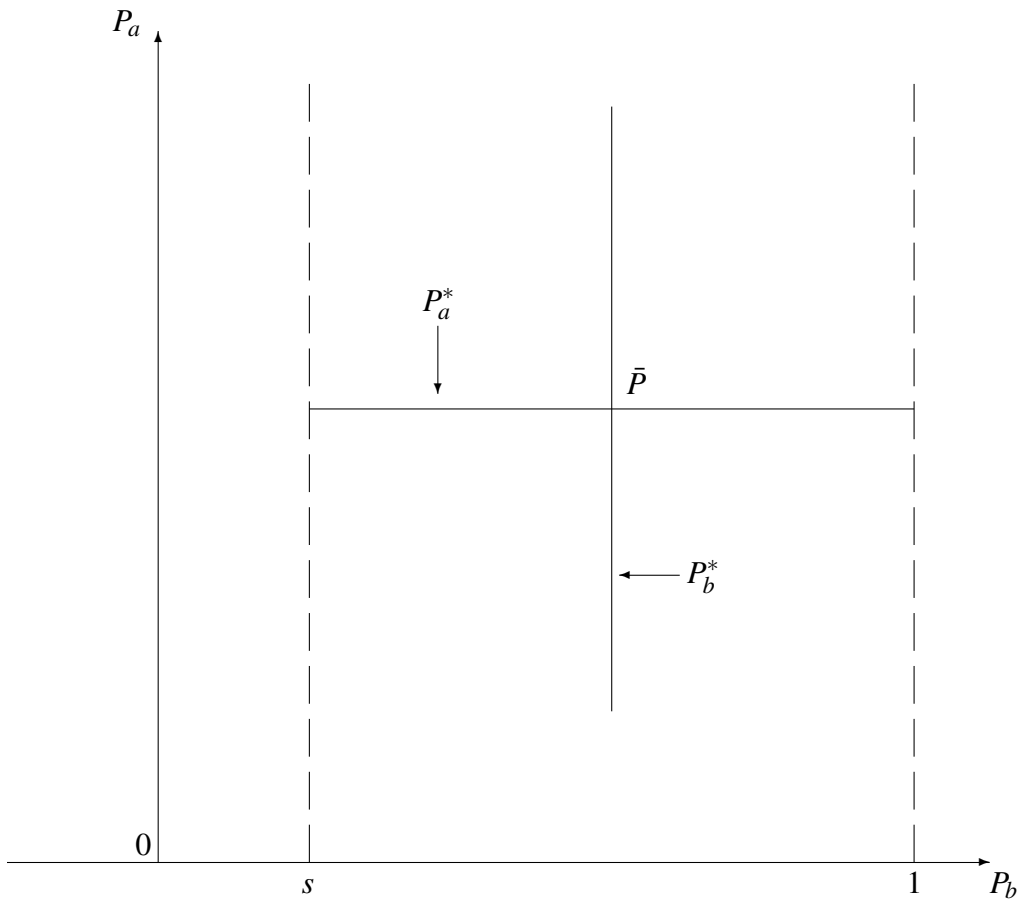
Figure A-8. Equilibrium prices when $\gamma > \frac{3}{2}(1-s)$. The solution is $\bar{P} = \frac{1+s}{2}$.
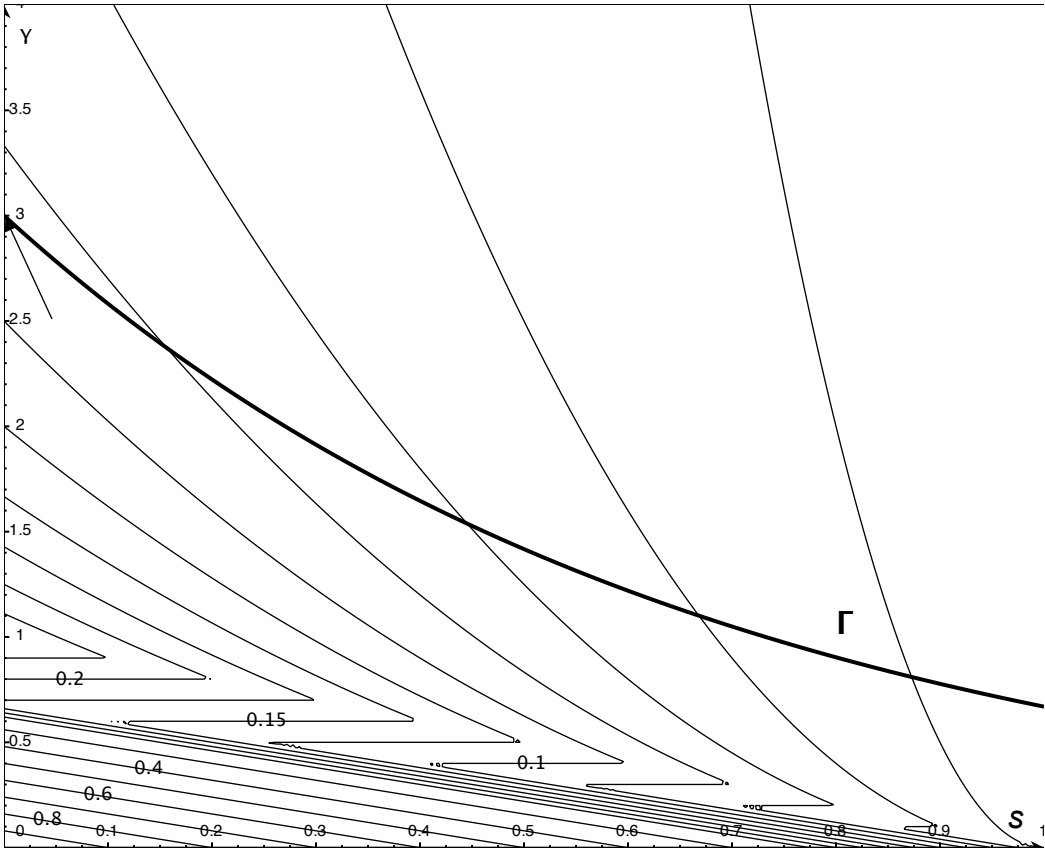
Figure A-9. Optimal assessment with $\Gamma(\sigma) = \frac{3}{e^{1.5\sigma}}$. The arrow points at the optimum $(\hat{s}, \hat{\gamma})$.
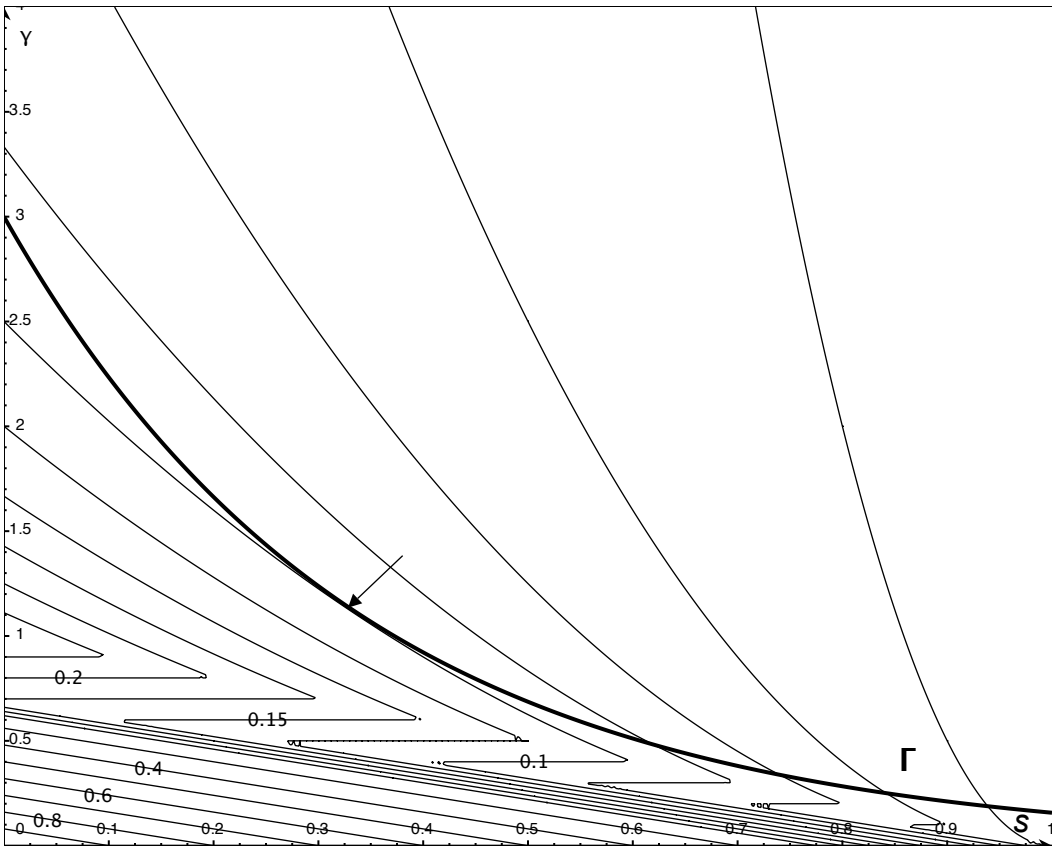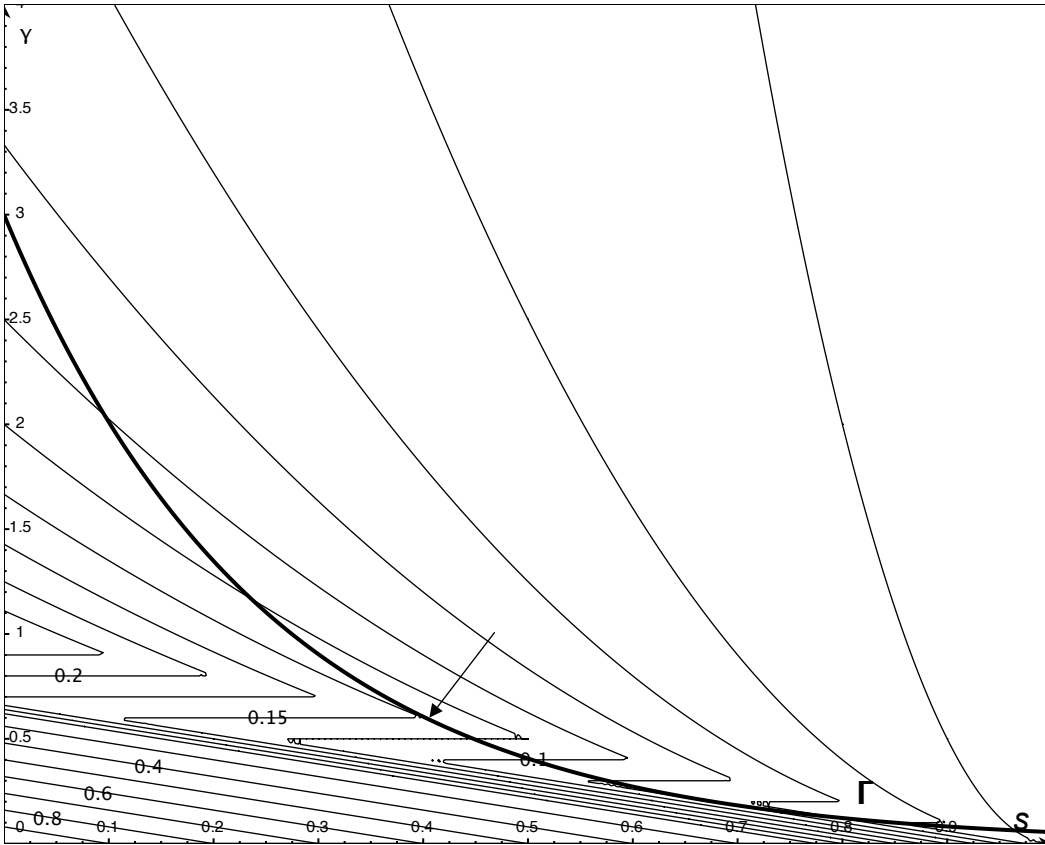
Figure A-10. Optimal assessment with $\Gamma(\sigma) = \frac{3}{e^{2.95\sigma}}$**. The arrow points at the optimum** $(\hat{s}, \hat{\gamma})$**.**

Figure A-11. Optimal assessment with $\Gamma(\sigma) = \frac{3}{e^{4\sigma}}$. The arrow points at the optimum $(\hat{s}, \hat{\gamma})$.
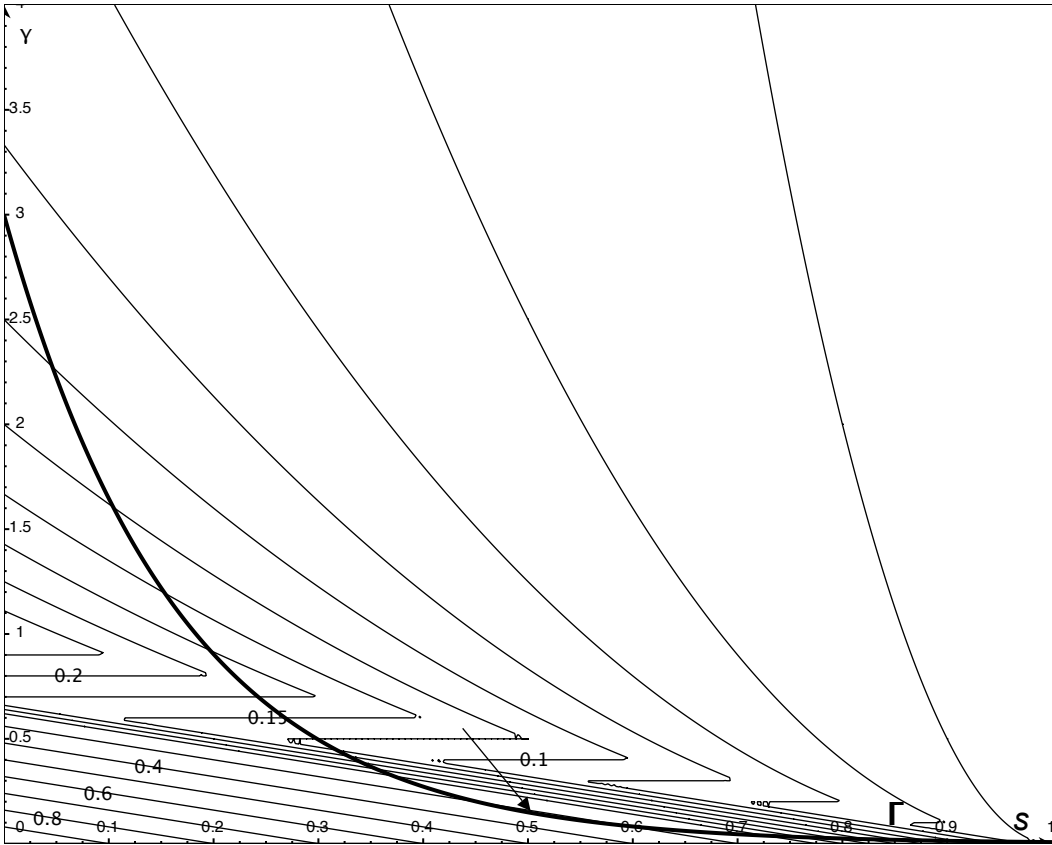
Figure A-12. Optimal assessment with $\Gamma(\sigma) = \frac{3}{e^{6\sigma}}$. The arrow points at the optimum $(\hat{s}, \hat{\gamma})$.