HEC MONTRĒAL

Institut
d'économie appliquée

The Simple Analytics of Price Signaling Quality

Leonard J. MIRMAN
Marc SANTUGINI

Cahier de recherche n° IEA-11-04

Version révisée
Février 2012

Institut d’économie appliquée
HEC Montréal
3000 chemin de la Côte-Sainte-Catherine
Montréal (Québec) H3T 2A7
Canada
http://www.hec.ca/iea
iea.info@hec.ca

ISSN : 0825-8643

The Simple Analytics of Price Signaling Quality

Leonard J. Mirman* Marc Santugini†

February 10, 2012

Abstract

We present a diagrammatic and step-by-step analysis of price signaling quality. Because quality is a continuum on the real positive line, out-of-equilibrium beliefs need not be specified, i.e., every positive price is a positive outcome in equilibrium. We first study the behavior of the monopoly when price conveys information about quality. We then show the effect of information flows on welfare, i.e., profit and consumer surplus.

Keywords: Asymmetric Information, Learning, Monopoly, Quality, Signaling.

JEL Classifications: D21, D42, D82, D83, D84, L12, L15.

*Department of Economics, University of Virginia. Email: lm8h@virginia.edu.

†Corresponding author. Institute of Applied Economics and CIRPÉE, HEC Montréal. Email: marc.santugini@hec.ca.

1 Introduction

We present a diagrammatic and step-by-step analysis of price signaling quality. Because quality is a continuum on the real positive line, out-of-equilibrium beliefs need not be specified, i.e., every positive price is a positive outcome in equilibrium. In this context, Mirman and Santugini (2011) provides conditions for the existence of an equilibrium in which the price signals quality, hereafter a *signaling* equilibrium. Moreover, Mirman and Santugini (2011) shows that when the signaling equilibrium exists, the price is linear in quality. In this paper, we take advantage of the linearity property to provide a simple analysis of price signaling quality when the unknown quality is a continuum on the real positive line.[1] The paper is organized as follows. Section 2 presents the model. Section 3 provides the analysis.

2 Model

Consider a market for a good of quality $\theta \in \mathbb{R}_+$ sold at price $P \geq 0$. The demand side is composed of informed and uninformed price-taking buyers. Informed buyers know θ and have demand $q^I = \theta - P$, and uninformed buyers do not know θ, but infer it from observing the price. Specifically, upon observing P, the uninformed buyers' beliefs about quality is represented by the updating rule $\chi(P)$, so that their demand is $q^U = \chi(P) - P$. The price plays the usual role of a parameter defining the feasible set of purchases as well as an informative role, due to the presence of uninformed buyers, about quality.

Let the mass of buyers be normalized to one and $\lambda \in [0, 1]$ be the fraction of informed buyers, then aggregate demand is

$$D(P, \theta, \chi(P)) = \lambda(\theta - P) + (1 - \lambda)(\chi(P) - P). \tag{1}$$

[1] See Bagwell and Riordan (1991) for the case in which the unknown quality takes on two values. See also Daughety and Reinganum (1995, 2005, 2007, 2008a,b) and Janssen and Roy (2010) when the good is not potentially valueless, i.e., unknown quality is never zero.

It is useful to refer to (1) as the *signaling* demand when $\lambda \in [0, 1)$ and the *full-information* demand when $\lambda = 1$.

On the supply side, there is a monopolist, who knows the quality θ, with marginal cost $c\theta$, $c \in [0, 1)$. The objective of the monopolist is to choose P to maximize profits

$$\pi = (P - c\theta)D(P, \theta, \chi(P)). \tag{2}$$

When $\lambda \in [0, 1)$, the learning activity of the uninformed buyers influences profit, and, thus, constitutes an informational externality to the monopolist.

The signaling equilibrium consists of a price as a function of quality and an updating rule as a function of price, such that profits are maximized and the price reveals θ.[2] In order to obtain full revelation, the updating rule must be *informationally consistent* with the price strategy of the firm, i.e., the updating rule is the inverse of the price function. The superscript S refers to *signaling*. Formally,

Definition 2.1. *The pair* $\{P^S(\theta), \chi^S(P)\}$ *is a signaling equilibrium if, for all* $\theta \in \mathbb{R}_+$,

1. *Given* $\chi^S(P)$,

$$P^S(\theta) = \arg\max_{P \geq 0} (P - c\theta)D(P, \theta, \chi^S(P)). \tag{3}$$

2. *Given* $P^S(\theta)$,

$$\chi^S(P^S(\theta)) = \theta. \tag{4}$$

Note that there exists a unique equilibrium, in which the updating rule is linear and increasing in the price. Because we consider a good that is potentially valueless, we focus on the case in which demand is composed of both informed and uninformed buyers.[3]

[2] Definition 2.1 focuses on a *separating* equilibrium. We do not analyze *pooling* or *non-signaling* equilibrium because, unless all buyers buyers are uninformed and the firm faces no cost, a non-signaling equilibrium in which the price is uninformative about quality does not exist. See Appendix A.

[3] When all buyers are uninformed and the good is potentially valueless, i.e., $\lambda = 0$ and $\theta = 0$ is the lower bound on quality, there exists a unique signaling equilibrium, however,

Assumption 2.2. $\lambda \in (0,1)$.

Proposition 2.3 is a special case of Propositions 3.4 and 3.5 in Mirman and Santugini (2011).

Proposition 2.3. *Under Assumption 2.2, given* (1) *and* (2)*, there exists a unique signaling equilibrium in which the updating rule is linear and increasing in* P*, i.e.,* $\chi^S(P) = \beta^S P$*,* $\beta^S \in (1, 1/(1-\lambda))$.

Proof. See the proof of Propositions 3.4 and 3.5 in Mirman and Santugini (2011). ☐

Before proceeding with the analysis, an important caveat is in order. Note that informed buyers' demand is $q^I = \theta - P$ and $q^U = \chi(P) - P$ rather than $q^I = \max\{\theta - P, 0\}$ and $q^U = \max\{\chi(P) - P, 0\}$. Expression (1) does not ignore the possibility of informed buyers exiting the market for prices above θ, rather it encompasses the idea that the monopoly faces competition from a fringe firm for high prices. This additional element is necessary for obtaining existence of a signaling equilibrium when the quality is on the real positive line. Indeed, Mirman and Santugini (2011) shows that there does not exist a signaling equilibrium when $q^I = \max\{\theta - P, 0\}$ and $q^U = \max\{\chi(P) - P, 0\}$ for $\theta \in \mathbb{R}_+$. If informed buyers can exit the market for prices above θ, then the monopolist (of any quality) has the incentive to sacrifice informed buyers in order to deceive uninformed buyers. Mirman and Santugini (2011) also shows that existence can be reestablished with the presence (or threat) of a fringe competition, since the fringe firm can remove the incentive for the monopolist to price above θ. One special case of fringe competition is equivalent to writing $q^I = \theta - P$ and $q^U = \chi(P) - P$. We retain this simple functional form. Using a more general (residual) demand would complicate the diagrammatic analysis without affecting the substance of the analysis.[4]

without trading. In other words, while the price is increasing in quality (and, thus, conveys information), the price is equal to the reservation price, which yields zero demand for any level of quality. See Bagwell and Riordan (1991) and Mirman and Santugini (2011). However, when the good is not potentially valueless, i.e., the lowest quality does generate positive demand, a signaling equilibrium with trading is possible with only uninformed buyers. See Daughety and Reinganum (1995, 2005, 2008a). However, out-of-equilibrium beliefs have to be selected.

[4]See Appendix B for a general exposition of the results presented in Mirman and

3 Analysis

Given Proposition 2.3, without loss of generality, the analysis can be restricted to the class of linear updating rules. We first derive the signaling demand given *any* linear updating rule and characterize the set of valid candidates for the equilibrium linear updating rule. Second, we solve the monopoly problem given a valid linear updating rule, i.e., the monopolist's optimal price given a valid linear updating rule is derived. Third, imposing informational consistency on the posterior beliefs of the uninformed buyers and the behavior of the firm, the updating rule is the inverse of the price function.

Signaling Demand. Let $\chi(P) = \beta^e P$ be an arbitrary linear updating rule, where $\beta^e > 0$ is the uninformed buyers' *expected* parameter regarding the relationship between the price and the unknown quality. Plugging $\chi(P) = \beta^e P$ into (1) yields

$$D(P, \theta, \chi(P)) = \lambda\theta - (1 - (1 - \lambda)\beta^e)P. \tag{5}$$

Solving (5) for P yields the inverse aggregate demand

$$P = \frac{\lambda\theta - q}{1 - (1 - \lambda)\beta^e}, \tag{6}$$

where q is quantity. Figure 1 depicts both signaling demand and full-information demand represented by solid lines and dashed lines, respectively.[5] As shown in Figure 1, the informational externality due to the learning activity of the uninformed buyers modifies the demand faced by the monopolist.[6] In particular, signaling demand is steeper than its full-information counterpart. This is due to the fact that, under signaling, an increase in the price has two (opposite) effects. While a higher price decreases the quantity demanded for both informed and uninformed buyers, it also raises the posterior mean beliefs of the uninformed buyers. Hence, updating beliefs dampens the decrease

Santugini (2011).

[5] Full-information inverse demand is $P = \theta - q$.

[6] To generate the figures, we set $\theta = 1$, $c = 0.3$, and $\lambda = 0.5$.

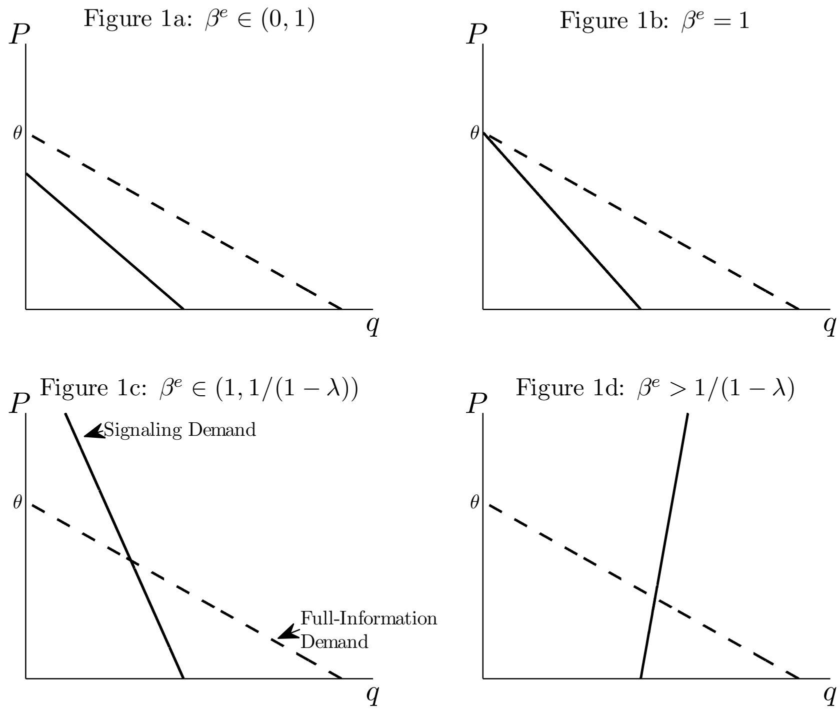

Figure 1: Full-Information and Signaling Demands

in quantity.

Figure 1 provides 4 different types of signaling demands depending on the value of $\beta^e > 0$, i.e., the expectations of the uninformed buyers. Figure 1a shows a signaling demand inward of the full-information demand when $\beta^e \in (0, 1)$. Figure 1b depicts the case in which both demands have the same intercept on the y-axis when $\beta^e = 1$. Figure 1c draws the signaling demand crossing the full-information demand from above when $\beta^e \in (1, 1/(1 - \lambda))$, while Figure 1d presents the case of an upward-sloping signaling demand when $\beta^e > 1/(1 - \lambda)$.

Some types of signaling demands can be immediately discarded. Indeed, a valid candidate for the equilibrium updating rule needs to meet two requirements. The first is that the signaling demand maintain a negative relationship between quantity and price in order for a solution to the monopoly problem to exist. That is, $\beta^e \in (0, 1/(1-\lambda))$. The second concerns the full-revelation nature of a signaling equilibrium. If there is a signaling equilibrium, full revelation implies that the signaling demand must cross the full-information demand. Indeed, since $\chi^S(P^S(\theta)) = \theta$, the equilibrium price-quantity pair must lie on both the full information and signaling demands. Both requirements are satisfied only in Figure 1c, i.e., $\beta^e \in (1, 1/(1-\lambda))$. Formally,

Remark 3.1. *In a signaling equilibrium, $\beta^e \in (1, 1/(1-\lambda)$.*

Monopolist's Optimal Price. From (5), the maximization problem of the monopolist given a valid linear updating rule, i.e., $\beta^e \in (1, 1/(1-\lambda))$, is

$$\max_{P \geq 0}(P - c\theta)(\lambda\theta - (1-(1-\lambda)\beta^c)P). \tag{7}$$

Equivalently, using (6),

$$\max_{q \geq 0} \frac{\lambda\theta - q}{1-(1-\lambda)\beta^e} q - c\theta q. \tag{8}$$

Because the diagrammatic analysis is easier in the context of a quantity-setting monopolist, we focus on (8). The analysis for the price-setting monopolist is equivalent and relegated to Appendix C.

The first-order condition corresponding to (8) is

$$\frac{\lambda\theta - 2q}{1-(1-\lambda)\beta^e} = c\theta. \tag{9}$$

Solving (9) for optimal output $\hat{q}$ yields

$$\hat{q} = \frac{\lambda\theta - (1-(1-\lambda)\beta^e)c\theta}{2}. \tag{10}$$

Plugging (10) into (6) yields

$$\hat{P} = \frac{\lambda + (1-(1-\lambda)\beta^e)c}{2(1-(1-\lambda)\beta^e)}\theta. \tag{11}$$

Since $\beta^e \in (1, 1/(1-\lambda))$ is a valid candidate for posterior beliefs, the second-

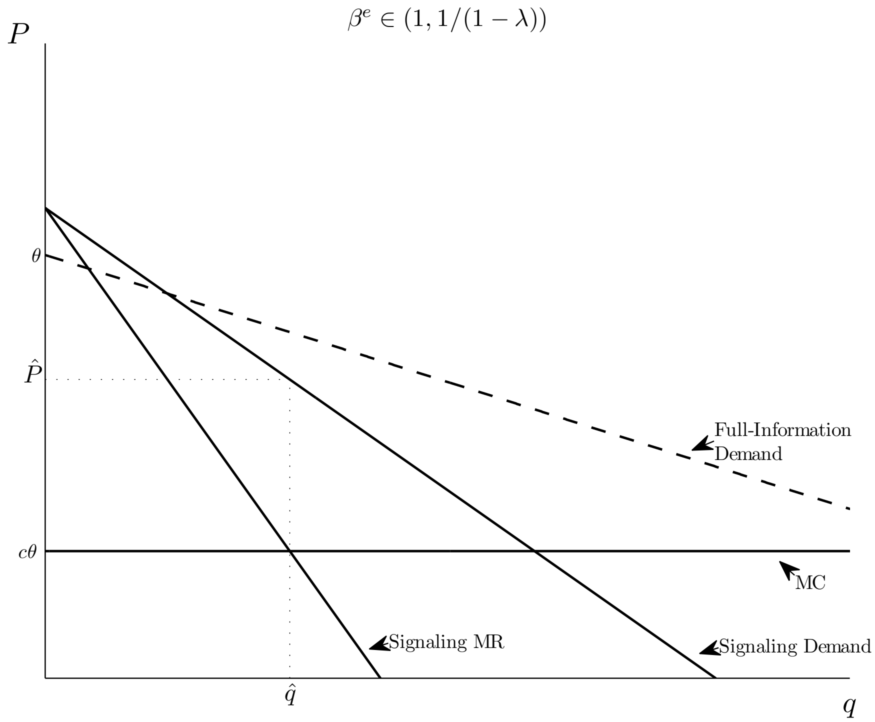

Figure 2: Optimal Behavior given Valid Linear Updating Rule

order condition is satisfied.

Optimal behavior for valid linear updating rule is depicted in Figure 2. Given $\beta^e \in (1, 1/(1-\lambda))$, the monopolist sets the price $\hat{P}$ and sells $\hat{q} = D(\hat{P}, \theta, \beta^e \hat{P})$ units which equates the marginal revenue corresponding to the signaling demand with the marginal cost. While Figure 2 depicts the best response of the monopolist under a signaling environment, it does not depict a signaling equilibrium. Indeed, since the equilibrium updating rule must yield full revelation of the unknown quality, the equilibrium price-quantity pair must lie on both the signaling and full-information demands. This is not the case in Figure 2 as the pair $\{\hat{P}, \hat{q}\}$ lies below the full-information demand. In other words, the solution depicted in Figure 2 is not informationally consistent.

Equilibrium. Informational consistency is shown in Figure 3, in which

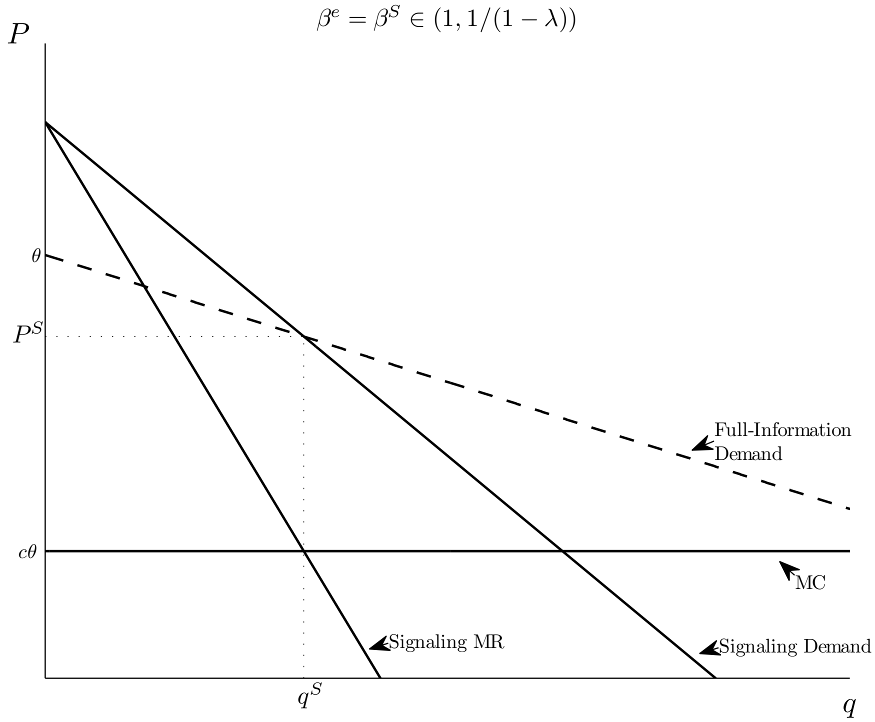

Figure 3: Optimal Behavior given Equilibrium Updating Rule

the signaling demand crosses the full information demand at the optimal quantity, $q^S = D(P^S(\theta), \theta, \chi^S(P^S(\theta))) = D(P^S(\theta), \theta, \theta) = \theta - P^S(\theta)$, i.e., where the marginal revenue corresponding to the signaling demand crosses the marginal cost. In other words, the updating rule implicit in the signaling demand in Figure 3 is consistent with the relationship between the price and the quality established by the monopolist.

To derive the situation in Figure 3 analytically , informational consistency must be imposed, i.e., the updating rule $\chi^S(P) = \beta^S P$ is the inverse of the price function $P^S(\theta)$. Formally, using (11)

$$\chi^S(P) = \frac{2(1-(1-\lambda)\beta^e)}{\lambda + (1-(1-\lambda)\beta^e)c} P, \tag{12}$$

$$= \beta^S P, \tag{13}$$

so that $\frac{2(1-(1-\lambda)\beta^e)}{\lambda+(1-(1-\lambda)\beta^e)c} = \beta^S$, $\beta^e = \beta^S$. Hence, the uninformed buyers' equilibrium parameter of the updating rule is the solution to the second-order polynomial

$$(1-\lambda)cx^2 - (2-\lambda+c)x + 2 = 0. \tag{14}$$

For $\lambda \in (0,1)$, the left-hand side of (14) is strictly convex in x and both roots are positive. If $x = 1$, the left-hand side of (14) is positive. If $x = 1/(1-\lambda)$, the left-hand side of (14) is negative. Hence, the smallest root of (14) is the only solution for equilibrium posterior beliefs, i.e.,

$$\beta^S = \begin{cases} \frac{2}{2-\lambda}, & c = 0 \\ \frac{2-\lambda+c-\sqrt{(2-\lambda+c)^2-8(1-\lambda)c}}{2(1-\lambda)c}, & c \in (0,1) \end{cases}, \tag{15}$$

$\beta^S \in (1, 1/(1-\lambda))$.

Using (10), (11), and (15), Proposition 3.2 follows.[7]

Proposition 3.2. *Suppose Assumption 2.2 holds. In a signaling equilibrium, given* (1) *and* (2)*, the monopolist sets the price*

$$P^S(\theta) = \theta/\beta^S, \tag{16}$$

and sells

$$q^S(\theta) = \frac{\beta^S - 1}{\beta^S}\theta \tag{17}$$

units, where $\beta^S \in (1, 1/(1-\lambda))$ *is defined by* (15).

Comparisons. Having fully characterized the equilibrium, we proceed with a comparison between the full-information and signaling environments. To simplify the welfare analysis, assume that the firm faces no cost. The results hold for $c \in (0,1)$, but the case of no cost yields a simple closed-form solution. Proposition 3.3 provides equilibrium price-quantity pairs under both signaling and full-information equilibrium.[8] Let $\{P^{FI}(\theta), q^{FI}(\theta)\}$ be

[7]Since the equilibrium pair lies on the full-information demand, $q^S(\theta) = \theta - P^S(\theta)$.

[8]Signaling equilibrium values for price and quantity can be recovered from (15) evaluated at $c = 0$. Full-information equilibrium values for price and quantity can be recovered from solving (14) evaluated at $\lambda = 1$ and $c = 0$, so that $x = 2$.

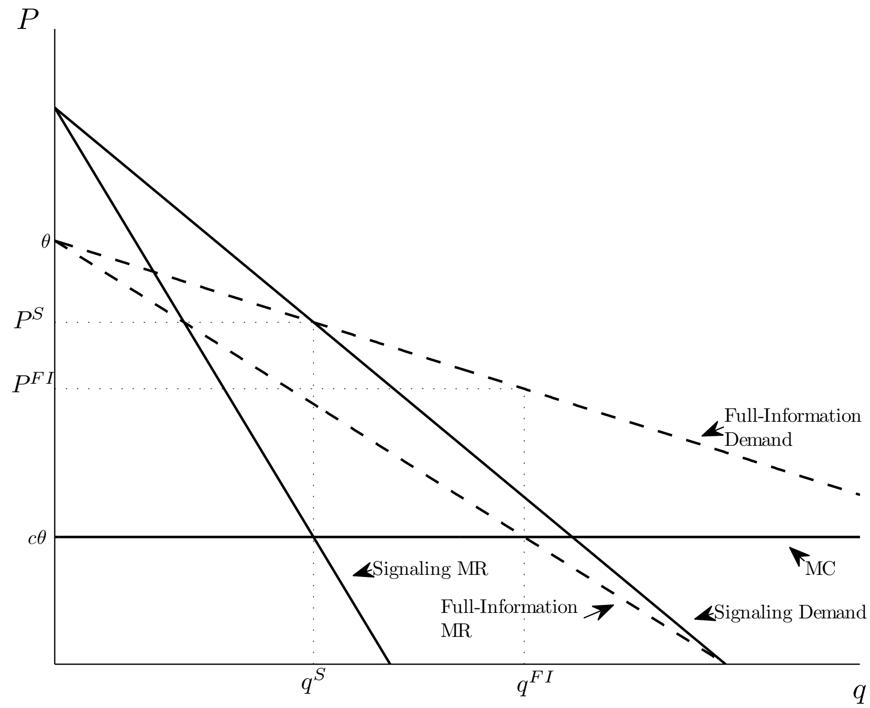

Figure 4: Full-Information vs. Signaling

the price-quantity pair corresponding to the monopoly problem under full-information, i.e., $\lambda = 1$.

Proposition 3.3. *Suppose $c = 0$. Then, the signaling equilibrium pair is*

$$\left\{P^S(\theta), q^S(\theta)\right\} = \left\{\frac{(2-\lambda)\theta}{2}, \frac{\lambda\theta}{2}\right\}, \tag{18}$$

while the full-information equilibrium pair is

$$\left\{P^{FI}(\theta), q^{FI}(\theta)\right\} = \left\{\frac{\theta}{2}, \frac{\theta}{2}\right\}. \tag{19}$$

Using Proposition 3.3, we compare strategies, profits and consumer surpluses under full-information and signaling environment.

From Figure 4, note that signaling demand always crosses the full-information

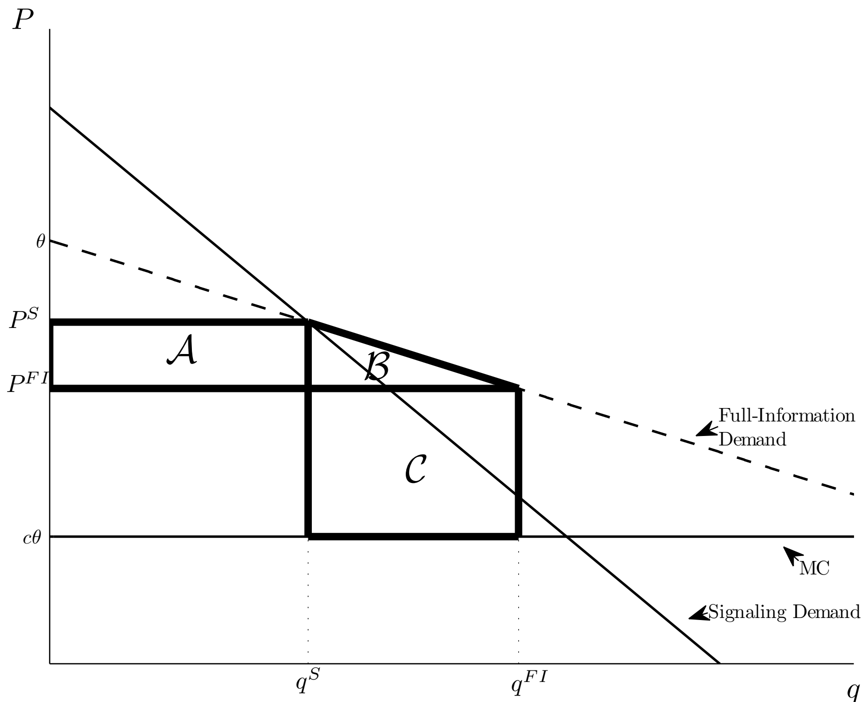

Figure 5: Welfare Loss

demand from above, so that $P^S > P^{FI}$ and $q^S < q^{FI}$.[9] The difference between the two prices depends on the steepness of the signaling demand. Specifically, from (18) and (19), the difference in the equilibrium prices is $P^S - P^{FI} = \lambda\theta/2$. The more uninformed buyers, the bigger the difference in equilibrium prices.

By increasing price, signaling yields a welfare loss as shown in Figure 5. Specifically, signaling profit is

$$\pi^S = \frac{\lambda(2-\lambda)\theta^2}{4}, \tag{20}$$

[9]To simplify the discussion, we drop the argument θ.

while full-information profit is

$$\pi^{FI} = \frac{\theta^2}{4}. \tag{21}$$

The difference $\pi^S - \pi^{FI}$ is always negative. The loss in profit is illustrated in Figure 5. While the firm gains the rectangular area $\mathcal{A}$, it loses the rectangular area $\mathcal{C}$.

Regarding consumer welfare, while it is beneficial for buyers to infer information from the price, the effect of signaling on the monopolist's behavior affects, in turn, consumer welfare. Specifically, the signaling consumer surplus is

$$CS^S = \frac{\lambda^2\theta^2}{8}, \tag{22}$$

while the full-information consumer surplus is

$$CS^{FI} = \frac{\theta^2}{8}. \tag{23}$$

Here, both (22) and (23) are computed from the full-information demand curve because there is full revelation. The presence of uninformed buyers imposes a cost in terms of consumer surplus. The loss in consumer surplus due to signaling is represented by the triangular area $\mathcal{B}$ in Figure 5.

A Non-Signaling Equilibrium

In this appendix, we consider the existence and characterization of an equilibrium in which the price is uninformative about quality. Such an equilibrium is hereafter referred to as a non-signaling equilibrium.

Definition A.1 presents the non-signaling equilibrium in which the price transmits no information about quality, and, thus, the uninformed buyers revert to prior mean beliefs $\mu \geq 0$ for any P. The superscript NS refers to *non-signaling.*

Definition A.1. *The pair* $\{P^{NS}(\theta), \chi^{NS}(P)\}$ *is a non-signaling equilibrium if, for all* $\theta \in \mathbb{R}_+$,

1. *Given* $\chi^{NS}(P)$,

$$P^{NS}(\theta) = \arg\max_{P \geq 0} (P - c\theta) D(P, \theta, \chi^{NS}(P)). \tag{24}$$

2. *Given* $P^{NS}(\theta)$,

$$\chi^{NS}(P^{NS}(\theta)) = \mu. \tag{25}$$

Proposition A.2 states the conditions for the existence of a non-signaling equilibrium.

Proposition A.2. *Given* (1) *and* (2), *there exists a non-signaling equilibrium if and only if* $\lambda = 0$ *and* $c = 0$.

Proof. Suppose that there exists a non-signaling equilibrium. Plugging $\chi^{NS}(P) = \mu$ and (1) into (24) yields

$$\max_{P \geq 0} (P - c\theta)(\lambda(\theta - P) + (1 - \lambda)(\mu - P). \tag{26}$$

From the first-order condition,

$$P^{NS}(\theta) = \frac{\lambda\theta + (1 - \lambda)\mu + c\theta}{2}. \tag{27}$$

If $\lambda \in (0,1)$ or $c \neq 0$, then (27) is increasing in θ, and, thus, informative about quality. However, if $\lambda = 0$ and $c = 0$, then a non-signaling equilibrium exists as the price is indeed uninformative about quality. □

B Fringe Competition

In this appendix, we first show that there cannot be an equilibrium when demands are $q^I = \max\{\theta - P, 0\}$ and $q^U = \max\{\chi(P) - P, 0\}$. We then provide an interpretation for demands $q^I = \theta - P$ and $q^U = \chi(P) - P$. See Mirman and Santugini (2011) for an extensive analysis.

Given Proposition 2.3,

$$\{P^S(\theta), q^S(\theta)\} = \left\{\frac{\theta}{\beta^S}, \frac{\beta^S - 1}{\beta^S}\theta\right\}, \tag{28}$$

is the equilibrium price-quantity pair in a signaling equilibrium, where $q^S(\theta) = \theta - P^S(\theta)$ is the equilibrium quantity.

When aggregate demand is of the form

$$q^D = \lambda \max\{\theta - P, 0\} + (1-\lambda)\max\{\chi(P) - P, 0\}, \tag{29}$$

there does not exist a signaling equilibrium. To see this, it suffices to show that (28) is not an equilibrium. Indeed, the solid line in Figure 6 depicts (29) evaluated at $\chi^S(P) = \beta^S P$. For $P < \theta$, both informed and uninformed buyers purchase the good. Although the demand of the uninformed buyers is upward-sloping, the aggregate demand is downward-sloping for prices below the reservation price.[10] However, for $P > \theta$, the informed buyers exit the market and the demand curve becomes upward-sloping, due to the informed buyers' upward-sloping demand curve. The isoprofit curve in Figure 6 is the locus of pairs $\{P, q\}$ yielding equilibrium profits $\pi^S(\theta)$.[11] The point $\{P^S, q^S\}$ is the signaling solution defined by (28), which yields profits $\pi^S(\theta)$.

[10] Indeed, signaling establishes a positive relationship between the price and the quantity demanded by the uninformed buyers, i.e., $\chi^S(P) - P > 0$ is increasing in $P \geq 0$.

[11] Equilibrium profits are $\pi^S(\theta) = (P^S(\theta) - c\theta)q^S(\theta)$. Hence, the isoprofit function is $P = c\theta + \pi^S(\theta)/q$.

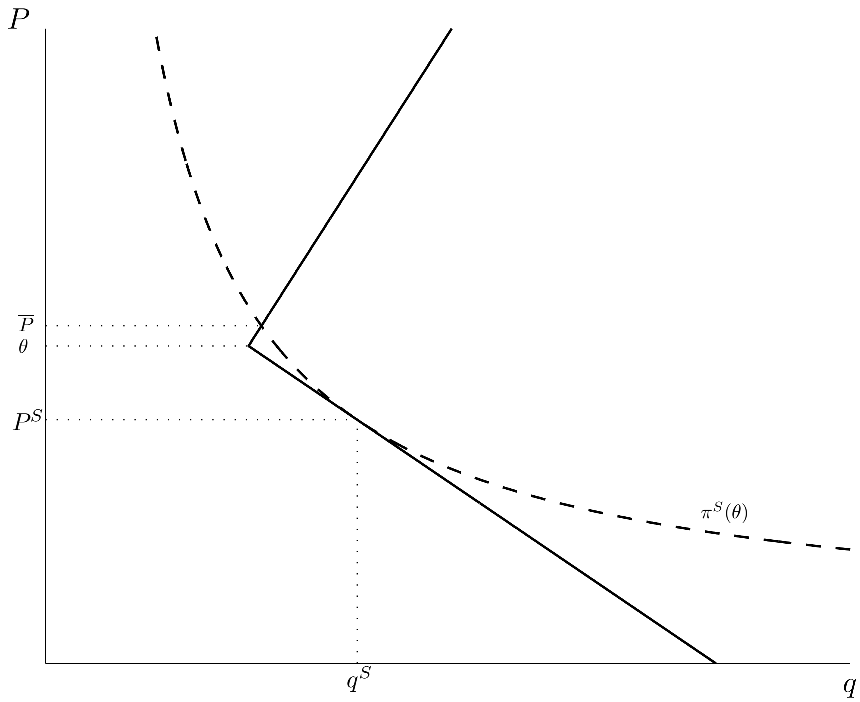

Figure 6: Monopoly without Threat of Competition

Figure 6 shows that there is always an incentive for the monopoly to deviate from $\{P^S, q^S\}$. Indeed, any price above $\overline{P}$ yields profits greater than $\pi^S(\theta)$ to the deviant firm. By charging a higher price, the monopoly sacrifices revenue from the informed buyers, but is able to deceive the uninformed buyers, making higher profits from them. Therefore, (28) cannot constitute a signaling equilibrium.

To obtain a signaling equilibrium in this environment, it is necessary to assume the existence of fringe competition. When the monopoly (i.e., the only price-setting firm) faces (residual) demand

$$q^{RD} = \lambda \max\{\theta - P, 0\} + (1-\lambda)\max\{\chi(P) - P, 0\} - \varphi \max\{P - c_F\theta, 0\}, \quad (30)$$

where $\varphi \max\{P - c_F\theta, 0\}$ is the quantity supplied by the competitive fringe

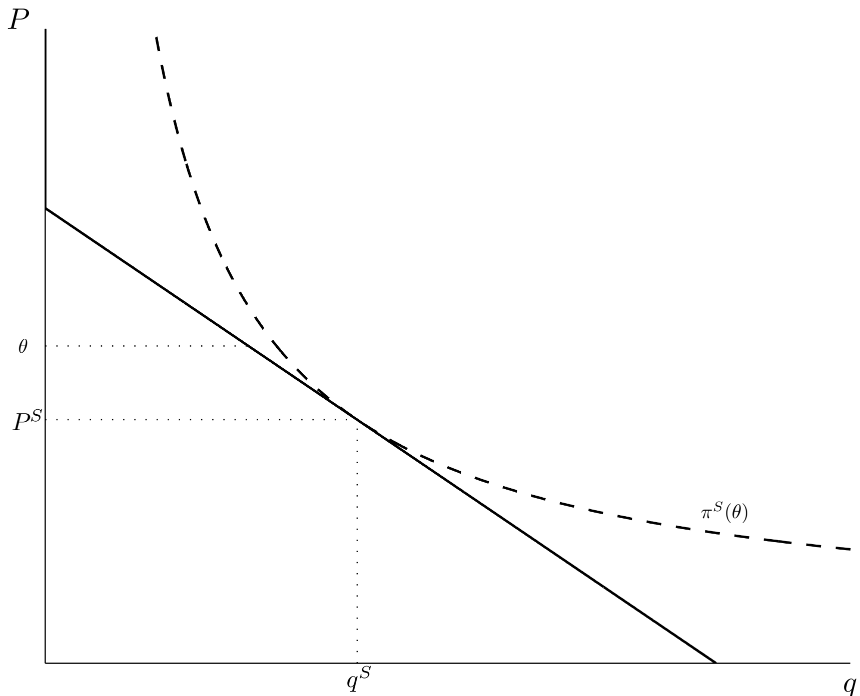

Figure 7: Monopoly with Threat of Competition

of size $\varphi \geq 0$ facing marginal cost $c_F\theta$, $c_F \in [0, 1]$, then there might exist a signaling equilibrium. See Propositions 3.4 and 3.5 in Mirman and Santugini (2011). We restate the proposition for the sake of clarity. Formally,

Proposition B.1. *Given* (1) *and* (2)*, there exists a signaling equilibrium provided that* $\varphi \geq \underline{\varphi}$ *where* $\underline{\varphi} \in (0, \lambda)$ *for* $\lambda \in (0, 1)$*.*

The presence of a competitive fringe changes the slope of the equilibrium residual demand. Indeed, when there is enough presence of the competitive fringe (i.e., $\varphi \geq \underline{\varphi}$), the equilibrium residual demand is downward-sloping enough so as to remove any incentive for the monopoly to deviate. In other words, the benefit of deceiving the uninformed buyers is reduced enough to be outweighed by the cost of facing competition. Graphically, for an equilibrium to exist, demand must never cross the isoprofit curve for prices above θ. Figure 7 considers a special case for which the incentive to deviate is blocked.[12] Indeed, when $\varphi = \lambda$ and $c_F = 1$, then the linearity of the demand remains for all prices. The special case is equivalent to using demands $q^I = \theta - P$ and $q^U = \chi(P) - P$.

C Price-Setting Monopoly

The first-order condition corresponding to (7) is

$$\lambda\theta - (1 - (1-\lambda)\beta^e P = (P - c\theta)(1 - (1-\lambda)\beta^e). \tag{31}$$

Solving (31) for $\hat{P}$ yields

$$\hat{P} = \frac{\lambda + (1 - (1-\lambda)\beta^e)c}{2(1 - (1-\lambda)\beta^e)}\theta, \tag{32}$$

which is identical to (11). The derivation provided in the body of the paper applies here as well.

References

K. Bagwell and M.H. Riordan. High and Declining Prices Signal Product Quality. *Amer. Econ. Rev.*, 81(1):224–239, 1991.

A.F. Daughety and J.F. Reinganum. Product Safety: Liability, R&D, and Signaling. *Amer. Econ. Rev.*, 85(5):1187–1206, 1995.

A.F. Daughety and J.F. Reinganum. Secrecy and Safety. *Amer. Econ. Rev.*, 95(4):1074–1091, 2005.

A.F. Daughety and J.F. Reinganum. Competition and Confidentiality: Signaling Quality in a Duopoly when there is Universal Private Information. *Games Econ. Behav.*, 58(1):94–120, 2007.

A.F. Daughety and J.F. Reinganum. Communicating Quality: A Unified Model of Disclosure and Signalling. *RAND J. Econ.*, 39(4):973–989, 2008a.

[12] For prices below θ, the equilibrium demand is identical in Figures 6 and 7.

A.F. Daughety and J.F. Reinganum. Imperfect Competition and Quality Signalling. *RAND J. Econ.*, 39(1):163–183, 2008b.

M.C.W. Janssen and S. Roy. Signaling Quality through Prices in an Oligopoly. *Games Econ. Behav.*, 68(1):192–207, 2010.

L.J. Mirman and M. Santugini. Monopoly Signaling: Existence and Non-Existence. Cahiers de Recherche 08-09, HEC Montréal, Institut d'économie appliquée, 2011.