Common Agency and Computational Complexity: Theory and Experimental Evidence[1]

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March 11, 1999

[1]We thank Eric van Damme, Simon Gächter, Richard McKelvey, Rebecca Morton, Jan Potters, Aldo Rustichini, Dolf Talman, Stef Tijs, and seminar audiences at Tilburg University, Tinbergen Institute (Amsterdam), and University College London for helpful comments. Financial support by the EU financed TMR-project on "Savings and Pensions" (Contract No. ERBFMRXCT 960016) was highly appreciated.

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Abstract

In a common agency game, several principals try to influence the behavior of an agent. Common agency games typically have multiple equilibria. One class of equilibria, called *truthful*, has been identified by Bernheim and Whinston and has found widespread use in the political economy literature.

In this paper we identify another class of equilibria, which we call *natural*. In a natural equilibrium, each principal offers a strictly positive contribution on at most one alternative. We show that a natural equilibrium always exists and that its computational complexity is much smaller than that of a truthful equilibrium. To compare the predictive power of the two concepts, we run an experiment on a common agency game for which the two equilibria predict a different equilibrium alternative. The results strongly reject the truthful equilibrium. The alternative predicted by the natural equilibrium is chosen in 65% of the matches, while the one predicted by the truthful equilibrium is chosen in less than 5% of the matches.

Keywords: lobbying, experimental economics, common agency, truthful equilibrium, natural equilibrium, computational complexity.

1 Introduction

Common agency games model a situation where several principals simultaneously try to influence the behavior of one agent. The agent must choose one alternative among a set of alternatives. Each of the principals cares about which alternative the agent chooses and can promise monetary contributions to the agent conditional on the agent's choice. Namely, each principal can promise a vector of monetary contributions, one for each possible alternative. Only the contribution on the alternative that is chosen will actually be paid. The agent observes all the monetary contributions offered by the principals and makes his choice.

Common agency provides a very general way of modeling the process of lobbying through campaign contributions. The agent is a politician who faces a set of policy alternatives. The politician cares both about monetary contributions, which he can spend on his electoral campaign, and directly about the policy alternative he chooses (either because he is genuinely concerned or because he wants to please voters). Each principal is a lobby who represents a special interest. Each lobby can offer – maybe implicitly – campaign contributions to the politician conditional on his policy stance.[1][2]

For instance, Grossman and Helpman [11] have applied common agency to trade policy. In an economy with many industries, a politician chooses the level of trade protection (tariff or subsidy) for each industry. Some of the industries are organized as lobbies and some are not. Through common agency, the authors arrive at a characterization of equilibrium trade protection in each industry as a function of import penetration, import elasticity, the preferences of the politician, and whether the industry is organized or not. Applying Grossman and Helpman's result, Goldberg and Maggi [9] have used data on trade protection in the US to estimate the preferences of US politicians.

All the political economy works cited above rely on theoretical foundations developed by Bernheim and Whinston [3], who were the first to study common agency. After noting that the typical common agency game has several equilibria, Bernheim and Whinston discuss a particular class of equilibria, which they name *truthful*. A truthful equilibria is

[1]A partial list of political economy papers that use common agency includes: Grossman and Helpman [11, 10], Dixit, Grossman, and Helpman [6], Rama and Tabellini [23], and Helpman and Persson [12].

[2]Note that other models besides common agency are used: e.g. all-pay auctions (See Potters, De Vries, and Van Winden [21] for experimental work on rent seeking modeled as an all-pay auction). The question of which model best captures the reality of lobbying is, to our knowledge, untackled and is outside the scope of the present paper.

called "truthful" because in it the contribution schedule of each principal follows the shape of the payoff function of that principal (the exact definition will be given later). Bernheim and Whinston show several striking properties: (1) A truthful equilibrium always exists (that is, the set of equilibria of a given common agency game always contains a truthful equilibrium); (2) An equilibrium is coalition-proof if and only if it is payoff-equivalent to a truthful equilibrium; and (3) In a truthful equilibrium the agent chooses an alternative which maximizes the sum of the payoffs of the agent and of the principals.[3]

In sum, truthful equilibria have several nice properties, which could make them focal. However, in this paper, we will argue that truthful equilibria also appear to be quite complex. We will formalize this idea through the theory of computational complexity and we will prove that the computation time needed to compute the truthful equilibrium of a generic common agency game increases exponentially with the number of principals. As a truthful equilibrium is not in dominant strategies, for each single principal the time needed to compute the optimal strategy is the same as the time needed to compute the whole equilibrium strategy. This implies that also the computation time for each principal grows exponentially. Problems with exponentially increasing computation time are considered hopeless in practice except for very small instances.

If common agency is to be applied to lobbying, computational complexity becomes an important issue. At the US federal level, thousands of lobbies make campaign contributions on interrelated issues (see Lehman, Schlozman and Tierney [26]). The fact that the time needed to compute truthful equilibria is exponential makes them non-computable for all practical purposes. For instance, it can take up to 35000 years to compute the truthful equilibrium of a game with 50 principals.[4] There are certainly more than 50 lobbies involved in the trade policy determination studied by Grossman and Helpman [11]).

Given the complexity of the situation, it seems plausible that principals behave in a

[3] Bergemann and Välimäki [2] extend Bernheim and Whinston's analysis to a multi-period common agency game. They define truthful equilibrium and coalition-proof equilibrium in a dynamic setting and show that (2) and (3) hold in this setting as well. Although here attention is restricted to one-period common agency, we conjecture that the gist of our results extends to a multi-period setting.

[4] The computing time of 35,000 year is obtained under the assumptions that a linear program with one constraint is solvable in a millisecond and that we use an algorithm for linear programming that is linear in the number of constraints. Two remarks are in order. First, the existing linear programming algorithms are worse than linear. Hence, the number 35,000 is by defect. Second, of course, the computation time does not only depend on the number of principals but also on the number of alternatives. However, as we shall see, the latter dependence is only linear and therefore has a less important effect on computation time.

more simple way than predicted by the concept of a truthful equilibrium. Such a simpler and a-priori plausible behavior is that a principal does not choose a whole contribution schedule, but rather makes a contribution only to one alternative that she hopes to get. Such strategies will be called *natural*, and an equilibrium in such strategies will be called a *natural* equilibrium. We show that a natural equilibrium exists (that is, the set of equilibria of a given common agency game always contains a natural equilibrium).[5]

Moreover, we show that a natural equilibrium can be computed in polynomial time. Problems for which computation time grows polynomially are considered to have good hopes of being solved in practice. Indeed, this is a particularly simple polynomial problem and, with 50 principals, it takes at most seconds to find the natural equilibrium.

However, a natural equilibrium does not enjoy the other nice properties of a truthful equilibrium. It need not be coalition-proof and it need not induce the alternative that maximizes the sum of the gross payoffs of the agent and the principals. This latter feature is of great importance for lobbying. If the chosen alternative is efficient (from the point of view of the participants to the common agency game), then inefficiencies arise if some lobbies are excluded from the lobbying process, or if there are transactions costs, or if some policy alternatives are exogenously excluded. Hence, with truthful equilibria, the policy goal would be to make lobbying as accessible and comprehensive as possible. Instead, with natural equilibria there may be inefficiencies in the lobbying process *in se*, even if everybody is represented and there are no transaction costs.

To establish which class of equilibria is a better predictor of actual play, we turned to an experiment. For this experiment we designed a simple common agency game with two principals and three alternatives, denoted by I, II, and III. The payoff functions are such that the efficient alternative II gives both principals a positive gross-payoff, whereas the inefficient alternatives I and III are desireable only for one of the principals. Furthermore, the natural equilibrium selects alternative I, whereas the truthful equilibrium selects alternative II.

The main result of the experiment is that alternative I is chosen in 65% of the matches, while the alternative II is chosen in 3.6% of the matches. This result is a clear-cut rejection of the hypothesis that subjects play according to the truthful equilibrium.

As we saw above, the alternative selected in the truthful equilibrium is always the efficient alternative. This property is used by the political economy literature which apply

[5] Besley and Coate [4] discuss the relation between truthfulness and efficiency in the context of lobbying. They also present the example of a nontruthful equilibrium which induces an inefficient action. According to our definition, the nontruthful equilibrium considered by Besley and Coate is a natural equilibrium.

common agency. Our experimental results suggest that the efficient alternative need not arise. This means that the lobbying process may be intrinsically inefficient and that welfare results obtained under truthful equilibrium are biased upwards.

We also look at the contribution schedules used by subjects. Compared to the contributions predicted by the truthful equilibrium, our subjects contribute too little on the 'compromise' alternative II and too much on the extreme alternatives I and III. For each type of principal, the difference between the contribution on the extreme alternative and the contribution on the compromise alternative is so high that it prevents the other principal from profitably inducing the compromise alternative.

After rejecting the truthful equilibrium, we ask whether instead the natural equilibrium is a good predictor of behavior. The answer is less straightforward since players did not coordinate at any equilibrium at all. We also hardly ever observe the choice of any contribution schedule that belongs to an equilibrium. However, the observed out of equilibrium play is consistent with the *spirit* of the natural contribution schedule. Each principal focusses on her preferred alternative and bids aggressively on it. The contribution on alternative II is positive most of the time, but it is only perfunctory in that it cannot induce alternative II under any reasonable assumption on the other principal's strategy.

In conclusion, our experimental evidence is clearly inconsistent with the truthful equilibrium and may be consistent with the natural equilibrium. We expect this result to hold a fortiori in games with more than two principals because the computational complexity of truthful equilibria increases faster than the computational complexity of natural equilibria.

A methodological contribution of the present work lies in the way we combine game theory and computational complexity. To our knowledge, this work represents the first test of a computational complexity measure as a predictor of behavior in strategic situations.[6] We consider two equilibria. One – the truthful – is supported by traditional game-theoretic refinements but has a high computational complexity, while the other is neither coalition-proof nor efficient but it is simpler to compute. In the game we consider, experimental

[6]Note that the sizeable literature on repeated games played by finite automata (such as Abreu and Rubinstein [1]) is related to *strategy implementation complexity* rather than computational complexity and is entirely different from the present work. This point will be made more explicit in Section 2. Computational complexity has seldom been applied to noncooperative game theory. The only examples we know of are Gilboa and Zemel [8] and Papadimitriou [18]. Gilboa and Zemel show that correlated equilibria are simpler to compute than Nash equilibria. Papadimitriou studies the computational complexity of repeated games. Outside noncooperative game theory, computational complexity has found application in cooperative games (see for instance Megiddo [17] or Faigle *et al* [7]) and in general equilibrium (see Rust [25]). Also, see Rubinstein [24] for a critical survey of bounded rationality models.

evidence suggests that computational complexity is better than traditional refinements at predicting actual play.

The plan of the paper is as follows. Section 2 contains the theory part. After reporting the main results by Bernheim and Whinston on truthful equilibria, we define natural equilibria and prove its properties. We also study the computational complexity of both truthful equilibria and natural equilibria. Section 3 describes the design of the experiment. Section 4 reports the results of the experiment with regard to the alternative chosen and to the contribution schedules used. Section 5 concludes. The Appendix contains the instructions of the experiment.

2 Theory

2.1 The model

In a common agency game, the players are one agent and m principals. The set of principals is denoted with $M = \{1, \ldots, m\}$. The agent chooses an alternative out of a finite set of alternatives S. Each principal tries to induce the agent to take a particular alternative rather than another by offering him a monetary payment which we denote as 'contribution'. Let t_s^j denote the contribution that principal j promises to make to the agent if the agent chooses alternative $s \in S$. The strategy of principal j is a *contribution schedule* t^j, namely a vector of contributions, one for each alternative in S. Contributions are restricted to be nonnegative. If the agent selects alternative s he receives a total monetary contribution $\sum_{j \in M} t_s^j$. Contributions promised on alternatives other than the chosen alternative are not paid (this is the difference between common agency and an all-pay auction).

The agent cares about how much money he receives and which alternative he chooses. His payoff is assumed to be separable in money and alternative. Let G_s^0 represent the utility that the agent derives from alternative s. The sum of contributions he receives by choosing s is $\sum_{j \in M} t_s^j$. Hence, the agent chooses s to maximize his net payoff $G_s^0 + \sum_{j \in M} t_s^j$.

Each principal cares about how much money she pays to the agent and which alternative the agent chooses. The separability assumption is made for principals too. Let G_s^j denote the utility (gross payoff) principal j derives from s. The net payoff of principal j if alternative s is chosen is $G_s^j - t_s^j$.[7]

[7]Dixit, Grossman, and Helpman [6] have shown that the main results of Bernheim and Whinston are still valid if the principals or the agent have nonseparable preferences.

The game is played in two stages. First, all principals simultaneously and noncooperatively choose their contribution schedules. Second, the agent observes the principals' contribution schedules and selects an alternative.[8]

2.2 Truthful Equilibria

Bernheim and Whinston [3] note that a typical common agency game has many equilibria[9]. They propose to focus on one type of equilibrium, which they call *truthful*, and they prove a number of important properties of truthful equilibria. This subsection reviews Bernheim and Whinston's results.

Definition 1 *The contribution schedule t^j of principal $j \in M$ is said to be truthful if it can be written as $t_s^j = \max(0, G_s^j - u^j)$ for all $s \in S$, where u^j is a constant. A truthful equilibrium is an equilibrium of the common agency game in which all principals offer truthful contribution schedules.*

A truthful contribution schedule follows the shape of the payoff function of the principal plus or minus a constant, except that, when the contribution would be negative, the nonnegativity constraint requires a zero contribution instead. The main feature of a truthful contribution schedule is that (but for the nonnegativity constraint) a principal who plays truthful is indifferent with regards of the alternative that the agent ends up choosing.

The properties of truthful equilibria that are relevant to our analysis can be summarized as follows:[10]

Theorem 1 (Bernheim and Whinston) *For any common agency game,*

(i) For any $j \in M$, given $\{t^i\}_{i \neq j}$, the set of best responses of principal j contains a truthful contribution schedule;

(ii) There exists a truthful equilibrium;

[8]Prat and Rustichini [22] have extended the analysis to common agency games where principals choose their contribution schedules sequentially. This paper will, however, focus exclusively on the simultaneous case.

[9]We will focus on subgame perfect Nash-equilibria, which for simplicity will be referred to as equilibria.

[10]Theorem 1 is not stated directly in that form in [3]. Part (i) corresponds to Bernheim and Whinston's Theorem 1. Part (ii) is an immediate consequence of Bernheim and Whinston's Theorem 2. Part (iii) is Bernheim and Whinston's Theorem 3. Part (iv) is Bernheim and Whinston's Theorem 2.

(iii) Every truthful equilibria is coalition-proof and every coalition-proof equilibrium is payoff-equivalent to a truthful equilibrium.

(iv) In a truthful equilibrium, the agent chooses $s^* \in argmax_{s\in S} \sum_{j\in M} G_s^j + G_s^0$. *The vector* $(n^j)_{j\in M}$ *is the vector of net payoffs for principals if and only if there exists positive numbers* $(a_j)_{j\in M}$ *such that* $(n^j)_{j\in M}$ *satisfies*

$$\max_{\{n^j\}_{j\in M}} \sum_{j\in M} a_j n^j \tag{1}$$

subject to

$$\forall J \subseteq M, \qquad \sum_{j\in J} n^j \leq \sum_{j\in M} G_{s^*}^j + G_{s^*}^0 - \max_{s\in S}(\sum_{j\in M/J} G_s^j + G_s^0) \tag{2}$$

Part (i) of Theorem 1 says that, given the contribution schedules of the other principals, a principal can restrict her attention without loss to truthful contribution schedules.

Note that (i) does not imply that a truthful equilibrium actually exists. Bernheim and Whinston do, however, show the existence of a truthful equilibrium (Part (ii)), that is, they prove that the set of equilibria of a given common agency game contains an equilibrium which is truthful.

Part (iii) links truthful equilibria to coalition-proofness. The definition of coalition-proofness for common agency can be found in Bernheim and Whinston's article. For the goal of the present paper, an informal definition will suffice. An equilibrium of a common agency game is coalition-proof if there exists no coalition of principals that can benefit by agreeing on a "self-enforcing" joint deviation from the equilibrium. The definition of self-enforcing deviation is recursive. A joint deviation for a given coalition is self-enforcing if there exists no coalition within the given coalition that can benefit from a (self-enforcing) deviation from the proposed joint deviation. Thus, Part (iii) of Theorem 1 says that there is an essential equivalence between the set of truthful equilibria and the set of coalition-proof equilibria. All truthful equilibria satisfy coalition-proofness and an equilibrium which is not truthful, or payoff-equivalent to a truthful equilibrium, does not satisfy coalition-proofness.[11]

[11] The fact that a truthful equilibrium is coalition-proof among the m principals does not imply that it is Pareto-efficient among the m principals if there are more than two principals. Indeed, Konishi, Le Breton and Weber [14] provide a simple three-principal example of common agency game in which there exists a non-coalition-proof equilibrium which gives each principal a strictly higher net payoff than every coalition-proof equilibrium of the same game.

Part (iv) supplies a complete and very useful characterization of truthful equilibria (developed by Bernheim and Whinston and investigated in detail by Laussel and Le Breton [16, 15]). The alternative chosen by the agent maximizes the sum of gross payoffs of all principals and of the agent. Note that (1) and (2) constitute a maximization problem in which the unknowns are the equilibrium net payoffs of the m principals.

2.3 Natural Equilibria

This subsection contains the original theoretical contribution of this paper. We introduce the concept of *natural* equilibrium and compare it to Bernheim and Whinston's truthful equilibrium.

As already explained in the Introduction, complexity reasons make it plausible that players use simpler strategies than those demanded by a truthful equilibrium. This is the case when a player just picks one alternative and goes only for this alternative, i.e. makes a serious contribution only to that alternative. We call such a strategy a *natural* contribution schedule, and an equilbrium is natural if it is in natural contribution schedules. A way to formalize this idea is

Definition 2 *The contribution schedule t^j of principal $j \in M$ is said to be natural if $t^j_s = 0$ for all $s \in S$ except, at most, one. A natural equilibrium is an equilibrium of the common agency game in which all principals offer natural contribution schedules.*

Notice that a natural equilibrium is an equilibrium of the game. Hence, in a natural equilibrium each principal has no incentive to use a more complicated strategy than the equilibrium natural contribution schedule. The next two results are the equivalents for natural equilibria of Parts (i) and (ii) of Theorem 1 discussed in the subsection on truthful equilibria.

Proposition 1 *For any $j \in M$, given $\{t^i\}_{i \neq j}$, the set of best responses of principal j contains a natural contribution schedule.*

Proof: Given $\{t^i\}_{i \neq j}$, let $\tilde{t}^j$ denote a best response contribution schedule for j. Let $\hat{s}$ be the alternative chosen by the agent. Consider the contribution schedule $\hat{t}^j$ such that $\hat{t}^j_s = \tilde{t}^j_s$ if $s = \hat{s}$ and $\hat{t}^j_s = 0$ otherwise. As $\hat{t}^j$ leaves j's net payoff unchanged, it belongs to the set of best responses of j given $\{t^i\}_{i \neq j}$. ∎

Proposition 1 shows that by offering a natural contribution schedule a principal is not worse off for any combination of contribution schedules of the other principals. Hence, whatever strategies the other principals choose (natural, or truthful, or all others), each principal has no incentive to deviate from a natural contribution schedule.

Of course Proposition 1 does not imply that a natural equilibrium actually exists. However, we will see that one can construct a natural equilibrium as follows. Assume that S contains at least two alternatives (If S contains only one alternative, all contribution schedules are natural by definitions). Let $\hat{S} = \{s' \in S : \exists j \in M, s' \in \text{argmax}_{s\in S} G_s^j + G_s^0\}$ and let

$$s^* \in \text{argmax}_{s\in\hat{S}} \sum_{j\in M} G_s^j + G_s^0 \tag{3}$$

and

$$\bar{s} \in \text{argmax}_{s\in\hat{S}/\{s^*\}} \sum_{j\in M} \max(0, G_s^j - G_{s^*}^j) + G_s^0 - G_{s^*}^0. \tag{4}$$

The set $\hat{S}$ comprises all the alternatives that are preferred by at least one principal, who must compensate the agent for changes in G_s^0. The alternative s^* maximizes the sum of payoffs of principals and agent within the set $\hat{S}$. Alternative $\bar{s}$ is the alternative to s^*, within $\hat{S}$, for which a coalition of principals is willing to pay the highest amount, after compensating the agent for changes in G_s^0. As we will see, alternative s^* is the alternative that the agent will select in the natural equilibrium.

Principals offer contribution schedules $\{\hat{t}^j\}_{j\in M}$ defined as follows:

(i) If $s \neq s^*$ and $s \neq \bar{s}$, then $\hat{t}_s^j = 0$ for all $j \in M$;

(ii) For all $j \in M$,

$$\hat{t}_{\bar{s}}^j = \max(0, G_{\bar{s}}^j - G_{s^*}^j); \tag{5}$$

(iii) The vector $\{\hat{t}_{s^*}^j\}_{J\in M}$ is such that, for all $j \in M$,

$$\hat{t}_{s^*}^j \in [0, \max(0, G_{s^*}^j - G_{\bar{s}}^j)] \tag{6}$$

and

$$\sum_{j\in M} \hat{t}_{s^*}^j + G_{s^*}^0 = \sum_{j\in M} \hat{t}_{\bar{s}}^j + G_{\bar{s}}^0. \tag{7}$$

Parts (i) and (ii) are clearly feasible, and, by the definitions of s^* and $\bar{s}$, it is possible to find $\{t_{s^*}^j\}_{j\in M}$ which satisfies Part (iii). Thus, contribution schedules $\{\hat{t}^j\}_{j\in M}$ are feasible.

For future reference, denote $\bar{M} = \{j \in M : G_{\bar{s}}^j \geq G_{s^*}^j\}$, i.e. $\bar{M}$ is the set of all principals who prefer $\bar{s}$ to s^*. Let M^* be the complement of $\bar{M}$ on M.

The contribution schedules defined in (5), (6), and (7) are natural. All alternatives receive zero contributions except two: s^* and $\bar{s}$. Principals in $\bar{M}$ prefer $\bar{s}$ to s^* and make positive offers on $\bar{s}$. Principals in M^* want s^* and offer just enough to make the agent indifferent between the two alternatives.

The following theorem shows that $(s^*, \{\hat{t}^j\}_{j\in M})$ is indeed an equilibrium of the common ageny game and, therefore, guarantees the existence of a natural equilibrium (that it, the set of equilibria of a given common agency game contains an equilibrium which is natural):

Theorem 2 *Every common agency game has a natural equilibrium.*

Proof: We want to show that alternative s^* and contribution schedules $\{\hat{t}^j\}_{j\in M}$ constitute a natural equilibrium of the common agency game. This will ensure existence of a natural equilibrium for every common agency game.

We have already argued that $\{\hat{t}^j\}_{j\in M}$ are feasible. It is left to prove that neither the agent nor any of the principals have a profitable deviation. As $s^* \in \text{argmax}_{s\in S} G^0_s + \sum_{j\in M} \hat{t}^j_s$, the agent has no incentive to deviate from s^*. Suppose that principal j deviates from $\hat{t}^j$ and plays $\tilde{t}^j$ instead. Given $\tilde{t}^j$, there are four possible cases:

(a) The agent still chooses s^*;

(b) The agent chooses $\bar{s}$;

(c) The agent chooses s', where $s' \neq s^*$, $s' \neq \bar{s}$, and $s' \in \hat{S}$;

(d) The agent chooses s', where s' does not belong to $\hat{S}$.

There are eight exhaustive cases of possible deviations corresponding to the combinations of (a), (b), (c), and (d) with $j \in \bar{M}$ and $j \in M^*$. For each of the eight cases, we prove that a deviation is not profitable for principal j.

The case in which $j \in \bar{M}$ and (a) is obvious. If $j \in \bar{M}$ and (b), (5) and (7) imply that $\tilde{t}^j_{\bar{s}} > G^j_{\bar{s}} - G^j_{s^*}$ and, therefore, a deviation is strictly not profitable. If $j \in \bar{M}$ and (c),

$$\begin{aligned}\tilde{t}^j_{s'} + G^0_{s'} &> \sum_{i\in M} \hat{t}^i_{s^*} + G^0_{s^*} = \sum_{i\in M} \hat{t}^i_{\bar{s}} + G^0_{\bar{s}} = \sum_{i\in M} \max(0, G^i_{\bar{s}} - G^i_{s^*}) + G^0_{\bar{s}} \\ &\geq \sum_{i\in M} \max(0, G^i_{s'} - G^i_{s^*}) + G^0_{s'} \geq G^j_{s'} - G^j_{s^*} + G^0_{s'},\end{aligned}$$

where the strict inequality makes the agent choose s' over s^*, the first equality is due to (7), the second equality comes from (5), the first weak inequality is implied by the definition of

$\bar{s}$, and the second weak inequality is immediate. Then, $\tilde{t}^j_{s'} > G^j_{s'} - G^j_{s^*}$ and the deviation is not profitable for j. If $j \in \bar{M}$ and (d), by the definition of $\hat{S}$, there exists an alternative $s'' \in \hat{S}$ such that $G^j_{s''} + G^0_{s''} > G^j_{s'} + G^0_{s'}$. Then, principal j increases her net payoff by substituting $\tilde{t}^j$ with $\tilde{\tilde{t}}^j$ in which $\tilde{\tilde{t}}^j{}_{s'} = 0$ and $\tilde{\tilde{t}}^j{}_{s''} = \tilde{t}^j_{s'} + G^0_{s'} - G^0_{s''}$. But, alternative s'' belongs to either (a), (b), or (c), and therefore this deviation is not profitable.

The cases in which $j \in M^*$ and (a) or (b) are obvious. If $j \in M^*$ and (c), it must be that

$$\begin{aligned} \tilde{t}^j_{s'} + G^0_{s'} &> \sum_{i \in M} \hat{t}^i_{\bar{s}} + G^0_{\bar{s}} = \sum_{i \in M} \max(0, G^i_{\bar{s}} - G^i_{s^*}) + G^0_{\bar{s}} \\ &\geq \sum_{i \in M} \max(0, G^i_{s'} - G^i_{s^*}) + G^0_{s'} \geq G^j_{s'} - G^j_{s^*} + G^0_{s'}, \end{aligned} \tag{8}$$

where the strict inequality makes the agent choose s' over $\bar{s}$, the equality comes from (5), and the first weak inequality is implied by the definition of $\bar{s}$. (8) implies

$$\tilde{t}^j_{s'} \geq G^j_{s'} - G^j_{\bar{s}}. \tag{9}$$

The deviation is profitable if

$$\tilde{t}^j_{s'} < \hat{t}^j_{s^*} + G^j_{s'} - G^j_{s^*}.$$

But, by (6), $\hat{t}^j_{s^*} \leq G^j_{s^*} - G^j_{\bar{s}}$. Then, the deviation is profitable only if $\tilde{t}^j_{s'} < G^j_{s'} - G^j_{\bar{s}}$, which contradicts (9). Finally, the case $j \in M^*$ and (d) is analogous to the case in which $j \in \bar{M}$ and (d) and is omitted. ∎

2.4 Computational Complexity

Next, we compare truthful equilibria and natural equilibria from the viewpoint of computational complexity. Natural equilibria have an extremely simple structure because a principal offers zero contributions for all alternatives but at most one. As we will see this makes natural equilibria for the players much easier to arrive at than truthful equilibria.

To establish this result, we need some basic concepts of computational complexity which are standard in computer science but may not be familiar to all economists.[12] Consider a class of well-defined mathematical problems. An *algorithm* for that class of problems is

[12]See Papadimitriou [19] for an introductory text on computational complexity. Note that the notion of computational complexity used here is radically different from that used by Abreu and Rubinstein [1] and other works in the literature on repeated games played by finite automata (See Rubinstein [24] for a survey and a discussion). In those works, players are bounded in their ability to *implement* strategies. In

a sequence of simple instructions that solve any instance of problem in that class. The *size* of a problem within a certain class is the dimension of the data input that defines the problem (the way to measure the dimension varies from class to class). In general the number of simple instructions executed by the algorithm is increasing with the size of the problem. For instance, a class of problems is matrix inversion. The size of the problem is given by the dimension of the matrix. It takes a higher number of simple instructions to invert a 3 by 3 matrix than a 2 by 2 matrix.[13]

The number of simple instructions determines the computation time necessary to execute the algorithm. The crucial question asked in computational complexity is: at what rate does computation time increase as the size of the problem increase? In particular a distinction is drawn between classes of problems for which computation time is a polynomial function of size (*solvable in polynomial time*) and classes problems for which computation time is a function that increases faster than any polynomial function (this is the case, for instance, when time grows exponentially). The distinction is of practical importance. Problems not solvable in polynomial time of large size have little hope of being solved, even with the fastest computers available.

Let TRUTHFUL denote the problem of finding a truthful equilibrium for a generic common agency game through maximization problem (1) and (2). Analogously, NATURAL is the problem of finding a natural equilibrium for a generic common agency game through (3), (4), (5), (6), and (7). Given a common agency game, both TRUTHFUL and NATURAL have the same input: a matrix of gross payoffs for the m principals and for the agent. Thus, we will take m to be the size of both TRUTHFUL and NATURAL.[14]

Theorem 3 *(i) NATURAL is solvable in polynomial time; (ii) TRUTHFUL is solvable in exponential time.*

Proof: In linear programming, the size of the input of a problem is proportional to the

a one-stage game such as ours, such a notion would be of little interest. Instead, our players are bounded in their ability to *find* optimal strategies (See also Papadimitriou [18] for discussion on the non-obvious relation between limits to implementation and limits to computation).

[13]The computation time refers to a generic instance of the problem. It can therefore be seen as the upper bound to the computation time. For instance, in the case of matrix inversion, the identity matrix is very easy to invert, independently of its size, but clearly it is not a generic matrix.

[14]Clearly, the size of the input of a common agency game also depends on the number of possible alternatives. However, it is easy to see that this not an important variable from the point of view of computational complexity because in the algorithm the only operation that is executed across alternatives is maximization, and maximization is linear in the number of alternatives.

product of the number of variables with the number of constraints. It can be shown that linear programming is solvable in polynomial time (see Papadimitriou [20, Theorem 8.5]).

Proof of (i): NATURAL includes two successive steps. First, $\hat{S}$, s^* and $\bar{s}$ are found through (3) and (4). Second, contribution schedules are computed through (5), (6), and (7). The first step involves an $m+2$ maximization problem. Its computation time is therefore linear (and hence polynomial) in m. Regarding the second step, we make the following:

Claim: The problem of finding a contribution matrix satisfying (5), (6), and (7) can be rewritten as a linear program with at most m variables and at most m constraints.

Proof of the Claim: The equalities in (5) can be substituted into (7), which, in turn, can be used as the objective function of the problem. The problem of finding a natural equilibrium can be rewritten as

$$\min_t \sum_{j \in M^*} \hat{t}^j_{s^*}$$

subject to

$$\sum_{j \in M^*} \hat{t}^j_{s^*} + G^0_{s^*} \geq \sum_{j \in \bar{M}} (G^j_{\bar{s}} - G^j_{s^*}) + G^0_{\bar{s}}$$
$$\hat{t}^j_{s^*} \leq G^j_{s^*} - G^j_{\bar{s}} \text{ for } j \in M^*.$$

This is a linear program with $\sharp M^*$ variables and $\sharp M^* + 1$ constraints. As $\sharp M^* \leq m$, the claim is proven.

By the Claim, the second step of NATURAL is a linear program of size at most $m(m+1)$. As linear programming is solvable in polynomial time with respect to the size of its input, also the second step of NATURAL is solvable in polynomial time and Part (i) is proven.

Proof of (ii): The number of possible coalitions among m principals is 2^m. Hence, the maximization problem in (1) and (2) is a linear program with m variables and 2^m constraints. Therefore, its size is $m2^m$. As linear programming is solvable in polynomial time with respect to the size of its input, the maximization problem in (1) and (2) is solvable in exponential time with respect to m. Hence, TRUTHFUL is solvable in exponential time. ∎

The proof of Theorem 3 relies on the fact that both NATURAL and TRUTHFUL are linear programs. They have the same number of variables: m. However, the first has $m + 1$ constraints while the second has 2^m constraints. The computation time of linear programming is known to be polynomial in the number of constraints. Hence, NATURAL is polynomial, while TRUTHFUL is exponential, and therefore not polynomial.

What we have found in Theorem 3 is the time necessary for a game-theorist to compute natural and truthful equilibria. Instead, what we are interested in is the time necessary for *a principal* to compute her optimal strategy and not the whole equilibrium. However, as neither truthful nor natural equilibria are in dominant strategies, the problem for one principal of finding her optimal strategy is equivalent to the problem of finding the whole equilibrium. A principal cannot know if the strategy she plays is optimal unless she knows what all other principals are doing. The computation time obtained in Theorem 3 is the computation time that the individual principals faces.

Theorem 3 does not exclude that there exists an algorithm that finds truthful equilibria in polynomial time. It only excludes that such an algorithm is based on the characterization provided in Theorem 1 Part (iv). Therefore, in principle one might find a simpler alternative characterization of truthful equilibria which results in a polynomial time algorithm. However, to the best of our knowledge, alternative characterizations are not known (and the present one seems already quite simple, given the difficulty of the problem).

In this section we compared the properties of truthful equilibria and natural equilibria. The two classes of equilibria share two properties: existence of best response within the class, and existence of equilibria. However, there are two important differences. One – coalition-proofness – is in favor of truthful equilibria: a natural equilibrium need not be coalition-proof. The other property – computational complexity – is in favor of natural equilibrium: finding truthful equilibria is harder than finding natural equilibria.[15]

3 Experimental Design

To evaluate the concepts of truthful and natural equilibria we implemented an experimental design with one agent and two principals, denoted by A and B. The agent had to choose between three alternatives, denoted by I, II, and III. The agent derived no utility from any of these alternatives. On the other hand, the principals cared about which alternative was chosen. Their gross payoffs derived from the alternatives were given by Table 1.

[15]Natural equilibria and truthful equilibria cannot be distinguished with respect to uniqueness. In common agency games with more than two principals, there can be multiple truthful equilibria as well as multiple natural equilibria.

		I	II	III
Table 1	A	17	11	0
	B	0	7	12

As described in Section 2, both principals had to choose simultaneously a contribution schedule in the first stage of the game. All contributions had to be nonnegative. To exclude the possibility of losses, the contribution to an alternative had to be not above the gross payoffs the principal received for that alternative.[16]

After the choice of the contribution schedules, the agent had to choose an alternative in the second stage of the game. Principals' net-payoffs were given by their gross payoff resulting from the chosen alternative minus their contributions to the chosen alternative.

Since the agent had no intrinsic interest in the alternatives, he should choose the alternative with the highest sum of contributions, the winning alternative. This prediction holds for any combination of contribution schedules of the principals. Truthful equilibria and natural equilibria differ in the contribution schedules of the principals, not in the behavior of the agent. Principals' strategy choices are the focus of interest of our experiments, not agent's behavior. Therefore, we substituted the agent by a rule stating that the winning alternative (i.e. the alternative with the highest sum of contributions) is choosen automatically.

In case of an equal sum of contributions for two or more alternatives, the tie was broken by rolling a die with equal probabilities for all winning alternatives (see the instructions in the appendix for a detailed description of the tie-breaking rule).[17] For a continuous strategy set of the principals this rule would lead to problems with the existence of an equilibrium. But strictly speaking a continuous strategy set is not available in experiments anyhow, since payments to the subjects have to be multiples of the smallest coin available, which was in our experiments 5 (Dutch) cents. Hence, we demanded all contributions to be multiples of 0.05.[18]

[16]Since losses are difficult to enforce it is common in experimental economics to restrict the strategy set such that losses are excluded. This only remove some dominated strategies that are not part of neither the truthful nor the natural equilibrium.

[17]If the contribution schedules chosen by the principals are part of an equilibrium, theory assumes that the agent breaks a tie such that the equilibrium is supported. To incorporate such a tie-breaking behavior into the rule substituting the agent would be very difficult to explain to the participants. Furthermore, theory is silent about how ties are broken out of equilibrium.

[18]Simon and Zame [28] consider a class of infinite games which comprise common agency games and

These changes in the game (substitution of the agent by a rule, probabilistic tie-breaking rule, finite strategy sets) led to inessential changes in the equilibrium predictions. Specifically, the truthful and the natural equilibrium contribution schedules were given by

	I	II	III
A	12	0	0
B	0	0	11.95

Table 2a: The Natural Equilibrium Contribution Schedules

	I	II	III
A	10.95	5	0
B	0	6	10.95

Table 2b: The Truthful Equilibrium Contribution Schedules

with I (in case of the natural equilibrium) and II (in case of the truthful equilibrium) being the chosen alternatives.[19]

The equilibrium net payoffs are 5 for A and 0 for B (for the natural equilibrium), and 6 for A and 1 for B (for the truthful equilibrium). Notice that the main features of the truthful equilibrium did not change: it is coalition-proof, and it depicts the efficient equilibrium.[20] The sum of net payoffs for both principals is 40% higher in the truthful equilibrium than in the natural equilibrium. Furthermore, both equilibria do not rest on the assumption that in case of a tie the agent makes the right (i.e. equilibrium supporting) decision - in both equilibria the rule just picks the alternative which would be agent's unique best choice.

The experiments were conducted in a classroom. In each session 16 subjects participated. Each subject played the game six times, three times in the role of principal A and

show that in this class the limit of the equilibrium of a discretized game as the discretization becomes finer is an equilibrium of the continuous game. Hence, our discretized game can be taken as a legitimate approximation of the original game.

[19]In the experiments we substituted the agent by a rule. Therefore, an alternative was in fact not chosen by the agent, but rather induced by the contribution schedules of both principals and the rule. Nonetheless, we refer to that alternative as the 'chosen' or 'winning' alternative.

[20]In this game the truthful equilibrium is the Pareto-efficient equilibrium for principals. Thus, we do not exploit the example of an common agency game in which coalition-proofness does not imply Pareto-efficiency (See Footnote 11).

three times in the role of B.[21] Each subject knew beforehand whether she was principal A or B in a certain round. Since it was common knowledge that each pair consisted of one principal A and one principal B, everyone also knew whether her partner was principal A or B. However, nobody knew the identity of her partner.

At the beginning of each session the instructions were read aloud (see Instructions in the Appendix). Then the subjects had time to privately ask questions, and after that the first round started. At the beginning of each round the principals had to choose simultaneously their contribution schedules by inserting them into their decision form in the line "your contributions" (see the decision form in the appendix). Then all contribution schedules were transferred to the experimenters' documentation. After that, we rolled a die. This was done irrespectively of whether a tie actually occurred or not[22]. Then we calculated for each pair of principals which alternative was chosen, and indicated it in the subjects' decision forms in the line "chosen alternative". We also inserted the contribution schedules of their partners in the decision form in the line "contributions of your partner". Hence each subject knew the alternative chosen as well as the strategy of the other principal. After inserting the chosen alternatives and the partner's contribution schedules into the subjects' decision forms, the next round started. After the last round, the net payoffs a subject made in all rounds were summed up and paid to her in cash.

The subjects were matched so that nobody played twice with the same partner. This was common knowledge. Furthermore, we used a matching protocol that maximized the number of independent observations in the later rounds under the constraint that nobody was matched twice with the same partner. Specifically, we applied the following procedure: In the first round the 16 subjects formed eight pairs in each session. At the beginning of the second round, two first round pairs were merged to form a group consisting of four subjects. Since this grouping remained the same in rounds 2 and 3 we refer to these groups as "$r_{2/3}$-groups". In rounds 2 and 3 each subject was matched with those members of her $r_{2/3}$-group with whom he had not been matched in the first round. This matching protocol guarantees that every member of a $r_{2/3}$-group did not experience any (previous or contemporary) decision of a non-member - any influence from a decision of a non-member on the behavior of a member can be excluded. Therefore, the decisions made within a

[21]This guaranteed that looking at the whole experiment all subjects were in a similar position. By that, the impact of distributional concerns (fairness, envy, altruism), which very often shape experimental results, was minimized.

[22]This excluded that subjects received any information about whether other pairs experienced ties.

$r_{2/3}$-group formed an independent observation[23].

At the beginning of the fourth round, two $r_{2/3}$-groups were merged into one group of eight participants. Since this grouping remained the same in rounds 4, 5, and 6 we refer to these groups as "r_{4-6}-groups". In rounds 4 to 6 each subject was matched with three of those members of her r_{4-6}-group with whom she had not been matched in a previous round. This matching procedures guarantees that the decisions of every member of a r_{4-6}-group were not influenced by any (previous or contemporary) decision of a non-member - the decisions made within a r_{4-6}-group formed an independent observation.

On the whole we conducted 2 sessions. Therefore, we observed 16 first round pairs, eight $r_{2/3}$-groups, and four r_{4-6}-groups. The experiments took place at the Center for Economic Research, Tilburg University, The Netherlands. The participants were students of different fields, mainly of business administration and law. None of them was a student of ours and none had knowledge in game theory or common agency theory.

A session lasted about 25 minutes net of going through the instructions. The average earnings of a participant was 15.12 Hfl, which was about 8.13 US$ at the time the experiments were conducted (October 1998). Principal A earned on average 4.26 Hfl per round, whereas B earned on average 0.78 Hfl per round. This brings us to the results of the experiments which will be discussed in detail in the next section.

4 Results

Truthful and natural equilibrium differ in two aspects: the alternative chosen and the strategies which leads to a particular alternative. Hence, we first examine which alternatives were chosen (Result 1). Then we analyze which contribution schedules were applied by the principals (Results 2 and 3).

4.1 Chosen alternative

On the whole, we observed 96 choices of alternatives. In 4 times, a tie between 2 alternatives occured, which was broken by using a die. In what follows we count these cases half for both

[23] Cooper, De Jong and Ross [5] introduced, and Kamecke [13] analyzed, a different matching protocol that preserves the best-reply structure of a one-shot game while maximizing the number of rounds. However, as also Kamecke [13, p. 411] explains, this does not imply that other, nonstrategic influences between the players (such as learning) are excluded. Hence, such a protocol does not maximise the number of independent observations, and it is, therefore, not helpful to increase the significance of statistical tests.

winning alternatives. In total, we observed 62.5 cases where I was the winning alternative (65% of all cases), 3.5 cases with II winning (3.6% of all cases), and 30 cases with III winning (31.4% of all cases). This already indicates:

Result 1

(a) Alternative II was hardly ever chosen.

(b) Alternative I was chosen in most of the cases.

To establish this result, one can use a binomial test on the hypothesis that the winning probability of II is larger than or equal to 10 %. This hypothesis has to be rejected at a 5% level.[24] On the other hand the hypothesis that the winning probability of I is 50% or less has to be rejected even at a 1% level.

Result 1 can also be inferred from Figure 1 which depicts the evolution of the relative frequences of the chosen alternatives during the course of the experiments.

Insert Figure 1

In all rounds the frequency of alternative II chosen was less than 10 %, and in 3 rounds we did not observe any case of II winning. In all rounds, alternative I as well as alternative III occured more often than II. In the last 2 rounds, however, the gap between II and III narrowed. On the other hand, alternative I won in more than half of the cases in all rounds except round 1, and there was no tendency of the frequency of I to decline.

Due to spillovers between partners and due to change of partner from round to round, the individual observations were of course not independent. Hence, tests based on individual observations like the binomial tests used above might be not appropriate. To construct independent observations, recall that we matched subjects such that they formed $r_{2/3}$-groups consisting of four persons each whose decisions in round 2 and 3 were independent of all decisions of all subjects not belonging to the same group. On the whole, we had 8 independent $r_{2/3}$-groups. For each group we calculated the frequency of the different alternatives winning.[25]

[24] See [27, pp38] for a description of the binomial test.

[25] Each group made four choices. These four choices together can be summarized by the frequency distribution over the chosen alternatives. Unlike the individual choices, frequencies of different groups were independent from each other. Hence, one can test whether the frequency of one altenative differs from that of another alternative.

$r_{2/3}$-groups	frequency of I	frequency of II	frequency of III
1	0.5	0	0.5
2	1	0	0
3	1	0	0
4	0.75	0	0.25
5	0.625	0	0.375
6	0.5	0	0.5
7	0.375	0	0.625
8	0.75	0	0.25

Table 3: The Relative Frequencies of the Chosen Alternatives in Rounds 2 and 3

As one can see from Table 3, in five groups I was chosen more often than III, in two groups I and III were equally often chosen, and in one group III was chosen more often than I. In two groups II as well as III never won, and in all other groups I as well as III were chosen more often than II. Applying a Wilcoxon signed ranks tests[26] for the hypothesis that the frequencies of I and III were equal, we have to reject this hypothesis at a 5% level (see Table 4).

	I versus II	I versus III	II versus III
p-values	0.047	0.004	0.016

Table 4: Wilcoxon signed rank tests: the p-values for rejecting the hypothesis that the frequencies of two alternatives in round 2 and 3 are equal.

Using the same test for the hypotheses that the frequencies of I and II, and II and III, respectively, were equal, we have to reject both hypotheses even at a 2% level. Hence, in round 2 and 3 we observe I significantly more often than the other alternatives, and II significantly less often than I and III.

In the last 3 rounds we formed two independent r_{4-6}-groups in each session. In three of these groups I won in 2/3 or more of all cases, and also in the forth group I was the most often observed alternative.

[26]See [27, pp87] for a description of this test.

$r_{4/6}$-groups	frequency of I	frequency of II	frequency of III
1	0.79	0	0.21
2	0.83	0	0.17
3	0.67	0.08	0.25
4	0.46	0.13	0.41

Table 5: The Relative Frequencies of the Chosen Alternatives in Rounds 4 to 6.

Using again a Wilcoxon signed ranks test the hypothesis that the frequency of I is equal to that of II (or III) has to be rejected at a 10% level. The p-value is 6.25%, which is the lowest possible level one can get with four observations. The same result holds for a comparison between II and III. Hence, also in the last 3 rounds the 'natural' alternative I was 'dominating', and the 'truthful' alternative II hardly ever won.

4.2 Contribution Schedules

We now turn to the contribution schedules chosen by the principals. The average contribution of A for I (II) was 9.96 (2.92), whereas B's contribution to II (III) was 3.53 (9.08)[27](see Table 6).

	I	II	III
principal A	9.96	2.92	0
principal B	0	3.53	9.08
sum of contributions	9.96	6.45	9.08

Table 6: Average Contribution Schedules

This implies that actual contributions for all alternatives were lower than the contributions of the truthful equilibrium. Compared with the natural equilibrium strategy, A's (B's) contributions to I (III) were too low, whereas their contributions to II were too high (compare Table 6 with Tables 2a and 2b). If we look at the development of the average contributions of the rounds, we find that A's contribution to I increases, whereas there is no clear trend for A's contributions to II. B tends to increase her contributions to II as well as to III (see Figure 2).

Insert Figure 2

[27]Recall that A's contribution to III as well as B's contribution to I had to be zero.

This already indicates that we hardly observe equilibrium play in the experiment. We define that a contribution schedule is a near equilibrium schedule if the contribution to any alternative differs no more than 5 cents from the actual strategy belonging to the equilibrium. Given this definition, we found 10 cases (out of 96) where principal A chose a near equilibrium schedule of the natural equilibrium. We never observed that A's strategy was a near equilibrium schedule of any other equilibrium. Furthermore, B's schedule was never a near equilibrium schedule of any equilibrium (natural, truthful, or any other). Consequently, we never observed that the chosen strategy combinations form an equlibrium of the game - all observed behavior was out of equilibrium. In most cases the out of equilibrium play was neither natural nor truthful. We say that a principal chooses a *nearly natural* contribution schedule if her contribution schedule differs from a natural contribution schedule by at most 5 cents on each alternative. An analogous definition is given for *nearly truthful* contribution schedules. Table 7 shows how often nearly natural and nearly truthful schedules were chosen.

	A - all rounds	A - last 3 rounds	B - all rounds	B - last 3 rounds
natural	24(25%)	17(35%)	14(15%)	6(13%)
truthful	16(17%)	6(13%)	17(18%)	9(19%)

Table 7: Number of Cases of Nearly Natural and Nearly Truthful Contribution Schedules Chosen by Principal A and B in All and Last 3 Rounds (percentages in parentheses).

This table shows that principal A chooses a nearly natural strategy more often than a nearly truthful, and this tendency was much stronger in the last 3 periods than in the beginning of the experiment. B chooses both types of strategies quite rarely, and there seems to be no change over time.

However, even principal A in the last three periods chooses in most of the cases a strategy that is neither natural nor truthful. Hence, we have to look whether actual schedules exhibit at least the main characteristics of either truthful or natural strategies. Recall at a truthful schedule is characterized by the feature that (but for the nonnegativity constraint) a principal who plays truthful is indifferent with regard to the chosen alternative. This implies for the game at hand that A's schedule should make him indifferent between I and II [28], whereas B's schedule should make him indifferent between III and II.[29] Hence, the difference between A's contribution to I and II should be 6, wheras B's contribution

[28]For alternative III the nonnegativity constraint is binding, anyhow.

[29]For alternative I the nonnegativity constraint is binding, anyhow.

to III and II should differ by 5. The actual average differences were 7.04 and 5.55. To see whether these numbers differ significantly from 6 and 5, respectively, we use the individual schedules to run a t-test for the hypothesis that $t_I^A - t_{II}^A = 6$ ($t_{III}^B - t_{II}^B = 5$). This hypothesis has to be rejected at the 1% (5%) level in favor of the counter-hypothesis that the difference is larger than 6 (5). Since the individual observations are not statistically independent, we also looked at the average schedules of the eight $r_{2/3}$-groups and the four r_{4-6}-groups. We found in 6 of the $r_{2/3}$-groups and in all r_{4-6}-groups that $t_I^A - t_{II}^A > 6$ and $t_{III}^B - t_{II}^B > 5$. This leads to

Result 2 *Player A's contribution schedules were not designed to make her indifferent between alternative I and alternative II.*

Result 3 *Player B's contribution schedules were not designed to make her indifferent between alternative III and alternative II.*

Hence, we can conclude that the actual strategies did not exhibit the main feature of truthful strategies. Do they exhibit the main feature of natural strategies, namely that the principals focus on one alternative and bid agressively on it? To answer this question, we first investigate the strategies employed by players A. Notice first that as long as the difference between A's contribution to I and to II was larger than or equal to 7, II could not win irrespectively of B's contributions. Hence, in these cases A's contribution to II did not matter.[30]. For example, whether A chose a schedule like (12,0,0), which was the natural strategy, or a strategy like (12,4,0) was completely irrelevant for A, since II could not defeat I in both cases. Hence, if the difference between t_I^A and t_{II}^A was larger than or equal to 7, we know for sure that A wanted I to be chosen, and the schedule clearly qualifies as an agressive bid on I. As already mentioned, the average difference between A's contribution to I and II was indeed slightly above 7. If we look at the individual strategies, we observe 38 (out of 96) cases were the difference was larger than or equal to 7 (see Table 8). In the last 3 rounds the difference was not below 7 in 26 (out of 48) cases.

	$t_I^A - t_{II}^A > 7$	$t_{III}^B - t_{II}^B > 11$
all rounds	38(40%)	19(20%)
last 3 rounds	26(54%)	15(31%)

[30]Recall that B cannot contribute more than 7 to II, since this is her payoff from II (see Table 1). Furthermore, we assume B does not contribute 7 to II, since otherwise B's net-payoff would be zero even if II wins. In fact we never observed that any B (A) chose a schedule such that she would have a zero net-payoff if II or III (II or I) would have won.

Table 8: Number of Cases in Which A's Schedule Excludes II ($t_I^A - t_{II}^A > 7$), and Number of Cases when B's Schedule Excludes II ($t_{III}^B - t_{II}^B > 11$) (in parentheses are the percentages of all cases).

When we looked at the four r_{4-6}-groups, we found that in 3 out of 4 cases the group average difference between A's contribution to I and II was larger than 7. Hence, in the last rounds in at least half of the cases A's strategies excluded II from winning for sure[31].

Up to now the discussion concentrated on whether A's strategy excluded the choice of II irrespectively of what B contributes. To calculate this, we have to look at B's most extreme possible contribution to II, namely 7. B's actual contributions, however, were never that extreme (no case of $t_{II}^B = 7$ was observed), and we can plausibly assume that A's expectations of what B will do were influenced by this experience. Hence, we assume that A expect B's contribution to II to be at the highest level A previously experienced. Then we calculate whether for these expectations A's contribution schedule excludes II from winning[32]. Notice that this approach is rather unfavorable for I, because it specifies the expectations such that II is most likely to defeat I. Nonetheless, A's actual contribution schedules jointly with these expectations about B's contributions implied that on average the sum of contributions for I exceeded the sum of contributions for II by 4.135. This is a quite substantial difference, much larger than A's as well as B's average contribution for II. Furthermore, for these expectations we find that in 60 individual cases (out of 64 cases[33]) A's schedule was such that alternative I would have defeated II. In 2 cases the sum of contributions would have been equal and only in 2 cases II would have defeated I. Taking the group average contributions of the eight independent $r_{2/3}$-groups and the average of the 4 independent r_{4-6}-groups, we found that in all cases A's contributions were such that I would have defeated II.

All this evidence indicates that A's strategy choices can be summarized by

Result 4 *Player A's contribution schedules were designed to get alternative I and to exclude alternative II.*

Players B had to decide whether they wanted to go for II or III. Their strategy choices were characterized by

[31]schedule to II was never observed - A's strategy never excluded I from being chosen.

[32]B's contribution to I had to be zero anyhow. Therefore, we do not have to make any assumptions about A's expectations about B's contribution to I.

[33]In the first two rounds, no subject had played the role of A previously. Hence, no subject has previous experience about B's contributions before the third round.

Result 5 *Player B's contribution schedules were designed to get alternative III and to exclude alternative II.*

Like player B excludes the choice of I whenever the difference between the contributions to III and II was not less than 11[34]. This happened 19 times, 15 times in the last 3 rounds (see Table 8). Hence, if we observed quite some cases that B contribited so agressively to III that I was excluded. This tendency, however, was less pronounced for B than for A.

If we take the highest contribution of A to II as B's expectation about A's behavior, the average expected sum of contributions for III exceeded that for II by 2.16. Furthermore, B's schedule was in 53 (out of 64) cases designed such that III would defeat II, and only in 7 cases II would defeat III; in 4 cases a tie would occur. Taking the group average of the eight independent $r_{2/3}$-groups and the group average of the four r_{4-6}-groups, we found that in all cases B's contributions were such that III would have defeated II.

The strategies employed by the players resemble neither truthful nor natural strategies. However, Results 2 and 3 show that players did not choose schedules which made them - as required by the concept of truthful contributions - indifferent between the alternatives. Results 4 and 5 indicate that players A as well as players B rather wanted to enforce their most preferred alternative which is in line with the spirit of of the concept of natural contributions. Since players A were in the better position, they succeeded to do so most of the time. Therefore, the natural alternative was chosen in most cases. If not, player B's most prefered alternative was chosen, whereas the truthful (and efficient) alternative was hardly ever observed.

5 Conclusions

We have introduced a new class of equilibria for common agency games – natural equilibria – and we have compared it with the class that is commonly used in the literature – truthful equilibria. By applying concepts from computational complexity, we show that playing a truthful equilibrium is computationally much more demanding than playing a natural equilibrium. Therefore, one is led to conjecture that natural equilibria may be more focal than truthful equilibria.

This conjecture is partly confirmed by an experiment we conducted on a two-principal common agency game in which the natural equilibrium and the truthful equilibrium predict different alternatives. We hardly ever observed equilibrium play, neither truthful nor

[34] B would have excluded III. But this case happened only 2 times.

natural nor any other equilibrium play. Out of equilibrium contribution schedules, however, were designed not in the spirit of truthful, but of natural strategies - players made a serious contribution only on their most prefered alternative. This resulted in the choice of the natural equilibrium in most of the matches, while the truthful alternative was almost never selected.

One criticism that can be moved to our experimental evidence is that real-world players, such as lobbies, have better computing resources than our experimental subjects. However, in real-world situations, it is also often true that the number of principals is much higher than two. For instance, in US federal politics the number of lobbies who make campaign contributions is in the order of thousands. As we have shown, the difficulty of reaching a truthful equilibrium is increasing at an exponential rate with the number of principals, and, with few tens of principals, it may already be out of the reach of existing computer technology. Hence, we strongly suspect that – not only in our experiment, but in many real-world situations as well – natural equilibrium may be a better predictor of behavior than truthful equilibrium.

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