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COMPETITION

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Competition

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Abstract

Competition has been modelled in the literature in a number of ways. What do these different parametrizations of competition have in common? For instance, it turns out that it is not always the case that a rise in competition reduces price cost margins, industry wide profits or concentration. All parametrizations of competition, considered here, have two features in common. First, the reallocation effect: a rise in competition raises the profits of a firm relative to the profits of a less efficient firm. Second, a rise in competition reduces the profits of the least efficient firm active in the industry.

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1 Introduction

What is competition? More than two hundred years after Adam Smith we still don't know. In many countries competition is at the top of the policy

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agenda. To illustrate, much is made of the rise in competition in the EU due to the single market and the single currency. Also, an argument often used in favor of privatization is that it intensifies competition between firms. In the economics literature, competition is a prominent concept. See for instance the literature on competition and managerial incentives (Hart (1983), Hermalin (1992), Schmidt (1997)), the theoretical literature on competition and innovation (Aghion et al. (1995), Martin (1993)) and the empirical literature on these subjects (Baily and Gersbach (1995), Blundell et al. (1995), Nickell (1996)). Yet, a coherent definition or robust measurement of competition does not exist.

The lack of a coherent framework can be illustrated by the following division in the literature. On the one hand, the theoretical literature has parametrized competition in a number of ways, including a reduction in entry barriers, a switch from Cournot to Bertrand competition and a rise in the substitution elasticity between firms' goods. On the other hand, the empirical literature has measured competition with variables like industry concentration, price cost margins and profits. As shown below for the realistic case where firms differ in their marginal cost levels, these two views of competition flatly contradict each other. As an illustration, increasing competition through a rise in the number of firms in the industry (as Martin (1993) does) reduces industry concentration. However, increasing competition through a switch from Cournot to Bertrand competition (as it is parametrized in Aghion et al. (1995)) raises industry concentration.

This division in the literature raises two questions. First, is there common ground in such theoretical parametrizations of competition? More precisely, is there a variable which is monotone in each of these parametrizations? Second, if so, can this variable be used to bridge the gap between the theoretical and empirical literature? That is, can this variable be used to measure competition empirically?

I show that in a number of simple examples (in total nine different parametrizations of competition), there is the following common ground. Competition always raises the profits of a firm relative to the profits of a less efficient firm. This is called the reallocation effect of competition. Also, a rise in competition reduces the profits of the least efficient firm active in the market.[1] These two effects together imply the selection effect of competition, which is described by Vickers (1995:p. 13) as 'when firms' cost differ, competition can play an important role in selecting more efficient firms from less

[1] In case competition is raised through increasing the number of firms in the market, the least efficient firm is identified as the least efficient firm before entry. I return to this below.

efficient ones'.[2] Moreover, in these examples a rise in competition can both increase and decrease variables like industry concentration, total industry profits and firms' price cost margins.

The idea behind examining relative profits is that a market maps marginal cost differentials between firms into profit differentials. The more competitive the market is, the steeper this mapping becomes in the sense that a given cost differential is mapped into a bigger profit differential. In a more competitive environment, firms are punished more severely (in terms of profits) for a drop in efficiency.

The intuition for these effects can be seen in terms of market transparency. As the market becomes more transparent, consumers see more clearly which firms offer high value at a low price. As a consequence, consumers buy more from these efficient firms at the expense of inefficient firms. Therefore, the profits of the least efficient firm decline. In other words, a lack of transparency or competition protects the least efficient firm from its opponents. Further, the rise in competition may or may not reduce the profits of the most efficient firms in the industry. If it reduces all firms' profits, high cost firms lose relatively more profits than low cost firms. But there is a possibility that the most efficient firms in the industry experience a rise in profits, because the rise in competition enables them to use their cost advantage more aggressively.

Given that relative profits are monotone in a number of well known theoretical parametrizations of competition, can this be used to bridge the gap between the theoretical and empirical literature? Compared to other empirical measures of competition used (reviewed in section 7), estimating the degree to which cost differentials are mapped into profit differentials has two main advantages and one disadvantage. First, this measure of competition is monotone in competition for different parametrizations of competition. Second, the measure can be estimated using observations of only a subset of the firms in the industry. This is useful as panel data sets (including only a subset of firms in the industry) are becoming more common in empirical industrial organization. Finally, the main disadvantage is that an estimate is needed of differences in firms' marginal cost levels.

The literature on measures of competition is reviewed below. Here I only mention Hay and Liu (1997) who use a measure which is based on the reallocation of revenues, instead of the reallocation of profits. Their idea is that in a more competitive industry, a given rise in relative marginal costs leads to a bigger fall in marketshare. Although I view their measure of

[2] Note that the reallocation effect is necessary but not sufficient for the selection effect because it does not exclude the case where a rise in competition raises each firm's profits. If this case is not excluded, more intense competition could invite inefficient firms to enter.

competition and the one proposed here as complementary, I show that some but not all parametrizations of competition satisfy this revenue reallocation effect.

The rest of this paper is organized as follows. Section 2 defines the common ground of different parametrizations of competition. Section 3 presents examples of parametrizations of competition and shows that these examples have indeed the reallocation and selection effects in common. In section 4, I argue that a necessary condition for a variable to measure competition is that it is monotone in well known parametrizations of competition. It turns out that some measures of competition used in the literature are not monotone in the competition parametrizations in section 3. Section 5 discusses why a rise in competition does not necessarily reduce concentration, industry wide profits or each firm's price cost margin. Section 6 discusses briefly some problems that one can encounter when using relative profits to measure competition in practice. Section 7 reviews the literature on measures of competition and section 8 concludes. All proofs of the results can be found in the appendix.

2 Competition: the common ground

Competition is a concept that is often used in economics. And economists attribute a number of properties to this concept. Examples are the following. A rise in competition is associated with a decrease in concentration in an industry, with a decrease in each firm's profits or total industry profits. These properties of competition turn out to be correct for some parametrizations of competition, but not for others.

This leaves open the question whether these parametrizations of competition have something in common at all. In this paper, I consider a number of simple examples with nine parametrizations of competition. Three effects of competition turn out to be robust over each of these parametrizations.

Clearly, I cannot prove that these effects hold for all possible parametrizations of competition. However, the intuition for these effects is compelling enough that my conjecture is that these effects do hold for each parametrization of competition. The conjecture introduces two conditions which are discussed at the end of this section.

Conjecture 1 *If firms are completely symmetric except for their marginal cost levels (only) and if firms choose their strategic variables simultaneously and independently, then a rise in competition*
(i) reduces the profits of the least efficient firm in the market,
(ii) increases the profits of any firm relative to profits of a less efficient firm

in the market and
(iii) (weakly) increases total variable costs of any firm relative to total variable costs of a less efficient firm.

Note that efficiency is used here in terms of marginal costs. The least efficient firm is the firm i with the highest marginal costs, c_i.[3] Further, profits are defined here as revenue minus total variable costs. Throughout this paper I assume that firms have constant marginal costs. Hence total variable costs equal marginal costs times output.

The first effect is familiar. It implies, for instance, that a rise in competition reduces each firm's profits if all firms have the same marginal costs. The second effect is the profit reallocation effect of competition. These two effects imply the selection effect of competition: a rise in competition separates efficient from inefficient firms by reducing inefficient firms' profits and thus forcing them to exit. The third effect is a reallocation effect in terms of total variable costs. A rise in competition raises total variable costs of a firm relative to the total variable costs of a less efficient firm.

A rise in competition reallocates output from inefficient to efficient firms because it allows efficient firms to use their cost advantage more aggressively. Low cost firms have lower prices and hence attract more customers as competition is intensified. This reallocation of output from inefficient to efficient firms raises variable costs of a firm relative to variable costs of a less efficient firm. Further, this reallocation of output raises the profits of efficient firms relative to inefficient firms. This does not necessarily imply that the profit level of an efficient firm rises. If, due to the more competitive environment, the profits of an efficient firm decrease, the profits of less efficient firms decrease faster.

It is important to distuingish three different ways in which competition can be increased in an industry. These are summarized in table 1, with their respective parametrizations in the examples below.

[3] Implicitly, I assume that there is no vertical product differentiation, that is consumers' valuation of product i, v_i, is the same for each product. If one would allow for vertical product differentiation, firm i is more efficient than j if and only if $v_i - c_i > v_j - c_j$. Although this extension is trivial theoretically, empirically it is hard to get estimates of v_i. This adds to the problem of estimating efficiency differences between firms, discussed below.

competition becomes more intense as:	parametrized as:
1. number of firms in the industry rises	* $f \downarrow$(and hence $n \uparrow$) in example 1
2. interaction between firms becomes more aggressive	* $\lambda \downarrow, \frac{b}{d} \downarrow, a \downarrow$ in example 1 * $\theta \uparrow$ in examples 2,4,5 * a switch from Cournot to Bertrand competition in example 3.
3. costs are reduced	* $c_j \downarrow$ in example 1

Table 1. parametrizations of competition

In example 1, the number of active firms n in the industry is raised through a fall in the exogenous entry cost f. Most economists would identify this as a rise in competition. It is useful to distinguish such a rise in competition from the case where competition is increased through more aggressive interaction between firms. Examples of more aggressive interaction are a decrease in the conjectural variation (see for instance Bresnahan (1989) and Hay and Liu (1997)), firms' products becoming closer substitutes and a switch from Cournot to Bertrand competition. A rise in competition through more aggressive interaction can lead to a fall in the number of firms in the market, through the selection effect. In other words, observing a fall in the number of firms in an industry can be caused by either a fall in competition (an increase in the entry cost) or a rise in competition (more aggressive interaction between firms). Finally, a firm is said to be in a more competitive environment if it faces opponents with lower costs. In other words, if a high cost firm is replaced by a low cost firm, then the industry is called more competitive. This is captured by the last form of competition related to a firm's opponents' cost levels. For instance, a rise in competition faced by domestic firms in their home market due to reductions in import tariffs (per unit of output) paid by their foreign competitors can be modelled in this way. A reduction in the tariff foreign firms have to pay is equivalent to a reduction in their marginal production costs.

In each of the examples below, competition is increased through one channel only. That is, I do not consider the case where more aggressive interaction or reducing a firm's cost level leads some firms to exit through the selection effect. The reason is that it is unclear what has happened to competition in this case. That is, the exit of firms may (or may not) outweigh the initial rise in competition through more aggressive interaction and overall competition may[4] (or may not) be reduced. Separating these effects turns out to be more

[4]A simple way to illustrate this is Sutton's (1991: 33) example. Consider a market with two potential entrants which are identical, that is the firms have the same constant marginal costs and produce the same product. To enter each has to pay a positive sunk

instructive. In particular, the separate analysis shows in which sense a rise in competition (1) through a rise in the number of active firms, (2) through more aggressive interaction and (3) through reducing a firm's opponents' cost levels are similar. In each case, the rise in competition reduces the profits of the least efficient firm in the market and features the reallocation effect in terms of profits and total variable costs.

Let me specify what the conjecture implies if competition is increased through a rise in the number of firms in the market and through a reduction in costs. Consider the case where the n most efficient firms are in the market. Entry barriers are lowered and firm $n+1$ enters the market, which is now the least efficient firm in the market. Clearly, as $n+1$'s profits and costs have increased from 0 to some positive value, claims (i)-(iii) in the conjecture do not apply to $n+1$. In this case the conjecture should be interpreted as applying to the ex ante least efficient firm, say I with $c_I \geq c_i$ for each $i \in \{1, 2, ..., n\}$. It is shown below that the addition of firm $n+1$ to the market, reduces firm I's profit level and raises profits and variable costs of more efficient firms relative to I. Similarly, if competition in the industry is increased through lowering the least efficient firm's marginal cost level, the conjecture applies to the one but least efficient firm in the industry.

The conditions in the conjecture that firms are symmetric except for their marginal cost levels and that they choose their strategic variables simultaneously and independently[5] are important, as the following counterexample shows.

Counterexample Consider two firms, denoted 1 and 2, where firm i $(= 1, 2)$ faces demand of the form $p(x_i, x_j) = 1 - x_i - \theta x_j$ with $i \neq j$ and $0 < \theta < \sqrt{2}$, where the upperbound on θ is needed to allow for an equilibrium with two active firms. Firm i has constant marginal costs c_i. Suppose firm 1 has a first mover advantage: firm 2 chooses output level x_2 after ob-

entry cost. If there is Cournot competition in the product market, both firms enter if the sunk cost is not too high. If competition intensifies through a switch from Cournot to Bertrand competition, at most one firm can profitably enter. This firm charges the monopoly price. In this case, the switch from Cournot to Bertrand competition reduces overall competition.

[5] Also, when considering collusion outcomes with side payments, the following model contradicts the reallocation effect. Consider the case of a duopoly where firm 1 is far more efficient than 2. They determine the output allocation that maximizes joint profits and (Nash) bargain to split the surplus. Suppose firm 1 has all the bargaining power. This implies that firm 2 receives a payoff equal to its outside option which equals the competitive outcome. In this case, a rise in competition caused by a move from the collusion outcome to the competitive outcome has no effect on the inefficient firm's profits. At the same time, the profits of the more efficient firm 1 fall due to the rise in competition.

serving 1's choice x_1. In this example, θ can be used as a parametrization of competition. That is, competition becomes more intense as goods become closer substitutes (θ rises). As shown in the appendix, it follows for $c_1 = 0.4$, $c_2 = 0.35$ and $\theta = 1.1$ that $\frac{d\pi_1(c_1,c_2;\theta)}{d\theta} > 0$ while $\frac{d\pi_2(c_2,c_1;\theta)}{d\theta} < 0$. In words, a rise in competition decreases the profits of the low cost firm while it increases the profits of the high cost firm because the latter has a first mover advantage. Hence results (i) and (ii) of the conjecture no longer hold.

The idea of the counterexample is that a rise in competition also affects the first mover advantage. Although more intense competition makes the cost disadvantage of firm 1 more pronounced, it also increases the benefit of 1's first mover advantage. Hence, marginal cost is no longer a sufficient statistic for the relative competitive position of a firm if there is an unlevel playing field, as here in the timing of firms' moves.

The asymmetries excluded by the conjecture are asymmetries that affect firms' behavior at the margin. So the conjecture still holds if, for instance, firms differ in their levels of fixed costs. A firm's level of fixed costs affects its entry decision. But once it is in the market, a change in competition will have the effects described in the conjecture.[6]

3 Examples

A parameter of competition is an exogenous variable which determines how competitive the industry is. It is the exogeneity of these variables that makes them a natural starting point of the analysis.[7] This section considers examples of parameters that the literature has interpreted as determinants of competition. It concludes with a proposition showing that conjecture 1 holds for each of these parametrizations of competition.

Empirical measures of competition are based on endogenous variables like firms' profits and revenues. The next section introduces measures like relative profits, the Herfindahl index and price cost margins. Then it is analyzed how the parametrizations of competition below affect these measures of com-

[6]Note, though, that with asymmetries in fixed costs, parts (i) and (ii) of the conjecture do not necessarily imply the selection effect of competition. Consider the case of a duopoly where firm 1 has lower marginal costs than firm 2 but higher fixed costs. Further, assume that 1's profits (gross of fixed costs) equal its fixed costs, while 2's profits exceed its fixed costs. Then a rise in competition which reduces both firms' profits forces the more efficient firm in terms of marginal costs (firm 1) to exit.

[7]More precisely, these variables are exogenous in the examples below. One could consider a meta game in which these variables are endogenised and determined by 'deeper' exogenous variables.

petition.

Example 1 Consider the following two stage game. In the first stage firms decide simultaneously and independently whether to enter an industry or not. Entering the industry costs a fixed entry fee f. In the second stage, the firms that have entered choose simultaneously and independently their output level. All firms know which firms enter and what each firm's constant marginal cost level is.
This two stage game can be solved by backward induction. First, the second stage is solved assuming n firms enter, then in the first stage n is determined. Assume that firm i in the second stage faces a demand curve of the form

$$p(x_i, x_{-i}) = a - bx_i - d\sum_{j\neq i} x_j$$

and marginal costs c_i. Then firm i chooses output x_i which solves

$$\max_{x\geq 0}\{(a - bx - d\sum_{j\neq i} x_j)x - c_i x\}$$

with $a > c_i > 0$ and $0 < d \leq b$. I write the first order condition as

$$a - 2bx_i - d\sum_{j\neq i} x_j - \lambda d x_i - c_i = 0 \tag{1}$$

where $\lambda \equiv \frac{d\sum_{j\neq i} x_j}{dx_i}$ is the conjectural variation. In words, λ measures the belief of firm i about how its opponents will react to a change in its own output level. For instance, the Cournot-Nash equilibrium assumes $\lambda = 0$.
This parameter λ can be seen as a parametrization of competition. The lower λ, the more softly a firm expects its opponents to react to a rise in its output level. Hence the more aggressively (in the sense of higher output levels) the firm will behave. Assuming all n firms produce positive output levels, one can solve the n first order conditions (1). This yields

$$x_i = \frac{\left(\frac{2b}{d} + \lambda - 1\right)a - \left(\frac{2b}{d} + \lambda + n - 1\right)c_i + \sum_{j=1}^{n} c_j}{(2b + d(\lambda + n - 1))\left(\frac{2b}{d} + \lambda - 1\right)} \tag{2}$$

Further, rewriting equation (1) as

$$a - bx_i - d\sum_{j\neq i} x_j - c_i = (b + \lambda d)x_i$$

allows one to write the profits of firm i as

$$\pi_i = (b + \lambda d)x_i^2 \tag{3}$$

and the price of firm i as

$$p_i = (b + \lambda d)x_i + c_i \tag{4}$$

Then moving back to the first stage, one can determine the number of firms in the industry n by the conditions that

$$\begin{aligned} \pi_n &\geq f \\ \pi_{n+1} &< f \end{aligned}$$

Within this model, there are five parameters that affect how competitively firms behave. First, as explained above, a lower value of λ implies more competitive behavior by firms. Second, the ratio $\frac{d}{b}$ measures how close substitutes the goods are. If $d = b$ then goods are perfect substitutes, for $d < b$ firms have some monopoly power due to product differentiation. Thus, a rise in $\frac{d}{b}$ is interpreted as a rise in competition. Third, a fall in $\sum c_j$ makes the industry more competitive as a firm faces competitors with lower costs. Fourth, a rise in the exogenous entry fee f reduces the number of active firms in the industry and hence reduces competition. Finally, for given number of firms in the market, a rise in a has the same effects as a rise in monopoly power. This can be seen as follows. Writing profits of firm $n+1$ minus the entry cost f as

$$c_{n+1}\frac{c_{n+1}}{d}(\frac{b}{d}+\lambda)\left(\frac{\left(\frac{2b}{d}+\lambda-1\right)\frac{a}{c_{n+1}} - \left(\frac{2b}{d}+\lambda+n\right) + \sum_{j=1}^{n+1}\frac{c_j}{c_{n+1}}}{\left(\frac{2b}{d}+(\lambda+n)\right)\left(\frac{2b}{d}+\lambda-1\right)}\right)^2 - f$$

it is clear that $\pi_{n+1} - f$ is homogenous of degree 1 in $(a, b, d, f, c_1, c_2, ...)$. That is, changing all these parameters with the same multiple leaves the entry decision of firm $n+1$ unchanged. Raising the intercept a while assuming that firm $n+1$ stays out is in this sense equivalent to assuming that $(b, d, f, c_1, c_2, ...)$ have all risen with the same factor. This leaves $\frac{b}{d}$ unchanged but raises f and $\sum c_j$. As explained above, this is interpreted as a fall in competition. In this sense, a rise in a for given number of firms in the industry decreases competition.

Example 2 Consider two firms, denoted 1 and 2, that face demand of the form $p_i(x_i, x_j) = \frac{1/x_i^{1-\theta}}{x_i^\theta + x_j^\theta}$ with $i \neq j$ and $0 < \theta < 1$. This demand function is derived from a CES utility function $u(x_1, x_2) = (x_1^\theta + x_2^\theta)^{\frac{1}{\theta}}$, where θ measures the degree of substitutability between the goods of firm 1 and 2. Firm i has marginal costs c_i. I say that competition is increased as goods become closer substitutes, that is θ rises. The Cournot Nash equilibrium output and price levels equal respectively

$$x_i = \frac{\theta}{c_i}\frac{\left(\frac{c_i}{c_j}\right)^{\theta}}{\left(1+\left(\frac{c_i}{c_j}\right)^{\theta}\right)^2} \tag{5}$$

$$p_i = \frac{c_i}{\theta}\left(1+\left(\frac{c_i}{c_j}\right)^{-\theta}\right) \tag{6}$$

Finally, the equilibrium profits of firm i can be written as $\pi(c_i, c_j; \theta) = \frac{1+(1-\theta)\left(\frac{c_i}{c_j}\right)^{\theta}}{\left(1+\left(\frac{c_i}{c_j}\right)^{\theta}\right)^2}$ with $i, j = 1, 2$ and $i \neq j$. Defining the cost gap between i and j as $c \equiv \frac{c_i}{c_j}$ the profits of firm i equal

$$\pi(c; \theta) = \frac{1+(1-\theta)c^{\theta}}{(1+c^{\theta})^2} \tag{7}$$

Example 3 Consider the same set up as in the previous example with homogenous goods, that is $\theta = 1$. I analyze the effects of a change in competition through a switch from Cournot to Bertrand competition in the market, where Bertrand competition is often seen as more competitive than Cournot competition. With Cournot competition the expressions for output x_i^C, price p_i^C and profits π_i^C can be found in the example above with $\theta = 1$. To derive the expressions for Bertrand competition, assume that $c_1 < c_2$. Then firm 1 chooses limit price $p^B = c_2$ to reduce firm 2's output level to 0. It follows that firm 1's output and profits equal

$$x_1^B = \frac{1}{c_2} \tag{8}$$

$$\pi_1^B = 1 - \frac{c_1}{c_2} \tag{9}$$

while firm 2's output and profit level are equal to 0.

Example 4 Consider the following Dixit-Stiglitz model of monopolistic competition. Firm i faces an inverse demand function of the form $p_i(x_i) = Ax_i^{-(1-\theta)}$, where it takes the value of $A = \frac{1}{\left(\sum_{j=1}^n p_j^{\frac{-\theta}{1-\theta}}\right)^{1-\theta}}$ as given. The idea is that the number of firms n is so big that the effect of firm i's actions on A are negligible. This is the main difference with example 2 where the duopolists do take the effects of their actions on A into account. This demand function is derived from a CES utility function $u(x_1, ..., x_n) = \left(\sum_{j=1}^{n} x_j^{\theta}\right)^{\frac{1}{\theta}}$ where $0 < \theta < 1$ measures the degree of substitutability between the goods $1, ..., n$. Hence firm i solves

$$\max_{x_i} Ax_i^{\theta} - c_i x_i$$

This yields $x_i = \left(\frac{\theta}{c_i} A\right)^{\frac{1}{1-\theta}}$ and

$$p_i = \frac{c_i}{\theta} \tag{10}$$

Substituting this expression for p_i into A allows one to write i's output as

$$x_i = \frac{\left(\frac{\theta}{c_i}\right)^{\frac{1}{1-\theta}}}{\sum_{j=1}^{n} \left(\frac{\theta}{c_j}\right)^{\frac{\theta}{1-\theta}}} \tag{11}$$

Finally, i's profits equal

$$\pi_i = (1-\theta) \frac{\left(\frac{\theta}{c_i}\right)^{\frac{\theta}{1-\theta}}}{\sum_{j=1}^{n} \left(\frac{\theta}{c_j}\right)^{\frac{\theta}{1-\theta}}} \tag{12}$$

As in example 2, competition is parameterized here through θ.

Example 5 Consider an Hotelling beach of length 1 with consumers distributed uniformly over the beach with density 1. Firm 1 is located on the far left of the beach and firm 2 on the far right. A consumer at position $x \in \langle 0, 1 \rangle$ who buys a product from firm 1 incurs a linear travel cost tx, and if she buys from firm 2 she incurs travel cost $t(1-x)$. Assume that each consumer buys one and only one product. Firm i has constant marginal costs c_i $(i = 1, 2)$. Then demand for the products of firm i equals $x_i(p_i, p_j; t) = \frac{1}{2} + \frac{p_j - p_i}{2t}$. As travel costs decrease, consumers are more inclined to buy from the cheapest firm rather than the closest one. So as travel costs decrease, firms' monopoly power is reduced and competition is higher. Parametrizing competition as $\theta = \frac{1}{t}$, the Nash equilibrium output and price levels equal respectively

$$x_i = \frac{1}{6}(3 + \theta(c_j - c_i)) \tag{13}$$

$$p_i = \frac{1}{3}(\frac{3}{\theta} + 2c_i + c_j) \tag{14}$$

Finally, the profits of firm i equal

$$\pi(c_i, c_j; \theta) = \frac{(3/\theta + c_j - c_i)^2}{18/\theta} \tag{15}$$

with $i, j = 1, 2$ and $i \neq j$.

Proposition 2 *Claims (ii) and (iii) in conjecture 1 hold for all examples above. Claim (i) holds for each example above, if upperbounds* $\bar{\lambda}, \bar{b} > 0$ *are imposed on* λ *and* b *respectively in example 1.*

This shows the sense in which conjecture 1 is robust to different parametrizations of competition. It holds for all examples in this section. The reason for the upperbounds $\bar{\lambda}, \bar{b}$ needed for claim (i) is the following. Recall that λ denotes firm i's expectation of its opponents' output response to a rise in i's own output level. Hence a high value of λ makes firm i reluctant to raise output. Equation (2) implies that $\lim_{\lambda \to +\infty} x_i = 0$. As a benchmark, consider a multi-product monopolist faced with the same demand structure and n plants where plant i produces product i with marginal costs c_i. For λ large enough and $d < b$, it is the case that each firm i produces less than the multi-product monopolist would produce with plant i. In this case a reduction in λ, which is interpreted as a rise in competition, raises each firm's output and profit levels. Hence this reduction in λ raises the least efficient firm's profits. To avoid this (extreme) case, where output is below the multi-plant monopolist's output, an upperbound on λ is needed.[8] Since the parameters b and λ enter symmetrically (for given d) in the expression for profits (3) (and output(2)), the same reasoning applies to the upperbound on b.

Although the proposition shows that the conjecture is fairly robust to different parametrizations of competition, it is important to note that all parametrizations have three important assumptions in common. First, each firm produces one good with constant marginal costs. This avoids problems

[8] Alternatively, one can argue that a reduction in λ (or b) which increases the least efficient firm's profits should not be interpreted as a rise in competition. In that case, λ is defined as a parameter of competition only for values satisfying $\lambda < \bar{\lambda}$.

like (dis)economies of scale and scope and how to allocate costs to certain activities. Second, firms choose their strategic variables simultaneously and independently. This excludes from the analysis issues like collusion,[9] mergers, predation and first mover advantages. Finally, firms compete with a one dimensional variable. That is, I do not consider the case where, for instance, firms choose both the price level and the amount spent on advertising. This makes the efficiency comparison of two firms unambiguous, because it excludes the possibility that one firm is more efficient in producing output while the other is more efficient in generating sales through advertising.

4 Measures of competition

Proposition 2 shows that relative profits, relative variable costs and the profits of the least efficient firm are monotone in competition for the examples in section 3. This section shows that this is not true for the Herfindahl index, total rents in an industry, relative revenues nor for the price cost margin, each of which has been used in the literature to measure competition (see section 7). Moreover, relative profits and relative variable costs are monotone in competition even if only a strict subset of firms in the industry is observed.

Within the simple framework of this paper, where each firm produces one good (only) with constant marginal costs, a common denominator for each of these variables is that they can be constructed from firms' revenues $p_i x_i$ and variable costs $c_i x_i$. Further, it is instructive to consider two cases: one in which all firms in the industry are observable, the other where only a sample of firms is observed. I use the following definition of a measure of competition and a measure of competition in panel data sets.

Definition 3 *A variable m is called a measure of competition if it satisfies both conditions (M) and (A) below. If m satisfies both (M) and (P) below it is called a measure of competition in panel data sets.*

[9] Allowing for both collusion and marginal costs which change with firm's output levels can cause the following problems. Consider a cartel model where collusive and punishment periods alternate, such as in Abreu (1986) or Porter (1983). The punishment phase is more competitive than the collusive phase. Suppose that firm i's total variable costs equal $c_i x_i + \gamma_i x_i^2$. Hence, i's marginal costs equal $c_i + 2\gamma_i x_i$. In the case where $c_i < c_j$ but $\gamma_i > \gamma_j$ it is conceivable that in the collusive (low output) phase, the marginal costs of firm j exceed those of firm i, while in the punishment (high output) phase it is the other way around. Comparing the relative profits of i and j in the two phases, which firm is the less efficient one? A related observation is that in the full collusion outcome with homogenous goods, firms' marginal costs will be equalized. Such extensions are left for future research.

(M) monotonicity: m is monotone in the parametrizations of competition in table 1;
(A) observability: m can be derived from revenues and total variable costs of each firm $\{p_i x_i, c_i x_i\}_{i \in A}$ where A is the set of all firms, $A = \{1, ..., n\}$;
(P) observability in panel data sets: m can be derived from revenues and total variable costs of each firm $\{p_i x_i, c_i x_i\}_{i \in P}$ where P is any strict subset of $\{1, ..., n\}$ consisting of at least two firms.

The first requirement is natural. A necessary condition for a variable to measure competition is that it is monotone in competition. That is, if one increases competition through a change in a parameter of the model, the measure of competition should always increase (or always decrease). This allows the interpretation of a change in this measure as a change in competition.[10]

The third requirement takes the following feature of panel data sets into account.[11] At each moment in time a panel data set only contains a subset of the firms in an industry. So one does not observe, say, total industry profits, but only the sum of profits of firms in the panel. Also, concentration can only be measured as concentration within the panel.[12] Although this seems a reasonable assumption for panel data sets, one may argue that for any data set it is the case that only a subset of the relevant firms is observed. This may be due to problems of the relevant market. Or due to the fact that firms or plants are classified by their main activity not by each separate activity. As shown below, the only variable that is a measure of competition but not a measure of competition in panel data sets is the profits of the least efficient firm in the market.

I analyze the effect of competition on the seven variables summarized in table 2. The notation used is the following. Firm i's Nash equilibrium price and output levels equal $p(c_i, c_{-i}, \theta)$ and $x(c_i, c_{-i}, \theta)$. Note that these functions are not indexed by i, since firms are completely symmetric except for their marginal cost levels. To ease notation, Nash equilibrium price and output levels are denoted by $p_i \equiv p(c_i, c_{-i}, \theta)$ and $x_i \equiv x(c_i, c_{-i}, \theta)$. Nash equilibrium profits of firm i are defined as $\pi(c_i, c_{-i}, \theta) \equiv p_i x_i - c_i x_i$. I use

[10] Ideally, one would like to have a variable which increases *if and only if* competition is raised. Strictly speaking, this is true for relative profits in the examples above. However, the examples are chosen for their analytical tractability and hence are parsimonious in parameters. So the claim of the paper is that relative profits rise *if* competition increases.

[11] Requirement (P) is stronger than (A), because under (P) one uses less information than is allowed under (A). In other words, under (A) one can always drop some observations and calculate m for only a subset of firms.

[12] Note that it is sometimes possible to calculate concentration indices in a panel data set using total industry revenues, because information on total revenues is available from a separate source.

the convention that a rise in θ denotes a rise in competition, as in example 2. If competition is parameterized as the conjectural variation λ in example 1, then θ is identified as $1/\lambda$.

RP	relative profits	$\frac{\pi(c_i,c_{-i},\theta)}{\pi(c_j,c_{-j},\theta)}$ where $c_i < c_j$
SP	sum of profits	$\sum_{i\in O}\pi(c_i,c_{-i},\theta)$
$PLEF$	profits of the least efficient firm	$\pi(c_I,c_{-I},\theta)$ where $c_I \geq c_i$ for each $i \in O$
RR	relative revenues	$\frac{p_i x_i}{p_j x_j}$ where $c_i < c_j$
RVC	relative variable costs	$\frac{c_i x_i}{c_j x_j}$ where $c_i < c_j$
PCM	price cost margin	$\frac{p_i x_i - c_i x_i}{p_i x_i}$
H	Herfindahl index	$\sum_{i\in O}\left(\frac{p_i x_i}{\sum_{j\in O} p_j x_j}\right)^2$

where O is the set of observed firms, that is $O = \{1, ..., n\}$ under requirement (A) and $O \subset \{1, ..., n\}$ under requirement (P).

Table 2. Seven variables

All of the variables in table 2 are defined with respect to a set O of observed firms. Under requirement (A), it is the case that $O = \{1, ..., n\}$, while under (P) the set O is a strict subset of all firms $\{1, ..., n\}$. Note that the ratios RP, RR and RVC are defined with the relatively efficient firm in the numerator and the relatively inefficient one in the denominator.

If competition is raised through a fall in entry barriers f which raises the number of firms in the market or a fall in a firm's marginal cost level, the variables in table 2 should be interpreted as follows. If the number of firms in the market rises from n to $n+1$, the relative measures should be calculated for any two firms, except $n+1$. If firm $n+1$ is the least efficient firm in the market, $PLEF$ should be calculated for the next least efficient firm. In the sum-measures and the Herfindahl index, firm $n+1$ should be included under requirement (A). Further, if competition is increased through a reduction in firm i's marginal costs, the relative measures should be calculated for any two firms, except firm i. Similarly, in this case the profits of the least efficient firm should be calculated for the least efficient firm, except i. Clearly, if i is the least efficient firm in the industry, reducing its marginal costs will raise its profits (in absolute value) and will raise its profits relative to any other firm. Whether one includes firm i or not in calculating the sum-measures turns out to be irrelevant. As the next result shows the sum variables are not monotone in competition anyway, irrespective of this decision.

Proposition 4 *Of the seven variables in table 2, only RP, $PLEF$ and RVC are measures of competition. Moreover, RP and RVC are measures of competition in panel data sets. RP is increasing and RVC is non-decreasing in competition.*

Of course, the observation that RP and RVC are measures of competition follows immediately from the proposition that conjecture 1 holds in the examples above. Since the conjecture is stated for any two firms, there is no need to observe all the firms in the industry. Thus RP and RVC are measures of competition in panel data sets as well. This is different for $PLEF$. Again, because conjecture 1 holds for all examples above,[13] $PLEF$ is a measure of competition. However, $PLEF$ is not a measure of competition in panel data sets. This follows from the fact that the sum of profits, SP, is not monotone in competition, which is proved in the appendix. Consequently, a rise in competition can raise the sum of profits. Thus it raises the profits of some firms in the market, and the panel may consist of exactly these firms. The proof in the appendix gives one counterexample for each of the variables that do not satisfy (M). That is, it shows by example that the variable can rise with competition and that it can fall with competition. The next section gives the intuition why the variables in table 2, with the exception of RP, RVC and $PLEF$, are not monotone in competition.

5 Existing measures and their drawbacks

Contrary to what is sometimes assumed, lemma 5 in the appendix shows that a rise in competition does not always reduce total industry profits. If all firms have the same marginal cost level, a rise in competition does indeed lower total industry profits. However, if firms differ in their marginal cost levels the rise in competition reallocates output from inefficient firms with a low price cost margin to efficient firms with a high price cost margin. In other words, output is reallocated from firms where output does not add much to industry profits to firms where it contributes a lot to industry profits. If the latter effect is strong enough, the rise in competition raises industry profits.

Although a rise in competition always raises RP and RVC, it raises relative revenues RR in some cases but not in all.[14] A rise in competition can raise RR by reallocating output from inefficient to efficient firms, while leaving relative prices between firms unchanged. For instance, if firms produce homogenous goods the relative price equals $\frac{p_i}{p_j} = 1$ and does not change with competition. The intuition why a rise in competition reduces RR, can be

[13] That is, with the upperbounds on λ and b.

[14] Recall from the introduction that Hay and Liu (1997) use RR as a way to measure competition.

seen by decomposing RR in the following way

$$\frac{p_i x_i}{p_j x_j} = \frac{\pi_i + c_i x_i}{\pi_j + c_j x_j} = \underbrace{\frac{\pi_j}{\pi_j + c_j x_j}}_{\alpha} \frac{\pi_i}{\pi_j} + \underbrace{\frac{c_j x_j}{\pi_j + c_j x_j}}_{1-\alpha} \frac{c_i x_i}{c_j x_j}$$

where it is the case that $c_i < c_j$ and π_i is shorthand notation for $\pi(c_i, c_{-i}; \theta)$. That is, relative revenues RR can be written as a weighted average of relative profits RP and relative variable costs RVC. The weight on relative profits α can be written as the price cost margin of the relatively inefficient firm j, $\frac{p_j - c_j}{p_j}$. It follows from conjecture 1 that $\frac{\pi_i}{\pi_j}$ and $\frac{c_i x_i}{c_j x_j}$ are increasing in competition. Hence if α does not vary (much), relative revenues rise as well with competition. Relative revenues fall with competition if the price cost margin of the relative high cost firm j falls because of the rise in competition, while at the same time $\frac{\pi_i}{\pi_j} > \frac{c_i x_i}{c_j x_j}$. This last condition can be rewritten as the price cost margin of the efficient firm exceeds that of the inefficient firm, $\frac{p_i - c_i}{p_i} > \frac{p_j - c_j}{p_j}$. So if the fall in the price cost margin of firm j is big enough to offset the increase in $\frac{\pi_i}{\pi_j}$ and $\frac{c_i x_i}{c_j x_j}$, a rise in competition reduces relative revenues RR. This non-monotonicity of RR is proved in lemma 6 in the appendix. Summarizing, a rise in competition can reallocate revenues or market share from inefficient to efficient firms, but this is not necessarily always the case.

Although in examples 3, 4 and 5 a rise in competition reduces each firm's price cost margin this is not always the case. As shown in lemma 7 in the appendix for a duopoly case, a rise in competition may raise the price cost margin of the most efficient firm, if its cost level is far lower than that of its opponent.[15,16] The intuition is that the rise in competition marginalizes the inefficient firm. This creates the opportunity for the efficient firm to raise its price level (while keeping it below the price of the less efficient firm), since customers are not buying from the inefficient firm anyway.

As noted by Tirole (1988: 223) and proved in lemma 8, concentration is not monotone in competition. This is clearly the case if competition is raised through more aggressive interaction between firms. Because such a rise in competition can reallocate revenues from the inefficient to the efficient

[15] The result that a rise in competition can raise prices is not new. Stiglitz (1989) gives an example where a rise in competition through a rise in the number of firms in the market raises firms' prices.

[16] There also seems to be some empirical support for this possibility. Goldberg and Knetter (1995) estimate the elasticity of residual demand faced by German beer exporters in five countries, using exchange rate shocks. This elasticity is interpreted as a measure of competition. They find that German beer exporters charge the highest prices in the market where they face the most intense competition.

firms, it raises concentration. This follows from the fact that efficient firms have already higher market shares to start with,[17] thus reallocating more revenues to them raises the Herfindahl index. A fall in concentration correctly indicates a rise in competition if there is entry into the market. However, lemma 8 also shows that this does not hold under requirement (P). That is, entry may lead to a rise in observed concentration in a panel data set. The intuition is again the reallocation effect. Suppose an additional firm enters into an industry, but this new firm is not (yet) in the panel of observed firms. This rise in competition reallocates market shares among the observed firms in the panel. In particular, the inefficient firms in the panel experience a bigger fall in revenues than the efficient firms. Consequently, observed concentration rises in the panel. In other words, in a panel data set a rise in competition through more aggressive interaction or through entry into the industry can lead to a rise in observed concentration.

6 Relative profits: discussion

As shown above, the variable RP is monotone in competition, at least for a number of parametrizations of competition. This is an important first step towards finding common ground in the different ways that economists have modelled competition. Next, given the observation in the introduction that the theoretical and empirical literature contradict each other on the topic of competition, can the relative profits measure bridge the gap?

In other words, given the theoretical robustness of relative profits and the result that it is a measure of competition in panel data sets, can it be used as an indicator of competition in an empirical sense? Here, I briefly mention three problems which one encounters when using RP as a measure of competition in practice. In particular, I discuss the consequences of the unobservability of marginal costs, an (unobserved) unlevel playing field and problems with defining the relevant market. The upshot of this discussion is that comparing competition between industries using relative profits is nonsense because of these problems. However, if one can argue that in a

[17]This was already noted in the discussion on the profitability-concentration relation. See for instance Clarke et al. (1984) and Tirole (1988: 223). One explanation of the positive correlation between concentration and profitability is that with fewer firms in the industry it is easier to collude and raise profits. Another explanation, put forward by Demsetz, is the following. If some firms have a strong efficiency advantage over their rivals, this will raise their profits and industry concentration. The point of my paper is that a rise in competition, say due to a reduction in the conjectural variation, can strengthen this Demsetz effect. In particular, such a rise in competition can raise the market share and profits of the most efficient firms at the expense of less efficient ones.

certain industry the extent of these problems has not changed over time, then relative profits can be used to monitor competition over time within this industry.

A simple way to implement relative profits RP as a measure of competition is the following. At a given moment in time, estimate the slope $\beta_{\pi c}$ of the following equation

$$\frac{\pi_i}{\pi_1} = \beta_{\pi 0} + \beta_{\pi c} \frac{c_i}{c_1} + \varepsilon_i \tag{16}$$

where firm 1 is used to normalize profits and marginal costs and $i \in O$ indexes the other firms that are observed in the industry. For concreteness, assume that firm 1 is the firm with the lowest marginal costs of the observed firms. That is, $c_i \geq c_1$ for each observed firm i. In general, this relation will be downward sloping, that is $\beta_{\pi c} < 0$. Firms with higher marginal costs have lower profits. As above, profits are defined as revenues minus variable costs, $\pi(c_i, c_{-i}, \theta) = p_i x_i - c_i x_i$ and π_i is used as short hand notation for $\pi(c_i, c_{-i}, \theta)$.

The intuition for the slope $\beta_{\pi c}$ is the following. It measures to which extent the cost gap $\frac{c_i}{c_1}$ is mapped into a profit differential $\frac{\pi_i}{\pi_1}$. In a very competitive (not competitive) industry, efficiency differences $\frac{c_i}{c_1}$ are mapped into big (small) profit differences; that is $\beta_{\pi c}$ is big (small) in absolute value.

Three problems one encounters when using (16) in practice are the following. First, as argued, for instance by Bresnahan (1989), a major problem in applied industrial economics is that relevant concepts are not readily observable. In particular, marginal costs c_i and profits defined as revenues minus variable costs, $\pi_i \equiv p_i x_i - c_i x_i$, are not readily observable. Measurement errors in these variables may cause biased estimation results in (16).[18]

Second, consider the following example of an (unobserved) unlevel playing field. As noted in Bishop (2000: 10), online retailers in the US do not charge their customers any sales tax, while traditional high street stores do collect the sales tax. Let firm a denote the online firm and b the high street store, and let c_a and c_b denote their respective marginal production costs. Now suppose that the researcher trying to estimate equation (16) does not add the sales tax t to the marginal costs of firm b; for instance, because he classifies the taxes paid by the firm as a fixed cost, or does not observe taxes at all. Then the effect of an unobserved change in the playing field t on the estimated slope $\beta_{\pi c}$ is the following. If $c_a < c_b$ then increasing t raises the relative profits $\frac{\pi_a}{\pi_b}$ for given observed marginal production costs. Hence the estimated slope $\beta_{\pi c}$ increases in absolute value. This would be interpreted as a rise in

[18] Another potential cause of bias is that firms' marginal cost levels are endogenously determined by firms' decisions on, say, R&D. If R&D decisions are affected by competition, simply estimating equation (16) yields biased results.

competition. However making the playing field less even is usually seen as a reduction in competition. If, on the other hand, $c_a > c_b$ then the (unobserved) rise in t reduces the estimated slope in absolute value. Summarizing, if the playing field is made unlevel in favour of the most efficient firms (while this is not observed), this gives the incorrect impression that competition has increased. If the playing field is made unlevel in favour of the least efficient firms, observed competition is reduced.

Finally, defining the relevant market is known to be a difficult problem. With a measure like concentration one can err in two directions. With too broad a definition of the market, concentration is too low and overestimates the degree of the competition. With too narrow a definition of the market, concentration is too high and underestimates the extent of competition. One can argue that with the indicator based on relative profits, the main problem is too broad a definition of the relevant market. In particular, condition (P) in definition 3 implies that relative profits still measure the extent of competition if only a subset of firms in the market is observed. In this sense, too narrow a definition of competition is not necessarily a problem. But if the definition of the market is too broad, the estimated indicator may overestimate or underestimate the true extent of competition depending on the relative cost and profit ratios of the added firms. Thus one can argue that the problem of defining the relevant market can be partly solved for the indicator $\beta_{\pi c}$ by choosing only the firms which are without doubt operating in the market under consideration.

Each of these problems prevents inter-industry comparisons of competition using relative profits. If the slope in (16) is steeper in industry A than in B, this may for instance be because (i) measurement errors with respect to marginal costs reduce the observed dispersion of marginal costs in A more than in B, (ii) the playing field is particularly unlevel in favour of the most efficient firm in A and (iii) some of the firms observed under B are in fact not on the relevant market and they reduce the estimated slope $\beta_{\pi c}$. Since these problems cannot be quantified, one cannot correct for them in a meaningful way.

However, more can be said about the change in competition over time within a certain industry. If relation (16) becomes steeper over time within industry A and if one can argue that the problems mentioned above have not changed over time in A, then this can be interpreted as evidence that competition in A rises over time.

7 Literature on measures of competition

In the empirical literature, competition is often measured with variables like concentration, rents, price cost margins and import penetration (see for instance, Shepherd (1982), Domowitz et al. (1986), Blundell et al. (1995, 1999) and Nickell (1996)). As argued above, the first three variables are not monotone in competition. The same is true for import penetration as can be seen as follows. A rise in import penetration, say through a reduction in trade barriers, increases competition on the domestic market. But more aggressive interaction on the domestic market may cause a fall in import penetration if home producers are more efficient than foreign ones. Although these variables are not monotone in competition, they have the advantage that they are relatively straightforward to calculate.

As surveyed by Bresnahan (1989), there are the structural and nonparametric approaches to measuring competition. Both approaches usually model the industry as a representative firm and try to identify a conjectural variation parameter. Bresnahan (1982) and Lau (1982) were the first to show that different hypotheses about conduct have different implications for the comparative statics of industry price and quantity with respect to demand and supply shocks. The structural approach estimates the demand curve a (representative) firm faces, its cost curve and a supply curve. Examples are Bresnahan (1987), Porter (1983) and Wolfram (1999). The main problem with this method is misspecification. The estimate of the conjectural variation parameter is sensitive to how, say, the cost curve is specified.

The non-parametric approaches of, for instance, Panzar and Rosse (1987) and Ashenfelter and Sullivan (1987), are more general in the sense that they use revealed preference arguments which hold for any demand and cost function. In particular, Panzar and Rosse (1987) estimate a reduced form revenue equation where among the independent variables are factor prices of the industry's inputs. Then they calculate a statistic ψ which equals the sum of elasticities of revenue with respect to factor prices. They show that monopoly implies ψ □ 0, (long run) monopolistic competition implies ψ □ 1 and perfect competition implies $\psi = 1$. This is a general approach, but cannot be used to see how competition changes. To illustrate, if in a certain industry ψ increases from -0.75 to 0.5 either competition increased from monopoly to monopolistic competition or it has stayed under monopolistic competition, in which case competition has not necessarily increased. In this approach there is the following misspecification problem. How should the relation between revenues and factor prices be specified; linear, log-linear or otherwise? The indicator proposed here has the same problem. For instance, equation (16) specifies a linear relation between relative profits and relative efficiency. But there is no reason to expect this relation to be linear in practice.

Also in this vein is Hall's (1988) instrument test of the joint hypothesis of perfect competition and constant returns to scale production technology. Hall shows that under this hypothesis, the Solow residual in an industry is uncorrelated with instrumental variables like military spending and the oil price. The problem is that in an industry where this hypothesis is rejected, nothing can be said about whether competition increases or decreases. To say more about competition in this case, Hall estimates the mark up ratio (price over marginal costs) for an industry and uses this as a measure of market power. One problem with this approach is that the mark up ratio is not monotone in competition. Second, as argued by Shapiro (1987), the mark up ratio should be corrected for the market elasticity of demand to derive a measure of competition. Third, as shown by Domowitz et al. (1988) the estimated mark ups are sensitive to the way marginal costs are measured. Similarly, the relative profits measure, proposed here, is sensitive to measurement problems with respect to marginal costs.

Summarizing, one can say that when using measures of competition in practice, one faces the following trade off. On the one hand, there are measures like concentration and industry profits which are relatively easy to calculate, but not monotone in competition. On the other hand, there are the structural and non-parametric approaches which are better grounded in theory but harder to calculate in practice. The relative profits measure clearly belongs to the latter category. However, it remains an empirical question to which extent these measures of competition yield different conclusions in practice.[19]

8 Conclusion

This paper has argued that relative profits are monotone in competition for a number of well known parametrizations of competition. The idea is that a given rise in marginal costs (relative to a firm's opponents) leads to a bigger fall in profits in a more competitive industry. This is a first step towards the theoretical goal of a better understanding of what different competition parametrizations have in common.

On the empirical goal of measuring competition in practice, estimating to which extent cost differences are translated into profit differences has the

[19] A simple way to test this is to examine how closely correlated different indicators are over industries and time periods. Alternatively, one could opt for a simulation approach, as in Hyde and Perloff (1995), to get a quantative feel of the differences between these measures of competition.

advantage that not all firms in the industry need to be observed. There are several problems with this measure which seem to preclude inter-industry comparisons of competition. I discussed an unlevel playing field and measurement problems with respect to profits and marginal costs. However, if one can argue that within a certain industry the bias caused by these problems has not changed over time, a change in the indicator can be interpreted as a change in competition in this industry.

Some people may wonder why nothing has been said here about the relation between competition and welfare. The reason is that there is no monotone relation between competition and welfare. This point has been formalized by Mankiw and Whinston (1986). They show that there are two externalities when firms decide whether or not to enter an industry. First, there is the appropriability effect which leads to too little entry in the private outcome. Second, there is the business stealing effect which leads to excessive entry. Using the number of firms as a parametrization of competition, one finds the following relation between competition and welfare. If the appropriability effect exceeds the business stealing effect, there are too few firms in the industry. Increasing the number of entrants raises both competition and welfare. However, if the business stealing effect exceeds the appropriability effect, reducing the number of entrants decreases competition and increases welfare.

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Appendix

This appendix contains the proofs of the claims made in the counterexample in section 2 and of proposition 2. Further it contains a number of lemmas showing that the variables in table 2, with the exception of RP, RVC and $PLEF$, are not monotone in competition.

Proof of counterexample in section 2

Firm 2, taking x_1 as given, chooses x_2 to solve

$$\max_{x_2}(1 - x_2 - \theta x_1 - c_2)x_2$$

It follows that $x_2 = \frac{1}{2}(1 - \theta x_1 - c_2)$ and $\pi_2 = \frac{1}{4}(1 - \theta x_1 - c_2)^2$. Firm 1, knowing 2's reaction function, solves

$$\max_{x_1}(1 - x_1 - \frac{\theta}{2}(1 - \theta x_1 - c_2) - c_1)x_1$$

Consequently $x_1 = \frac{1}{2-\theta^2}(1 - \frac{\theta}{2} + \frac{\theta}{2}c_2 - c_1)$ and $\pi_1 = \frac{1}{4-2\theta^2}(1 - \frac{\theta}{2} + \frac{\theta}{2}c_2 - c_1)^2$. It follows that

$$\frac{\partial \pi_1}{\partial \theta} = 4\frac{1 - c_1 - \frac{\theta}{2}(1-c_2)}{(4-2\theta^2)^2}(\theta(1-c_1) - (1-c_2))$$

Further

$$\frac{\partial \pi_2}{\partial \theta} = -\frac{1}{2}(1 - \theta x_1 - c_2)\left(x_1 + \theta\frac{\partial x_1}{\partial \theta}\right)$$

where

$$\begin{aligned} x_1 + \theta\frac{\partial x_1}{\partial \theta} &= \frac{1 - c_1 - \frac{\theta(1-c_2)}{2}}{2-\theta^2} + \theta\frac{-\frac{(1-c_2)(2-\theta^2)}{2} + 2\theta(1 - c_1 - \frac{\theta(1-c_2)}{2})}{(2-\theta^2)^2} \\ &= \frac{(2+\theta^2)(1-c_1) - 2\theta(1-c_2)}{(2-\theta^2)^2} \end{aligned}$$

Hence

$$\frac{\partial \pi_2}{\partial \theta} = -\frac{1}{2}(1 - \theta x_1 - c_2)\frac{(2+\theta^2)(1-c_1) - 2\theta(1-c_2)}{(2-\theta^2)^2}$$

It follows at $c_1 = 0.4, c_2 = 0.35$ and $\theta = 1.1$ that $\frac{\partial \pi_1}{\partial \theta} > 0$ while $\frac{\partial \pi_2}{\partial \theta} < 0$.

Proof of proposition 2

First, I prove that the profits of the least efficient firm are decreasing in competition for all of the examples (part (i) of conjecture 1). Then I prove that relative profits and relative variable costs are monotone in competition for all examples (parts (ii) and (iii) of the conjecture).

Proof of part (i) of conjecture 1:

♠Example 1:

If λ falls then the profits of the least efficient firm in the market, denoted I, fall. It follows from equation (3) that

$$\frac{\partial \pi_I}{\partial \lambda} = dx_I^2 + 2(b + \lambda d)x_I\frac{\partial x_I}{\partial \lambda}$$

Using equation (2), this can be written as

$$\frac{\partial \pi_I}{\partial \lambda} = dx_I^2 + 2(b+\lambda d)x_I\left(\frac{-d(\frac{2b}{d} + \lambda - 1)x_I}{D} + \frac{d(nc_I - \sum_{j=1}^n c_j)(\frac{2b}{d} + \lambda - 1 + n)}{D^2}\right)$$

where the denominator D is defined as $D \equiv (\frac{2b}{d} + \lambda - 1)(2b + d(\lambda + n - 1))$. It is the routine to verify that

$$sign\left(\frac{\partial \pi_I}{\partial \lambda}\right) = sign\left(\frac{2(b+\lambda d)(nc_I - \sum_{j=1}^{n} c_j)}{(\frac{2b}{d} + \lambda - 1)^2(2b + d(\lambda + n - 1))} + x_I\left(d - \frac{2(b+\lambda d)}{2b + d(\lambda + n - 1)}\right)\right)$$

The first term on the right hand side is positive because firm I is the least efficient firm in the market, and thus has above average costs. The second term is positive if λ is not too big. This defines the upperbound $\bar{\lambda}$ for λ.

For the analysis of the parametrization $\frac{b}{d}$ on the variable $PLEF$, it turns out to be instructive to look at the effects of b and d separately. The effect of b on the profits of the least efficient firm, π_I, is the same as the effect of λ. This can be seen as follows. For given value of d, the parameters λ and b enter the expressions for profits (3) and output (2) symmetrically.

If $\frac{1}{d}$ falls then the profits of the least efficient firm, π_I, decrease.

It follows from equations (3) and (2) that

$$\frac{\partial \pi_I}{\partial\left(\frac{1}{d}\right)} = \frac{x_I}{D}\left\{\overbrace{Dx_I\left(\frac{2b}{d} + \lambda\right)}^{.>0} + \left(\frac{b}{d} + \lambda\right)\frac{4b}{d}\frac{1}{D}\left[\overbrace{-(a - c_I)\left(\frac{2b}{d} + \lambda - 1\right)^2}^{.<0} \right.\right.$$
$$\left.\left.\underbrace{(nc_I - \sum_{j=1}^{n} c_j)\left(\frac{4b}{d} + 2\lambda + n - 2\right)}_{.>0}\right]\right\}$$

where the denominator is again defined as $D \equiv (\frac{2b}{d} + \lambda - 1)(2b + d(\lambda + n - 1))$. To prove that the overall effect of $\frac{1}{d}$ on π_I is positive, I use the following substitution, introducing a parameter ϕ.

$$a - c_I = \phi \frac{nc_I - \sum_{j=1}^{n} c_j}{\frac{2b}{d} + \lambda - 1}$$

where $\phi \geq 1$ because firm I's output level is nonnegative. This follows from the observation that x_I can be written as

$$x_I = \frac{(\phi - 1)(nc_I - \sum_{j=1}^{n} c_j)}{D}$$

Substituting these expressions for $a - c_I$ and x_I into $\frac{\partial \pi_I}{\partial\left(\frac{1}{d}\right)}$ above yields

$$sign\left(\frac{\partial \pi_I}{\partial\left(\frac{1}{d}\right)}\right) = sign\left((\phi-1)\left(\frac{2b}{d}+\lambda\right)D + \frac{4b}{d}\left(\frac{b}{d}+\lambda\right)\left[\left(\frac{2b}{d}+\lambda-1\right)(1-\phi)+\frac{2b}{d}+\lambda+n-1\right]\right)$$

This can be rewritten as

$$sign\left(\frac{\partial \pi_I}{\partial\left(\frac{1}{d}\right)}\right) = sign\left(\overbrace{(\phi-1)\left(\frac{2b}{d}+\lambda-1\right)}^{.>0}\left[\overbrace{\left(\frac{2b}{d}+\lambda\right)(n-1)+\lambda^2}^{.>0}\right] + \underbrace{+\frac{4b}{d}\left(\frac{b}{d}+\lambda\right)\left(\frac{2b}{d}+\lambda+n-1\right)}_{>0}\right)$$

Hence I find that $\frac{\partial \pi_I}{\partial\left(\frac{1}{d}\right)} > 0$.

The observation that profits of the least efficient firm I fall as a falls, is immediate by looking at equations (3) and (2).

The observation that profits of the least efficient firm I fall as c_j (with $j \neq I$) falls, is immediate by looking at equations (3) and (2).

Finally, I show that a rise in the number of firms in the market from n to $n+1$ reduces the profits π_I of the least efficient firm I in the market. Where I is defined by $c_I \geq c_j$ for each $j \in \{1, ..., n\}$ but it is not necesarily the case that $c_I \geq c_{n+1}$. It follows from the expression for profits (3) that the profits of I with $n+1$ firms are lower than with n firms if and only if the output level of I is lower with $n+1$ firms. Using equation (2), output of firm I is lower with $n+1$ firms than with n firms, if and only if

$$\frac{\left(\frac{2b}{d}+\lambda-1\right)a - \left(\frac{2b}{d}+\lambda+n\right)c_I + \sum_{j=1}^{n} c_j + c_{n+1}}{(2b+d(\lambda+n))\left(\frac{2b}{d}+\lambda-1\right)} < \frac{\left(\frac{2b}{d}+\lambda-1\right)a - \left(\frac{2b}{d}+\lambda+n-1\right)c_I + \sum_{j=1}^{n} c_j}{(2b+d(\lambda+n-1))\left(\frac{2b}{d}+\lambda-1\right)}$$

It is routine to verify that this condition can be rewritten as

$$\left(\frac{2b}{d}+\lambda-1\right)a - \left(\frac{2b}{d}+\lambda+n\right)c_{n+1} + \sum_{j=1}^{n+1} c_j > 0$$

which is satisfied because firm $n+1$ produces a positive output level. Hence the profits of the least efficient firm I fall as the number of firms in the market rise.

♠Example 2:

The least efficient firm, is the one with the cost gap $c > 1$. Hence I have to show that $\frac{\partial \pi(c,\theta)}{\partial \theta} < 0$ for $c > 1$. Differentiating the expression for $\pi(c, \theta)$ in equation (7) yields

$$\frac{\partial \pi(c,\theta)}{\partial \theta} = \frac{-c^{\theta}}{(1+c^{\theta})^3} \left(([\ln c][1+\theta+(1-\theta)c^{\theta}] + 1 + c^{\theta} \right)$$

which is clearly negative for $c > 1$.

♠Example 3:

With $c_2 > c_1$ it is the case that the least efficient firm 2 makes zero profits under Bertrand competition, while it makes positive profits under Cournot competition. Hence raising competition by switching from Cournot to Bertrand competition reduces the profits of the least efficient firm.

♠Example 4:

Write the profits of the least efficient firm I as

$$\pi_I = \frac{1-\theta}{\sum_{j=1}^{n} \left(\frac{c_I}{c_j} \right)^{\frac{\theta}{1-\theta}}}$$

Then it follows, from the observation that $\frac{c_I}{c_j} \geq 1$ for each j, that a rise in competition θ reduces I's profits.

♠Example 5:

It follows from the expression for profits (15) that

$$\frac{\partial \pi}{\partial \theta} = -\frac{(3+\theta(c_j - c_i))(3+\theta(c_i - c_j))}{18\theta^2} < 0$$

because both firms produce a positive output level. Hence a rise in competition θ reduces both firms' profits.

Proof of parts (ii) and (iii) of conjecture:

♠Example 1:

(ii): Using equation (3) one can write the ratio of profits as

$$\frac{\pi(c_i, c_{-i}, \theta)}{\pi(c_k, c_{-k}, \theta)} = \left(\frac{\left(\frac{2b}{d} + \lambda - 1\right) a + \sum_{j=1}^{n} c_j - \left(\frac{2b}{d} + \lambda + n - 1\right) c_i}{\left(\frac{2b}{d} + \lambda - 1\right) a + \sum_{j=1}^{n} c_j - \left(\frac{2b}{d} + \lambda + n - 1\right) c_k} \right)^2 \quad (17)$$

It is routine to verify that

$$sign\left(\frac{\partial\left(\frac{\pi(c_i,c_{-i},\theta)}{\pi(c_k,c_{-k},\theta)}\right)}{\partial\lambda}\right)=sign\left(\frac{(c_i-c_k)\left(\sum_{j=1}^{n}(a-c_j)\right)}{\left[\left(\frac{2b}{d}+\lambda-1\right)a+\sum_{j=1}^{n}c_j-\left(\frac{2b}{d}+\lambda+n-1\right)c_k\right]^2}\right)$$

hence $\frac{\partial\left(\frac{\pi(c_i,c_{-i},\theta)}{\pi(c_k,c_{-k},\theta)}\right)}{\partial\lambda}<0$ if and only if $(c_i-c_k)<0$. In words, a rise in competition through a fall in λ increases firm i's relative profits if and only if i's cost level is below k's cost level.

Now consider the effect of a rise in competition through a fall in $\frac{b}{d}$.

$$sign\left(\frac{\partial\left(\frac{\pi(c_i,c_{-i},\theta)}{\pi(c_k,c_{-k},\theta)}\right)}{\partial\left(\frac{b}{d}\right)}\right)=sign\left(\frac{(c_i-c_k)\left(\sum_{j=1}^{n}(a-c_j)\right)}{\left[\left(\frac{2b}{d}+\lambda-1\right)a+\sum_{j=1}^{n}c_j-\left(\frac{2b}{d}+\lambda+n-1\right)c_k\right]^2}\right)$$

Then one finds that a rise in competition raises firm i's relative profits if and only if i's cost level is below k's cost level.

Consider the effects of a for given number of firms in the industry n. Then one can verify that

$$sign\left(\frac{\partial\left(\frac{\pi(c_i,c_{-i},\theta)}{\pi(c_k,c_{-k},\theta)}\right)}{\partial a}\right)=sign\left(\frac{(c_i-c_k)\left(\frac{2b}{d}+\lambda-1\right)\left(\frac{2b}{d}+\lambda+n-1\right)}{\left[\left(\frac{2b}{d}+\lambda-1\right)a+\sum_{j=1}^{n}c_j-\left(\frac{2b}{d}+\lambda+n-1\right)c_k\right]^2}\right)$$

In words, a rise in a, for given n, increases the relative profits of firm i if it has higher costs than k.

Finally decreasing $\sum_{j=1}^{n}c_j$ through a reduction in c_j affects the ratio of profits of firms $i,k\neq j$ in the following way

$$sign\left(\frac{\partial\left(\frac{\pi(c_i,c_{-i},\theta)}{\pi(c_k,c_{-k},\theta)}\right)}{\partial\sum_{j=1}^{n}c_j}\right)=sign\left(\frac{(c_i-c_k)\left(\frac{2b}{d}+\lambda+n-1\right)}{\left[\left(\frac{2b}{d}+\lambda-1\right)a+\sum_{j=1}^{n}c_j-\left(\frac{2b}{d}+\lambda+n-1\right)c_k\right]^2}\right)$$

Hence a rise in competition through a fall in $\sum_{j=1}^{n}c_j$ raises i's relative profits if and only if i's costs are lower than k's costs.

The entryfee f has only an effect on market shares if the reduction in f leads to more entry. So the effect of a reduction in f is considered directly as the effect of a rise in n where (with a slight abuse of notation) $\frac{\partial \sum_{j=1}^{n} c_j}{\partial n} \equiv c_{n+1}$, the cost level of the new entrant. The relative profits of two firms $i, k \neq n+1$ are affected by f in the following way.

$$sign\left(\frac{\partial\left(\frac{\pi(c_i,c_{-i},\theta)}{\pi(c_k,c_{-k},\theta)}\right)}{\partial n}\right) = sign\left(\frac{(c_k - c_i)\left[\left(\frac{2b}{d}+\lambda-1\right)a + \sum_{j=1}^{n+1} c_j - \left(\frac{2b}{d}+\lambda+n\right)c_{n+1}\right]}{\left[\left(\frac{2b}{d}+\lambda-1\right)a + \sum_{j=1}^{n} c_j - \left(\frac{2b}{d}+\lambda+n-1\right)c_k\right]^2}\right)$$

Since $\left[\left(\frac{2b}{d}+\lambda-1\right)a + \sum_{j=1}^{n+1} c_j - \left(\frac{2b}{d}+\lambda+n\right)c_{n+1}\right] > 0$ (because $n+1$'s output level is positive), it follows that a rise in competition, through a rise in n, raises firm i's relative profits if and only if i's cost level is below k's cost level.

(iii): Using equation (2) the relative variable costs of i and k can be written as

$$\frac{c_i x_i}{c_k x_k} = \frac{c_i}{c_k}\frac{\left(\frac{2b}{d}+\lambda-1\right)a + \sum_{j=1}^{n} c_j - \left(\frac{2b}{d}+\lambda+n-1\right)c_i}{\left(\frac{2b}{d}+\lambda-1\right)a + \sum_{j=1}^{n} c_j - \left(\frac{2b}{d}+\lambda+n-1\right)c_k}$$

It is routine to verify that the signs of the partial derivatives of $\frac{c_i x_i}{c_k x_k}$ with respect to $\lambda, \frac{b}{d}, a, \sum_{j=1}^{n} c_j$ and f equal the signs of the corresponding partial derivatives of $\frac{\pi(c_i,c_{-i},\theta)}{\pi(c_k,c_{-k},\theta)}$.

♠Example 2:

(ii): Using equation (7) the ratio of profits can be written as

$$\frac{\pi(c,\theta)}{\pi(\frac{1}{c},\theta)} = \left(\frac{1+c^{-\theta}}{1+c^{\theta}}\right)^2 \frac{1+(1-\theta)c^{\theta}}{1+(1-\theta)c^{-\theta}} = \frac{c^{-\theta}+1-\theta}{c^{\theta}+1-\theta}$$

It is routine to verify that

$$\frac{\partial\left(\frac{\pi(c,\theta)}{\pi(\frac{1}{c},\theta)}\right)}{\partial\theta} = \frac{c^{-\theta} - c^{\theta} - \ln(c)[2+(1-\theta)(c^{-\theta}+c^{\theta})]}{(c^{\theta}+1-\theta)^2}$$

which is positive if and only if $c < 1$. Hence a rise in competition increases the relative profits of the most efficient firm.

(iii): From equation (5), it follows that

$$\frac{c_i x_i}{c_j x_j} = 1$$

thus competition has no effect on the relative variable cost shares.
♠Example 3:
(ii): Holds trivially as $\pi_2^B = 0$.
(iii): Holds trivially as $x_2^B = 0$.
♠Example 4:
(ii): It follows from equation (12) that

$$\frac{\pi_i}{\pi_j} = \left(\frac{c_j}{c_i}\right)^{\frac{\theta}{1-\theta}}$$

This implies that a rise in competition θ raises the relative profits of the low cost firm.
(iii): It follows from equation (11) that

$$\frac{c_i x_i}{c_j x_j} = \left(\frac{c_j}{c_i}\right)^{\frac{\theta}{1-\theta}}$$

which is increasing in θ if and only if i is the low cost firm.
♠Example 5:
(ii): Using equation (15) one can write the relative profits as

$$\frac{\pi(c_i, c_j, \theta)}{\pi(c_j, c_i, \theta)} = \frac{3 + \theta\,(c_j - c_i)}{3 + \theta\,(c_i - c_j)}$$

Hence it is immediately clear that a rise in competition θ raises the relative profits of the low cost firm.
(iii): Using equation (13) it follows that

$$\frac{c_i x_i}{c_j x_j} = \frac{c_i}{c_j}\frac{3 + \theta(c_j - c_i)}{3 - \theta(c_j - c_i)}$$

which is increasing in θ if $c_i < c_j$.■

Proof of proposition 4:
The proof that RP and RVC satisfy (M), (A) and (P) follows from proposition 2. Similarly, proposition 2 implies that $PLEF$ satisfies (M) and (A). Finally the proof that the other variables in table 2 do not satisfy (M) follows from lemma 5 - lemma 8 below.■

Lemma 5 *In example 3 SP is both increasing and decreasing in competition modelled as a switch from Cournot to Bertrand competition.*

Remark: The same is true for competition as parametrized by θ in example 2.

Proof. Using equation (7) with $\theta = 1$ the sum of profits for Cournot competition can be written as

$$\pi_1^C + \pi_2^C = \frac{c_1^2 + c_2^2}{(c_1 + c_2)^2}$$

while the sum of profits for Bertrand competition equal

$$\pi_1^B = 1 - \frac{c_1}{c_2}$$

It is routine to verify that $\pi_1^C + \pi_2^C > \pi_1^B$ if c_1 is close to c_2 and that $\pi_1^C + \pi_2^C < \pi_1^B$ if c_1 is far smaller than c_2. ■

Lemma 6 *In example 1 RR is both increasing and decreasing in competition as parameterized by λ.*

Proof. Consider the following parameter values: $c_1 = 1$, $c_2 = 2$, $b = 1$, $d = 0.4$, $n = 5$ and $\sum_{j=1}^{5} c_j = 5$. Then one can show, using equations (2) and (4), that for $a = 4$ it is the case that

$$\left.\frac{p_1 x_1}{p_2 x_2}\right|_{\lambda=-0.5} > \left.\frac{p_1 x_1}{p_2 x_2}\right|_{\lambda=0.5}$$

while for $a = 14$ it is the case that

$$\left.\frac{p_1 x_1}{p_2 x_2}\right|_{\lambda=-0.5} < \left.\frac{p_1 x_1}{p_2 x_2}\right|_{\lambda=0.5}$$

Note that in both cases the profits of the least efficient firm 2 are increasing in λ at $\lambda = 0.5$. That is, $\lambda = 0.5$ is below the upperbound $\bar{\lambda}$ derived in the proof of proposition 2 ■

Lemma 7 *In example 2 PCM is both increasing and decreasing in competition as parametrized by θ.*

Proof. From equation (6) it follows that

$$\frac{p_i}{c_i} = \frac{1 + c^{-\theta}}{\theta}$$

where $c = \frac{c_i}{c_j}$ measures the cost gap between firms i and j. Since the price cost margin is defined as $pcm_i = \frac{p_i - c_i}{p_i}$, one finds

$$sign\left(\frac{\partial pcm_i}{\partial \theta}\right) = sign\left(c^{-\theta}\left(-\theta\left(\ln c\right) - 1\right) - 1\right)$$

Hence $\frac{\partial pcm_i}{\partial \theta} < 0$ for the high cost firm ($c > 1$) but $\frac{\partial pcm_i}{\partial \theta} > 0$ is possible for the low cost firm ($c < 1$). ■

Lemma 8 *In example 1 H is increasing in competition as measured by λ. H can be decreasing in competition as measured by n if all firms can be observed, that is under requirement (A). However, H can be increasing in n under requirement (P).*

Remark: H is also increasing in competition as parameterized by $\frac{b}{d}$ in example 1, by a switch from Cournot to Bertrand competition in example 3 and as parameterized by θ in examples 4 and 5.

Proof. For simplicity, consider the case where goods are perfect substitutes, that is $b = d$. Then all firms charge the same price and firm i's market share equals $\frac{x_i}{\sum_{j=1}^n x_j}$. It follows from equation (2) that

$$\frac{x_i}{\sum_{j=1}^n x_j} = \frac{1}{n}\frac{(2+\lambda-1)\,a + \sum_{j=1}^n c_j - (2+\lambda+n-1)\,c_i}{(2+\lambda-1)\,a + \sum_{j=1}^n c_j - (2+\lambda+n-1)\,\bar{c}} \tag{18}$$

where $\bar{c} \equiv \frac{\sum_{j=1}^n c_j}{n}$ equals the average cost level in the industry. Now consider the effect of λ on the market share of firm i, $\frac{x_i}{\sum_{j=1}^n x_j}$, for given number of firms in the industry n. Then it is routine to verify that

$$\frac{\partial\left(\frac{x_i}{\sum_{j=1}^n x_j}\right)}{\partial \lambda} = \frac{1}{n}\frac{(c_i - \bar{c})\left(\sum_{j=1}^n (a - c_j)\right)}{\left[(2+\lambda-1)\,a + \sum_{j=1}^n c_j - (2+\lambda+n-1)\,\bar{c}\right]^2} \tag{19}$$

Hence, a rise in competition, through a fall in λ, increases firm i's market share if and only if i's cost level is below the average cost level, $c_i < \bar{c}$. That is, the rise in competition increases the market share of the big firms and reduces the market share of the small firms. This leads to a rise in H. Note that this holds for any λ, so in particular for values of λ below the upperbound $\bar{\lambda}$ derived in the proof of proposition 2.

To show that H can be decreasing in competition as parameterized by n, consider the case where all firms have the same marginal cost level. Then firm i's market share equals $\frac{1}{n}$. Consequently the Herfindahl index equals

$H = \frac{1}{n}$ which is indeed decreasing in n.
Now consider the effect of n on H in a panel P of firms. Increasing n leads to an additional firm with cost level c_{n+1}. Assume that the new firm $n+1$ is not in the panel, that is the set P is unchanged. Then the observed market share of firm i equals $\frac{x_i}{\sum_{j \in P} x_j}$. And the effect of n on the observed market share equals

$$\frac{\partial \left(\frac{x_i}{\sum_{j \in P} x_j} \right)}{\partial n} = \frac{1}{|P|} \frac{(\bar{c}_P - c_i) \left[(2 + \lambda - 1)\, a + \sum_{j=1}^{n+1} c_j - (2 + \lambda + n)\, c_{n+1} \right]}{\left[(2 + \lambda - 1)\, a + \sum_{j=1}^{n} c_j - (2 + \lambda + n - 1)\, \bar{c}_P \right]^2} \tag{20}$$

where $|P|$ is the number of firms in the panel and $\bar{c}_P$ equals the average cost level of firms in the panel, $\bar{c}_P = \frac{1}{|P|} \sum_{j \in P} c_j$. Since

$$\left[\left(\frac{2b}{d} + \lambda - 1 \right) a + \sum_{j=1}^{n+1} c_j - \left(\frac{2b}{d} + \lambda + n \right) c_{n+1} \right] > 0$$

(because $n+1$'s output level is positive), it follows that a rise in competition, through a rise in n, raises firm i's observed market share if and only if i's cost level is below the average cost level in the sample P. This is again an instance of the reallocation effect of competition. Hence the rise in n reallocates market share from inefficient firms in the panel to the efficient ones and thus raises the observed Herfindahl index in the panel. ■