THE BANK'S CHOICE OF FINANCING AND THE CORRELATION STRUCTURE OF LOAN RETURNS

By Vasso P. Ioannidou, Yiannos Pierides

TILBURG ◆ UNIVERSITY

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Vasso P. Ioannidou[*] Yiannos Pierides[†]

Tilburg University University of Cyprus

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Abstract

This paper examines how the correlation structure of loan returns within a bank's loan portfolio affects its choice of financing when the bank faces binding capital constraints and there is asymmetric information about the quality of its loans. The paper uses an asymmetric information model similar to Myers and Majluf (1984), where a bank must raise its equity-to-assets ratio either by issuing equity or by selling loans in the secondary market. The results suggest that the correlation structure of loan returns can have significant influence on the cost of issuing equity since it affects the variance of a bank's loan portfolio. However, it is shown that a bank will always prefer to sell loans instead of equity if it has favorable inside information for some of its loans and unfavorable information for some of its other loans.

Keywords: Loans Sales; Bank Capital Requirements; Asymmetric Information

JEL classification: G21; G10

[*]Vasso P. Ioannidou (corresponding author), Tilburg University, Department of Economics and CentER, 5000 LE Tilburg, The Netherlands. Tel.: +31-13-466-3097; Fax: +31-13-466-3042; Email: ioannido@uvt.nl.

[†]Yiannos Pierides, University of Cyprus, Department of Public and Business Administration, P.O. Box 20537, CY-1678 Nicosia, Cyprus. Tel.: +357-22-892464; Fax: +357-22-892460; Email: bapierid@ucy.ac.cy

1 Introduction

Financial institutions have sold loans among themselves for over 100 years. Even though this market existed for many years, it grew slowly until the early 1980s, when it entered a period of spectacular growth. For example, the volume of loans sold by U.S. banks grew from less than $20 billion in 1980 to $280 billion in 1989 (Saunders, 1999). Between 1990 and 1994 the volume of loan sales fell almost equally dramatically, as a result of the credit-crunch associated with the 1990-1991 recession. In recent years, however, the volume of loan sales has expanded again.

The dramatic increase in loan sales during the 1980s gave raise to a voluminous literature on why banks sell loans, what type of loans they sell, and why did we observe this increase in loan sales during the 1980s. In general, loan sales occur in many forms (e.g., with or without recourse[1]) and for many reasons (e.g., to avoid regulatory taxes, for diversification and liquidity reasons, as a response to market-based capital requirements, for regulatory capital arbitrage, etc.). Berger and Udell (1993) present a comprehensive review of this literature up to 1993. More recent studies on this phenomenon include, among others, Carlstrom and Samolyk (1995), Gordon and Pennacchi (1995), Demsetz (2000), Jones (2000), Dahiya et al. (forthcoming), and Cebenoyan and Strahan (forthcoming). A detail review of this literature is beyond the scope of this paper. The discussion here, instead, will focus on studies that motivate loan sales as a cheaper source of finance compared to traditional sources like equity and deposits.

The cost of equity is generally perceived to be much greater than the cost of deposits due to various capital market imperfections. For example, corporate tax benefits, high transaction costs of issuing equity, and the "bank safety net" (e.g., access to deposit insurance and the discount window) are among the many corporate and regulatory induced imperfections. However,

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[1]If the loan is sold without recourse, not only it is removed from the bank's balance sheet, but the bank has no explicit liability if the loan eventually goes bad. Thus, the buyer bears all the credit risk. Instead, if the loan is sold with recourse, under certain conditions the buyer can put the loan back to the selling bank. Thus, the bank retains a contingent credit risk liability. In practice, most loans are sold without recourse because a loan sale is technically removed from the balance sheet only when the buyer has no future credit claim on the bank (Saunders, 1999).

asymmetric information and agency costs are probably the most popular explanations. The seminal contribution by Myers and Majluf (1984) shows that internally generated funds (which have no issue costs and no information problems) are generally preferred to externally generated funds. If external funds are needed, deposits are usually preferred to equity. This is because in the presence of asymmetric information, the bank's existing shareholders may be reluctant to issue equity since it may sell at a discount. The more recent literature on loan sales, however, suggests that in some cases loan sales are cheaper than deposits (and thus equity). For example, a bank can reduce its regulatory tax burden by selling assets without recourse. Loan sales without recourse provide a funding source that is not subject to deposit insurance premiums or reserve requirements. Also, by shrinking the balance sheet, loan sales allow a bank to reduce its capital requirement (e.g., Pavel and Phillis, 1987 and Pennacchi, 1988). Instead, loan sales with recourse or backed by standby letters of credit could provide a cheaper source of funds than risky-debt, since they have payoff characteristics similar to secured debt (James, 1988).

This paper considers a bank that must raise its equity-to-assets ratio either by selling loans or by issuing equity. In general, a bank can boost its equity-to-assets ratio by increasing the numerator of this ratio (i.e., by issuing equity) and/or by decreasing the denominator (i.e., by selling loans, not initiating new loans, not renewing old loans, etc.). Here, we consider a situation where the bank can decrease the denominator of its capital ratio only by selling loans without recourse.[2] The paper differs from the ones discussed above in an important way: its focus is on describing how the correlation structure of loan returns affects the bank's choice of financing. In the above mentioned papers, only the expected value (and not the correlation structure) of loan returns enters the analysis. However, in the presence of asymmetric information the correlation structure of loan returns could have important implications since it affects the cost of issuing equity. Everything else equal, the more diversified a loan portfolio is, the less risky it is, and therefore the higher the price investors are willing to pay for equity.

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[2]From now on, the term loans sales will be used to refer to loan sales without recourse unless otherwise stated.

The model presented here is similar to the Myers and Majluf (1984) model of equity issues under asymmetric information. Myers and Majluf (1984) consider the choice between debt and equity as the source of funding for a new project. The new project is one that will be undertaken by a firm whose managers have inside information about its returns; this information is not available to prospective debt or equity investors. Similarly, in this paper, we consider a bank that must increase its equity-to-assets ratio either by issuing equity or by selling loans. As in Myers and Majluf (1984), the bank's managers have inside information about loan returns that is not available to prospective loan or equity buyers. It is shown that in a Myers and Majluf (1984) setup the correlation structure of loan returns has significant influence on the cost of issuing equity. Everything else equal, the more diversified a loan portfolio is, the smaller its variance and thus the higher the price investors are willing to pay for equity (i.e., the less costly an equity issue is for the bank's existing shareholders). However, the analysis suggests that as far as a bank has favorable inside information for some of its loans and unfavorable information for some of its other loans, it will always prefer to sell loans instead of equity.

Theories of financial intermediation predict that loan sales should not be possible, since such a market would be a "lemons" market (e.g., Boyd and Prescott, 1986 and Diamond, 1984). However, there are a number of possible explanations why this market exists and why it has not shut down. For example, there is evidence suggesting that banks offer implicit contract features that make loan sales incentive compatible (see Gorton and Pennacchi, 1995). In addition, the loan sales market would not shut down if not all loans supplied in this market are lemons. For example, loan sales would still occur if some constrained banks are forced to supply profitable projects. This reasoning is similar to Akerlof's (Akerlof, 1970) argument that, as long as some individuals must sell their cars every year, the used car market will not shut down because not all cars will be lemons. Here, the loan sales market is not a lemons market because constrained banks (i.e., banks that know that the low quality scenario will prevail in the next period) are forced to supply loans at a price below their true value.

The paper is organized as follows. Section 2 presents the assumptions underlying the model. Section 3 describes the expected payoffs from each strategy assuming that the bank is willing to issue equity or sell any of its loans regardless of which state occurs (i.e., the bank's choice of security to sell involves no signaling). Section 4 describes the model's equilibrium after taking into account signaling effects, while section 5 concludes. It is important to point out that in order to keep the analysis tractable the model is highly stylized. However, the paper's main point would survive more complicated setups. For this matter, the intuition of the main results is discussed extensively in a broader context.

2　A model of bank financing choice

The model has two periods, period 1 and period 2.[3] Consider a bank that has only two loans outstanding, loan 1 and loan 2. In period 1 we define L_{iB} to be the book value of loan i and L_{iM} to be the market value of loan i, where $i = 1, 2$. D_B and E_B are the book values of debt and equity respectively, while D_M and E_M are the corresponding market values. Clearly,

$$D_B + E_B = L_{1B} + L_{2B} \tag{1}$$

$$D_M + E_M = L_{1M} + L_{2M}, \tag{2}$$

where the market value of each loan is equal to its expected market value in period 2 conditional on whatever information the market has in period 1. For simplicity, we assume that in period 1 the two loans have the same expected future value

$$L_{1M} = L_{2M}, \tag{3}$$

and that the bank's portfolio is balanced with respect to each loan

$$L_{1B} = L_{2B}. \tag{4}$$

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[3]The two periods are meant to capture the state of information available to the market, and not calendar time.

It is assumed that *market* and *regulatory constraints* force the bank to increase its equity-to-assets ratio from its current level $d = \frac{E_B}{L_{1B}+L_{2B}}$ to a new level $c > d$. This must be accomplished in period 1 (i.e., the bank cannot wait until period 2). In general, a bank can increase its capital ratio by increasing the numerator of its capital ratio (i.e., by issuing equity) and/or by decreasing the denominator (i.e., by selling loans, not initiating new loans, not renewing old loans). Here, the bank can decrease the denominator of its capital ratio only by selling loans.

We assume that the bank has enough of each loan as to be able to sell only one type of loan and satisfy its new capital requirement.[4] Hence, the bank can satisfy its new capital requirement by pursuing one of the following strategies: (i) issue E of new equity, (ii) sell proportion b of loan 1, (iii) sell proportion b of loan 2, and (iv) sell some riskless combination of the two loans.

If strategy (i) is pursued, E must be such that

$$c = \frac{E + E_B}{L_{1B} + L_{2B}}. \tag{5}$$

Instead, if strategy (ii) is pursued, b must be such that

$$c = \frac{E_B + b(L_{1M} - L_{1B})}{L_{2B} + (1-b)L_{1B}}. \tag{6}$$

Note that in the numerator, $b(L_{1M} - L_{1B})$ is the gain (if $L_{1M} > L_{1B}$) or loss (if $L_{1M} < L_{1B}$) on the sale of loan 1 which is reflected in the book value of equity. The cash obtained from the sale is kept on the balance sheet until period 2, when it is used to repay debt.[5] Finally, if strategy (iii) is pursued, b must be such that

$$c = \frac{E_B + b(L_{2M} - L_{2B})}{L_{1B} + (1-b)L_{2B}}. \tag{7}$$

The last five equations result in the following relationship between E and b

$$E = \frac{b(2L_{1M} - D_B)}{(2-b)}. \tag{8}$$

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[4]Since the purpose of the paper is to study the bank's choice of loan to sell under different assumptions about the correlation structure of loan returns, this assumption guarantees that the bank's choice will not be constrained by factors other than those the paper attempts to analyze. Nonetheless, we will discuss later on how our results would be affected if we were to relax this assumption.

[5]The bank will not use the cash obtained from selling existing loans to acquire new loans since this would defy the reason for which the loans were sold in the first place (i.e., to increase its equity-to-assets ratio).

As in Myers and Majluf (1984), the bank's managers and the investors are assumed to be risk neutral.[6] The maximization problem of the bank's managers and the investors will be described in turn below. In period 1, the managers must decide which strategy to pursue. We assume that the management acts for the best interest of the existing shareholders (i.e., those who hold shares at the beginning of period 1 prior to any new equity issue). Moreover, the existing shareholders are assumed to be passive (i.e., they "sit tight" if stock is issued; thus the issue goes to a different group of investors). This is a standard Myers and Majluf (1984) setup.

To formalize the bank's managers maximization problem, we define V_2 as the market value of existing shareholder's wealth in period 2 and V_{2j} as the market value of V_2 given the choice of strategy j in period 1. The optimal strategy is chosen from the set $I = \{I_1, I_2, I_E, I_w\}$ of possible strategies in period 1, where I_1 is the strategy of selling proportion b of loan 1, I_2 is the strategy of selling proportion b of loan 2, I_E is the strategy of selling equity, and I_w is the mixed strategy. Hence, the bank manager's maximization problem can be written as

$$\max_{j \in I} V_2. \tag{9}$$

If two or more strategies result in the same V_2, we assume that the bank management chooses the strategy that maximizes the wealth of existing shareholders in period 1. To formalize this, we define V_1 to be the market value of existing shareholders wealth in period 1 and V_{1j} to be the value of V_1 given the choice of strategy j in period 1. Hence, when more than one strategy solves (9), the chosen strategy should also solve

$$\max_{j \in I^*} V_1, \tag{10}$$

where I^* is the set of strategies that solves (9).

We will now describe the investors maximization problem. In period 1, the investors purchase loans or equity in order to maximize their wealth in period 2. However, they are at an

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[6]The bank's stockholders and the investors will behave as if they are risk neutral if we assume that outside of this model they hold sufficiently large and diversified portfolios as to achieve perfect risk-pooling.

informational disadvantage. In period 1, the bank's managers know the values of the two loans in period 2, while the investors do not observe these values until period 2. In period 1, the investors know what are the *possible* values in period 2 and thus they form expectations upon which they base their decisions in period 1. Note that the paper takes this information asymmetry as given and side-steps the question of how much information managers should release. Hence, the underlying assumption is that transmitting information is prohibitively costly. Obviously, the distortions introduced because of the asymmetric information would disappear if the managers could convey their special information to the market.

Suppose that in period 2 there are two equally possible states of nature for each loan: the market value of loan 1 can increase or decrease by $\delta\alpha$ and the market value of loan 2 can increase or decrease by α, where $0 < \delta < 1$ and $\alpha > 0$. This implies that in period 1 the expected future value of each loan is equal to its market value in period 1 (which is consistent with equation 2) and that the two loans have the same expected future value (which is consistent with equation 3). If we assume that the correlation structure of loan returns is *negative* (i.e., in period 2, one of the two loans appreciates whereas the other loan depreciates), then Table 1 below describes the possibilities with which investors are faced in period 2.

Table 1: The correlation structure of loan returns is negative

| | Market Value in Period 1 | Market Value in Period 2 | |
		High Quality Scenario	Low Quality Scenario
Loan 1	L_{1M}	$L_{1M}(1 - \delta\alpha)$	$L_{1M}(1 + \delta\alpha)$
Loan 2	L_{2M}	$L_{2M}(1 + \alpha)$	$L_{2M}(1 - \alpha)$

According to Table 1, there are two equally possible states of nature in period 2: the "high quality scenario" and the "low quality scenario". The high (low) quality scenario is one in which the average market value of the bank's loan portfolio increases (decreases) from period 1 to period 2. In period 1, the investors know what are the two possible scenarios described above,

but they do not know which one will be realized so they assign a 0.5 probability to each one of them. Note, however, that whether it will be the high or the low quality scenario that will prevail, it depends on whether loan 2 appreciates or depreciates in period 2, since it is the riskier of the two loans and $L_{1M} = L_{2M}$.

If, instead, we assume that the correlation structure of loan returns is *positive* (i.e., in period 2, both loans appreciate or depreciate), then Table 2 below describes the possibilities with which investors are faced in period 2.

Table 2: The correlation structure of loan returns is positive

	Market Value in Period 1	Market Value in Period 2	
		High Quality Scenario	Low Quality Scenario
Loan 1	L_{1M}	$L_{1M}(1 + \delta\alpha)$	$L_{1M}(1 - \delta\alpha)$
Loan 2	L_{2M}	$L_{2M}(1 + \alpha)$	$L_{2M}(1 - \alpha)$

Note that in Table 1 the correlation coefficient (ρ) between the markets values of loan 1 and loan 2 is equal to -1, while in Table 2 it is equal to 1. Since the bank has only two loans, the correlation coefficient between loan 1 and loan 2 fully characterizes the correlation structure of the bank's loan portfolio.[7] In particular, $\rho = 1$ represents a case where all loans (two in this case) are positively correlated with each other: they all appreciate or they all depreciate. Instead, $\rho = -1$ represents a case where at each possible scenario some loans appreciate and some depreciate. Using the cases described in Table 1 and Table 2, it will be shown that the bank's optimal choice of financing is different. It is important to point out that the portfolios described in Table 1 and Table 2 differ only with respect to their correlation structure; all other properties regarding the bank's portfolio are the same.

The risk neutral investors are willing to purchase a security offered by the bank if the

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[7]If there were more than two loans, we would have to take into account the correlation of each pair in order to fully characterize the correlation structure of a bank's portfolio.

security's expected return is at least equal to the expected return from their best alternative. Here, if the investors do not purchased the security offered by the bank, they can invest their funds in a riskless asset that pays a real interest rate r, where r is normalized to 0. In addition, it is assumed that capital markets are perfect and efficient with respect to publicly available information and that there are no transaction costs in issuing equity or selling loans. Hence, at the equilibrium the investors will buy the security offered by the bank at a price that will imply an expected return of zero (i.e., equal to the return from their best alternative).

Finally, we assume that bank debt is risky (i.e., there is a positive probability of bankruptcy). In particular, it is assumed that all bank debt matures in period 2 and that in the worst case scenario the bank will default on its debt. Since $r = 0$, the amount of debt that has to be repaid in period 2 is equal to the face value of debt in period 1, D_B. Thus, when $\rho = -1$

$$L_{1M}(1 - \delta\alpha) + L_{2M}(1 + \alpha) > D_B \tag{11}$$

$$L_{1M}(1 + \delta\alpha) + L_{2M}(1 - \alpha) < D_B. \tag{12}$$

The first inequality states that if the high quality scenario is realized, the market value of the two loans in period 2 exceeds the face value of the bank's debt. Hence, the bank can repay its debt in full. The second inequality states that if the low quality scenario is realized, the market value of the bank's portfolio in period 2 is lower than the face value of its debt. The bank, in this case, will go bankrupt. Likewise, when $\rho = 1$ the equivalents of equations (11) and (12) are

$$L_{1M}(1 + \delta\alpha) + L_{2M}(1 + \alpha) > D_B \tag{13}$$

$$L_{1M}(1 - \delta\alpha) + L_{2M}(1 - \alpha) < D_B. \tag{14}$$

Given equations (12) and (14), the bank will be bankrupt in period 2 if the low quality scenario prevails. However, the choice of security to sell in period 1 could improve or worsen the bank's ability to repay its debt. For example, suppose that $\rho = 1$ and that the bank's managers know that the low quality scenario will prevail in period 2 and they choose to sell proportion b

of loan 2. If the investors *do not know* which scenario will prevail, they are willing to buy loan 2 at its expected value (i.e., L_{2M}). Hence, V_{2I_2} is given by

$$V_{2I_2} = \max\left\{0,\ bL_{2M} + L_{1M}(1 - \delta\alpha) + (1 - b)L_{2M}(1 - \alpha) - D_B\right\},$$

which might be positive or zero depending on the parameter values. If $V_{2I_2} > 0$, selling loan 2 prevents the bank from a sure bankruptcy. The intuition is simple: because of asymmetric information the bank is able to sell loan 2 at a favorable price before a major loss in its market value. However, if the investors *knew* that the low quality scenario will prevail they would be willing to pay only $bL_{2M}(1 - \alpha)$. Hence, V_{1I_2} and thus V_{2I_2} would immediately drop to zero since it is common knowledge that under these conditions the bank will be bankrupt in period 2. In particular, V_{2I_2} is given by

$$V_{2I_2} = \max\left\{0,\ bL_{2M}(1 - \alpha) + L_{1M}(1 - \delta\alpha) + (1 - b)L_{2M}(1 - \alpha)\right\} \Rightarrow$$

$$V_{2I_2} = \max\left\{0,\ L_{1M}(1 - \delta\alpha) + L_{2M}(1 - \alpha)\right\},$$

which is equal to zero since $L_{1M}(1 - \delta\alpha) + L_{2M}(1 - \alpha) < D_B$.

In some cases, selling a certain security might worsen the bank's ability to repay its debt. For example, suppose that $\rho = 1$ and the bank's managers know that the high quality scenario will prevail and they choose to sell loan 2. Since the investors *do not know* which scenario will prevail, they are willing to buy loan 2 at its expected value (i.e., L_{2M}). Hence, V_{2I_2} is given by

$$V_{2I_2} = \max\left\{0,\ bL_{2M} + L_{1M}(1 - \delta\alpha) + (1 - b)L_{2M}(1 + \alpha) - D_B\right\},$$

which might be positive or zero depending on the parameter values. If $V_{2L_2} = 0$, selling loan 2 worsens the bank's ability to repay its debt (i.e., before the sale the bank was able to repay its debt for sure, while after the sale it is not always able). The intuition is simple: because of the asymmetric information the bank has to sell loan 2 at an unfavorable price (i.e., a price that takes into account the possibility of the low quality scenario).

3   Expected payoffs

In this section we calculate the existing shareholder's gain/loss from each strategy assuming that the bank is willing to issue equity or sell any of the two loans regardless of whether the favorable or unfavorable state occurs. This implies that investors are willing to buy any of the three securities at their expected values (e.g., when investors buy proportion b of loan 1 they are willing to pay up to bL_{1M} since there is a fifty percent chance that the loan they are buying will have a value equal to $bL_{1M}(1 + \delta\alpha)$ and a fifty percent chance that it will have a value equal to $bL_{1M}(1 - \delta\alpha)$). As it will become clear later on this assumption does not hold at the equilibrium since the bank will find it profitable to supply each security in some states and not in some others. Hence, the bank's choice will involve signaling, which in turn will affect the bank's choice of security to sell at the equilibrium. However, this assumption is maintained in this section in order to calculate the gain/loss from each strategy and examine what the bank would like to sell under each scenario. When in section four we study the model's equilibrium we will take into account the effect of signaling in the bank's choice of security to sell.

Before proceeding it is important to point out that the existing shareholder's gain/loss from selling equity will be calculated under two *alternative* assumptions, depending on whether the new equity issue saves the bank under the low quality scenario or not:

Assumption 1: In period 2 the bank can repay its debt in full under the low quality scenario if it issues an amount E of new equity in period 1. This implies that $L_{1M}(1+\delta\alpha)+L_{2M}(1-\alpha)+E > D_B$ when $\rho = -1$ and $L_{1M}(1 - \delta\alpha) + L_{2M}(1 - \alpha) + E > D_B$ when $\rho = 1$.

Assumption 2: In period 2 the bank can not repay its debt in full under the low quality scenario if it issues an amount E of new equity in period 1. This implies that $L_{1M}(1 + \delta\alpha) + L_{2M}(1 - \alpha) + E < D_B$ when $\rho = -1$ and $L_{1M}(1 - \delta\alpha) + L_{2M}(1 - \alpha) + E < D_B$ when $\rho = 1$.

This distinction is necessary because the price investors are willing to pay for equity depends

on whether assumption 1 or 2 holds, since in the event of bankruptcy equity holders receive whatever is left after the bank repays its debt in full. In the next subsection we will explain in detail how the price investors are willing to pay for equity is determined.

3.1 Negative correlation

Table 3 below describes the existing shareholder's gains/losses under each possible scenario when the correlation structure of loan returns is negative.

Table 3: Existing Shareholder's Gains/Losses when $\rho = -1$

	High Quality Scenario		Low Quality Scenario	
Strategy	Assumption 1	Assumption 2	Assumption 1	Assumption 2
I_1	$bL_{1M}\delta\alpha$	$bL_{1M}\delta\alpha$	$-bL_{1M}\delta\alpha$	$-bL_{1M}\delta\alpha$
I_2	$-bL_{1M}\alpha$	$-bL_{1M}\alpha$	$bL_{1M}\alpha$	$bL_{1M}\alpha$
I_E	$-bL_{1M}\alpha(1-\delta)/2$	$-E$	$bL_{1M}\alpha(1-\delta)/2$	E
I_w	0	0	0	0

The calculations for strategies I_1 and I_2 are straightforward. For example, suppose that under the **high quality scenario** the bank sells proportion b of loan 1 for its expected value in period 2 (i.e., bL_{1M}). In this case, the bank's existing shareholders will gain $bL_{1M}\delta\alpha$, since the market value of loan 1 will decrease by $\delta\alpha$ in period 2. Instead, if the bank sells proportion b of loan 2 for its expected value, bL_{2M}, the bank's existing shareholders will forgo the increase in the value of loan 2 in the next period. Hence, their loss from strategy I_2 will equal $-bL_{2M}\alpha$. Since $L_{1M} = L_{2M}$, $-bL_{2M}\alpha$ can be written as $-bL_{1M}\alpha$ to facilitate comparison with loss/gain from I_1. The existing shareholder's gain/loss under the **low quality scenario** are derived in a similar way since the two scenarios are symmetric.

The expected gains/losses from strategies I_1 and I_2 are feasible, since investors are willing to buy loan 1 or loan 2 at their expected prices. For example, buying loan 1 is a fair deal for

investors since their expected payoff is equal to the opportunity cost of their money. Since the existing shareholder's gain is the investor's loss and vice versa, the investor's expected gain is

$$Investor's\ Expected\ Gain = 0.5\left[bL_{1M}\alpha\delta\right] + 0.5\left[-bL_{1M}\alpha\delta\right] = 0.$$

Given that $r = 0$ and that investors are risk neutral, an expected gain of zero is one that satisfies investors.[8] It is straightforward to show that the same is true for loan 2.

The calculations of the shareholder's gains/losses from an equity issue are more complicated and deserve some discussion. Recall from the definitions that V_{1I_E} is the market value of existing shareholders shares in period 1 if the bank sells new equity in period 1. Clearly, V_{1I_E} will be determined by the price at which the new investors are willing to buy new equity. New shareholders realize that the market value of debt increases when new equity is issued and they adjust the price at which they are willing to buy.[9] However, since in period 1 investors do not know which scenario will be realized, they calculate the expected value of debt after the equity issue, D_{Mnew}, by assigning a probability of 0.5 to each of the two possible scenarios.

Under **assumption 1**, the market value of debt in period 1 increases from its initial level D_M to its risk free level D_B, since the equity issue enables the bank to repay its debt in full under the low quality scenario. Hence, D_{Mnew} is given by

$$D_{Mnew} = 0.5D_B + 0.5D_B = D_B.$$

The new shareholders want this increase in debt value to be paid by the existing shareholders (i.e., they want V_{1I_E} to be worth its pre-issue market value E_M less the increase in the market value of debt $D_B - D_M$). Consequently, V_{1I_E} is given by

$$V_{1I_E} = E_M - (D_B - D_M) \tag{15}$$

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[8]These assumptions could be relaxed to allow for a positive interest rate and risk averse investors. However, the model's equilibrium and economic intuition would remain unchanged. Hence, the assumptions are retained in the interest of simplicity.

[9]The market value of debt increases because an equity issues makes more funds available to repay the bank's debt. This is because debt holders are senior claimants and equity holders are junior claimants (i.e., equity holders receive whatever is left after a bank repays its debt in full).

Given that $D_M = 05D_B + 0.5\left[L_{1M}(1+\delta\alpha) + L_{2M}(1-\alpha)\right]$, we can express V_{1E} as

$$V_{1I_E} = E_M - 0.5\left\{D_B - \left[L_{1M}(1+\delta\alpha) + L_{2M}(1-\alpha)\right]\right\}, \tag{16}$$

where $0.5\left\{D_B - \left[L_{1M}(1+\delta\alpha) + L_{2M}(1-\alpha)\right]\right\}$ measures the expected shortfall of existing funds that will be covered with funds from the new equity issue.

The new shareholders will pay E for new shares in period 1 if they receive a proportion k of the total shares outstanding after the new equity issue, where k is given by

$$k = \frac{E}{V_{1I_E} + E}. \tag{17}$$

In this case, the existing shareholders own a proportion $1 - k = \frac{V_{1I_E}}{V_{1I_E}+E}$ of the total shares outstanding after the new equity issue in period 1. Since the total market value of all shares after the new equity issue is $V_{1I_E} + E$, the market value of the existing shareholders shares is $(1-k)(V_{1I_E} + E) = V_{1I_E}$. Hence, the new shareholders achieve their objective.

Note that for a given amount E, equations (15) and (17) determine the price of each share. In particular, the higher the increase in the market value of debt $D_B - D_M$, the higher the proportion k of the total shares that the new shareholders get for a given amount E, and thus the lower the price of each share. In addition, it is clear from equation (16) that the price of each share depends on the market value of the bank's portfolio in the low quality scenario. In particular, the lower the market value of the bank's loan portfolio in the worst case scenario the larger the shortfall and thus the lower the price the new shareholders are willing to pay for each share. This means that the correlation structure of loan returns affects the price of equity. Everything else equal, the value of the bank's portfolio (in the low quality scenario) is lower when $\rho = 1$ than when $\rho = -1$. In particular, when $\rho = 1$ both loans depreciate in value, while when $\rho = -1$ one loan depreciates and the other appreciates mitigating the total drop.

If new equity is issued under these conditions, the gain to the bank's existing shareholders will be given by the difference between the value of the new equity in period 1 and the value of the new equity in period 2. This implies that under the **high quality scenario** the existing

shareholder's loss is given by

$$Existing\ Shareholder's\ Loss = E - k\left[L_{1M}(1 - \delta a) + L_{2M}(1 + a) - D_B + E\right], \qquad (18)$$

substituting equations (2), (3), (8), (15) and (17) into (18) we get

$$Existing\ Shareholder's\ Loss = -\frac{b}{2}L_{1M}\alpha(1 - \delta).$$

Similarly, under the **low quality scenario** the existing shareholder's gain is

$$Existing\ Shareholder's\ Gain = \frac{b}{2}L_{1M}\alpha(1 - \delta).$$

We will now calculate the existing shareholder's gain/loss from selling equity under assumption 2. Under **assumption 2**, new shareholders know that in period 1 the market value of debt after an equity issue will increase from D_M (the original level) to some new level D_{Mnew}. In this case, $D_{Mnew} < D_B$ because the bank will not be able to repay its debt in full if the low quality scenario prevails. Hence, V_{1I_E} is given by

$$V_{1I_E} = E_M - (D_{Mnew} - D_M), \qquad (19)$$

where

$$D_{Mnew} = 0.5D_B + 0.5\left[L_{1M}(1 + \delta\alpha) + L_{2M}(1 - \alpha) + E\right] \qquad (20)$$

$$D_M = 0.5D_B + 0.5\left[L_{1M}(1 + \delta\alpha) + L_{2M}(1 - \alpha)\right]. \qquad (21)$$

Using equations (20) and (21) we can express V_{1I_E} as

$$V_{1I_E} = E_M - 0.5E. \qquad (22)$$

In this case, the expected shortfall that will be covered with funds from the new equity issue is fixed and equal to the amount they contributed (i.e., if the low quality scenario prevails, the bank will go bankrupt and they will loose all their funds). Hence, the price of each share does not depend on the value of the bank's portfolio in the worst case scenario. This implies that under assumption 2 the correlation of loan returns does not affect the cost of issuing equity.

Under these conditions, the existing shareholder's loss under the **high quality scenario** is

$$Existing\ Shareholder's\ Loss = E - k\left[L_{1M}(1 - \delta a) + L_{2M}(1 + a) - D_B + E\right] \quad (23)$$

substituting equations (2), (3), (8), (19), (20), and (17) into (23) we get

$$Existing\ Shareholder's\ Loss = -E.$$

Similarly, under the **low quality scenario** the existing shareholder's gain is given by

$$Existing\ Shareholder's\ Gain = E.$$

Buying equity is a fair deal for investors since their expected payoff is equal to the opportunity cost of their money. In particular, under **assumption 1**, the investor's expected gain is

$$Investor's\ Expected\ Gain = 0.5\left[\frac{b}{2}L_{1M}\alpha(\delta + 1)\right] + 0.5\left[-\frac{b}{2}L_{1M}\alpha(\delta + 1)\right] = 0,$$

while under **assumption 2** is

$$Investor's\ Expected\ Gain = 0.5(E) + 0.5(-E) = 0.$$

Finally, we consider the expected gain/loss from mixed strategy I_w, which involves selling a riskless combination of the two loans. Such a combination must be considered because it eliminates the asymmetric information problems and thus it could potentially dominate the other strategies. When $\rho = -1$, there exists a combination of loans which involves no risk (i.e., a combination that has the same value in period 1 and period 2 regardless of which scenario prevails). Specifically, a combination wb of loan 1 and $(1 - w)b$ of loan 2 involves no risk if

$$w = \frac{1}{1 + \delta}.$$

To prove that this combination is riskless, suppose that the bank sells this combination of loans in period 1 and that in period 2 the **low quality scenario** prevails. In that case, the

value of this combination in period 2 is given by

$$wbL_{1M}(1 + \delta\alpha) + (1 - w)bL_{M2}(1 - \alpha).$$

Substituting into the above expression the definition of w and equation (3) we get

$$wbL_{1M} + (1 - w)bL_{2M},$$

which clearly shows that the value of this combination in period 2 is equal to the value for which it is sold in period 1. The same is true under the **high quality scenario**. In particular,

$$wbL_{1M}(1 - \delta\alpha) + (1 - w)bL_{2M}(1 + a) = wbL_{1M} + (1 - w)bL_{2M}.$$

Hence, this combination is indeed riskless and thus it involves no gain or loss for the bank's existing shareholders.

3.2 Positive correlation

Table 4 describes the existing shareholder's gains/losses under each possible scenario when $\rho = 1$. The calculations are not presented since they are similar to those described in the previous section for $\rho = -1$. All expected gains reported in Table 4 are feasible for the same reasons explained in the previous section. Note, however, that Table 4 does not report the results for a mixed strategy. This is because no such riskless combination exists when $\rho = 1$.

Table 4: Existing Shareholder's Gains/Losses when $\rho = 1$

Strategy	High Quality Scenario		Low Quality Scenario	
	Assumption 1	Assumption 2	Assumption 1	Assumption 2
I_1	$-bL_{1M}\delta\alpha$	$-bL_{1M}\delta\alpha$	$bL_{1M}\delta\alpha$	$bL_{1M}\delta\alpha$
I_2	$-bL_{1M}\alpha$	$-bL_{1M}\alpha$	$bL_{1M}\alpha$	$bL_{1M}\alpha$
E	$-bL_{1M}\alpha(\delta + 1)/2$	$-E$	$bL_{1M}\alpha(\delta + 1)/2$	E

4   Equilibrium

We now derive the bank's optimal choice of security to sell at the equilibrium assuming that the bank's managers and the investors are rational economic agents maximizing their objective functions conditional on whatever information they have in period 1.

4.1   Negative correlation

Comparing the gains/losses reported in Table 3 we can rank the bank's preferences. Table 5 below shows the bank's preferences in descending order.

Table 5: Descending order of preferences when $\rho = -1$.

	High Quality Scenario	Low Quality Scenario
Assumption 1	I_1, I_w, I_E, I_2	I_2, I_E, I_w, I_1
Assumption 2	I_1, I_w, I_E, I_2	I_2, I_E, I_w, I_1 if $bL_{1M}\delta\alpha > E$
		I_E, I_2, I_w, I_1 if $bL_{1M}\delta\alpha < E$

Given the above ranking of preferences, if the bank's managers know that the high quality scenario will prevail they would like to sell loan 1, irrespective of whether assumption 1 or 2 holds for equity. This is because I_1 is the only strategy that results in a gain for the bank's existing shareholders. On the contrary, if the bank's managers know that the low quality scenario will prevail and assumption 1 holds, they would like to sell loan 2 since it is the strategy with the largest gain. Instead, if they know that the low quality scenario will prevail and assumption 2 holds, they would like to sell loan 2 or equity depending on whether $bL_{1M}\delta\alpha \gtrless E$.

However, this can not be the equilibrium solution. In particular, the bank's choice under the high quality scenario is *always different* form its choice under the low quality scenario. Hence, if the bank chooses anything else other than its optimal strategy under the high quality scenario it will give the signal to investors that it is the low quality scenario that will prevail. As mentioned earlier, investors observe the values reported in Table 1 (i.e., they know what are the various

possibilities; what they do not know is which one will be realized). Hence, they can solve the bank's optimization problem and if they observe the bank selling anything else other than its optimal strategy under the high quality scenario, they think that it is the low quality scenario that will prevail. In that case, they are willing to buy the security offered by the bank only at the price they think it will have in period 2. Under these circumstances, it is straightforward to show that the bank will always choose the optimal strategy under the high quality scenario in order to postpone bankruptcy for one period. This leads to proposition Proposition 1.

Proposition 1 *When the correlation structure of loan returns is negative the unique equilibrium is one in which proportion b of loan 1 is sold.*

Proof: It will be shown that under each possible scenario the bank will not deviate from the postulated equilibrium. The proof is divided in two steps.

Step 1: *It will be shown that if the bank's managers know that the high quality scenario will prevail, they have no incentive to deviate from the postulated equilibrium.* Suppose that the bank's managers sell proportion b of loan 1, V_{2I_1} is given by

$$V_{2I_1} = \max\left\{0,\ bL_{1M} + (1-b)L_{1M}(1-\delta\alpha) + L_{2M}(1+\alpha) - D_B\right\}, \tag{24}$$

which is positive since equation (11) holds.

Given the out of equilibrium belief, if the bank chooses to sell anything other than I_1 it will give the signal to investors that it is the low quality scenario that will prevail. In particular, if the bank decides to sell proportion b of loan 2 it will be able to sell it only at $bL_{2M}(1-a)$ (i.e., since investors think that the low quality scenario will prevail, they are not willing to pay more than the value of loan 2 under the low quality scenario). Investors also believe that the bank will be bankrupt in period 2, since they think that the bank's condition is described by

$$bL_{2M}(1-\alpha) + L_{1M}(1+\delta\alpha) + (1-b)L_{2M}(1-\alpha) - D_B \Rightarrow$$

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$$L_{1M}(1 + \delta\alpha) + L_{2M}(1 - \alpha) - D_B,$$

which according to equation (12) is negative. Since investors believe that the bank will be bankrupt in period 2, the market value of its equity will immediately drop to zero (i.e., $V_{1I_1} = 0$. and thus, $V_{2I_2} = 0$). Given that $V_{2I_1} > V_{2I_2}$, the bank will not deviate by selling loan 2.

We will now show that the bank's managers will also not deviate by issuing equity. Given the out of equilibrium belief, an equity issue signals to investors that the low quality scenario will prevail. Under such circumstances, investors are unwilling to buy equity because they think that their payoff in period 2 will be less than the amount they contributed in period 1. To prove this, note that if the bank issues new equity, the market value of both old and new stake in period 2 under the low quality scenario is

$$L_{1M}(1 + \delta\alpha) + L_{2M}(1 - \alpha) + E - D_B.$$

Investors believe that this value is less than E because under the low quality scenario

$$L_{2M}(1 + \delta\alpha) + L_{2M}(1 - \alpha) < D_B.$$

Thus, investors will not buy new equity worth E in period 1 even if they are offered 100% ownership. In other words, if investors think that equity in period 2 will have zero market value, they are not willing to buy equity for any positive price. Hence, selling is not feasible.

So far we proved that under the high quality scenario the bank will not deviate from the postulated equilibrium. We will now turn to the low quality scenario.

Step 2: *It will be shown that if the bank's managers know that low quality scenario will prevail, they do not have an incentive to deviate from the postulated equilibrium* because selling anything other than loan 1 will give the signal to investors that the low quality scenario will prevail. For the same reasons as in step 1, selling loan 2 when investors think that the low quality scenario will prevail results in $V_{1I_1} = V_{2I_2} = 0$. If, instead, the bank chooses to sell proportion b of loan

1, V_{2I_2} is given by

$$V_{2I_2} = \max\left\{0,\ bL_{1M} + (1-b)L_{1M}(1+\delta\alpha) + L_{2M}(1-\alpha)\right\},$$

which is equal to zero (using equation (12) it is easy to show that $bL_{1M} + (1-b)L_{1M}(1+\delta\alpha) + L_{2M}(1-\alpha) < 0$). However, since I_1 does not signal to investors that the low quality scenario will prevail, $V_{1I_1} > 0$. Given that $V_{2I_2} = V_{2I_1} = 0$ and $V_{1I_1} > V_{1I_2}$, the bank will not deviate from the postulated equilibrium by selling loan 2. Finally, for the same reasons as in step 1, selling equity at any positive price is not feasible. ∎

4.2   Positive Correlation

Comparing the gains/losses reported in Table 4 we can rank the bank's preferences. Table 6 below shows the preferences of each bank in descending order of preference.

Table 6: Descending order of preferences when $\rho = 1$.

	High Quality Scenario	Low Quality Scenario
Assumption 1	$I_1,\ I_E,\ I_2$	$I_2,\ I_E,\ I_1$
Assumption 2	$I_E,\ I_1,\ I_2$ if $bL_{1M}\delta\alpha > E$	$I_2,\ I_1,\ I_E$ if $bL_{1M}\delta\alpha > E$
	$I_1,\ I_E,\ I_2$ if $bL_{1M}\delta\alpha < E$	I_2 or $I_E,\ I_1$ if $bL_{1M}\delta\alpha < E$

Given the above ranking of preferences, if assumption 1 holds and the bank's managers know that the high quality scenario will prevail, they would like to sell loan 1 in order to minimize the loss for their existing shareholders. On the contrary, if they know that the low quality scenario will prevail, they would like to sell loan 2 in order to maximize the gain for their existing shareholders. Similarly, if assumption 2 holds and the bank's managers know that the high quality scenario will prevail, they would like to issue equity if $bL_{1M}\delta\alpha > E$ and they would like to sell loan 1 if $bL_{1M}\delta\alpha < E$. If, instead, they know that the low quality scenario will prevail they would like to sell loan 2 if $bL_{1M}\delta\alpha > E$ and they would like to sell loan 2 or to issue equity if $bL_{1M}\delta\alpha < E$, where their choice between loan 2 and equity depends on whether $bL_{1M}\alpha \gtrless E$.

It is clear that also in this case the bank's choice under the high quality scenario is *always different* form its choice under the low quality scenario. Hence, if the bank deviates from its optimal strategy under the high quality scenario it will give the signal that it is the low quality scenario that will prevail and the investors will adjust the price for which they are willing to buy. It is straightforward to show that under these conditions the bank will not find it profitable to deviate from its optimal strategy under the high quality scenario, even if they know that it is the low quality scenario that will prevail in period 2. This leads to Propositions 2 and 3.

Proposition 2 *When the correlation stucture of loan returns is positve and assumption 1 holds for equity, the unique equilibrium is one which proportion b of loan 1 is sold.*

Proposition 3 *When the correlation stucture of loan returns is positive and assumption 2 holds for equity, the unique equilibrium depends on the relationship between $bL_{M1}\delta\alpha$ and E. If $bL_{M1}\delta\alpha < E$, then proportion b of loan 1 is sold at the equilibrium. Instead, if $bL_{M1}\delta\alpha > E$ then equity will be issued at the equilibrium.*

Formal proofs of proposition 2 and 3 are not presented because they are very similar to the proof of proposition 1. The rational of all three propositions is straightforward. At the equilibrium the bank's managers will always choose to follow the optimal strategy under the high quality scenario because they will otherwise give the signal to investors that it is the low quality scenario that will prevail in period 2. This out of equilibrium belief is possible because the optimal strategy under the high quality scenario is always different from the optimal strategy under the low quality scenario.

5   Conclusions

This paper considered a setting where a bank can sell loans or issue equity in order to increase its equity-to-assets ratio. The paper's focus is on describing the bank's choice of financing

under conditions of asymmetric information, given different assumptions about the correlation structure of loan returns. Previous work on this issue considered only the expected value (and not the correlation structure) of loan returns.

The model presented here considered two cases: one in which there is perfect negative correlation between loan 1 and loan 2 ($\rho = -1$) and one in which there is perfect positive correlation ($\rho = 1$). Since the bank has only two loans, the correlation coefficient, ρ, between loan 1 and loan 2 fully characterizes the correlation structure of the bank's loan portfolio. If there were more than two loans, we would have to take into account the correlation of each pair in order to fully characterize the correlation structure of a bank's portfolio. Hence, in more general terms $\rho = 1$ should be thought as a case where *all* loans are positively correlated with each other: they all appreciate or they all depreciate. Instead, $\rho = -1$ should be thought as a case where at each possible scenario *some* loans appreciate and some depreciate.

As demonstrated by Propositions 1 to 3, the equilibrium in each case is different. In particular, when $\rho = -1$ there exists a loan that the bank can sell so that its existing shareholders *gain* under the high quality scenario. No such loan exists when $\rho = 1$. In particular, when $\rho = 1$ selling any of the available securities involves a *loss* for the bank's existing shareholders. Thus, the bank in this case tries to minimize the loss for its existing shareholders. According to Propositions 2 and 3 this choice depends on whether assumption 1 or 2 holds for equity (i.e., on whether issuing equity allows (or not) the bank to avoid bankruptcy under worst case scenario).

When assumption 1 holds, the bank finds it optimal to sell its less risky loan (i.e., loan 1) instead of loan 2 or equity. The rationale is simple. Loan 1 clearly dominates loan 2 because it varies less. However, loan 1 also dominates equity. In particular, when assumption 1 holds the loss from an equity issue depends positively on the variance of the bank's total loan portfolio. Instead, the loss from selling a loan depends only on its own variance. Since in this case the variance of the bank's portfolio is greater than the variance of its less risky loan, the bank finds it optimal to sell its less risky loan. When assumption 2 holds, the bank will either choose loan 1

or equity depending on which one involves the smallest loss for the bank's existing shareholders (i.e., $bL_{1M}\delta\alpha \lessgtr E$). Note that an equity issue could be preferred in this case, because the loss from equity does not depend on the variance of the bank's portfolio. Since the bank will be bankrupt for sure if the low quality scenario prevails, the loss for the bank's existing shareholders is fixed and equal to E. Hence, if the loss from selling loan 1 ($bL_{1M}\delta\alpha$) is higher than the loss from equity (E) the bank will find it optimal to sell loan 1 and vice versa.

Let us now discuss which case is more realistic (i.e., $\rho = 1$ or -1). In practice, it is unlikely that a bank will have either favorable or unfavorable inside information about all of its loans. This might happen occasionally but is unlikely to happen on a consistent basis; if it did, it would imply that some banks are consistently undervalued or overvalued. It is more likely that a bank will have favorable inside information for some of its loans and unfavorable inside information for some of its other loans. As far as this is true, the results suggest that the bank will always prefer to sell loans instead of equity.

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