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Chapter 12

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INTRODUCTION

In *spatial dispersion games* the agents' common goal is to choose distinct locations. Such games have been used to study congestion problems, habitat selection, and networking issues, e.g., Alpern and Reyniers [2002] and Alpern and Gal [2003]. More generally, dispersion incentives in location games appear in models of product differentiation, e.g., Salop [1979], and variants of the voting models of Hotelling [1929] and Downs [1957], e.g., Palfrey [1984].¹

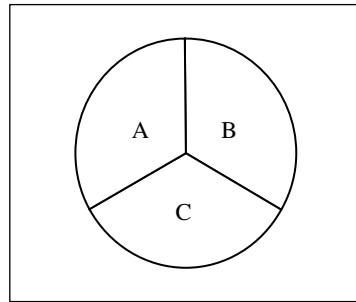
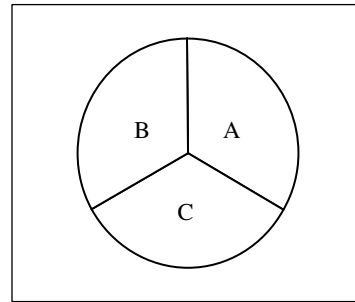


We experimentally investigate the role of cognition in such games and compare it with the role of cognition in spatial matching games, where the common goal of the agents is to choose the same location. In our setup cognition matters because agents may be differentially aware of the dispersion opportunities that are created by the history of the game. Once agents achieve dispersion in a repeated spatial dispersion game and if they can remember past choices, they have the option to maintain dispersion by simply maintaining their previous choices. When agents do not have a simple record of their own past choices there may be other ways of sustaining dispersion. Cognitive issues arise when agents do not have a simple record of their own past choices, but there is a procedure for inferring own past choices. Some agents may be aware of this procedure while other agents may be unaware of it.

Unawareness of this sort requires more than simple lack of knowledge. In addition to not knowing the procedure the agent must not know that he does not know the procedure, i.e., he must lack *negative introspection*. Unawareness seems commonplace in everyday life, and yet has only recently attracted attention in the literature. One likely reason is that unawareness does not easily fit into conventional models of information economics. Violations of negative introspection are not compatible with the standard partitioned state space model of knowledge, Aumann [1976], as pointed out by Geanakoplos [1992]. More recently, Dekel, Lipman and Rustichini [1998] have demonstrated that *any* standard state space model precludes unawareness. They suggest that one way to avoid this conundrum is to make a distinction between the agent's and the analyst's description of the state space, and to treat the state space as "representing the agent's view of possibilities." Recently, there have been a few proposals of models of knowledge that permit unawareness, e.g., Li [2003] and Schipper [2002]. Furthermore, there have been suggestions that properly incorporating unawareness into our models may shed light on issues related to contractual incompleteness and no-trade theorems.

Our objective is more modest. We accept unawareness as a simple empirical phenomenon and ask what happens when agents differ in their awareness in a simple strategic setting, i.e., when there is interactive unawareness. Common-interest games are attractive for this purpose because they help us focus on the central issue of how unawareness affects players' strategic reasoning about others. We need not worry for example about how differential awareness interacts with signaling motives, bargaining motives, deception, threats, punishments, or other-regarding preferences. Location games with a spatial structure are appealing because agents may differ in how much of this structure and its possible uses they perceive.

For a formal model of interactive unawareness in our games we follow Bacharach [1993]. He calls for a model of games in which "one specifies the way players conceive the situation and how this varies." He provides details of such a model of *variable universe games* for the case where the players' aim is to choose a common action, i.e., for matching games. In Bacharach's model, a player's perception is essentially given by a partition of the set of actions. Blume and Gneezy [2002] extend Bacharach's approach to permit a more general structure on the sets of actions than partitions, or collections of partitions. It permits the spatial (circular)

*Player 1**Figure 1a.**Player 2**Figure 1b.*

structure that is used in Blume and Gneezy [2002], Blume, DeJong and Maier [2003] and that will be used in the present paper to address spatial dispersion games.²

A basic version of the dispersion game (that we expand upon and fully develop later in the paper) consists of two players who are randomly paired together for a one-shot game. The two players simultaneously and independently choose one of three identical unlabeled sectors of a disc, as illustrated in Figure 1. One player sees a disc whose labels have the directional order indicated in Figure 1a. The other player sees a disc with the directional order of the labels reversed, as in Figure 1b. The locations are randomized at the beginning of the one-shot game and neither player sees the labels A, B, and C themselves. In a spatial dispersion game, the payoffs are one if both players choose different sectors, A and B, B and C, or C and A, and zero if they choose the same sector, A, B, or C. For a simple spatial matching game the payoffs are just the reverse.

Blume and Gneezy [2002] have experimentally demonstrated that there are differences in awareness in spatial matching games. Blume and Gneezy consider one-shot spatial matching games in which players simultaneously choose a single sector from a disc with five sectors. All sectors are identical in size and shape, three are white, and two are black. They compare two scenarios, one in which a single individual plays against him- or herself, and one in which two distinct players play against each other. In either case, given the symmetry constraints imposed by the task, there is a unique optimal way to play the game. Success is only guaranteed if both choices correspond to the midpoint of the odd distance between the two black sectors. Cognitive differences can be shown to exist by having players play against themselves. When playing against themselves, players who are aware of the guaranteed success strategy will use it, while others will be attracted to the obvious alternative, to choose one of the black sectors. Blume and Gneezy find that a significant percentage of participants do not solve the game when playing against themselves.

In the matching games of Blume and Gneezy [2002], cognitive differences prevent players from coordinating on the unique optimal solution. Cognitive differences are likely to play a different role in dispersion games. Even though in both kinds





of games agents have a common objective, the structure of equilibria is different. Unlike in matching games, in dispersion games typically none of the equilibria are strict: As long as there are more locations than agents, an agent can always switch to an unused location and still maintain dispersion. Also, while the matching games of Blume and Gneezy [2002] have a unique optimal solution, there are multiple ways in which dispersion can be achieved in our games. This makes the questions of whether any equilibrium is attained and, if so, which one will be selected important.

The present paper has agents interact repeatedly in spatial dispersion games. Repeated interaction in spatial matching games with a circular structure has been investigated by Blume, DeJong and Maier [2003]. There, players are randomly paired each period. The stage game played in each period consists of two rounds. In the first round of the stage game two players simultaneously and independently choose one of n identical sectors of a disc, where n is odd.³ In the second round, after observing first round choices, but without being able to distinguish one's own from one's partner's choice, both players choose again. In both rounds, payoffs are one if both players choose identical sectors and zero otherwise. Note that the second round induces essentially the same choice problem as the task in Blume and Gneezy [2002] and therefore has a unique optimal solution.

In the repeated spatial matching games of Blume, DeJong and Maier, learning can occur at two levels. At one level, in each period, agents can learn by labeling actions in the first round and using these labels in the second round.⁴ At the other level, agents can learn across periods about how to learn within a period. This type of learning, which we call *cognitive learning*, has to the best of our knowledge of the literature only been addressed in the Blume, DeJong and Maier [2003] paper. Initially, there may be agents who are unaware of the fact that the labels introduced by first-round choices can always be used to identify a unique distinct sector. Other agents may be aware of this possibility. In the course of the multi-period interaction, agents may *become aware* of this possibility, i.e., engage in *aha learning*, Bühler [1907, 1908], Köhler [1925] and Weber [2003].⁵ The results from our matching games support coordination outcomes and we find evidence for cognitive learning. That is, in simple environments agents learn across periods to make better use within a period of labels created in that period. We observe transfer of cognitive learning from simple environments to more complicated environments.

As previously noted, the structures of the action space that agents may or may not be aware of have different uses in dispersion games than matching games. For example, the circular structure of the matching game of Blume, DeJong and Maier [2003] enables agents to identify a unique candidate for a common action. The same circular structure in a dispersion game generates a "coordination problem" characterized by multiple, non-strict equilibria. This difference in the possible use of structures suggests that the learning may also be different.

Our main finding in the present paper is that in spatial dispersion games, strategic interaction magnifies the role of cognitive constraints. Specifically, with cognitive constraints, pairs of agents fail to solve a dispersion problem that poses little or no problem for individual agents playing against themselves. When we remove the



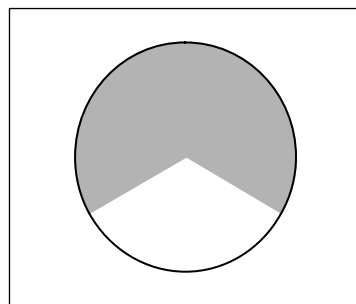
cognitive constraints in our design, pairs of agents solve the same problem just as well as individuals do. In addition, we find that when playing against themselves agents do not change the mode by which they solve the dispersion problem when our design removes the cognitive constraints.

GAME AND EXPERIMENTAL DESIGN

We study a repeated dispersion game in which two players are randomly paired together and stay paired for twenty-one periods. In the first period, the two players simultaneously and independently choose one of three identical unlabeled sectors of a disc, as illustrated in Figure 1.⁶ One player sees a disc whose labels have the directional order indicated in Figure 1a. The other player sees a disc with the directional order of the labels reversed, as in Figure 1b. Neither player sees the labels A, B, and C themselves. The payoffs are one if both players choose different sectors, A and B, B and C, or C and A, and zero if they choose the same sector, A, B, or C. At the end of period one, the two players are informed about the sectors that were chosen.

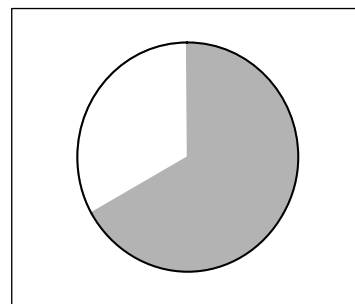
At the beginning of the second period, players observe the previous period's choices but without being able to distinguish one's own from one's partner's choice, for example see Figure 2 where the players achieved a dispersion outcome and where the discs with the first period choices have been randomly spun and presented to the players, Figure 2a and 2b respectively, at the beginning of period two. Both players then choose again. The payoffs are again one if both players choose different sectors and zero if they choose the same sector. At the end of period two, the two players are informed about the sectors that were chosen. Specifically they see the choices made in period 2, marked by red dots, on the background of the choices made in the previous period, marked by shaded sectors. Each of the subsequent periods follows the same sequence outlined for the second period.

We implement a two-by-two design. The first dimension is the information provided to players about their choices. The *relative-location information condition* is described above. In the theory for dispersion games, it is common practice to assume



Player 1

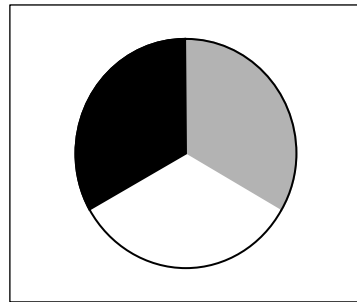
Figure 2a.



Player 2

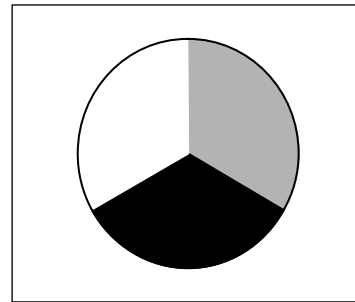
Figure 2b.





Player 1

Figure 3a.



Player 2

Figure 3b.

that agents know their present location and the location of other agents when taking their future choice of action. This describes the *precise-location information condition* and is illustrated in Figure 3a and 3b, where the choice of one player is noted in dark shading and the other player's choice is lightly shaded.

The second dimension of the design is the pairing of the players. The first condition is *fixed pairing*, as describe above. The second condition is *self pairing* where a player is paired with him or herself for the duration of the repeated spatial dispersion game. The purpose of this dimension is to separate the cognition problem from the coordination problem. Thus, there are four treatments in our design; fixed-pairing with relative and precise location information, and self-pairing with relative and precise information.

The experiment was conducted using a series of six cohorts; two cohorts or replications each for the two information treatments with fixed-pairing and one replication each for the two information treatments with self-pairing. A cohort consisted of twelve participants. Such a design provides the same number of pair observations in each of the four treatments. All participants were recruited from undergraduate (sophomore and above) and graduate classes at the University of Iowa. None of the participants had previously taken part in or otherwise gained experience with this series of treatments. Upon arrival, participants were seated at separate computer terminals and given a copy of the instructions.⁷ Before each replication, instructions were read aloud and participants individually filled out questionnaires confirming their knowledge and understanding of the instructions. We then went over the questionnaire orally and answered questions. Since these instructions were read aloud, we assume that the information contained in them was mutual knowledge.

Each cohort played a repeated spatial dispersion game for twenty-one periods from one of the four treatments in the design. Each period had the following structure. Prior to the beginning of the first period, participants were paired using a random-matching procedure or paired with themselves. In the first period, participants chose a sector from a symmetric disc with 3 identical sectors. At the beginning of the first





period, the discs were randomly rotated, independently across participants or across the two computer screens used by a participant in the self-pairing treatments, to eliminate all possibilities for *a priori* coordination. Then, participants made their choices by using a mouse to click on their chosen sector. They were given an opportunity to either revise or confirm their choices. At the end of the period, when all participants had made and confirmed their choices, they were informed about which sectors were chosen in their match.

At the beginning of period two, each disc was randomly rotated and period-one choices were displayed in the new configurations. In the display for the relative information treatments no distinction was made between one's own choice and one's partner's choice, see Figure 2a and 2b. This procedure ensured that in the second period, participants only had information about the configuration of choices. In the precise information treatments, each player's choice was indicated for both players and for the self-pairing treatments the choices made on each computer screen were indicated for the player, see Figure 3a and 3b. In the second period, participants once more chose one of the three sectors from the same disc as before with the prior choices displayed as just described. At the end of the period, when all participants had made and confirmed their choices, they were informed about which sectors were chosen in their pair along with the relative (precise) locations of the previous period's choices. Each subsequent period through period twenty-one followed the same sequence detailed for period two.

Each replication lasted from one-half to one hour. Participants' earnings ranged from \$7.50 to \$15.75 plus a "show up" payment of \$5.

THEORY

A solution for our relative information fixed-pairing treatment, must acknowledge two fundamental characteristics of the game. These are the symmetries that are built into the game, and potential differences in players' abilities to recognize when these symmetries have been broken.

Our design ensures that in the first period of our game all three sectors are completely symmetric. Players could not guarantee dispersion even if we permitted them to talk before the game. The fact that we rotate the disc independently across players guarantees that players *de facto* randomize by assigning equal probabilities to all sectors in the first period.

In the second period, players observe which sectors were chosen in the first period. Consider the case where players achieved dispersion in the first period (the other case, in which their choices resulted in congestion, is analyzed analogously). The fact that we spin the disc and that both players' choices are marked identically ensures that players cannot distinguish between their own choice and their partner's choice. Therefore, players are *de facto* precluded from guaranteeing dispersion in the second period by maintaining their first-period choices in the second period.

However, unlike in the first period, the absence of communication is a binding constraint here. If they could communicate, they could agree on one player playing



the *odd sector*, the sector not chosen by either player in the first period, and the other player playing one of the first-period choices. In the absence of communication, the fact that players' positions are identical prevents them from coordinating on such asymmetric behavior. Therefore we look for equilibria where in the second period both players put the same probability on the odd sector.

Before the third period (and similarly for subsequent periods) smart players will remember whether in the second period they chose the odd sector, the sector to the left of the odd sector (as viewed from the center of the disc), or the sector to the right of the odd sector. Then, if they manage to achieve dispersion in the third period, they can achieve dispersion in every subsequent period by following the rule of choosing the same sector in relation to the odd sector as in the previous period.

A problem arises because not all players need be smart, in the sense of realizing the possibility of making left-right distinctions on the disc. Players who can only distinguish chosen and unchosen sectors can only guarantee future dispersion if the dispersion realized was such that one player in the previous period chose the odd, unchosen, sector and the other chose one of the two previously chosen sectors. We formalize this problem by allowing for different types of players, who are endowed with different languages, a coarse-language and a fine-language, in which they describe the choice set to themselves.

The distinction between coarse- and fine-language players is as follows. Coarse-language players can only distinguish *chosen* and *unchosen* sectors in any period after the first period. Fine-language players can use the circular structure to enumerate all sectors after the first period. Further, fine-language players can commonly distinguish all sectors in a period after the second period. The reason is that for period three and after, fine-language players can describe each others' choices relative to the odd sector. Already, in period two, a fine-language player can for example choose "the sector to the left of the odd sector." At the beginning of period three, a fine-language player can also see his partner's period-two choice in reference to the odd sector of period one. As a result, fine-language players can maintain dispersion in period three and all subsequent periods.⁸

Player symmetry requires that players use identical strategies. Accordingly, we will focus on equilibria in which players use identical strategies and in which they employ efficient symmetric continuation strategies.

Denote by V_D a player's continuation payoff after players have achieved sustainable dispersion (dispersion in period three or later for fine-language players, and chosen-unchosen dispersion for coarse-language players) and by V_O the continuation payoff otherwise. Denote by p and q the probabilities of each player choosing the odd sector before there is sustainable dispersion, either the sector not chosen if players chose different sectors or the sector chosen if players chose the same sector. Note that the probabilities assigned to the two remaining sectors have to equal $(1 - p)/2$ each for one player and $(1 - q)/2$ each for the other. Of course in a symmetric equilibrium p and q must be the same. Consider the two cases were all players are fine-language players, $\lambda = 1$, or all players are coarse-language players, $\lambda = 0$. Then the payoff from using probability q against probability p equals:





$$\begin{aligned} \pi(q, p) = & pq[0 + V_o] + (p(1 - q) + q(1 - p))[1 + V_D] \\ & + (1 - p)(1 - q) \left[\frac{1}{2}[1 + \lambda V_D + (1 - \lambda)V_o] + \frac{1}{2}[0 + V_o] \right]. \end{aligned}$$

In equilibrium, the player choosing q must be indifferent among all q . Hence the derivative with respect to q must be zero.

$$\begin{aligned} \frac{\partial \pi(q, p)}{\partial q} = & pV_o + (1 - 2p)[1 + V_D] - (1 - p) \left[\frac{1}{2}[1 + \lambda V_D + (1 - \lambda)V_o] + \frac{1}{2}V_o \right] \\ = & 0. \end{aligned}$$

Solving for p , we obtain

$$p = \frac{2V_D + 1 - 2V_o - \lambda[V_D - V_o]}{4V_D + 3 - 4V_o - \lambda[V_D - V_o]}.$$

Hence, if all players are fine-language players, $\lambda = 1$, then

$$p_f = \frac{1}{3}.$$

Fine-language players uniformly randomize across all three sectors through period two and continue to randomize in period three and subsequent periods until dispersion is achieved. Once dispersion is achieved, players coordinate by both choosing left or right of the odd sector or by selecting chosen and unchosen.

If all players are coarse-language players, $\lambda = 0$, then

$$p_c = \frac{2[V_D - V_o] + 1}{4[V_D - V_o] + 3}.$$

Note that p_c is increasing in $V_D - V_o$. We conclude that coarse-language players put more probability on the odd sector than fine-language players. After period one, coarse-language players randomize until they achieve the dispersion outcome of chosen and unchosen sectors. Observe that cognitive differences only matter in the repeated game with at least three periods.

More generally, we can consider the incomplete information game where a player is a fine-language player with probability μ and a coarse-language player with probability $1 - \mu$.⁹ Coarse-language players being unaware of their cognitive constraint attach no probability to other players being fine-language players. They play under the presumption that the other player is a coarse-language player with certainty. Therefore, in the incomplete information game, regardless of μ , coarse-language players use the strategy derived for the complete information game above in which all players are coarse-language players.





In contrast, fine-language players are aware of the fact that both types are present and accordingly have beliefs about the type of the player they are facing. Thus, in general, optimal behavior of fine-language players could depend on their beliefs and potentially require complicated updating of beliefs. Fortunately, in the present context, the previously noted strategy for fine-language players, derived above under the assumption that it is common knowledge that all players are fine-language players, remains optimal for *any* belief β by fine-language players that their partner is a fine-language player. To see this, simply note that this strategy is optimal against both fine-language players and coarse-language players. The optimality against fine-language players is immediate.

The optimality against coarse-language players follows from the following facts: (1) against a coarse-language player one can not do better than a coarse-language player; (2) in periods in which a coarse-language player randomizes, *any* form of randomization, including playing the odd sector with probability p_f or repeating an action that led to dispersion the last period is optimal; and (3), trying to maintain dispersion by repeating last period's action is optimal in periods where a coarse-language partner is doing the same.

In the precise information fixed-pairing treatment, all players are fine-language players unless they ignore the information given to them. They can all distinguish among the sector they chose, the sector chosen by the player they are paired with, and the odd sector. All players uniformly randomize until a dispersion outcome is achieved. Once achieved, the dispersion outcome is played for the remainder of the game, both play left or right of the odd sector. As long as there are coarse-language players here, the probability of picking the odd sector is greater than or equal to one-third and the dispersion outcome can also be achieved by the chosen and unchosen selection.

In the self-pairing treatments, relative and precise information, all players uniformly randomize in period one. In period two, all players should achieve a dispersion outcome because there is no coordination problem after the first period. Fine-language players have the option of choosing to the left or right of the odd sector; coarse-language players can only coordinate by focusing on chosen and unchosen sectors.

RESULTS

Dispersion Outcomes

We first present the proportion of dispersion outcomes achieved by period for the four treatments, fixed-pairing with precise and relative information and self-pairing with precise and relative information, Figure 4. First, note that the self-pairing precise information treatment reaches full coordination first. Second, the proportion of dispersion outcomes for the fixed-pairing precise information and self-pairing precise and relative information treatments are indistinguishable. In these three treatments, all players are either fine-language players (fixed-pairing) or should not have a coordination problem when selecting a dispersion outcome (self-pairing). Third,





Figure 4.

while the self-pairing precise and relative information treatments do not reach coordination in period two, as predicted, the treatments are well on their way by period three. Finally, the proportion of dispersion outcomes in the fixed-pairing relative information treatment is indistinguishable from the expectation that behavior is random, .67. This result contrasts sharply with the result in Blume and Gneezy [2002], where relative information increased coordination relative to precise information.

Individual Player Choices

Regarding individual player choices, our theory suggests that for fixed-pairing, prior to achieving a dispersion outcome, the probability of selecting the Odd sector is higher in the relative information treatment ($p > 1/3$) than in the precise information treatment ($p = 1/3$). Unfortunately, there are very few observations here, sixteen in period two to be exact, too few for any meaningful analysis across the two treatments.¹⁰ However, aggregating across the two treatments, $p > 1/3$, which is the prediction from theory in the presence of coarse-language players in both treatments.

Paired Player Choices

Paired choices of players are presented in Table 1 for the four treatments and as a basis for comparison, the expectation that behavior is random. The relationship

Table 1. Paired Player Choices

The relationship between paired choices in period t and outcomes in period $t - 1$ is presented by treatment for periods two to twenty-one. Paired choices in period t are broken down by whether the paired choices are Odd/Not Odd or Both Not Odd with the Dispersed outcome in period t , or whether the paired choices are Other combinations that all imply the Matched outcome in period t . Outcomes are broken down by Dispersed and Matched in period $t - 1$. For comparison purposes, outcomes are also presented under the expectation that behavior is random.

<i>Treatment</i>	<i>Choices Period t:</i>	<i>Paired Choices</i>			<i>Total</i>
		<i>Dispersed</i>		<i>Matched</i>	
		<i>Odd/ Not Odd</i>	<i>Both Not Odd</i>	<i>Other</i>	
	<i>Outcome Period $t - 1$:</i>				
Fixed-Pair					
Precise					
	Dispersed	39	150	13	202
	Matched	15	6	17	38
		54	156	30	240
Relative					
	Dispersed	69	44	52	165
	Matched	44	9	22	75
		113	53	74	240
Random		107	53	80	240
Self-Pair					
	Dispersed	323	71	36	430
	Matched	27	11	12	50
		350	82	48	480
Random		214	106	160	480



between paired choices in period t and outcomes in period $t - 1$ is presented for periods two to twenty-one. Table 1 also presents the outcomes, Dispersed and Matched, for the paired choices for period t . Paired choices in period t are broken down by whether the choices are Odd/Not Odd or Both Not Odd with the Dispersed outcome in period t , or whether the paired choices are Other combinations that all imply the Matched outcome in period t . Outcomes in period $t - 1$ are broken down by Dispersed and Matched.

In the fixed-pairing precise information treatment, all players should be fine-language players and therefore should have access to playing left or right of the odd sector. How successful were the players in achieving a dispersed outcome and in coordinating their Not Odd choices, both choose Left or both Right, to achieve a dispersed outcome? From Table 1, out of a possible 240 outcomes, 210 are dispersed. For the 210 dispersed outcomes, 156 choices in period t were Both Not Odd (which implies both chose Left or both Right) and 54 were Odd/Not Odd (which from our theory implies chosen and unchosen).

Figure 4 suggests a difficult coordination problem in the fixed pair relative information treatment. Table 1 documents this problem. For the 165 dispersed outcomes achieved in period $t - 1$, players failed to capitalize on this success 52 times in period t . Further, for the successes achieved in period t , sometimes players coordinated on Odd/Not Odd, 69, and sometimes Both Not Odd, 44. A similar conclusion holds for the analysis of the 75 matched outcomes in period $t - 1$. Given either a dispersed or matched outcome in period $t - 1$, players face the coordination task in period t of choosing over Odd/Not Odd or Not Odd (with Not Odd presenting a secondary coordination problem of how to coordinate over the two sectors). Player choices are consistent with the expectation that behavior is random.

In the self-pairing treatments, players do not face such a coordination problem. A player can decide him or herself between Odd/Not Odd and Not Odd (both Right or both Left), regardless of the prior period's outcome. Players were very successful at achieving a dispersion outcome, but it is difficult to distinguish between coarse and fine-language players. The results implied by Figure 4 and shown in Table 1 (the two information treatments are combined in Table 1 because of their similar play) document that the number of matched outcomes is lowest in the self-pairing treatments despite the large number of Odd choices by players. Some players coordinated in period t by choosing Right or Left of the Odd sector on both screens, 82 out of 480. However, most players coordinated by Odd/Not Odd, 350 out of 480. This choice, Odd/Not Odd (or from theory, chosen and unchosen) appears to be the "least costly" way to coordinate rather than a statement about coarse and fine-language players.

Frequency of Paired Choices by Period

We next consider how many times player pairs chose a particular set of choices in each period. Figure 5 presents the results for the fixed-pairing precise information treatment. The graph documents the frequency of the paired choices made, Both Not Odd, Odd/Not Odd and Other. To read this graph, note that for Both Not Odd, eight

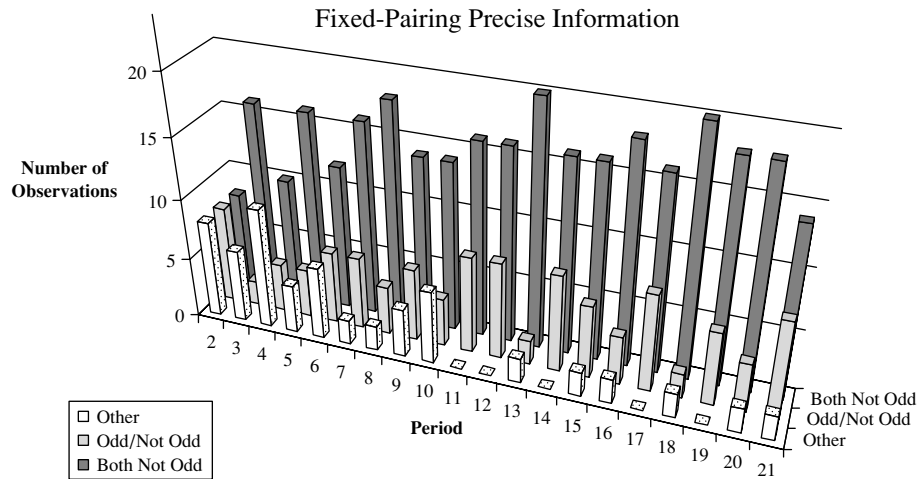


Figure 5. Frequency of Paired Choices.

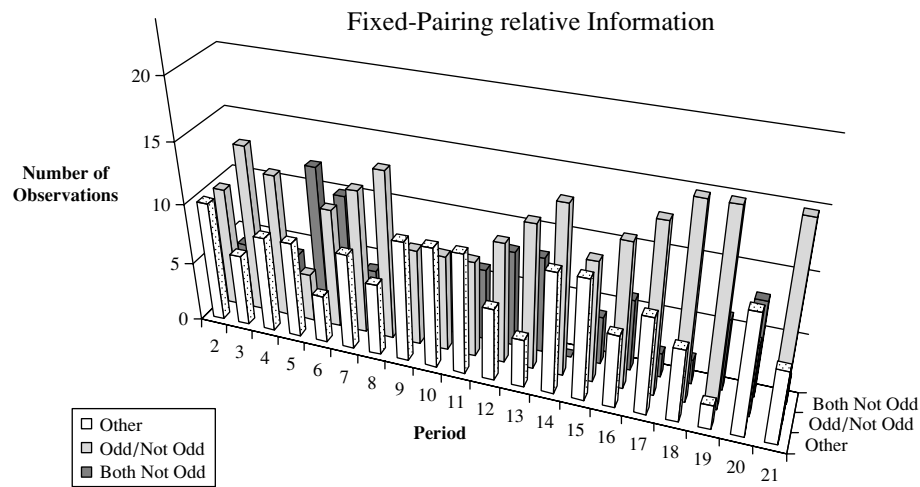


Figure 6. Frequency of Paired Choices.

such paired choices were made in period two and thirteen such choices were made in period twenty-one with the frequencies of Both Not Odd choices similarly graphed for the periods in between. The graph documents not only the high frequency of the Both Not Odd choice and its sustainability but also the demise of the Other category of paired choices.

Figure 6 describes the frequency of paired choices in the fixed-pairing relative information treatment. Again, the figure documents the coordination problem in this



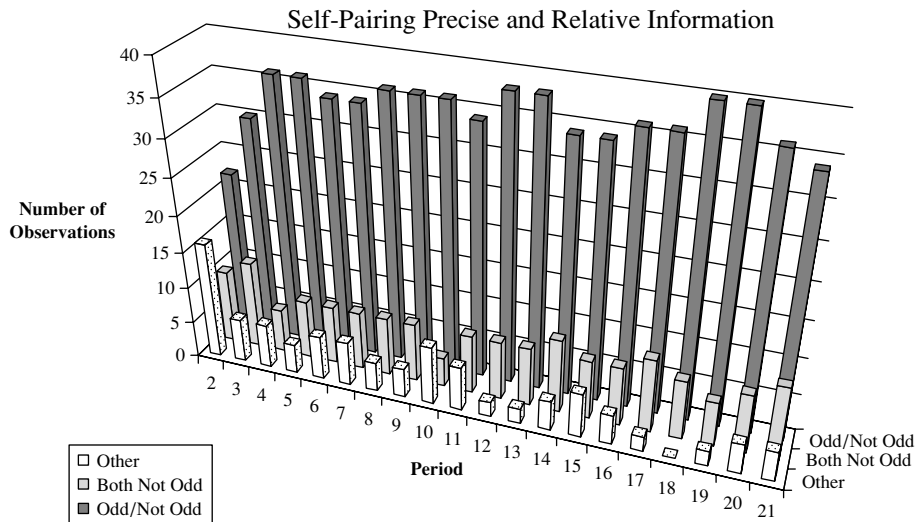


Figure 7. Frequency of Paired Choices.

treatment. All three paired choices, Both Not Odd, Odd/Not Odd and Other, were chosen throughout the treatment.

The self-pairing treatments of precise and relative information are presented in Figure 7; the two information treatments are again combined because of their similar play. The graph documents the high frequency and sustainability of the paired choice of Odd/Not Odd (from theory, chosen and unchosen). The Both Not Odd choice occurs with less frequency but is sustained throughout the treatments. The Other category of choices tends to die off over the treatments.

SUMMARY

Spatial dispersion games are characterized by multiple, non-strict equilibria. It is an open question whether players can select and attain an equilibrium in a spatial dispersion game. If equilibrium can be achieved, how long will it take and what are its characteristics. A natural question to also ask is whether the insights from matching games extend to dispersion games?

Our principal finding is that in spatial dispersion games, strategic interaction magnifies the role of cognitive constraints when compared to matching games. Players in the fixed-pairing relative information treatment had a difficult time coordinating their actions in order to achieve a dispersion outcome. This result contrasts with the result in Blume and Gneezy [2002], where in matching games relative information increased coordination compared to precise information, and Blume, DeJong and Maier [2003], where three sector matching games with relative information achieved a high level of coordination.





With these cognitive constraints in the fixed-pairing relative information treatment, pairs of agents failed to solve the dispersion problem that posed little or no problem for individual agents. In the self-pairing treatments, players were very successful in achieving dispersion outcomes. While some players coordinated by choosing right or left of the odd sector on both screens, most players coordinated by selecting the “least costly” way to coordinate, selecting the odd and not odd sectors. Thus, in both information treatments with self-pairing, we find that the mode used by individual agents to solve the dispersion problem is the same, odd and not odd.

When we remove the cognitive constraints in our design, pairs of agents solve the same problem just as well as individuals do. The frequency of dispersion outcomes in the fixed-pairing precise information treatment is comparable to both self-pairing treatments. However, the dispersion outcomes were different. Consistent with theory, players essentially coordinated by both players choosing left or right of odd in the fixed-pairing precise information treatment. In the self-pairing treatments, the majority of players picked the least costly way to coordinate, selecting the odd and not odd sectors.

NOTES

- ¹ The issue of coordination via dispersion is extensively studied; other examples include Rapoport, Lo and Zwick [2002] and Zwick, Rapoport and Lo [2002] in which agents must disperse across several “locations” where the probability of success is inversely related to the number of agents at a location. Ochs [1999] is another example of spatial coordination in a market entry game.
- ² As well as other structures that incorporate relative position, temporal order, size, brightness, modularity, etc.
- ³ For the sectors to be identical, it is important that the orientation of the disc is not common to both players. This can be achieved by spinning the disc before presenting it to each player pair. Furthermore, we wish to eliminate asymmetries arising from a directional structure on the disc (clockwise vs. counter-clockwise). This can be achieved by having agents in a match choose from opposite sides of the disc, which is presented to each player before each choice.
- ⁴ This type of optimal learning has been analyzed by Crawford and Haller [1990] and Blume [2000]; other applications of this idea can be found in Alpern and Reyniers [2002], Bhaskar [2000] and Kramarz [1996].
- ⁵ First introduced into the literature by cognitive and language psychologist Karl Bühler [1907, 1908] as Aha-Erlebnis; literally described by the situation in which one encounters a difficult foreign thought, hesitates and then suddenly attains the insight. Köhler [1925] studied the aha-experience experimentally with chimpanzees and Weber [2003] is an application applied to human psychology.
- ⁶ For the sectors to be identical, it is important that the orientation of the disc is not common to both players. This is achieved by spinning the disc before presenting it to each player. Furthermore, we wish to eliminate asymmetries arising from a directional structure on the disc (clockwise versus counter-clockwise and up versus down). This can be achieved by having agents in a match choose from opposite sides of the disc, or by randomizing over the side which is presented to each player before each choice. The randomizing scheme is more powerful in preserving symmetries, but for our purposes the opposite-side scheme suffices.
- ⁷ Instructions are available from the authors upon request.
- ⁸ To appreciate the difference between a coarse- and fine-language player, note that there can be two unchosen sectors when players match by selecting the odd sector or there can be one unchosen sector when players achieve a dispersion outcome by not selecting the odd sector.



⁹ Others have studied differences in cognition in games; however, in the games considered players interests are not necessarily perfectly aligned. Nagel [1995] studied players' ability to reason through iterative dominance in the guessing game; Stahl and Wilson [1995] and Costa-Gomes, Crawford and Broseta [2001] studied players' varying abilities in dominance solvable games and games with unique equilibria.

¹⁰ Choices in period one are *de facto* random. Thus, period two is the first period in which to observe player choices.

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