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Location Choice by Households and Polluting Firms: An Evolutionary Approach*

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Abstract

This paper examines several policy regimes to deal with the problem that households suffer from environmental damage by firms in the same region. We employ an evolutionary framework to analyze migration movements in the course of time, since firms and households will not relocate immediately in response to payoff differentials. We show that taxation gives firms and households an incentive to stay away from each other. Laissez faire (compensation) only gives households (firms) an incentive to stay away from firms (households). We find that taxation creates the right incentives to reach a local welfare maximum. However, compensation may lead to a better outcome than taxation.

Keywords: environmental policy, location choice, evolutionary game theory
JEL classification: Q28; R20; R30

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1 Introduction

Environmental problems occur on different geographical scales: local, national, transboundary and even global. Depending on the nature of the problem, there may be several ways of dealing with it: reducing emissions, taking protective measures to limit the damage, or moving polluters and victims away from each other. Our paper focuses on the latter remedy. Needless to say, we do not wish to suggest that segregation is the only or the preferable answer to all environmental problems, but in some cases where economic activities cause local damage to other activities (like agriculture or recreation) or to human health, segregation may be effective.

We study the equilibria that will occur when both polluters and victims of pollution (which we call firms and households respectively) can choose their location in a two-region model. We examine whether laissez faire, taxation of pollution or compensation of damage to the victims is most likely to lead to the optimal outcome.

One may wonder why we wish to compare taxation with laissez faire and compensation at all. After all, it is well known that marginal damage taxation is efficient. On the other hand, one of the first things we learn in environmental economics is that laissez faire leads to too much pollution. Compensation might also be regarded with suspicion, as it gives households no incentive to stay away from polluting firms. However, we shall see that when we look at location choice, laissez faire and compensation are no longer necessarily inferior to taxation. This is because all welfare optima and all stable policy equilibria are corner solutions with at least partial segregation. For instance, the optimum may be to have all households in the larger region A and all firms in region B. This optimum, which we will call (h, f), can be sustained with taxation. Under laissez faire, region A becomes more attractive to firms, whereas under compensation region B becomes more attractive to households. But there are cases in which (h, f) can be sustained with laissez faire and/or compensation as well. In those cases, these instruments clearly perform as well as taxation.

In this paper, we will show that compensation can actually outperform taxation. This is because the existence of corner solutions leads to multiplicity of solutions. In the above example, where there is a welfare optimum with all households in A and all firms in B, there may be another local optimum (with lower welfare) with all firms in A and all households in B. For certain initial location patterns, compensation will lead to the global optimum while taxation only leads to the suboptimal local optimum.

Two qualifications must be borne in mind. First, the superiority of compensation derives from our assumption that households have a higher demand for land than firms. When firms have a higher demand for land, it would be laissez faire rather than compensation that would improve upon taxation. Second, the model does not allow for side payments across the two sectors, i.e., we do not explore the issue of Coasean bargaining (Coase [3]).

In order to examine the dynamics and the stability of equilibria, we employ evolutionary game theory as applied in economics by Friedman [6, 7], Young [24], Kandori et al. [11] and Samuelson [18]. We regard an evolutionary approach as quite suitable for location decisions. When there is a payoff differential between regions, firms and households do not immediately move to the higher-payoff region. There will rather be a stream of migration in the course of time. For example, firms that recently made location-specific investments would have relatively low incentives to move. Likewise for households; they can be attached to their region of residence because of the close vicinity of their family and friends, and jobs and schools.

The literature on polluter mobility (Motta and Thisse [14], Hoel [9], Dijkstra [5], Rauscher [16, 17]) and victim mobility (Wellisch [23], Hoel and Shapiro [10]) has mainly focused on the question of whether and how the noncooperative policy equilibrium deviates from the optimum. In our paper, however, the regions do not set the policy that maximizes their own welfare, but they follow particular policy rules.

Several authors have previously noted that victim defence activities like migrating introduce nonconvexities, which may result in multiple welfare optima. Baumol [1] was the first to note that when there are multiple local welfare maxima, taxation will lead toward a local optimum, but may lead away from the global optimum. Unlike the present paper, Baumol [1] does not explore the possibility that other instruments may improve upon taxation.

Shibata and Winrich [20] show that when it is (locally) optimal to have only victim defence measures and no abatement by firms, laissez faire also implements the optimum.[1] Helfand and Rubin [8] argue that nonconvexities may lead to the geographic concentration of damage and that a tax may not be able to implement the optimum. However, they only consider uniform taxation, whereas we allow different tax rates in different regions. Moreover, Helfand and Rubin [8] only analyze the case of a unique welfare optimum, whereas we are interested in the existence of multiple local optima.

McKitrick and Collinge [12] derive a condition under which there is a unique local op-

[1] They claim that taxation does not lead to the optimum in this case, but Oates [15] shows that it does.

timum in the presence of nonconvexities. They discuss two strategies for implementation: an iterative tax rule and a non-iterative demand-revealing mechanism. In the same vein, Sandholm [19] determines the conditions under which the iterative tax rule of marginal damage taxation converges globally to the unique optimum under asymmetric information. Like Sandholm [19], we use the iterative tax rule of marginal damage taxation. However, we also examine laissez faire and compensation. Further, unlike McKitrick and Collinge [12] and Sandholm [19], we are mainly concerned with the existence of multiple local optima, and the possibility that the iterative tax rule leads only to a local and not to the global optimum.

Two other contributions on the stability of equilibria with location choice by producers and victims of externalities are Copeland and Taylor [4] and Miyao [13]. Both studies find, as we do, that the interior equilibrium is unstable and the corner equilibria are stable.

Copeland and Taylor [4] consider a two-country two-industry world with free trade and without environmental policy. The "Smokestack" industry adds to the national stock of pollution and the "Farming" industry suffers from pollution.[2] Copeland and Taylor [4] find that the diversified autarky equilibrium is unstable and there may be two stable equilibria with at least partial specialization. When one country needs to be diversified, this should be the country with the lowest labour endowment or the largest regenerative capacity. However, Copeland and Taylor [4] show that the world does not necessarily evolve to the desired specialization pattern.[3]

Miyao [13] considers two populations, the members of which can either live inside or outside a city. An agent's payoff in the city is decreasing in the size of the other group (negative intergroup externalities) or increasing in the size of the own group (positive intragroup externalities). In both cases, a mixed-city Nash equilibrium is unstable and the two Nash equilibria with a single population in the city are stable.

The rest of the paper is organized as follows. Section 2 outlines the model. In Section 3, we determine the Nash and evolutionary equilibria of the three policy regimes. We show that the corner Nash equilibria are stable and the interior Nash equilibria are not. In Section 4, we assess the outcomes of the policy regimes on welfare and we derive our main result that compensation can improve upon taxation. In the concluding Section 5 we offer some remarks on Coasean bargaining.

[2] Unteroberdoerster [21] adds transboundary pollution to Copeland and Taylor's [4] model.

[3] As Copeland and Taylor [4] already note, even when the optimal specialization pattern is achieved, it may not implement the first best. Due to the absence of environmental policy, the Smokestack industry will emit too much in the diversified country.

2 The model

2.1 Introduction

We have seen that there are no contributions in the literature that relate directly to our subject of location decisions by households and polluting firms under different policy regimes. We shall therefore set up our own model in this section, keeping it as simple as possible. As we shall see in subsection 2.3, pollutio is a factor that influences location decisions of some populations under some policy regimes. However, pollution is not always a driving force. We therefore need at least one other factor that influences the location decision. This will be the land market: Households derive utility from the amount of land they occupy and firms need land to produce.

There are two populations: households h and firms f. The number of households is normalized to one. There are n_f firms. For the sake of concreteness, we only analyze the case where $n_f < 1$: there are less firms than households. Further, there are two regions: A and B. Denote by s_k^z the fraction of population k ($k = h, f$) located in region z ($z = A, B$). There is a fixed area of land in either region that can be occupied. The size of region A is normalized to one. The size of region B is $\beta < 1$.

The market for the consumption good is competitive, as is the land market in both regions. The latter implies that the whole area will always be rented out to occupants.

In this section we discuss production by firms and consumption by households (subsection 2.2), pollution and environmental policy (2.3), and the land market (2.4). Finally, we present the expressions for the household's and firm's total payoff (2.5).

2.2 Firms and households

The firms f in region z produce a homogeneous good x with land g_f^z and a firm-specific indivisible factor F_f, with $F_f = 1$. The production technology features constant returns to scale and thus diminishing marginal returns to land. The production x_f^z of x by a firm in region z is then given by:

$$x_f^z = \phi(F_f, g_f^z) \qquad \frac{\partial \phi}{\partial g^z} > 0 \qquad \frac{\partial^2 \phi}{(\partial g^z)^2} < 0. \tag{1}$$

Since good x is chosen as the numeraire, $\phi(F_f, g^z)$ is also the firm's revenue.

Utility from consumption for household H in region z is given by:

$$U_H^z = x_H + m_H u(g_H^z / m_H) \qquad m_H > 0 \qquad u' > 0 \qquad u'' < 0, \tag{2}$$

where x_H is the household's consumption of x and $m_H u(g^z/m_H)$ its utility from land. To simplify the analysis, we assume that the payoff function $u(g^z)$ from land equals the firm's production function $\phi(F_f, g^z)$ of land with $F_f = 1$. Thus, the firm's production function (1) becomes:

$$x_f^z = u(g_f^z). \tag{3}$$

The assumption implies that one firm's demand for land equals the demand by m_H households. We can then scale household size such that one firm's demand for land equals one household's demand for land. Let there be M_H households H with utility from consumption given by (2). Then there are $N_h \equiv M_H/m_H$ households h (with N_h normalized to one) with utility from consumption given by:

$$u_h^z = x_h + u(g_h^z). \tag{4}$$

Household h derives income y_h from the ownership of land and firm shares and possibly from the redistribution of tax revenues or compensation of environmental damage. While all households have identical tastes, they may differ in their incomes. However, we assume that each household has enough income to consume at least some x. Then all households in region z will occupy the same amount of land g_h^z.

2.3 Pollution and environmental policy

Households suffer from pollution by firms in the same region. A household's payoff is separable in environmental damage and consumption of land and the consumption good. Emissions are caused by the firms' use of the fixed production factor F_f. The emission damage that one firm inflicts upon one household is normalized to one. Firms cannot decrease their emissions by reducing output or by taking abatement measures.

We consider three environmental policy regimes ρ:

- Laissez-faire λ: no environmental policy. Households tend to stay away from firms, because they are harmed by the firms' pollution. Firms, on the other hand, have no incentive to stay away from households;

- Taxation τ: firms are taxed for the total damage they inflict upon households.[4] Tax revenues are distributed in lump-sum fashion among households. Households tend to stay away from firms (because of the pollution) and firms tend to stay away from households (because they are taxed for the damage they cause). Thus, households' and firms' incentives are symmetric.

[4]Note that average damage equals marginal damage, so that this tax can be seen as a Pigouvian tax.

- Compensation κ: firms fully compensate households for the damage they inflict upon them. Firms tend to stay away from households, because otherwise they have to compensate households for the damage they cause. On the other hand, households have no incentive to stay away from firms. They are fully compensated for the pollution damage they suffer from firms. Thus, compensation is the mirror image of laissez-faire, with interchanged incentives for households and firms.

2.4 The land market

Recall from subsection 2.3 that a household's payoff is separable in environmental damage and consumption of land and the consumption good. Pollution and environmental policy therefore do not affect the choice of land size g_h^z by household h in region z. The household maximizes its utility (4) under the budget constraint $p^z g_h^z + x_h = y_h$, with p^z the rental price of land in area z and y_h household h's income. The household thus maximizes its consumer surplus Π_h^z from land:

$$\Pi_h^z \equiv \left\{ \max_{g_h^z} u(g_h^z) - p^z g_h^z \right\}. \tag{5}$$

Since firms cannot decrease their emissions by reducing output or by taking abatement measures, pollution and environmental policy do not affect the choice of land size g_f^z by firm f in region z. The firm maximizes its operating profits Π_f^z (revenue minus cost of renting land), which from (3) and the choice of x as the numeraire are given by:

$$\Pi_f^z \equiv \left\{ \max_{g_f^z} u(g_f^z) - p^z g_f^z \right\}. \tag{6}$$

The solution to both the household's (5) and the firm's (6) maximization problem is:

$$p^z = u'(g^z). \tag{7}$$

A firm and a household occupy the same amount of land in region z: $g_f^z = g_h^z = g^z$. In both regions, the total area is distributed equally among all occupants:

$$g^A = \frac{1}{n_f s_f^A + s_h^A} \qquad g^B = \frac{\beta}{n_f(1-s_f^A) + (1-s_h^A)}. \tag{8}$$

Substituting (7) and (8) into (5) and (6), we find for the net payoff from land in region z:

$$\begin{aligned} \Pi_f^A(s_f^A, s_h^A) &= \Pi_h^A(s_f^A, s_h^A) = \Pi^A(s_f^A, s_h^A) = u\left(g^A\right) - \frac{u'\left(g^A\right)}{n_f s_f^A + s_h^A}, \\ \Pi_f^B(s_f^A, s_h^A) &= \Pi_h^B(s_f^A, s_h^A) = \Pi^B(s_f^A, s_h^A) = u\left(g^B\right) - \frac{\beta u'\left(g^B\right)}{n_f(1-s_f^A) + (1-s_h^A)}, \end{aligned} \tag{9}$$

with g^A and g^B given by (8). It is easily seen that the function $\Pi^A(s_f^A, s_h^A)$ is decreasing in both arguments and $\Pi^B(s_f^A, s_h^A)$ is increasing in both arguments.

Table 1: Value of σ_k^ρ in equation (9)

	Laissez faire ($\rho = \lambda$)	Taxation ($\rho = \tau$)	Compensation ($\rho = \kappa$)
Firms ($k = f$)	0	1	1
Households ($k = h$)	1	1	0

2.5 Total payoff

In this subsection we will put together the elements that make up a household or firm's total payoff V when located in region z ($z = A, B$). We have seen in subsection 2.4 that payoff depends on the amount of land g^z the household or firm occupies in region z, with g^z decreasing in the number of firms and households in z (equation (9)). Depending on the policy regime ρ ($\rho = \lambda, \tau, \kappa$), a household or firm may also care about pollution in z, which is increasing in the number of firms or households (respectively) in z (subsection 2.3). Finally, we have assumed that a household's payoff is separable in land, the consumption good and pollution. Therefore, total location-specific payoff V of a firm and a household respectively, can be written as:

$$\begin{aligned} V_f^z(s_f^A, s_h^A, \rho) &= \Pi^z(s_f^A, s_h^A) - \sigma_f^\rho s_h^z, \\ V_h^z(s_f^A, s_h^A, \rho) &= \Pi^z(s_f^A, s_h^A) - \sigma_h^\rho s_f^z n_f, \end{aligned} \tag{10}$$

with Π^z given by (9) and the value of σ_k^ρ given by Table 1.

3 Policy Equilibria

In this section, we determine the Nash and evolutionary equilibria of the three policy regimes. A Nash equilibrium is characterized by market equilibrium: the goods market and the regional land markets are in equilibrium. There is also policy equilibrium: tax and compensation payments equal actual damage. Finally, there is interregional equilibrium: no one has an incentive to move.

It seems reasonable to assume that the market and policy equilibria are more quickly reached than interregional equilibrium. It must be easier to adjust the amount of land occupied and to adjust the required tax or compensation payments, than it is to move from the low-payoff region to the high-payoff region. A Nash equilibrium is only stable if the system returns to it after a small perturbation of the interregional equilibrium.

In order to model this formally, let us define the overall state space as $\mathbf{s} \equiv (s_h \; s_f)'$ with $s_h \equiv (s_h^A, 1-s_h^A)$ and $s_f \equiv (s_f^A, 1-s_f^A)$. We assume that the state evolves deterministically in continuous time according to the differential equation:

$$\dot{\mathbf{s}} \equiv \frac{d\mathbf{s}}{dt} = H(\mathbf{s}, n_f, \beta, \rho). \tag{11}$$

The evolutionary process described by the dynamic H is assumed to be payoff monotonic, implying that fitter strategies crowd out less fit strategies (e.g. Weibull [22]). In our case, monotonicity implies that $\dot{s}_k^A$ should be compatible with $\Delta V_k \equiv V_k^A - V_k^B$ with V_k^z given by (10). If $\Delta V_k > 0$, population k will migrate to region A; if $\Delta V_k < 0$ they will migrate to region B.[5]

Figures 1 to 3 show the phase diagrams for the three policy regimes from which the Nash and evolutionary equilibria can be derived.[6] The lines ff', hh' and dd' are the loci at which either firms or households are indifferent between the two locations. When a population is off its loci, it will move towards it. Formally, the loci and the dynamics as derived from (10) are given by:

- ff' : firms, taking environmental damage into account, are indifferent between A and B. This applies to taxation and compensation.

$$\Pi^A(s_f^A, s_h^A) - s_h^A = \Pi^B(s_f^A, s_h^A) - (1 - s_h^A). \tag{12}$$

  When the LHS is larger (smaller) than the RHS, firms move to A (B).

- hh' : households, taking environmental damage into account, are indifferent between A and B. This applies to laissez faire and taxation.

$$\Pi^A(s_f^A, s_h^A) - n_f s_f^A = \Pi^B(s_f^A, s_h^A) - n_f(1 - s_f^A). \tag{13}$$

  When the LHS is larger (smaller) than the RHS, households move to A (B).

- dd' : the population that only takes density into account, is indifferent between A and B. This applies to firms under laissez faire and to households under compensation. The condition is $\Pi^A(s_f^A, s_h^A) = \Pi^B(s_f^A, s_h^A)$, or substituting (9):

$$n_f(1 - s_f^A) + 1 - s_h^A = \beta\left(n_f s_f^A + s_h^A\right). \tag{14}$$

  When the LHS is larger (smaller) than the RHS, the population moves to A (B).

[5] See subsection 6.1 in the Appendix for the formal compatibility conditions.

[6] The locus dd' is always linear. For simplicity, the lines ff' and hh' are also drawn as linear. The specifications on which the Figures are based are available from the authors upon request.

For each policy regime there are two possibilities: either the loci do not intersect, or they intersect once. These possibilities are shown in the Figures 1 to 3a and b respectively.[7] If the loci do not intersect for a given policy regime, there is one Nash equilibrium, which is in a corner. If the loci intersect, there are two additional Nash equilibria: one in the opposite corner and one interior.

Let us now state this result formally.[8] Qualitatively, there are seven different location configurations, which we shall denote by shorthand notation like, for example, (f, fh). This means: firms in A and firms and households in B. The configurations can be classified into three types. Type FH comprises $(f, h), (fh, h)$ and (f, fh). Type HF comprises $(h, f), (fh, f)$ and (h, fh). The mixed type consists of (fh, fh) only.

Proposition 1 *Under laissez faire, there are three Nash equilibria (one FH, one HF and one mixed), if and only if:*

$$n_f < \beta(2 + n_f). \tag{15}$$

If and only if (15) is not satisfied, there is a unique Nash equilibrium which is FH.

Proposition 2 *Under taxation, there are three Nash equilibria (one FH, one HF and one mixed), if and only if:*

$$\Pi^A\left(1, \frac{1}{2} + \frac{1}{2}n_f\right) - n_f < \Pi^B\left(1, \frac{1}{2} + \frac{1}{2}n_f\right). \tag{16}$$

If and only if (16) is not satisfied, there is a unique Nash equilibrium which is HF.

Proposition 3 *Under compensation, there are three Nash equilibria (one FH, one HF and one mixed), if and only if:*

$$\beta(1 + 2n_f) > 1. \tag{17}$$

If and only if (17) is not satisfied, there is a unique Nash equilibrium which is HF.

As we can see from the phase diagrams, the corner Nash equilibria are evolutionarily stable and the interior Nash equilibria are not:

Proposition 4 *For all policy regimes, a Nash equilibrium is an evolutionary equilibrium if and only if it is a corner Nash equilibrium.*

[7] In these Figures, a black dot indicates an evolutionary equilibrium and a white dot an unstable Nash equilibrium. A square indicates the global welfare optimum and a diamond indicates a local welfare optimum that is not globally optimal.

[8] All proofs are in the Appendix.

Let us now discuss the outcome for laissez faire with the aid of Figure 1. The discussion for taxation and compensation is similar.[9] From Figure 1 it is clear that there is an FH equilibrium (on the top edge) when point h is to the left of point d. At point d, firms are indifferent between A and B. All firms are in A and households are in A and B so that density is equal. At point d, households prefer the clean region B. For households to be indifferent, there should be more households in B and less in A. Thus, h is always to the left of d, and an FH equilibrium always exists.

There is an HF equilibrium (on the right or bottom edge) under laissez faire when point h' is above or to the right of point d'. When point d' is on the $s_f^A = 0$ edge, applying the above reasoning reveals that point h' is to the right of point d'. In that case, an HF equilibrium exists. When point d' is on the $s_h^A = 1$ edge (as in Figure 1b), all households are in A and firms are in A and B so that density is equal. When there are more (less) firms in A than in B, households prefer B (A) and h' is above (below) d'. Thus, there is not always an HF equilibrium under laissez faire.

It is clear from Figure 1 that there is an interior Nash equilibrium if and only if there is an HF equilibrium. The interior equilibrium is unstable because the hh' curve is flatter than the dd' curve. To understand why hh' is flatter than dd', let us consider what happens when, starting from the interior Nash equilibrium, we move a number of households from A to B. That is, we move to the left from the intersection of hh' and dd'. The question is how many firms we have to move from B to A in order to make households and firms indifferent between A and B again. In other words: how far do we have to move up to reach hh' or dd' again. For firms, it is just a matter of restoring equal density in both regions. For households, however, the movement of firms does not only increase density in A compared to B, but it also increases pollution in A compared to B. Thus less firms need to move from B to A to make households indifferent than to make firms indifferent. In terms of Figure 1, this means that hh' is flatter than dd'.

4 Welfare analysis

4.1 Introduction

In this section, we shall analyze the welfare performance of the three policy regimes. The question is which policy regime leads to the best equilibrium in terms of welfare. In subsection 4.2, we determine the welfare optima. We shall see that the first order

[9] Following the reasoning outlined for laissez faire, it can be shown that f' is always below or to the left of d' and h'. Furthermore, at the point of intersection, ff' is steeper than dd'.

conditions are identical to the loci for taxation. Thus, taxation always results in at least a local welfare optimum. However, as we shall see in subsection 4.3, laissez faire and compensation can also lead to a local welfare optimum in some cases. As long as the population that is in both regions takes environmental damage into account, an evolutionary equilibrium is a local welfare optimum. Finally, in subsection 4.4 we address the question whether another policy regime can lead to a better outcome than taxation. We shall see that compensation may outperform taxation.

4.2 Welfare optima

In this subsection we shall identify the welfare optima. Aggregate welfare is:

$$\begin{aligned} W(s_f^A, s_h^A) &= s_h^A u(g^A) + (1-s_h^A)u(g^B) + n_f\left[s_f^A u(g^A) + (1-s_f^A)u(g^B)\right] \\ &\quad -n_f s_f^A s_h^A - n_f(1-s_f^A)(1-s_h^A). \end{aligned} \tag{18}$$

The first two terms on the RHS of (18) represent household utility of land in regions A and B respectively. The third term is household utility from the consumption good x, which by (4) equals its total production. Production per firm is given by (3). The last two terms reflect household damage from emissions by firms in regions A and B respectively. Differentiating W with respect to s_f^A and s_h^A yields, from (18), (8) and (9):

$$\frac{\partial W}{\partial s_f^A} = n_f\left[\Pi^A(s_f^A, s_h^A) - s_h^A - \Pi^B(s_f^A, s_h^A) + (1-s_h^A)\right] = 0, \tag{19}$$

$$\frac{\partial W}{\partial s_h^A} = \Pi^A(s_f^A, s_h^A) - n_f s_f^A - \Pi^B(s_f^A, s_h^A) + n_f(1-s_f^A)u(g^A) = 0. \tag{20}$$

We see that the first order conditions for s_f^A and s_h^A correspond to the conditions (12) for firms and (13) for households to be indifferent between A and B under taxation. Furthermore, when welfare is increasing in s_f^A (s_h^A), firms (households) move to A under taxation. Our findings from section 3 about the Nash and evolutionary equilibria under taxation can then be translated into welfare terms as follows:

Proposition 5 *1. When there is an interior solution to the first order conditions (19) and (20), it is a saddle point.*

2. There is always a (local) welfare maximum HF.

3. There may be an additional local welfare maximum FH.

The close connection between taxation and welfare can be interpreted as follows. Environmental damage occurs when firms and households are in the same location. Thus, firms and households should have an incentive to stay away from each other. These are exactly the incentives that taxation provides.

When both HF and FH are local welfare maxima, it is best to have the largest group in the largest region. Therefore households should be in A and firms in B:

Proposition 6 *HF is the global welfare maximum.*

4.3 Local assessment of policy regimes

In this subsection, we shall see whether the evolutionary equilibria of the policy regimes correspond to a local welfare maximum. We already know that taxation always results in a local welfare maximum. However, laissez faire and compensation may also lead to a welfare maximum. This will happen when either the two populations are completely separated or the population that is in both regions takes environmental damage into account.

With laissez faire (illustrated in Figure 1), only households take environmental damage into account. Laissez faire always has an equilibrium FH, which is either (f, h) or (fh, h). This equilibrium corresponds to the local (suboptimal) welfare maximum FH, if it exists. There may also be an evolutionary HF equilibrium. When this is (h, f) or (h, fh), it will correspond to the global welfare maximum. When the laissez faire equilibrium is (fh, f), it deviates from the global optimum. This is because firms are in both locations, and they do not take environmental damage into account. Thus s_f^A is higher than optimal.

With compensation (illustrated in Figure 3), only firms take environmental damage into account. Compensation always has an equilibrium HF. This corresponds to the global optimum when the equilibrium is (h, f) or (fh, f). When the compensation equilibrium is (h, fh), it deviates from the global optimum. This is because the households are in both locations, and they do not take environmental damage into account. Compensation may also have an evolutionary FH equilibrium, which is always (fh, h). This equilibrium never corresponds to the local optimum, because the households do not take environmental damage into account.

4.4 Global assessment of policy regimes

We now know that taxation always results, and compensation and laissez faire may result in a welfare maximum. The question we address in this subsection is: Are there cases in which laissez faire or compensation outperforms taxation? This would happen when taxation leads to the suboptimal local welfare maximum FH and the other instrument results in a better HF.

It can be shown that taxation is always better than laissez faire.[10] The interesting comparison is between compensation and taxation. We know from Propositions 2 and 3 that taxation and compensation have either one or two evolutionary equilibria. We can also derive:

Lemma 1 *When there is a single evolutionary equilibrium (EE) under compensation, there can be two EE under taxation. However, when there is a single EE under taxation, there is also a single EE under compensation.*

There are then three cases to discuss. *Case 1* occurs when (16) does not hold. Then taxation and compensation have only one equilibrium. Taxation yields the global welfare maximum. Compensation results in the global welfare maximum when $\beta < n_f$. Thus:

Proposition 7 *When both taxation and compensation have one evolutionary equilibrium, taxation is at least as good as compensation.*

Case 2 occurs when (16) holds and (17) does not. Then there are two evolutionary equilibria with taxation (the global HF and the local FH optimum), whereas there is only one (the global optimum HF) with compensation. This implies that there are initial states that lead to the local optimum FH under taxation, but to HF under compensation. As we know from subsection 4.3, compensation implements the global optimum if households are only located in region A. But even if compensation does not implement the global optimum, welfare in HF under compensation may still be higher than in FH under taxation. In the following, we will assume that this is the case:

Condition 1 *In the HF equilibrium under compensation, welfare is higher than in the FH equilibrium under taxation.*

In that case:

[10] This can be checked by going through the steps detailed below for the comparison between compensation and taxation. Note that the difference between compensation and laissez faire is due to our assumption that there are more households than firms (or more precisely, households have higher aggregate demand for land). If there were more firms than households, laissez faire rather than compensation could outperform taxation.

Proposition 8 *Let there be one evolutionary equilibrium under taxation and two evolutionary equilibria under compensation and let Condition 1 hold. Then compensation is at least as good as taxation.*

Intuitively, the reason why there may be an FH equilibrium under taxation, but not under compensation, is the following. Because points d and f in Figures 2 and 3 are always on the $s_f^A = 1$ axis,[11] the FH equilibrium is (fh, h) under compensation and (f, h) or (fh, h) under taxation. Thus FH is an equilibrium if and only if all firms want to remain in A. With compensation, households are not bothered by the firms' pollution, so that there will be more households in A than with taxation. This makes A a less pleasant place for firms. Thus, the conditions for an FH equilibrium to exist are stricter under compensation.

Finally, *Case 3* occurs when (17) holds. Then both taxation and compensation have two evolutionary equilibria. The question is whether there are initial states that evolve toward FH under taxation and toward HF under compensation.

Combining the relevant Figures 2b and 3b, we obtain Figure 4. The crucial points in this Figure are the interior unstable Nash equilibria c under compensation and t under taxation. The saddle paths through t and c are drawn as Ttt' and Ccc', respectively. All points below (above) the saddle path evolve to the (sub)optimal equilibrium g (w) under the respective policy regimes. In Figure 4, point c is above point t, so that path Ccc' is above path Ttt' in the area around points c and t. If we could prove that in general point c is always above point t and path Ccc' always above path Ttt', all points between Ccc' and Ttt' would lead to the good equilibrium under compensation and the bad equilibrium under taxation. Then compensation would always be at least as good as taxation, and sometimes better. Let us first establish:

Lemma 2 *The mixed equilibrium under compensation features a lower s_h^A and a higher s_f^A than the mixed equilibrium under taxation.*

The proof follows from the proofs of Propositions 2 and 3.[12] Intuitively, the reason is as follows. Both points c and t are on ff', so we only have to show that s_h^A is lower in c.

First, consider point c. To keep households indifferent between the two locations, they should have the same density. Firms are interested in density and compensation

[11]If d or f would be on the $s_h^A = 0$ axis, all households would be in B. Then firms would prefer the low-density, zero-compensation area A. See also subsections 6.3.3 and 6.4.3 for the FH equilibria under taxation and compensation.

[12]Subsections 6.3.2 and 6.4.2.

payments. Since both regions have the same density, they should also have the same level of compensation payment per firm. This means that the number of households should be the same in both regions. Thus, in the mixed equilibrium c under compensation, half the households are in A.

Under taxation, both firms and households take density and pollution into account. In terms of density, one region will be more attractive than the other, and equally so to firms and households. In terms of pollution, this region must be equally unattractive for firms and households. The difference in the number of firms between A and B thus should equal the difference in the number of households. There must be more households and firms in A than in B, since density as well as pollution would be higher in B otherwise. Thus, in the mixed equilibrium t under taxation, more than half of the households are in A.

We now know that saddle path Ccc' is above saddle path Ttt' in the area around c and t. The next question is whether it is possible for Ccc' and Ttt' to intersect. Because we do not want to specify the dynamics too much and undermine the generality of the analysis, we do not know exactly where the saddle paths are. However, with compatibility condition 4 of Section 6.1 in the Appendix, we can say something about the relative location of the two saddle paths.

Compatibility condition 4 says that in any point in Figure 4, a population will move faster to the higher-payoff region under the regime with the higher payoff difference. Since firms have the same payoffs under taxation and compensation, their speed of migration does not depend on the policy regime. Households, on the other hand, look at density only under compensation and at density and pollution under taxation. When $s_f^A < \frac{1}{2}$, there are more firms and there is more pollution in B than in A. Therefore, B becomes less attractive under taxation and households will be less inclined to move there or more inclined to leave from there than under compensation. The opposite holds for $s_f^A > \frac{1}{2}$. That is, in Figure 4, below (above) the line mm', households move slower (faster) from A to B or faster (slower) from B to A under taxation than under compensation.

We shall now determine whether the saddle paths Ccc' and Ttt' can intersect, given compatibility condition 4. Let us first look at the area to the upper right of c and t, where households and firms are moving from A to B. Note that this whole area is above the mm' line, since $s_f^A > \frac{1}{2}$ in point t as we saw in our discussion of why point c is above point t. Under taxation, households are moving faster to B than under compensation, because pollution is lower in B. This means that the compensation paths that cross path tt' are steeper than path tt' itself. Path cc' cannot cross path tt' in this area, because it must go through point c, which is above point t.

Now we move to the area to the lower left of c and t, where households and firms are moving from B to A. Above (below) the line mm', households are moving slower (faster) from B to A under taxation. Therefore, above mm' path Tt is crossed by flatter compensation paths. Thus, path Cc cannot cross path Tt above mm', because it must go through point c, which is above point t. Below mm' however, path Tt is crossed by steeper compensation paths, and path Cc might well cross path Tt here (but only once).

From this we conclude that:

Lemma 3 *Let there be a mixed equilibrium c (t) under compensation (taxation). Either the saddle paths through c and t do not intersect, or they intersect once at point* $\bar{s}$ *with* $\bar{s}_k^A < \frac{1}{2}$ $(k = f, h)$.

Now we can state:

Proposition 9 *Let there be two evolutionary equilibria under taxation and under compensation, so that there is a mixed equilibrium c (t) under compensation (taxation), and let Condition 1 hold.*

1. *If the saddle paths through c and t do not intersect, compensation leads to at least as good an equilibrium as taxation.*

2. *If the saddle paths through c and t intersect, then for all starting points with at least one* $s_k^A \geq \bar{s}_k^A$, *with* $\bar{s}_k^A$ *defined by Lemma 3, compensation leads to at least as good an equilibrium as taxation. However, there are starting points with both* $s_k^A < \bar{s}_k^A$ *for which taxation leads to the better equilibrium.*

Figure 4 illustrates the Proposition. Saddle paths Ccc' and Ttt' intersect once, in point $\bar{s}$ which is below the mm' line. For all points in the areas above vv' and to the right of yy', compensation leads to at least as good an equilibrium as taxation. Both regimes lead to the suboptimal equilibrium w from the area $vwc'c\bar{s}$ above the two saddle paths and to the optimal equilibrium g from the area $y'\bar{s}tt'g$ below the two saddle paths. Starting points in the area $\bar{s}cc'O't't$ between the two saddle paths evolve to the bad equilibrium w under taxation and to the good equilibrium g under compensation. When saddle paths Ccc' and Ttt' cross, as they do in Figure 4, there are also starting points for which taxation leads to the better equilibrium. These are the starting points in the area $OT\bar{s}C$ between the two saddle paths in Figure 4.

Finally, note that while we do not know the exact location of point $\bar{s}$, we know that it is in the area $Omkn$. Thus, at least for the area $mwO'gnk$ (three quarters of the whole area), compensation leads to at least as good an equilibrium as taxation.

5 Conclusion

In this paper we have modelled location choice by households and firms, where households suffer from pollution by firms in the same region. Since households and firms will not move immediately in response to payoff differences, we have used an evolutionary approach. We have studied three policy regimes. Under taxation, firms and households have an incentive to stay away from each other. Under laissez faire, households have an incentive to stay away from firms, but firms do not have an incentive to stay away from households. Finally, under compensation, firms have an incentive to stay away from households, but households do not have an incentive to stay away from firms.

There is a close association between taxation and welfare. This is because damage arises when firms and households are in the same region. Thus, one would want firms to stay away from households and households to stay away from firms. These are exactly the incentives that taxation creates.

Although taxation always leads to a local welfare maximum, this does not imply that taxation is always the best instrument. Laissez faire and compensation may also lead to welfare maxima. This is because the maximum is always a corner solution: at least one population locates in only one region. Only the population that is in both regions should take environmental damage into account. In fact, it is possible that compensation leads to the global welfare maximum, whereas taxation only leads to the local optimum.

The policy implication is that when designing instruments to regulate pollution damage, policy makers should look ahead to make sure the economy does not end up in an inferior equilibrium. This, of course, requires a lot from policy makers: They have to be both interested in and able to determine the long-run effects of their policy. However, if they do not look ahead, they may find themselves one day on the path to the bad equilibrium. At that point, it may be very difficult to turn around.

Building our model, we have had to make some special assumptions that it would be worthwhile to relax. We could allow for firms to reduce their emissions and for households to take other defensive measures than moving away from pollution. Firms would take the optimal abatement measures with taxation and compensation. Households would take the optimal defensive measures with taxation and laissez faire. Compensation still implements the global optimum as long as firms are only located in one region. Also, compensation could still be better than taxation as long as compensation resulted in the optimal segregation pattern and taxation did not.

Other simplifying assumptions include the additive separability of the utility function and the equality of firm and household demand for land. In addition, there are no commuting costs for workers or transportation costs for the consumption good. These assumptions simplify the analysis, but relaxing them would not affect our central result that in some circumstances, compensation (or laissez faire) leads to a better outcome than taxation. Indeed, we would conjecture that in any situation with multiple local welfare optima due to externalities, there are always circumstances under which Pigouvian taxation (or subsidization) of the externality leads to a worse outcome than an appropriately designed alternative instrument.

An interesting further development of the model would be the application of the Coase [3] theorem. According to the Coase theorem, welfare is be maximized as long as property rights have been assigned. The result does not depend on who has the property rights.

In our laissez faire scenario, firms have the right to pollute. With compensation, households have the right to clean air. However, these scenarios did not always lead to the local welfare optimum. This is of course because Coase [3] envisaged that voluntary payments would take place in order to implement the optimum. Under laissez faire, households would have to pay firms to stay away from them. Under compensation, firms would have to pay households. Until now, we have not taken this possibility into account.

One might wonder how voluntary transfers arise and develop in an evolutionary world. It seems plausible that they can only arise in an evolutionary equilibrium, for two reasons. The first is that it facilitates looking forward. In order to understand the use of voluntary transfers, an agent must be able to predict what will happen without transfers, and how transfers can improve upon that. Out of equilibrium, the world is constantly changing. That makes it difficult to look forward. In an evolutionary equilibrium nothing changes, so that conditions are favourable for someone to get the idea that nothing will change in the future without voluntary transfers. Secondly, in an evolutionary equilibrium agents will start to identify with their current residence and start wondering how they can make it a more pleasant place. On the other hand, when an agent has recently moved and observes others moving, she will not feel great attachment to his residence.

6 Appendix

6.1 Compatibility conditions

Define $\mathbf{s} \equiv (s_h\ s_f)'$ with $s_k \equiv (s_k^A, 1 - s_k^A),\ k = f, h$. Compatibility of the dynamic H in (11) with the payoff function ΔV_k^ρ, also called payoff monotonicity, holds if the following four conditions apply:

1. $sign\ H(\mathbf{s}, n_f, \beta, \rho) = sign\ \Delta V_k^\rho(\mathbf{s}, n_f, \beta)$;
2. $|H(\mathbf{s}, n_f, \beta, \rho)| \geq \varepsilon\, |\Delta V_k^\rho(\mathbf{s}, n_f, \beta)|$ for some $\varepsilon > 0$;
3. $\Delta V_k^\rho(0, n_f, \beta) < 0 \Rightarrow H(0, n_f, \beta, \rho) = 0$, $\Delta V_k^\rho(1, n_f, \beta) > 0 \Rightarrow H(1, n_f, \beta, \rho) = 0$;
4. $H(\mathbf{s}, n_f^*, \beta^*, \bar{\rho}) > H(\mathbf{s}, n_f^*, \beta^*, \hat{\rho})$ iff $\Delta V_k^{\bar{\rho}}(\mathbf{s}, n_f^*, \beta^*) > \Delta V_k^{\hat{\rho}}(\mathbf{s}, n_f^*, \beta^*)$ for all $\bar{\rho}$ and $\hat{\rho}$.

The first condition refers to "survival of the fittest"; the choice for a region that yields a higher payoff will grow over time. The second condition is necessary for not letting trajectories leave the state space. The third condition simply states that both regions cannot claim more than 100% of households and firms. Condition 4 says that when everything but the policy regime is equal, a population will move faster to the high-payoff region under the regime with the higher difference in payoffs.

6.2 Proof of Proposition 1

6.2.1 FH equilibrium

The FH equilibrium cannot be (f, fh) because density would be higher in B and firms would want to move to A. The FH equilibrium is (f, h) if households want to stay in B:

$$\Pi^A(1,0) - n_f < \Pi^B(1,0). \tag{21}$$

Firms stay in A, because density is lower there. If (21) does not hold, there is an equilibrium (fh, h) with households indifferent between A and B. As density is higher in B, firms do not want to move there. This implies that there is always an FH equilibrium under laissez faire, and it is unique.

6.2.2 Mixed equilibrium

Firms and households are indifferent between A and B if (14) and (13) hold, respectively. Both equations can only be satisfied simultaneously if $s_f^A = \frac{1}{2}$. Substituting this into (14) and solving for s_h^A:

$$s_h^A = \frac{2 + (1-\beta)\,n_f}{2(1+\beta)}. \tag{22}$$

We see that $s_h^A < 1$ only if condition (15) holds. Obviously, if there is a mixed equilibrium, it is unique, with $s_f^A = \frac{1}{2}$ and s_h^A given by (22).

6.2.3 HF equilibrium

The HF equilibrium is (h, f) if households prefer to stay in A and firms in B, respectively:

$$\Pi^A(0,1) > \Pi^B(0,1) - n_f, \tag{23}$$
$$\Pi^B(0,1) > \Pi^A(0,1). \tag{24}$$

If (23) is not satisfied, there is an equilibrium (h, fh). When households are indifferent between A and B, density must be lower in B, and therefore firms prefer to stay in B. If (24) is not satisfied, there may be an equilibrium (fh, f) where firms are indifferent between A and B and households prefer A, respectively:

$$\Pi^A(s_f^A, 1) - s_f^A n_f > \Pi^B(s_f^A, 1) - (1 - s_f^A) n_f, \tag{25}$$
$$\Pi^A(s_f^A, 1) = \Pi^B(s_f^A, 1). \tag{26}$$

Substituting (26), we see that (25) can only hold for $s_f^A < \frac{1}{2}$. Applying the equal-density condition (26):

$$\beta(s_f^A n_f + 1) = (1 - s_f^A) n_f,$$

this means that condition (15) must hold. Thus, if there is an HF equilibrium, it is unique. We see that condition (15) must hold for an HF as well as for a mixed equilibrium to exist, which is also clear from Figure 1.

6.3 Proof of Proposition 2

6.3.1 HF equilibrium

The HF equilibrium is (h, f) if households will stay in A and firms will stay in B, respectively:

$$\Pi^A(0,1) > \Pi^B(0,1) - n_f, \tag{27}$$
$$\Pi^B(0,1) > \Pi^A(0,1) - 1. \tag{28}$$

If (27) is not satisfied, there will be an equilibrium (h, fh) with households indifferent between A and B:

$$\Pi^A(0, s_h^A) = \Pi^B(0, s_h^A) - n_f. \tag{29}$$

Note that $s_h^A > \frac{1}{2}$, because if $s_h^A \leq \frac{1}{2}$, density would be lower in the larger region A, so that households would prefer the low-density, clean region A.

Firms prefer to stay in B in (h, fh) if:

$$\Pi^A(0, s_h^A) - s_h^A < \Pi^B(0, s_h^A) - (1 - s_h^A). \tag{30}$$

Substituting (29), inequality (30) holds if $s_h^A > \frac{1}{2} - \frac{1}{2}n_f$. Since $s_h^A > \frac{1}{2}$, this inequality is always satisfied. If (28) is not satisfied, there will be an equilibrium (fh, f) with firms indifferent between A and B:

$$\Pi^A(s_f^A, 1) - 1 = \Pi^B(s_f^A, 1). \tag{31}$$

Households prefer to stay in A in (fh, f) if:

$$\Pi^A(s_f^A, 1) - n_f s_f^A > \Pi^B(s_f^A, 1) - n_f(1 - s_f^A).$$

Substituting (31), this inequality becomes $1+n_f > 2n_f s_f^A$, which is always satisfied. Thus, there is always an HF equilibrium with taxation, and it is unique.

6.3.2 Mixed equilibrium

Firms and households are indifferent between A and B if (12) and (13) hold, respectively. Then:

$$n_f(2s_f^A - 1) = 2s_h^A - 1. \tag{32}$$

It is clear from (32) that s_h^A is between s_f^A and $\frac{1}{2}$. However, when $s_f^A \leq s_h^A \leq \frac{1}{2}$, the LHSs of (12) and (13) exceed the RHSs. In this case, A would be the low-density, low-pollution region. Thus, in the mixed equilibrium, $s_f^A > s_h^A > \frac{1}{2}$. Then A is the high-pollution, low-density region. For a mixed equilibrium to exist, firms should prefer to move to B when they are all in A. From (12) and (32) with $s_f^A = 1$, this will happen if and only if condition (16) holds. The mixed equilibrium is defined by (12) and (32). In (12), s_f^A decreasing in s_h^A, while in (32) s_f^A is increasing in s_h^A. Thus, if there is a mixed equilibrium, it must be unique.

6.3.3 FH equilibrium

The FH equilibrium is (f, h) if firms will stay in A and households will stay in B, respectively:

$$\Pi^A(1, 0) > \Pi^B(1, 0) - 1, \tag{33}$$
$$\Pi^B(1, 0) > \Pi^A(1, 0) - n_f. \tag{34}$$

Condition (33) always holds, because density is lower in A by (34). Thus, there is no (f, fh) equilibrium. If (34) does not hold, there may be an (fh, h) equilibrium where firms prefer to stay in A and households are indifferent between A and B respectively:

$$\Pi^A(1, s_h^A) - s_h^A > \Pi^B(1, s_h^A) - (1 - s_h^A), \quad (35)$$

$$\Pi^A(1, s_h^A) - n_f = \Pi^B(1, s_h^A). \quad (36)$$

Substituting (36) into (35), we see that the inequality holds if and only if $s_h^A < \frac{1}{2} + \frac{1}{2}n_f$. Substituting into (36), with Π^A decreasing and Π^B increasing in s_h^A, yields condition (16). Thus, if there is an FH equilibrium, it is unique. We see that condition (16) must hold for an FH as well as for a mixed equilibrium to exist, which is also clear from Figure 2.

6.4 Proof of Proposition 3

6.4.1 HF equilibrium

The HF equilibrium is (h, f) if households will stay in A and firms will stay in B respectively:

$$\Pi^A(0,1) > \Pi^B(0,1), \quad (37)$$

$$\Pi^B(0,1) > \Pi^A(0,1) - 1. \quad (38)$$

If (37) does not hold, there is an equilibrium (h, fh) with households indifferent and thus density equal in A and B:

$$\Pi^A(0, s_h^A) = \Pi^B(0, s_h^A). \quad (39)$$

The solution is:

$$s_h^A = \frac{1 + n_f}{1 + \beta} > \frac{1}{2}.$$

The inequality follows from $n_f < 1$ and $\beta < 1$. Firms prefer to stay in B:

$$\Pi^A(0, s_h^A) - s_h^A < \Pi^B(0, s_h^A) - (1 - s_h^A).$$

Substituting (39), the inequality holds when $s_h^A > \frac{1}{2}$, which is the case. If (38) does not hold, there is an equilibrium (fh, f). Households will remain in A in (fh, f), because density is lower in A as firms are indifferent between A and B. Thus, there is always an HF equilibrium with compensation, and it is unique.

6.4.2 Mixed equilibrium

Households and firms are indifferent between A and B if (13) and (14) hold, respectively. Both equations can only be satisfied simultaneously if $s_h^A = \frac{1}{2}$. Substituting this into (14) and solving for s_f^A:

$$s_f^A = \frac{1 - \beta + 2n_f}{2n_f(1+\beta)}. \tag{40}$$

We see that $s_f^A < 1$ only if condition (17) holds. Obviously, if there is a mixed equilibrium, it is unique, with $s_h^A = \frac{1}{2}$ and s_f^A given by (40).

6.4.3 FH equilibrium

The FH equilibrium cannot be (f, h) or (f, fh), because households would want to go to A where density is lower. There may be an equilibrium (fh, h) where firms prefer to stay in A and households are indifferent between A and B, respectively:

$$\Pi^A(1, s_h^A) - s_h^A \;>\; \Pi^B(1, s_h^A) - (1 - s_h^A), \tag{41}$$

$$\Pi^A(1, s_h^A) \;=\; \Pi^B(1, s_h^A). \tag{42}$$

Substituting (42), we see that (41) can only hold if $s_h^A < \frac{1}{2}$. Substituting this into the equal-density condition (42):

$$\beta(n_f + s_h^A) = 1 - s_h^A,$$

which implies that condition (17) must hold. Thus, if there is an FH equilibrium, it is unique, but there is an FH equilibrium if and only if condition (17) holds. We see that condition (17) must hold for an FH as well as for a mixed equilibrium to exist, which is also clear from Figure 3.

6.5 Proof of Proposition 4

Let us first look at stability of the interior NE. As is clear from Figures 1 to 3, the interior NE is unstable when the loci for firms is steeper than the loci for households.[13] The loci are determined by $\Delta V_k^\rho \equiv V_k^A - V_k^B$ with V_k^ρ given in (10) for $k = f, h$ and regimes ρ of laissez faire λ, taxation τ and compensation κ. The slope of the loci ds_f^A/ds_h^A follows from the total differentiation with respect to s_h^A:

$$\frac{\partial V_k^A}{\partial s_f^A}\frac{ds_f^A}{ds_h^A} + \frac{\partial V_k^A}{\partial s_f^A} = \frac{\partial V_k^B}{\partial s_f^A}\frac{ds_f^A}{ds_h^A} + \frac{\partial V_k^B}{\partial s_f^A},$$

[13]See Chiang [2], 640-3, for a rigorous underpinning.

or:

$$\frac{ds_f^A}{ds_h^A} = -\frac{\partial \Delta V_k^\rho / \partial s_h^A}{\partial \Delta V_k^\rho / \partial s_f^A}.$$

Taking into account that the loci always have negative slope, the loci for firms is steeper than the loci for households in regime ρ when:

$$\delta^\rho \equiv \frac{\partial \Delta V_f^\rho}{\partial s_f^A}\frac{\partial \Delta V_h^\rho}{\partial s_h^A} - \frac{\partial \Delta V_f^\rho}{\partial s_h^A}\frac{\partial \Delta V_h^\rho}{\partial s_f^A} < 0.$$

The payoff differences between the two locations are, from (10):

$$\begin{aligned}
\Delta V_h^\lambda &= \Delta V_h^\tau = \pi(g^A) - \pi(g^B) + n_f(1 - 2s_f^A), \\
\Delta V_f^\tau &= \Delta V_f^\kappa = \pi(g^A) - \pi(g^B) + 1 - 2s_h^A, \\
\Delta V_h^\kappa &= \Delta V_f^\lambda = \pi(g^A) - \pi(g^B),
\end{aligned}$$

where $\pi(g^z) = \Pi^z(s_f^A, s_h^A)$ with $\pi' > 0$. The derivatives of the payoff differences with respect to s_f^A and s_h^A are, using (8):

$$\begin{aligned}
\frac{\partial \Delta V_f^\rho}{\partial s_f^A} &= \frac{\partial \Delta V_h^\kappa}{\partial s_f^A} = -n_f D\pi' < 0, \\
\frac{\partial \Delta V_h^\rho}{\partial s_h^A} &= \frac{\partial \Delta V_f^\lambda}{\partial s_h^A} = -D\pi' < 0, \\
\frac{\partial \Delta V_h^\lambda}{\partial s_f^A} &= \frac{\partial \Delta V_h^\tau}{\partial s_f^A} = -n_f D\pi' - 2n_f < 0, \\
\frac{\partial \Delta V_f^\tau}{\partial s_h^A} &= \frac{\partial \Delta V_f^\kappa}{\partial s_h^A} = -D\pi' - 2 < 0,
\end{aligned} \tag{43}$$

with $D \equiv 1 + 1/\beta$. We then find:

$$\delta^\lambda = \delta^\kappa = -2n_f D\pi' < 0, \qquad \delta^\tau = -4n_f[D\pi' + 4] < 0.$$

This implies that any interior NE is unstable. Let us now look at the stability of NE at the corners and the edges of the state space. It is easily seen that all corner NE (f, h) and (h, f), with complete segregation, are stable. This is because the conditions for a corner NE already stipulate that both populations prefer to stay in their present location. For NE at the edge, with partial segregation, the population that is in one location should prefer to stay there. For the population k that is in both locations, we have to check whether $\partial \Delta V_k^\rho / \partial s_k^A \leq 0$. It is clear from (43) that this is always the case.

6.6 Proof of Lemma 1

When there are two EE under taxation and one under compensation, condition (16) for an FH equilibrium under taxation:

$$\Pi^A\left(1, \frac{1}{2} + \frac{1}{2}n_f\right) - n_f < \Pi^B\left(1, \frac{1}{2} + \frac{1}{2}n_f\right)$$

is satisfied, whereas the condition (17) for an FH equilibrium under compensation is not:

$$\Pi^A\left(1, \frac{1}{2}\right) > \Pi^B\left(1, \frac{1}{2}\right). \tag{44}$$

The LHS of (44) exceeds the LHS of (16), whereas the RHS of (44) is below the RHS of (16). Thus, it is possible for (16) and (44) to be satisfied simultaneously. However, it is not possible for neither (16) nor (44) to be satisfied simultaneously, i.e. for there to be one EE under taxation and two under compensation.

6.7 Proof of Proposition 6

First, we will show that HF is better than (f, h). We shall see that:

$$u(1) + n_f u(\beta/n_f) > u(\beta) + n_f u(1/n_f). \tag{45}$$

The LHS of (45) denotes aggregate welfare in (h, f) and the RHS aggregate welfare in (f, h). Inequality (45) can be rewritten as:

$$u(1) - n_f u(1/n_f) > u(\beta) - n_f u(\beta/n_f).$$

The inequality follows from $\beta < 1$ and

$$\frac{\partial\left[u(g) - n_f u(g/n_f)\right]}{\partial g} = u'(g) - u'(g/n_f) > 0,$$

where the inequality follows from $n_f < 1$ and $u''(g) < 0$.

Secondly, we will see that there are always constellations with higher welfare than (fh, h). Note that with (fh, h), there are n_f firms in A, s_h^A households in A and $(1 - s_h^A)$ households in B. Damage from pollution is $n_f s_h^A$. There are three possibilities:

1. $n_f + s_h^A < 1 - s_h^A$. In this case, density is higher in B. Welfare can then be increased by exchanging the occupants of A and B. Pollution damage will remain the same. As our previous discussion of (45) has shown, aggregate welfare from land will increase, because the large group is brought to the large region.

2. $1 - s_h^A < n_f + s_h^A < 1$. Welfare can be increased by having $(n_f + s_h^A)$ households in A and $(1 - n_f - s_h^A)$ households and all n_f firms in B. Note that density in both regions remains the same. However, pollution damage has decreased from $n_f s_h^A$ to $n_f(1 - n_f - s_h^A)$.

3. $n_f > 1 - s_h^A$. Welfare is higher in an alternative scenario with all households in A, $(n_f - 1 + s_h^A)$ firms in A and $(1 - s_h^A)$ firms in B. Again, density remains unchanged, but pollution damage has decreased by:

$$n_f s_h^A - n_f + 1 - s_h^A = (1 - n_f)(1 - s_h^A) > 0.$$

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7 Key to the graphs

For the Figures, specify $u(g^z) = -\theta/2g^z$, and define $b \equiv 1/\beta$. Then, from (8) and (9):

$$\Pi^A = -\frac{\theta}{g^A} = -\theta(n_f s_f^A + s_h^A) \qquad \Pi^B = -\frac{\theta}{g^B} = -\theta b[n_f(1-s_f^A) + (1-s_h^A)].$$

ff' is, from (12):

$$\theta n_f s_f^A + (\theta+1)s_h^A = b\theta n_f(1-s_f^A) + (\theta+1)(1-s_h^A).$$

hh' is, from (13):

$$(\theta+1)n_f s_f^A + \theta s_h^A = (b\theta+1)\, n_f(1-s_f^A) + b\theta\left(1-s_h^A\right).$$

dd' is, from (14):

$$n_f s_f^A + s_h^A = b\left[n_f(1-s_f^A) + 1 - s_h^A\right].$$

There are three equilibria for laissez faire iff:

$$bn_f < 2 + n_f,$$

for taxation iff:

$$b\theta\,(1-n_f) < \theta + (3\theta+1)\, n_f,$$

and for compensation iff:

$$b < 1 + 2n_f.$$

The following table gives the parameter values to draw Figures 1 to 3, respectively, and the resulting endpoints of the loci.

Fig. 1-3	n_f	θ	b	f	f'	h	h'	d	d'
panel a	0.5	$\frac{5}{6}$	8	$(0.76; 1)$	$(1, 0.4)$	$(0.77; 1)$	$(1; 0.63)$	$(0.83; 1)$	$(1; 0.67)$
panel b	0.8	$\frac{5}{8}$	$\frac{5}{3}$	$(0.42; 1)$	$(0.78; 0)$	$(0; 0.91)$	$(1; 0.34)$	$(0.33; 1)$	$(1; 0.16)$

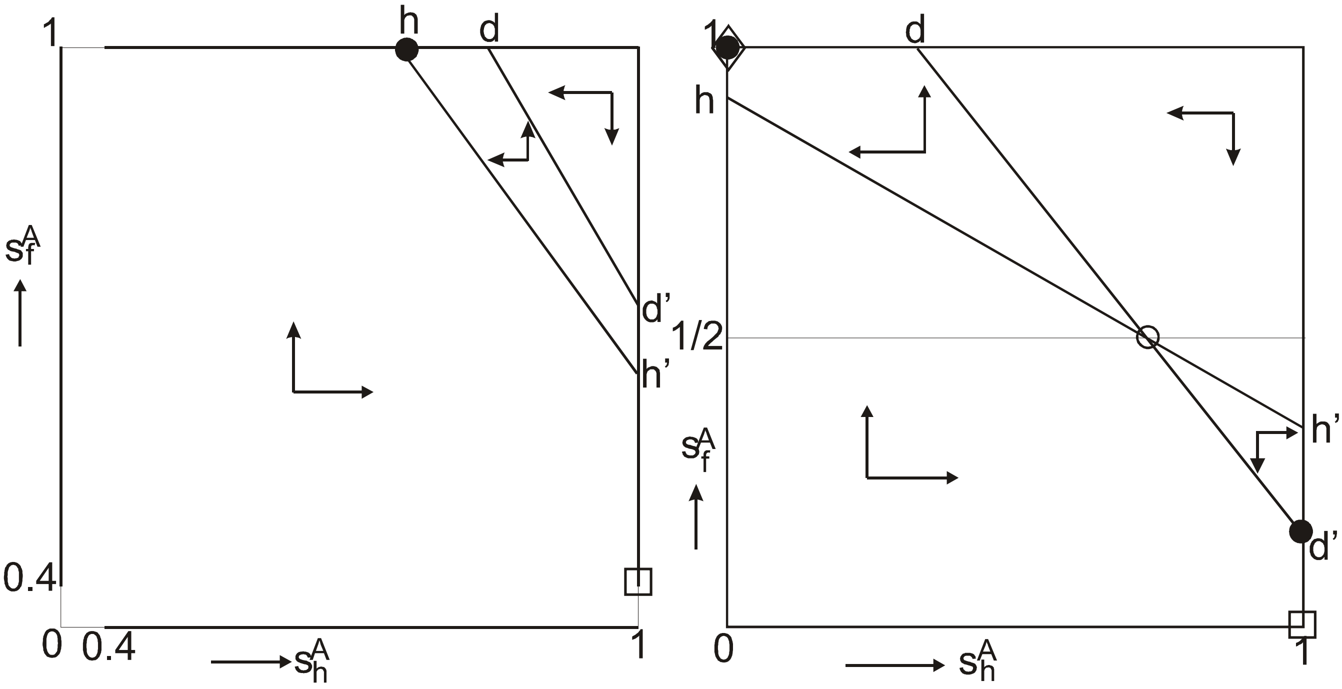

Figure 1: Phase portraits laissez faire. Figure 1a (left): One equilibrium, Figure 1b (right): Multiple equilibria.

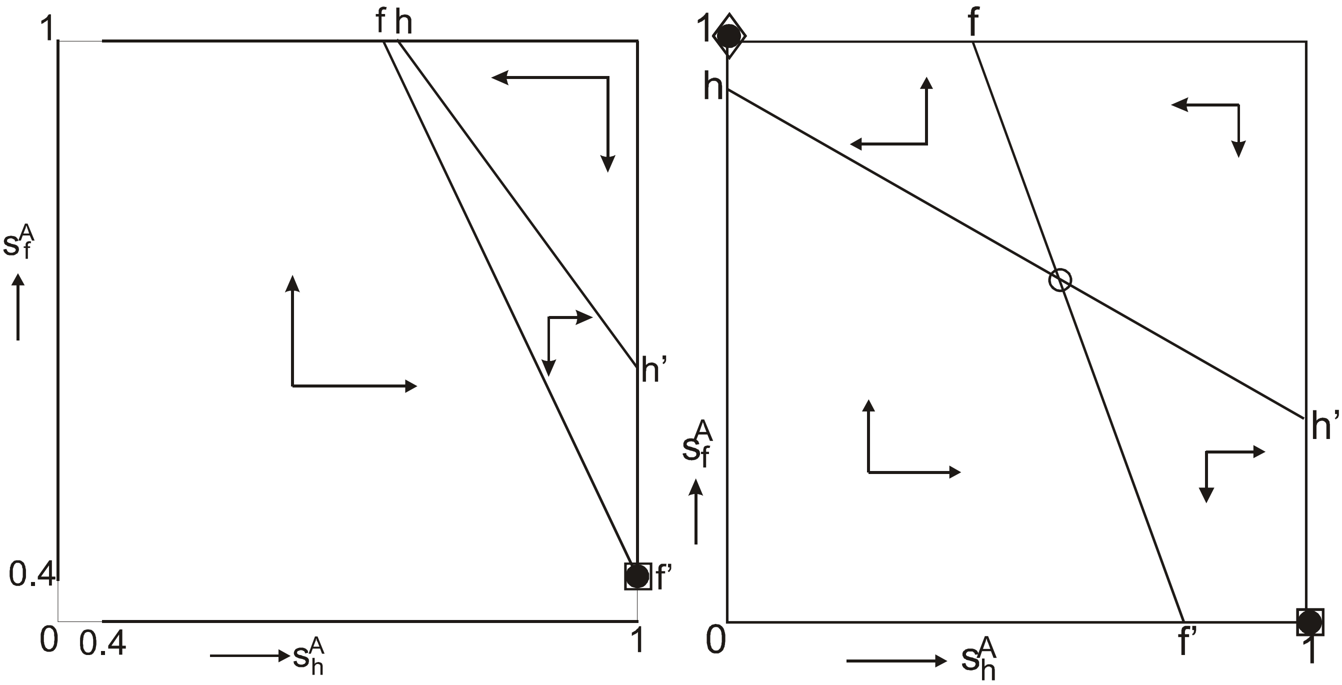

Figure 2: Phase portraits taxation. Figure 2a (left): One equilibrium, Figure 2b (right): Multiple equilibria.

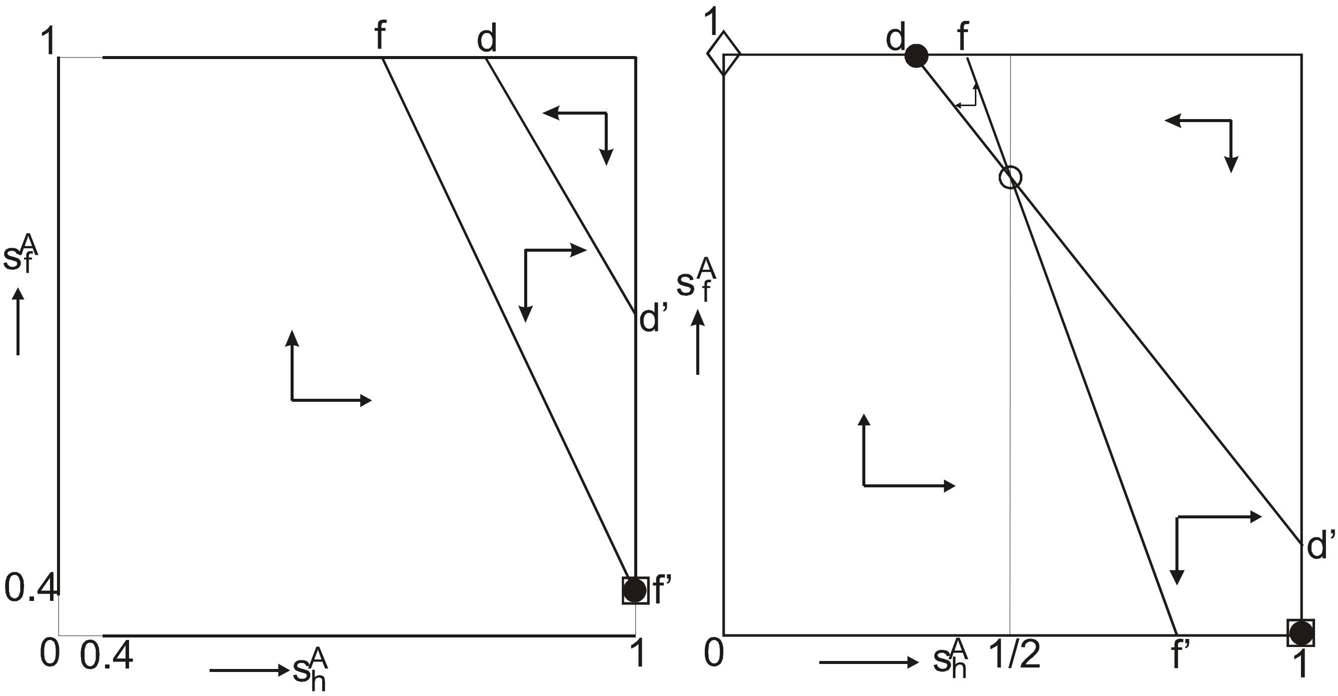

Figure 3: Phase portraits compensation. Figure 3a (left): One equilibrium, Figure 3b (right): Multiple equilibria.

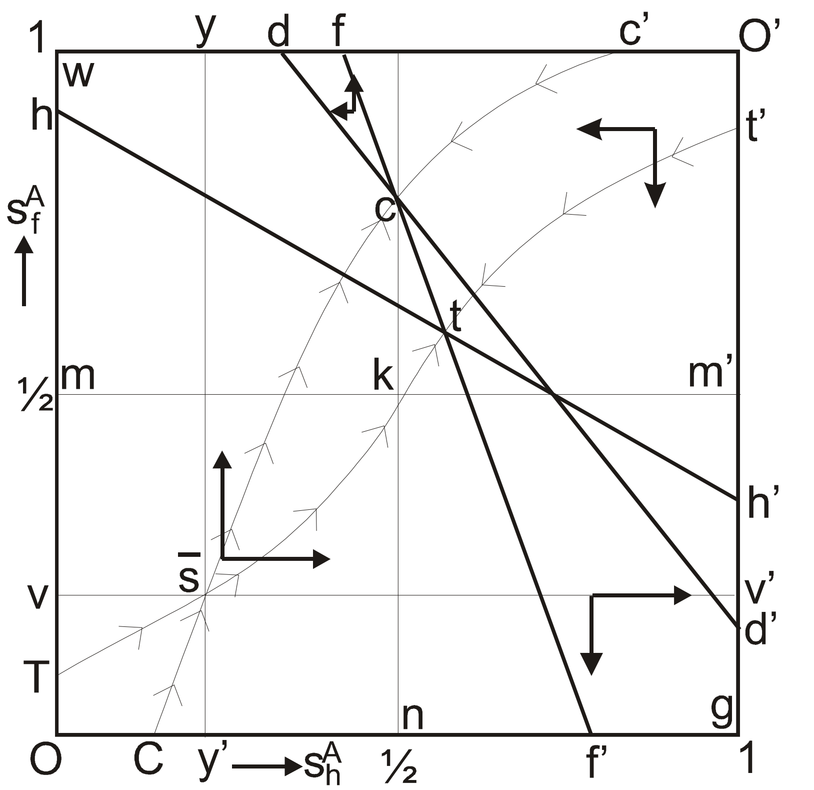

Figure 4: Overall phase portrait