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THE INEFFICIENCY OF THE STOCK MARKET EQUILIBRIUM UNDER MORAL HAZARD

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Inefficiency of the Stock Market Equilibrium with Moral Hazard*

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Abstract

This paper studies the efficiency of a stock market equilibrium. We extend a standard general equilibrium framework with moral hazard (Magill and Quinzii 1999, 2002) to allow for a more general initial ownership distribution of firms. We show that the market allocation is constrained-efficient only when in each firm the entrepreneur who generates payoffs through unobservable effort has the full initial property rights to his firm.

Keywords: stock market equilibrium, moral hazard, general equilibrium, constrained optimality

JEL Classification: D51, D52, D62, D82, G14

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1 Introduction

Can stock markets perform their role of allocating resources efficiently in the presence of moral hazard? Obviously, when production depends on entrepreneurial effort (which is neither verifiable nor contractible), one cannot expect the first best to be achieved. This is because risk-sharing and financing motives require that an entrepreneur sells parts of his firm but in doing this he reduces his incentives to exert effort in the firm, thus creating an inefficiency (Jensen and Meckling, 1976). A more appropriate question is therefore to ask whether a stock market operates efficiently relative to the moral hazard problem (this is the concept of constrained-efficiency dating back to Diamond, 1967), i.e., whether a social planner who cannot create new assets and who cannot observe individual efforts is unable to improve the market allocation.

Several authors have addressed this issue (e.g. Kihlstrom and Mathews, 1990, Kocherlakota, 1998, Lisboa, 2001, Magill and Quinzii, 1999, 2002), showing that a stock market can be indeed (constrained)-efficient. The intuition of this result is that investors are aware of the entrepreneur's moral hazard problem and lower their valuation of the firm in anticipation of a lower effort choice when an entrepreneur sells a stake in his firm. Thus, the inefficiencies stemming from a lower effort choice are internalized by the entrepreneur through the lower price he obtains from selling the firm.

In particular, Magill and Quinzii (1999, 2002) prove the (constrained)-efficiency of the stock market in a standard general equilibrium framework with moral hazard under the assumption that investors correctly infer an entrepreneur's effort from his (observable) financing decisions and that an entrepreneur cannot influence equilibrium state prices (i.e., price perceptions are *rational* and *competitive*).

In their analysis, entrepreneurs are all full owners of their firm before trading. In this paper we show that full initial ownership is in fact a necessary condition to obtain constrained efficiency. Intuitively, the reason for this result is a simple externality: an entrepreneur does not internalize the inefficiencies imposed on other initial owners when selling his firm.

The condition of full initial ownership before trade may not be fulfilled in practice for several reasons. First, venture capitalists typically own a stake in the venture at the time this goes public. Furthermore, stock markets offer several opportunities for an entrepreneur to sell shares of his firm: besides at the IPO, an entrepreneur can sell through secondary market trading or with seasoned offerings. Hence, even though he may be the full owner of the firm at the time of going public, he may not be so at each time he trades. Moreover, there may be several owners with a stake in the firm to begin with (e.g., several entrepreneurs or managers) that trade their stakes in the firm independently. Thus, under quite plausible circumstances, our results imply that a stockmarket will not operate efficiently.

The remainder of the paper is organized as follows. In the next section we define a stock market equilibrium in a general equilibrium model under moral hazard by extending the framework of Magill and Quinzii (1999) and (2002, Example 1) (for brevity, MQ from now on) for a generalized initial ownership in firms. Subsequently, we show that the market equilibrium is (constrained)-inefficient whenever there is not full initial inside ownership.

2 Stock Market Equilibrium with Moral Hazard

A one-good production economy runs for two periods.[1] There are two types of agents in the economy: entrepreneurs and investors. The set of entrepreneurs and investors is denoted by I_1 and I_2, respectively. Both sets are assumed to be non-empty and finite. $I = I_1 \cup I_2$ is the set of all agents. Each agent $i \in I$ has an initial wealth $\omega_0^i > 0$ at $t = 0$. If agent i is an entrepreneur, then he can obtain an (uncertain) income stream at $t = 1$ by investing

[1] The exposition in this section is condensed. For a more detailled discussion of the framework and the issues involved we refer to MQ.

an amount of capital $z^i \in \mathbb{R}_+$ and exercising effort $e^i \in \mathbb{R}_+$. The income stream is

$$F_s^i(z^i, e^i) = f^i(z^i, e^i)\eta_s^i$$

where $s = 1, ..., S$ denotes the state of nature and $\boldsymbol{\eta}^i = (\eta_1^i, ..., \eta_S^i) \in \mathbb{R}_+^S$ characterizes the risk structure of the firm. When agent i is an investor we set $\boldsymbol{\eta}^i = \mathbf{0}$. Each agent has a separable utility function

$$U^i(\mathbf{x}^i, e^i) = u_0^i(x_0^i) + u_1^i(x_1^i, ..., x_S^i) - c^i(e^i)$$

where $\mathbf{x}^i = (x_0^i, x_s^i, ..., x_S^i)$ is the vector of consumption at $t = 0$ and in all states at $t = 1$. For an investor $i \in I_2$, we set $e^i = z^i = 0$ since $F_s^i = 0$.

The economy satisfies the following additional assumptions: u_0^i and u_1^i are differentiable, strictly concave and increasing; c^i is differentiable, convex and increasing, with $c^i(0) = 0$. These functions satisfy the boundary conditions

$$u_0^{i\prime}(x_0^i) \longrightarrow \infty \text{ if } x_0^i \longrightarrow 0; \text{ for all } s = 1, ..., S\text{: } \partial u_1^i/\partial x_s^i \longrightarrow \infty \text{ if } x_s^i \longrightarrow 0; \text{ and } c^{i\prime}(0) = 0$$

i.e., consumption is essential in all states and effort is essentially costless for small levels of effort. $f^i(z^i, e^i)$ is assumed to be differentiable and increasing in (z^i, e^i) and concave in z^i, with $f^i(0, e^i) = f^i(z^i, 0) = 0$ (both inputs are essential).

Entrepreneurs can sell claims to their firms on a stock market (but their effort is still required for production). There is further a single bond traded, which is riskless because we assume that the penalty for bankruptcy is infinite. We deviate from MQ by allowing for a more general initial ownership structure that does not require that an entrepreneur has full ownership in his firm. More formally, we denote an entrepreneur's i initial ownership in his own firm by $\theta_{0,i}^i$ ($0 < \theta_{0,i}^i \leq 1$) and agent j's, $j \neq i$, ownership in the firm by $\theta_{0,i}^j$ ($\theta_{0,i}^j \geq 0$). Feasibility requires that $\sum_{k \in I} \theta_{0,i}^k = 1$ for all $i \in I$.

The agents' choices and their timing are as follows. At $t = 0$, an agent i decides on the amount of capital to invest in his firm z^i, the amount to borrow b^i, the share of his own firm to sell $(\theta^i_{0,i} - \theta^i_i)$, and shares in other firms $k \neq i$ to buy $(\theta^i_k - \theta^i_{0,k})$. It is assumed that firms' income streams $\boldsymbol{\eta}^i \in \mathbb{R}^S_+$ are linearly independent to ensure that entrepreneurs cannot replicate the income stream of their firm by trading other firms.[2] Denote with q_0 the price of the bond and with Q_i the price of firm i (the price for full ownership of firm i). The accountability of agent i requires the following budget equations to be satisfied

$$\begin{aligned} x^i_0 &= \omega^i_0 + q_0 b^i - \sum_{k \neq i} (\theta^i_k - \theta^i_{0,k}) Q_k + (\theta^i_{0,i} - \theta^i_i) Q_i - z^i & (1) \\ x^i_s &= -b^i + \sum_{k \neq i} \theta^i_k f^k(z^k, e^k) \eta^k_s + \theta^i_i f^i(z^i, e^i) \eta^i_s, \ \ s = 1, ..., S & (2) \end{aligned}$$

After the financial decisions have been made, the agent chooses his optimal effort if he is an entrepreneur. Since effort is unobservable, the effort choice has only an impact on the agent's income at $t = 1$ (through a change in the output of his firm). Hence, optimal effort solves

$$\max_{e^i \geq 0} \{ u^i_1(x^i_1, ..., x^i_S) - c^i(e^i) \} \tag{E}$$

Under some regularity conditions the optimal effort choice is unique and differentiable, and exercising effort is always worthwhile (see assumption MCMP and Proposition 1 in Magill and Quinzii, 1999). Because of the uniqueness result we refer to the solution to (E) by $\widetilde{e} = \widetilde{e}(b^i, z^i, \boldsymbol{\theta}^i)$.

All financial decisions are assumed to be mutually observable and agents' preferences are common knowledge. Hence, although effort is not observable, investors can infer from entrepreneur's i financing decisions $(b^i, z^i, \boldsymbol{\theta}^i)$ the (unique) effort that he will exercise $\widetilde{e}^i(b^i, z^i, \boldsymbol{\theta}^i)$. Furthermore, we denote with q^i the price of firm i's unit income stream $\boldsymbol{\eta}^i$. Competitiveness of agents implies that they take q^i as given.[3] Thus, for a given financial portfolio

[2] In MQ, this is ensured by ruling out short sales. This is not sufficient here since entrepreneurs can have positive endowments in other firms.

[3] Competitive behavior is consistent with our model assumptions: because of the multiplicative structure of the production technology ($\mathbf{F}^i = f^i(z^i, e^i)\boldsymbol{\eta}^i$), the unit income stream generated by firm i cannot be changed by varying the production plan (z^i, e^i).

$(b^i, z^i, \boldsymbol{\theta}^i)$, rational investors will price firm i at

$$Q_i = q_i f(z^i, \widetilde{e}^i(b^i, z^i, \boldsymbol{\theta}^i)) \tag{3}$$

which is in turn the price an entrepreneur i expects to receive if he chooses $(b^i, z^i, \boldsymbol{\theta}^i)$ (price perceptions are both competitive and rational). This leads to the following definition of a stock market equilibrium.

Definition 1 *A stock market equilibrium in an economy with generalized ownership* $\boldsymbol{\theta}_0$ *consists of actions* $(\overline{\mathbf{x}}, \overline{\mathbf{e}}, \overline{\mathbf{z}}, \overline{\mathbf{b}}, \overline{\boldsymbol{\theta}})$ *and prices* $\overline{q}_0$ *and* $\overline{q}_i$ *($i \in I_1$) such that*

(i) for each agent, $(\overline{\mathbf{x}}^i, \overline{e}^i)$ *maximizes* $U^i(\mathbf{x}^i, e^i)$ *subject to (1), (2), with* Q_i *in (1) given by* $Q_i = \overline{q}_i f(z^i, \widetilde{e}^i(b^i, z^i, \boldsymbol{\theta}^i))$,

(ii) $\sum_{i \in I} \overline{b}^i = 0$, *(iii) for all* $k = 1, ..., I : \sum_{i \in I} \overline{\theta}^i_k = 1$.

3 Inefficiency of the Stock Market Equilibrium

An allocation is constrained Pareto optimal (CPO) if a social planner that has to respect the limited availability of assets (bonds and stocks) and the unobservability of effort cannot improve the allocation. Or, in other words, there are no prices and feasible reallocation of financial assets that lead to a pareto-improvement in the economy. MQ have shown that in an economy where entrepreneurs have full ownership in their firms before trading at the stockmarket, rational and competitive price perceptions induce the entrepreneur to choose an optimal capital structure, and thus guarantee the CPO of the market equilibrium. The following proposition shows that CPO breaks down exactly when there is at least one entrepreneur that has not full initial ownership in his firm (alternatively, we can interpret Proposition 1 as saying that the market equilibrium is CPO only if entrepreneurs are allowed to trade once at the stock market, namely at the first stage of the economy when they have full ownership).

Proposition 2 *The stock market equilibrium is (constrained)-efficient* *if and only if* *there is full initial inside ownership ($\theta^i_{0,i} = 1$ for all $i \in I_1$).*

Proof. *"if" part*: $\theta^i_{0,i} = 1$ for all $i \in I_1$. This is exactly the case of MQ. For the proof we refer to them.

"only if": Let $(\overline{\mathbf{x}}, \overline{\mathbf{e}}, \overline{\mathbf{z}}, \overline{\mathbf{b}}, \overline{\boldsymbol{\theta}})$ with $\overline{q}_0$ and $\overline{q}_i$ $(i \in I_1)$ be an arbitrary stock market equilibrium. We show that there exists a feasible reallocation of assets that is pareto-improving if there exists at least one entrepreneur $i \in I_1$ with $\theta^i_{0,i} < 1$. This proves the necessity of full initial inside ownership for CPO.

Let $\theta^i_{0,i} < 1$ for one $i \in I_1$. Consider the following redistribution of shares of firm i: entrepreneur $i's$ stake in his own firm is increased by an (infinitesimal) small amount $d\theta^i_i$, each entrepreneur's $j \neq i$ stake in firm i is changed by

$$d\theta^j_i = -\overline{\theta}^j_i \frac{\partial f^i(\overline{z}^i, \overline{e}^i)}{\partial e^i} \frac{\partial \widetilde{e}^i(\overline{b}^i, \overline{z}^i, \overline{\boldsymbol{\theta}}^i)}{\partial \theta^i_i} \frac{1}{f^i(\overline{z}^i, \overline{e}^i)} d\theta^i_i \tag{4}$$

and an arbitrary investor $k's$ stake is changed by $d\theta^k_i = -d\theta^i_i - \sum_{j \neq i} d\theta^j_i$. All other financial decisions $(\mathbf{z}, \mathbf{b}, \boldsymbol{\theta}_{\neq i})$ and prices q_0 and q_i, $i \in I_1$, are not changed. For brevity, we suppress the functions' arguments from now on whenever the function is evaluated at the original market equilibrium value.

Since $d\theta^i_i + \sum_{j \in I_1 \setminus \{i\}} d\theta^j_i + d\theta^k_i = 0$, the reallocation is feasible. Furthermore, the reallocation does not change the effort choice of an entrepreneur $j \neq i$. We show this by demonstrating that if all effort choices e^k, $k \neq i, j$ do not change, the f.o.c.'s for effort e^j is still fulfilled. From the uniqueness of the effort choice follows then that $de^j = 0$ for $j \neq i$. The f.o.c. for the effort choice e^j (from 2 and E) is

$$\overline{\theta}^j_j \sum_{s \in S} \frac{\partial u^j_1(x^j_s)}{\partial x^j_s} \frac{\partial f^j(\overline{z}^j, e^j)}{\partial e^j} \eta^j_s - c^{j\prime}(e^j) = 0 \tag{5}$$

Thus, if the reallocation does not affect income x^j_s at $t = 1$ for all $s = 1, ...S$, (5) is still fulfilled. From (2) and (4) we have under the assumption that all effort choices e_k, $k \neq i$, do not change that

$$dx_s^j(e^j) = \overline{\theta}_i^j \frac{\partial f^i}{\partial e^i}\frac{\partial \widetilde{e}^i}{\partial \theta_i^i}\eta_s^i d\theta_i^i + f^i \eta_s^i d\theta_i^j$$
$$= \overline{\theta}_i^j \frac{\partial f^i}{\partial e^i}\frac{\partial \widetilde{e}^i}{\partial \theta_i^i} d\theta_i^i \eta_s^i - f^i \eta_s^i \overline{\theta}_i^j \frac{\partial f^i}{\partial e^i}\frac{\partial \widetilde{e}^i}{\partial \theta_i^i}\frac{1}{f^i} d\theta_i^i = 0 \tag{6}$$

i.e., the impact of higher effort by i on $j's$ income at $t = 1$ (first term) is exactly offset by a reduction in $j's$ stake in firm i (second term). Hence, for all $j \neq i$, the f.o.c. for effort is fulfilled and thus $de^j = 0$ for all $j \neq i$.

Next we show that the reallocation is pareto-improving. We know that the f.o.c. for the choice of the optimal amount of shares in firm i for entrepreneur i and an investor m must be fulfilled at the market equilibrium. From (1), (2) and (3) these are

$$\theta_i^{i*} : \begin{array}{l} u'(\overline{x}_0^i)[-\overline{q}_i f^i + (\theta_{0,i}^i - \overline{\theta}_i^i)\overline{q}_i \frac{\partial f^i}{\partial e^i}\frac{\partial \widetilde{e}^i}{\partial \theta_i^i}] + \\ \sum_{s\in S} \frac{\partial u_1^i(\overline{x}_s^i)}{\partial x_s^i}[f^i \eta_s^i] + \left(\sum_{s\in S} \frac{\partial u_1^i(\overline{x}_s^i)}{\partial x_s^i}\overline{\theta}_i^i \frac{\partial f^i}{\partial e^i}\eta_s^i - c^{i\prime}(\overline{e}^i)\right)\frac{\partial \widetilde{e}^i}{\partial \theta_i^i} = 0 \end{array} \tag{7}$$

$$\theta_i^{m*} : u'(\overline{x}_0^m)[-\overline{q}_i f^i] + \sum_{s\in S} \frac{\partial u_1^m(\overline{x}_s^m)}{\partial x_s^m}[f^i \eta_s^i] = 0 \tag{8}$$

Now we can compute the utility change for an entrepreneur $j \neq i$. Since $dx_s^j = 0$ and $de^k = 0$ for $k \neq i$, we have

$$dU^j = u'(\overline{x}_0^j)[-(\overline{\theta}_i^j - \theta_{0,i}^j)\overline{q}^i \frac{\partial f^i}{\partial e^i}\frac{\partial \widetilde{e}^i}{\partial \theta_i^i} d\theta_i^i - \overline{q}^i f^i d\theta_i^j]$$
$$= u'(\overline{x}_0^j)\theta_{0,i}^j \overline{q}^i \frac{\partial f^i}{\partial e^i}\frac{\partial \widetilde{e}^i}{\partial \theta_i^i} d\theta_i^i \geq 0 \tag{9}$$

where the last line is obtained by using (4) to substitute $d\theta_i^j$. For an investor k we get with (8)

$$dU^k = u'(\overline{x}_0^k)[(\theta_{0,i}^k - \overline{\theta}_i^k)\overline{q}^i \frac{\partial f^i}{\partial e^i}\frac{\partial \widetilde{e}^i}{\partial \theta_i^i} d\theta_i^i - d\theta_i^k \overline{q}^i f^i] + \sum_{s\in S} \frac{\partial u_1^k(\overline{x}_s^k)}{\partial x_s^k}[\overline{\theta}_i^k \frac{\partial f^i}{\partial e^i}\frac{\partial \widetilde{e}^i}{\partial \theta_i^i} d\theta_i^i \eta_s^i + d\theta_i^k f^i \eta_s^i]$$
$$= u'(\overline{x}_0^k)\theta_{0,i}^k \overline{q}^i \frac{\partial f^i}{\partial e^i}\frac{\partial \widetilde{e}^i}{\partial \theta_i^i} d\theta_i^i \geq 0 \tag{10}$$

and for any investor $l \neq k$ we have

$$dU^l = u'(\overline{x}_0^l)[(\theta_{0,i}^l - \overline{\theta}_i^l)\overline{q}^i \frac{\partial f^i}{\partial e^i}\frac{\partial \widetilde{e}^i}{\partial \theta_i^i} d\theta_i^i] + \sum_{s\in S} \frac{\partial u_1^l(\overline{x}_s^l)}{\partial x_s^l}[\overline{\theta}_i^l \frac{\partial f^i}{\partial e^i}\frac{\partial \widetilde{e}^i}{\partial \theta_i^i} d\theta_i^i \eta_s^i]$$
$$= u'(\overline{x}_0^l)\theta_{0,i}^l \overline{q}^i \frac{\partial f^i}{\partial e^i}\frac{\partial \widetilde{e}^i}{\partial \theta_i^i} d\theta_i^i \geq 0 \tag{11}$$

Finally, for entrepreneur i we have

$$dU^i = u'(\overline{x}_0^i)[(\theta_{0,i}^i - \overline{\theta}_i^i)\overline{q}^i \frac{\partial f^i}{\partial e^i}\frac{\partial \widetilde{e}^i}{\partial \theta_i^i} d\theta_i^i - \overline{q}^i f^i d\theta_i^i] +$$

$$\sum_{s \in S} \frac{\partial u_1^i\left(\overline{x}_s^i\right)}{\partial x_s^i}[\overline{\theta}_i^i \frac{\partial f^i}{\partial e^i}\frac{\partial \widetilde{e}^i}{\partial \theta_i^i} d\theta_i^i \eta_s^i + f^i \eta_s^i d\theta_i^i - c^{i'}(\overline{e}^i)\frac{\partial \widetilde{e}^i}{\partial \theta_i^i} = 0$$

because of (7). Hence, no agent is worse off and because of $\theta_{0,i}^i < 1$ there exists at least one agent $m \neq i$ with $\theta_{0,i}^m > 0$, who is strictly better off by (9), (10) or (11). △

The intuition for the existence of a Pareto-improving reallocation if there is initial outside ownership is straightforward. Since the entrepreneur does not internalize the inefficiency losses on initial outside owners when selling his firm, an increase in his stake in the firm reduces this externality and makes all initial outside owners better off while having only a second order impact on all other agents.

4 Conclusions

In this paper we show that the contrained-efficiency of decentralized stock markets breaks down when entrepreneurs do not have full ownership in their firms before market trading. Hence, a stock market generally does not fulfill its function of allocating resources optimally across agents. The question arises of how efficiency can be ensured. Government intervention, through appropriate taxes, may help to restore efficiency (see, for example, Geanakoplos and Polemarchakis, 2003). Alternatively, retaining the decentralization of the economy, other organizational forms, like the competitive pools of Dubey and Geanakoplos (2002) can possibly overcome the externalities posed by entrepreneurs on initial owners. One specific way to eliminate the externality shown in this paper would be to use contracts that limit the possibilities for entrepreneurs to divest their stake in the firm through the stock market without the previous agreement of the initial owners (as analyzed in Wagner, 2002).

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