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Abstract

As becomes apparent from the standard text books in industrial organization (cf. Tirole, 1988, The Theory of Industrial Organization), the analysis of the effects of uncertainty within this field is yet underdeveloped. This paper shows that the new theory of strategic real options can be used to fill this "empty hole". Based on the work by Smets (1991) standard models are identified, and they are analyzed by applying a method involving symmetric mixed strategies. As an illustration, extensions regarding asymmetry, technology adoption and decreasing uncertainty over time are reviewed. Among others, it is found that the value of a high cost firm can increase in its own cost. Furthermore, it is established to what extent investments are delayed when technologial progress is anticipated, and it is found that competition can be bad for welfare.

1 Introduction

The main difference between financial options and real options is that in most cases real options are not exclusive. Exercising a given option by one party results in the termination of corresponding options held by other parties. For example, an option to open an outlet in an attractive location is alive only until a competitive firm opens its own store there.

However, as it is now the real option theory mainly considers single decision maker problems of firms operating in monopoly or perfect competition markets. But capital budgeting decisions can be strongly influenced by existing as well as potential competitors. The creation of the European Union and the growing internationalization has increased interdependencies among firms in European industries. Former domestic market leaders now have to deal with competition. The conclusion is that there is a strong need to consider a situation where several firms have the option to invest in the same project. This brand new topic requires a merge between game theory and real options.

At present, only a few contributions deal with the effects of strategic interactions on the option value of waiting associated with investments under uncertainty (see Grenadier (2000) for a survey). One of the main reasons is that the application of game theory to continuous-time models is not well developed and often quite tricky. However, due to the importance of studying the topic of investment under uncertainty in an oligopolistic setting, it can be expected that more publications will appear in the immediate future.

This paper provides an overview of the state of the art, where we mainly concentrate on identical firms in a duopoly context. We begin by discussing two standard models. One model is a new market model (Dixit/Pindyck (1996)) and the other one considers a framework where the firms can enlarge an existing profit flow (Smets (1991)). Since firms are identical it seems natural to consider symmetric strategies. However, it can be expected that coordination problems arise in situations where investment is optimal only if the other firm refrains from doing so. While discussing the standard models we apply an approach which shows that imposing mixed strategies can deal with this coordination problem in an economically meaningful way. This approach, being inspired by the deterministic analysis in Fudenberg/Tirole (1985), was developed in Huisman (2001) (see also Huisman/Kort (2003)) and formalized in Thijssen/Huisman/Kort (2002b). A similar attempt can be found in Boyer/Lasserre/Mariotti/Moreaux (2001). We show that Smets (1991) and Dixit/Pindyck (1996) conclude wrongly when they say that, in case it is only optimal for one firm to invest, joint investment never occurs.

As an illustration of the applicability of this framework we proceed by reviewing some of our own work. The first model incorporates asymmetry in the investment cost function. Among other things, a surprising result is that the value of the high cost firm can increase in its own investment cost. In the second model firms take into account the occurrence of future technologies when deciding about investment. A scenario is identified where this results in a game with second mover advantages. Finally, the third model extends the existing real option literature by studying a framework where over time information arrives in the form of signals. This information reduces uncertainty. In analyzing a new market model it is found that the mode of the game depends on the first mover advantage relative to the value of information free riding of the second mover.

In Section 2 we present the basic models, while in Section 3 some recent literature is reviewed that makes use of this framework. Section 4 concludes.

2 Standard Models

The first paper dealing with a multiple decision maker model in a real option context is Smets (1991). He considers an international duopoly where both firms can increase their revenue stream by investing. Like in Fudenberg/Tirole (1985) two equilibria arise: a preemption equilibrium, where one of the firms invests early, and a simultaneous one, where both firms delay their investment considerably. A simplified version was discussed in Dixit/Pindyck (1996) in the sense that the firms are not active before the investment is undertaken. The resulting new market model only has the preemption equilibrium. In this section our symmetric mixed strategy approach is applied to both models. Section 2.1 treats the new market model (Dixit/Pindyck (1996), for a more thorough analysis see Thijssen/Huisman/Kort (2002b)), and the Smets (1991)-model is discussed in Section 2.2 (see Huisman (2001) for a complete analysis).

2.1 New Market Model

This model considers an investment project with sunk costs $I > 0$. After the investment is made the firm can produce one unit of product at any point in time. Since the number of firms is two, market supply is $Q \in \{0, 1, 2\}$. It is assumed that the firms are risk neutral, value maximizing, discount with constant rate r, and variable costs of production are absent. The market demand curve is subject to shocks that follow a geometric Brownian motion process. In particular, it is assumed that the unit output price is given by

$$P_t = Y_t D(Q), \tag{1}$$

in which

$$dY_t = \mu Y_t dt + \sigma Y_t d\omega, \tag{2}$$

$$Y_0 = y, \tag{3}$$

where $y > 0$, $0 < \mu < r$, $\sigma > 0$, and the $d\omega$'s are independently and identically distributed according to a normal distribution with mean zero and variance dt. Furthermore, $D(Q)$ is a decreasing function, comprising the non-stochastic part of the inverse demand curve.

Given the stochastic process $(Y_t)_{t \geq 0}$ we can define the payoff functions for the firms. If there is a firm that invests first while the other firm does not, the firm that moves first is called the leader. When it invests at time t its discounted profit stream is given by $L(Y_t)$. The other firm is called the follower. When the leader invests at time t the optimal investment strategy of the follower leads to a discounted profit stream $F(Y_t)$. If both firms invest simultaneously at time t, the discounted profit stream for both firms is given by $M(Y_t)$. In Figure 1 the three value functions are plotted. If the leader invests at $Y < Y_F$, the follower's value is maximized when the follower invests at Y_F. The follower's profit flow will be $YD(2)$. Following familiar steps (cf. Dixit and Pindyck (1996)), we can find Y_F. It satisfies

$$Y_F = \frac{\beta_1}{\beta_1 - 1} \frac{[r - \mu] I}{D(2)}, \tag{4}$$

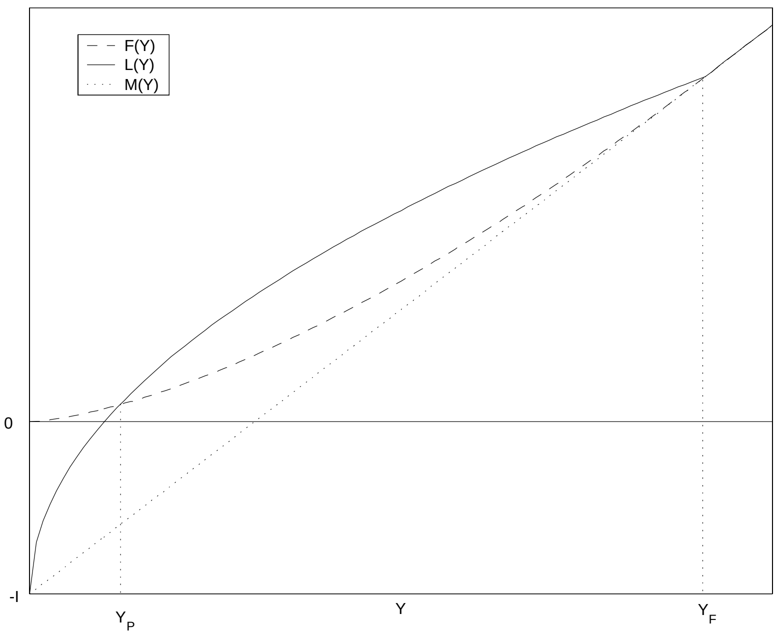

Figure 1: Value functions in the standard new market model.

where β_1 is given by

$$\beta_1 = \frac{\frac{1}{2}\sigma^2 - \mu + \sqrt{\left[\frac{1}{2}\sigma^2 - \mu\right]^2 + 2\sigma^2 r}}{\sigma^2} > 1.$$

Since firms are identical there seems to be no reason why one of these firms should be given the leader role beforehand. The fact that firms are rational and identical also implies that it is hard to establish coordination on a non-symmetric equilibrium. Therefore, we concentrate on equilibria that are supported by symmetric strategies. We use the subgame perfect equilibrium concept for timing games as formalized in Thijssen/Huisman/Kort (2002b). This approach extends the perfect equilibrium concept of Fudenberg/Tirole (1985) to stochastic games. It is argued there that in continuous time preemption games a closed loop strategy of Firm i consists of a collection of simple strategies $\left(G_i^\tau\left(.\right), \alpha_i^\tau\left(.\right)\right)$. $G_i^\tau\left(t\right)$ is the probability that Firm i has invested in between $[\tau, t)$. The function $\alpha_i^\tau\left(t\right)$ is the measure of the intensity of atoms in the interval $[t, t+dt]$. In other words it can be said that G determines when something happens, while α determines what exactly happens in case of a coordination problem.

To describe the equilibrium, first define the preemption point

$$Y_P = \min_Y \{Y \mid L(Y) = F(Y)\},$$

see also Figure 1. This point is called preemption point because to the right of this point the leader value, $L(Y_t)$, exceeds the follower value, $F(Y_t)$, and this results in strategic behavior of the firms trying to preempt each other with investing as will become apparent from the description below. The equilibrium under consideration is therefore called a preemption equilibrium.

From Thijssen/Huisman/Kort (2002b) it is obtained that the equilibrium strategy of Firm i equals

$$G_i^\tau(t) = \begin{cases} 0 & \text{if} \quad Y_t < Y_P \\ \frac{L(Y_t)-M(Y_t)}{L(Y_t)-2M(Y_t)+F(Y_t)} & \text{if} \quad Y_P \leq Y_t < Y_F \\ 1 & \text{if} \quad Y_t \geq Y_F \end{cases}, \tag{5}$$

and

$$\alpha_i^\tau(t) = \begin{cases} 0 & \text{if} \quad Y_t < Y_P \\ \frac{L(Y_t)-F(Y_t)}{L(Y_t)-M(Y_t)} & \text{if} \quad Y_P \leq Y_t < Y_F \\ 1 & \text{if} \quad Y_t \geq Y_F \end{cases}. \tag{6}$$

From this result it is clear that three regions have to be distinguished. The first region is defined by $Y_t \geq Y_F$. According to equation (6) the outcome exhibits immediate joint investment. Here the unit output price is large enough for both firms to enter the market.

In the second region it holds that $Y_P \leq Y_t < Y_F$. Immediate joint investment gives a payoff $M(Y_t)$. this is not a Nash equilibrium since if one of the firms deviates by waiting with investment until $Y_t = Y_F$, it obtains the follower value $F(Y_t)$ which exceeds $M(Y_t)$ as long as $Y_P \leq Y_t < Y_F$, cf. Figure 1.

In case both firms refrain from investment and wait until $Y_t = Y_F$, they get the follower payoff $F(Y_t)$. Again this is not a Nash equilibrium, because if one of the firms deviates by investing this firm receives a payoff $L(Y_t)$ which is more than $F(Y_t)$ on this interval.

Since we restrict ourselves to symmetric strategies the only possibility left is to apply mixed strategies. Denote the probability that Firm i invests by $\alpha_i(t)$. As already said the function $\alpha_i(t)$ is the measure of the intensity of atoms in the interval $(t, t+dt]$. It can be interpreted as the probability that firm i chooses row 1 in the matrix game depicted in Figure 2.

Playing the game costs no time and if Firm i chooses row 2 and Firm j column 2 the game is repeated. If necessary the game will be repeated infinitely often.

Since $\alpha_i(\alpha_j)$ is the probability that Firm $i(j)$ invests, they are the control variables of both firms that need to be optimally determined. To do so, define V_i as the value of Firm i, which is given by

$$V_i = \max_{\alpha_i} [\alpha_i [1-\alpha_j] L(Y) + [1-\alpha_i] \alpha_j F(Y) + \alpha_i \alpha_j M(Y) + [1-\alpha_i][1-\alpha_j] V_i]. \tag{7}$$

	Invest	Not invest
Invest	$\left(M(Y_t), M(Y_t)\right)$	$\left(L(Y_t), F(Y_t)\right)$
Not invest	$\left(F(Y_t), L(Y_t)\right)$	repeat game

Figure 2: Matrix game.

Since Firm i invests with probability α_i and Firm j with probability α_j, the probability that Firm i obtains the leader role, and thus receives $L(Y)$, is $\alpha_i [1-\alpha_j]$. Similarly, with probability $[1-\alpha_i]\,\alpha_j$ Firm i is the follower, $\alpha_i\alpha_j$ is the joint investment probability, and with probability $[1-\alpha_i][1-\alpha_j]$ nothing happens and the game is repeated. After writing down the first order conditions for Firm i and Firm j, and imposing symmetric strategies, i.e. $\alpha_i = \alpha_j = \alpha$, it is obtained that

$$\alpha = \frac{L(Y) - F(Y)}{L(Y) - M(Y)}. \tag{8}$$

From Figure 1 we learn that $M(Y) < F(Y) \leq L(Y)$ on the relevant Y-interval $[Y_P, Y_F)$, so that we are sure that the probability α lies between zero and one. From (8) it is obtained that, given the difference $L(Y) - M(Y)$, the firm is more eager to invest when the difference between the payoffs associated with investing first and second is large.

After substitution of $\alpha = \alpha_i = \alpha_j$ into (7), the value of Firm i can be expressed as

$$V_i = \frac{\alpha[1-\alpha]\,L(Y) + [1-\alpha]\,\alpha F(Y) + \alpha^2 M(Y)}{2\alpha - \alpha^2}. \tag{9}$$

Of course, both firms do not want to invest at the same time, because it leaves them with the lowest possible payoff $M(Y)$. From (9) it can be derived that the probability of occurrence of such a mistake is

$$\frac{\alpha}{2-\alpha}, \tag{10}$$

which naturally increases with α. We also see that, whenever $\alpha > 0$ which is in fact the case for $Y \in (Y_P, Y_F)$, the probability that the firms invest simultaneously is strictly positive. This is not in accordance with many contributions in the literature. For instance, Smets (1991, p. 12) and Dixit/Pindyck (1996), p. 313) state that "if both players move simultaneously, each of them becomes leader with probability one half and follower with probability one half".

Similarly, it can be obtained that the probability of a firm being the first investor equals

$$\frac{1-\alpha}{2-\alpha}. \tag{11}$$

Due to symmetry this is also the probability of ending up being the follower. Since the probability of simultaneous investment increases with α, it follows that the probability of being the first investor decreases with α, which is at first sight a strange result. But it is not that unexpected, because if one firm increases its probability to invest, the other firm does the same. This results in a higher probability of investing jointly, which leaves less room for the equal probabilities of being the first investor.

In the third region it holds that $Y_t < Y_P$. From Figure 1 it can be concluded that the follower value exceeds the leader value. Hence, investing first is not optimal so that both firms refrain from investing and wait until $Y = Y_P$. Then the second region is entered, and it can be obtained from (8) and the fact that $L(Y_P) = F(Y_P)$, that $\alpha = 0$. From (11) we get that the probability for a firm to become leader is one half, and with the same probability this firm will be the second investor. Furthermore, from (10) it can be concluded that the probability of simultaneous investment at Y_P is zero. All this implies that one of the firms will invest at Y_P and the other one, being the follower, will wait with investment until Y equals Y_F. Since the values of leader and follower are equal at Y_P, the firms have equal preferences of becoming the first or the second investor in this case. This is called rent equalization.

2.2 Existing Market Model

Contrary to the previous section, here two identical firms are already active on the market. They have the possibility to make an irreversible investment which results in a higher output price. A possible interpretation is that both firms have the possibility to adopt a new technology which after adoption increases the quality of the firm's product. The resulting model is similar to the one of the previous section with the exception that expression (1) is replaced by

$$P_t = Y_t D_{N_\mathrm{i} N_\mathrm{j}},$$

where, for $k \in \{i, j\}$:

$$N_k = \begin{cases} 0 & \text{if firm } k \text{ has not invested,} \\ 1 & \text{if firm } k \text{ has invested.} \end{cases}$$

Keeping in mind that (i) the investment increases the unit output price and (ii) the demand for the firm's product is higher if the competitor still produces the old quality products (thus not having invested (yet)), the following restrictions on $D_{N_\mathrm{i} N_\mathrm{j}}$ are implied:

$$D_{10} > D_{11} > D_{00} > D_{01}. \tag{12}$$

Further we assume that there is a first mover advantage to investment:

$$D_{10} - D_{11} > D_{11} - D_{01}.$$

As expected, the resulting equilibria of this game also depend here on the payoffs of the leader (L), the follower (F) and immediate joint investment (M), but, in addition to the analysis of the previous section, the equilibria also depend on the optimal joint investment payoff, which we denote by J. In the latter case the firms invest at a threshold level

$$Y_J = \frac{\beta_1}{\beta_1 - 1} \frac{[r-\mu] I}{D_{11} - D_{00}}. \tag{13}$$

When firms invest simultaneously they increase their profit flow from YD_{00} to YD_{11}. For the follower it holds that investing changes the profit flow from YD_{01} to YD_{11}. Consequently, the follower threshold is

$$Y_F = \frac{\beta_1}{\beta_1 - 1} \frac{[r-\mu] I}{D_{11} - D_{01}}. \tag{14}$$

Since $D_{01} < D_{00}$ (cf. (12)), before the investment takes place the follower's profits are lower than those of the simultaneous investors. Therefore, for the follower the incentive to invest is greater which explains why $Y_F < Y_J$.

It is important to note that if in the new market model the firms decide to invest simultaneously, their optimal threshold will be the same as the one of the follower. Thus it equals Y_F, as defined by (4). This is the case because for the follower as well as for simultaneous investment it holds that a profit flow of zero is replaced by a profit flow of $YD(2)$. Consequently, in the new market model the follower payoff curve coincides with the payoff curve of optimal simultaneous investment, and for this reason the latter plays no role in the determination of the new market equilibrium.

If we again choose for symmetric strategies two cases can be distinguished in the existing market model. Depending on whether or not the optimal joint investment curve lies above the leader curve on the interval $[Y_P, Y_F)$, one of them will occur. In the first case the leader curve lies above the optimal joint investment curve for some $Y \in [Y_P, Y_F)$, see Figure 2. Here the equilibrium strategy of Firm i is also given by (5) and (6). For a description of this preemption equilibrium we therefore refer to the previous section.

In the second case the optimal joint investment curve lies above the leader curve on the interval $[Y_P, Y_F)$, as can be seen in Figure 3. Besides the still existing preemption equilibrium, there exists a continuum of simultaneous investment equilibria from which simultaneous investment at $Y = Y_J$ Pareto dominates all other equilibria including the preemption equilibrium. The Pareto dominant equilibrium is given by

$$G_i^\tau(t) = \begin{cases} 0 & \text{if } Y_t \le Y_J \\ 1 & \text{if } Y_t > Y_J \end{cases},$$

and

$$\alpha_i^\tau(t) = \begin{cases} 0 & \text{if } Y_t \le Y_J \\ 1 & \text{if } Y_t > Y_J \end{cases}.$$

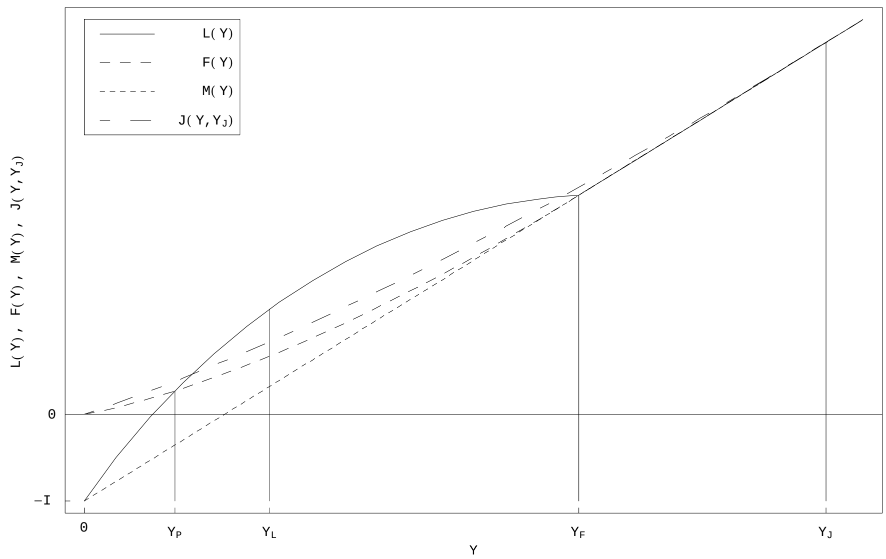

Figure 3: First Case: Preemption equilibrium in the standard existing market model.

In this Pareto dominant equilibrium the firms tacitly collude by refraining from investment until Y becomes that large that it equals Y_J, which is beneficial to both of them. Therefore, in Boyer/Lasserre/Mariotti/Moreaux (2001) this equilibrium is called a tacit collusion equilibrium. Note that in Figure 3 this simultaneous equilibrium does not exist for Y sufficiently small, since at the moment that Y is such that $L(Y) > J(Y)$, Firm i can gain by deviating in the form of investing immediately.

Thijssen (2003) shows that the Pareto dominant equilibrium is also risk dominant, which makes selection of the Pareto dominant equilibrium more likely than selection of the preemption equilibrium.

Now the question remains under which scenario which case occurs. In Huisman (2001) it is proved that, no matter the degree of uncertainty, the equilibrium is always of the preemption type if D_{10} is large enough, i.e. if the incentives to become leader are large enough.

3 Extensions

This section treats three direct extensions to the standard models of Section 2. In Section 3.1 we incorporate some asymmetry in the sense that one of the firms can invest in a cheaper way than the other one (see Pawlina/Kort

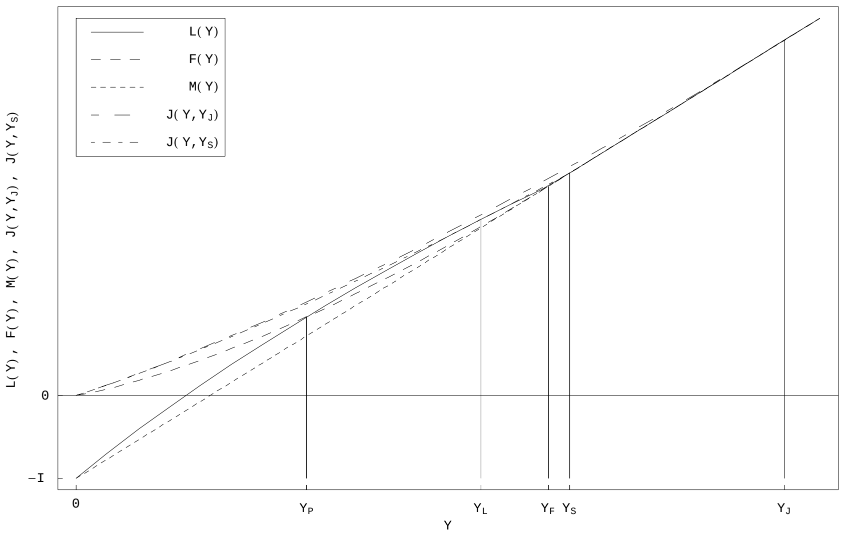

Figure 4: Second Case: Tacit collusion equilibrium in the standard existing market model.

(2001a) for a more thorough analysis). Section 3.2 considers firm investment behavior in a scenario where with some probability a better technology will become available in the future. This technology adoption problem is analyzed in depth in Huisman/Kort (2000). Finally in Section 3.3, which summarizes Thijssen (2002), another type of uncertainty is considered, namely uncertainty that reduces because of information that becomes available over time.

3.1 Asymmetric Firms

The asymmetric model is a direct extension to the standard existing market model presented in Section 2.2 (see Huisman (2001) for incorporating asymmetry in a new market model). Also here we analyze the situation where two firms have an opportunity to invest in a profit enhancing investment project, but the difference is that they face different (effective) investment costs. Sources of potential costs asymmetry are, for instance, access to capital markets, organizational flexibility, and regulation.

For the model we can thus refer to Section 2.2 with the exception of the irreversible investment cost. We now have a low cost firm, say Firm 1, having investment cost I, and a high cost Firm 2 with investment cost κI, where $\kappa \in [1, \infty)$.

Contrary to the model of Section 2.2, now there are three types of equilibria that can occur. The first type of equilibrium is the **preemption equilibrium**. It occurs in the situation in which both firms have an incentive to become the leader, i.e. when the cost disadvantage of Firm 2 is relatively small. Therefore, Firm 1 has to take into account the fact that Firm 2 will aim at preempting Firm 1 as soon as a certain threshold is reached. This threshold, denoted by Y_{21}^{P}, is the lowest realization of the process Y_t for which the leader and follower curve of Firm 2 are equal. As a consequence, when the initial value of Y is sufficiently small, Firm 1 invests at

$$\min\left\{Y_{21}^{P}, Y_{1}^{L}\right\},$$

where Y_1^L is Firm 1's optimal leader threshold equal to

$$Y_1^L = \frac{\beta_1}{\beta_1 - 1}\frac{I\,(r-\mu)}{D_{10} - D_{00}}.$$

Firm 2 invests at the follower threshold Y_2^F. The corresponding figure is qualitatively similar to Figure 2.

The second type of equilibrium is the **sequential equilibrium.** This one occurs when Firm 2 has no incentive to become the leader, thus when the follower curve of Firm 2 always lies above the leader curve. Then Firm 1 simply maximizes the value of the investment opportunity, which, provided that the initial level of Y is sufficiently low, always leads to investment at the optimal threshold Y_1^L. In other words, Firm 1 acts as if it has exclusive rights to invest in a profit enhancing project but, of course, Firm 2's investment still affects Firm 1's payoff. As in the previous case, Firm 2 invests at its follower threshold Y_2^F. Figures 5 and 6 illustrate the firms' payoffs associated with the sequential investment equilibrium.

The last type of equilibrium is the **simultaneous equilibrium.** The difference with the simultaneous equilibrium in Section 2.2 is that here the optimal joint investment thresholds differ for the firms. Since the optimal threshold of Firm 1 is lower than that of Firm 2, the firms will jointly invest at that threshold. The corresponding figures are qualitatively similar as Figure 4, to which the reader is referred.

An important question of course is which equilibrium when occurs. It turns out that two elements are crucial: the (relative) first mover advantage, D_{10}/D_{11}, and the investment cost asymmetry, κ. Figure 6 depicts the investment strategies as a function of these two variables. When the investment cost asymmetry is relatively small and there is no significant first mover advantage, the firms invest jointly. When the first mover advantage becomes significant, Firm 1 prefers being the leader to investing simultaneously, which results in the preemption equilibrium. Finally, if the asymmetry between the firms is significant, the firms invest sequentially.

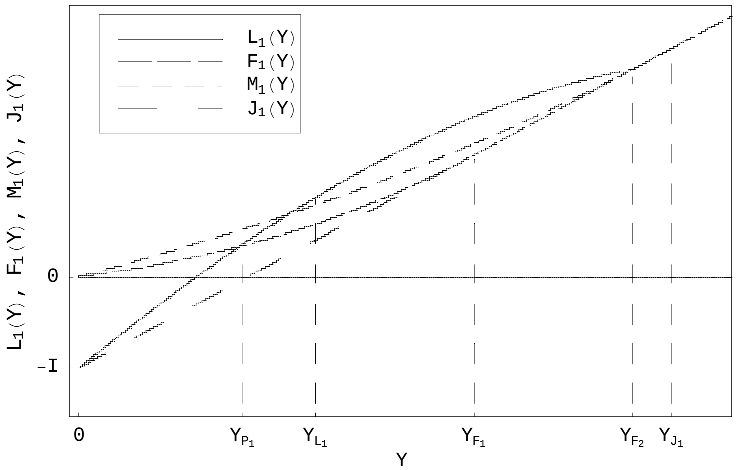

Figure 5: Firm 1's value functions when the resulting equilibrium is of the sequential type.

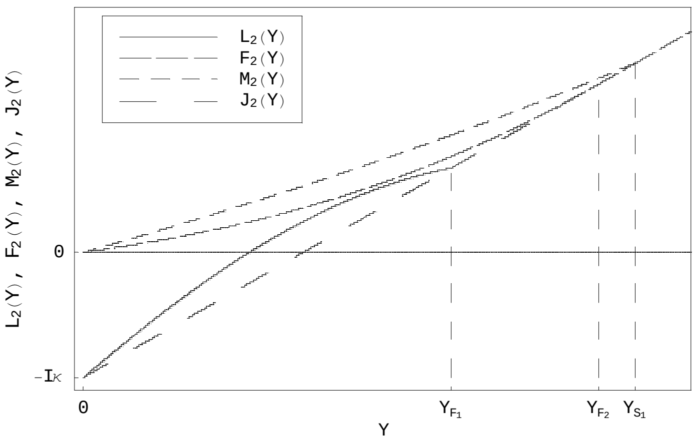

Figure 6: Firm 2's value functions when the resulting equilibrium is of the sequential type.

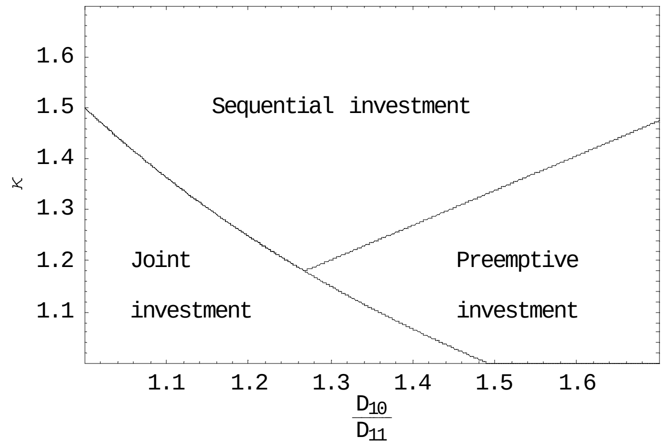

Figure 7: Regions of sequential, preemptive and joint investment equilibria for the set of parameter values: $r = 0.05$, $\mu = 0.015$, $\sigma = 0.1$, $D_{00} = 0.5$, $D_{01} = 0.25$, and $D_{11} = 1$.

Interesting observations can be made if the values of the firms are depicted as a function of the asymmetry parameter κ; see Figure 8. Here the parameter values are chosen in such a way that for different values of the cost asymmetry parameter all three types of equilibria are possible (cf. Figure 7).

The first interesting observation is that in the region where the preemption equilibrium prevails, the value of Firm 2 is increasing in its own investment cost. This surprising result is caused by the fact that increasing κ makes Firm 2 a 'weaker' competitor. This implies that the preemption threat of Firm 2 declines in the investment cost asymmetry, so that Firm 1 will invest later. This is beneficial for the cash flow of Firm 2 since, due to the fact that $D_{00} > D_{10}$, Firm 2 can enjoy a higher cash flow for a longer period. In this case the non-strategic, i.e. increasing investment cost for Firm 2, and strategic effects work in the opposite direction and the latter effect dominates.

The second counterintuitive observation is that at κ^{**} the value of Firm 1 jumps downward if the investment cost of the other firm is increased marginally. The reason is that this increase makes sequential investment for Firm 1 more attractive because of the increasing Firm 2's follower threshold. However, Firm 2 anticipates this and is willing to invest an instant before Firm 1 does. Again, Firm 1 reacts on this and this preemption mechanism leads to a, from the perspective of value maximization, too early investment of Firm 1.

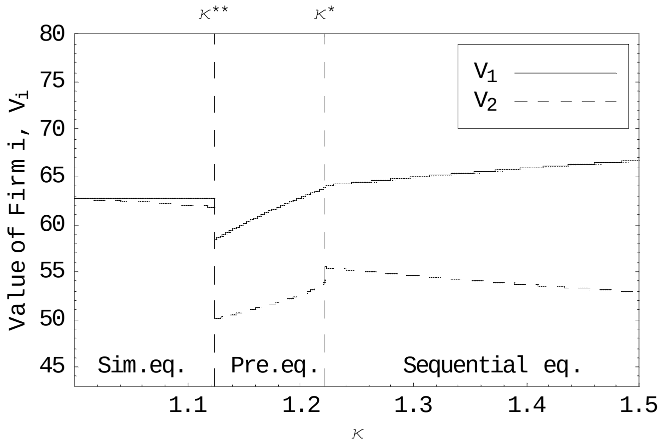

Figure 8: The value of Firm i (V_i) corresponding to the regions of the sequential, preemptive and joint investment equilibria for the set of parameter values: $r = 0.05$, $\mu = 0.015$, $\sigma = 0.1$, $D_{00} = 0.5$, $D_{01} = 0.25$, $D_{10} = 1.33$, $D_{11} = 1$, $I = 100$, and $Y = 4$.

3.2 Technology Adoption

The model extends the new market model of Section 2.1. At the beginning of the game, entering the market means producing with an existing technology 1. However, the decision to invest in technology 1 will be influenced by technological progress. Adopting technology 1 would have been a bad decision if a little later a much better technology becomes available. In the model technological progress is included as follows. At the stochastic time T a new and better technology 2 becomes available for the firms. Time T is distributed according to an exponential distribution with mean $1/\lambda$, so that the arrival of technology 2 follows a Poisson process with parameter λ.

It is assumed that firms can invest only once and that the investment costs of both technologies are equal to I. Concerning the profit flow of Firm i we replace expression (1) by

$$\pi_{it} = Y_t D_{N_i N_j},$$

where N_k denotes the technology that firm k ($\in \{i, j\}$) is using. Hence, $N_k \in \{0, 1, 2\}$, where 0 means that the firm is not active.

We make the following assumptions on the D's. First, a firm makes the highest amount of profits with a given technology if the other firm is not active (monopoly). It also holds that, given its own technology, profits are lowest when the other firm is a strong competitor, thus producing with the efficient technology 2. Second, given the technology of the competitor, the firm's profits are higher when it produces with the modern technology 2. In this way the following inequalities are obtained:

$$\begin{array}{ccc} D_{20} > & D_{21} > & D_{22} \\ \vee & \vee & \vee \\ D_{10} > & D_{11} > & D_{12} \end{array}.$$

As can be expected, the outcome of the game heavily depends on the arrival rate of the new technology. If $\lambda \in [0, \lambda_1^*)$, with

$$\lambda_1^* = \frac{[r-\mu] D_{11}}{D_{21} - D_{11}},$$

then the probability of the arrival of a new technology is that low that the type of the resulting equilibrium is the same as in Section 2.1. Both firms are expected to invest in technology 1. But an increase of λ leads to a higher threshold value. Therefore it can be concluded that the probability that both leader and follower will invest in technology 1 decreases with λ. This is the case since, if it happens that technology 2 arrives before (one of) the firms have invested, they (it) will invest in technology 2 instead.

The "Section 2.1 solution", but then with respect to investing in technology 2, also occurs if the arrival rate is large enough. This holds for $\lambda \in [\lambda_3^*, \infty)$, with

$$\lambda_3^* = \frac{[r-\mu] D_{10}}{D_{22} - D_{12}}.$$

Here the probability that the new technology will become available soon is that high that investing in the current technology is not considered. The firms wait for the arrival of the new technology after which the preemption game of Section 2.1 is played.

For $\lambda \in [\lambda_1^*, \lambda_2^*)$, with

$$\lambda_2^* = \frac{[r-\mu] D_{10}}{D_{21} - D_{12}},$$

the outcome is also a preemption equilibrium, but now the leader will invest in technology 1 and the follower in technology 2. As before the leader's adoption of technology 1 is conditional on technology 2 not arriving before the investment timing of the leader, and the probability that the leader will invest in technology 1 decreases with λ.

The only case left is when $\lambda \in [\lambda_2^*, \lambda_3^*)$. Also here the leader will invest in technology 1 and the follower in technology 2, but the difference with the previous case is that now the arrival rate of the second technology is that high that the value of the follower is higher. The leader has the advantage of monopoly profits until the time that the follower invests in technology 2, but the disadvantage of producing with a less efficient technology after this date. Apparently here this disadvantage outweighs the monopoly profits.

A timing game with a higher payoff of the follower is called a war of attrition. In this particular case it can be shown that there does not exist a symmetric equilibrium. There are two asymmetric equilibria, where in each of them a different firm has the leader role. Here neither firm would like to be the first investor, but if they both keep on waiting, their payoff will be even less than the payoff of the one that decides immediately to invest first.

3.3 Uncertainty Being Reduced over Time

The real option literature mainly considers intrinsic uncertainty that will always exist regardless of the firm having invested or not. This is the kind of uncertainty we dealt with in the paper until now. However, there exists also another kind of uncertainty, which is uncertainty that reduces because of information that becomes available over time. A key feature of the latter kind of uncertainty is that the information is imperfect. For example, consider the introduction of a new communication technology by a telecom firm. There will always be uncertainty about the demand for the new service, depending on e.g. the business cycle, the unemployment rate and so on. On the other hand, there is uncertainty about the level of structural demand for the new service. Due to marketing research the firm gains more insight about structural demand, which decreases uncertainty. Since a marketing survey consists of a sample and not the entire population, the signals that it provides on the profitability of the investment are imperfect.

The model treated in this section deals with the kind of uncertainty that reduces over time due to the arrival of new information. Consider two identical firms that face the choice of investing in a certain project. The project can be either good (denoted by H), leading to high revenues, U_L^H for the leader, $U_F^H < U_L^H$ for the follower or $U_M^H \in \left(U_F^H, U_L^H\right)$ in the case of simultaneous investment, or bad (denoted by L), leading to no revenue. As an example we can think of a duopoly game of quantity competition. Then in case the project is good the leader captures a Stackelberg advantage, and simultaneous investment implies a Cournot-Nash equilibrium. The sunk costs involved in investing in the project are given by I.

After investment has taken place by at least one firm the state of the project becomes immediately known to both firms. Hence, this creates a second mover advantage. If the firms do not invest simultaneously, the follower decides on investing immediately after the true state of the project is revealed.

When the firms receive the option to invest, they have a prior belief about the investment project being good or bad. The *ex ante* probability of high revenues is p_0. Occasionally the firms receive a signal indicating the project to be good (an h-signal) or bad (an l-signal). The probabilities with which these signals occur depend on the true state of the project. To model the imperfectness of signals, it is assumed that a correct signal always occurs with probability $1/2 < \lambda < 1$, see Table 1 (note that the signal is uninformative if $\lambda = 1/2$). In this table the first row (column) lists the probabilities in case of a good project (good signal) and the second row (column) in case of a bad project (bad signal).

	h	l
H	λ	$1-\lambda$
L	$1-\lambda$	λ

Table 1: Probability of a signal indicating a good or bad project, given the true state of the project.

The signals' arrivals are modelled via a Poisson process with parameter μ.Both firms have an identical prior belief $p_0 \in [0,1]$ in the project being good that is common knowledge. Let g and b be the number of h-signals and l-signals, respectively. Then it is shown in Thijssen/Huisman/Kort (2002a) that

$$p = p(g,b) = \frac{\lambda^{g-b}}{\lambda^{g-b} + \frac{1-p_0}{p_0}[1-\lambda]^{g-b}}.$$

At the moment of its investment, the leader's ex ante expected payoff equals

$$L(p) = p\left[U_L^H - I\right] + [1-p][-I] = pU_L^H - I.$$

The follower only invests in case of a good project. Therefore, if the leader invests when the belief in a good project equals p, the ex ante expected payoff for the follower equals

$$F(p) = p\left[U_F^H - I\right].$$

In case of mutual investment at belief p, each firm has an ex ante expected payoff that equals

$$M(p) = pU_M^H - I.$$

Beforehand it is not clear whether this is a game of first mover or second mover advantages. If the Stackelberg advantage, i.e. $U_L^H - U_F^H$, is sufficiently large, the firms prefer to be the first investor and a preemption game results. On the other hand the follower is able to free ride on the investment decision taken by the leader since immediately after this investment all uncertainty is resolved. Then by refraining from investment the follower does not incur any losses in case the project turns out to be bad. If the value of this information spillover exceeds the Stackelberg advantage a war of attrition results. Thijssen (2002) finds that the game is a first mover game if

$$\Psi < \frac{U_L^H - U_F^H}{U_L^H - I}, \tag{15}$$

in which

$$\Psi = \frac{\beta_1[r+\mu][r+\mu[1-\lambda]] - \mu\lambda[1-\lambda][r+\mu[1+\beta_1-\lambda]]}{\beta_1[r+\mu][r+\mu\lambda] - \mu\lambda[1-\lambda][r+\mu[\beta_1+\lambda]]},$$

where

$$\beta_1 = \frac{r+\mu}{2\mu} + \frac{1}{2}\sqrt{\left[\frac{r}{\mu}+1\right]^2 - 4\lambda\left[1-\lambda\right]}.$$

If the inequality in (15) is reversed, the game is a second mover game.

In case (15) holds the usual preemption game results. The analysis of this game is qualitatively similar to what we have seen in Section 2. On the other hand, when the game is a second mover game, firms eventually face the for these games usual dilemma that by investing immediately the leader value is obtained which is below the follower value, while waiting is bad for both firms if the other firm also waits. In this case Thijssen (2002) finds a mixed strategy equilibrium where the investment probability is a function of the difference between the number of good and bad signals. During the time where this war of attrition goes on it happens with positive probability that both firms refrain from investment. It can then be the case that so many bad signals arrive that the belief in a good project becomes so low that the war of attrition is ended and that no firm invests for the time being. On the other hand, it can happen that so many positive signals in excess of bad signal arrive that at some time the Stackelberg advantage starts to exceed the value of the information spillover. This then implies that the war of attrition turns into a preemption game.

In Thijssen (2002) also some welfare results are reported. From the industrial organization literature it is known that a monopoly is bad for social welfare. Indeed, in the framework under consideration it is possible to find examples where a duopoly does better than a monopoly in terms of ex ante expected total surplus. However, within a duopoly it is also possible that in the case of a preemption equilibrium the first investor is tempted by the Stackelberg advantage to undertake the investment too soon from a social welfare perspective, i.e. when the environment is too risky. Moreover, there are two investing firms so that sunk costs are higher. As a result it happens that welfare is lower than in the monopoly case.

4 Epilogue

Besides our own extensions presented in Section 3, the framework being presented in Section 2 is used for many different applications. Grenadier (1996) applies it to the real estate market, Weeds (2002) and Miltersen/Schwartz (2002) study R&D investments, Pennings (2002) and Pawlina/Kort (2002) analyze the product quality choice, Mason/Weeds (2002) study merger policy and entry, Boyer/Lasserre/Mariotti/Moreaux (2001) look at incremental indivisible capacity investments, Lambregt (2001), Morellec (2001) and Pawlina (2002) take into account debt financing, Nielsen (2002) and Mason/Weeds (2001) analyze the effects of positive externalities, Lambrecht/Perraudin (2003) consider incomplete information, Pawlina/Kort (2001b) explicitly model demand uncertainty, while Sparla (2002) considers the decision to close down.

Application of our method to the standard models in Section 2 showed that mixed strategy equilibria can be handled in a very tractable fashion. Nevertheless, in the literature the prevailing method is to rule out simultaneous exercise beforehand (besides our own work, an exception is Boyer/Lasserre/Mariotti/Moreaux (2001)). This is either done by (i) assumption or by (ii) avoiding cases where suboptimal simultaneous investment can occur. Examples of (i) are, for instance, Grenadier (1996, pp. 1656-1657) who assumes that "if each tries to build first, one will randomly (i.e. through the toss of a coin) win the race", or Dutta/Lach/Rustichini (1995, p. 568) where it is assumed that "If both i and j attempt to enter at any period t, then only one of them succeeds in doing so" (for a similar argument, see Nielsen (2002)). Examples of (ii) are Weeds (2002) who in a new market model assumes that the initial value lies below the preemption point, so that sequential investment is the only equilibrium outcome (cf. Section 2), or Pennings (2002), Mason/Weeds (2002) and Pawlina/Kort (2002), where the leader and follower roles are exogenously assigned.

Overall, with this contribution we attempted to show that the strategic real option framework is a suitable tool to extend the industrial organization literature in a dynamic stochastic direction. By reviewing some existing research in this field, this paper proves that the interplay of game theory and real option valuation is a fascinating area that can generate economic results being significantly different from what is known from the existing industrial organization literature.

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