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THE HARTWICK RULE: MYTHS AND FACTS

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The Hartwick Rule: Myths and Facts *

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Abstract. We shed light on the Hartwick rule for capital accumulation and resource depletion by providing semantic clarifications and investigating the implications and relevance of this rule. We extend earlier results by establishing that the Hartwick rule does *not* indicate sustainability and does *not* require substitutability between man-made and natural capital. We use a new class of simple counterexamples *(i)* to obtain the novel finding that a negative value of net investments need not entail that utility is unsustainable, and *(ii)* to point out deficiencies in the literature.

Keywords: the Hartwick rule, natural resources, sustainability

JEL codes: D9, Q01, Q3

1. Introduction

In resource economics two intertemporal allocation rules have attracted particular attention: the *Hotelling rule* and the *Hartwick rule*. The Hotelling rule is the fundamental no-arbitrage condition that every efficient resource utilisation path has to meet. In its basic form it implies that the net price of an exhaustible resource must grow at a rate that equals the interest rate. Although the Hotelling rule is in principle relevant for all models of non-renewable resource use, its simplest application is that of a cake-eating economy where consumption results from depleting a given resource stock. The Hartwick rule, in contrast,

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was formulated for a production economy where consumption at any point of time depends not only on the resource extraction but also on the stock of man-made capital available at that point in time. In such a *Dasgupta-Heal-Solow model*, Hartwick (1977) showed that a zero value of net investments entails constant consumption over time, provided the Hotelling rule holds as a condition for local efficiency. This result was the heart of what later on was called the Hartwick rule.

Hartwick's result reinforced a basic message of neoclassical resource economics (cf. Solow, 1974): Man-made capital can substitute for raw material extracted from a non-renewable resource in such a way that resource depletion does not harm future generations. Hence substitutability between natural and man-made capital may, in spite of the exhaustibility of natural resources, allow for equitable consumption for all generations, and Hartwick (1977) seemed to have found the investment policy that would bring about sustainability.

Doubts have since been raised concerning the true status of Hartwick's results and thus of the Hartwick rule. Following Asheim (1994) and Pezzey (1994) it has been claimed that the Hartwick rule is, contrary to the first impression, not a prescriptive but rather a descriptive rule (cf. Toman, Pezzey and Krautkraemer, 1995, p. 147). The original formulation of the Hartwick rule sounds, however, more like a prescription than a description. And even if one tends to see the Hartwick rule as a description, it is still not clear – twenty-five years after Hartwick's pioneering work – what exactly is described by it.

The ambiguous status of the Hartwick rule has led to false beliefs concerning the content of the rule. There are two myths on the Hartwick rule that are pertinent in the literature.

Myth 1: *The Hartwick rule indicates sustainability.*

This myth was already suggested by Hartwick (1977, pp. 973-974) himself when he stated that "investing all net returns from exhaustible resources in reproducible capital . . . implies intergenerational equity", but it lives on in recent contributions.

Myth 2: *The Hartwick rule requires substitutability between man-made and natural capital.*

This myth is implicit in many contributions on the Hartwick rule. An explicit formulation can be found in, e.g., Spash and Clayton (1997, p. 146): "... the... Hartwick rule depends upon man-made capital ... being a substitute for, rather than a complement to, natural capital."

We will demonstrate that neither of these two assertions is true, showing that an adequate understanding of the Hartwick rule is still pending. The structure of our argument will be as follows. After introducing the general technological framework in section 2, we give some semantic clarifications in section 3 where we distinguish among the Hartwick investment rule, the Hartwick result, and its converse. In sections 4 and 5 we will deal separately with the two myths described above. In section 4 we use the Dasgupta-Heal-Solow model to illustrate that consumption may exceed or fall short of the maximum sustainable level even if capital management is guided by the Hartwick investment rule in the short run. By considering a new class of simple counterexamples, *(i)* we settle an open question, showing that a negative value of net investments need not indicate that the current consumption is unsustainable, and *(ii)* we point to deficiencies in Hamilton's (1995) analysis of the Hartwick rule. In section 5 we show how the Hartwick rule applies even in models with no possibility for substitution between man-made and natural capital. Based on the analysis of the previous sections we then discuss in the concluding section 6 whether the Hartwick rule should be viewed as a prescription or a description.

2. The Setting

While Hartwick (1977) had used the Dasgupta-Heal-Solow model to formulate his rule, Dixit, Hammond, and Hoel (1980) applied a general framework to establish its broad applicability. We adopt their more general approach here and use the following notation.

At time t (≥ 0) the vector of consumption flows is denoted $\mathbf{c}(t)$, the vector of capital stocks is denoted $\mathbf{k}(t)$, and the vector of investment flows is denoted $\dot{\mathbf{k}}(t)$. Here, consumption includes both ordinary material consumption goods, as well as environmental amenities, while the vector of capital stocks comprises not only different kinds of man-made capital, but also stocks of natural capital and stocks of accumulated knowledge. Let $\mathbf{k}^0$ denote the initial stocks at time 0.

We describe the *technology* by a time-independent set $\mathcal{F}$. The triple $(\mathbf{c}(t), \mathbf{k}(t), \dot{\mathbf{k}}(t))$ is *attainable* at time t if $(\mathbf{c}(t), \mathbf{k}(t), \dot{\mathbf{k}}(t)) \in \mathcal{F}$, and the path $\{\mathbf{c}(t), \mathbf{k}(t), \dot{\mathbf{k}}(t)\}_{t=0}^{\infty}$ is *feasible given* $\mathbf{k}^0$ if $\mathbf{k}(0) = \mathbf{k}^0$ and $(\mathbf{c}(t), \mathbf{k}(t), \dot{\mathbf{k}}(t))$ is attainable at all $t \geq 0$. We assume that

- The set $\mathcal{F}$ is a smooth, closed, and convex.
- Consumption flows are non-negative: $(\mathbf{c}, \mathbf{k}, \dot{\mathbf{k}}) \in \mathcal{F}$ implies $\mathbf{c} \geq 0$.
- Capital stocks are non-negative: $(\mathbf{c}, \mathbf{k}, \dot{\mathbf{k}}) \in \mathcal{F}$ implies $\mathbf{k} \geq 0$.
- Free disposal of investment flows: $(\mathbf{c}, \mathbf{k}, \dot{\mathbf{k}}) \in \mathcal{F}$ and $\dot{\mathbf{k}}' \leq \dot{\mathbf{k}}$ imply $(\mathbf{c}, \mathbf{k}, \dot{\mathbf{k}}') \in \mathcal{F}$.

The latter assumption means, e.g., that stocks of environmental resources are considered instead of stocks of pollutants.

We assume that there is a *constant population*, where each generation lives for one instance. Hence, generations are not overlapping nor infinitely lived, implying that any intertemporal issue is of an intergenerational nature. Issues concerning distribution within each generation will not be discussed. The vector of consumption goods generates utility, $u(\mathbf{c})$, where u is a time-invariant, strictly increasing, concave, and differentiable function. Write $u(t) = u(\mathbf{c}(t))$ for utility at time t.

We assume that there are market prices for all consumption goods and capital goods. The discussion of the Hartwick rule is facilitated by using *present value* prices; i.e., deflationary nominal prices that correspond to a zero nominal interest rate. Hence, prices of future deliveries are measured in a numeraire at the present time. The vector of present value prices of consumption flows at time t is denoted $\mathbf{p}(t)$, and the vector of present value prices of investment flows at time t is denoted $\mathbf{q}(t)$. It follows that $-\dot{\mathbf{q}}(t)$ is the vector of rental prices for capital stocks at time t, entailing that $\mathbf{p}(t)\mathbf{c}(t)+\mathbf{q}(t)\dot{\mathbf{k}}(t)+\dot{\mathbf{q}}(t)\mathbf{k}(t)$ can be interpreted as the instantaneous profit at time t.

Competitiveness of a path is defined in the following way:

Definition 1. Let $T > 0$ be given. A feasible path $\{\mathbf{c}(t), \mathbf{k}(t), \dot{\mathbf{k}}(t)\}_{t=0}^{\infty}$ is *competitive* during $(0, T)$ at discount factors $\{\mu(t)\}_{t=0}^{T}$ and prices $\{\mathbf{p}(t), \mathbf{q}(t)\}_{t=0}^{T}$ if, for all $t \in (0, T)$, $\mu(t) > 0$, $(\mathbf{p}(t), \mathbf{q}(t)) \geq 0$, and the following conditions are satisfied:

$$\begin{gathered}\text{Instantaneous } \textit{utility} \text{ is maximised:}\\ \mathbf{c}(t) \text{ maximises } \mu(t)u(\mathbf{c}) - \mathbf{p}(t)\mathbf{c}\,.\end{gathered} \tag{1a}$$

$$\begin{gathered}\text{Instantaneous } \textit{profit} \text{ is maximised: } (\mathbf{c}(t), \mathbf{k}(t), \dot{\mathbf{k}}(t))\\ \text{maximises } \mathbf{p}(t)\mathbf{c} + \mathbf{q}(t)\dot{\mathbf{k}} + \dot{\mathbf{q}}(t)\mathbf{k} \text{ subject to } (\mathbf{c}, \mathbf{k}, \dot{\mathbf{k}}) \in \mathcal{F}\,.\end{gathered} \tag{1b}$$

In the sequel we will refer to (1a) and (1b) as the *competitiveness* conditions. Competitive paths have the following property that is at the heart of the analysis of the Hartwick rule.

LEMMA 1. *Let $T > 0$ be given. Suppose $\{\mathbf{c}(t), \mathbf{k}(t), \dot{\mathbf{k}}(t)\}_{t=0}^{\infty}$ is competitive during $(0, T)$ at $\{\mu(t)\}_{t=0}^{T}$ and $\{\mathbf{p}(t), \mathbf{q}(t)\}_{t=0}^{T}$. Then:*

1. *For all $t \in (0, T)$, $\mu(t)\partial u(\mathbf{c}(t))/\partial c_i = p_i(t)$ if $c_i(t) > 0$.*

2. *For all $t \in (0, T)$, $\mathbf{p}(t)\dot{\mathbf{c}}(t) + d(\mathbf{q}(t)\dot{\mathbf{k}}(t))/dt = 0$.*

Proof. Part 1 follows directly from (1a). For the proof of part 2, we follow Dixit et al. (1980). Since $\mathcal{F}$ is time-invariant, (1b) implies that

$$\begin{aligned}&\mathbf{p}(t)\mathbf{c}(t + \Delta t) + \mathbf{q}(t)\dot{\mathbf{k}}(t + \Delta t) + \dot{\mathbf{q}}(t)\mathbf{k}(t + \Delta t)\\ &\quad \leq \mathbf{p}(t)\mathbf{c}(t) + \mathbf{q}(t)\dot{\mathbf{k}}(t) + \dot{\mathbf{q}}(t)\mathbf{k}(t)\,.\end{aligned}$$

Divide by Δt, and let $\Delta t \to 0$ from both directions. This yields

$$0 = \mathbf{p}(t)\dot{\mathbf{c}}(t) + \mathbf{q}(t)\ddot{\mathbf{k}}(t) + \dot{\mathbf{q}}(t)\dot{\mathbf{k}}(t) = \mathbf{p}(t)\dot{\mathbf{c}}(t) + d(\mathbf{q}(t)\dot{\mathbf{k}}(t))/dt$$

as the right-hand derivative cannot lie above zero and the left-hand derivative cannot lie below zero and both have to coincide. □

Some results on the Hartwick rule require that the path is not only competitive, but also regular.

Definition 2. A feasible path $\{\mathbf{c}(t), \mathbf{k}(t), \dot{\mathbf{k}}(t)\}_{t=0}^{\infty}$ is *regular* if it is competitive during $(0, \infty)$ at discount factors $\{\mu(t)\}_{t=0}^{\infty}$ and prices $\{\mathbf{p}(t), \mathbf{q}(t)\}_{t=0}^{\infty}$, and the following conditions are satisfied:

$$\int_0^{\infty} \mu(t)u(\mathbf{c}(t))dt < \infty \,, \tag{2a}$$

$$\mathbf{q}(t)\mathbf{k}(t) \to 0 \text{ as } t \to \infty \,. \tag{2b}$$

It can be shown that a regular path maximises $\int_0^{\infty} \mu(t)u(\mathbf{c}(t))dt$ over all feasible paths, implying that any regular path is efficient.

In the real world environmental externalities are not always internalised. This is one of many causes that prevent market economies from being fully efficient. Furthermore, for many capital stocks (e.g., stocks of natural and environmental resources or stocks of accumulated knowledge) it is hard to find market prices (or to calculate shadow prices) that can be used to estimate the value of such stocks. Since the Hartwick rule is formulated in terms of efficiency prices, we must abstract from these problems in our analysis.

The time-independency of the set $\mathcal{F}$ is an assumption of *constant technology.* It means that all technological progress is *endogenous*, being captured by accumulated stocks of knowledge. If there is *exogenous* technological progress in the sense of a time-dependent technology, we may capture this within our formalism by including time as an additional stock. Since the time-derivative of time equals 1, this can be done as follows: The triple $(\mathbf{c}(t), \mathbf{k}(t), \dot{\mathbf{k}}(t))$ is *attainable* at time t if

$$(\mathbf{c}(t), (\mathbf{k}(t), t), (\dot{\mathbf{k}}(t), 1)) \in \mathcal{F} \,.$$

This formulation, which is used by e.g. Cairns and Long (2001) and Pezzey (2002), does, however, lead to the challenge of calculating the present value price associated with the passage of time. Vellinga and Withagen (1996) show how this price in principle can be derived through a forward-looking term.

3. What Is the Hartwick Rule?

The term 'the Hartwick rule' has been used in different meanings. E.g., Dixit et al. (1980) in their first paragraph (p. 551) associated this term with both the investment rule of keeping "the total value of net investment under competitive pricing equal to zero" and the result that following such an investment rule "yields a path of constant consumption". It will be clarifying to differentiate between

- *The Hartwick investment rule*, which is associated with the prescription of holding the value of net investments $\mathbf{q}(t)\dot{\mathbf{k}}(t)$ (also known as "genuine savings") constant and equal to zero, and
- *The Hartwick result* that we will associate with the finding that following such a prescription leads to constant utility.

In this section, we formally state the definitions that we will suggest, present the results that follow from the analysis of section 2, and provide a partial review of the relevant literature.

Definition 3. Let $T > 0$ be given. The *Hartwick investment rule* is followed along a path $\{\mathbf{c}(t), \mathbf{k}(t), \dot{\mathbf{k}}(t)\}_{t=0}^{\infty}$ for $t \in (0, T)$ if the path is competitive during $(0, T)$ at discount factors $\{\mu(t)\}_{t=0}^{T}$ and prices $\{\mathbf{p}(t), \mathbf{q}(t)\}_{t=0}^{T}$, and $\mathbf{q}(t)\dot{\mathbf{k}}(t) = 0$ for all $t \in (0, T)$.

We first show the result that Hartwick (1977) originally showed in a special model, but which – as established by Dixit et al. (1980) – carries over to our general setting.

PROPOSITION 1 (The Hartwick result). *Let $T > 0$ be given. If the Hartwick investment rule is followed for $t \in (0, T)$ in an economy with constant population and constant technology, then utility is constant for all $t \in (0, T)$.*

Proof. For all $t \in (0, T)$ we have that

$$\begin{aligned} \mu(t)\dot{u}(t) &= \mathbf{p}(t)\dot{\mathbf{c}}(t) && \text{(by Lemma 1(i))} \\ &= -d(\mathbf{q}(t)\dot{\mathbf{k}}(t))/dt && \text{(by Lemma 1(ii))} \\ &= 0 && \text{(since } \mathbf{q}(t)\dot{\mathbf{k}}(t) = 0\text{),} \end{aligned}$$

noting that prices of consumption flows that remain equal to zero, and thus are constant, do not matter for the first equality. □

Dixit et al. (1980) made the observation that the Hartwick result can be generalised. For the statement of this more general result we first need to define 'the generalised Hartwick investment rule', which is the prescription of holding the *present* value of net investments $\mathbf{q}(t)\dot{\mathbf{k}}(t)$ constant, but not necessarily equal to zero.

Definition 4. Let $T > 0$ be given. The *generalised Hartwick investment rule* is followed along a path $\{\mathbf{c}(t), \mathbf{k}(t), \dot{\mathbf{k}}(t)\}_{t=0}^{\infty}$ for $t \in (0, T)$ if the path is competitive during $(0, T)$ at discount factors $\{\mu(t)\}_{t=0}^{T}$ and prices $\{\mathbf{p}(t), \mathbf{q}(t)\}_{t=0}^{T}$, and $\mathbf{q}(t)\dot{\mathbf{k}}(t)$ is constant for all $t \in (0, T)$.

PROPOSITION 2 (The generalised Hartwick result). *Let $T > 0$ be given. If the generalised Hartwick investment rule is followed for $t \in (0, T)$ in an economy with constant population and constant technology, then utility is constant for all $t \in (0, T)$.*

Proof. The proof of Proposition 1 applies even if $\mathbf{q}(t)\dot{\mathbf{k}}(t) = \nu$ for all $t \in (0, T)$, with ν constant. □

Dixit et al. (1980) posed the question of whether the converse of the Hartwick result can be established. It is instructive to observe that the converse of the (ordinary) Hartwick result is not correct.

INCORRECT CLAIM 1 (The converse of the Hartwick result). *Let $T > 0$ be given. If utility is constant for all $t \in (0, T)$ in an economy with constant population and constant technology, then the Hartwick investment rule is followed for $t \in (0, T)$.*

We provide a counterexample using the *Ramsey model*. Here there is a single consumption good, and one capital good. To denote these scalars we use symbols in italics instead of boldface. The stock of the aggregate capital good ($k(t)$) leads to production $f(k(t))$ that can either contribute to the quality of life of generation t or be used to accumulate capital. Hence, $(c(t), k(t), \dot{k}(t))$ is attainable if and only if $c(t) + \dot{k}(t) \le f(k(t))$. The initial stock equals k^0. The production function f is twice continuously differentiable, with $f' > 0$ and $f'' < 0$. Furthermore, $f(0) = 0$, $\lim_{k \to 0} f'(k) = \infty$, and $\lim_{k \to \infty} f'(k) = 0$. For this model the competitiveness condition (1b) implies that $c(t) + \dot{k}(t) = f(k(t))$, $p(t) = q(t)$, and $q(t)f'(k(t)) = -\dot{q}(t)$. Hence, omitting the time variable,

$$p\dot{c} = q\dot{c} = -q\ddot{k} + qf'(k)\dot{k} = -q\ddot{k} - \dot{q}\dot{k} = -d(q\dot{k})/dt\,.$$

Suppose there exist $T > 0$ with $\dot{c}(t) = 0$ for all $t \in (0, T)$. This is compatible with $q(t)\dot{k}(t) = \nu \neq 0$ for all $t \in (0, T)$. In particular, if $\nu < 0$, then $c = c(t) > f(k(t))$, which is feasible in the short run.

However, as shown by Dixit et al. (1980), the converse of the generalised Hartwick result can be established.

PROPOSITION 3 (The converse of the generalised Hartwick result). *Let $T > 0$ be given. If utility is constant for all $t \in (0, T)$ in an economy with constant population and constant technology, then the generalised Hartwick investment rule is followed for $t \in (0, T)$.*

Proof. Since (1a) and (1b) imply that

$$\mu(t)\dot{u}(t) = \mathbf{p}(t)\dot{\mathbf{c}}(t) = -d(\mathbf{q}(t)\dot{\mathbf{k}}(t))/dt\,,$$

as shown in the proof of Proposition 1, it follows from the constancy of utility that $\mathbf{q}(t)\dot{\mathbf{k}}(t)$ is constant. □

Applying these results at all times along infinite horizon paths yields some observations concerning the relationship between the (generalised) Hartwick result and the concept of sustainable development, as a precursor to the discussions of sections 4 and 5. For the statement of these results, we introduce the following definition.

Definition 5. A utility path $\{u(t)\}_{t=0}^{\infty}$ is *egalitarian* if utility is constant for all t.

The following two results are direct consequences of Propositions 1 and 2 established above.

COROLLARY 1 (The Hartwick rule for sustainability). *If the Hartwick investment rule is followed for all t in an economy with constant population and constant technology, then the utility path is egalitarian.*

COROLLARY 2 (The generalised Hartwick rule for sustainability). *If the generalised Hartwick investment rule is followed for all t in an economy with constant population and constant technology, then the utility path is egalitarian.*

One may wonder whether Corollary 2 is an empty generalisation of Corollary 1, in the sense that any feasible competitive path with constant utility does in fact satisfy the (ordinary) Hartwick investment rule. This is not the case since in the Ramsey model there exist feasible competitive paths with constant utility for which $q(t)\dot{k}(t) = \nu > 0$ for all t, provided that $\nu < q(0)f(k^0)$. Then $c = c(t) < f(k(t))$ for all t, so that the path is inefficient since capital is over-accumulated. It is, however, true – as suggested by Dixit et al. (1980) and shown under general assumptions by Withagen and Asheim (1998) – that the (ordinary) Hartwick investment rule must be satisfied for all t if the egalitarian utility path is efficient. This is stated next.

PROPOSITION 4. (The converse of the Hartwick rule for sustainability). *If the utility path is egalitarian along a regular path in an economy with constant population and constant technology, then the Hartwick investment rule is followed for all t.*

The proof by Withagen and Asheim (1998) is too extensive to be reproduced here. The result means that a regular path with constant utility satisfies $\mathbf{q}(T)\dot{\mathbf{k}}(T) \to 0$ as $T \to \infty$.[1] Combining this transversality condition with the results of Lemma 1 means that $\mathbf{q}(t)\dot{\mathbf{k}}(t) = \int_t^\infty \mu(t)\dot{u}(t)dt$ (cf. Aronsson et al., 1997, p. 105). Thus, the value of net investments at time t measures the present value of future changes in utility. From this it can be easily seen that the Hartwick investment rule is satisfied for all t if the utility path is egalitarian.

The fact that there exist egalitarian, but inefficient, utility paths in the Ramsey model means that Proposition 4 does not hold if regularity is not assumed. If only the competitiviness conditions (1a) and (1b) are assumed to hold at any t, then the following weaker result – due to Dixit et al. (1980) – follows from Proposition 3.

COROLLARY 3. (The converse of the generalised Hartwick rule for sustainability). *If the utility path is egalitarian along a competitive path in an economy with constant population and constant technology, then the generalised Hartwick investment rule is followed for all t.*

When discussing the significance and applicability of the Hartwick rule, the results on sustainability (i.e., Corollaries 1–3 and Proposition 4) are of particular interest. In the following two sections we will discuss what significance the Hartwick rule may have for sustainability along two dimensions. Firstly, we note that these results are weak since they are based on strong premises involving the properties of the entire paths. In section 4 we therefore pose the question: can stronger results be obtained by weakening the premises – i.e., by relating sustainability of a path to only the current value of net investment – thereby addressing Myth 1. Secondly, in section 5 we discuss whether the Hartwick rule for sustainability requires substitutability between man-made and natural capital, thereby addressing Myth 2.

4. Myth 1: The Hartwick Investment Rule Indicates Sustainability

What makes Hartwick's investment rule so appealing in the framework of resource economics is its alleged relationship with intergenerational fairness. Hartwick himself purported to have found a prescription how

[1] This follows from optimality (and hence, from regularity) of the path if there is a constant discount rate; i.e., if $\mu(t) = \mu(0)e^{-\rho t}$ (cf. Dasgupta and Mitra, 1999). However, if $\mu(t)$ is not an exponentially decreasing function, then it is not immediate that regularity implies this condition (cf. Cairns and Long, 2001, footnote 4).

"to solve the ethical problem of the current generation short-changing future generations by 'overconsuming' the current product, partly ascribable to current use of exhaustible resources" (Hartwick, 1977, p. 972). By invoking Hartwick's result the Hartwick investment rule then seemed to provide a sufficient condition for intergenerational justice. Such an interpretation carries over to some recent text books, e.g., Tietenberg (2001, p. 91) and Hanley et al. (2001, p. 137).

Although the result proven by Hartwick (1977) is undoubtedly correct, it does not follow that one can draw a close link between Hartwick's result and intergenerational equity without taking notice of additional conditions. There are more or less sophisticated versions of such precipitate interpretations. The first one only makes weak assumptions on the path under consideration and is rather easy to refute.

INCORRECT CLAIM 2 (Trivial version). *Let $T > 0$ be given. Suppose a path $\{\mathbf{c}(t), \mathbf{k}(t), \dot{\mathbf{k}}(t)\}_{t=0}^{\infty}$ is competitive during $(0, T)$ in an economy with constant population and constant technology. If the value of net investments $\mathbf{q}(t)\dot{\mathbf{k}}(t)$ is non-negative for $t \in (0, T)$, then, for any $t \in (0, T)$, $u(\mathbf{c}(t))$ can be sustained forever given $\mathbf{k}(t)$.*

Whether this claim, which combines short-term considerations with long-term results, is correct or incorrect crucially depends on the underlying technology. It is certainly correct in case of the Ramsey model.

The claim, however, is not true in the Dasgupta-Heal-Solow model (see, e.g., Dasgupta and Heal, 1974, and Solow, 1974). In this model there are two capital stocks: man-made capital, denoted by k_M, and a non-renewable natural resource, the stock of which is denoted by k_N. So, $\mathbf{k} = (k_M, k_N)$. The initial stocks are given by $\mathbf{k}^0 = (k_M^0, k_N^0)$. The technology is described by a Cobb-Douglas production function $F(k_M, -\dot{k}_N) = k_M^a(-\dot{k}_N)^b$ depending on two inputs, man-made capital k_M and the raw material $-\dot{k}_N$ that can be extracted without cost from the non-renewable resource. The output from the production process is used for consumption and for investments in man-made capital $\dot{k}_M$. Hence, $(c(t), \mathbf{k}(t), \dot{\mathbf{k}}(t))$ is attainable at time t if and only if

$$c(t) + \dot{k}_M(t) \leq k_M(t)^a(-\dot{k}_N(t))^b \text{ where } a > 0,\ b > 0 \text{ and } a + b \leq 1\,,$$

and $c(t) \geq 0$, $k_M(t) \geq 0$, $k_N(t) \geq 0$, and $-\dot{k}_N(t) \geq 0$. With $r(t) := -\dot{k}_N(t)$ denoting the flow of raw material, these assumptions entail

$$\int_0^\infty r(t)dt \leq k_N^0 \quad \text{and} \quad r(t) \geq 0 \text{ for all } t \geq 0\,.$$

The competitiveness condition (1b) requires that

$$c(t) + \dot{k}_M(t) = k_M(t)^a r(t)^b \tag{3a}$$

$$r(t) + \dot{k}_N(t) = 0 \tag{3b}$$

$$p(t) = q_M(t) \tag{3c}$$

$$q_M(t) \cdot b \cdot k_M(t)^a r(t)^{b-1} = 1 \tag{3d}$$

$$q_M(t) \cdot a \cdot k_M(t)^{a-1} r(t)^b = -\dot{q}_M(t)\,, \tag{3e}$$

where (3d) follows from $q_M(t) \cdot b \cdot k_M(t)^a r(t)^{b-1} = q_N(t)$ and $0 = \dot{q}_N(t)$ by choosing extracted raw material as numeraire: $q_N(t) \equiv 1$. Note that (3d) and (3e) entail that the growth rate of the marginal product of raw material equals the marginal product of man-made capital; thus, the Hotelling rule is satisfied.

Assume that $a > b > 0$. Then there is a strictly positive maximum constant rate of consumption c^* that can be sustained forever given $\mathbf{k}^0$ (see, e.g., Dasgupta and Heal, 1974, p. 203). It is well known that this constant consumption level can be implemented along a competitive path where net investment in man-made capital $\dot{k}_M(t)$ is at a constant level $i^* = bc^*/(1-b)$. To give a counterexample to the claim above, fix a consumption level $c > c^*$. Set $i = bc/(1-b)$ and define T by

$$\int_0^T (i/b)^{\frac{1}{b}} (k_M^0 + it)^{-\frac{a}{b}} dt = k_N^0\,. \tag{4}$$

For $t \in (0, T)$, consider the path described by $\mathbf{k}(0) = \mathbf{k}^0$ and

$$\begin{aligned} c(t) &= c \\ \dot{k}_M(t) &= i \\ -\dot{k}_N(t) = r(t) &= (i/b)^{\frac{1}{b}} (k_M^0 + it)^{-\frac{a}{b}}\,, \end{aligned}$$

which by (4) implies that the resource stock is exhausted at time T. This feasible path is competitive during $(0, T)$ at prices $p(t) = q_M(t) = r(t)/i$ and $q_N(t) = 1$, implying that the value of net investments $q_M(t)i - q_N(t)r(t)$ is zero, and thus the Hartwick investment rule is followed. Hence, even though the competitiveness condition (1b) is satisfied (while (1a) does not apply) and the value of net investments is non-negative during the interval $(0, T)$, the constant rate of consumption during this interval is not sustainable forever.

Hartwick (1977) does not say much about efficiency requirements going beyond competitiveness conditions, i.e., the Hotelling rule, other than remarking that the entire stock of the non-renewable resource has to be used up in the long run to achieve an optimal solution. It seems appropriate, however, to consider efficiency requirements going beyond competitiveness on a finite interval when looking for counterexamples. The path described above for the Dasgupta-Heal-Solow model is in fact not efficient. At time T a certain stock of man-made capital, $k_M(T) =$

$k_M^0 + iT$, has been accumulated, while the flow of extracted raw material falls abruptly to zero due the exhaustion of the resource. In our Cobb-Douglas case the marginal productivity of r is a strictly decreasing function of the flow of raw material for a given positive stock of man-made capital. This implies that profitable arbitrage opportunities can be exploited by shifting resource extraction from right before T to right after T, implying that the Hotelling rule is not satisfied at that time.

As the path in this counterexample is inefficient, it might be possible that the Hartwick investment rule does not indicate sustainability in the example due to this lack of efficiency. However, this is not true either. The claim above does not become valid even if we refer to regular – and thus efficient – paths for which competitiveness holds throughout and transversality conditions are satisfied.

INCORRECT CLAIM 3 (Sophisticated version). *Let $T > 0$ be given. Suppose a path $\{\mathbf{c}(t), \mathbf{k}(t), \dot{\mathbf{k}}(t)\}_{t=0}^{\infty}$ is regular in an economy with constant population and constant technology. If the value of net investments $\mathbf{q}(t)\dot{\mathbf{k}}(t)$ is non-negative for $t \in (0, T)$, then, for any $t \in (0, T)$, $u(\mathbf{c}(t))$ can be sustained forever given $\mathbf{k}(t)$.*

Again, counterexamples can be provided in the framework of the Dasgupta-Heal-Solow model. Asheim (1994) and Pezzey (1994) independently gave a counterexample by considering paths where the sum of utilities discounted at a constant utility discount rate is maximised. If, for some discount rate, the initial consumption level along such a discounted utilitarian optimum exactly equals the maximum sustainable consumption level given k_M^0 and k_N^0, then there exists an initial interval during which the value of net investments is strictly positive, while consumption is unsustainable given the current capital stocks $k_M(t)$ and $k_N(t)$. It is not quite obvious, however, that the premise of this statement can be fulfilled, i.e., that there exists some discount rate such that initial consumption along the optimal path is barely sustainable. This was subsequently established for the Cobb-Douglas case by Pezzey and Withagen (1998). The fact that their proof is quite intricate indicates, however, that this is not a trivial exercise.

Consequently, we wish to provide another type of counterexample here, which is also within the Dasgupta-Heal-Solow model and resembles the one given above to refute Incorrect Claim 2. We will show that a path identical to that described in our first counterexample during an initial phase can always be extended to an efficient path. Moreover, this second counterexample can be used to show that there exist regular paths with a non-negative value of net investments during an initial phase even if $a \leq b$, entailing that a positive and constant rate of consumption cannot be sustained indefinitely.

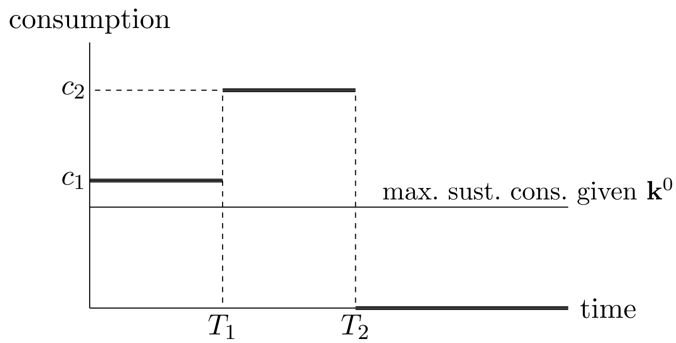

Figure 1. Counterexample to Incorrect Claim 3.

The example, illustrated in Figure 1, consists of three separate phases with constant consumption, constructed so that there are no profitable arbitrage opportunities at any time, not even at the two points in time, T_1 and T_2, where consumption is not continuous. Both capital stocks are exhausted at T_2, implying that consumption equals zero for (T_2, ∞).

In the construction of the example, k_M^0 is given, while k_N^0 is treated as a parameter. Fix some consumption level $c_1 > 0$ and some terminal time T_1 of the first phase of the path. In the interval $(0, T_1)$ the path is – as in our first example – described by $\mathbf{k}(0) = \mathbf{k}^0$ and

$$\begin{aligned} c(t) &= c_1 \\ \dot{k}_M(t) &= i_1 \\ -\dot{k}_N(t) = r(t) &= (i_1/b)^{\frac{1}{b}} (k_M^0 + i_1 t)^{-\frac{a}{b}} \,, \end{aligned}$$

where $i_1 = bc_1/(1-b)$, but with the difference that the resource stock will *not* be exhausted at time T_1. As in the first example, the Hartwick investment rule applies during this phase.

The second phase starts at time T_1. Consumption jumps upward discontinuously to $c_2 > c_1$, but we ensure that the flow of raw material is continuous to remove profitable arbitrage opportunities. Consumption is constant at the new and higher level c_2, and Proposition 3 implies that there exists $\nu_2 < 0$ such that, for all $t \in (T_1, T_2)$, the generalised Hartwick investment rule, $q_M(t)\dot{k}_M(t) = r(t) + \nu_2$, is observed. By (3a) and (3d), this rule may (for any c and ν) be written as

$$k_M(t)^a r(t)^b - c = b \cdot k_M(t)^a r(t)^{b-1} (r(t) + \nu) \tag{5}$$

(cf. Hamilton, 1995). As $k_M^a r^b - b \cdot k_M^a r^{b-1} r = (1-b) \cdot k_M^a r^b$, this implies

$$c = (1-b) \cdot k_M(t)^a r(t)^b \left(1 - \frac{b}{1-b} \cdot \frac{\nu}{r(t)}\right) . \tag{6}$$

Since both $k_M(t)$ and $r(t)$ are continuous at time T_1, we can now use (6) to determine ν_2 as follows:

$$c_2 = (1-b) \cdot k_M(T_1)^a r(T_1)^b \left(1 - \frac{b}{1-b} \cdot \frac{\nu_2}{r(T_1)}\right) . \tag{7}$$

By choosing $c_2 > k_M(T_1)^a r(T_1)^b$ ($> c_1$) and fixing ν_2 according to (7), the generalised Hartwick investment rule combined with (3a)–(3b) determines a competitive path along which investment in man-made capital becomes increasingly negative.[2] Determine T_2 as the time at which the stock of man-made capital reaches 0, and determine k_N^0 such that the resource stock is exhausted simultaneously. With both stocks exhausted, consumption equals 0 during the third phase (T_2, ∞).

The Hotelling rule holds for $(0, T_1)$ and (T_1, T_2), and by the construction of ν_2, a jump of the marginal productivity of the natural resource at T_1 is avoided such that the Hotelling rule obtains even at T_1. Thus, the path is competitive throughout. By letting $u(c) = c$ and, for all $t \in (0, T_2)$, $\mu(t) = p(t)$, it follows that both regularity conditions (2a) and (2b) are satisfied, implying that the path is regular.

Note that the above construction is independent of whether $a > b$. If $a \le b$, so that no positive and constant rate of consumption can be sustained indefinitely, we have thus shown that having a non-negative value of net investments during an initial phase of a regular path is compatible with consumption exceeding the sustainable level.

However, even if $a > b$, so that the production function allows for a positive level of sustainable consumption, we obtain a counterexample as desired. For this purpose, increase c_2 beyond all bounds to that ν_2 becomes more negative. Then T_2 decreases and converges to T_1, and the aggregate input of raw material in the interval (T_1, T_2) – being bounded above by $r(T_1) \cdot (T_2 - T_1)$ since $r(t)$ is decreasing (cf. footnote 2) – converges to 0. This in turn means that, for large enough c_2, c_1 cannot be sustained forever given the choice of k_N^0 needed to achieve exhaustion of the resource at time T_2.

This example shows that a non-negative value of net investments on an open interval is not a *sufficient* condition for having consumption be sustainable. However, it has up to now been an open question whether it is a *necessary* condition: Does a negative value of net investments during a time interval imply that consumption exceeds the sustain-

[2] By differentiating $k_M(t)^a r(t)^b - c_2 = b \cdot k_M(t)^a r(t)^{b-1}(r(t) + \nu_2)$ w.r.t. time and observing that c_2 is constant, it follows that the growth rate of the marginal product of r equals the marginal product of k_M, i.e., the Hotelling rule is satisfied and the path is competitive during this phase. By totally differentiating the same equation, it can be seen that a falling k_M leads to a falling r and thus a falling rate of output and – due to the constant c_2 – a falling $\dot{k}_M$.

able level? The following result due to Pezzey (2002) shows such an implication when utilities are discounted at a constant rate.

PROPOSITION 5. *Let $T > 0$ be given. Suppose a path $\{\mathbf{c}(t), \mathbf{k}(t), \dot{\mathbf{k}}(t)\}_{t=0}^{\infty}$ is regular in an economy with constant population and constant technology, with the supporting utility discount factor satisfying, for all t, $\mu(t) = \mu(0)e^{-\rho t}$. If the value of net investments $\mathbf{q}(t)\dot{\mathbf{k}}(t)$ is negative for $t \in (0, T)$, then, for any $t \in (0, T)$, $u(\mathbf{c}(t))$ cannot be sustained forever given $\mathbf{k}(t)$.*

Proof. It follows from Lemma 1 that $\mu(t)\dot{u}(t) + d(\mathbf{q}(t)\dot{\mathbf{k}}(t))/dt = 0$, implying $d(\mu(t)u(t))/dt = \dot{\mu}(t)u(t) - d(\mathbf{q}(t)\dot{\mathbf{k}}(t))/dt$. When this observation is combined with $\mu(t) = \mu(0)e^{-\rho t}$ and $\mathbf{q}(T)\dot{\mathbf{k}}(T) \to 0$ as $T \to \infty$ (cf. footnote 1), Weitzman's (1976) main result can be established:

$$\int_t^{\infty} \mu(s)\left(u(\mathbf{c}(t)) + \mathbf{q}(t)\dot{\mathbf{k}}(t)/\mu(t)\right) ds = \int_t^{\infty} \mu(s)u(\mathbf{c}(s))ds\,. \qquad (8)$$

Since the path is regular, it maximises $\int_t^{\infty} \mu(s)u(\mathbf{c}(s))ds$ over all feasible paths. This combined with (8) implies that the maximum sustainable utility level given $\mathbf{k}(t)$ cannot exceed $u(\mathbf{c}(t)) + \mathbf{q}(t)\dot{\mathbf{k}}(t)/\mu(t)$. Suppose $\mathbf{q}(t)\dot{\mathbf{k}}(t) < 0$ for $t \in (0, T)$. Then $u(\mathbf{c}(t)) > u(\mathbf{c}(t)) + \mathbf{q}(t)\dot{\mathbf{k}}(t)/\mu(t)$. Hence, $u(\mathbf{c}(t))$ exceeds the maximum sustainable utility level and cannot be sustained forever given $\mathbf{k}(t)$. □

It is *not*, however, a general result that a negative value of net investments implies non-sustainability, as we establish below.

INCORRECT CLAIM 4. *Let $T > 0$ be given. Suppose a path $\{\mathbf{c}(t), \mathbf{k}(t), \dot{\mathbf{k}}(t)\}_{t=0}^{\infty}$ is regular in an economy with constant population and constant technology. If the value of net investments $\mathbf{q}(t)\dot{\mathbf{k}}(t)$ is negative for $t \in (0, T)$, then, for any $t \in (0, T)$, $u(\mathbf{c}(t))$ cannot be sustained forever given $\mathbf{k}(t)$.*

Also in this case we will provide a counterexample in the framework of the Dasgupta-Heal-Solow model. Assume that $a > b$ so that the production function allows for a positive level of sustainable consumption. Again, the example (which is illustrated in Figure 2) consists of three separate phases with constant consumption, constructed so that there are no profitable arbitrage opportunities at any time, not even at the two points in time, T_1 and T_2, where consumption is not continuous.

As before, k_M^0 is given, while k_N^0 is treated as a parameter. Fix some consumption level $c_1 > 0$ and some terminal time T_1 of the first phase of the path. Construct a path that has constant consumption c_1 and obeys the generalised Hartwick investment rule (5) with $\nu_1 < 0$ in the

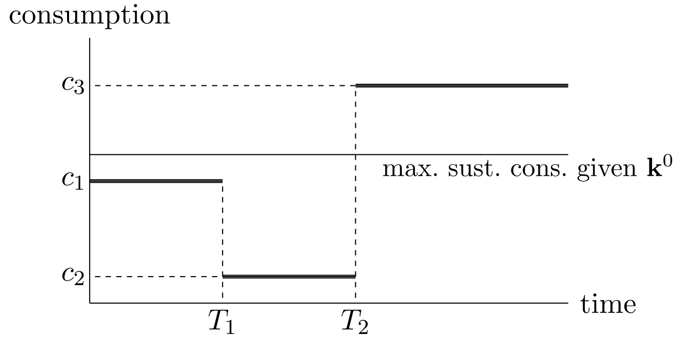

Figure 2. Counterexample to Incorrect Claim 4.

interval $(0, T_1)$, where T_1 is small enough to ensure that $k_M(T_1) > 0$. Let the path have, as its second phase, constant consumption $c_2 > 0$ and a constant (present) value of net investments $\nu_2 > 0$ in the interval (T_1, T_2). To satisfy the Hotelling rule at time T_1, c_2 and ν_2 must fulfill (7); hence, by choosing $c_2 < (1-b) \cdot k_M(T_1)^a r(T_1)^b$ it follows that $\nu_2 > 0$. Let $k_M(T_2)$ and $r(T_2)$ be the stock of man-made capital and the flow of raw material, respectively, at time T_2. At this point in time the path switches over to the third phase where the (ordinary) Hartwick investment rule is followed with $c_3 = (1-b) \cdot k_M(T_2)^a r(T_2)^b$.

Since $a > b$, the production function allows for a positive level of sustainable consumption, and there exists an appropriate choice of k_N^0 that ensures resource exhaustion as $t \to \infty$ and makes the path regular. This initial resource stock depends on T_1 and T_2, but it is finite in any case. Keep T_1 fixed and increase T_2. As T_2 goes to infinity, then the stock k_N^0 needed will also tend to infinity.[3] The same holds true for the maximum sustainable consumption level c^* that is feasible given k_M^0 and the initial resource stock k_N^0 determined in this way. Hence, by shifting T_2 far enough into the future, it follows that $c_1 < c^*$. Thus, a regular path can be constructed which has a first phase with a negative value of net investments even though the rate of consumption during this phase is sustainable given the initial stocks.

Both our counterexamples are consistent with the result for regular paths that the value of net investments measures the present value of all future changes in of utility, which is a consequence of Lemma 1. It follows directly from that result that if along an efficient path utility is monotonically decreasing/increasing indefinitely, then the value of net investments will be negative/positive, while utility will exceed/fall short of the sustainable level. The value of net investments thus indi-

[3] It follows from (6) and $c_2 > 0$ that $r(t) > b\nu_2/(1-b)$ (> 0) for all $t \in (T_1, T_2)$.

cates sustainability correctly along such monotone utility paths. Hence, the counterexamples above are minimal by having consumption (and thus utility) constant except at two points in time.

Moreover, paths with piecewise constant consumption would not yield counterexamples if constant consumption is associated with a constant consumption interest rate (as it is in the Ramsey model). In the Dasgupta-Heal-Solow model, however, the consumption interest rate, $-\dot{p}(t)/p(t)$, which equals the marginal productivity of man-made capital along a competitive path, is decreasing whenever consumption is constant. It is therefore the non-monotonicity of the paths, combined with the property that the consumption interest rate is decreasing when consumption is constant, that leads to the negative results established above concerning the connection between the value of net investments (the "genuine savings") and the sustainability of utility.

It is also worth emphasising the point made in Asheim (1994) and elsewhere that the relative equilibrium prices of different capital stocks *today* depend on the properties of the whole future path. The counterexamples above show how the relative price of natural capital depends positively on the consumption level of the generations in the distant future. Thus, the future development – in particular, the distribution of consumption between the intermediate and the distant future – affects the value of net investments today and, thereby, the usefulness of this measure as an indicator of sustainability today.

Hence, to link the (generalised) Hartwick investment rule to sustainability we cannot avoid letting this rule apply to investment behaviour at all points in time. We present a correct claim concerning the value of net investments and the sustainability of utility by restating the generalised Hartwick rule for sustainability (Corollary 2).

CORRECT CLAIM 1. *Suppose a path* $\{\mathbf{c}(t), \mathbf{k}(t), \dot{\mathbf{k}}(t)\}_{t=0}^{\infty}$ *is competitive during* $(0, \infty)$ *in an economy with constant population and constant technology. If the value of net investments* $\mathbf{q}(t)\dot{\mathbf{k}}(t)$ *is constant for all* $t \in (0, \infty)$*, then the rate of utility realised at any time* t *can be sustained forever given* $\mathbf{k}(t)$.

Proof. From the generalised Hartwick rule for sustainability, it follows that the utility path is egalitarian. Hence, at any time, utility is sustainable. □

If the path is regular, it follows from Proposition 4 that an egalitarian utility path implies that $\mathbf{q}(t)\dot{\mathbf{k}}(t)$ equals zero for all t. In the Ramsey model, it is feasible but not efficient to have $\mathbf{q}(t)\dot{\mathbf{k}}(t)$ constant and positive for all t. It follows from footnote 3 that this is not even feasible in the Dasgupta-Heal-Solow model, since the integral of extracted raw material would become infinite as time goes to infinity. In both models, feasibility rules out $\mathbf{q}(t)\dot{\mathbf{k}}(t)$ being constant and negative for all t.

Hamilton (1995) also analyses paths satisfying the generalised Hartwick rule (i.e., our equation (5)) in different versions of the Dasgupta-Heal-Solow model. For the version that overlaps with the one treated here ($\sigma = 1$), he incorrectly claims (1995, pp. 397–398 and Table 1) that if $\nu > 0$, then the rate of consumption has to become negative at a finite point in time, which contradicts Proposition 2. This as well as many other inaccuracies seem to be caused by his implicit and inappropriate assumption that variables are continuous functions of time throughout, even in the case when a constant consumption path cannot be sustained indefinitely. A competitive path with constant consumption and positive and constant (present) value of net investments (i.e., $\nu > 0$) can be sustained up to the point when the resource stock has been exhausted. The path from then on must be a completely different path, which cannot be governed by the generalised Hartwick rule with $\nu > 0$. E.g., it is not correct, as claimed by Hamilton (1995, pp. 397-398), that resource extraction goes continuously to zero as the stock of natural capital approaches exhaustion.

It is an open question whether Correct Claim 1 can be strengthened to the following statement for competitive paths in an economy with constant population and constant technology: "If $\mathbf{q}(t)\dot{\mathbf{k}}(t)$ is non-negative for all $t \in (0, \infty)$, then, for any $t \in (0, T)$, $u(\mathbf{c}(t))$ can be sustained forever given $\mathbf{k}(t)$." We cannot prove this under general assumptions, but neither do we have a counterexample.

5. Myth 2: The Hartwick Rule for Sustainability Requires Substitutability Between Man-made and Natural Capital

Hartwick (1977) concentrated his attention on economies where substitution of man-made capital and resource extraction is possible. In the wake of his contribution an impression appears to have been formed to the effect that the Hartwick rule for sustainability requires that man-made capital can substitute for natural capital; i.e., that the production possibilities are consistent with the beliefs held by the proponents of 'weak sustainability' (cf. the citation from Spash and Clayton, 1997, reproduced in the Introduction). If, on the other hand, natural capital has to be conserved in order for utility to be sustained (i.e., the world is as envisioned by the proponents of 'strong sustainability'), then – it is claimed – the Hartwick rule for sustainability does not apply.

The *relevance* of the Hartwick rule for sustainability is related to the question of whether a constant utility path exists. Since a false premise does not falsify an implication, the Hartwick rule for sustainability as an implication is true even if, in some specific model, there does not exist any path where $\mathbf{q}(t)\dot{\mathbf{k}}(t)$ equals zero for all t. What the Hartwick rule for sustainability entails is that if no constant utility path exists, then there cannot exist any path where $\mathbf{q}(t)\dot{\mathbf{k}}(t)$ equals zero for all t. Still, even though the non-existence of an egalitarian path does not falsify the Hartwick rule for sustainability, it is interesting to discuss in what kind of technologies there exists an egalitarian utility path, implying that the result is relevant (i.e., not empty).

It turns out that substitutability is not necessary for the relevance of the Hartwick rule for sustainability.

INCORRECT CLAIM 5. *The Hartwick rule for sustainability is relevant only if man-made capital can substitute for natural capital.*

We provide a counterexample in a model which combines the restriction of the Ramsey model that available output must be produced, $c(t) + \dot{k}_M(t) \leq f(k_M(t))$, with the restriction that available output requires raw material in fixed proportion, $c(t) + \dot{k}_M(t) \leq r(t)$. Hence, we refer to it as the *complementarity model*.[4] The production function f satisfies the assumptions of the Ramsey model (cf. section 3), and the raw material is extracted without cost from a stock of a *renewable* natural resource, $k_N(t)$, with a rate of natural regeneration that equals $k_N(t) \cdot (\bar{k}_N - k_N(t))$, where $\bar{k}_N > 0$. Together with the restrictions that $c(t) \geq 0$, $r(t) \geq 0$, $k_M(t) \geq 0$, and $k_N(t) \geq 0$, this determines what triples $(c(t), \mathbf{k}(t), \dot{\mathbf{k}}(t))$ are attainable at time t, where $\mathbf{k}(t) = (k_M(t), k_N(t))$. The initial stocks are given by $\mathbf{k}^0 = (k_M^0, k_N^0)$.

As long as output does not exceed the maximal rate of natural regeneration, $(\bar{k}_N)^2/4$, this model behaves as the Ramsey model. However, if one tries to sustain production above such a level, then the resource stock will be exhausted in finite time, undermining the productive capabilities. The competitiveness condition (1b) implies that

$$c(t) + \dot{k}_M(t) = \min\{f(k_M(t)), r(t)\} \tag{9a}$$

$$r(t) + \dot{k}_N(t) = k_N(t) \cdot (\bar{k}_N - k_N(t)) \tag{9b}$$

$$p(t) = q_M(t) \tag{9c}$$

$$(q_M(t) - q_N(t)) \cdot f'(k_M(t)) = -\dot{q}_M(t)\,, \tag{9d}$$

$$q_N(t) \cdot (\bar{k}_N - 2k_N(t)) = -\dot{q}_N(t)\,, \tag{9e}$$

Any competitive path with constant consumption forever will satisfy the (ordinary) Hartwick investment rule by having the stock of man-made capital remain constant and the value of investment in natural

[4] Variants of this model appear in Asheim (1978) and Hannesson (1986).

capital equal to zero. Hence, constant consumption along a competitive path is characterised by $c^* = f(k_M^*)$, implying that $\dot{k}_M(t) = 0$, while $q_N(t)\dot{k}_N(t) = 0$. If, along such a path, the resource stock converges to a size larger than the one corresponding to the maximal level of natural regeneration, then $q_N(t) \equiv 0$ and the productivity of man-made capital measures the consumption interest rate: $f'(k_M^*) = -\dot{q}_M(t)/q_M(t)$. If, on the other hand, the resource stock is constant and smaller than the size corresponding to the maximal level of natural regeneration, then $c^* = f(k_M^*) = r^* = k_N^* \cdot (\bar{k}_N - k_N^*)$ and $q_N(t) > 0$. And the productivity of natural regeneration measures the consumption interest rate: $f'(k_M^*) > -\dot{q}_M(t)/q_M(t) = -\dot{q}_N(t)/q_N(t) = \bar{k}_N - 2k_N^*$. In this latter case, the application of the Hartwick investment rule leads to a feasible egalitarian path by keeping both stocks constant. Hence, the model is consistent with the world as envisioned by the proponents of 'strong sustainability'; still, the Hartwick rule for sustainability applies.

To state a correct claim concerning the relevance of the Hartwick rule for sustainability, we define the concept of 'eventual productivity'.

Definition 6. A model satisfies *eventual productivity given the initial stocks* $\mathbf{k}^0$ if starting from $\mathbf{k}^0$ there exists a regular path with constant utility forever.

CORRECT CLAIM 2. *The Hartwick rule for sustainability is relevant in an economy with constant population and constant technology if eventual productivity is satisfied given the initial stocks* $\mathbf{k}^0$.

Proof. From eventual productivity and the converse of the Hartwick rule for sustainability (Proposition 4), it follows that there exists a path with $\mathbf{q}(t)\dot{\mathbf{k}}(t)$ being constant and equal to zero for all t. □

The question of whether man-made capital can substitute for natural capital is important for the relevance of the Hartwick rule for sustainability only to the extent that a lack of such substitutability means that eventual productivity cannot be satisfied.

6. Prescription or Description?

The preceding analysis leads to the following questions: Can the Hartwick investment rule be used as a prescription? Or is the Hartwick rule for sustainability (and its converse) a description of an egalitarian utility path; i.e., a characterisation result?

The new class of simple counterexamples presented in section 4 yields the finding that *(i)* a generation may well obey the Hartwick investment rule but nevertheless consume more than the maximum sustainable consumption level, as well as the novel result that *(ii)* a generation with a negative value of net investments will not necessarily undermine the consumption possibilities of its successors. The analysis of section 4 thus reinforces the message of Toman et al. (1995, p. 147), namely that the Hartwick investment rule cannot serve as a prescription for sustainability. It is not enough to know whether the current investment in man-made capital in value makes up for the current depletion of natural capital, since the Hartwick result (Proposition 1) only says that following the Hartwick investment rule will entail constant consumption for an interval of time. This is neither sufficient nor necessary for development to be sustainable. Rather, a judgement on whether short-run behaviour is compatible with sustainable development must be based on the long-run properties of the path and the technological environment. By the generalised Hartwick rule for sustainability (Corollary 2) these long-run properties are:

1. *Competitiveness conditions.* The generalised Hartwick rule for sustainability requires that the economy realises a perfectly competitive equilibrium indefinitely. In particular, this entails that all externalities will be internalised. How can we know *now* that competitiveness conditions will be followed at any future point in time?

2. *Constant present value of net investments.* The generalised Hartwick rule for sustainability requires that $\mathbf{q}(t)\dot{\mathbf{k}}(t)$ is constant indefinitely. It is not sufficient to have current price-based information about the path in order to prescribe sustainable behaviour; rather such information has to be available at all future points in time. How can we know *now* that $\mathbf{q}(t)\dot{\mathbf{k}}(t)$ will be constant for all t?

3. *Feasibility.* The generalised Hartwick rule for sustainability is relevant only if positive and constant consumption can be sustained indefinitely. How can we know *now* that a path with constant consumption during some interval of time can be sustained forever? The Dasgupta-Heal-Solow model illustrates these problems; e.g., our counterexample to Incorrect Claim 2 shows how feasibility breaks down simply due to an overestimation of the resource stock.

4. *No exogenous technological progress.* The generalised Hartwick rule for sustainability is valid only if all technological progress is endogenous, being captured by accumulated stocks of knowledge. How can we know *now* that we will be able to attribute any future technological progress to accumulated stocks of knowledge? Similar problems arise for an open economy facing changing terms-of-trade (e.g., a resource exporter facing increasing resource prices).

Moreover, if all this information about the long-run properties of paths as well as the future technological environment were available, and a constant consumption path is desirable, then the price-based information entailed in Hartwick's rule would hardly seem necessary nor convenient for social planning. Therefore, it is our opinion that the Hartwick investment rule has little prescriptive value for decision-makers trying to ensure that development is sustainable.

The Hartwick investment rule is, however, of interest when it comes to describing an efficient path with constant utility. It follows from the converse of the Hartwick rule for sustainability (Proposition 4) that any such egalitarian path will be characterised by the Hartwick investment rule being satisfied at all points in time. Note that the importance of this result is not that it tells decision-makers anything concerning how to steer the economy along such a path; rather, it describes how the path would look if it were followed. Hence, in line with the view of Toman et al. (1995, p. 147), it seems more natural to consider $\mathbf{q}(t)\dot{\mathbf{k}}(t) = 0$ for all t as a descriptive result, characterising an efficient and egalitarian utility path. What we have added here is to point out that this characterisation result follows from the converse of Hartwick's rule for sustainability and that it is general: While its relevance relies on the assumption of eventual productivity and its validity on the assumption that all technological progress can be attributed to accumulated stocks of knowledge, it does *not* impose any particular requirements on the possibility of substitution between man-made and natural capital, as was seen in section 5. The Dasgupta-Heal-Solow model is only one application among many.

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