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PRICE RECALL, BERTRAND PARADOX AND PRICE DISPERSION WITH ELASTIC DEMAND

By Miguel Carvalho

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TILBURG UNIVERSITY

Price Recall, Bertrand Paradox and Price Dispersion with Elastic Demand

Miguel Carvalho
CentER,
Tilburg University,
Netherlands
m.carvalho@uvt.nl

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Abstract

This paper studies the consequence of an imprecise recall of the price by the consumers in the Bertrand price competition model for a homogeneous good. It is shown that firms can exploit this weakness and charge prices above the competitive price. This markup increases for rougher recall of the price.

If firms have different production costs, those with higher costs are not driven out of the market. However they choose to have a higher price in equilibrium, therefore price dispersion arises. It is shown that firms behave on average as a monopolist with stricter demand and that price dispersion increases with the price recall errors. If bigger recall errors happen, then both consumers and firms on the aggregate level are worse off, for some parameter choices. Furthermore being given the irrational choice that some consumers make, there are situations where the protection of a monopolist against entrants is a welfare maximizing policy.

The introduction of more firms in the market does not have a significant impact on the prices. Even though the presented model is static, it can be interpreted as a stage game of an infinitely repeated game where a Nash Equilibrium is played in every stage. The intuition is that consumers do not actually seek information before every purchase, but have a vague idea of the price they faced in previous purchases.

Keywords: Behavioral Industrial Organization, Bounded Rationality, Price Recall, Price Dispersion, Bertrand Paradox

JEL classification: D03, D43, D83, L11, L13

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1 Introduction

There is hardly someone that knows the precise price of a given consumption good in the closest stores. Take a retail store, one may have a reasonable idea on the average price level at different stores but one does not recall the precise price of golden apples. The textbook model of price competition for homogeneous goods assumes however that consumers are fully informed about the prices posted by the firms. Everything else equal this means that not all consumers head to the store actually having the lowest prices, as commonly assumed.

While the issue is absent from the Industrial Organization literature, the Marketing literature has handled it for decades. Monroe and Lee (1999) present a summary on price-awareness research, stating that previous studies have found that the average absolute recall error ranges from 6% to 19.45% of the correct price[1]. Neglecting this limited rationality of consumers cannot be seen as a merely theoretical simplification, for it implies that in the basic price competition model with a homogeneous good firms charge a markup over the cost and that profits are therefore non-zero. Furthermore equilibrium prices will depend on the number of firms and do not equal marginal costs.

In this paper a model for the pricing behavior of firms in a Bertrand setting facing consumers with imperfect recall is proposed, abstracting from other (classical) deviations as heterogeneous goods, search costs, spatial competition, product differentiation, price discrimination, etc. The imperfect recall on prices is modeled as a random shock (with mean zero) that is added to the real price. Consumers decide where to shop following their wrongly recalled prices, but at the store the demanded quantity is a function of the real price. It is shown that firms charge a markup, following a pricing strategy equivalent to that of a monopolist facing a stricter price elasticity. The markup increases with the incorrect recall of prices by consumers.

Once the full awareness of the price is dropped, price dispersion becomes a possibility because consumers do not fully react to the price differences. In the present model it arises due to cost differentials, which also means that firms with higher costs are not driven out of the market. Instead of mixed strategies or random strategy pricing equilibria, this model proposes the inability of consumers to screen between high and low cost firms as an explanation for the existence of price dispersion on a homogeneous good. The monopoly analogy is still robust for different costs. Intuitively one might foresee that larger recall errors would imply lower equilibrium price gaps because of diminishing incentives to price differentiation among firms. In reality the low cost firm will choose to make its price advantage more salient, increasing price dispersion.

Introducing more firms has a weak competition pressure on prices, which does not lead to marginal cost pricing even with infinitely many firms. In fact increasing the number of firms can at most have the same effect as reducing the price recall errors of consumers to half. It is shown that the equilibrium price dispersion has a sensitive dependence on the cost structure of the firms in the market.

Both exogenous and endogenous shocks are considered. In the latter consumers are aware of their limitations and can choose to make a higher costly effort in order to improve the accuracy of their price recall. The firms are however fully rational and maximize their profits anticipating the errors of the consumers.

While the proposed model is a static game, it can be interpreted as the (constant) outcome of a repeated game where there is no learning process by the consumers.

The paper is structured as follows. Section 2 reviews the literature in the field, Section 3 introduces the basic model and discusses the main implications, Section 4 examines the introduction of more firms and the importance of the price structure, Section 5 endogenizes

[1]The authors argue in the mentioned paper that consumers do recall more than what they explicitly acknowledge, but there is no doubt that price recall is not perfect.

the price recall error committed by the consumers, Section 6 compares the results of the present paper with closely related models in the literature and Section 7 concludes.

2 Related Literature

There is a closely related literature to this paper on pricing with boundedly rational consumers. Hehenkamp (2002) proposes an evolutionary game where consumers only receive information about the prices of the firms with some given probability. Sellers on the other hand have a probability of learning about the other sellers' prices and profits, mimicking the one with higher profit. Depending on the level of sluggishness, i.e. frequency with which they receive new information, the equilibrium price will fall between the marginal cost and monopoly pricing regimes. Chen, Iyer, and Pazgal (2005) use the limited memory model of Dow (1991), where consumers do not recall the exact price but only a price range to which it belongs, the price ranges being optimally chosen. These consumers constitute a fraction of all consumers in a price competition setting, the remaining being either fully informed or fully uninformed. In equilibrium they choose to have finer partitions in the low prices range, and firms choose to have a degenerate random price strategy, where the number of possible prices equals the number of memory partitions. In Gabaix and Laibson (2004) and Gabaix, Laibson, and Li (2005) consumers make errors when evaluating their inherent value of a product. Having firms competing for an homogeneous good leads in this case to a markup. The authors show that increasing competition, i.e. more firms in the market, has almost no consequence in terms of a markup decrease. If allowed to increase noise of the product evaluation, e.g. through confusing characteristics of the good, they choose to do so in an inefficient way.

Another branch of pricing models where rationally bounded consumers are exploited is add-on pricing. In Gabaix and Laibson (2006) consumers are unable to fully take into account the add-on charge, so that firms have positive profits.

Search costs are another source of imperfect competition leading to similar outcomes. The seminal paper by Diamond (1971) assumes that consumers do not know the prices of firms, having to visit different stores, only purchasing when a price below a given cutoff price is found. Prices will be adjusted to a unique equilibrium price in finite time, namely the monopoly price. Stahl (1989) shows that assuming two types of consumers (zero and positive search costs) leads to intermediate results, i.e. equilibrium prices between the marginal cost and the monopoly pricing.

Starting with Varian (1980) there is a literature mainly interested in price dispersion, where it is assumed that firms choose a random pricing strategy, in opposition to a fixed price. That is the strategy space is a set of probability distrubions, not the the positive real numbers. In Varian (1980) a fraction of the consumers is persistently uniformed about the prices, but having a reservation price. If stores are allowed to choose a random price distribution, they choose to do so in equilibrium balancing the probability of having the lowest price (and therefore getting the informed consumers) and maximizing profits with the uninformed consumers. Spiegler (2006) comes to a similar conclusion when all consumers are unable to take the random pricing strategy into account, and thus sample the prices in the stores taking thereafter that sample as the final price and picking the lowest price. In this context bounded rationality can also be attributed to firms as in Baye and Morgan (2004). Consumers are fully informed and rational whereas the firms choose random pricing strategies playing either Nash equilibrium, quantal-response equilibrium or ε-equilibrium. It is shown that the last two are closer to the results obtained in experiments (where subjects choose prices and rational consumers are played by the computer). In Alos-Ferrer, Ania, and Schenk-Hoppe (2000) firms play a pricing oligopoly evolutionary game, following a simple behavior of imitation and experimentation.

While formally close to the present paper, the product differentiation paper by Perloff and Salop (1985) has a different motivation. Their paper suggests a model with differentiated goods where consumers have heterogeneous preferences over the available products. Firms exploit that by charging a markup, which is increasing in the variance of the preferences. In the limit case of fixed demand that model is formally equivalent to the models where consumers make mistakes about the value of the good (as in Gabaix and Laibson (2004) and Gabaix, Laibson, and Li (2005)) and to the present model, where consumers have imperfect recall of prices. This is discussed in Section 6.1.

On the empirical side Monroe and Lee (1999) show that consumers do not perfectly recall the prices when explicitly asked to do so. Baye and Morgan (2004) and Pan, Ratchford, and Shankar (2004) indicate that price dispersion exists in settings which are very close to the textbook Bertrand competition. In the experiment of Kalayci and Potters (2009) individuals playing the firms choose to make product comparisons more complex to the individuals acting as consumers, so that they do not choose optimally allowing firms to charge a markup. This markup is increasing in the confusion caused in consumers.

While having a different motivation, the present model can be thought of one of horizontal differentiation with fixed positions as a first intuition. The value of the good is the same across consumers, but each one is biased towards one of them. Therefore having a higher price does not imply zero demand. Firms compete for the indifferent consumer (here the one recalling prices as being equal) at the margin, etc. But as it shall be seen the results differ, because the 'gap' between one consumer and one firm, in the horizontal differentiation models, decreases the value of the good to the consumer either due to transportation costs or preferences in the product space. That is not the case for imperfect price recall. This is discussed thoroughly in Section 6.2.

3 Model

3.1 Basic Setup

Consider two risk neutral firms, A and B, selling one homogeneous good whose cost of production is zero. Firms announce their price simultaneously, p_A by firm A and p_B by firm B.

Consumer $\alpha \in [0, 1]$, recalls prices $p_i'^{\alpha} = p_i + \epsilon_i^{\alpha}$, for $i = A, B$, where ϵ_A^{α} and ϵ_B^{α} are independently and identically distributed shocks for each α with non-degenerate probability density function $f(\cdot)$ and cumulative distribution function $F(\cdot)$ with an expected value of zero[2]. The consumers then do their shopping at the firm with the lowest recalled price[3] (it can be assumed that they randomize in case of a tie, but this happens with probability zero). At the store they learn the real price so the demand curve is therefore given by the real and not the recalled price. It is assumed that transportation costs between the two firms are high enough, in the sense that the consumer does not visit the second store if she learns that the real price of that firm is higher than the recalled price of the other firm. This may be pictured as having the consumer at home wondering where to buy a product without accurately remembering the prices. When the consumer gets to the chosen store and learns the real price, her cost of going to the other store and checking its price as well is higher than the (possible) expected gain.

[2] For a more detailed model of memory related bounded rationality see Mullainathan (2002).

[3] An equivalent interpretation of the same setting is to consider that the consumer visits the firms sequentially. On the arrival at the second firm, she does not correctly recall the first price p_1 but recalls it plus a shock given by $p_1 + \epsilon_1 - \epsilon_2$. If she considers the second price as lower, she buys there instantly, otherwise she goes back to the first firm. This interpretation is very close to the model in Chen, Iyer, and Pazgal (2005), but it relies on a unintuitive assumption. It must be assumed that in those cases where the consumer decides to return to the first firm, she never returns to the second if she learns that she made a mistake. In other words the transportation costs of the two first trips are negligible, but the third trip is infinitely costly.

Firms anticipate this behavior and play best response to their competitor's strategy, resulting in a Nash Equilibrium in pure strategies. Notice that for a degenerate distribution shock with zero variance and zero mean this model reduces to the basic Bertrand model. Firms are risk neutral.

Let $D(p)$ be the demand curve of each consumer, where p is the real price of the firm at which the consumer is buying the good. Firms are assumed to be aware of this demand function. When facing the prices $p_A'^{\alpha}$ and $p_B'^{\alpha}$, consumer α will find price p_i lower than p_j with the following probability:

$$\begin{aligned} P\left(p_i'^{\alpha} < p_j'^{\alpha} \middle| p_i, p_j\right) &= P\left(p_i + \epsilon_i^{\alpha} < p_j + \epsilon_j^{\alpha}\right) = P\left(\epsilon_i^{\alpha} < \epsilon_j^{\alpha} + p_j - p_i\right) \\ &= \int_{-\infty}^{\infty} f(y)F(y + p_j - p_i)dy \\ &= g(p_j - p_i), \end{aligned}$$

where $g(x)$ is the market share of a firm whose price is lower than its competitor price by x. It is given by $g(x) = \int_{-\infty}^{\infty} f(y)F(y+x)dy$. Notice that $g'(x) = \int_{-\infty}^{\infty} f(y)f(y+x)dy$. Moreover $g(x)$ is independent of α because all consumers have the same price recall shock distribution. Now given the behavior of consumers, firm i maximizes expected profits $\Pi_i(p_i, p_j) = \mu p_i g(p_j - p_i)D(p_i)$ over $p_i \geq 0$ with $i, j = A, B$, $i \neq j$, where μ is the number of consumers. Without loss of generality it is assumed that $\mu = 1$. The first order condition for a maximum is

$$\begin{aligned} \frac{\partial \Pi_i}{\partial p_i} = g(p_j - p_i)D(p_i) - p_i D(p_i)g'(p_j - p_i) + p_i g(p_j - p_i)D'(p_i) = 0 \\ \Leftrightarrow -p_i \frac{D'(p_i)}{D(p_i)} + p_i \frac{g'(p_j - p_i)}{g(p_j - p_i)} = 1 \\ \Leftrightarrow \varepsilon(p_i) - p_i \frac{d}{dx} \ln g(p_j - p_i) = -1, \end{aligned} \tag{1}$$

where $\varepsilon(p) \equiv \frac{d \ln D(p)}{d \ln p}$ is the price elasticity of demand. Notice that this equation can be rewritten as

$$\varepsilon(p_i) + \varepsilon_i^g(p_i, p_j) = -1 \tag{1'}$$

with $\varepsilon_i^g(p_i, p_j) \equiv p_i \frac{\partial}{\partial p_i} \ln g(p_j - p_i) = -p_i \frac{d}{dx} \ln g(p_j - p_i)$ being the own price elasticity of the market share of firm i. This is just the usual result of a profit maximization but it will help giving some insight to the model later in this Section. Now further assumptions on functional forms are stated.

Assumption 1 *The random shocks ϵ_A and ϵ_B are iid with mean 0, variance σ^2 and full support on the real line with probability density function $f(.)$ and cumulative distribution function $F(.)$. Moreover the distribution of ϵ_A and ϵ_B is such that $g(\cdot)$ is logconcave, i.e. $\frac{d}{dx} \ln g(x)$ is non-increasing in x.*

Assumption 2 *The price elasticity $\varepsilon(p)$ of demand is continuous, non-increasing and $-1 \leq \varepsilon(0) \leq 0$.*

Notice that Assumption 2 is rather weak, linear demand is an example that satisfies it. Compare it with the necessary conditions for the classic monopolistic price setting model to be well-defined, i.e. unique profit maximizing price, namely that $\varepsilon(p)$ takes the value -1 for one and only one $\hat{p}$ and that it is bigger (smaller) than -1 before (after) $\hat{p}$[4]. Assuming $\varepsilon(0) \geq -1$ guarantees that the equilibrium will not be a corner solution, having firms outside the market. The following results will be used.

[4]These conditions guarantee that the derivative of the profit function equals zero for one unique price, and that it is quasi-concave.

Lemma 1 *For each $p_j \geq 0$, $j \neq i$, $i = A, B$, equation (1) has a unique positive solution.*

Proof
First it is shown that the left-hand side (LHS) of (1) is decreasing in p_i. The first term, $\varepsilon(p_i)$, is non-increasing according to Assumption 2. In the second term the part $\frac{d}{dx}\ln g(p_j - p_i)$ is positive, because

$$\frac{d}{dx}\ln g(p_j - p_i) = \frac{g'(p_j - p_i)}{g(p_j - p_i)} > 0.$$

From Assumption 1 it is non-decreasing in p_i since

$$\frac{\partial}{\partial p_i}\left[\frac{d}{dx}\ln g(p_j - p_i)\right] = -\frac{d^2}{dx^2}\ln g(p_j - p_i) \geq 0,$$

where $\frac{\partial x}{\partial p_i} = -1$ was used. Because firm i only considers $p_i \geq 0$, p_i is obviously positive and increasing in p_i. Therefore minus their product, i.e. the second term on the LHS is negative and decreasing in p_i. Hence, the LHS is decreasing in p_i.
Now, for $p_i = 0$ the LHS equals $\varepsilon(0) \geq -1$ because $\frac{\partial}{\partial p_i}\ln g(p_j - p_i)|_{p_i=0}$ is finite due to the random variables' full support. For $p_i > 0$ the first term of the LHS $\varepsilon(p_i)$ is negative and non-increasing in p_i. In the second term $\lim_{p_i \to \infty}\frac{d}{dx}\ln g(p_j - p_i)$ is strictly positive because $\frac{d}{dx}\ln g(p_j - p_i)$ is strictly positive and non-decreasing in p_i. This part of the second term is multiplied by p_i, therefore the second term goes to $-\infty$ as $p_i \to \infty$. This implies that the limit of the LHS is $-\infty$ as $p_i \to \infty$. Hence the LHS must equal -1 for a unique p_i, for any given $p_j \geq 0$. ■

The sufficiency of the condition for a maximum is implied by the above mentioned uniqueness.

Lemma 2 *The first order conditions are sufficient for the profit maximization problem.*

Proof
It follows from the proof of Lemma 1 that the solution for $\frac{\partial \Pi_i}{\partial p_i} = 0$ for $p_i \geq 0$ is unique and that $\frac{\partial \Pi_i}{\partial p_i}$ switches its sign, from positive to negative, at that point. These properties imply that $\Pi_i(p_i, p_j)$ is quasi-concave on p_i for any p_j, so that the FOC are sufficient. ■

The next step is to see that this game has a unique Nash equilibrium. It was shown in the proof of lemma 2 that the reaction curves are properly defined, the next step is to prove that they cross. The symmetry of the problem indicates the possible existence of a symmetric equilibrium. It is known that for each p_j there is a unique $p_i(p_j)$ solving (1), so it can be checked in (1) whether some $p_i = p_j = p$ is a possible solution. For firm i, $i = A, B$, (1) becomes

$$\varepsilon(p) - p\frac{g'(0)}{g(0)} = -1.$$

Since $g(0) = \frac{1}{2}$,

$$2pg'(0) = 1 + \varepsilon(p). \tag{2}$$

From Assumption 2 which implies that $\varepsilon(0) \geq -1$ and $\varepsilon(p)$ is nonincreasing, and the fact that $g'(0)$ is positive it is concluded that equation (2) has just one solution $p^* \geq 0$.

Lemma 3 *The reaction curves implied by the first order conditions yield a unique (subgame perfect) equilibrium price p^* for both prices.*

Proof
From (1) it is possible to prove that the slope of the reaction curve $p_i(p_j)$ lies between 0 and 1. To see this consider an increase Δ in $p_j \geq 0$, which increases the LHS of (1).

An increase in p_i (to be precise in $p_i(p_j)$) of Δ would offset the decrease of $g(\cdot)$ (because g contains the term $p_j - p_i$) but would increase the factor by which $\frac{\partial}{\partial p_i} \ln g(p_j - p_i)$ is multiplied, that is p_i, and decrease $\varepsilon(p_i)$, yielding a total decrease in the LHS. Because the LHS is strictly monotonous in p_i, the price p_i implicitly defined by (1) (that is the best reaction of firm i) must increase less than Δ. In other words the slope of the reaction curve satisfies $\frac{\partial p_i(p_j)}{\partial p_j} < 1$ for all $p_j \geq 0$, and by symmetry $\frac{\partial p_j(p_i)}{\partial p_i} < 1$ for all $p_i \geq 0$. This means that $p_i(p_j)$ is always flatter in the (p_j, p_i) plane than $p_j(p_i)$, implying that they only cross once. There is therefore no other equilibrium besides $p_A = p_B = p^*$.■

Proposition 1 *In the basic model where consumers suffer a price recall shock and firms face zero costs, the firms are able to charge a nonnegative price in equilibrium* p^* *defined by*

$$2p^* g'(0) = 1 + \varepsilon(p^*) \tag{3}$$

Moreover they charge $p^* = 0$ *if and only if a monopolist also would do it.*

Proof
The last statement follows from setting $p = 0$ in (2) and noticing that it is only a solution if $\varepsilon(0) = -1$, which is the case where a monopolist is indifferent between selling and exiting the market.■

Thus in this model firms are able to exploit the bounded rationality of consumers. The intuition of the result, in opposition to the zero profit solution, is that announcing a price lower than that of the opponent is not enough to attract all consumers, because some of them will still recall the price as higher. This means that charging a price marginally below, equal to or marginally above the opponent is irrelevant. The pressure of price competition is eroded by the inability of consumers to reward firms with lower prices, that is failing to give the right incentives for firms competing on the same good.

Here there are two trade-offs playing a role in the price-setting decision. One is a Hotelling type of trade-off, higher revenue per consumer vs higher market share, the other one is that of a monopolist, higher revenue per unit sold vs more goods sold per consumer. The equilibrium price depends on the strength of both.

3.1.1 Recall Mistake and Hotelling Trade-off

The firm opts between the increase of market share achieved through lower price, and the revenue per consumer achieved through higher price. This appears in equation (3) in the term $g'(0)$, which stands for the marginal decrease in the market share due to a price increase. In other words, it is the marginal change of the indifferent consumer as in Hotelling models.

The term $g'(0)$ is lower for higher variance. The intuition is straightforward, the more difficulties the consumers have in remembering and therefore comparing the prices, the smaller the marginal change in the market share due to a price variation. Suppose the change of the variance is achieved through the "spreading" of the possible random values, that is changing x to σx with $\sigma > 0$. The new density function f satisfies $\sigma f(\sigma x) = f_0(x)$, with $f_0(x)$ standing for $f(x)$ with $\sigma = 1$, so that $\int f(y)dy = \int \sigma^{-1} f_0(\sigma^{-1} y)dy = \int f_0(x)dx = 1$, where $x = \sigma^{-1} y$ and $dy/dx = \sigma$ was used.

The new variance is given by $var(y) = \sigma^2 var(x)$. The term appearing in equation (3) changes according to

$$\int_{-\infty}^{\infty} f^2(y)dy = \int_{-\infty}^{\infty} \sigma^{-2} f_0^2(\sigma^{-1} y)dy = \frac{1}{\sigma} \int_{-\infty}^{\infty} f_0^2(x)dx, \tag{4}$$

or simply $g'(0) = \sigma^{-1} g_0'(0)$, where $g_0'(0)$ is the $g'(0)$ for $\sigma = 1$. The variation in the new equilibrium price is seen in the new version of equation (3)

$$\begin{aligned} 2p^* g'(0) &= 1+\varepsilon(p^*) \\ \Leftrightarrow 2p^* g_0'(0) &= \sigma\left(1+\varepsilon(p^*)\right). \end{aligned} \tag{5}$$

Implicit differentiation of (5) yields

$$\frac{dp^*}{d\sigma} = \frac{1+\varepsilon(p^*)}{2g_0'(0)+\sigma\varepsilon'(p^*)} > 0.$$

As expected, higher price uncertainty σ means higher price markup. Compared to the fixed demand case (take $\varepsilon(p) = \varepsilon'(p) = 0$) this influence is however smaller, because the denominator is now bigger and the numerator smaller (remember that $0 \le 1+\varepsilon(p^*) < 1$). The intuition is that firms must also take the diminishing demand into account. It is not true that the marginal increase of the price due to σ is always diminishing for it depends on the value of $\varepsilon'(p^*)$. Two extreme results can however be established.

Corollary 1 *For a decreasing price recall error variance the equilibrium price goes to the Bertrand price.*

Corollary 2 *If the demand allows for a monopoly price, i.e. there is a $\hat{p}$ such that $\varepsilon(\hat{p}) = -1$, then for an increasing price recall error variance the equilibrium price goes to the monopoly price.*

The first result follows easily from (5) with $\sigma \to 0$ which forces the first term to be zero, and because the derivative of $g_0(\cdot)$ is a strictly positive constant, p^* must be zero. The second result follows from the fact that the first term in (5) is bounded (because p^* is bounded) so as $\sigma \to \infty$ it must be that $1+\varepsilon(p^*) \to 0$.

3.1.2 Monopolist Trade-off

Because each consumer's demanded quantity also depends on the stated (in opposition to recalled) price, firms face a monopolist-type of price setting trade-off. High price means higher per unit revenue but also less units being sold. Because of non-increasing elasticity of demand this monopolist type of decision also decreases the equilibrium price. This can be seen in (1'), where both non-increasing elasticities are added up and set equal to -1. Notice that the left-hand side of (3) is positive at p^*, so in equilibrium it must be that $\epsilon(p^*) > -1$, that is the equilibrium price has the monopoly price (if it exists) as an upper bound.

3.1.3 Explicit Equilibrium Solutions

To gain some more insight several specific functional forms are considered. Suppose demand $D(p)$ has a linear elasticity over all $p \ge 0$ range, with $\varepsilon(p) = -(a+bp)$ for some $a \in [0,1]$ and $b \ge 0$. Equation (3) simplifies to

$$2pg'(0) = 1-(a+bp),$$

so that in equilibrium

$$p^* = \frac{1-a}{b+2g'(0)}. \tag{6}$$

As a comparison, for the same price elasticity a monopolist would charge price $p_M = \frac{1-a}{b}$. If the recall errors ϵ_A and ϵ_B follow a normal distribution with mean 0 and variance

σ^2, which satisfies Assumption 1[5], then $g'(0) = (2\sqrt{\pi}\sigma)^{-1}$. If they follow a Gumbel distribution[6] with cumulative distribution function $F(x) = e^{-e^{-\tau(x-\mu)}}$ for some $\mu \in \mathbb{R}$, $\tau > 0$ and variance $\frac{\pi^2}{6\tau^2}$, then $g(x) = \frac{1}{1+e^{-\tau x}}$[7]. Now it holds $g'(0) = \frac{\tau}{4}$.

Some comments can be made for this closed form solutions. When firms face consumers with a wrong price recall, they charge a price which is related to the monopolist price. Formally it is equivalent to an increase in the slope of the demand elasticity, the worse the price recall is the flatter the slope. While this slope is not an intuitive concept, it may help to recall that the monopolist price does not depend on some point elasticity but on the price at which this elasticity crosses some threshold, namely -1. The higher (in absolute terms) the slope, the smaller the price the monopolist firm, as well as the competitive firms here, will choose.

The term related to the price recall depends on the standard deviation of the shock. For $\sigma \to 0$, the price goes to the usual Bertrand price equilibrium, that is $p^* = 0$.

Taking the linear term of the Taylor series expansion in respect to σ, at $\sigma = 0$, gives the first order impact of introducing a price shock compared to usual Bertrand, namely $\frac{1-a}{2g'(0)}$.

For small b the monopolist is able to charge a high price, because demand is very inelastic in the low prices range. Here competition among the firms compensates for the inelasticity of demand and sets a lower price.

3.1.4 Strategic Complements

The proof of Lemma 3 shows that the choices of the two players are strategic complements. Because the demand of consumers at the firm is not affected by the other firm's price, the market share is the only variable at stake. The actions are therefore strategic complements. While it is beyond the scope of this paper to model the dynamics of the equilibrium process, it is known from experimental economics (see Fehr and Tyran (2008) for instance) that games with strategic complements tend to converge faster to the equilibrium and to be more stable. The bounded rationality of the agents playing the firms is therefore a minor issue here.

3.1.5 Other Comments

Equation (3) provides a testable prediction of the model, provided that the price recall variance and the slope of the elasticity of demand is known. If valid, it can be used to estimate one from the other.

While having different motivations and intuitions the present model is formally equivalent for the basic case to one of horizontal differentiation with unitary demand and bounded support of the recall errors. It is easy to see that for each choice of parameters in this model, there is a distribution of consumers on the horizontal line and a distance (transportation or preference) cost that yield the same problem and therefore the same

[5]The sum of two independent random variables following $N(0,\sigma^2)$ is a normal random variable with $N(0,2\sigma^2)$. So $g(p_j - p_i) = \Phi\left(\frac{1}{\sqrt{2}\sigma}(p_j - p_i)\right)$ where $\Phi(\cdot)$ is the standard normal cumulative distribution function. Therefore $\frac{d}{dx}\ln g(x) = \frac{1}{\sqrt{2}\sigma}\left(\Phi'(\frac{x}{\sqrt{2}\sigma})\right)\left(\Phi(\frac{x}{\sqrt{2}\sigma})\right)^{-1}$. The ratio $\frac{\Phi'(x)}{\Phi(x)}$ is similar to the so-called hazard function $\frac{\Phi'(x)}{\Phi(-x)}$, which is strictly positive and strictly increasing for the normal distribution. Because $\Phi'(x) = \Phi'(-x)$, $\frac{\Phi'(-x)}{\Phi(-x)}$ must be strictly increasing, therefore $\frac{\Phi'(x)}{\Phi(x)}$ is strictly positive and strictly decreasing. Hence Assumption 1 applies to the normal distribution.

[6]Also known as Fisher-Tippet or log-Weibull, this distribution is important in Order and Extreme-value Statistics. It is also widely used in the literature on random utility models and on quantal response equilibria because of its mathematical tractability.

[7]Again it is easy to check that Assumption 1 is satisfied, for $\frac{d}{dx}\ln g(x) = \frac{\tau}{1+e^{\tau x}}$, which is decreasing in x.

result. The position on the line represents the price recall bias[8] towards one of the firms, the distance cost represents the dispersion of recall biases, the firms being located at the extremes of the line[9]. The intuition is straightforward, in both cases there are consumers that are inherently inclined to one of the firms, and this hinders perfect competition.

This similarity while giving helpful insights to the intuition in the present model, only holds for the basic setup. See Section 6.2 for further discussion.

3.2 Firms with Symmetric Costs

In this Section firms face a positive constant cost c for the production of the good. Because of the non-linearity of the model, the results will be more complex but with simple and intuitive limit cases. The objective function of firm i can be written as

$$\Pi_i(p_i, p_j) = (p_i - c)g(p_j - p_i)D(p_i),$$

for $i = A, B$. To maximize profit firm i solves the first order condition

$$\frac{1}{p_i - c} + \frac{D'(p_i)}{D(p_i)} - \frac{g'(p_j - p_i)}{g(p_j - p_i)} = 0.$$

In this Section the previous assumptions are also made so that the earlier lemmas will also be applicable here and these conditions will be sufficient.

Proposition 2 *If the firms face the same strictly positive unitary cost c, Assumptions 1 and 2 imply the existence and uniqueness of a symmetric equilibrium, where the equilibrium price p^* is implicitly defined by*

$$2p^* g'(0) = \frac{p^*}{p^* - c} + \varepsilon(p^*).$$

Proof
Rewriting the maximization condition as

$$\begin{aligned} \frac{p_i}{p_i - c} &= -p_i \frac{D'(p_i)}{D(p_i)} + p_i \frac{g'(p_j - p_i)}{g(p_j - p_i)} \\ \Leftrightarrow \frac{p_i}{p_i - c} &= -\varepsilon(p_i) - p_i \frac{\partial}{\partial p_i} \ln g(p_j - p_i), \end{aligned} \tag{7}$$

enables a similar proof to the one of Proposition 1. The LHS in (7) decreases monotonically from ∞ at $p_i = c$ to 1 when $p_i \to \infty$, and the RHS in (7) increases monotonically from $-\varepsilon(c) + c\frac{d}{dx} \ln g(p_j - c)$ at $p_i = c$ to ∞ as $p_i \to \infty$, so the equation is satisfied for only one $p_i \geq c$ for every p_j. This single crossing implies quasi-concavity of Π_i in p_i for any p_i given $p_j \geq c$, and therefore sufficiency of the first order conditions above. The focus is again on symmetric equilibria, so that $p_A = p_B = p$ and therefore $g(0) = \frac{1}{2}$, the equilibrium condition being

$$\frac{p}{p - c} = -\varepsilon(p) + 2g'(0)p. \tag{8}$$

[8]It is not strictly speaking a bias, because it is drawn from the difference of two random variables, but visualizing it as a bias may help the intuition here.

[9]In the bounded support case firms must be placed in the extremes, otherwise the consumers that would be placed outside the section between the two firms, would have exactly the same price recall bias. One of the firms could then have a discontinuous market share increase if its (negative) price difference in comparison to the other firm would be bigger than the distance cost between the two firms. In the case of full support the firms should also be located at the "extremes", otherwise there would not be consumers with all possible values of recall biases.

Again it is easy to see that there is such a p and it is unique. The reasoning in the proof of Lemma 3 can also be applied here, noting that a hypothetical increase in p_i of the same amount as a hypothetical increase in p_j would not only increase the RHS but also decrease the LHS. This implies that $p_i(p_j)$ increases less than p_j, in other words, the reaction curve slope is below 1. The equilibrium price p^* is therefore unique. ■

The unique symmetric equilibrium p^* can also be defined by

$$\begin{aligned}\frac{p^*-c}{p^*} &= \frac{1}{-\varepsilon(p^*)+2g'(0)p^*} \\ \Leftrightarrow \frac{p^*-c}{p^*} &= -\frac{1}{\bar{\varepsilon}(p^*)},\end{aligned} \tag{9}$$

where $\bar{\varepsilon}(p) \equiv \varepsilon(p) - 2g'(0)p$. Equation (9) is comparable to the usual first order condition $\frac{p-c}{p} = -\frac{1}{\varepsilon(p)}$ for the monopolist. The introduction of the consumer errors leads to the situation where firms have a monopolist like behavior but their Lerner index is reduced by a term that depends on the variance of the consumers' errors. The intuition is that once the consumer is at the firm, that is when the demanded quantity is set, the firm can act as a monopolist. But because of the a priori price competition, prices cannot be set too high.

To see that the equilibrium price and Lerner index are indeed smaller than those of the monopoly, notice that introducing $2g'(0)p$ on the right side of (8) yields a lower right hand side. Because the left (right) hand side of (8) decreases (increases) with p, the equilibrium price must be strictly smaller. Notice also that the additional term $2g'(0)p^*$ depends on the price, indicating that there is a variance-price level interaction in the equilibrium Lerner index[10].

The variation of the equilibrium price, or the Lerner index or the markup, as a function of the cost c depends on the two terms in the denominator. For low c and consequently low p, the first term may prevail so that the Lerner index is close to constant and the markup is increasing in c. However for high costs the second term becomes predominant if $-\varepsilon(p)$ does not increase indefinitely (or at least it increases less than proportional to p), meaning that the Lerner index will be proportional to p^{-1} and the markup will be constant. In the first case the recall errors are so high compared to the costs that consumers are shopping randomly. While for high c the $-\varepsilon(p)$ can be neglected so that the markup will be proportional to the standard deviation of the errors, namely given by

$$p^* - c \approx \frac{1}{2g'(0)}. \tag{10}$$

Considering higher prices in the denominator is equivalent to lower variances (recall that $g'(0)$ is inversely proportional to the standard deviation), the intuition being that the price recall error is relatively smaller when compared to the absolute value of the price. Once again the price and therefore the markup increases with the price recall errors magnitude

[10] The same analysis concerning the equilibrium price is not straightforward, because it is impossible to distinguish the changes due to the cost from those due to the diminishing demand. If the demand function is normalized, that is if the new demand function $\tilde{D}(\cdot)$ is chosen such that $\tilde{D}(p+c) = D(p)\ \forall p$ with $p \geq c \geq 0$, there will be no change in the equilibrium markup. The FOC is formally the same:

$$\begin{aligned}\frac{1}{p_i-c} + \frac{\tilde{D}'(p_i)}{\tilde{D}(p_i)} - \frac{\int_{-\infty}^{\infty} f(u)f(u+p_j-p_i)du}{\int_{-\infty}^{\infty} f(u)F(u+p_j-p_i)du} &= 0 \\ \Leftrightarrow \frac{1}{r_i} + \frac{D'(r_i)}{D(r_i)} - \frac{\int_{-\infty}^{\infty} f(u)f(u+r_j-r_i)du}{\int_{-\infty}^{\infty} f(u)F(u+r_j-r_i)du} &= 0,\end{aligned}$$

so that the optimal markup r^* equals the no-cost equilibrium price and $p^* = r^* + c$. The markup is therefore the same.

by consumers. This can be seen from equation (8) (by implicit differentiation) that shows that p^* decreases with $g'(0)$ and therefore increases with the error variance,

$$\frac{dp^*}{dg'(0)} = -\frac{2p^*}{\frac{c}{(p^*-c)^2} + 2g'(0)} < 0.$$

3.3 Firms with Different Costs and Price Dispersion

Introducing asymmetric costs for the firms leads to two interesting results. Firms with higher costs for the same homogeneous good do participate in the market, with higher equilibrium prices, for there will be always consumers recalling its price as smaller. The second feature is already implicit in the previous statement: the model predicts a price dispersion situation in a homogeneous good market.

Assumptions 1 and 2 are once again taken. Firm A faces unitary cost $c_A > 0$ and firm B faces $c_B > 0$. The profit of firm $i = A, B$ is given by $\Pi_i(p_i, p_j) = (p_i - c_i)g(p_j - p_i)D(p_i)$. In order to maximize it, firm i solves $\frac{\partial \Pi_i}{\partial p_i} = 0$, that is

$$\frac{p_i}{p_i - c_i} = -\varepsilon(p_i) - p_i \frac{\partial}{\partial p_i} \ln g(p_j - p_i), \tag{11}$$

with $i, j = A, B$ and $i \neq j$. Proposition 2 can be applied here with two changes. First the issue of existence of one equilibrium, i.e. an intersection of the reaction curves implicitly defined by (11), must be put differently because the system of equations is now asymmetric. Take $c_B > c_A$ without loss of generality. The best response $p_B(c_A)$ of firm B to the minimum price that firm A may offer, c_A, satisfies $p_B(c_A) \geq c_B > c_A$. So $(c_A, p_B(c_A))$ lies 'above' (taking p_B as the vertical axis) or on the $(p_A, p_B) = (c_A + t, c_B + t)$, $t \in \mathbb{R}$, diagonal. On the other hand, $p_A(c_B)$ satisfies $p_A(c_B) \geq c_A$ so $(p_A(c_B), c_B)$ lies below or on that diagonal. From Proposition 2 it is known that the best response curves always cross the main diagonal $p_A = p_B$[11], and they do that only once because $\frac{\partial p_i(p_j)}{\partial p_j} < 1$ for all $p_j \geq 0$ with $i, j = A, B$, $i \neq j$. So either $p_A(c_B)$ lies on the same side of the main diagonal as $p_B(c_A)$ (the crosspoint with the main diagonal lies then below c_B, representing a price combination that is never considered by the firms) so that $p_B(p_A)$ must cross $p_A(p_B)$ only once to get to the main diagonal, or $p_A(c_B)$ lies on the other side which means that $p_A(p_B)$ will also cross the main diagonal, crossing therefore $p_B(p_A)$ as well.

The second difference is that the equilibrium will not be symmetric, being thus implicitly defined by a system of equations with equation (11) and its counterpart, instead of being simply defined by (8). The FOC do not have here a closed solution because $p_A = p_B$ does not hold. It is however possible to determine how the equilibrium prices change depending on the costs through implicit differentiation of the FOC, when costs depart from the symmetric case $c = c_A = c_B$. In Section A.1 in the Appendix it is derived how both firms "react to changes" in one of the costs, which is shown in formulas (22) and (23). Some comments on those formulas are as follows.

3.3.1 Cost Price Relation

In the symmetric case it was clear that a higher cost implies a higher equilibrium price. While the trade-off here is quite more intricate it is the case that a cost increase of one firm leads to a price increase by both firms.

Proposition 3 *An increase in cost c_i of firm i leads in equilibrium to both an increase of p_i^* and p_j^* satisfying*

$$\frac{\partial p_i^*}{\partial c_i} > \frac{\partial p_j^*}{\partial c_i} > 0$$

[11] Take p_j to be the equilibrium price in the symmetric cost scenario with costs being $c = c_i$. By definition the best reply of firm i in this case is to set $p_i = p_j$, that is $p_i(p_j) = p_j$.

Proof
Inspecting equation (11), which defines the best strategy p_i given p_j and c_i, it can be seen that the left-hand side of equation (strictly) increases in c_i and the right-hand-side (strictly) decreases with p_j. Now for a given p_j an increase in c_i leads to an increase in the response p_i defined by (11). From Section 3.1.4 it is known that the price strategies in this game are strategic complements, so the best-response p_j of firm j shall also (strictly) increase. This decreases the right-hand side of equation (11), which further increases the implied best response p_i. Both changes point in the same direction and that implies the result $\frac{\partial p_i^*}{\partial c_i} > 0$. From strategic complementarity it is obtained that $\frac{\partial p_j^*}{\partial c_i} > 0$. At last, from $\frac{\partial p_i(p_j)}{\partial p_j} < 1$, which is shown in the proof of lemma 3 (notice that the FOC of firm j are not altered with the change in c_i), it follows that $\frac{\partial p_i^*}{\partial c_i} > \frac{\partial p_j^*}{\partial c_i}$. ■

One might have foreseen that a smaller cost gap would mean a more competitive market, meaning lower prices of both. Consider the case where the low cost firm suffers a cost increase. The high cost firm could choose to take benefit of the competitor handicap by trying to attract more consumers, that is by lowering the price. But it appears that the revenue increase that is attainable by a higher price of the high cost firm, dominates the benefits from trying to attract more consumers.

3.3.2 Monopolistic Aggregate Behavior

On the aggregate level, the firms act (locally) as a monopolist. It is known that the monopolist reaction to a cost change at marginal cost c equals

$$\frac{\partial p_M}{\partial c} = \frac{1}{1 - \left(\frac{p_M - c}{p_M}\right)^2 (p_M \varepsilon'(p_M) - \varepsilon(p_M))}. \tag{12}$$

Recall that the monopolist price p_M and the symmetric cost equilibrium price p^* are defined in the same way given $\varepsilon(\cdot)$ and $\bar{\varepsilon}(\cdot)$, respectively. It was already shown in equation (9) that the competitive price follows the cost in the same way as the monopolist price, when the cost of both firms change. To see that this is also the case for a change in just one of the firms' costs, say firm i, the following term must be examined $2 \times \frac{1}{2}\left(\frac{\partial p_i^*}{\partial c_i} + \frac{\partial p_j^*}{\partial c_i}\right)$, the factor 2 appearing because an increase in c_i yields just a half increase in the average cost and the factor $\frac{1}{2}$ is needed to have the average price change (and not their sum). From formulas (22) and (23) it must be that at $c = c_A = c_B$

$$\begin{aligned} \frac{\partial p_i^*}{\partial c_i} + \frac{\partial p_j^*}{\partial c_i} &= \frac{1}{1 - \left(\frac{p^* - c}{p^*}\right)^2 (p^* \varepsilon'(p^*) - \varepsilon(p^*))} \\ &= \frac{1}{1 - \left(\frac{p^* - c}{p^*}\right)^2 [p^*(\varepsilon'(p^*) + 2g'(0)) - (\varepsilon(p^*) + 2p^* g'(0))]} \\ &= \frac{1}{1 - \left(\frac{p^* - c}{p^*}\right)^2 (p^* \bar{\varepsilon}'(p^*) - \bar{\varepsilon}(p^*))}. \end{aligned} \tag{13}$$

Equation (13) tells us that the average price of the firms follows the corresponding average cost as a monopolist price follows its cost, just performing the custom substitution from $\varepsilon(.)$ to $\bar{\varepsilon}(.)$. In other words, the costs of the individual firms are irrelevant for the average market price determination once the average cost is known.

3.3.3 Recall Error Amplitude and Price Dispersion

Price dispersion, defined here as the price difference, is driven in this model by cost dispersion. Thus to analyze it, it should be checked how the price difference depends on one of the costs. This is given by

$$\frac{\partial p_i^*}{\partial c_i} - \frac{\partial p_j^*}{\partial c_i} = \frac{1}{1 - \left(\frac{p^*-c}{p^*}\right)^2 (p^*\varepsilon'(p^*) - \varepsilon(p^*)) + 8(p^*-c)^2 g'^2(0)}$$

as shown in Section A.2.

The demand and the recall error effects must now be disentangled. If demand falls sharply at some given price, firms will not choose to price their goods above (or largely above) that level, no matter what the cost structure and recall error level are. The best way to study the recall error effect is to fix elasticity of demand at p^*, and this is so for a strong reason. The equilibrium price in the symmetric case is a function of elasticity (not of demand or the derivative of demand alone). Therefore fixing elasticity does not change the symmetric equilibrium outcome as it would happen if $D(p)$ or $D'(p)$ were fixed, even if just locally.

Proposition 4 *Price dispersion, defined here as the price difference, is ceteris paribus an increasing function of the recall error standard deviation.*

Proof
Price dispersion with $\varepsilon(p) = \varepsilon(p^*) \equiv \varepsilon^*$ for any p is

$$\frac{\partial p_i^*}{\partial c_i} - \frac{\partial p_j^*}{\partial c_i} = \frac{1}{1 + \left(\frac{p^*-c}{p^*}\right)^2 \varepsilon^* + 8g'^2(0)(p^*-c)^2}. \tag{14}$$

In Section A.3 it is shown that the whole denominator is decreasing in σ meaning that $\frac{\partial p_i^*}{\partial c_i} - \frac{\partial p_j^*}{\partial c_i}$ at $c_i = c_j$ (and therefore the price dispersion when the firms' costs are similar) is locally an increasing function of the recall error.■

Note that as consumers become more inattentive, it could be thought that the diminishing competitive pressure on the firms would lead to lower price dispersion, as it leads to higher markups, because consumers are less responsive to price gaps. It turns out that it is optimal for the low cost firm to increase this gap.

3.3.4 Welfare Analysis

Price recall errors can be regarded as a weakening of competition, given that the incentives of a fierce price competition are smaller. There are however some counterintuitive results with important welfare and policy implications. To have some insights a comparison between the extreme cases, perfect price recall and random shopping, is made. In the first case the basic textbook equilibrium arises. The low cost firm will charge the lowest of the following two, the price it would charge as a monopolist and the cost of the other firm. The high cost firm simply takes price equal to cost. At the other extreme, the two firms are monopolists on half of the market, so they simply both charge their monopolist prices.

Profits of the high cost firm go up (not necessarily monotonically) from zero to half of the monopolist profits, from one extreme to the other. If the low cost firm charges his monopolist price in the perfect competition situation, then its profits decrease (not necessarily monotonically). Otherwise it depends.

On the aggregate level a surprising result occurs.

Lemma 4 *There are instances where firms are worse off, on the aggregate level, when facing higher standard deviations of the recall errors.*

Proof
First it is shown that a degenerate distribution can be taken without loss of generality for the recall errors ϵ_i, $i = A, B$, so that the setting is the basic Bertrand model. From

equation (11) it is concluded that the full support case can be arbitrarily close to this solution, because the last term goes to zero as the recall error standard deviation goes to zero, given that $g'(\cdot) \to 0$, while the other terms are bounded. Furthermore, if demand is bounded then profits also converge to the degenerate case. Considering the degenerate case is thus a simplification.

Take any c_A, $D(p)$ and corresponding $\varepsilon(p)$ such that the monopolist problem of firm A has a solution, call it p_A^M, yielding strictly positive profits. Assume that $c_B > p_A^M$. In equilibrium the market will be completely covered by firm A setting p_A^M, getting profits $\Pi_A^M = p_A^M D(p_A^M)$ and $\Pi_B = 0$.

Now consider a new recall error distribution with full support. It follows directly from the definition of monopolist price that the new equilibrium price of firm A, p_A^*, satisfies $p_A^* D(p_A^*) \leq p_A^M D(p_A^M)$. Moreover, because $c_B > c_A$ it must also hold for firm B that $p_B^* D(p_B^*) < p_A^M D(p_A^M)$. The aggregate profits are now given by $\Pi^{\text{agg}} \equiv p_A^* D(p_A^*) g(p_B^* - p_A^*) + p_B^* D(p_B^*) \left(1 - g(p_B^* - p_A^*)\right)$ satisfying

$$\Pi^{\text{agg}} < p_A^M D(p_A^M) \left[g(p_B^* - p_A^*) + 1 - g(p_B^* - p_A^*)\right] = \Pi_A^M.$$

This example is sufficient to prove the lemma.■

While the proof mentions an extreme case, it is clear that worse price recall can decrease the total profits in a broad set of parameters. Larger recall errors push more consumers to the firm with higher price, that is the one with higher cost, whose profits are typically lower.

Lemma 5 *Worse price recall decreases the welfare of both the firms and the consumers in some instances.*

Proof
Notice that the welfare decrease for the consumers alone, follows directly from the symmetric cost case. To prove the lemma it must be shown that it happens in cases where the aggregate profits also decrease, which is not the case for symmetric costs. Recall the cases from the previous proof. As the standard deviation increases, the share of consumers buying at firm B can be arbitrarily close to one half, given that an increasing number of consumers will recall the price of the high cost firm as lower. This implies further that p_A^* can be arbitrarily close to p_A^M. Taking a linear demand function for instance, it becomes clear that the extra consumer surplus that the consumers still at firm A get due to existence of two firms in the market, i.e. firm A is not a monopolist, is smaller than the welfare cost of the other half, that switched from monopolist A to duopolist B. ■

This result is striking given the previous qualification of price recall errors as a cause of weaker competition. In fact, if consumers shop (close to) randomly then goods are being bought at high price which does not imply higher profits since consumption is being shifted from a low to a high cost firm. Whilst this observation is obvious, the counterintuitive nature of the above result is simply a consequence of it. In further extensions of this model, where the standard deviation of the price recall is somehow manipulated by the firms, it may happen that firms choose strategies that make them worse off.

Furthermore, for fixed price recall error amplitude, one may wonder if competition is itself welfare decreasing. In other words, may a duopoly be something that should be avoided in comparison to a monopoly? This is indeed the case, again because consumers do not make optimal choices. If the possibility of buying at higher prices is somehow not there, in some cases the welfare is higher with a monopoly in comparison to the duopoly.

Proposition 5 *In a market where consumers do not perfectly recall the prices, protecting a monopoly from entrant firms is in some instances optimal from the consumer welfare point of view, as well as from the social welfare point of view.*

Proof
Straightforward conclusion from the proof of Lemma 5, just considering the degenerate

distribution in the proof simply as the monopoly, which is then compared with duopoly. ■

Notice that the proofs of the above two lemmas use the extreme case of recall errors with degenerate distribution, for simplicity. Given the continuity of the equilibrium prices (therefore profits and surpluses) as a function of the error standard deviation, the results apply for other non-extreme parameter choices. The statement in Proposition 5 is not related to parameter choices but to different market structure under the same conditions, and should not be confused with the lemmas. The proposition compares monopoly with duopoly, where the monopolist case happens to be equivalent to the duopoly with degenerate errors.

The many firms case is not discussed in this Section, but the result in the proposition can be easily extended to that case.

3.3.5 Some Examples

Take again a linear elasticity, $\varepsilon(p) = -(a + bp)$, and the Gumbel distribution for the recall error. In the end of Section A.1 in the Appendix, the following formulas are worked out.

$$\frac{\partial p_i^*}{\partial c_i} = \frac{1}{1 - a(\frac{p-c}{p})^2} \frac{1 - a(\frac{p-c}{p})^2 + \left[\frac{\tau}{2}(p-c)\right]^2}{1 - a(\frac{p-c}{p})^2 + 2\left[\frac{\tau}{2}(p-c)\right]^2}$$

and

$$\frac{\partial p_j^*}{\partial c_i} = \frac{1}{\left[1 - a(\frac{p-c}{p})^2\right]^2 (\frac{\tau}{2}(p-c))^{-2} + 2\left[1 - a(\frac{p-c}{p})^2\right]}.$$

Only the two limiting cases, $c_A = c_B = 0$ and $c_A = c_B \to \infty$ will be discussed because other cases yield intermediate results.

For $c_A = c_B = 0$ the price is driven by the recall error of consumers, and is therefore of the same magnitude. Departing from $c_A = c_B = 0$ the equilibrium prices react in the following way:

$$\frac{\partial p_i^*}{\partial c_i} = \frac{1}{1-a} - \frac{1}{[2b/\tau + 1]^2 + 2[1-a]}$$

and

$$\frac{\partial p_i^*}{\partial c_j} = \frac{1}{[2b/\tau + 1]^2 + 2[1-a]},$$

which are positive because $0 < 1 - a$. For $c_A = c_B \to \infty$ the other extreme case occurs:

$$\frac{\partial p_i^*}{\partial c_i} = 1 - \frac{1}{[2b/\tau + 1]^2 + 2}$$

and

$$\frac{\partial p_j^*}{\partial c_i} = \frac{1}{[2b/\tau + 1]^2 + 2},$$

which are also positive. Notice that as in Section 3.1.3, as costs (and therefore equilibrium prices) increase, the level of the demand elasticity, which is defined by the parameter a, becomes irrelevant (it enters the first order conditions as a/p_i). Only the slope of the elasticity b is maintained. It is thus easy to see that the derivatives at $c \to \infty$ are smaller than at $c = 0$.

In equilibrium the prices will rise with an increase of any of the two costs. Firm's pricing behavior lies between the perfect competition lower bound and the monopolist upper bound, depending on the size of the recall error (which fosters towards the upper bound) and the relative cost advantage (also towards the upper bound).

Figure 1 shows some examples how incorrect price recall affects the market, in the case of different costs of firms. The variance of the price error increases from left to right. In the first column consumers are able to recall the price with very good precision, so that the outcome is close to the classic setting. A firm sets the monopolist price for low production cost, switches to a price slightly below the other firm's price for low intermediate cost, and follows his own cost for high costs. Its market share is almost 100% for low own cost and almost 0% for costs higher than the other firm's cost. In the last column the consumers are very unaware of the chosen prices so that they almost shop randomly, as can be seen in the slowly decreasing market share of firm i on the second row. Because consumers almost shop randomly, firms set a price close to the monopolist price given their cost.

Figure 2 illustrates Lemma 4.

4 Three or More Firms

In the Bertrand pricing model with no product differentiation, the number of firms is irrelevant as long as it is more than one. In this extension with more than two firms the equilibrium will present however an intuitive outcome: the price will decrease as the number of firms increases, though only slightly. Moreover this result does not depend on the number of consumers in the market, so it is entirely driven by the competition among the firms. To see this notice that the number of consumers μ just appears as a multiplicative constant in the profit function. Thus its maximization is independent of the value of μ.

Let there be n firms, $n \geq 3$. The number of consumers heading to firm $i = 1, \cdots, n$ is given by

$$\begin{aligned} P(p'_i < p'_j, \forall j \neq i) &= P(p_i + \epsilon_i < p_j + \epsilon_j, \forall j \neq i) \\ &= \int_{-\infty}^{\infty} f(y) \prod_{j \neq i} F(y + p_j - p_i) dy \\ &\equiv G_i(p_1 - p_i, \ldots, p_{i-1} - p_i, p_{i+1} - p_i \ldots, p_n - p_i), \end{aligned}$$

where

$$G_i(x_{-i}) \equiv \int_{-\infty}^{\infty} f(y) \prod_{j \neq i} F(y + x_j) dy.$$

The profit of firm $i = 1, \ldots, n$ is

$$\Pi_i(p_1, \cdots, p_n) = (p_i - c_i) G_i(p_1 - p_i, \cdots, p_n - p_i) D(p_i). \tag{15}$$

and the first order condition for its maximization is

$$\begin{aligned} \frac{\partial \Pi_i(p_1, \cdots, p_n)}{\partial p_i} &= 0 \\ \Leftrightarrow \frac{1}{p_i - c_i} + \frac{D'(p_i)}{D(p_i)} + \frac{\partial}{\partial p_i} \ln G_i(x_{-i}) &= 0, \end{aligned} \tag{16}$$

where $x_j = p_j - p_i$.

Assumption 1 is now generalized as follows.

Assumption 3 *The random shocks $\epsilon_1, \ldots, \epsilon_n$ are iid with mean 0 and full support on the real line with probability density function $f(.)$ and cumulative distribution function*

$F(.)$. Moreover the distribution of $\epsilon_1, \ldots, \epsilon_n$ is such that, for each $i = 1, \ldots, n$ and x_{-i}, $\frac{\partial}{\partial p_i} \ln G_i(x_{-i})$ is non-decreasing in p_i, i.e. $\sum_{j\neq i}^{n} \frac{\partial^2}{\partial x_j^2} \ln G_i(x_{-i}) \leq 0$.

Assumption 3 assures the existence of the equilibrium, whose proof is easily obtained from the $n = 2$ case. For uniqueness it must be assured that the best response hypersurfaces only cross once. Again this is guaranteed if $\frac{\partial p_i^*(p_{-i})}{\partial p_j} < 1$ for any $p_{-i} \geq c_{-i}$, i and $j \neq i$.

Consider a Δp_j increase in p_j. Notice that an equal increase in p_i offsets the increase in x_j, and furthermore it decreases all other x_k with $k \neq i, j$, meaning that $\frac{\partial}{\partial p_i} \ln G(x_{-i})$ is increased since $\sum_{j\neq i}^{n} \frac{\partial^2}{\partial x_j^2} \ln G_i(x_{-i}) \leq 0$. Following the same reasoning steps as in the $n = 2$ case, it is concluded that the change in the best response to Δp_j is some Δp_i satisfying $\Delta p_i < \Delta p_j$ which proves the uniqueness of equilibrium.

Lemma 6 *The imperfect price recall model for many firms, has one and only one equilibrium if Assumptions 2 and 3 are satisfied. This equilibrium is implicitly defined by equation (16).*

4.1 Symmetric Costs

For $c_i = c$ for all $i = 1, \cdots, n$ the equilibrium price is given by

$$\frac{p^* - c}{p^*} = \frac{1}{-\varepsilon(p^*) + np^* \sum_{j\neq i}^{n} \frac{\partial}{\partial x_j} G_i(0)},$$

where $p^* n \frac{\partial}{\partial p_i} G_i(x_{-i})\big|_{x_j = p_j - p_i = 0, \forall j \neq i} = -np^* \sum_{j\neq i}^{n} \frac{\partial}{\partial x_j} G_i(0)$ was used.

For the Gumbel distribution it is known that

$$G_i(x_{-i}) = \frac{1}{1 + \sum_{j\neq i} e^{-\tau x_j}}$$

so that

$$\begin{aligned} \frac{\partial}{\partial p_i} \ln G(x_{-i}) &= -\sum_{j\neq i}^{n} \frac{\partial}{\partial x_j} \ln G_i(x_{-i}) \\ &= (-\tau) \left[1 - \frac{1}{1 + \sum_{j\neq i} e^{-\tau x_j}} \right], \end{aligned}$$

which is increasing in p_i. In the symmetric equilibrium

$$\sum_{j\neq i}^{n} \frac{\partial}{\partial x_j} \ln G_i(0) = \tau \frac{n-1}{n}$$

and the equilibrium price is defined by

$$\frac{p^* - c}{p^*} = \frac{1}{-\varepsilon(p^*) + \tau \frac{n-1}{n} p^*}.$$

The competition pressure due to an increase in the number of firms has a small impact. Observe that an increase in n from 2 to ∞ is equivalent, in terms of markup setting, to just halving the standard deviation of the recall error (which is proportional to $\frac{1}{\tau}$). As a corollary, note that the Lerner index does not go to zero as $n \to \infty$. Gabaix and Laibson (2004) also conclude that the number of firms has a low impact in decreasing the charged markup, while in Stahl (1989) it even brings the equilibrium price closer to the monopoly price.

4.2 Asymmetric Costs

Due to analytical complexity, the results in the $n = 2$ case cannot be extended. It is however possible to obtain numerically the equilibrium prices for a given cost structure. The graphs in the Appendix show the equilibrium prices and different price dispersion measures for cases with $n = 3$, Gumbel distributed shocks and $\varepsilon(p) = -\frac{1}{2}$ for any $p \geq 0$. Two price dispersion measures are presented, namely the difference between maximum and minimum price as well as its ratio. The standard deviation of prices seems to follow very closely the pattern of Max-Min in all cases that were checked, whereas the standard deviation/mean ratio follows Max/Min.

In the first case, depicted in Figure 3, the firms' costs are given by the cost vector $(c_1, c_2, c_3) = (1, 2, 3)$. Except for a minor exception (see discussion below in the $(c_1, c_2, c_3) = (1, 2, 2)$ case), all prices grow with the price recall error. The behavior of price dispersion depends on the chosen measure.

More interesting is the comparison between $(c_1, c_2, c_3) = (1, 1, 2)$ and $(c_1, c_2, c_3) = (1, 2, 2)$, presented in Figures 4 and 5. They differ significantly, while one might have thought the opposite given that both cases have the same cost values, the only difference being the number of firms having the different costs. If there are two low-cost firms they compete among them setting the competitive price, i.e. price marginally above cost, for small recall errors. As errors grow larger, their market share becomes less dependent on the price and they opt for a price close to that of the high-cost firm. The price dispersion is therefore decreasing in the errors standard deviation.

If there is only one low-cost firm, it is sufficient for it to charge a price slightly below the high cost. As higher errors are considered there are two effects that become clear. Initially the need to differentiate its price from that of the high-cost firms is predominant, so that it actually chooses a lower price. But then the effect of having a market share which is less dependent on the price starts to act and the low-cost firm raises the price again. This explains the steepest price dispersion increase among the three considered cases. While a deeper analytical investigation is beyond the scope of this paper, it becomes clear from the above examples that the cost structure of the firms in the market have a strong influence on the equilibrium prices and price dispersion.

5 Price Dependent Error Variance

A shortcoming of the previous Sections is the arbitrariness of the shock variance. In this Section it is assumed that the recall error is in some way related to the value of the good. As a simple example one may think of the uncertain price of a coffee in a bar comparing to the uncertain price of an expensive computer. While in the first the consumers may have a ±0.20€ 'confidence interval' in the second case this would be around ±100€[12].

5.1 Exogenous Variance

Setting the shock proportional to the equilibrium price would lead to an implicit definition problem. The cost of the good is however a good proxy for it. In this Section it is assumed that firms face cost c to produce the good and the price observation of the consumers suffers a shock whose standard deviation is an increasing function of c.

Derivation is very similar to above and Lemmas 1, 2 and 3 can be applied here. Profits are still given by $\Pi_i(p_i, p_j) = p_i g(p_j - p_i) D(p_i)$, but the standard deviation is now

[12] See Vanhuele, Laurent, and Drze (2006) for empirical evidence on the harder memorability of lengthier prices.

multiplied by c. In other words $g'(0)$ becomes $\frac{g_0'(0)}{c}$ where g_0 is just the benchmark $g(\cdot)$ function for $c = 1$. Equation (8) becomes now

$$\frac{p}{p-c} = -\varepsilon(p) + \frac{2}{c}g_0'(0)p.$$

Introducing this cost dependence modifies the competition pressure term in the equilibrium Lerner index. Higher costs mean lower attention (in absolute terms) paid by consumers. If constant elasticity is assumed, to abstract from demand driven changes, the above formula can be written as

$$\frac{1}{1-c/p} = -\varepsilon + \frac{2p}{c}g_0'(0),$$

which can be regarded as an equation on $\frac{p}{c}$. This means that in equilibrium the ratio $\frac{p}{c}$ is the same for any c, meaning that the markup is proportional to cost.

5.2 Endogenous Shock Variance

While the previous Section has an intuitive outcome regarding the dependence of markup on the cost of a good, it would be more realistic to allow consumers to choose an effort according to the amount of money involved. For instance Sorensen (2001), while related to search costs, states that consumers put a higher search effort for pharmaceutical products that they buy more often. Here the standard deviation of the recall error will be associated to the effort they will be putting in remembering the exact prices, which is denoted by Σ. It is assumed that for a given effort level Σ the recall error will be proportional do c, in other words the effort is not related to recalling the last digits of the price but the first digits. Thus the standard deviation of a given distribution $g_0(\cdot)$ is here multiplied by Σc (in the above Section $\Sigma = 1$). The consumer faces here a trade-off between mental effort and overspending.

As proxy for the costs of overspending the following is used $\beta\Sigma c$, that is the standard deviation of the recall error chosen by the consumer times β, a constant related to the error distribution.[13] The idea is that higher recall errors imply higher expected losses from choosing the firm with the highest price.

As stated before consumers will face a mental effort cost of reducing Σ, $h(\Sigma)$. It is assumed that decreasing the recall error - lower Σ - is increasingly costly, $h'(\cdot) < 0, h''(\cdot) >$

[13]This choice can be motivated by a more complex modeling as follows.

Let the consumer be concerned about the worst expected loss that may happen for a given standard deviation. In other words, for a given price dispersion $x \equiv \max p_i - \min p_i$ there is an associated expected loss (when the consumer mistakenly chooses the firm with the highest price) and the consumer will take into account the highest value of the expected losses across all x. While this choice is not very intuitive, it must be noticed that a priori the consumer is not aware of the price dispersion distribution x, so she cannot calculate the expected loss.

Consider the case with $p_A < p_B$. The probability of shopping at B by mistake is $1 - g(x)$. Hence the expected loss due to false recall is $x(1 - g(x))$. The consumer is not aware of the real prices and therefore not aware of x, so that she can only evaluate what the maximum expected loss is for a given Σ. Call it $\Delta(\Sigma)$:

$$\Delta(\Sigma) \equiv \max_{x \geq 0} x\left(1 - g_\Sigma(x)\right),$$

where $g_\Sigma(x)$ is the probability of choosing the lowest price given effort Σ. While Assumption 1 does not guarantee the uniqueness of this maximum, it exists and is unique for many distributions (like Normal and Gumbel). Now, as seen in the discussion of equation (4) the standard deviation enters $g(\cdot)$ as a number by which x is divided. So for a given recall error distribution, the above maximization has the same solution if solved for $\frac{x}{\Sigma c}$, a solution which is independent of Σ. In other words the maximum expected loss $\Delta(\Sigma)$ is just (linearly) proportional to Σc, which motivates the choice for the overspending proxy. The proportionality constant is denoted by β.

0 so that $\frac{\partial h(\Sigma)}{\partial(-\Sigma)} > 0$ and $\frac{\partial h^2(\Sigma)}{\partial(-\Sigma)^2} > 0$. Consumers will thus compare the benefit $\beta\Sigma c$ and the cost $h(\Sigma)$ of making a memory effort Σ. Equalizing marginal benefit βc and marginal cost $h'(\Sigma)$ yields the optimal recall error of consumers.

Take for instance $h(\Sigma) = \frac{1}{\Sigma}$ as the effort cost. Then consumers minimize the losses from the recall errors given by $\beta\Sigma c + h(\Sigma)$. The minimum is attained at

$$\Sigma = \sqrt{\frac{1}{\beta c}}$$

or $\Sigma c = \sqrt{\frac{c}{\beta}}$. Recall that while the consumers choose Σ the shock standard deviation will be Σc. The idea behind this result is that consumers do make bigger mistakes when recalling a price of a more expensive good, but this mistake is only proportional to the root of it. In other words, when buying the outfit consumers exert a bigger effort in price comparison because the stakes are higher.

Once again Lemmas 1, 2 and 3 regarding the equilibrium in the firms' price game are applicable here, because firms take the consumers' effort as given. The equilibrium price will be characterized by

$$\frac{1}{p^* - c} = -\frac{\varepsilon(p^*)}{p^*} + 2\sqrt{\frac{c}{\beta}} g_0'(0).$$

For costs and thus prices close to zero the first term on the right hand side overweighs the second, so that the equilibrium price is close to the monopoly price. But for higher costs (and assuming that $-\varepsilon(p)$ increases less than proportionally to p for high prices) the markup will be close to $\sqrt{\frac{\beta}{c}} \frac{1}{2g_0'(0)}$. This case lies thus between the basic case where markup is constant as $c \to \infty$ and the exogenous recall variance case where it is proportional to c.

6 Related Models

In this Section the price recall model is compared with the price competition model with utility uncertainty and the classic horizontal differentiation model.

6.1 Comparison to Utility Uncertainty

The literature related to this paper takes an approach close to the quantal response equilibrium concept, as in Gabaix and Laibson (2004) and Gabaix, Laibson, and Li (2005). That is, consumers[14] are not able to compare two utility levels, corresponding to two alternatives, perfectly. Mathematically, instead of comparing U_1 and U_2, they compare the utility plus a shock, $U_1 + \epsilon_1$ with $U_2 + \epsilon_2$.

For the simple case of fixed demand, where consumers are willing to buy one and only one unit of the good, this is equivalent to the present model where the random component is added to the price. In a utility shock setting, consumers compare $(u - p_1) + \epsilon_1$ with $(u - p_2) + \epsilon_2$, where u is the monetary utility of having the good. This is equivalent to the comparison of $u - (p_1 + \epsilon_1')$ with $u - (p_2 + \epsilon_2')$ where $\epsilon_i' = -\epsilon_i$, $i = 1, 2$.

But once the assumption of fixed demand is relaxed, the models yield different predictions.

Let $V(p)$ be the total indirect utility function of a consumer from buying the good at price p, that is the indirect utility minus the cost in utility units. The consumers when facing p_A and p_B compare $V(p_A) + \epsilon_A$ with $V(p_B) + \epsilon_B$, where ϵ_A and ϵ_B are i.i.d. random

[14]This bounded rationality argument can also be applied to the firms, as Baye and Morgan (2004) do. The rationale for this option is however not so clear. The possible gains and losses at stake for the firms are clearly higher, as is their availability to compute the problem lengthily through.

errors. Define the probability of a given consumer to consider firm i as having the best option as $q(V(p_j) - V(p_i))$ with $j \neq i$, where $q(\cdot) = 1 - g(\cdot)$ as now consumers opt for the highest $V(\cdot)$ instead of the lowest p. Now the procedure to find the equilibrium price follows closely that of previous Sections[15]. Equation (8) becomes now

$$\frac{p}{p-c} = -\varepsilon(p) + 2q'(0)V'(p)p,$$

where $g'(0)$ became $q'(0)V'(p)$ since this term follows from $-\frac{\partial}{\partial p_i} q(V(p_j) - V(p_i))$ at $p_i = p_j$.

Consider the simple case of constant elasticity of demand $-1 < \varepsilon(p) < 0$ for a comparison of the implications of the two approaches. Demand is given by $D(p) = D_0 p^{-a}$ so that $\varepsilon(p) = -a$ for any $p \geq 0$, where $D_0 > 0$ is some constant. Assuming that the expenditure on the product on focus is small compared to the total income of the consumer, one can take the marginal utility of income λ as fixed when setting the consumer utility function for this purchasing behavior. Thus the (separable) utility of the good that yields the desired demand function is simply $U = -U_0 x^{-\frac{1-a}{a}}$ with $U_0 > 0$ and x being the amount of consumed good. This leads to the above mentioned demand function with $D_0 = \left(\frac{1-a}{a}\frac{U_0}{\lambda}\right)^a$. The total indirect utility function as a function of price is

$$\begin{aligned} V(p) &= -U_0 D(p)^{-\frac{1-a}{a}} - \lambda D(p) p \\ &= -\left(\frac{U_0}{a}\right)^a \left(\frac{\lambda}{1-a}\right)^{1-a} p^{1-a}. \end{aligned}$$

If ϵ_i are Gumbel i.i.d. then a given consumer chooses firm A with probability $q(V(p_B) - V(p_A))$ defined by

$$q(V(p_B) - V(p_A)) = \frac{1}{1 + e^{\tau(V(p_B) - V(p_A))}} = \frac{1}{1 + e^{-\tau\left(\frac{U_0}{a}\right)^a \left(\frac{\lambda}{1-a}\right)^{1-a} (p_B^{1-a} - p_A^{1-a})}}.$$

The symmetric equilibrium price p^* is given by

$$\frac{p^* - c}{p^*} = \frac{1}{a + \frac{\tau}{2}\left(\frac{1-a}{a} U_0\right)^a \lambda^{1-a} {p^*}^{1-a}},$$

where $q'(0) = -\frac{\tau}{4}$ and $V'(p) = \left(\frac{1-a}{a} U_0\right)^a \lambda^{1-a} {p^*}^{-a}$ were used. It is hard to compare the two models, because the utility uncertainty has more degrees of freedom. But two observations can be made that distinguish them, both related to the change from $g'(0)$ to $q'(0)V'(p)$. First, the equilibrium price equation now contains U_0 (or D_0) meaning that products with the same elasticity of demand may have different equilibrium prices. In the price recall model, both demand and market share depend solely on price so the firm incentives only depend on price. Now the market share, related to the probability of correctly choosing the good according to its utility, depends on U_0 (or D_0). Consumers make less mistakes for products with higher U_0 (that is with a higher demand parameter D_0) so the competition pressure will be stronger and the prices lower.

Second, the new term contains now ${p^*}^{1-a}$ instead of p^* implying a non-linear response to price. Recall that the $g'(0)$ or $q'(0)V'(p)$ term can be interpreted as an increase in the (absolute value) of the elasticity of demand when compared to the monopolistic price setting. If a is close to 1, this increase is almost independent of p.

Therefore the new bounded rationality concept not only offers a more intuitive and tractable model, but its predictions depart from the common model in the literature. Assuming a shock in the utility, instead of in the price, changes the new term showing up

[15] The assumption needed to guarantee existence and uniqueness of equilibrium differs. Here it is sufficient that $\frac{\partial}{\partial p_i} \ln q(V(p_j) - V(p_i))$ is non-increasing in p_i for any $p_j \geq 0$.

in the equilibrium Lerner index of the firms. The price in the utility shock model depends on many parameters, namely on the demand level D_0, the functional form of the indirect utility function, the price elasticity of demand and the error distribution, whereas only the last parameters is to be found on the price shock model.

6.2 Price Recall as a Horizontal Differentiation Model

As argued in Section 3.1.5 the price recall model also resembles the horizontal differentiation model in the basic cases with fixed unitary demand. This equivalence is however not true for distributions with full support. If the firms are placed in the "extremes" of the infinite horizontal line, then for any strictly positive distance cost, both total prices (good plus distance cost) will be infinite for all consumers. The full support cases can however be approximated by distributions with bounded support, assuring that the latter yields an interior solution. The problem with the approximation is that in the latter case a firm can have the whole market if it chooses a sufficiently low price. Again this can be avoided by assuming high distance costs and equivalently high utility of the good to guarantee that all the market is covered. Assigning a high utility may however distort further applications of the model, for instance in welfare analysis.

In the price recall model it is natural to assume that the difference between the price recall errors follows some usual symmetric probability distribution, but it is intuitively not so clear why the consumers in the horizontal differentiation framework should be densely concentrated in the center of the horizontal line, a feature which is necessary for the formal equivalence. But the models diverge more once one moves away from the basic case and considers elastic demand. The reason is that in the horizontal model, the distance decreases the willingness to buy the goods, either because the total price is higher (transportation costs interpretation) or the value of the good is lower (preferred variety interpretation). Put simple, in a horizontal differentiation model the distance represents a worse alternative. In the price recall model the distance only reflects a lower probability of buying the good. In the first case the consumers choose to buy less, in the latter demand remains constant. Mathematically, the demand in the horizontal differentiation model is given by the integral of different demands along the line, here it is simply demand as a function of price times the market share. In equation (1') the market share term remains unchanged in horizontal differentiation models, but the demand term becomes an analytically complex term[16]. This also makes the bounded support approximation problem more salient. Assuming a value of the good which is sufficiently high to guarantee a covered market and an interior solution, is not compatible with a low demand. Put differently, in the limit case where recall error variance goes to infinity, the market in the price recall model is split between the firms irrespective of their prices, acting both as a monopolist in their half. The demand they get is not affected. That is not so for horizontal differentiation. A robust market split only occurs for distance costs increasing towards infinite, but that would have an impact in actual demand.

The distinctness is also more evident when extensions are considered. In Section 5 the recall error variance is endogenized, being it a choice of the individual consumers (could be a choice of firms as well). In the horizontal differentiation model this would be equivalent to having the individual consumer choosing the distribution (or the distance cost) of the whole market. Risk aversion of firms' managers would also damage the analogy, because the focus here has been expected (therefore random) market share. Moreover, in horizontal differentiation the positioning of firms can conceptually be a decision of the agents, but here that cannot be the case.

[16]The Hotelling model with elastic demand is only mathematically tractable for very specific parameter functions. See for instance Puu (2002).

7 Conclusions

In this paper a model of bounded rationality of the demand side in price competition settings is presented. Assuming a simple and intuitive extension of the classic Bertrand model, some of its paradoxes are solved. Competitive firms do charge above the competitive equilibrium price, having therefore a positive profit. There is no benefit (in equilibrium) in lowering the price for some consumers will still recall the other firm's price as lower. This has obvious welfare damaging effects, which can be easily obtained given the proposed characterization of the equilibrium which is equivalent to that of a monopoly price model. Moreover it was shown that in some cases with asymmetric costs both the firms and the consumers incur welfare costs compared to the classic case.

Firms with different costs do coexist in the market. Price dispersion does persist in competitive frameworks. In the simple setting, price dispersion increases with the recall errors. If the effort choice is endogenized, consumers put more effort when purchasing a valuable product.

While these results are quite interesting by itself, this extension may turn many models more realistic on the demand side. For instance in multi-product retail pricing, the capacity of consumers of comparing price vectors of baskets of goods seems to t a central issue. It also leads to continuous demands and continuous best response functions, which are more realistic and sometimes analytically more tractable.

A final word on the rationality of consumers. Here it was assumed that all consumers had the same type and degree of bounded rationality. That is actually a weak assumption because all consumers act rationally once at the store. If one were to consider consumers with heterogeneous recall errors, the trade-off of the firms would still be consisted of the two trade-offs, the same monopoly-like trade-off and a similar marginal market share dispute. The former is independent of the type of consumers and the later can be mimicked with homogeneous consumers with a different recall error distribution.

A Appendix

A.1 Price Cost Partial Derivatives

Equation (11) and its counterpart can be rewritten as

$$H_i(p_i, p_j, c_i) \equiv (p_i - c_i)(-\varepsilon(p_i) - p_i \frac{\partial}{\partial p_i} \ln g(p_j - p_i)) - p_i = 0, \tag{17}$$

for $i, j = A, B$ and $i \neq j$.

The partial derivatives of the equilibrium prices with respect to costs (both that of the considered firm and that of the opponent) are obtained by implicit differentiation of the first order conditions of the two firms

$$\begin{pmatrix} \frac{\partial p_A^*}{\partial c_A} & \frac{\partial p_A^*}{\partial c_B} \\ \frac{\partial p_B^*}{\partial c_A} & \frac{\partial p_B^*}{\partial c_B} \end{pmatrix} = - \begin{pmatrix} \frac{\partial H_A}{\partial p_A} & \frac{\partial H_A}{\partial p_B} \\ \frac{\partial H_B}{\partial p_A} & \frac{\partial H_B}{\partial p_B} \end{pmatrix}^{-1} \begin{pmatrix} \frac{\partial H_A}{\partial c_A} & \frac{\partial H_A}{\partial c_B} \\ \frac{\partial H_B}{\partial c_A} & \frac{\partial H_B}{\partial c_B} \end{pmatrix}. \tag{18}$$

From (17) it follows that

$$\begin{aligned}
\frac{\partial H_i}{\partial p_i} &= (p_i - c_i)\left(-\varepsilon'(p_i) - \frac{\partial}{\partial p_i}\ln g(p_j - p_i) - p_i\frac{\partial^2}{\partial p_i^2}\ln g(p_j - p_i)\right) + \\
&\quad -\varepsilon(p_i) - p_i\frac{\partial}{\partial p_i}\ln g(p_j - p_i) - 1, \\
\frac{\partial H_i}{\partial p_j} &= -(p_i - c_i)p_i\frac{\partial^2}{\partial p_i \partial p_j}\ln g(p_j - p_i), \\
\frac{\partial H_i}{\partial c_i} &= p_i\frac{\partial}{\partial p_i}\ln g(p_j - p_i) + \varepsilon(p_i), \\
\frac{\partial H_i}{\partial c_j} &= 0
\end{aligned}$$

for $i, j = A, B$, $i \neq j$. Because the partial derivatives will be taken at $c_A = c_B = c$, and therefore at $p_A = p_B = p$, one can perform the change $\frac{\partial}{\partial p_j} = -\frac{\partial}{\partial p_i}$ so that

$$\frac{\partial H_i}{\partial p_j} = (p_i - c_i)p_i\frac{\partial^2}{\partial p_i^2}\ln g(p_j - p_i).$$

Pricing behavior depending on own cost at $c_i = c_j$

From equation (18),

$$\frac{\partial p_i^*}{\partial c_i} = -\frac{\frac{\partial H_i}{\partial c_i}\frac{\partial H_j}{\partial p_j}}{\frac{\partial H_i}{\partial p_i}\frac{\partial H_j}{\partial p_j} - \frac{\partial H_i}{\partial p_j}\frac{\partial H_j}{\partial p_i}}.$$

Using $x = \frac{\partial H_i}{\partial c_i}$, $y = -\frac{\partial H_i}{\partial p_j}$ and $z = \frac{\partial H_i}{\partial p_i} + \frac{\partial H_i}{\partial p_j}$, and noting that $\frac{\partial H_i}{\partial p_j} = \frac{\partial H_j}{\partial p_i}$ and $\frac{\partial H_i}{\partial p_i} = \frac{\partial H_j}{\partial p_j}$, this can be written as

$$\frac{\partial p_i^*}{\partial c_i} = -\frac{x(y+z)}{(y+z)^2 - (-y)^2} = -\frac{x}{z}\frac{z+y}{z+2y}.$$

Simplifying the first fraction leads to

$$\begin{aligned}
-\frac{x}{z} &= \frac{p}{(p-c)(-\varepsilon(p) - 1 + 2(2p-c)g'(0) - (p-c)\varepsilon'(p))} && (19)\\
&= \frac{1}{1 - \left(\frac{p-c}{p}\right)^2(p\varepsilon'(p) - \varepsilon(p))}, && (20)
\end{aligned}$$

where $2g'(0) = \frac{1}{p-c} + \frac{\varepsilon(p)}{p}$ from (9) was used. To see that the denominator is positive, substitute $\frac{p-c}{p}\varepsilon(p)$ to $2(p-c)g'(0) - 1$ following (9). It then becomes

$$1 - \left(\frac{p-c}{p}\right)^2 p\varepsilon'(p) - \frac{p-c}{p} + 2\frac{p-c}{p}(p-c)g'(0) =$$

$$= \left[1 - \frac{p-c}{p}\right] + \left[-\left(\frac{p-c}{p}\right)^2 p\varepsilon'(p)\right] + \left[2\frac{p-c}{p}(p-c)g'(0)\right]$$

where all square brackets are strictly positive given that the markup $p - c$ is strictly positive, due to the distribution full support. For the simplification of the other fraction the following will be needed

$$
\begin{aligned}
-\frac{d^2}{dx^2}\ln g(0) &= -\frac{g(0)g''(0)-g'^2(0)}{g^2(0)} \\
&= -\frac{\frac{1}{2}g''(0)-g'^2(0)}{\left(\frac{1}{2}\right)^2} \\
&= 4g'^2(0). \qquad (21)
\end{aligned}
$$

The second derivative of $g(.)$ at 0 equals zero because ϵ_A and ϵ_B have the same distribution, so that $g(x) = 1 - g(-x)$, therefore $g''(x) = -g''(-x)$, which implies $g''(0) = 0$. Now the second fraction of the partial derivative becomes similarly

$$
\frac{z+y}{z+2y} = \frac{1-\left(\frac{p-c}{p}\right)^2(p\varepsilon'(p)-\varepsilon(p))+4(p-c)^2g'^2(0)}{1-\left(\frac{p-c}{p}\right)^2(p\varepsilon'(p)-\varepsilon(p))+8(p-c)^2g'^2(0)}.
$$

Notice that both the denominator and the numerator are strictly positive, so that this fraction belongs to the interval $\left(\frac{1}{2},1\right)$. Concluding,

$$
\begin{aligned}
\frac{\partial p_i^*}{\partial c_i} &= \frac{1}{1-\left(\frac{p-c}{p}\right)^2(p\varepsilon'(p)-\varepsilon(p))}\times \\
&\quad \times\frac{1-\left(\frac{p-c}{p}\right)^2(p\varepsilon'(p)-\varepsilon(p))+4(p-c)^2g'^2(0)}{1-\left(\frac{p-c}{p}\right)^2(p\varepsilon'(p)-\varepsilon(p))+8(p-c)^2g'^2(0)}. \qquad (22)
\end{aligned}
$$

Using the Gumbel distribution, so that $g'(0) = \frac{\tau}{4}$, and the linear elasticity the above implicit derivative turns to

$$
\frac{\partial p_i^*}{\partial c_i} = \frac{1}{1-a(\frac{p-c}{p})^2}\,\frac{1-a(\frac{p-c}{p})^2+\left[\frac{\tau}{2}(p-c)\right]^2}{1-a(\frac{p-c}{p})^2+2\left[\frac{\tau}{2}(p-c)\right]^2}.
$$

At $c = 0$ the equilibrium prices will be $p^* = \frac{1-a}{b+\tau/2}$ and so

$$
\begin{aligned}
\frac{\partial p_i^*}{\partial c_i} &= \frac{1}{1-a}\left[1-\frac{1}{(1-a)\left(\frac{2}{\tau}\frac{b+\tau/2}{1-a}\right)^2+2}\right] \\
&= \frac{1}{1-a}-\frac{1}{\left[\frac{2b}{\tau}+1\right]^2+2\,[1-a]}.
\end{aligned}
$$

At $c_A = c_B = c \to \infty$ the equilibrium prices will be $p^* - c = \frac{1}{b+\tau/2}$, the term $\frac{p^*-c}{p^*}$ goes thus to zero, so that

$$
\begin{aligned}
\frac{\partial p_i^*}{\partial c_i} &= 1-\frac{1}{\left[\frac{\tau}{2(b+\tau/2)}\right]^{-2}+2} \\
&= 1-\frac{1}{\left[\frac{2b}{\tau}+1\right]^2+2}.
\end{aligned}
$$

Pricing behavior depending on other firm's cost at $c_i = c_j$

The other firm responds in the following manner according to equation (18),

$$
\frac{\partial p_j^*}{\partial c_i} = \frac{\frac{\partial H_j}{\partial p_i}\frac{\partial H_i}{\partial c_i}}{\frac{\partial H_j}{\partial p_j}\frac{\partial H_i}{\partial p_i}-\frac{\partial H_j}{\partial p_i}\frac{\partial H_i}{\partial p_j}}
$$

$$= \frac{-4(p-c)^2 p^4 g'^2(0)}{[-(p-c)^2(p\varepsilon'(p)-\varepsilon(p))+p^2]\,[(p-c)^2(p\varepsilon'(p)-\varepsilon(p))-p^2-8p^2g'^2(0)]}$$

$$= \frac{1}{1-\left(\frac{p-c}{p}\right)^2(p\varepsilon'(p)-\varepsilon(p))}\,\frac{4(p-c)^2g'^2(0)}{1-\left(\frac{p-c}{p}\right)^2(p\varepsilon'(p)-\varepsilon(p))+8(p-c)^2g'^2(0)}. \quad (23)$$

While the first fraction is the same as above (therefore positive), the second one is different. It is however easy to see that it is also positive, because the denominator is again the same and the numerator is positive. Moreover this partial derivative is smaller than $\frac{\partial p_i^*}{\partial c_i}$, for the disappearing terms in the numerator are positive.

With linear ε and Gumbel distributed recall error it turns to

$$\begin{aligned}
\frac{\partial p_j^*}{\partial c_i} &= \frac{\frac{\tau^2}{4}\frac{1}{(p-c)^2}}{\left(\frac{a}{p^2}-\frac{1}{(p-c)^2}-\frac{\tau^2}{4}\right)^2-\left(\frac{\tau^2}{4}\right)^2} \\
&= \frac{\frac{\tau^2}{4}(p-c)^2}{\left(1-a(\frac{p-c}{p})^2+(\frac{\tau}{2}(p-c))^2\right)^2-\left(\frac{\tau}{2}(p-c)\right)^4} \\
&= \frac{\frac{\tau^2}{4}(p-c)^2}{\left[1-a(\frac{p-c}{p})^2\right]^2+\left[1-a(\frac{p-c}{p})^2\right]\frac{\tau^2}{2}(p-c)^2} \\
&= \frac{1}{\left[1-a(\frac{p-c}{p})^2\right]^2(\frac{\tau}{2}(p-c))^{-2}+2\left[1-a(\frac{p-c}{p})^2\right]}.
\end{aligned}$$

For prices and price errors of comparable magnitude, that is $c=0$ and $p^*=\frac{1-a}{b+\tau/2}$, this response simplifies to

$$\begin{aligned}
\frac{\partial p_j^*}{\partial c_i} &= \frac{1}{[1-a]^2\,(\frac{\tau}{2}\frac{1-a}{b+\tau/2})^{-2}+2\,[1-a]} \\
&= \frac{1}{[1-a]^2\,(\frac{\tau}{2}\frac{1-a}{b+\tau/2})^{-2}+2\,[1-a]} \\
&= \frac{1}{[2b/\tau+1]^2+2\,[1-a]}.
\end{aligned}$$

The limit for $c_A=c_B=c\to\infty$, which implies $\frac{p^*-c}{p^*}\to 0$ and $p^*-c=\frac{1}{b+\tau/2}$, will be

$$\frac{\partial p_j^*}{\partial c_i} = \frac{1}{[2b/\tau+1]^2+2}.$$

A.2 Price Dispersion with Asymmetric Costs

As it follows from Section A.1 the price difference as a function of one of the costs can be approximated by

$$\frac{\partial p_i^*}{\partial c_i}-\frac{\partial p_j^*}{\partial c_i}=\frac{-\frac{\partial H_j}{\partial p_j}\frac{\partial H_i}{\partial c_i}-\frac{\partial H_j}{\partial p_i}\frac{\partial H_i}{\partial c_i}}{\frac{\partial H_j}{\partial p_j}\frac{\partial H_i}{\partial p_i}-\frac{\partial H_j}{\partial p_i}\frac{\partial H_i}{\partial p_j}}. \quad (24)$$

Taken at $c_A=c_B=c$, so that $p_i=p_j=p^*$, $\frac{\partial H_i}{\partial p_i}=\frac{\partial H_j}{\partial p_j}$, $\frac{\partial H_i}{\partial p_j}=\frac{\partial H_j}{\partial p_i}$, $\frac{\partial H_i}{\partial c_i}=\frac{\partial H_j}{\partial c_j}$ and $\frac{\partial H_i}{\partial c_j}=\frac{\partial H_j}{\partial c_i}$, equation (24) becomes

$$\begin{aligned}
\frac{\partial p_i^*}{\partial c_i} - \frac{\partial p_j^*}{\partial c_i} &= -\frac{\frac{\partial H_i}{\partial c_i}\left(\frac{\partial H_j}{\partial p_j} + \frac{\partial H_j}{\partial p_i}\right)}{\left(\frac{\partial H_i}{\partial p_i}\right)^2 - \left(\frac{\partial H_i}{\partial p_j}\right)^2} \\
&= \frac{\frac{\partial H_i}{\partial c_i}}{\frac{\partial H_i}{\partial p_j} - \frac{\partial H_i}{\partial p_i}} \\
&= \frac{-2p^* g'(0) + \varepsilon(p^*)}{1 + \varepsilon(p^*) + (p^* - c)\varepsilon'(p^*) - 2g'(0)(2p^* - c) - 8(p^* - c)p^* g'^2(0)} \\
&= \frac{1}{1 - \left(\frac{p^*-c}{p^*}\right)^2 (p^*\varepsilon'(p^*) - \varepsilon(p^*)) + 8(p^* - c)^2 g'^2(0)},
\end{aligned}$$

where $2g'(0) = \frac{\varepsilon(p^*)}{p^*} + \frac{1}{p^*-c}$ and equation (21) were used in the simplification.

A.3 Proof of Proposition 4

Since p^* increases with the standard deviation σ of the recall error, $\left(\frac{p^*-c}{p^*}\right)^2 \varepsilon^*$ in equation (14) is decreasing in σ. Write now $g_0'(0)\sigma^{-1}$ for $g'(0)$, following equation (4). The last term in the denominator of equation (14) depends on σ according to

$$\frac{d}{d\sigma}(p^* - c)^2 g_0'^2(0)\sigma^{-2} = 2(p^* - c)g_0'^2(0)\sigma^{-3}\left[\sigma\frac{dp^*}{d\sigma} - (p^* - c)\right]. \tag{25}$$

The sign of the derivative equals the sign of the term between the brackets, because all other terms are strictly positive. Because a constant elasticity is being considered, equation (8) is a quadratic equation with a closed form solution, namely

$$p^* = \frac{c}{2} + \frac{\sigma}{4g_0'(0)}\left[1 + \varepsilon^* + \sqrt{\left(1 + \varepsilon^* + \frac{2}{\sigma}cg_0'(0)\right)^2 - \frac{8}{\sigma}cg_0'(0)\varepsilon^*}\right].$$

The term $\sigma\frac{dp^*}{d\sigma} - (p^* - c)$ in (25) is therefore

$$\sigma\frac{dp^*}{d\sigma} - (p^* - c) = \frac{c}{2}\left[1 - \frac{\frac{2}{\sigma}cg_0'(0) + 1 - \varepsilon^*}{\sqrt{\left(1 + \varepsilon^* + \frac{2}{\sigma}cg_0'(0)\right)^2 - \frac{8}{\sigma}cg_0'(0)\varepsilon^*}}\right].$$

Now notice that

$$\begin{aligned}
\left(1 + \varepsilon^* + \frac{2}{\sigma}cg_0'(0)\right)^2 - \frac{8}{\sigma}cg_0'(0)\varepsilon^* &= \left(\frac{2}{\sigma}cg_0'(0) + 1 - \varepsilon^*\right)^2 + 4\varepsilon^* \\
&\leq \left(\frac{2}{\sigma}cg_0'(0) + 1 - \varepsilon^*\right)^2,
\end{aligned}$$

implying that the square root is smaller than the numerator which shows that the above expression is negative. Therefore the term $\sigma\frac{dp^*}{d\sigma} - (p^* - c)$ is negative and the last term in the denominator of equation (14) is decreasing in σ. Hence the whole denominator is decreasing in σ.

A.4 Figures

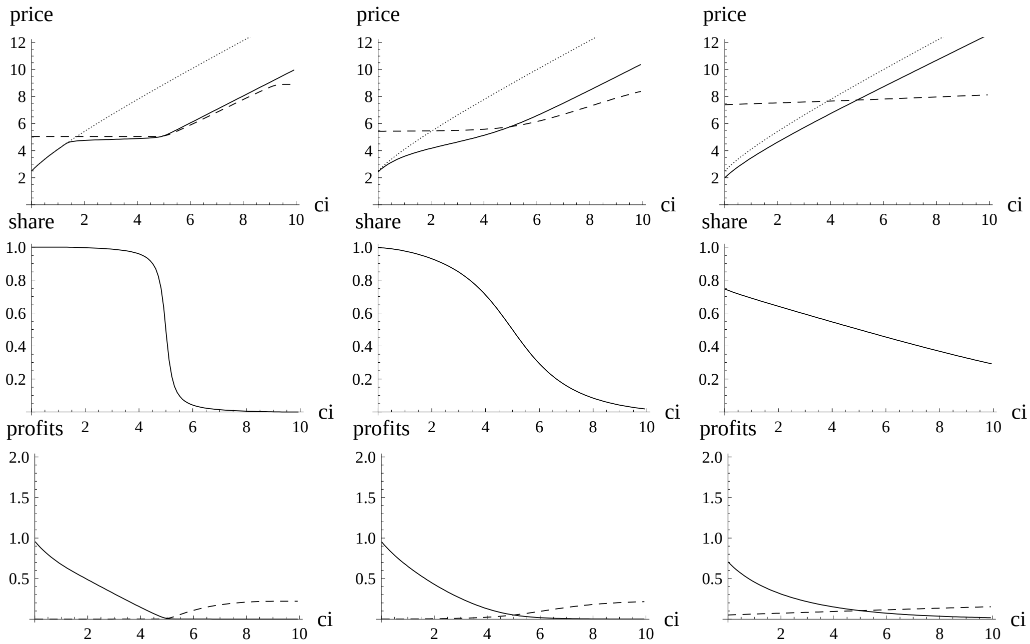

Figure 1: Equilibrium prices, market share of firm i and profits depending on the cost of firm i, c_i. The straight lines denote firm i, the dashed lines firm j and the dotted the monopolist with c_i. Chosen parameters $c_j = 5$, $D(p) = e^{-\frac{1}{2}\log p - \frac{p}{5}}$ so that $\varepsilon(p) = -\frac{1}{2} - \frac{p}{5}$, Gumbel distribution. Left column: $\tau = 20$, $\mathrm{var}(\epsilon) = \frac{\pi^2}{2400}$, center column: $\tau = 2$, $\mathrm{var}(\epsilon) = \frac{\pi^2}{24}$, right column: $\tau = 0.2$, $\mathrm{var}(\epsilon) = \frac{25\pi^2}{6}$.

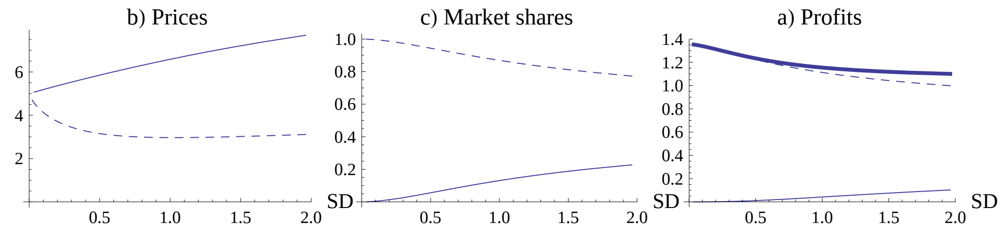

Figure 2: Aggregate profit decrease as a function of the standard deviation of the recall error, with $D(p) = e^{-\frac{1}{2}\log p - \frac{p}{10}}$ so that $\varepsilon(p) = -\frac{1}{2} - \frac{p}{10}$, and Gumbel distribution. Firm i with $c_i = 0$ and straight line, firm j with $c_j = 5$ and dashed line. Total profits represented by the thick line.

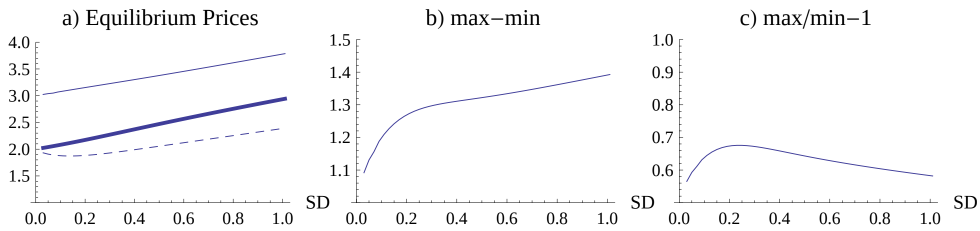

Figure 3: Equilibrium prices and two measures of price dispersion for $(c_1, c_2, c_3) = (1, 2, 3)$ depending on the standard deviation of the Gumbel distributed recall error. Firm 1, 2 and 3 represented by the dashed, the thick and the normal line. Demand is $D(p) = 10p^{-\frac{1}{2}}$ so that its price elasticity is $\varepsilon(p) = -\frac{1}{2}$.

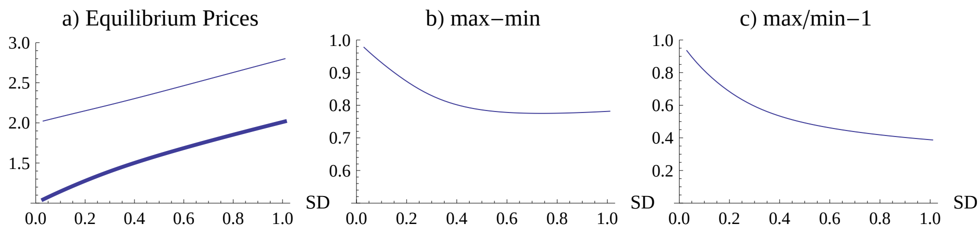

Figure 4: Equilibrium prices and two measures of price dispersion for $(c_1, c_2, c_3) = (1, 1, 2)$ depending on the standard deviation of the Gumbel distributed recall error. Firm 1 and 2 represented by the dashed line, firm 3 by the thick line. Demand is $D(p) = 10p^{-\frac{1}{2}}$ so that its price elasticity is $\varepsilon(p) = -\frac{1}{2}$.

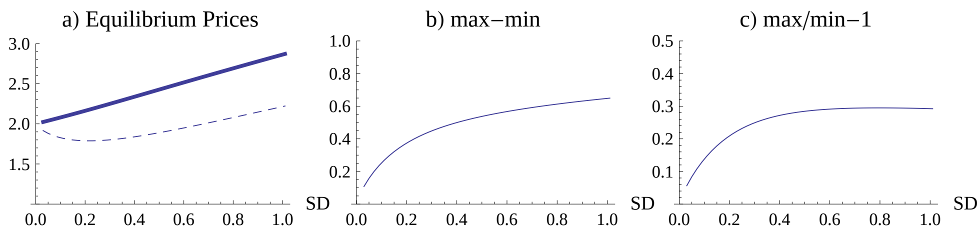

Figure 5: Equilibrium prices and two measures of price dispersion for $(c_1, c_2, c_3) = (1, 2, 2)$ depending on the standard deviation of the Gumbel distributed recall error. Firm 1 represented by the dashed line, firms 2 and 3 by the thick line. Demand is $D(p) = 10p^{-\frac{1}{2}}$ so that its price elasticity is $\varepsilon(p) = -\frac{1}{2}$.

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