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By Grzegorz Pawlina and Peter M. Kort

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Real Options in an Asymmetric Duopoly: Who Benefits from Your Competitive Disadvantage?[*]

Grzegorz Pawlina[†] and Peter M. Kort[‡]

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Abstract

This paper considers the impact of investment cost asymmetry on the value and optimal real option exercise strategies of firms under imperfect competition. Both firms have an opportunity to invest in a project enhancing (ceteris paribus) the profit flow. We show that three types of equilibria exist and derive critical levels of cost asymmetry separating the regions in which they prevail. The presence of strategic interactions leads to counter-intuitive results. First, depending on the level of asymmetry, a marginal increase in the investment cost of the firm with the cost disadvantage can increase this firm's own value. Second, such a cost increase can result in a decrease in value of the competitor. Moreover, we discuss the welfare implications of the optimal exercise strategies and show that the presence of identical firms can result in a socially less desirable outcome than if one of the competitors has a significant investment cost disadvantage. Finally, we prove that profit uncertainty always delays investment, even in the presence of a strategic option of becoming the first investor.

Keywords: real options, capital budgeting, strategic investment, social welfare

JEL classification: C61, D81, G31

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[†] Corresponding author, Department of Econometrics and Operations Research, and CentER, Tilburg University, P.O. Box 90153, 5000 LE Tilburg, the Netherlands, email: g.pawlina@kub.nl, phone: + 31 13 4663178, fax: + 31 13 4663066.

[‡] Department of Econometrics and Operations Research, and CentER, Tilburg University, the Netherlands.

1 Introduction

The aim of this paper is to study the effects of imperfect competition on the optimal real option exercise strategies in a situation where the costs of exercising options differ among firms. The need for a separate treatment of real, as opposed to financial, options results from the non-exclusivity of the former. In particular, the optimal exercise decision of a firm competing in an oligopolistic market depends not only on the value of the underlying process but also on the actions undertaken by its competitor(s) (cf. Zingales [22]). For example, the investment opportunity to set up the production line of personal cars in place of the existing assembly line may be represented as a real option to exchange a fixed amount of money for an incremental stream of uncertain cash flows. The cost of launching the production net of the scrap value of the assembly line can be viewed as the strike price of the option, whereas the increase in the expected present value of the stochastic profit flow corresponds to the underlying asset. The value of the investment opportunity, as well as the optimal exercise strategy associated with it, highly depends on the actions taken by the competing car manufacturer. Consequently, neither the value of the firm nor the optimal exercise strategy resembles any longer the situation where the firm has an exclusive option to invest.

The last decade's research results in a number of contributions dealing with the non-exclusivity of real options. The basic continuous-time model of strategic real option exercise under product market competition is presented by Smets [18]. He considers a duopolistic firm's decision to (costly) switch the production from a developed to an emerging economy where production costs are lower. Applications and extensions of the strategic real option exercise model include Grenadier [5], [6], Williams [21], Lambrecht and Perraudin [12], Décamps and Mariotti [2], Perotti and Rossetto [16], and Mason and Weeds [15], whereas Reinganum [17], and Fudenberg and Tirole [4] provide the game-theoretical foundations within a deterministic framework.[1]

In this paper we analyze the situation where two firms have an opportunity to invest in a profit enhancing investment project and face different (effective) investment costs. This framework, which allows for a departure from the unrealistic assumption that the duopolistic rivals are identical, reflects a number of factors.[2] First, investment cost asymmetry is present when the firms have a different access to the capital markets. In such a case, the cost of capital of a liquidity-constrained firm is higher than of its counterpart with an access to a credit line or with substantial cash reserves (Lensink, Bo and Sterken [13]). Consequently, the investment cost of the firm facing capital market imperfections is higher as well. Moreover, cost asymmetry occurs when the firms exhibit a different degree of organizational flexibility at implementing a new production technology. This flexibility, known as absorptive capacity (cf. Cohen and

[1] The discrete time analysis of a strategic options exercise is presented, among others, by Smit and Ankum [19], and Kulatilaka and Perotti [11].

[2] It is expected that other forms of asymmetry (such as profitability of the project, profit uncertainty characteristics etc.) lead to similar results.

Levintal [1]), measures the ¯rm's ability to adopt external technologies, to assimilate to a changing economic environment, and to commercialize newly invented products. A higher absorptive capacity is therefore equivalent to a lower cost associated with an investment project. Di®ering real options embedded in the existing assets of the ¯rms are another source of the investment cost asymmetry. This re°ects the possibility that the ¯rms currently exploit two di®erent but equally e±cient technologies. After an arrival of a new invention it may appear that one of the existing technologies is more easily extendable than the other. Finally, the di®erence in the investment costs is often a consequence of purely exogenous factors, resulting e.g. from di®erent regulation. Those legal and ¯scal factors are discussed later in the section concerning social welfare.

We consider the optimal real option exercise strategy of duopolistic ¯rms already competing in a product market. Both ¯rms have an investment opportunity enhancing (ceteris paribus) the pro¯t °ow. If one ¯rm invests, the other ¯rm's payo® is reduced. This is, for example, the case when the investment gives the ¯rm the possibility to produce more e±ciently and thus cheaper, which leads to a higher market share. The ¯rms di®er ex ante only with respect to the required sunk cost associated with the investment. Our framework most directly generalizes Smets [18] and Grenadier [5], who restrict the analysis to a game between symmetric ¯rms and of Huisman and Nielsen [10], who consider a new market entry of asymmetric ¯rms. This generalization results in the presence of three di®erent equilibrium strategies. First, when the asymmetry among ¯rms is relatively small and so is the ¯rst-mover advantage, the ¯rms invest jointly. When the ¯rst-mover advantage is su±ciently large, the lower-cost ¯rm preempts the higher-cost ¯rm. In the situation where both the ¯rst-mover advantage and asymmetry between ¯rms are signi¯cant, the ¯rms exercise their investment options sequentially and their investment timing do not a®ect each other directly. The two latter equilibria are also present in Perotti and Rossetto [16].

Subsequently, we determine the ¯rms' values and present welfare implications of the strategic option exercise. We ¯nd that, when an increase in the investment expenditure of the higher-cost ¯rm results in a switch from joint investment to preemption equilibrium, the value of both ¯rms decrease. Moreover, in a preemption equilibrium, an increase in the higher-cost ¯rm's investment expenditure results in increasing its value due to its commitment not to invest before the market is su±ciently large. Then the low cost ¯rm knows that it could delay the investment without bearing the risk of being preempted. This investment delay raises the value of the higher cost ¯rm. Using an example of a duopoly in which after the investment the ¯rms can o®er a good with a higher quality, we show the relationship between the type of equilibrium and the level of consumer surplus. This analysis indicates that an equal access of competitors to a new technology (or a new market) may not be socially optimal. Finally, we analyze the impact of uncertainty on the optimal investment thresholds. We ¯nd that the value of waiting option increases in the pro¯t volatility despite the presence of strategic interactions.

The paper is organized as follows. In Section 2 we present the model.

Section 3 contains the derivation of value functions and optimal investment thresholds. The discussion of the resulting equilibrium strategies is presented in Section 4 and the analysis of the impact of strategic interactions on the value of the firms is included in Section 5. In Section 6 we analyze the relationship between firms' investment strategies and social welfare whereas Section 7 discusses the impact of uncertainty on the timing of investment. Section 8 concludes.

2 Framework of the Model

Essentially, the basic framework of the non-strategic model of McDonald and Siegel [14] is adapted here, with the difference that we consider two firms rather than one. The two firms are risk neutral, compete in the product market, and realize a non-negative stochastic profit flow.[3] The uncertainty in each of the firms' profits is introduced via a geometric Brownian motion $x = \{x_t : t \geq 0\}$ such that

$$dx_t = \alpha x_t dt + \sigma x_t dw_t, \tag{1}$$

where α and σ are constants corresponding to the instantaneous drift and, respectively, to the instantaneous standard deviation, dt is the time increment and dw_t is the Wiener increment. In order to obtain finite valuations, we impose $\alpha < r$, where r is a risk-free rate. The uncertainty in the profit function is included in a multiplicative way. The instantaneous profit of Firm i can be expressed as

$$\pi_{t,N_iN_j} = x_t D_{N_iN_j}, \tag{2}$$

where, for $i, j \in \{0, 1\}$, $i \neq j$:

$$N_i = \begin{cases} 0 & \text{if firm } i \text{ has not invested,} \\ 1 & \text{if firm } i \text{ has invested.} \end{cases}$$

$D_{N_iN_j}$ stands for the deterministic contribution to the profit function, and it holds that

$$\begin{array}{ccc} D_{10} & > & D_{00} \\ D_{11} & > & D_{01}. \end{array}$$

$D_{10} > D_{00}$ implies that the profit of the firm that invests as first exceeds ceteris paribus the initial (symmetric) profit. Moreover, this investment leads to the deterioration of the profit of the firm that did not undertake the project yet, i.e. $D_{00} > D_{01}$. Finally, the 'catch-up' investment made by the lagging firm enhances its profit, so $D_{11} > D_{01}$, but, at the same time, it reduces the profit

[3] Alternatively, the risk neutrality assumption may be replaced by the replicating portfolio argument.

of the first mover, so that $D_{11} < D_{10}$: The last inequality implies that there are negative network externalities among the firms.[4]

The investment opportunity is assumed to last forever and the structure of the associated payoff can only change as a result of the competitor's action. Therefore, the opportunity can be modelled as a perpetual American option with a payoff determined endogenously. Consequently, we denote the investment cost of Firm i; $i \in \{1; 2\}$ by I_i: Without loss of generality I_1 is normalized to I; which is the investment cost of the low-cost firm, and I_2 is set equal to ρI; where $\rho \in [1; \infty)$:

Finally, we assume that the initial realization of the process underlying both firms' profits is low enough, so that an immediate investment is not optimal.[5]

3 Value Functions and Investment Thresholds

There are three possibilities concerning the timing of Firm i's investment relatively to the decision of the competitor (Firm j). First, Firm i may invest before Firm j does and, therefore, become the leader. Alternatively, Firm j may invest sooner and Firm i becomes the follower. Finally, Firm i and Firm j may invest simultaneously.

In this section we establish the payoffs associated with the three situations described above. As in the standard approach used to solve dynamic games, we analyze the problem backwards. First, we derive the optimal strategy of the follower, who takes the strategy of the leader as given. Subsequently, we analyze the decision of the leader. Finally, the case of joint investment is discussed.

3.1 Follower

Consider an investment decision of Firm i and define τ_j to be the moment in time at which its competitor (Firm j) invests as the leader. Firm i will undertake the investment when profits are sufficiently large, i.e. when x_t exceeds a certain threshold level denoted by x_i^F. Determining x_i^F is equivalent to finding the optimal option exercise strategy. At x_t, where $t \geq \tau_j$, the value of Firm i as the

[4] Mason and Weeds [15] allow for $D_{11} > D_{10}$ to reflect the positive network externalities that can arise among the competitors. In our setting (firms already compete in a product market) such an assumption would be more difficult to justify. Moreover, $D_{10} > D_{11}$ does not preclude the presence of positive network externalities among the firms' customers (for example, the profits generated by Microsoft Corp. in the office software segment are not likely to be positively affected by technological improvements made by Corel).

[5] Immediate investment is optimal in case of a sufficiently high initial realization of the stochastic process. Then mixed strategies equilibria can occur, as discussed for identical firms in Huisman and Kort [9].

follower equals

$$
\begin{aligned}
V_i^F(x_t) &= E\left[\int_{\min[t,T_i^F]}^{T_i^F} x_s D_{01} e^{-r(s-t)} ds\right] \\
&\quad + E\left[e^{-r\max[T_i^F - t,0]} \left(\int_{\max[t,T_i^F]}^{\infty} x_s D_{11} e^{-r(s-\max[t,T_i^F])} ds - I_i \right)\right],
\end{aligned} \tag{3}
$$

where

$$
T_i^F = \inf\left(t \mid x_t \geq x_i^F\right). \tag{4}
$$

The realization x_i^F corresponds to the follower's optimal investment threshold[6]

$$
x_i^F = \frac{\beta}{\beta - 1} \frac{I_i}{D_{11} - D_{01}} (r - \alpha), \tag{5}
$$

and $\beta (> 1)$ is the larger root of the quadratic equation

$$
\frac{1}{2}\sigma^2 \beta(\beta - 1) + \alpha\beta - r = 0. \tag{6}
$$

The first integral in (3) corresponds to the present value of profits obtained before the investment is undertaken. The second part of (3) reflects the present value of profits after the investment is made minus the associated sunk cost.

The value of the firm as well as the optimal investment threshold can be calculated explicitly by applying the well-known standard dynamic programming methodology (see Dixit and Pindyck [3]). By solving the Bellman equation with corresponding value-matching, smooth-pasting and no-bubbles conditions, we arrive at the following expression for the value of Firm i as the follower

$$
V_i^F(x_t) = \begin{cases} \frac{x_t D_{01}}{r-\alpha} + \left(\frac{x_i^F (D_{11} - D_{01})}{r - \alpha} - I_i\right)\left(\frac{x_t}{x_i^F}\right)^{\beta} & \text{if } x_t \leq x_i^F, \\ \frac{x_t D_{11}}{r-\alpha} - I_i & \text{if } x_t > x_i^F, \end{cases} \tag{7}
$$

The interpretation of (7) is as follows. The first row is the present value of profits when the follower does not invest immediately. The first term is the payoff in case the follower refrains from investing forever, whereas the second term is the value of the option to invest. The second row corresponds to the present value of enhanced cash flows resulting from immediate investment minus its cost.

---

[6] It is worth pointing out that the Marshallian investment threshold, x_i^{FM}, which is based on the static NPV criterion, equals

$$
x_i^{FM} = \frac{I_i}{D_{11} - D_{01}} (r - \alpha).
$$

As explained in Dixit and Pindyck [3], the difference between x_i^F and x_i^{FM} reflects the value of the option to wait, which raises the optimal threshold by factor $\frac{\beta}{\beta-1} > 1$.

3.2 Leader

Following a similar reasoning as in the previous subsection, we determine the payoff of Firm i when it invests first, thus Firm i is the leader. Then the value function of Firm i, evaluated at x_{τ_i}, where τ_i is the moment of investing, equals

$$V_i^L(x_{\tau_i}) = E\left[\int_{\tau_i}^{T_j^F} x_s D_{10} e^{-r(s-\tau_i)} ds - I_i + \int_{T_j^F}^{\infty} x_s D_{11} e^{-r(s-T_j^F)} ds\right]. \quad (8)$$

The first two components of (8) correspond to the present value of the leader's profits realized until the moment of the follower's investment net of the leader's sunk cost. The second integral corresponds to the discounted perpetual stream of profits obtained after the investment of the follower.

Using the results of the follower problem, we can express the value of Firm i as the leader in the following way

$$V_i^L(x_{\tau_i}) = \begin{cases} \frac{x_{\tau_i} D_{10}}{r-\alpha} - I_i + \frac{x_j^F (D_{11}-D_{10})}{r-\alpha}\left(\frac{x_{\tau_i}}{x_j^F}\right)^{\beta} & \text{if } x_{\tau_i} \le x_j^F; \\ \frac{x_{\tau_i} D_{11}}{r-\alpha} - I_i & \text{if } x_{\tau_i} > x_j^F. \end{cases} \quad (9)$$

The first row of (9) is the net present value of profits before the follower made the investment minus the present value of future profits lost due to the follower's investment. The second row corresponds to the net present value of profits in a situation where it is optimal for the follower to invest immediately.

3.3 Simultaneous Investment

It is possible that the firms, despite the asymmetry in the investment cost, decide to invest simultaneously. The value function of Firm i investing at its optimal threshold simultaneously with Firm j is

$$\begin{aligned} V_i^S(x_t) &= E\left[\int_{\min[t;T_i^S]}^{T_i^S} x_s D_{00} e^{-r(s-t)} ds\right] + \quad (10) \\ & E\left[\int_{\max[t;T_i^S]}^{\infty} x_s D_{11} e^{-r(s-\max[t;T_i^S])} ds - I_i e^{-r\max[T_i^S-t;0]}\right], \end{aligned}$$

where

$$T_i^S = \inf\left(t \mid x_t \ge x_i^S\right) \quad (11)$$

and

$$x_i^S = \frac{\beta}{\beta-1}\frac{I_i}{D_{11}-D_{00}}(r-\alpha). \quad (12)$$

Expression (10) is interpreted analogously to (3) and (8). Consequently, the value of Firm i when the investment is made simultaneously equals

$$V_i^S(x_t) = \begin{cases} \frac{x_t D_{00}}{r-\alpha} + \left(\frac{x_i^S(D_{11}-D_{00})}{r-\alpha} - I_i\right)\left(\frac{x_t}{x_i^S}\right)^{\beta} & \text{if } x_t \leq x_i^S; \\ \frac{x_t D_{11}}{r-\alpha} - I_i & \text{if } x_t > x_i^S; \end{cases} \tag{13}$$

The second row equals the value of Firm i when the simultaneous investment is made immediately. In such a case, we denote the value of Firm i by $V_i^J(\cdot)$: From (12) it can be seen that x_i^S differs among the firms. As it is shown in the next section, this divergence does not preclude the simultaneous investment strategy.

4 Equilibria

There are three types of equilibria that can occur in the choice of strategies, namely the preemptive, sequential and simultaneous equilibrium. In this section we discuss the characteristics of each type of equilibrium and present the conditions under which each of them occurs.

4.1 Preemptive Equilibrium

The first type of equilibrium we consider is the preemptive equilibrium.[7] It occurs in the situation in which both firms have an incentive to become the leader, i.e. when the cost disadvantage of Firm 2 is relatively small. Therefore, Firm 1 has to take into account the fact that Firm 2 will aim at preempting Firm 1 as soon as a certain threshold is reached. This threshold, denoted by x_{21}^P, is the lowest realization of the process x_t for which Firm 2 is indifferent between being the leader and the follower. Formally, x_{21}^P is the smallest solution to

$$\xi_2(x_t) = 0; \tag{14}$$

where $\xi_i(x_t)$ is defined as

$$\xi_i(x_t) = V_i^L(x_t) - V_i^F(x_t); \tag{15}$$

where $V_i^F(x_t)$ and $V_i^L(x_t)$ are given by (7) and (9), respectively. As a consequence, Firm 1 invests at

$$\min\left[x_{21}^P; x_1^L\right];$$

where x_1^L is Firm 1's optimal leader threshold equal to

$$x_1^L = \frac{\beta}{\beta-1}\frac{I}{D_{10}-D_{00}}(r-\alpha): \tag{16}$$

[7] For an elaborate treatment of this type of equilibrium the reader is referred to Fudenberg and Tirole [4].

Figure 1 shows an example of the firms' payoffs associated with being the leader, both investing at Firm 1's optimal simultaneous investment threshold and both investing immediately. The follower payoff of Firm i is set as a reference level.

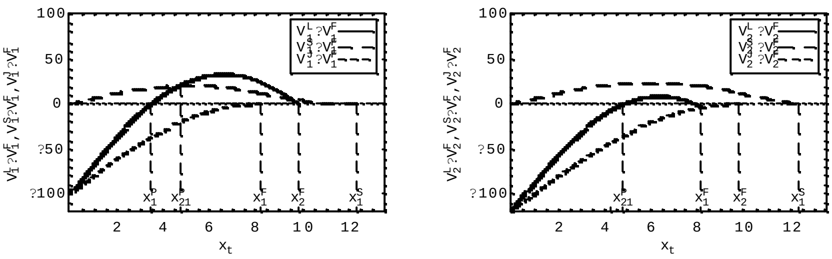

Figure 1

Figure 1. The relative (to the follower value) value functions of Firm i, $i \in \{1; 2\}$, as the leader, $V_1^L - V_1^F$, in case of optimal simultaneous investment, $V_1^S - V_1^F$, and when simultaneous investment is made immediately, $V_1^J - V_1^F$, for the set of parameter values: $\rho = 1{:}2$; $r = 0{:}05$; $\alpha = 0{:}015$; $\sigma = 0{:}1$; $I = 100$; $D_{00} = 0{:}5$; $D_{10} = 1{:}5$; $D_{01} = 0{:}25$; and $D_{11} = 1$.

Firm 1 invests as soon as the process reaches the smaller of two values: x_{21}^P at which Firm 2 is indifferent between being the leader and the follower, and x_1^L at which it is optimal for Firm 1 to invest given that Firm 2 does not invest until x_1^L is reached. It can be seen in Figure 1 (right) that to the left of x_{21}^P the value of Firm 2 as the leader is lower than the value as the follower, while to the right the opposite is true. Consequently, Firm 1 uses the fact that Firm 2 has no incentive to invest before x_{21}^P and preempts it by just an instant. For ρ tending to 1, i.e. when firms become symmetric, x_{21}^P gets closer to the Firm 1's preemption point, x_1^P, at which Firm 1 itself is indifferent between being the leader and the follower.

The presence of cost asymmetry implies the following corollary.

Corollary 1 Firm 1 extracts a relative surplus from becoming the leader vs. being the follower, i.e.

$$\xi_1\left(\min\left[x_{21}^P; x_1^L\right]\right) = V_1^L\left(\min\left[x_{21}^P; x_1^L\right]\right) - V_1^F\left(\min\left[x_{21}^P; x_1^L\right]\right) > 0. \quad (17)$$

Proof. The proof directly follows from the definition of the preemption point and the observation that $x_1^P < \min\left[x_{21}^P; x_1^L\right]$. ■

4.2 Sequential Equilibrium

The sequential equilibrium occurs when Firm 2 has no incentive to become the leader, i.e. when (14) does not have a real solution. In this case, Firm 1 simply maximizes the value of the investment opportunity, what always leads to investment at the optimal threshold x_1^L: In other words, Firm 1 acts as if it had exclusive rights to invest in a profit-enhancing project. Figure 2 describes the firms' payoffs corresponding to the sequential investment equilibrium.

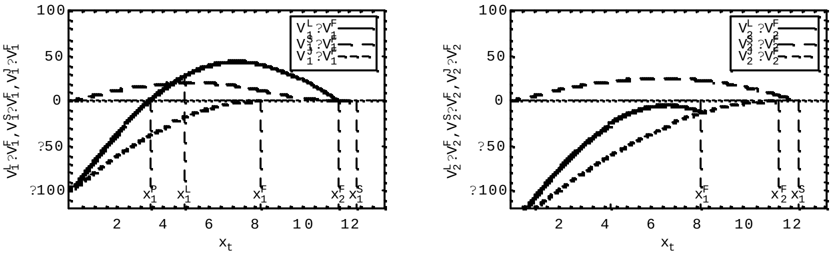

Figure 2

Figure 2. The relative (to the follower value) value functions of Firm i, $i \in \{1; 2\}$, as the leader, $V_1^L - V_1^F$, in case of optimal simultaneous investment, $V_1^S - V_1^F$, and when simultaneous investment is made immediately , $V_1^J - V_1^F$, for the set of parameter values: $\rho = 1.4$; $r = 0.05$; $\alpha = 0.015$; $\sigma = 0.1$; $I = 100$; $D_{00} = 0.5$; $D_{10} = 1.5$; $D_{01} = 0.25$; and $D_{11} = 1$.

From Figure 2 (right) it can be concluded that Firm 2 is never better off by becoming the leader compared to being the follower. Therefore Firm 1 does not need to take into account the possibility of being preempted by Firm 2. As a result, Firm 1 is able to invest at its unconditional threshold, x_1^L (see Figure 2, left diagram). At x_1^L the value of the investment opportunity smooth-pastes to the net present value of incremental benefits from making the investment (cf. Dixit and Pindyck [3]). As in the previous case, Firm 2 invests at its follower threshold x_2^F:

Proposition 2 There exists a unique value of $\rho > 1$; denoted by ρ^*, which is equal to

$$\rho^* = \frac{1}{D_{11} - D_{01}} \left(\frac{(D_{10} - D_{01})^{\beta} - (D_{11} - D_{01})^{\beta}}{\beta (D_{10} - D_{11})} \right)^{\frac{1}{\beta - 1}}, \qquad (18)$$

that separates the regions of the preemptive and the sequential equilibrium. For $\rho < \rho^*$ Firm 1 needs to take into account possible preemption by Firm 2, whereas $\rho \geq \rho^*$ implies that firms always invest sequentially at their optimal thresholds.

Proof. See Appendix. ■

Intuitively, Proposition 2 states that there is a cut-off level for the cost disadvantage of Firm 2 above which Firm 1 can act as a monopolist in exercising its investment option.

4.3 Simultaneous Equilibrium

Another type of equilibrium is the simultaneous (or: joint investment) equilibrium. In this case, the ¯rms invest at the same point in time. Figure 3 depicts both ¯rms' payo®s associated with the simultaneous equilibrium.

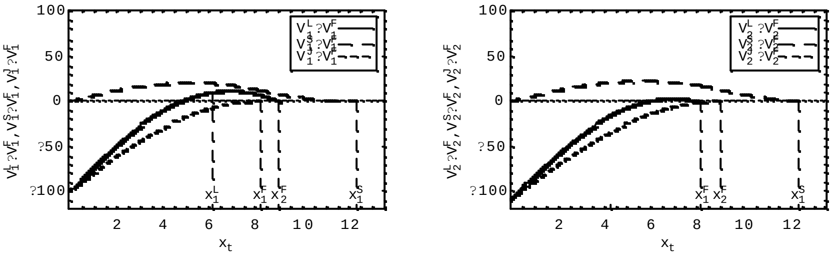

Figure 3

Figure 3. The relative (to the follower value) value functions of Firm i, i 2 f1; 2g, as the leader, V_1^L ¡ V_1^F, in case of optimal simultaneous investment, V_1^S ¡ V_1^F, and when simultaneous investment is made immediately , V_1^J ¡ V_1^F, for the set of parameter values: ½ = 1:1; r = 0:05; ® = 0:015; ¾ = 0:1; I = 100; D_{00} = 0:5; D_{10} = 1:25; D_{01} = 0:25; and D_{11} = 1. The set of input parameters results in the optimality of a simultaneous investment at x_1^S.

In the simultaneous investment equilibrium one of the ¯rms has to adopt a strategy that does not optimize its payo® unconditionally (since the optimal joint investment thresholds di®er). Since the optimal threshold of Firm 1 is lower than of Firm 2, the only candidate for a simultaneous investment threshold is x_1^S; de¯ned by (12). For simultaneous investment to occur, the payo® of Firm 1 associated with being the leader has to be lower than the payo® resulting from simultaneous investment. Otherwise, Firm 1 will invest either at x_1^L or at x_2^P (depending on the level of cost asymmetry). Moreover, Firm 2's follower threshold must be lower than x_1^S: In other words, Firm 2 has to ¯nd it more pro¯table to respond to Firm 1's investment at x_1^S immediately than to wait. Otherwise, Firm 2 would invest as the follower at x_2^F: It turns out that wherever it is optimal for Firm 1 to invest simultaneously, Firm 2 prefers simultaneous investment to being the follower (see the proof of Proposition 3 below).

In the subsequent section we analyze under which circumstances the simultaneous equilibrium occurs.

4.4 Conditions for Equilibria

The occurrence of a particular type of equilibrium is determined by the relationship between the relative payo®s, which in turn depend on the level of cost asymmetry, ¯rst-mover advantage and market parameters such as volatility, the growth rate and the interest rate. From Proposition 1 we already know the cut-off value of the cost asymmetry that separates the preemptive and the sequential equilibrium. Now, we concentrate on determining the region in which the simultaneous equilibrium occurs. In order to do so, let us define

$$\zeta_i(x_t) = V_i^S(x_t) - V_i^L(x_t). \tag{19}$$

$\zeta_i(x_t)$ can be interpreted as the change in Firm i's value associated with refraining from an immediate (at x_t) investment as the leader in favor of the simultaneous investment strategy. If the minimum of $\zeta_1(x_t)$ on the interval $[x_0; x_1^F]$ is larger than zero, the change is positive, and thus a simultaneous equilibrium occurs. In other words, the simultaneous equilibrium requires that Firm 1 is always better off by investing jointly at its optimal threshold x_1^S compared to becoming the leader.[8] Otherwise, either the sequential or the preemption equilibrium occurs.

Proposition 3 There exists a unique value of $\rho \geq 1$; denoted by ρ^{**}, which is equal to

$$\rho^{**} = \max\left[(D_{11} - D_{01})\left(\frac{\beta(D_{10} - D_{11})}{(D_{10} - D_{00})^{\beta} - (D_{11} - D_{00})^{\beta}}\right)^{\frac{1}{\beta-1}}; 1\right], \tag{20}$$

that determines the regions of the simultaneous and the sequential/preemptive investment equilibria. For $\rho < \rho^{**}$ the resulting equilibrium is of the joint investment type, whereas for $\rho \geq \rho^{**}$ the sequential/preemptive investment equilibrium occurs.

Proof. See Appendix. ■

Proposition 3 implies that for a relatively high degree of asymmetry between firms (for a given set of D_{ij}s and β), simultaneous investment is not optimal and either a sequential or preemption equilibrium occurs. Moreover, there exists a set of parameter values for which simultaneous investment is not optimal even when the firms are symmetric. In this case ρ^{**} is equal to 1. We present an illustration of when the resulting equilibria occur in a two-dimensional graph. In Figure 4 we depict the investment strategies as a function of the first-mover advantage, D_{10}/D_{11}, and the investment cost asymmetry, ρ.

[8] Strictly speaking, the equilibrium with sequential/preemptive investment still exists in this case but is Pareto-dominated by the simultaneous entry equilibrium (cf. Fudenberg and Tirole [4]).

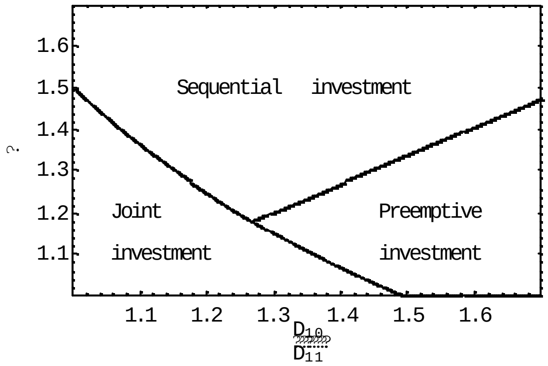

Figure 4

Figure 4. The regions of sequential, preemptive and joint investment equilibria for the set of parameter values: $r = 0{:}05$; ® $= 0{:}015$; ¾ $= 0{:}1$; $D_{00} = 0{:}5$; $D_{01} = 0{:}25$; and $D_{11} = 1$:

When the investment cost asymmetry is relatively small and there is no signi¯cant ¯rst-mover advantage, the ¯rms invest jointly (a triangular area in the south-west). When the ¯rst-mover advantage becomes signi¯cant, Firm 1 prefers being the leader to investing simultaneously. This results in the preemption equilibrium (area in the south-east). Finally, if the asymmetry between ¯rms is signi¯cant (for the set of parameter values in the upper part of the Figure 4), the ¯rms invest sequentially and Firm 1 can act as a monopolist.

5 Cost Asymmetry and Value of the Firm

In this section we discuss the impact of the degree of investment cost asymmetry on the value of each ¯rm and, in particular, on the net present value (NPV) of the investment opportunities. We show that, in the presence of strategic interactions, the relationship between the magnitude of the investment cost asymmetry and the value of the ¯rm is, in general, discontinuous and non-monotonic.

In the absence of strategic interactions among the ¯rms the value-asymmetry relationship is relatively straightforward. An increase in the investment cost of Firm 2 a®ects its value via i) a higher present value of the investment expenditure that has to be incurred and ii) a delay in the optimal timing of investment which results in postponing the moment of the pro¯t °ow increase. Consequently, the value of Firm 2 decreases monotonically in ½. Conversely, the value of Firm 1 remains una®ected by a change in ½ since the ¯rms do not interact with each other.

Introducing competition changes the way the asymmetry a®ects the values of both ¯rms. In such a case, the value of Firm 2 is a®ected not only by an increase in its investment cost but also by the fact that Firm 1 can ¯nd it optimal to revise its reaction curve in response to the changing characteristics of

Firm 2. Consequently, the value of Firm 2 will be also a®ected by the change of Firm 1's investment timing in°uencing cash °ow of the former. We illustrate the impact of strategic interactions with an example in which parameter values are chosen in such a way that all three types of equilibria are possible (cf. Figure 4). The ¯rms' values resulting from their optimal strategies are depicted in Figure 5 below.

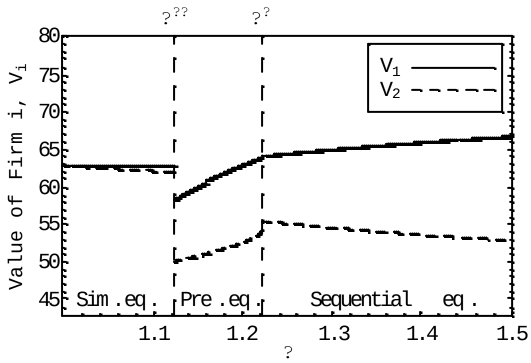

Figure 5

Figure 5. The value of Firm i (V_i) corresponding to the regions of the joint investment, preemptive and sequential equilibria for the set of parameter values: r = 0:05; ® = 0:015; ¾ = 0:1; D_{00} = 0:5; D_{01} = 0:25; D_{10} = 1:33; D_{11} = 1, I = 100 and x_t = 4.

The lowest degree of asymmetry between the ¯rms corresponds to the simultaneous investment equilibrium. In the simultaneous equilibrium the outcome closely resembles the case where the strategic interactions are absent. A marginal increase in ½ does not a®ect the value of Firm 1 and has a negative impact on the value of Firm 2.

As ½ increases, the sequential investment becomes more attractive for Firm 1 because of the increasing Firm 2's follower threshold. This means that Firm 2 will invest later so that Firm 1's sequential investment pro¯t goes up. Consequently, for ½ exceeding ½¤¤, Firm 1 would optimally invest at its leader threshold x_1^L. However, Firm 2 anticipates this and, since its leader value at x_1^L is larger than its follower value, it is willing to invest an instant before Firm 1 does. In such a situation the shift in Firm 1's reaction curve is discontinuous and a preemption equilibrium resulting in lower values of both ¯rms occurs. The implication is that a marginal increase in the investment cost of Firm 2 that changes the equilibrium from simultaneous to preemptive, results in both ¯rms' payo®s jumping downward.

Once the ¯rms are in the preemption region, the values of both ¯rms increase in ½. The at ¯rst sight surprising positive relationship between Firm 2's investment cost and its value is caused by the fact that increasing ½ makes Firm 2 a 'weaker' competitor. This implies that the preemption threat of Firm 2 declines in the investment cost asymmetry, so that x_{21}^P increases with ρ. Therefore, Firm 1 invests later, and this is beneficial for the cash flow of Firm 2 since it can enjoy a higher cash flow for a longer period. In this case, the non-strategic and strategic effects work in the opposite direction and the latter dominates. As far as Firm 1 is concerned, its value increases because its investment threshold moves closer to x_1^L. Moreover, it benefits from the delayed investment of Firm 2.

When the asymmetry between the firms reaches the critical level ρ^*, above which it is not optimal anymore for Firm 2 to become the leader, the sequential equilibrium occurs. Upon the switch to the sequential equilibrium the values of both firms jump upward. This jump is in both cases caused by the discontinuous change, from x_{21}^P to x_1^L, of Firm 1's investment threshold. By investing at x_1^L Firm 1 maximizes its value, and lets Firm 2 enjoy a higher cash flow for a longer period.

In the sequential equilibrium region the changes in the firms' values result entirely from the sunk cost asymmetry and its impact on Firm 2's investment timing. Consequently, Firm 1 benefits from the delayed investment of Firm 2 and the value of the latter decreases for the same reason as in the non-strategic case.

In order to provide better intuition about the nature of the non-monotonic relationship between the value of the firm, V_i, and the investment cost asymmetry, ρ, we decompose V_i into three components. First, we calculate the expected value of discounted future profits in case no investment is made, which reflects the value of assets in place, A/P_i. Further, we derive the value of the firm's own investment opportunity given that the other firm does not invest, $PVGO_i^O$, and the impact of the competitor's investment on the firm's profits, $PVGO_i^C$. The sum of $PVGO_i^O$ and $PVGO_i^C$ can be interpreted as the strategic NPV of the investment opportunity of Firm i.

Table 1a contains the decomposition of Firm 1's value for different levels of the cost-asymmetry.

ρ	1.1	1.15	1.2	1.25	1.33	1.5
A/P_1	57.14	57.14	57.14	57.14	57.14	57.14
$PVGO_1^O$	14.19	15.14	17.16	18.15	18.15	18.15
$PVGO_1^C$	-8.58	-12.17	-11.51	-10.91	-10.05	-8.57
V_1	62.75	60.12	62.80	64.39	65.24	66.72
			$\rho^* = 1.222$		$\rho^{**} = 1.124$	

Table 1a. Decomposition of Firm 1's value into the expected present value of the perpetual cash flow stream from assets in place, A/P_1, the option to invest, $PVGO_1^O$, short the competitor's option to invest and the value reduction due to the competitor's investment, $PVGO_1^C$, for the set of parameter values $r = 0.05$; $\alpha = 0.015$; $\sigma = 0.1$; $D_{00} = 0.5$; $D_{01} = 0.25$; $D_{10} = 1.33$; $D_{11} = 1$; and $I = 100$. The value of the firm, V_1, equals $A/P_1 + PVGO_1^O + PVGO_1^C$.

From Table 1a a number of conclusions can be drawn. First, we notice that the value attributed to assets in place does not change with the investment cost asymmetry. This is understandable since the existing production assets of the firms are identical. Second, the value of Firm 1's investment opportunity rises with ρ. This reflects the fact that the growing competitive advantage allows Firm 1 to keep its investment strategy closer to the unconditional optimum, x_1^L (at which the value of $PVGO_1^O$ in the example equals 18.15). Consequently, the only source of non-monotonicity is the interaction of Firm 2's investment decision with Firm 1's profit (see $PVGO_1^C$ in Table 1a). When the cost-asymmetry becomes larger, i.e. when $\rho \geq \rho^{**} = 1.124$, then Firm 1 has no longer an incentive to wait until the optimal simultaneous threshold is reached and is aiming at preempting Firm 2. As discussed above, the resulting preemption game deteriorates both firm's payoffs and, as a direct consequence, their values.

Table 1b contains an analogous decomposition of the value of Firm 2.

ρ	1.1	1.15	1.2	1.25	1.33	1.5
A/P_2	57.14	57.14	57.14	57.14	57.14	57.14
$PVGO_2^O$	13.45	11.94	11.29	10.70	9.80	7.88
$PVGO_2^C$	-8.58	-18.25	-15.85	-12.67	-12.67	-12.67
V_2	62.01	50.83	52.59	55.18	54.61	52.36

$\rho^* = 1.222$ $\quad\quad$ $\rho^{**} = 1.124$

Table 1b. Decomposition of Firm 2's value into the expected present value of the perpetual cash flow stream from assets in place, A/P_2, the option to invest, $PVGO_2^O$, short the competitor's option to invest and recapture the part of the market share, $PVGO_2^C$, for the set of parameter values $r = 0.05$; $\alpha = 0.015$; $\sigma = 0.1$; $D_{00} = 0.5$; $D_{01} = 0.25$; $D_{10} = 1.33$; $D_{11} = 1$; and $I = 100$. The value of the firm, V_2, equals $A/P_2 + PVGO_2^O + PVGO_2^C$.

Upon analyzing Table 1b it can be concluded that increasing investment cost asymmetry has two effects on the value of Firm 2. First, it results in the drop in the value of Firm 2's investment opportunity, $PVGO_2^O$. This relationship is monotonic irrespective from the type of the prevailing equilibrium and results from the increase in the investment expenditure that has to be incurred. Second, it influences the way the competitor's option to invest, $PVGO_2^C$, affects the value of the firm. In the region of the preemptive equilibrium, i.e. for $\rho \in [1.124; 1.222]$, the value of Firm 2 lost due to the exercise of the investment opportunity by Firm 1, $PVGO_2^C$, is inversely related to the investment cost asymmetry. In other words, when Firm 2's cost becomes higher, the investment of its competitor has a smaller negative impact on its value since the competitor invests later. This is the result of the strategic effect of the marginal increase in investment cost, via $PVGO_2^C$, which is stronger than the direct effect of the increase in ρ on the net present value of the project, $PVGO_2^O$.

So far, we considered the impact of a difference in the investment cost on the value of the firms. We have shown that there exists a non-monotonic and discontinuous relationship between the cost asymmetry and the firms' values resulting from the switches among the different types of equilibrium strategies. In the next section we discuss the impact of ρ on social welfare by showing how particular types of strategies affect consumer surplus.

6 Welfare Analysis

In order to assess the desirability of policies affecting the firms' access to new market segments and technologies, we investigate how investment cost asymmetry affects social welfare. The investment cost that has to be incurred by the firm can be influenced by the regulator, for instance, via fiscal measures, governmental guarantees resulting in a lower cost of capital, a different treatment of foreign vs. domestic investors, and the possibility of influencing the speed of knowledge spillovers.[9]

Desirability of such a policy can be measured by the way it affects social welfare, which is the sum of the consumer surplus and the firms' values.[10] Since in the previous sections we already established the firms' payoffs, here we begin the analysis with deriving the consumer surplus. Subsequently, we discuss how this surplus is influenced by the firms investment strategies. After having done this, we are ready to present the relationship between the investment strategies and social welfare. Finally, we provide some conclusions.

In order to derive the consumer surplus, we specify the way investment is beneficial to the consumers. To do so, we introduce a simple setting in which after the investment Firm i is offering a product of quality $b_1 > b_0$, where b_0 denotes the initial quality of the product. As long as the firms offer the same quality b_k, $k \in \{0; 1\}$, they compete à la Cournot, whereas after making the investment first, Firm 1 achieves a Stackelberg advantage in the differentiated product market. The Cournot outcome is restored after Firm 2 has invested. Then both firms compete in the market with a higher quality.

The market we consider has a continuum of consumers with utility function

$$U_{it} = E_t\left[\int_t^{\infty} (\theta_{s;m} b_k - p_{s;kl})\, e^{-rs} ds\right]; \tag{21}$$

where $\theta_{t;m}$ is a time-varying consumer-specific parameter that is uniformly distributed over the interval $[0; A_t]$; and $p_{t;kl}$ is the time t price of the product of quality b_k while the product offered by the other firm is of quality b_l. Parameter A_t follows the geometric Brownian motion

$$dA_t = \frac{1}{2}\left(\alpha - \frac{1}{4}\sigma^2\right) A_t dt + \frac{1}{2}\sigma A_t dw_t; \tag{22}$$

where $\alpha; \sigma$ and dw_t are the same as in (1). It is useful to observe (by applying Ito's lemma) that A_t^2 can be replaced by x_t since it exactly follows the process (1).

Let us derive the expressions for the instantaneous consumer surplus, denoted by $cs_{t;kl}$, where again k and l relate to the quality offered by the firms.

[9] For instance, lowering the cost of capital via a third-party guarantee is documented by Kleimeier and Megginson [8]. They find that in the sample of 1,803 syndicated project finance loans an average reduction of the spread due to the guarantees amounts to 43 basis points.

[10] Tirole [20], Ch. 5-8, provides an excellent introduction to oligopoly theory.

We have to consider three cases. In the first case only quality b_0 is provided. In the second case one firm provides quality b_0 and the other b_1 and, finally, both firms offer b_1. In the first and third case maximizing the firm's profits and calculating the residual surplus (see Appendix) yields

$$cs_{t;kk} = \int_0^{Q_t} (P_{t;k}(q) - p_{t;kk})\,dq = \frac{1}{2}Q_t(A_t b_k - p_{t;kk}) = \frac{2}{9}b_k x_t, \tag{23}$$

where $Q_t = 2q_{t;kl}$ is the total quantity offered, $q_{t;kl}$ and $p_{t;kl}$ are the equilibrium time t quantity and price of the product of quality b_k, respectively, where the other firm's product is of quality b_l, and $P_{t;k}(q)$ is a time t inverse demand function corresponding to the quality b_k.

The formulation of $cs_{t;10}$ (the second mentioned case) is slightly more involved and it corresponds to a Stackelberg equilibrium with second degree price discrimination. Consequently, $cs_{t;10}$ consists of two components: the surplus of consumers purchasing the good of quality b_1 and the surplus of those who choose b_0. Solving the Stackelberg game yields

$$\begin{aligned} cs_{t;10} &= \int_0^{q_{t;10}} (P_{t;1}(q) - p_{t;10})\,dq + \int_{q_{t;10}}^{Q} (P_{t;0}(q) - p_{t;01})\,dq \\ &= \frac{4b_1 + 5b_0}{32} x_t. \end{aligned} \tag{24}$$

To find out in what way the consumer surplus is related to the firms' investment strategies, we analyze the changes in the consumer surplus across the equilibria. If the resulting equilibrium is of the simultaneous type the consumer surplus, CS_t^S, equals

$$CS_t^S = E\left[\int_t^{T_1^S} e^{-rs} cs_{s;00}\,ds + \int_{T_1^S}^{\infty} e^{-rs} cs_{s;11}\,ds\right], \tag{25}$$

where T_1^S is given by (11). When the resulting equilibrium is of the preemption type, the consumer surplus, CS_t^P, amounts to

$$CS_t^P = E\left[\int_t^{\min[T_{21}^P;T_1^L]} e^{-rs} cs_{s;00}\,ds + \int_{\min[T_{21}^P;T_1^L]}^{T_2^F} e^{-rs} cs_{s;10}\,ds + \int_{T_2^F}^{\infty} e^{-rs} cs_{s;11}\,ds\right], \tag{26}$$

where

$$T_{21}^P = \inf\left(t \mid x_t \geq x_{21}^P\right), \text{ and} \tag{27}$$

$$T_1^L = \inf\left(t \mid x_t \geq x_1^L\right). \tag{28}$$

The consumer surplus in the sequential equilibrium is the same as (26), with the exception that $\min\left[T_{21}^P; T_1^L\right]$ is replaced by T_1^L.

After taking into account that the firms invest later in the simultaneous equilibrium, a comparison of (25) and (26) enables us to formulate the following proposition.

Proposition 4 Under the preemptive/sequential equilibrium the consumer surplus is always larger than in the joint investment equilibrium.

Proof. See Appendix. ■

Consequently, from the consumers' viewpoint, the situation in which the ¯rms invest simultaneously is undesirable. This is easy to understand since in this case the ¯rms invest later so that during a longer period of time the product with a higher quality is not available.

Now, let us investigate social welfare, which equals, as mentioned earlier, the consumer surplus plus the value of the ¯rms. In order to relate the latter to the analyzed market, we can make the following substitution, where the expressions at the RHS of each equality result from the maximization of the ¯rms' pro¯ts:

$$D_{00} ´ \frac{b_0}{9} \qquad D_{01} ´ \frac{b_0}{16} \qquad D_{10} ´ \frac{2b_1 ¡ b_0}{8} \qquad D_{11} ´ \frac{b_1}{9}$$

For a particular example, the consumer surplus and the ¯rms' values are depicted as functions of the asymmetry in the investment cost in Figure 6.

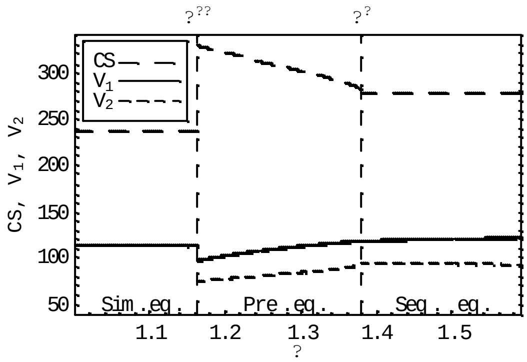

Figure 6

Figure 6. The value of Firm i (V_i) and consumer surplus (CS) corresponding to the regions of the joint investment, preemptive and sequential equilibria for the set of parameter values: $r = 0{:}05$; $® = 0{:}015$; $¾ = 0{:}1$; $b_0 = 5$; $b_1 = 7$; $I = 100$; and $x_t = 7$.

From Figure 6 it can be concluded that low asymmetry in the investment costs results in a relatively low consumer surplus and higher values of the ¯rms. Increasing the asymmetry among the ¯rms, such that the simultaneous equilibrium is superseded by the preemption equilibrium, leads to a downward jump in the ¯rms' values and, at the same time, to an upward jump in the consumer surplus. As seen before, the decline in the ¯rms' values mainly results from the need to incur the investment expenditure, I, earlier. The increase in the consumer surplus is the consequence of an earlier provision of the higher quality product. When the cost is large compared to the increase in the consumer surplus associated with higher quality, it is optimal from a welfare perspective to postpone the investment. Therefore, in such a case an increase in ½ leading to a switch from simultaneous to preemption equilibrium has a detrimental effect on welfare. Conversely, when the required sunk cost is relatively small, the resulting preemption equilibrium is socially desirable.

The impact of increasing ½ on social welfare is summarized in the following corollary.

Corollary 5 There exists a critical level of investment expenditure below which social welfare is always larger in the preemptive/sequential equilibrium than in the joint investment equilibrium.

Consequently, if the investment expenditure is small relatively to the consumer surplus, social welfare is highest under the preemption equilibrium. In this case, the loss in the ¯rms' values resulting from the preemption game is outweighed by the e®ect on the consumer surplus of an earlier provision of the high quality product. This implies that in the case of a relatively low investment expenditure, a relative cost disadvantage of one of the competitors results in the strategies yielding a socially preferred outcome.

Conversely, a relatively high investment expenditure implies the social optimality of the simultaneous equilibrium. This results from the fact that the simultaneous equilibrium is associated with a later investment outlay. Since the increase in consumer surplus resulting from providing a higher quality product earlier is not su±cient to fully compensate for the higher present value of an early investment, postponing the investment is socially desirable. Therefore, in the presence of a high sunk cost of the project, investment strategies resulting in the simultaneous equilibrium maximize social welfare. This, in turn, implies that the cost asymmetry is not desirable.

We conclude that an equal access of two ¯rms to a new market segment does not maximize the consumer surplus. Moreover, after taking into account the values of the ¯rms, it is not always socially desirable. If the ¯rms' investment costs are not excessively high, the presence of asymmetry among them yields a socially more desirable outcome.

However, it is important to notice that these conclusions do not carry over to the case where the ¯rst-mover advantage is large, which would occur when the product quality di®erence is higher. Then, as illustrated in Figure 4 the preemption equilibrium prevails even if ¯rms are symmetric. Consequently, from a welfare perspective, asymmetry is not desirable even if the investment is associated with a relatively low sunk cost.

7 Uncertainty and Investment Thresholds

From e.g. Dixit and Pindyck [3] it is known that in a non-strategic real options framework increasing uncertainty leads to a higher optimal investment threshold. As we show below, this observation also holds in strategic models as long as the firms' investment thresholds are solutions to the optimization problem. The follower's threshold, the leader's threshold in the sequential equilibrium, and the critical value triggering simultaneous investment satisfy this condition. Conversely, in the preemptive equilibrium the leader (Firm 1) does not invest at the threshold that solves its optimization problem, but instead it invests at the follower's (Firm 2's) preemption point.

From (5), (16) and (12) it is concluded that the optimal thresholds can be expressed as

$$x_i^{opt} = \frac{\beta}{\beta - 1}\frac{I_i}{D_{after} - D_{before}}(r - \alpha); \qquad (29)$$

where D_{after} and D_{before} are the deterministic contributions to the profit function corresponding to a given threshold. Consequently

$$\frac{\partial x^{opt}}{\partial(\sigma^2)} = -\frac{r - \alpha}{(\beta - 1)^2}\frac{I_i}{D_{after} - D_{before}}\frac{\partial\beta}{\partial(\sigma^2)} > 0; \qquad (30)$$

i.e. the optimal follower's threshold, optimal leader's threshold and the critical value corresponding to simultaneous investment increase with uncertainty.[11]

The impact of volatility on Firm 2's preemption point, x_{21}^P, at which Firm 1 invests, requires slightly more attention. Let us recall that x_{21}^P is the smallest root of $\xi_2(x_t) = 0$. Consequently, we calculate the derivative of $\xi_2(x_t)$ with respect to the market uncertainty. The change of (15), calculated for Firm 2, resulting from a marginal increase in σ^2 can be decomposed as follows:

$$\frac{d\xi_2(x_t)}{d(\sigma^2)} = \left(\frac{\partial\xi_2(x_t)}{\partial\beta} + \frac{\partial\xi_2(x_t)}{\partial x_1^F}\frac{dx_1^F}{d\beta}\right)\frac{\partial\beta}{\partial(\sigma^2)}. \qquad (31)$$

The derivative $\frac{\partial\xi_2(x_t)}{\partial\beta}\frac{\partial\beta}{\partial(\sigma^2)}$ measures the direct influence of uncertainty on the net benefit of being the leader. The product $\frac{\partial\xi_2(x_t)}{\partial x_1^F}\frac{dx_1^F}{d\beta}\frac{\partial\beta}{\partial(\sigma^2)}$ reflects the impact on the net benefit of being the leader of the fact that the follower investment threshold increases with uncertainty.

It can be shown that

$$\frac{\partial\xi_2(x_t)}{\partial\beta}\frac{\partial\beta}{\partial(\sigma^2)} < 0; \qquad (32)$$

$$\frac{\partial\xi_2(x_t)}{\partial x_1^F}\frac{dx_1^F}{d\beta}\frac{\partial\beta}{\partial(\sigma^2)} > 0. \qquad (33)$$

[11] Numerical simulations indicate that ρ^{**} increases and, respectively, ρ^{*} decreases in σ. This results in additional positive impact of uncertainty on the firms' investment thresholds.

Apparently, the joint impact of both e®ects is ambiguous. The ¯rst e®ect is (32), which represents the simple value of waiting argument: if uncertainty is large, it is more valuable to wait for new information before undertaking the investment (Dixit and Pindyck [3]) As we have just seen, this also holds for the follower. The implication for the leader of the follower investing later is that the leader has a cost advantage for a longer time. This makes an earlier investment of the leader more bene¯cial. This e®ect is captured by (33), which can thus be interpreted as an increment in the strategic value of becoming the leader vs. the follower resulting from the delay in the follower's entry. Obviously, the latter e®ect is not present in the monopolistic/perfectly competitive markets, where the impact of uncertainty is unambiguous.

However, it is possible to show that the direct e®ect captured by (32) dominates, irrespective of the values of the input parameters.

Proposition 6 *When uncertainty of the product market increases, the leader investment threshold increases as well.*

Proof. See Appendix. ■

Our conclusions concerning the relationship between the investment timing and uncertainty are consistent with recent empirical evidence. The negative investment-uncertainty relationship for ¯rms operating in an imperfectly competitive environment is documented, for example, by Guiso and Parigi [7].

8 Conclusions

In this paper we analyze the impact on the ¯rms' optimal investment strategies of the di®erence in the costs associated with their pro¯t-enhancing investments. Since the ¯rms operate in an imperfectly competitive market, the profitability of each ¯rm's project is a®ected by the other ¯rm's decision to invest. We show that when the asymmetry among ¯rms is relatively small and so is the ¯rst-mover advantage, the ¯rms invest jointly. When the ¯rst-mover advantage is signi¯cant, the lower-cost ¯rm preempts the higher-cost ¯rm. In the situation where the asymmetry between ¯rms becomes su±ciently large, the ¯rms exercise their investment options sequentially and their mutual decisions do not a®ect each other directly.

Asymmetry has remarkable implications for the value of both ¯rms, since the value-asymmetry relationship for both ¯rms is non-monotonic and discontinuous. Consequently, we obtain a number of counter-intuitive results. For reasonable parameter values, deepening the ¯rm's competitive disadvantage due to a marginal rise in its irreversible cost may decrease the value of its competitor. This situation results when a switch from simultaneous to preemptive equilibrium occurs upon the marginal change in the cost asymmetry. Another interesting e®ect of strategic interactions is present when the ¯rms are engaged in the preemption game. Then increasing the extent to which the ¯rms is set at cost-disadvantage leads to an appreciation of its value due to the strategic effect on the competitor's investment timing.

Moreover, there are significant welfare effects of strategic interactions between the firms. In an example where the investment increases product quality, we show that the relationship between cost asymmetry and social welfare depends on the cost of investment. If it is relatively high and the first-mover advantage is not too large, social welfare is maximized when none of the firms suffers from competitive disadvantage. However, if the investment cost is low, an increase of the consumer surplus resulting from the early investment in the preemption equilibrium exceeds the loss of the firms' joint value associated with such an investment. Therefore, the preemption equilibrium, occurring when the costs sufficiently differ, is in this case desirable. This observation allows for the conclusion that an equal access of competitors to a new technology or market segment may not be socially optimal.

Finally, the impact of uncertainty is analyzed. Despite the presence of strategic interactions, increasing uncertainty always results in a higher investment threshold. This holds not only for the optimal investment thresholds but also for the case when the lower-cost firm faces the threat of being preempted by its higher-cost opponent.

9 Appendix

Proof of Proposition 2. The sequential equilibrium occurs when Firm 2 has no incentive to invest as the leader. Formally, this requires that $\xi_2(x_t)$ is negative for all $x_t \in [x_0, x_1^L)$. Therefore, we are interested in finding a pair (x^*, ρ^*) that satisfies the following system of equations

$$\begin{cases} \xi_2(x^*, \rho^*) = 0 \\ \left.\frac{\partial \xi_2(x^*, \rho^*)}{\partial x}\right|_{x=x^*} = 0. \end{cases} \tag{34}$$

In other words, we are interested in a point (x^*, ρ^*) in which Firm's 2 leader function is tangent to the follower function. After substituting (7) and (9) into (15), all defined for Firm 2, and rearranging we obtain

$$\begin{cases} \frac{x^*(D_{10} - D_{01})}{r - \alpha} - I\rho^* + \frac{x_1^F (D_{11} - D_{10})}{r - \alpha}\left(\frac{x^*}{x_1^F}\right)^{\beta} - \frac{x_2^F(\rho^*)(D_{11} - D_{01})}{\beta(r - \alpha)}\left(\frac{x^*}{x_2^F(\rho^*)}\right)^{\beta} = 0 \\ \frac{D_{10} - D_{01}}{r - \alpha} + \beta\frac{D_{11} - D_{10}}{r - \alpha}\left(\frac{x^*}{x_1^F}\right)^{\beta - 1} - \frac{D_{11} - D_{01}}{r - \alpha}\left(\frac{x^*}{x_2^F(\rho^*)}\right)^{\beta - 1} = 0. \end{cases} \tag{35}$$

After multiplying both sides of the second equation in (35) by $\frac{x^*}{\beta}$, subtracting it from the first equation, and rearranging, we obtain

$$x^* = \frac{\beta}{\beta - 1}\frac{I\rho^*}{D_{10} - D_{01}}(r - \alpha). \tag{36}$$

Substituting (36) into the first equation in (35) and (5) for x_1^F yields

$$\frac{\beta}{\beta-1} I\rho^* - I\rho^* + \left(\frac{D_{11}-D_{10}}{D_{10}-D_{01}}\rho^*\right)^{\beta} \frac{\beta}{\beta-1}\frac{D_{11}-D_{10}}{D_{11}-D_{01}} I - \left(\frac{D_{11}-D_{01}}{D_{10}-D_{01}}\right)^{\beta} \frac{I\rho^*}{\beta-1} = 0. \tag{37}$$

Rearranging (37) leads to the expression (18).

In the remaining part of the proof, we demonstrate that $\rho^* > 1$. It holds that

$$\rho^* > 1 \iff \frac{(D_{10}-D_{01})^{\beta} - (D_{11}-D_{01})^{\beta}}{\beta(D_{10}-D_{11})} - (D_{11}-D_{01})^{\beta-1} > 0. \tag{38}$$

Consequently

$$\begin{aligned} &\frac{(D_{10}-D_{01})^{\beta} - (D_{11}-D_{01})^{\beta}}{\beta(D_{10}-D_{11})} - (D_{11}-D_{01})^{\beta-1} \\ =\ &\frac{(D_{10}-D_{01})^{\beta} - (D_{11}-D_{01})^{\beta} - \beta(D_{10}-D_{11})(D_{11}-D_{01})^{\beta-1}}{\beta(D_{10}-D_{11})}. \end{aligned} \tag{39}$$

By substituting

$$a = D_{11} - D_{01}, \tag{40}$$
$$b = D_{10} - D_{01}, \tag{41}$$

and rearranging, we conclude that (39) is equivalent to

$$\frac{a^{\beta}}{\beta(b-a)}\left(\left(\frac{b}{a}\right)^{\beta} - 1 - \beta\frac{b}{a} + \beta\right). \tag{42}$$

After observing that $b > a$ and $\frac{a^{\beta}}{\beta(b-a)} > 0$, we still have to prove that the second factor of (42) is positive. Let us denote $w = \frac{b}{a}$ and $g(w) = w^{\beta} - 1 - \beta w + \beta$. Consequently, we have

$$g(1) = 0; \text{ and} \tag{43}$$
$$\frac{\partial g(w)}{\partial w} = \beta w^{\beta-1} - \beta > 0. \tag{44}$$

This completes the proof. ■

Proof of Proposition 3. Firm 1 prefers simultaneous investment unless for some x_t its leader payoff, $V_1^L(x_t)$, exceeds the optimal joint investment payoff, $V_1^S(x_t)$. Formally, simultaneous equilibrium occurs only if $\phi_1(x_t)$ is positive for all $x_t \in \left[x_1^P; x_2^F\right]$. Therefore, we are interested in finding a pair $(x^{**}; \rho^{**})$ that satisfies the following system of equations

$$\begin{cases} \phi_1(x^{**}; \rho^{**}) = 0 \\ \left.\frac{\partial \phi_1(x; \rho^{**})}{\partial x}\right|_{x = x^{**}} = 0. \end{cases} \tag{45}$$

In other words, we are interested in a point $(x^{**};\rho^{**})$ in which Firm's 1 simultaneous investment function is tangent to its leader function. After substituting (9) and (13) evaluated at (12) into (19), all defined for Firm 1, and rearranging, we obtain

$$\begin{cases} \frac{x^{**}(D_{00}-D_{10})}{r-\alpha} + I + \frac{x_1^S(D_{11}-D_{00})}{\beta(r-\alpha)}\left(\frac{x^{**}}{x_1^S}\right)^{\beta} - \frac{x_2^F(D_{11}-D_{10})}{r-\alpha}\left(\frac{x^{**}}{x_2^F(\rho^{**})}\right)^{\beta} = 0 \\ \frac{D_{00}-D_{10}}{r-\alpha} + \frac{D_{11}-D_{10}}{r-\alpha}\left(\frac{x^{**}}{x_1^S}\right)^{\beta-1} - \frac{\beta(D_{11}-D_{10})}{r-\alpha}\left(\frac{x^{**}}{x_2^F(\rho^{**})}\right)^{\beta-1} = 0. \end{cases} \tag{46}$$

After multiplying both sides of the second equation in (46) by $\frac{x^{**}}{\beta}$, subtracting it from the first equation, and rearranging, we obtain

$$x^{**} = \frac{\beta}{\beta-1}\frac{I}{D_{10}-D_{00}}(r-\alpha). \tag{47}$$

Substituting (47) into the first equation in (46) and (12) and (5) for x_1^S and x_2^F yields

$$-\frac{I}{\beta-1} + \left(\frac{D_{11}-D_{00}}{D_{10}-D_{00}}\right)^{\beta}\frac{I}{\beta-1} - \left(\frac{D_{11}-D_{01}}{(D_{10}-D_{00})\rho^{**}}\right)^{\beta}\frac{\beta I\rho^{**}}{\beta-1}\frac{D_{11}-D_{10}}{D_{11}-D_{01}} = 0. \tag{48}$$

Given that we only consider the case that $\rho^{**} \neq 1$, rearranging (48) leads to the expression (20).

In the remaining part of the proof we show that the optimality of the simultaneous investment for Firm 1 implies that Firm 2 is better off by investing simultaneously as well. Consequently, we prove that as long as it is optimal for Firm 1 to invest simultaneously, Firm 2's follower threshold is always smaller than Firm 1's optimal joint investment threshold (since if this is true, then it is always optimal for Firm 2 to invest immediately when Firm 1 invests). First, we determine $\hat{\rho}$ which solves

$$x_2^F(\hat{\rho}) = x_1^S(\hat{\rho}). \tag{49}$$

For $\rho < \hat{\rho}$ it holds that $x_2^F(\hat{\rho}) < x_1^S(\hat{\rho})$. After substituting (5) for Firm 2 and (12) for Firm 1 into (49), and rearranging, we obtain

$$\hat{\rho} = \frac{D_{11}-D_{01}}{D_{11}-D_{00}}. \tag{50}$$

Now, we show that $\hat{\rho} > \rho^{**}$, i.e. that

$$\frac{D_{11}-D_{01}}{D_{11}-D_{00}} - (D_{11}-D_{01})\left(\frac{\beta(D_{10}-D_{11})}{(D_{10}-D_{00})^{\beta}-(D_{11}-D_{00})^{\beta}}\right)^{\frac{1}{\beta-1}} > 0 \tag{51}$$

holds. After substituting

$$\begin{aligned} c &= D_{11}-D_{00}; \\ d &= D_{10}-D_{00}; \end{aligned}$$

and rearranging, we obtain that condition (51) is equivalent to

$$\frac{1}{c} - \left(\frac{\beta(d-c)}{d^{\beta}-c^{\beta}}\right)^{\frac{1}{\beta-1}} > 0. \tag{52}$$

This implies

$$\left(\frac{d}{c}\right)^{\beta} - 1 - \beta\left(\frac{d}{c} - 1\right) > 0.$$

Let us denote $z = \frac{d}{c}$ and $h(z) = z^{\beta} - 1 - \beta(z-1)$. Consequently, we have

$$\begin{aligned} h(1) &= 0; \text{ and} && (53)\\ \frac{\partial h(z)}{\partial z} &= \beta z^{\beta-1} - \beta > 0, && (54)\end{aligned}$$

since $z > 1$ and $\beta > 1$. This completes the proof. ■

Proof of Proposition 4. Since $T_{21}^{P} < T_2^{F} < T_1^{S}$, subtracting the value of consumer surplus in preemptive equilibrium from the value corresponding to the joint investment yields

$$\Delta CS_t^{P-S} = \int_{T_{21}^{P}}^{T_2^{F}} e^{-rs}(cs_{s;10} - cs_{s;00})\,ds + \int_{T_2^{F}}^{T_1^{S}} e^{-rs}(cs_{s;11} - cs_{s;00})\,ds > 0.$$

An identical reasoning can be applied while comparing the simultaneous equilibrium with the sequential exercise strategy. ■

Proof of Proposition 6. The difference of Firm 2's payoffs as the leader and the follower can be expressed as

$$\begin{aligned}\pi_2(x_t) = {} & \frac{x_t(D_{10}-D_{01})}{r-\alpha} - I\rho + {} \\ & + \frac{I}{D_{11}-D_{01}} \frac{\left(x_t \frac{\beta-1}{\beta} \frac{D_{11}-D_{01}}{I(r-\alpha)}\right)^{\beta}}{\beta-1} \left(\beta(D_{11}-D_{10}) - \frac{D_{11}-D_{01}}{\rho^{\beta-1}}\right).\end{aligned} \tag{55}$$

We are interested in the direction in which uncertainty affects, x_{21}^{P}, i.e. the smallest root of (55). The derivative of (55) with respect to β equals

$$\begin{aligned}\frac{\partial \pi_2(x_t)}{\partial \beta} = {} & \frac{I}{D_{11}-D_{01}} \frac{\left(x_t \frac{\beta-1}{\beta} \frac{D_{11}-D_{01}}{I(r-\alpha)}\right)^{\beta}}{\beta-1} \times \\ & \times \left(\ln\left(x_t \frac{\beta-1}{\beta} \frac{D_{11}-D_{01}}{I(r-\alpha)}\right)\left(\beta(D_{11}-D_{10}) - \frac{D_{11}-D_{01}}{\rho^{\beta-1}}\right)\right. \\ & \left. + D_{11} - D_{10} + \frac{D_{11}-D_{01}}{\rho^{\beta-1}} \ln\rho\right).\end{aligned} \tag{56}$$

It is straightforward to observe that for sufficiently small x_t (56) is positive. This can be generalized into the statement that there exists $\bar{x}$ satisfying

$$\operatorname{sgn}\frac{\partial \pi_2(x_t)}{\partial \beta} = \begin{cases} 1; & x_t \in (0;\bar{x}); \\ 0; & x_t = \bar{x}; \\ -1; & x_t \in \big(\bar{x}; x_2^F\big): \end{cases} \tag{57}$$

Since (in general) it is not possible to obtain an analytical formula for x_{21}^P, we evaluate the sign of the derivative (56) at such a realization of x_t for which the corresponding sign is the same as at x_{21}^P. Consequently, we are interested in the realization of x_t that satisfies the following two properties

$$\pi_2(x^*) < 0 \Rightarrow \pi_2(x_t) < 0 \ \forall x_t; \text{ and} \tag{58}$$
$$\exists x_{21}^P \Rightarrow x_{21}^P < x^*: \tag{59}$$

Properties (57) and (59) imply that

$$\left.\frac{\partial \pi_2(x_t)}{\partial \beta}\right|_{x_t = x^*} > 0 \Rightarrow \left.\frac{\partial \pi_2(x_t)}{\partial \beta}\right|_{x_t = x_{21}^P} > 0: \tag{60}$$

The realization x^* equal to (cf. (36))

$$x^* = \frac{\beta}{\beta - 1}\frac{l\rho}{D_{10} - D_{01}}(r - \alpha) \tag{61}$$

satisfies (58) and (59). Property (58) can be verified by examining the definition of x^* (cf. (34)) and by observing that

$$\frac{\partial \pi_2(x_t)}{\partial \rho} < 0:$$

Property (59) follows directly. Subsequently, we determine the sign of the derivative (56) at x^*:

$$\begin{aligned} \left.\frac{\partial \pi_2(x_t)}{\partial \beta}\right|_{x_t = x^*} = {} & \frac{l}{D_{11} - D_{01}}\frac{\left(\frac{D_{11}-D_{01}}{D_{10}-D_{01}}\rho\right)^{\beta}}{\beta - 1} \times \\ & \times \left(\ln\left(\frac{D_{11}-D_{01}}{D_{10}-D_{01}}\rho\right)\left(\beta(D_{11} - D_{10}) - \frac{D_{11}-D_{01}}{\rho^{\beta}-1}\right)\right. \\ & \left. + D_{11} - D_{10} + \frac{D_{11}-D_{01}}{\rho^{\beta}-1}\ln\rho\right): \end{aligned} \tag{62}$$

Let us denote

$$\begin{aligned} \varphi(\rho) = {} & \ln\left(\frac{D_{11}-D_{01}}{D_{10}-D_{01}}\rho\right)\left(\beta(D_{11} - D_{10}) - \frac{D_{11}-D_{01}}{\rho^{\beta}-1}\right) \\ & + D_{11} - D_{10} + \frac{D_{11}-D_{01}}{\rho^{\beta}-1}\ln\rho: \end{aligned} \tag{63}$$

Positive $\varphi(\rho)$ for $\forall\rho \in [1;\rho^*]$; where ρ^* is defined by (18), would imply the positive relationship between uncertainty and the leader threshold. First, we show that $\varphi(\rho^*)$ is positive. Subsequently, we prove that

$$\varphi(\rho^*) > 0 \Rightarrow \varphi(\rho) > 0 \ \forall\rho \in [1;\rho^*]: \tag{64}$$

We prove that $\varphi(\rho^*) > 0$ in three steps. First, we change the variables and factorize the function $\varphi(\rho^*)$, what yields the product of two factors: one with negative and one with unknown sign. Second, we show that the factor with the unknown sign is increasing in the relevant variable. Finally, we show that the value of the factor with a priori unknown sign approaches zero when the underlying variable approaches the upper limit of its domain. The last two steps imply that the sign of the analyzed factor is negative what is equivalent to the positive sign of $\varphi(\rho^*)$.

Consequently, we substitute (37) into (63) and obtain

$$\begin{aligned}\varphi(\rho^*) = {} & \ln\left(\frac{D_{11}-D_{01}}{D_{10}-D_{01}}\right)\times \\ & \times\left(\beta(D_{11}-D_{10}) - (D_{11}-D_{01})^{\beta}\frac{\beta(D_{10}-D_{11})}{(D_{10}-D_{01})^{\beta}-(D_{11}-D_{01})^{\beta}}\right) \\ & + \ln\left(\frac{1}{D_{11}-D_{01}}\left(\frac{(D_{10}-D_{01})^{\beta}-(D_{11}-D_{01})^{\beta}}{\beta(D_{10}-D_{11})}\right)^{\frac{1}{\beta-1}}\right)\beta(D_{11}-D_{10}) \\ & + D_{11}-D_{10}:\end{aligned} \tag{65}$$

Now, we change the variables in order to simplify the expression for $\varphi(\rho^*)$. Substitution of (40) and (41) into (65) yields

$$\begin{aligned}\varphi(\rho^*) &= \ln\left(\frac{a}{b}\right)\left(\beta(a-b) - a^{\beta}\frac{\beta(b-a)}{b^{\beta}-a^{\beta}}\right) + \\ &\quad + \ln\left(\frac{1}{a}\left(\frac{b^{\beta}-a^{\beta}}{\beta(b-a)}\right)^{\frac{1}{\beta-1}}\right)\beta(a-b) + a - b:\end{aligned} \tag{66}$$

Consequently, we divide (66) by b; and define

$$p = \frac{a}{b}: \tag{67}$$

As an immediate result we get

$$\begin{aligned}\frac{\varphi(\rho^*)}{b} &= \ln p\left(\beta(p-1) - p^{\beta}\frac{\beta(1-p)}{1-p^{\beta}}\right) \\ &\quad + \ln\left(\left(\frac{1-p^{\beta}}{\beta(p^{\beta-1}-p^{\beta})}\right)^{\frac{1}{\beta-1}}\right)\beta(p-1) + p - 1:\end{aligned} \tag{68}$$

Factorization of (68) yields

$$\frac{\varphi(\rho^*)}{b} = -\frac{(1-p)}{(\beta-1)(1-p^\beta)} \times \left[(\beta-1)\left(1-p^\beta+\ln p^\beta\right) - \left(1-p^\beta\right)\beta \ln\left(\frac{\beta p^{\beta-1}(1-p)}{1-p^\beta}\right)\right]. \tag{69}$$

Since it always holds that

$$-\frac{(1-p)}{(\beta-1)(1-p^\beta)} < 0, \tag{70}$$

we are interested in the sign of the second factor of (69). Therefore, we define

$$\Psi(p;\beta) \equiv (\beta-1)\left(1-p^\beta+\ln p^\beta\right) - \left(1-p^\beta\right)\beta \ln\left(\frac{\beta p^{\beta-1}(1-p)}{1-p^\beta}\right) \tag{71}$$

Now, we determine the sign of the derivative of (71) calculated with respect to p. Consequently, we obtain

$$\frac{\partial\Psi(p;\beta)}{\partial p} = -\frac{\beta}{p(1-p)}\left(\beta p^\beta(1-p)\left(1-\ln\left(\frac{\beta p^{\beta-1}(1-p)}{1-p^\beta}\right)\right) - p\left(1-p^\beta\right)\right). \tag{72}$$

This can be expressed as

$$\frac{\partial\Psi(p;\beta)}{\partial p} = -\frac{\beta\left(1-p^\beta\right)}{(1-p)}\left(\left(\frac{\beta p^{\beta-1}(1-p)}{1-p^\beta}\right)\left(1-\ln\left(\frac{\beta p^{\beta-1}(1-p)}{1-p^\beta}\right)\right)-1\right). \tag{73}$$

The first factor of (73) is always negative. After the following substitution

$$z = \frac{\beta p^{\beta-1}(1-p)}{1-p^\beta}, \tag{74}$$

the second factor of (73) can be expressed as

$$-z\left(\frac{1}{z} - 1 - \ln\frac{1}{z}\right), \tag{75}$$

which is negative for every $z \in \mathbb{R}_{++}$. This implies

$$\frac{\partial\Psi(p;\beta)}{\partial p} > 0. \tag{76}$$

In the last step we show that $\forall\beta \; \lim_{p\uparrow 1}\Psi(p;\beta) = 0$. The limit of (71) can be decomposed as

$$\lim_{p\uparrow 1}(\beta-1)\left(1-p^\beta+\ln p^\beta\right) - \lim_{p\uparrow 1}\left(1-p^\beta\right)\beta\ln\left(\frac{\beta p^{\beta-1}(1-p)}{1-p^\beta}\right). \tag{77}$$

The first part can be determined directly

$$\lim_{p\uparrow 1}(\beta-1)\left(1-p^{\beta}+\ln p^{\beta}\right)=0. \tag{78}$$

The second part requires a slightly closer examination

$$\begin{aligned}
&\lim_{p\uparrow 1}-\left(1-p^{\beta}\right)\ln\left(\frac{p^{\beta-1}(1-p)}{1-p^{\beta}}\right)= && (79)\\
&=\lim_{p\uparrow 1}\left[-\left(1-p^{\beta}\right)\ln\left(\frac{1-p}{1-p^{\beta}}\right)-\left(1-p^{\beta}\right)\ln\left(p^{\beta-1}\right)\right]= \\
&=\lim_{p\uparrow 1}-\left(1-p^{\beta}\right)\ln\left(\frac{1-p}{1-p^{\beta}}\right)=0
\end{aligned}$$

The last equality holds since (by de l'Hôpital rule) we know that

$$\lim_{p\uparrow 1}\ln\left(\frac{1-p}{1-p^{\beta}}\right)=1. \tag{80}$$

Consequently, substituting (78) and (79) into (77) yields

$$\lim_{p\uparrow 1}\psi(p;\beta)=0. \tag{81}$$

(81) together with (76) imply that (71) is negative and, as a consequence, (65) is positive.

Having proven the positive sign of $\varphi(\rho^{*})$, now we show that (64) holds. Differentiating (63) with respect to ρ yields

$$\frac{\partial\varphi(\rho)}{\partial\rho}=\frac{1}{\rho}\left(\beta(a-b)-\frac{a}{\rho^{\beta-1}}\right)+\ln\left(\frac{a}{b}\rho\right)\frac{(\beta-1)a}{\rho^{\beta}}-((\beta-1)\ln\rho-1)\frac{a}{\rho^{\beta}}; \tag{82}$$

where a and b are defined by (40) and (41). Defining

$$\tilde{\varphi}(\rho)\equiv\frac{\varphi(\rho)}{b}; \tag{83}$$

and substitution of (67) result in

$$\frac{\partial\tilde{\varphi}(\rho)}{\partial\rho}=\frac{\beta(p-1)}{\rho}-\frac{p}{\rho^{\beta}}+\ln(p\rho)\frac{(\beta-1)p}{\rho^{\beta}}+\frac{p}{\rho^{\beta}}-\ln\rho\frac{(\beta-1)p}{\rho^{\beta}}<0. \tag{84}$$

This completes the proof. ∎

Derivation of the consumer surplus and profit functions. When both firms offer a product of quality, the resulting equilibrium is symmetric. The prices and quantities are equal to

$$p_{t;kk}=\frac{b_kA_t}{3}\qquad q_{t;kk}=\frac{A_t}{3};$$

what yield the instantaneous profit

$$\pi_{t;kk} = \frac{b_k A_t^2}{9}.$$

The consumer surplus equals

$$cs_{t;kk} = \frac{1}{2}\left(b_k A_t - \frac{b_k A_t}{3}\right)\frac{2A_t}{3} = \frac{2b_k}{9}A_t^2.$$

After Firm 1 achieves a Stackelberg advantage by investing, the prices and quantities equal

$$p_{t;10} = \frac{2b_1 - b_0}{4}A_t \qquad p_{t;01} = \frac{b_0}{4}A_t \qquad q_{t;10} = \frac{A_t}{2} \qquad q_{t;01} = \frac{A_t}{4}.$$

The instantaneous profits are therefore equal

$$\pi_{t;10} = \frac{2b_1 - b_0}{8}A_t^2 \qquad \pi_{t;10} = \frac{b_0}{16}A_t^2,$$

and the consumer surplus is

$$cs_{t;10} = \frac{1}{2}\left(b_1 A_t - \frac{2b_1 - b_0}{4}A_t\right)\frac{A_t}{2} + \frac{1}{2}\left(b_0 A_t - \frac{b_0}{4}A_t\right)\frac{A_t}{4} = \frac{4b_1 + 5b_0}{32}A_t^2.$$

The observation that $A_t^2 = x_t$ allows for an immediate calculation of the consumer surplus in terms of (1) and for an identification of the deterministic contributions of the profit functions.

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